PART THREE

ELECTRODYNAMICS

3.1. CONSTANT ELECTRIC FIELD IN VACUUM

- Strength and potential of the field of a point charge q:

$$\mathbf{E}=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\mathbf{r}, \qquad \varphi=\frac{1}{4\pi\varepsilon_0}\frac{q}{r}. \tag{3.1a}$$

- Relation between field strength and potential:

$$\mathbf{E}=-\nabla\varphi, \tag{3.1b}$$

i.e. field strength is equal to the antigradient of the potential.

- Gauss's theorem and circulation of the vector $\mathbf{E}$:

$$\oint \mathbf{E}\,d\mathbf{S}=q/\varepsilon_0, \qquad \oint \mathbf{E}\,d\mathbf{r}=0. \tag{3.1c}$$

- Potential and strength of the field of a point dipole with electric moment $\mathbf{p}$:

$$\varphi=\frac{1}{4\pi\varepsilon_0}\frac{\mathbf{pr}}{r^3}, \qquad E=\frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{1+3\cos^2\theta}, \tag{3.1d}$$

where θ is the angle between the vectors $\mathbf{r}$ and $\mathbf{p}$.

- Energy W of the dipole $\mathbf{p}$ in an external electric field, and the moment $\mathbf{N}$ of forces acting on the dipole:

$$W=-\mathbf{pE}, \quad \mathbf{N}=[\mathbf{pE}]. \tag{3.1e}$$

- Force $\mathbf{F}$ acting on a dipole, and its projection F_x:

$$\mathbf{F}=p\frac{\partial\mathbf{E}}{\partial l}, \qquad F_x=\mathbf{p}\cdot\nabla E_x, \tag{3.1f}$$

where $\partial\mathbf{E}/\partial l$ is the derivative of the vector $\mathbf{E}$ with respect to the dipole direction, ∇E_x is the gradient of the function E_x.

3.1. Calculate the ratio of the electrostatic to gravitational interaction forces between two electrons, between two protons. At what value of the specific charge q/m of a particle would these forces become equal (in their absolute values) in the case of interaction of identical particles?

3.2. What would be the interaction force between two copper spheres, each of mass 1 g, separated by the distance 1 m, if the total electronic charge in them differed from the total charge of the nuclei by one per cent?

3.3. Two small equally charged spheres, each of mass m, are suspended from the same point by silk threads of length l. The distance between the spheres $x \ll l$. Find the rate dq/dt with which the charge leaks off each sphere if their approach velocity varies as $v = a/\sqrt{x}$, where a is a constant.

3.4. Two positive charges q_1 and q_2 are located at the points with radius vectors $\mathbf{r}_1$ and $\mathbf{r}_2$. Find a negative charge q_3 and a radius vector $\mathbf{r}_3$ of the point at which it has to be placed for the force acting on each of the three charges to be equal to zero.

3.5. A thin wire ring of radius r has an electric charge q. What will be the increment of the force stretching the wire if a point charge q_0 is placed at the ring's centre?

3.6. A positive point charge 50 μC is located in the plane xy at the point with radius vector $\mathbf{r}_0 = 2\mathbf{i} + 3\mathbf{j}$, where $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors of the x and y axes. Find the vector of the electric field strength $\mathbf{E}$ and its magnitude at the point with radius vector $\mathbf{r} = 8\mathbf{i} - 5\mathbf{j}$. Here r_0 and r are expressed in metres.

3.7. Point charges q and $-q$ are located at the vertices of a square with diagonals $2l$ as shown in Fig. 3.1. Find the magnitude of the electric field strength at a point located symmetrically with respect to the vertices of the square at a distance x from its centre.

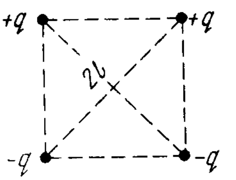

Fig. 3.1.

3.8. A thin half-ring of radius $R = 20$ cm is uniformly charged with a total charge $q = 0.70$ nC. Find the magnitude of the electric field strength at the curvature centre of this half-ring.

3.9. A thin wire ring of radius r carries a charge q. Find the magnitude of the electric field strength on the axis of the ring as a function of distance l from its centre. Investigate the obtained function at $l \gg r$. Find the maximum strength magnitude and the corresponding distance l. Draw the approximate plot of the function $E(l)$.

3.10. A point charge q is located at the centre of a thin ring of radius R with uniformly distributed charge $-q$. Find the magnitude of the electric field strength vector at the point lying on the axis of the ring at a distance x from its centre, if $x \gg R$.

3.11. A system consists of a thin charged wire ring of radius R and a very long uniformly charged thread oriented along the axis of the ring, with one of its ends coinciding with the centre of the ring. The total charge of the ring is equal to q. The charge of the thread (per unit length) is equal to λ. Find the interaction force between the ring and the thread.

3.12. A thin nonconducting ring of radius R has a linear charge density $\lambda = \lambda_0 \cos\varphi$, where λ_0 is a constant, φ is the azimuthal angle. Find the magnitude of the electric field strength

(a) at the centre of the ring;

(b) on the axis of the ring as a function of the distance x from its centre. Investigate the obtained function at $x \gg R$.

3.13. A thin straight rod of length $2a$ carrying a uniformly distributed charge q is located in vacuum. Find the magnitude of the electric field strength as a function of the distance r from the rod's centre along the straight line

(a) perpendicular to the rod and passing through its centre;

(b) coinciding with the rod's direction (at the points lying outside the rod).

Investigate the obtained expressions at $r \gg a$.

3.14. A very long straight uniformly charged thread carries a charge λ per unit length. Find the magnitude and direction of the electric field strength at a point which is at a distance y from the thread and lies on the perpendicular passing through one of the thread's ends.

3.15. A thread carrying a uniform charge λ per unit length has the configurations shown in Fig. 3.2 a and b. Assuming a curvature

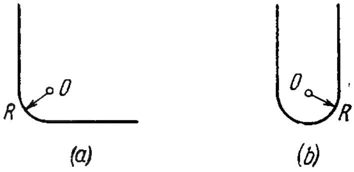

Fig. 3.2.

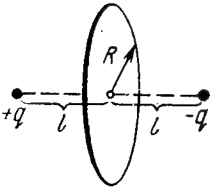

Fig. 3.3.

radius R to be considerably less than the length of the thread, find the magnitude of the electric field strength at the point O.

3.16. A sphere of radius r carries a surface charge of density $\sigma = \mathbf{ar}$, where $\mathbf{a}$ is a constant vector, and $\mathbf{r}$ is the radius vector of a point of the sphere relative to its centre. Find the electric field strength vector at the centre of the sphere.

3.17. Suppose the surface charge density over a sphere of radius R depends on a polar angle θ as $\sigma = \sigma_0 \cos\theta$, where σ_0 is a positive constant. Show that such a charge distribution can be represented as a result of a small relative shift of two uniformly charged *balls* of radius R whose charges are equal in magnitude and opposite in sign. Resorting to this representation, find the electric field strength vector inside the given sphere.

3.18. Find the electric field strength vector at the centre of a ball of radius R with volume charge density $\rho = \mathbf{ar}$, where $\mathbf{a}$ is a constant vector, and $\mathbf{r}$ is a radius vector drawn from the ball's centre.

3.19. A very long uniformly charged thread oriented along the axis of a circle of radius R rests on its centre with one of the ends. The charge of the thread per unit length is equal to λ. Find the flux of the vector $\mathbf{E}$ across the circle area.

3.20. Two point charges q and $-q$ are separated by the distance $2l$ (Fig. 3.3). Find the flux of the electric field strength vector across a circle of radius R.

3.21. A ball of radius R is uniformly charged with the volume density ρ. Find the flux of the electric field strength vector across the ball's section formed by the plane located at a distance $r_0 < R$ from the centre of the ball.

3.22. Each of the two long parallel threads carries a uniform charge λ per unit length. The threads are separated by a distance l. Find the maximum magnitude of the electric field strength in the symmetry plane of this system located between the threads.

3.23. An infinitely long cylindrical surface of circular cross-section is uniformly charged lengthwise with the surface density $\sigma = \sigma_0 \cos \varphi$, where φ is the polar angle of the cylindrical coordinate system whose z axis coincides with the axis of the given surface. Find the magnitude and direction of the electric field strength vector on the z axis.

3.24. The electric field strength depends only on the x and y coordinates according to the law $\mathbf{E} = a(x\mathbf{i} + y\mathbf{j})/(x^2 + y^2)$, where a is a constant, $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors of the x and y axes. Find the flux of the vector $\mathbf{E}$ through a sphere of radius R with its centre at the origin of coordinates.

3.25. A ball of radius R carries a positive charge whose volume density depends only on a separation r from the ball's centre as $\rho = \rho_0 (1 - r/R)$, where ρ_0 is a constant. Assuming the permittivities of the ball and the environment to be equal to unity, find:

(a) the magnitude of the electric field strength as a function of the distance r both inside and outside the ball;

(b) the maximum intensity E_{max} and the corresponding distance r_m.

3.26. A system consists of a ball of radius R carrying a spherically symmetric charge and the surrounding space filled with a charge of volume density $\rho = \alpha/r$, where α is a constant, r is the distance from the centre of the ball. Find the ball's charge at which the magnitude of the electric field strength vector is independent of r outside the ball. How high is this strength? The permittivities of the ball and the surrounding space are assumed to be equal to unity.

3.27. A space is filled up with a charge with volume density $\rho = \rho_0 e^{-\alpha r^3}$, where ρ_0 and α are positive constants, r is the distance from the centre of this system. Find the magnitude of the electric field strength vector as a function of r. Investigate the obtained expression for the small and large values of r, i.e. at $\alpha r^3 \ll 1$ and $\alpha r^3 \gg 1$.

3.28. Inside a ball charged uniformly with volume density ρ there is a spherical cavity. The centre of the cavity is displaced with respect to the centre of the ball by a distance $\mathbf{a}$. Find the field strength $\mathbf{E}$ inside the cavity, assuming the permittivity equal to unity.

3.29. Inside an infinitely long circular cylinder charged uniformly with volume density ρ there is a circular cylindrical cavity. The distance between the axes of the cylinder and the cavity is equal to $\mathbf{a}$. Find the electric field strength $\mathbf{E}$ inside the cavity. The permittivity is assumed to be equal to unity.

3.30. There are two thin wire rings, each of radius R, whose axes coincide. The charges of the rings are q and $-q$. Find the potential difference between the centres of the rings separated by a distance a.

3.31. There is an infinitely long straight thread carrying a charge with linear density $\lambda = 0.40\ \mu C/m$. Calculate the potential difference between points *1* and *2* if point *2* is removed $\eta = 2.0$ times farther from the thread than point *1*.

3.32. Find the electric field potential and strength at the centre of a hemisphere of radius R charged uniformly with the surface density σ.

3.33. A very thin round plate of radius R carrying a uniform surface charge density σ is located in vacuum. Find the electric field potential and strength along the plate's axis as a function of a distance l from its centre. Investigate the obtained expression at $l \to 0$ and $l \gg R$.

3.34. Find the potential φ at the edge of a thin disc of radius R carrying the uniformly distributed charge with surface density σ.

3.35. Find the electric field strength vector if the potential of this field has the form $\varphi = \mathbf{ar}$, where $\mathbf{a}$ is a constant vector, and $\mathbf{r}$ is the radius vector of a point of the field.

3.36. Determine the electric field strength vector if the potential of this field depends on x, y coordinates as

a) $\varphi = a(x^2 - y^2)$; (b) $\varphi = axy$,

where a is a constant. Draw the approximate shape of these fields using lines of force (in the x, y plane).

3.37. The potential of a certain electrostatic field has the form $\varphi = a(x^2 + y^2) + bz^2$, where a and b are constants. Find the magnitude and direction of the electric field strength vector. What shape have the equipotential surfaces in the following cases:

(a) $a > 0$, $b > 0$; (b) $a > 0$, $b < 0$?

3.38. A charge q is uniformly distributed over the volume of a sphere of radius R. Assuming the permittivity to be equal to unity throughout, find the potential

(a) at the centre of the sphere;

(b) inside the sphere as a function of the distance r from its centre.

3.39. Demonstrate that the potential of the field generated by a dipole with the electric moment $\mathbf{p}$ (Fig. 3.4) may be represented as $\varphi = \mathbf{pr}/4\pi\varepsilon_0 r^3$, where $\mathbf{r}$ is the radius vector. Using this expression, find the magnitude of the electric field strength vector as a function of r and θ.

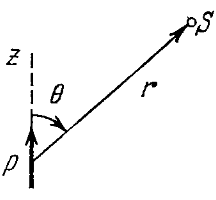

Fig. 3.4.

3.40. A point dipole with an electric moment p oriented in the positive direction of the z axis is located at the origin of coordinates. Find the projections E_z and $E_\perp$ of the electric field strength vector (on the plane perpendicular to the z axis at the point S (see Fig. 3.4)). At which points is $\mathbf{E}$ perpendicular to $\mathbf{p}$?

3.41. A point electric dipole with a moment $\mathbf{p}$ is placed in the external uniform electric field whose strength equals $\mathbf{E}_0$, with

$\mathbf{p} \uparrow\uparrow \mathbf{E}_0$. In this case one of the equipotential surfaces enclosing the dipole forms a sphere. Find the radius of this sphere.

3.42. Two thin parallel threads carry a uniform charge with linear densities λ and $-\lambda$. The distance between the threads is equal to l. Find the potential of the electric field and the magnitude of its strength vector at the distance $r \gg l$ at the angle θ to the vector $\mathbf{l}$ (Fig. 3.5).

3.43. Two coaxial rings, each of radius R, made of thin wire are separated by a small distance l ($l \ll R$) and carry the charges q and $-q$. Find the electric field potential and strength at the axis of the system as a function of the x coordinate (Fig. 3.6). Show in the same drawing the approximate plots of the functions obtained. Investigate these functions at $|x| \gg R$.

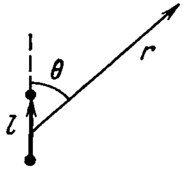

Fig. 3.5.

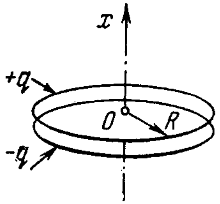

Fig. 3.6.

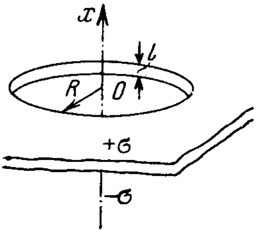

Fig. 3.7.

3.44. Two infinite planes separated by a distance l carry a uniform surface charge of densities σ and $-\sigma$ (Fig. 3.7). The planes have round coaxial holes of radius R, with $l \ll R$. Taking the origin O and the x coordinate axis as shown in the figure, find the potential of the electric field and the projection of its strength vector E_x on the axes of the system as functions of the x coordinate. Draw the approximate plot $\varphi(x)$.

3.45. An electric capacitor consists of thin round parallel plates, each of radius R, separated by a distance l ($l \ll R$) and uniformly charged with surface densities σ and $-\sigma$. Find the potential of the electric field and the magnitude of its strength vector at the axes of the capacitor as functions of a distance x from the plates if $x \gg l$. Investigate the obtained expressions at $x \gg R$.

3.46. A dipole with an electric moment $\mathbf{p}$ is located at a distance r from a long thread charged uniformly with a linear density λ. Find the force $\mathbf{F}$ acting on the dipole if the vector $\mathbf{p}$ is oriented

(a) along the thread;

(b) along the radius vector $\mathbf{r}$;

(c) at right angles to the thread and the radius vector $\mathbf{r}$.

3.47. Find the interaction force between two water molecules separated by a distance $l = 10$ nm if their electric moments are oriented along the same straight line. The moment of each molecule equals $p = 0.62 \cdot 10^{-29}$ C·m.

3.48. Find the potential $\varphi(x, y)$ of an electrostatic field $\mathbf{E} = a(y\mathbf{i} + x\mathbf{j})$, where a is a constant, $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors of the x and y axes.

3.49. Find the potential $\varphi(x, y)$ of an electrostatic field $\mathbf{E} = 2axy\mathbf{i} + a(x^2 - y^2)\mathbf{j}$, where a is a constant, $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors of the x and y axes.

3.50. Determine the potential $\varphi(x, y, z)$ of an electrostatic field $\mathbf{E} = ay\mathbf{i} + (ax + bz)\mathbf{j} + by\mathbf{k}$, where a and b are constants, $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are the unit vectors of the axes x, y, z.

3.51. The field potential in a certain region of space depends only on the x coordinate as $\varphi = -ax^3 + b$, where a and b are constants. Find the distribution of the space charge $\rho(x)$.

3.52. A uniformly distributed space charge fills up the space between two large parallel plates separated by a distance d. The potential difference between the plates is equal to $\Delta\varphi$. At what value of charge density ρ is the field strength in the vicinity of one of the plates equal to zero? What will then be the field strength near the other plate?

3.53. The field potential inside a charged ball depends only on the distance from its centre as $\varphi = ar^2 + b$, where a and b are constants. Find the space charge distribution $\rho(r)$ inside the ball.

3.2. CONDUCTORS AND DIELECTRICS IN AN ELECTRIC FIELD

- Electric field strength near the surface of a conductor in vacuum:

$$E_n = \sigma/\varepsilon_0. \tag{3.2a}$$

- Flux of polarization $\mathbf{P}$ across a closed surface:

$$\oint \mathbf{P}\, d\mathbf{S} = -q', \tag{3.2b}$$

where q' is the algebraic sum of *bound* charges enclosed by this surface.

- Vector $\mathbf{D}$ and Gauss's theorem for it:

$$\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}, \qquad \oint \mathbf{D}\, d\mathbf{S} = q, \tag{3.2c}$$

where q is the algebraic sum of *extraneous* charges inside a closed surface.

- Relations at the boundary between two dielectrics:

$$P_{2n} - P_{1n} = -\sigma', \qquad D_{2n} - D_{1n} = \sigma, \qquad E_{2\tau} = E_{1\tau}, \tag{3.2d}$$

where σ' and σ are the surface densities of bound and extraneous charges, and the unit vector $\mathbf{n}$ of the normal is directed from medium 1 to medium 2.

- In isotropic dielectrics:

$$\mathbf{P} = \varkappa\varepsilon_0\mathbf{E}, \quad \mathbf{D} = \varepsilon\varepsilon_0\mathbf{E}, \quad \varepsilon = 1 + \varkappa. \tag{3.2e}$$

- In the case of an isotropic uniform dielectric filling up all the space between the equipotential surfaces:

$$\mathbf{E} = \mathbf{E}_0/\varepsilon. \tag{3.2f}$$

3.54. A small ball is suspended over an infinite horizontal conducting plane by means of an insulating elastic thread of stiffness k. As soon as the ball was charged, it descended by x cm and its separation from the plane became equal to l. Find the charge of the ball.

3.55. A point charge q is located at a distance l from the infinite conducting plane. What amount of work has to be performed in order to slowly remove this charge very far from the plane.

3.56. Two point charges, q and $-q$, are separated by a distance l, both being located at a distance $l/2$ from the infinite conducting plane. Find:

(a) the modulus of the vector of the electric force acting on each charge;

(b) the magnitude of the electric field strength vector at the midpoint between these charges.

3.57. A point charge q is located between two mutually perpendicular conducting half-planes. Its distance from each half-plane is equal to l. Find the modulus of the vector of the force acting on the charge.

3.58. A point dipole with an electric moment $\mathbf{p}$ is located at a distance l from an infinite conducting plane. Find the modulus of the vector of the force acting on the dipole if the vector $\mathbf{p}$ is perpendicular to the plane.

3.59. A point charge q is located at a distance l from an infinite conducting plane. Determine the surface density of charges induced on the plane as a function of separation r from the base of the perpendicular drawn to the plane from the charge.

3.60. A thin infinitely long thread carrying a charge λ per unit length is oriented parallel to the infinite conducting plane. The distance between the thread and the plane is equal to l. Find:

(a) the modulus of the vector of the force acting on a unit length of the thread;

(b) the distribution of surface charge density $\sigma(x)$ over the plane, where x is the distance from the plane perpendicular to the conducting surface and passing through the thread.

3.61. A very long straight thread is oriented at right angles to an infinite conducting plane; its end is separated from the plane by a distance l. The thread carries a uniform charge of linear density λ. Suppose the point O is the trace of the thread on the plane. Find the surface density of the induced charge on the plane

(a) at the point O;

(b) as a function of a distance r from the point O.

3.62. A thin wire ring of radius R carries a charge q. The ring is oriented parallel to an infinite conducting plane and is separated by a distance l from it. Find:

(a) the surface charge density at the point of the plane symmetrical with respect to the ring;

(b) the strength and the potential of the electric field at the centre of the ring.

3.63. Find the potential φ of an uncharged conducting sphere outside of which a point charge q is located at a distance l from the sphere's centre.

3.64. A point charge q is located at a distance r from the centre O of an uncharged conducting spherical layer whose inside and outside radii are equal to R_1 and R_2 respectively. Find the potential at the point O if $r < R_1$.

3.65. A system consists of two concentric conducting spheres, with the inside sphere of radius a carrying a positive charge q_1. What charge q_2 has to be deposited on the outside sphere of radius b to reduce the potential of the inside sphere to zero? How does the potential φ depend in this case on a distance r from the centre of the system? Draw the approximate plot of this dependence.

3.66. Four large metal plates are located at a small distance d from one another as shown in Fig. 3.8. The extreme plates are inter-

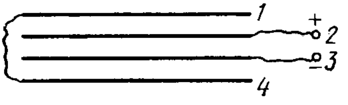

Fig. 3.8.

connected by means of a conductor while a potential difference $\Delta\varphi$ is applied to internal plates. Find:

(a) the values of the electric field strength between neighbouring plates;

(b) the total charge per unit area of each plate.

3.67. Two infinite conducting plates *1* and *2* are separated by a distance l. A point charge q is located between the plates at a distance x from plate *1*. Find the charges induced on each plate.

3.68. Find the electric force experienced by a charge reduced to a unit area of an arbitrary conductor if the surface density of the charge equals σ.

3.69. A metal ball of radius $R = 1.5$ cm has a charge $q = 10$ μC. Find the modulus of the vector of the resultant force acting on a charge located on one half of the ball.

3.70. When an uncharged conducting ball of radius R is placed in an external uniform electric field, a surface charge density $\sigma = \sigma_0 \cos\theta$ is induced on the ball's surface (here σ_0 is a constant, θ is a polar angle). Find the magnitude of the resultant electric force acting on an induced charge of the same sign.

3.71. An electric field of strength $E = 1.0$ kV/cm produces polarization in water equivalent to the correct orientation of only one out of N molecules. Find N. The electric moment of a water molecule equals $p = 0.62 \cdot 10^{-29}$ C·m.

3.72. A non-polar molecule with polarizability β is located at a great distance l from a polar molecule with electric moment **p**. Find the magnitude of the interaction force between the molecules if the vector **p** is oriented along a straight line passing through both molecules.

3.73. A non-polar molecule is located at the axis of a thin uniformly charged ring of radius R. At what distance x from the ring's centre is the magnitude of the force F acting on the given molecule

(a) equal to zero; (b) maximum?

Draw the approximate plot $F_x(x)$.

3.74. A point charge q is located at the centre of a ball made of uniform isotropic dielectric with permittivity ε. Find the polarization $\mathbf{P}$ as a function of the radius vector $\mathbf{r}$ relative to the centre of the system, as well as the charge q' inside a sphere whose radius is less than the radius of the ball.

3.75. Demonstrate that at a dielectric-conductor interface the surface density of the dielectric's bound charge $\sigma' = -\sigma(\varepsilon - 1)/\varepsilon$, where ε is the permittivity, σ is the surface density of the charge on the conductor.

3.76. A conductor of arbitrary shape, carrying a charge q, is surrounded with uniform dielectric of permittivity ε (Fig. 3.9).

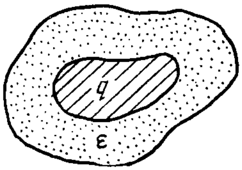

Fig. 3.9.

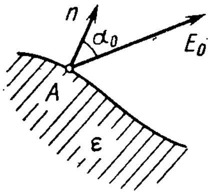

Fig. 3.10.

Find the total bound charges at the inner and outer surfaces of the dielectric.

3.77. A uniform isotropic dielectric is shaped as a spherical layer with radii a and b. Draw the approximate plots of the electric field strength E and the potential φ vs the distance r from the centre of the layer if the dielectric has a certain positive extraneous charge distributed uniformly:

(a) over the internal surface of the layer; (b) over the volume of the layer.

3.78. Near the point A (Fig. 3.10) lying on the boundary between glass and vacuum the electric field strength in vacuum is equal to $E_0 = 10.0$ V/m, the angle between the vector $\mathbf{E}_0$ and the normal n of the boundary line being equal to $\alpha_0 = 30°$. Find the field strength E in glass near the point A, the angle α between the vector $\mathbf{E}$ and $\mathbf{n}$, as well as the surface density of the bound charges at the point A.

3.79. Near the plane surface of a uniform isotropic dielectric with permittivity ε the electric field strength in vacuum is equal to E_0, the vector $\mathbf{E}_0$ forming an angle θ with the normal of the dielectric's surface (Fig. 3.11). Assuming the field to be uniform both inside and outside the dielectric, find:

(a) the flux of the vector $\mathbf{E}$ through a sphere of radius R with centre located at the surface of the dielectric;

(b) the circulation of the vector **D** around the closed path Γ of length l (see Fig. 3.11) whose plane is perpendicular to the surface of the dielectric and parallel to the vector $\mathbf{E}_0$.

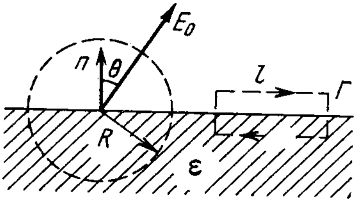

Fig. 3.11.

3.80. An infinite plane of uniform dielectric with permittivity ε is uniformly charged with extraneous charge of space density ρ. The thickness of the plate is equal to $2d$. Find:

(a) the magnitude of the electric field strength and the potential as functions of distance l from the middle point of the plane (where the potential is assumed to be equal to zero); having chosen the x coordinate axis perpendicular to the plate, draw the approximate plots of the projection $E_x(x)$ of the vector **E** and the potential $\varphi(x)$;

(b) the surface and space densities of the bound charge.

3.81. Extraneous charges are uniformly distributed with space density $\rho > 0$ over a ball of radius R made of uniform isotropic dielectric with permittivity ε. Find:

(a) the magnitude of the electric field strength as a function of distance r from the centre of the ball; draw the approximate plots $E(r)$ and $\varphi(r)$;

(b) the space and surface densities of the bound charges.

3.82. A round dielectric disc of radius R and thickness d is *statically* polarized so that it gains the uniform polarization **P**, with the vector **P** lying in the plane of the disc. Find the strength **E** of the electric field at the centre of the disc if $d \ll R$.

3.83. Under certain conditions the polarization of an infinite uncharged dielectric plate takes the form $\mathbf{P} = \mathbf{P}_0(1 - x^2/d^2)$, where $\mathbf{P}_0$ is a vector perpendicular to the plate, x is the distance from the middle of the plate, d is its half-thickness. Find the strength **E** of the electric field inside the plate and the potential difference between its surfaces.

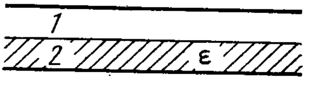

Fig. 3.12.

3.84. Initially the space between the plates of the capacitor is filled with air, and the field strength in the gap is equal to E_0. Then half the gap is filled with uniform isotropic dielectric with permittivity ε as shown in Fig. 3.12. Find the moduli of the vectors **E** and **D** in both parts of the gap (*1* and *2*) if the introduction of the dielectric

(a) does not change the voltage across the plates;

(b) leaves the charges at the plates constant.

3.85. Solve the foregoing problem for the case when half the gap is filled with the dielectric in the way shown in Fig. 3.13.

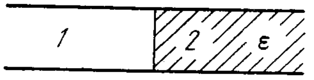

Fig. 3.13.

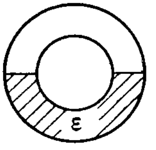

Fig. 3.14.

3.86. Half the space between two concentric electrodes of a spherical capacitor is filled, as shown in Fig. 3.14, with uniform isotropic dielectric with permittivity ε. The charge of the capacitor is q. Find the magnitude of the electric field strength between the electrodes as a function of distance r from the curvature centre of the electrodes.

3.87. Two small identical balls carrying the charges of the same sign are suspended from the same point by insulating threads of equal length. When the surrounding space was filled with kerosene the divergence angle between the threads remained constant. What is the density of the material of which the balls are made?

3.88. A uniform electric field of strength $E = 100$ V/m is generated inside a ball made of uniform isotropic dielectric with permittivity $\varepsilon = 5.00$. The radius of the ball is $R = 3.0$ cm. Find the maximum surface density of the bound charges and the total bound charge of one sign.

3.89. A point charge q is located in vacuum at a distance l from the plane surface of a uniform isotropic dielectric filling up all the half-space. The permittivity of the dielectric equals ε. Find:

(a) the surface density of the bound charges as a function of distance r from the point charge q; analyse the obtained result at $l \to 0$;

(b) the total bound charge on the surface of the dielectric.

3.90. Making use of the formulation and the solution of the foregoing problem, find the magnitude of the force exerted by the charges bound on the surface of the dielectric on the point charge q.

3.91. A point charge q is located on the plane dividing vacuum and infinite uniform isotropic dielectric with permittivity ε. Find the moduli of the vectors **D** and **E** as well as the potential φ as functions of distance r from the charge q.

3.92. A small conducting ball carrying a charge q is located in a uniform isotropic dielectric with permittivity ε at a distance l from an infinite boundary plane between the dielectric and vacuum. Find the surface density of the bound charges on the boundary plane as a function of distance r from the ball. Analyse the obtained result for $l \to 0$.

3.93. A half-space filled with uniform isotropic dielectric with permittivity ε has the conducting boundary plane. Inside the dielectric, at a distance l from this plane, there is a small metal ball possessing a charge q. Find the surface density of the bound charges at the boundary plane as a function of distance r from the ball.

3.94. A plate of thickness h made of uniform *statically* polarized dielectric is placed inside a capacitor whose parallel plates are interconnected by a conductor. The polarization of the dielectric is equal

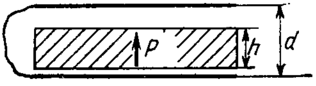

Fig. 3.15.

to **P** (Fig. 3.15). The separation between the capacitor plates is d. Find the strength and induction vectors for the electric field both inside and outside the plates.

3.95. A long round dielectric cylinder is polarized so that the vector $\mathbf{P} = \alpha\mathbf{r}$, where α is a positive constant and $\mathbf{r}$ is the distance from the axis. Find the space density ρ' of bound charges as a function of distance r from the axis.

3.96. A dielectric ball is polarized uniformly and *statically*. Its polarization equals **P**. Taking into account that a ball polarized *in this way* may be represented as a result of a small shift of all positive charges of the dielectric relative to all negative charges,

(a) find the electric field strength **E** inside the ball;

(b) demonstrate that the field outside the ball is that of a dipole located at the centre of the ball, the potential of that field being equal to $\varphi = \mathbf{p}_0\mathbf{r}/4\pi\varepsilon_0$, where $\mathbf{p}_0$ is the electric moment of the ball, and $\mathbf{r}$ is the distance from its centre.

3.97. Utilizing the solution of the foregoing problem, find the electric field strength $\mathbf{E}_0$ in a spherical cavity in an infinite statically polarized uniform dielectric if the dielectric's polarization is **P**, and far from the cavity the field strength is **E**.

3.98. A uniform dielectric ball is placed in a uniform electric field of strength $\mathbf{E}_0$. Under these conditions the dielectric becomes polarized uniformly. Find the electric field strength **E** inside the ball and the polarization **P** of the dielectric whose permittivity equals ε. Make use of the result obtained in Problem 3.96.

3.99. An infinitely long round dielectric cylinder is polarized uniformly and statically, the polarization **P** being perpendicular to the axis of the cylinder. Find the electric field strength **E** inside the dielectric.

3.100. A long round cylinder made of uniform dielectric is placed in a uniform electric field of strength $\mathbf{E}_0$. The axis of the cylinder is perpendicular to vector $\mathbf{E}_0$. Under these conditions the dielectric becomes polarized uniformly. Making use of the result obtained in the foregoing problem, find the electric field strength **E** in the cylinder and the polarization **P** of the dielectric whose permittivity is equal to ε.

3.3. ELECTRIC CAPACITANCE. ENERGY OF AN ELECTRIC FIELD

• Capacitance of a parallel-plate capacitor:

$$C = \varepsilon\varepsilon_0 S/d. \tag{3.3a}$$

• Interaction energy of a system of point charges:

$$W = \frac{1}{2}\sum q_i\varphi_i. \tag{3.3b}$$

• Total electric energy of a system with continuous charge distribution:

$$W = \frac{1}{2}\int \varphi\rho\, dV. \tag{3.3c}$$

• Total electric energy of two charged bodies 1 and 2:

$$W = W_1 + W_2 + W_{12}, \tag{3.3d}$$

where W_1 and W_2 are the self-energies of the bodies, and W_{12} is the interaction energy.

• Energy of a charged capacitor:

$$W = \frac{qV}{2} = \frac{q^2}{2C} = \frac{CV^2}{2} \tag{3.3e}$$

• Volume density of electric field energy:

$$w = \frac{\mathbf{ED}}{2} = \frac{\varepsilon\varepsilon_0 E^2}{2} \tag{3.3f}$$

3.101. Find the capacitance of an isolated ball-shaped conductor of radius R_1 surrounded by an adjacent concentric layer of dielectric with permittivity ε and outside radius R_2.

3.102. Two parallel-plate air capacitors, each of capacitance C, were connected in series to a battery with emf $\mathscr{E}$. Then one of the capacitors was filled up with uniform dielectric with permittivity ε. How many times did the electric field strength in that capacitor decrease? What amount of charge flows through the battery?

3.103. The space between the plates of a parallel-plate capacitor is filled consecutively with two dielectric layers *1* and *2* having the thicknesses d_1 and d_2 and the permittivities ε_1 and ε_2 respectively. The area of each plate is equal to S. Find:

(a) the capacitance of the capacitor;

(b) the density σ' of the bound charges on the boundary plane if the voltage across the capacitor equals V and the electric field is directed from layer *1* to layer *2*.

3.104. The gap between the plates of a parallel-plate capacitor is filled with isotropic dielectric whose permittivity ε varies linearly from ε_1 to ε_2 ($\varepsilon_2 > \varepsilon_1$) in the direction perpendicular to the plates. The area of each plate equals S, the separation between the plates is equal to d. Find:

(a) the capacitance of the capacitor;

(b) the space density of the bound charges as a function of ε if the charge of the capacitor is q and the field **E** in it is directed toward the growing ε values.

3.105. Find the capacitance of a spherical capacitor whose electrodes have radii R_1 and $R_2 > R_1$ and which is filled with isotropic dielectric whose permittivity varies as $\varepsilon = a/r$, where a is a constant, and r is the distance from the centre of the capacitor.

3.106. A cylindrical capacitor is filled with two cylindrical layers of dielectric with permittivities ε_1 and ε_2. The inside radii of the layers are equal to R_1 and $R_2 > R_1$. The maximum permissible values of electric field strength are equal to E_{1m} and E_{2m} for these dielectrics. At what relationship between ε, R, and E_m will the voltage increase result in the field strength reaching the breakdown value for both dielectrics simultaneously?

3.107. There is a double-layer cylindrical capacitor whose parameters are shown in Fig. 3.16. The breakdown field strength values for these dielectrics are equal to E_1 and E_2 respectively. What is the breakdown voltage of this capacitor if $\varepsilon_1 R_1 E_1 < \varepsilon_2 R_2 E_2$?

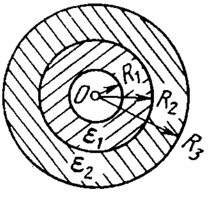

Fig. 3.16.

3.108. Two long straight wires with equal cross-sectional radii a are located parallel to each other in air. The distance between their axes equals b. Find the mutual capacitance of the wires per unit length under the condition $b \gg a$.

3.109. A long straight wire is located parallel to an infinite conducting plate. The wire cross-sectional radius is equal to a, the distance between the axis of the wire and the plane equals b. Find the mutual capacitance of this system per unit length of the wire under the condition $a \ll b$.

3.110. Find the capacitance of a system of two identical metal balls of radius a if the distance between their centres is equal to b, with $b \gg a$. The system is located in a uniform dielectric with permittivity ε.

3.111. Determine the capacitance of a system consisting of a metal ball of radius a and an infinite conducting plane separated from the centre of the ball by the distance l if $l \gg a$.

3.112. Find the capacitance of a system of identical capacitors between points A and B shown in

(a) Fig. 3.17a; (b) Fig. 3.17b.

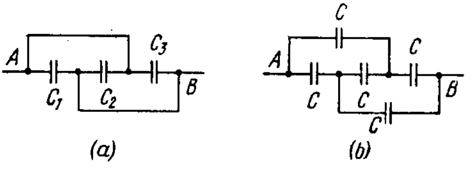

Fig. 3.17.

3.113. Four identical metal plates are located in air at equal distances d from one another. The area of each plate is equal to S. Find the capacitance of the system between points A and B if the plates are interconnected as shown
(a) in Fig. 3.18a; (b) in Fig. 3.18b.

Fig. 3.18.

3.114. A capacitor of capacitance $C_1 = 1.0$ μF withstands the maximum voltage $V_1 = 6.0$ kV while a capacitor of capacitance $C_2 = 2.0$ μF, the maximum voltage $V_2 = 4.0$ kV. What voltage will the system of these two capacitors withstand if they are connected in series?

3.115. Find the potential difference between points A and B of the system shown in Fig. 3.19 if the emf is equal to $\mathscr{E} = 110$ V and the capacitance ratio $C_2/C_1 = \eta = 2.0$.

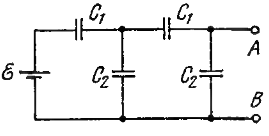

Fig. 3.19.

3.116. Find the capacitance of an infinite circuit formed by the repetition of the same link consisting of two identical capacitors, each with capacitance C (Fig. 3.20).

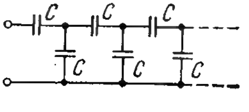

Fig. 3.20.

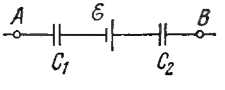

Fig. 3.21.

3.117. A circuit has a section AB shown in Fig. 3.21. The emf of the source equals $\mathscr{E} = 10$ V, the capacitor capacitances are equal to $C_1 = 1.0$ μF and $C_2 = 2.0$ μF, and the potential difference $\varphi_A - \varphi_B = 5.0$ V. Find the voltage across each capacitor.

3.118. In a circuit shown in Fig. 3.22 find the potential difference between the left and right plates of each capacitor.

3.119. Find the charge of each capacitor in the circuit shown in Fig. 3.22.

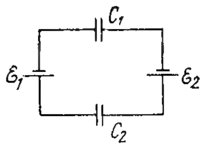

Fig. 3.22.

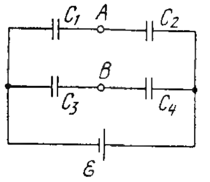

Fig. 3.23.

3.120. Determine the potential difference $\varphi_A - \varphi_B$ between points A and B of the circuit shown in Fig. 3.23. Under what condition is it equal to zero?

3.121. A capacitor of capacitance $C_1 = 1.0$ μF charged up to a voltage $V = 110$ V is connected in parallel to the terminals of a circuit consisting of two uncharged capacitors connected in series and possessing the capacitances $C_2 = 2.0$ μF and $C_3 = 3.0$ μF. What charge will flow through the connecting wires?

3.122. What charges will flow after the shorting of the switch *Sw* in the circuit illustrated in Fig. 3.24 through sections *1* and *2* in the directions indicated by the arrows?

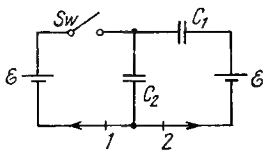

Fig. 3.24.

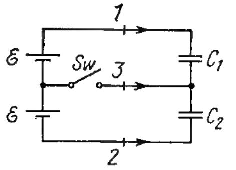

Fig. 3.25.

3.123. In the circuit shown in Fig. 3.25 the emf of each battery is equal to $\mathscr{E} = 60$ V, and the capacitor capacitances are equal to $C_1 = 2.0$ μF and $C_2 = 3.0$ μF. Find the charges which will flow after the shorting of the switch *Sw* through sections *1*, *2* and *3* in the directions indicated by the arrows.

3.124. Find the potential difference $\varphi_A - \varphi_B$ between points A and B of the circuit shown in Fig. 3.26.

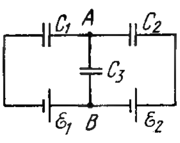

Fig. 3.26.

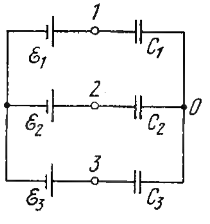

Fig. 3.27.

3.125. Determine the potential at point *1* of the circuit shown in Fig. 3.27, assuming the potential at the point O to be equal to zero.

Using the symmetry of the formula obtained, write the expressions for the potentials. at points *2* and *3*.

3.126. Find the capacitance of the circuit shown in Fig. 3.28 between points A and B.

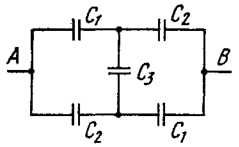

Fig. 3.28.

3.127. Determine the interaction energy of the point charges located at the corners of a square with the side a in the circuits shown in Fig. 3.29.

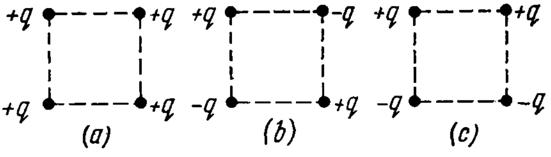

Fig. 3.29.

3.128. There is an infinite straight chain of alternating charges q and $-q$. The distance between the neighbouring charges is equal to a. Find the interaction energy of each charge with all the others.

Instruction. Make use of the expansion of $\ln(1+\alpha)$ in a power series in α.

3.129. A point charge q is located at a distance l from an infinite tonducting plane. Find the interaction energy of that charge with chose induced on the plane.

3.130. Calculate the interaction energy of two balls whose charges q_1 and q_2 are spherically symmetrical. The distance between the centres of the balls is equal to l.

Instruction. Start with finding the interaction energy of a ball and a thin spherical layer.

3.131. A capacitor of capacitance $C_1 = 1.0\ \mu F$ carrying initially a voltage $V = 300$ V is connected in parallel with an uncharged capacitor of capacitance $C_2 = 2.0\ \mu F$. Find the increment of the electric energy of this system by the moment equilibrium is reached. Explain the result obtained.

3.132. What amount of heat will be generated in the circuit shown in Fig. 3.30 after the switch *Sw* is shifted from position *1* to position *2*?

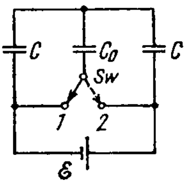

Fig. 3.30.

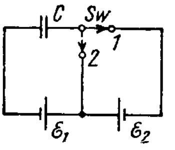

Fig. 3.31.

3.133. What amount of heat will be generated in the circuit shown in Fig. 3.31 after the switch *Sw* is shifted from position *1* to position *2*?

3.134. A system consists of two thin concentric metal shells of radii R_1 and R_2 with corresponding charges q_1 and q_2. Find the self-energy values W_1 and W_2 of each shell, the interaction energy of the shells W_{12}, and the total electric energy of the system.

3.135. A charge q is distributed uniformly over the volume of a ball of radius R. Assuming the permittivity to be equal to unity, find:

(a) the electrostatic self-energy of the ball;

(b) the ratio of the energy W_1 stored in the ball to the energy W_2 pervading the surrounding space.

3.136. A point charge $q = 3.0\ \mu C$ is located at the centre of a spherical layer of uniform isotropic dielectric with permittivity $\varepsilon = 3.0$. The inside radius of the layer is equal to $a = 250$ mm, the outside radius is $b = 500$ mm. Find the electrostatic energy inside the dielectric layer.

3.137. A spherical shell of radius R_1 with uniform charge q is expanded to a radius R_2. Find the work performed by the electric forces in this process.

3.138. A spherical shell of radius R_1 with a uniform charge q has a point charge q_0 at its centre. Find the work performed by the electric forces during the shell expansion from radius R_1 to radius R_2.

3.139. A spherical shell is uniformly charged with the surface density σ. Using the energy conservation law, find the magnitude of the electric force acting on a unit area of the shell.

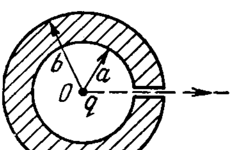

Fig. 3.32.

3.140. A point charge q is located at the centre O of a spherical uncharged conducting layer provided with a small orifice (Fig. 3.32). The inside and outside radii of the layer are equal to a and b respectively. What amount of work has to be performed to slowly transfer the charge q from the point O through the orifice and into infinity?

3.141. Each plate of a parallel-plate air capacitor has an area S. What amount of work has to be performed to slowly increase the distance between the plates from x_1 to x_2 if

(a) the capacitance of the capacitor, which is equal to q, or (b) the voltage across the capacitor, which is equal to V, is kept constant in the process?

3.142. Inside a parallel-plate capacitor there is a plate parallel to the outer plates, whose thickness is equal to $\eta = 0.60$ of the gap width. When the plate is absent the capacitor capacitance equals $c = 20$ nF. First, the capacitor was connected in parallel to a constant voltage source producing $V = 200$ V, then it was disconnected from it, after which the plate was slowly removed from the gap. Find the work performed during the removal, if the plate is

(a) made of metal; (b) made of glass.

3.143. A parallel-plate capacitor was lowered into water in a horizontal position, with water filling up the gap between the plates $d = 1.0$ mm wide. Then a constant voltage $V = 500$ V was applied to the capacitor. Find the water pressure increment in the gap.

3.144. A parallel-plate capacitor is located horizontally so that one of its plates is submerged into liquid while the other is over its surface (Fig. 3.33). The permittivity of the liquid is equal to ε, its density is equal to ρ. To what height will the level of the liquid in the capacitor rise after its plates get a charge of surface density σ?

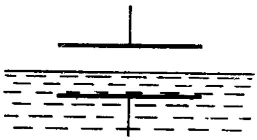

Fig. 3.33

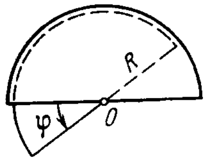

Fig. 3.34.

3.145. A cylindrical layer of dielectric with permittivity ε is inserted into a cylindrical capacitor to fill up all the space between the electrodes. The mean radius of the electrodes equals R, the gap between them is equal to d, with $d \ll R$. The constant voltage V is applied across the electrodes of the capacitor. Find the magnitude of the electric force pulling the dielectric into the capacitor.

3.146. A capacitor consists of two stationary plates shaped as a semi-circle of radius R and a movable plate made of dielectric with permittivity ε and capable of rotating about an axis O between the stationary plates (Fig. 3.34). The thickness of the movable plate is equal to d which is practically the separation between the stationary plates. A potential difference V is applied to the capacitor. Find the magnitude of the moment of forces relative to the axis O acting on the movable plate in the position shown in the figure.

- Ohm's law for an inhomogeneous segment of a circuit:

$$I=\frac{V_{12}}{R}=\frac{\varphi_1-\varphi_2+\mathscr{E}_{12}}{R}, \tag{3.4a}$$

where V_{12} is the voltage drop across the segment.

- Differential form of Ohm's law:

$$\mathbf{j}=\sigma(\mathbf{E}+\mathbf{E}^*), \tag{3.4b}$$

where $\mathbf{E}^*$ is the strength of a field produced by extraneous forces.

- Kirchhoff's laws (for an electric circuit):

$$\sum I_k=0, \quad \sum I_k R_k=\sum \mathscr{E}_k. \tag{3.4c}$$

- Power P of current and thermal power Q:

$$P=VI=(\varphi_1-\varphi_2+\mathscr{E}_{12})\,I, \quad Q=RI^2. \tag{3.4d}$$

- Specific power P_{sp} of current and specific thermal power Q_{sp}:

$$P_{sp}=\mathbf{j}(\mathbf{E}+\mathbf{E}^*), \quad Q_{sp}=\rho j^2 \tag{3.4e}$$

- Current density in a metal:

$$\mathbf{j}=en\mathbf{u}, \tag{3.4f}$$

where $\mathbf{u}$ is the average velocity of carriers.

- Number of ions recombining per unit volume of gas per unit time:

$$\dot{n}_r=rn^2, \tag{3.4g}$$

where r is the recombination coefficient.

3.147. A long cylinder with uniformly charged surface and cross-sectional radius $a = 1.0$ cm moves with a constant velocity $v = 10$ m/s along its axis. An electric field strength at the surface of the cylinder is equal to $E = 0.9$ kV/cm. Find the resulting convection current, that is, the current caused by mechanical transfer of a charge.

3.148. An air cylindrical capacitor with a dc voltage $V = 200$ V applied across it is being submerged vertically into a vessel filled with water at a velocity $v = 5.0$ mm/s. The electrodes of the capacitor are separated by a distance $d = 2.0$ mm, the mean curvature radius of the electrodes is equal to $r = 50$ mm. Find the current flowing in this case along lead wires, if $d \ll r$.

3.149. At the temperature 0 °C the electric resistance of conductor *2* is η times that of conductor *1*. Their temperature coefficients of resistance are equal to α_2 and α_1 respectively. Find the temperature coefficient of resistance of a circuit segment consisting of these two conductors when they are connected

(a) in series; (b) in parallel.

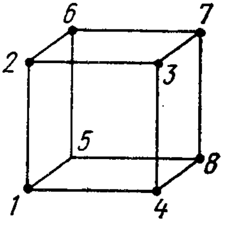

Fig. 3.35.

3.150. Find the resistance of a wire frame shaped as a cube (Fig. 3.35) when measured between points

(a) *1-7*; (b) *1-2*; (c) *1-3*.

The resistance of each edge of the frame is R

3.151. At what value of the resistance R_x in the circuit shown in Fig. 3.36 will the total resistance between points A and B be independent of the number of cells?

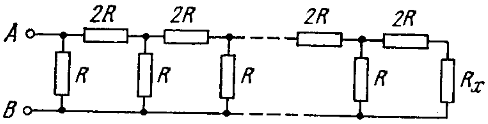

Fig. 3.36.

3.152. Fig. 3.37 shows an infinite circuit formed by the repetition of the same link, consisting of resistance $R_1 = 4.0\ \Omega$ and $R_2 = 3.0\ \Omega$. Find the resistance of this circuit between points A and B.

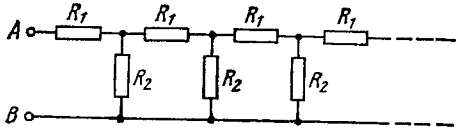

Fig. 3.37.

3.153. There is an infinite wire grid with square cells (Fig. 3.38). The resistance of each wire between neighbouring joint connections is equal to R_0. Find the resistance R of the whole grid between points A and B.

Instruction. Make use of principles of symmetry and superposition.

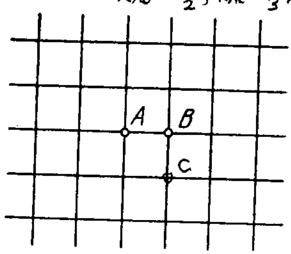

Fig. 3.38.

3.154. A homogeneous poorly conducting medium of resistivity ρ fills up the space between two thin coaxial ideally conducting cylinders. The radii of the cylinders are equal to a and b, with $a < b$, the length of each cylinder is l. Neglecting the edge effects, find the resistance of the medium between the cylinders.

3.155. A metal ball of radius a is surrounded by a thin concentric metal shell of radius b. The space between these electrodes is filled up with a poorly conducting homogeneous medium of resistivity ρ. Find the resistance of the interelectrode gap. Analyse the obtained solution at $b \to \infty$.

3.156. The space between two conducting concentric spheres of radii a and b ($a < b$) is filled up with homogeneous poorly conducting medium. The capacitance of such a system equals C. Find the resistivity of the medium if the potential difference between the spheres, when they are disconnected from an external voltage, decreases η-fold during the time interval Δt.

3.157. Two metal balls of the same radius a are located in a homogeneous poorly conducting medium with resistivity ρ. Find the resistance of the medium between the balls provided that the separation between them is much greater than the radius of the ball.

3.158. A metal ball of radius a is located at a distance l from an infinite ideally conducting plane. The space around the ball is filled with a homogeneous poorly conducting medium with resistivity ρ. In the case of $a \ll l$ find:

(a) the current density at the conducting plane as a function of distance r from the ball if the potential difference between the ball and the plane is equal to V;

(b) the electric resistance of the medium between the ball and the plane.

3.159. Two long parallel wires are located in a poorly conducting medium with resistivity ρ. The distance between the axes of the wires is equal to l, the cross-section radius of each wire equals a. In the case $a \ll l$ find:

(a) the current density at the point equally removed from the axes of the wires by a distance r if the potential difference between the wires is equal to V;

(b) the electric resistance of the medium per unit length of the wires.

3.160. The gap between the plates of a parallel-plate capacitor is filled with glass of resistivity $\rho = 100$ GΩ·m. The capacitance of the capacitor equals $C = 4.0$ nF. Find the leakage current of the capacitor when a voltage $V = 2.0$ kV is applied to it.

3.161. Two conductors of arbitrary shape are embedded into an infinite homogeneous poorly conducting medium with resistivity ρ and permittivity ε. Find the value of a product RC for this system, where R is the resistance of the medium between the conductors, and C is the mutual capacitance of the wires in the presence of the medium.

3.162. A conductor with resistivity ρ bounds on a dielectric with permittivity ε. At a certain point A at the conductor's surface the electric displacement equals D, the vector $\mathbf{D}$ being directed away from the conductor and forming an angle α with the normal of the surface. Find the surface density of charges on the conductor at the point A and the current density in the conductor in the vicinity of the same point.

3.163. The gap between the plates of a parallel-plate capacitor is filled up with an inhomogeneous poorly conducting medium whose conductivity varies linearly in the direction perpendicular to the plates from $\sigma_1 = 1.0$ pS/m to $\sigma_2 = 2.0$ pS/m. Each plate has an area $S = 230$ cm², and the separation between the plates is $d = 2.0$ mm. Find the current flowing through the capacitor due to a voltage $V = 300$ V.

3.164. Demonstrate that the law of refraction of direct current lines at the boundary between two conducting media has the form

$\tan\alpha_2/\tan\alpha_1 = \sigma_2/\sigma_1$, where σ_1 and σ_2 are the conductivities of the media, α_2 and α_1 are the angles between the current lines and the normal of the boundary surface.

3.165. Two cylindrical conductors with equal cross-sections and different resistivities ρ_1 and ρ_2 are put end to end. Find the charge at the boundary of the conductors if a current I flows from conductor *1* to conductor *2*.

3.166. The gap between the plates of a parallel-plate capacitor is filled up with two dielectric layers *1* and *2* with thicknesses d_1 and d_2, permittivities ε_1 and ε_2, and resistivities ρ_1 and ρ_2. A dc voltage V is applied to the capacitor, with electric field directed from layer *1* to layer *2*. Find σ, the surface density of extraneous charges at the boundary between the dielectric layers, and the condition under which $\sigma = 0$.

3.167. An inhomogeneous poorly conducting medium fills up the space between plates *1* and *2* of a parallel-plate capacitor. Its permittivity and resistivity vary from values ε_1, ρ_1 at plate *1* to values ε_2, ρ_2 at plate *2*. A dc voltage is applied to the capacitor through which a steady current I flows from plate *1* to plate *2*. Find the total extraneous charge in the given medium.

3.168. The space between the plates of a parallel-plate capacitor is filled up with inhomogeneous poorly conducting medium whose resistivity varies linearly in the direction perpendicular to the plates. The ratio of the maximum value of resistivity to the minimum one is equal to η The gap width equals d. Find the volume density of the charge in the gap if a voltage V is applied to the capacitor. ε is assumed to be 1 everywhere.

3.169. A long round conductor of cross-sectional area S is made of material whose resistivity depends only on a distance r from the axis of the conductor as $\rho = \alpha/r^2$, where α is a constant. Find:

(a) the resistance per unit length of such a conductor;

(b) the electric field strength in the conductor due to which a current I flows through it.

3.170. A capacitor with capacitance $C = 400$ pF is connected via a resistance $R = 650\ \Omega$ to a source of constant voltage V_0. How soon will the voltage developed across the capacitor reach a value $V = 0.90\ V_0$?

3.171. A capacitor filled with dielectric of permittivity $\varepsilon = 2.1$ loses half the charge acquired during a time interval $\tau = 3.0$ min. Assuming the charge to leak only through the dielectric filler, calculate its resistivity.

3.172. A circuit consists of a source of a constant emf $\mathscr{E}$ and a resist ance R and a capacitor with capacitance C connected in series. The internal resistance of the source is negligible. At a moment $t = 0$ the capacitance of the capacitor is abruptly decreased η-fold. Find the current flowing through the circuit as a function of time t.

3.173. An ammeter and a voltmeter are connected in series to a battery with an emf $\mathscr{E} = 6.0$ V. When a certain resistance is connected in parallel with the voltmeter, the readings of the latter decrease $\eta = 2.0$ times, whereas the readings of the ammeter increase the same number of times. Find the voltmeter readings after the connection of the resistance.

3.174. Find a potential difference $\varphi_1 - \varphi_2$ between points *1* and *2* of the circuit shown in Fig. 3.39 if $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, $\mathscr{E}_1 = 5.0$ V, and $\mathscr{E}_2 = 2.0$ V. The internal resistances of the current sources are negligible.

3.175. Two sources of current of equal emf are connected in series and have different internal resistances R_1 and R_2 ($R_2 > R_1$). Find the external resistance R at which the potential difference across the terminals of one of the sources (which one in particular?) becomes equal to zero.

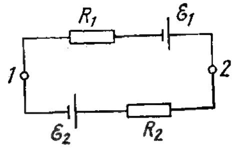

Fig. 3.39.

3.176. N sources of current with different emf's are connected as shown in Fig. 3.40. The emf's of the sources are proportional to

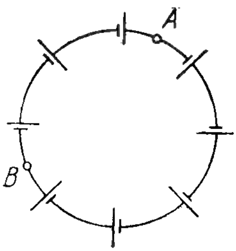

Fig. 3.40.

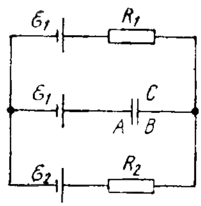

Fig. 3.41.

their internal resistances, i.e. $\mathscr{E} = \alpha R$, where α is an assigned constant. The lead wire resistance is negligible. Find:

(a) the current in the circuit;

(b) the potential difference between points A and B dividing the circuit in n and $N - n$ links.

3.177. In the circuit shown in Fig. 3.41 the sources have emf's $\mathscr{E}_1 = 1.0$ V and $\mathscr{E}_2 = 2.5$ V and the resistances have the values $R_1 = 10\ \Omega$ and $R_2 = 20\ \Omega$. The internal resistances of the sources are negligible. Find a potential difference $\varphi_A - \varphi_B$ between the plates A and B of the capacitor C.

3.178. In the circuit shown in Fig. 3.42 the emf of the source is equal to $\mathscr{E} = 5.0$ V and the resistances are equal to $R_1 = 4.0\ \Omega$ and $R_2 = 6.0\ \Omega$. The internal resistance of the source equals $R = 0.10\ \Omega$. Find the currents flowing through the resistances R_1 and R_2.

3.179. Fig. 3.43 illustrates a potentiometric circuit by means of which we can vary a voltage V applied to a certain device possessing a resistance R. The potentiometer has a length l and a resistance

R_0, and voltage V_0 is applied to its terminals. Find the voltage V fed to the device as a function of distance x. Analyse separately the case $R \gg R_0$.

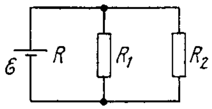

Fig. 3.42.

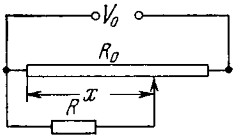

Fig. 3.43.

3.180. Find the emf and the internal resistance of a source which is equivalent to two batteries connected in parallel whose emf's are equal to $\mathscr{E}_1$ and $\mathscr{E}_2$ and internal resistances to R_1 and R_2.

3.181. Find the magnitude and direction of the current flowing through the resistance R in the circuit shown in Fig. 3.44 if the

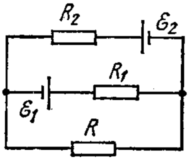

Fig. 3.44.

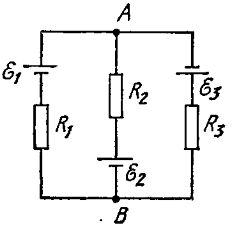

Fig. 3.45.

emf's of the sources are equal to $\mathscr{E}_1 = 1.5$ V and $\mathscr{E}_2 = 3.7$ V and the resistances are equal to $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, $R = 5.0\ \Omega$. The internal resistances of the sources are negligible.

3.182. In the circuit shown in Fig. 3.45 the sources have emf's $\mathscr{E}_1 = 1.5$ V, $\mathscr{E}_2 = 2.0$ V, $\mathscr{E}_3 = 2.5$ V, and the resistances are equal to $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, $R_3 = 30\ \Omega$. The internal resistances of the sources are negligible. Find:

(a) the current flowing through the resistance R_1;

(b) a potential difference $\varphi_A - \varphi_B$ between the points A and B.

3.183. Find the current flowing through the resistance R in the circuit shown in Fig. 3.46. The internal resistances of the batteries are negligible.

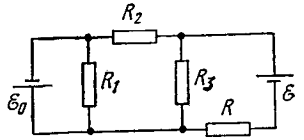

Fig. 3.46.

3.184. Find a potential difference $\varphi_A - \varphi_B$ between the plates of a capacitor C in the circuit shown in Fig. 3.47 if the sources have emf's $\mathscr{E}_1 = 4.0$ V and $\mathscr{E}_2 = 1.0$ V and the resistances are equal to $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, and $R_3 = 30\ \Omega$. The internal resistances of the sources are negligible.

3.185. Find the current flowing through the resistance R_1 of the circuit shown in Fig. 3.48 if the resistances are equal to $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, and $R_3 = 30\ \Omega$, and the potentials of points *1*, *2*, and *3* are equal to $\varphi_1 = 10$ V, $\varphi_2 = 6$ V, and $\varphi_3 = 5$ V

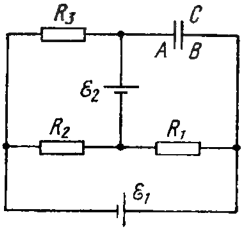

Fig. 3.47.

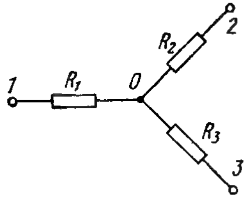

Fig. 3.48.

3.186. A constant voltage $V = 25$ V is maintained between points A and B of the circuit (Fig. 3.49). Find the magnitude and direction of the current flowing through the segment CD if the resistances are equal to $R_1 = 1.0\ \Omega$, $R_2 = 2.0\ \Omega$, $R_3 = 3.0\ \Omega$, and $R_4 = 4.0\ \Omega$.

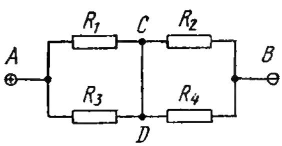

Fig. 3.49.

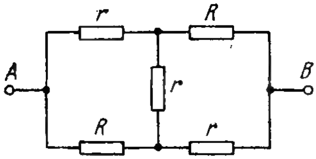

Fig. 3.50.

3.187. Find the resistance between points A and B of the circuit shown in Fig. 3.50.

3.188. Find how the voltage across the capacitor C varies with time t (Fig. 3.51) after the shorting of the switch Sw at the moment $t = 0$.

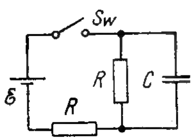

Fig. 3.51.

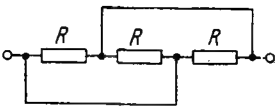

Fig. 3.52.

3.189. What amount of heat will be generated in a coil of resistance R due to a charge q passing through it if the current in the coil

(a) decreases down to zero uniformly during a time interval Δt;

(b) decreases down to zero halving its value every Δt seconds?

3.190. A dc source with internal resistance R_0 is loaded with three identical resistances R interconnected as shown in Fig. 3.52.

At what value of R will the thermal power generated in this circuit be the highest?

3.191. Make sure that the current distribution over two resistances R_1 and R_2 connected in parallel corresponds to the minimum thermal power generated in this circuit.

3.192. A storage battery with emf $\mathscr{E} = 2.6$ V loaded with an external resistance produces a current $I = 1.0$ A. In this case the potential difference between the terminals of the storage battery equals $V = 2.0$ V. Find the thermal power generated in the battery and the power developed in it by electric forces.

3.193. A voltage V is applied to a dc electric motor. The armature winding resistance is equal to R. At what value of current flowing through the winding will the useful power of the motor be the highest? What is it equal to? What is the motor efficiency in this case?

3.194. How much (in per cent) has a filament diameter decreased due to evaporation if the maintenance of the previous temperature required an increase of the voltage by $\eta = 1.0\%$? The amount of heat transferred from the filament into surrounding space is assumed to be proportional to the filament surface area.

3.195. A conductor has a temperature-independent resistance R and a total heat capacity C. At the moment $t = 0$ it is connected to a dc voltage V. Find the time dependence of a conductor's temperature T assuming the thermal power dissipated into surrounding space to vary as $q = k(T - T_0)$, where k is a constant, T_0 is the environmental temperature (equal to the conductor's temperature at the initial moment).

3.196. A circuit shown in Fig. 3.53 has resistances $R_1 = 20\,\Omega$ and $R_2 = 30\,\Omega$. At what value of the resistance R_x will the thermal power generated in it be practically independent of small variations of that resistance? The voltage between the points A and B is supposed to be constant in this case.

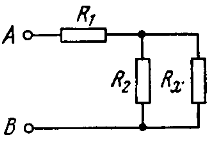

Fig. 3.53.

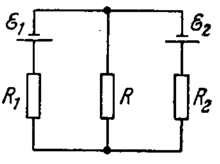

Fig. 3.54.

3.197. In a circuit shown in Fig. 3.54 resistances R_1 and R_2 are known, as well as emf's $\mathscr{E}_1$ and $\mathscr{E}_2$. The internal resistances of the sources are negligible. At what value of the resistance R will the thermal power generated in it be the highest? What is it equal to?

3.198. A series-parallel combination battery consisting of a large number $N = 300$ of identical cells, each with an internal resistance

$r = 0.3\ \Omega$, is loaded with an external resistance $R = 10\ \Omega$. Find the number n of parallel groups consisting of an equal number of cells connected in series, at which the external resistance generates the highest thermal power.

3.199. A capacitor of capacitance $C = 5.00\ \mu F$ is connected to a source of constant emf $\mathscr{E} = 200$ V (Fig. 3.55). Then the switch *Sw* was thrown over from contact *1* to contact *2*. Find the amount of heat generated in a resistance $R_1 = 500\ \Omega$ if $R_2 = 330\ \Omega$.

3.200. Between the plates of a parallel-plate capacitor there is a metallic plate whose thickness takes up $\eta = 0.60$ of the capacitor

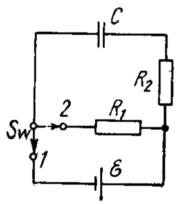

Fig. 3.55.

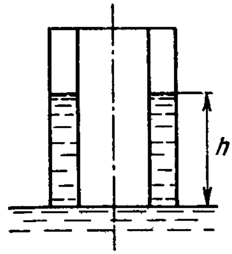

Fig. 3.56.

gap. When that plate is absent the capacitor has a capacity $C = 20$ nF. The capacitor is connected to a dc voltage source $V = 100$ V. The metallic plate is slowly extracted from the gap. Find:

(a) the energy increment of the capacitor;

(b) the mechanical work performed in the process of plate extraction.

3.201. A glass plate totally fills up the gap between the electrodes of a parallel-plate capacitor whose capacitance in the absence of that glass plate is equal to $C = 20$ nF. The capacitor is connected to a dc voltage source $V = 100$ V. The plate is slowly, and without friction, extracted from the gap. Find the capacitor energy increment and the mechanical work performed in the process of plate extraction.

3.202. A cylindrical capacitor connected to a dc voltage source V touches the surface of water with its end (Fig. 3.56). The separation d between the capacitor electrodes is substantially less than their mean radius. Find a height h to which the water level in the gap will rise. The capillary effects are to be neglected.

3.203. The radii of spherical capacitor electrodes are equal to a and b, with $a < b$. The interelectrode space is filled with homogeneous substance of permittivity ε and resistivity ρ. Initially the capacitor is not charged. At the moment $t = 0$ the internal electrode gets a charge q_0. Find:

(a) the time variation of the charge on the internal electrode;

(b) the amount of heat generated during the spreading of the charge.

3.204. The electrodes of a capacitor of capacitance $C = 2.00$ μF carry opposite charges $q_0 = 1.00$ mC. Then the electrodes are interconnected through a resistance $R = 5.0$ MΩ. Find:

(a) the charge flowing through that resistance during a time interval $\tau = 2.00$ s;

(b) the amount of heat generated in the resistance during the same interval.

3.205. In a circuit shown in Fig. 3.57 the capacitance of each capacitor is equal to C and the resistance, to R. One of the capacitors was connected to a voltage V_0 and then at the moment $t = 0$ was shorted by means of the switch Sw. Find:

(a) a current I in the circuit as a function of time t;

(b) the amount of generated heat provided a dependence $I(t)$ is known.

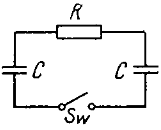

Fig. 3.57.

3.206. A coil of radius $r = 25$ cm wound of a thin copper wire of length $l = 500$ m rotates with an angular velocity $\omega = 300$ rad/s about its axis. The coil is connected to a ballistic galvanometer by means of sliding contacts. The total resistance of the circuit is equal to $R = 21$ Ω. Find the specific charge of current carriers in copper if a sudden stoppage of the coil makes a charge $q = 10$ nC flow through the galvanometer.

3.207. Find the total momentum of electrons in a straight wire of length $l = 1000$ m carrying a current $I = 70$ A.

3.208. A copper wire carries a current of density $j = 1.0$ A/mm^2. Assuming that one free electron corresponds to each copper atom, evaluate the distance which will be covered by an electron during its displacement $l = 10$ mm along the wire.

3.209. A straight copper wire of length $l = 1000$ m and cross-sectional area $S = 1.0$ mm^2 carries a current $I = 4.5$ A. Assuming that one free electron corresponds to each copper atom, find:

(a) the time it takes an electron to displace from one end of the wire to the other;

(b) the sum of electric forces acting on all free electrons in the given wire.

3.210. A homogeneous proton beam accelerated by a potential difference $V = 600$ kV has a round cross-section of radius $r = 5.0$ mm. Find the electric field strength on the surface of the beam and the potential difference between the surface and the axis of the beam if the beam current is equal to $I = 50$ mA.

3.211. Two large parallel plates are located in vacuum. One of them serves as a cathode, a source of electrons whose initial velocity is negligible. An electron flow directed toward the opposite plate produces a space charge causing the potential in the gap between the plates to vary as $\varphi = ax^{4/3}$, where a is a positive constant, and x is the distance from the cathode. Find:

(a) the volume density of the space charge as a function of x;

(b) the current density.

3.212. The air between two parallel plates separated by a distance $d = 20$ mm is ionized by X-ray radiation. Each plate has an area $S = 500$ cm^2. Find the concentration of positive ions if at a voltage $V = 100$ V a current $I = 3.0$ μA flows between the plates, which is well below the saturation current. The air ion mobilities are $u_0^+ = 1.37$ cm^2/(V·s) and $u_0^- = 1.91$ cm^2/(V·s).

3.213. A gas is ionized in the immediate vicinity of the surface of plane electrode *1* (Fig. 3.58) separated from electrode *2* by a distance l. An alternating voltage varying with time t as $V = V_0 \sin \omega t$ is applied to the electrodes. On decreasing the frequency ω it was observed that the galvanometer G indicates a current only at $\omega < \omega_0$, where ω_0 is a certain cut-off frequency. Find the mobility of ions reaching electrode *2* under these conditions.

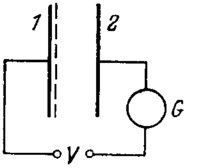

Fig. 3.58.

3.214. The air between two closely located plates is uniformly ionized by ultraviolet radiation. The air volume between the plates is equal to $V = 500$ cm^3, the observed saturation current is equal to $I_{sat} = 0.48$ μA. Find:

(a) the number of ion pairs produced in a unit volume per unit time;

(b) the equilibrium concentration of ion pairs if the recombination coefficient for air ions is equal to $r = 1.67 \cdot 10^{-6}$ cm^3/s.

3.215. Having been operated long enough, the ionizer producing $\dot{n}_i = 3.5 \cdot 10^9$ cm^{-3}·s^{-1} of ion pairs per unit volume of air per unit time was switched off. Assuming that the only process tending to reduce the number of ions in air is their recombination with coefficient $r = 1.67 \cdot 10^{-6}$ cm^3/s, find how soon after the ionizer's switching off the ion concentration decreases $\eta = 2.0$ times.

3.216. A parallel-plate air capacitor whose plates are separated by a distance $d = 5.0$ mm is first charged to a potential difference $V = 90$ V and then disconnected from a dc voltage source. Find the time interval during which the voltage across the capacitor decreases by $\eta = 1.0\%$, taking into account that the average number of ion pairs formed in air under standard conditions per unit volume per unit time is equal to $\dot{n}_i = 5.0$ cm^{-3}·s^{-1} and that the given voltage corresponds to the saturation current.

3.217. The gap between two plane plates of a capacitor equal to d is filled with a gas. One of the plates emits ν_0 electrons per second which, moving in an electric field, ionize gas molecules; this way each electron produces α new electrons (and ions) along a unit length of its path. Find the electronic current at the opposite plate, neglecting the ionization of gas molecules by formed ions.

3.218. The gas between the capacitor plates separated by a distance d is uniformly ionized by ultraviolet radiation so that n_i electrons per unit volume per second are formed. These electrons moving in the electric field of the capacitor ionize gas molecules, each electron producing α new electrons (and ions) per unit length of its path. Neglecting the ionization by ions, find the electronic current density at the plate possessing a higher potential.

3.5. CONSTANT MAGNETIC FIELD. MAGNETICS

- Magnetic field of a point charge q moving with non-relativistic velocity $\mathbf{v}$:

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{q\,[\mathbf{vr}]}{r^3}. \tag{3.5a}$$

- Biot-Savart law:

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{[\mathbf{jr}]}{r^3}\,dV, \quad d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I\,[d\mathbf{l}, \mathbf{r}]}{r^3}. \tag{3.5b}$$

- Circulation of a vector $\mathbf{B}$ and Gauss's theorem for it:

$$\oint \mathbf{B}\,d\mathbf{r} = \mu_0 I, \quad \oint \mathbf{B}\,d\mathbf{S} = 0. \tag{3.5c}$$

- Lorentz force:

$$\mathbf{F} = q\mathbf{E} + q\,[\mathbf{vB}]. \tag{3.5d}$$

- Ampere force:

$$d\mathbf{F} = [\mathbf{jB}]\,dV, \quad d\mathbf{F} = I\,[d\mathbf{l}, \mathbf{B}]. \tag{3.5e}$$

- Force and moment of forces acting on a magnetic dipole $\mathbf{p}_m = IS\mathbf{n}$:

$$\mathbf{F} = p_m \frac{\partial \mathbf{B}}{\partial n}, \quad \mathbf{N} = [\mathbf{p}_m \mathbf{B}], \tag{3.5f}$$

where $\partial\mathbf{B}/\partial n$ is the derivative of a vector $\mathbf{B}$ with respect to the dipole direction.

- Circulation of magnetization $\mathbf{J}$:

$$\oint \mathbf{J}\,d\mathbf{r} = I', \tag{3.5g}$$

where I' is the total molecular current.

- Vector $\mathbf{H}$ and its circulation:

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{J}, \quad \oint \mathbf{H}\,d\mathbf{r} = I, \tag{3.5h}$$

where I is the algebraic sum of macroscopic currents.

- Relations at the boundary between two magnetics:

$$B_{1n} = B_{2n}, \quad H_{1\tau} = H_{2\tau}. \tag{3.5i}$$

- For the case of magnetics in which $\mathbf{J} = \chi\mathbf{H}$:

$$\mathbf{B} = \mu\mu_0\mathbf{H}, \quad \mu = 1 + \chi. \tag{3.5j}$$

3.219. A current $I = 1.00$ A circulates in a round thin-wire loop of radius $R = 100$ mm. Find the magnetic induction

(a) at the centre of the loop;

(b) at the point lying on the axis of the loop at a distance $x = 100$ mm from its centre.

3.220. A current I flows along a thin wire shaped as a regular polygon with n sides which can be inscribed into a circle of radius R. Find the magnetic induction at the centre of the polygon. Analyse the obtained expression at $n \to \infty$.

3.221. Find the magnetic induction at the centre of a rectangular wire frame whose diagonal is equal to $d = 16$ cm and the angle between the diagonals is equal to $\varphi = 30°$; the current flowing in the frame equals $I = 5.0$ A.

3.222. A current $I = 5.0$ A flows along a thin wire shaped as shown in Fig. 3.59. The radius of a curved part of the wire is equal to $R = 120$ mm, the angle $2\varphi = 90°$. Find the magnetic induction of the field at the point O.

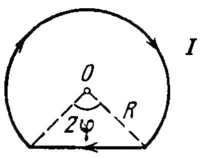

Fig. 3.59.

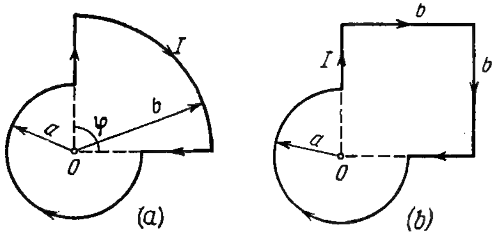

Fig. 3.60.

3.223. Find the magnetic induction of the field at the point O of a loop with current I, whose shape is illustrated

(a) in Fig. 3.60a, the radii a and b, as well as the angle φ are known;

(b) in Fig. 3.60b, the radius a and the side b are known.

3.224. A current I flows along a lengthy thin-walled tube of radius R with longitudinal slit of width h. Find the induction of the magnetic field inside the tube under the condition $h \ll R$.

3.225. A current I flows in a long straight wire with cross-section having the form of a thin half-ring of radius R (Fig. 3.61). Find the induction of the magnetic field at the point O.

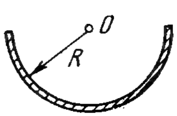

Fig. 3.61.

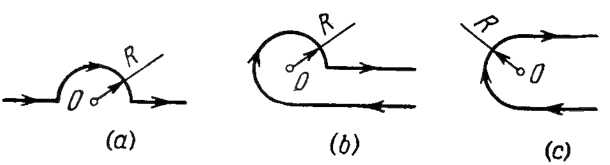

Fig. 3.62.

3.226. Find the magnetic induction of the field at the point O if a current-carrying wire has the shape shown in Fig. 3.62 a, b, c. The radius of the curved part of the wire is R, the linear parts are assumed to be very long.

3.227. A very long wire carrying a current $I = 5.0$ A is bent at right angles. Find the magnetic induction at a point lying on a perpendicular to the wire, drawn through the point of bending, at a distance $l = 35$ cm from it.

3.228. Find the magnetic induction at the point O if the wire carrying a current $I = 8.0$ A has the shape shown in Fig. 3.63 *a, b, c*.

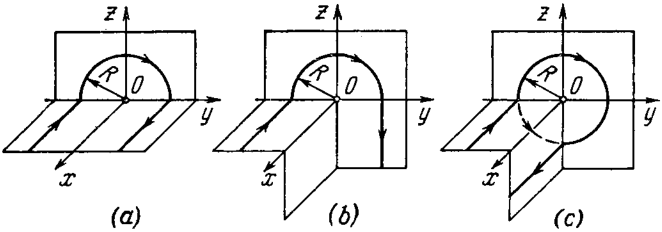

Fig. 3.63.

The radius of the curved part of the wire is $R = 100$ mm, the linear parts of the wire are very long.

3.229. Find the magnitude and direction of the magnetic induction vector **B**

(a) of an infinite plane carrying a current of linear density **i**; the vector **i** is the same at all points of the plane;

(b) of two parallel infinite planes carrying currents of linear densities **i** and —**i**; the vectors **i** and —**i** are constant at all points of the corresponding planes.

3.230. A uniform current of density j flows inside an infinite plate of thickness $2d$ parallel to its surface. Find the magnetic induction induced by this current as a function of the distance x from the median plane of the plate. The magnetic permeability is assumed to be equal to unity both inside and outside the plate.

3.231. A direct current I flows along a lengthy straight wire. From the point O (Fig. 3.64) the current spreads radially all over an infinite conducting plane perpendicular to the wire. Find the magnetic induction at all points of space.

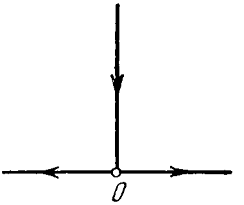

Fig. 3.64.

3.232. A current I flows along a round loop. Find the integral $\int \mathbf{B}\, d\mathbf{r}$ along the axis of the loop within the range from $-\infty$ to $+\infty$. Explain the result obtained.

3.233. A direct current of density **j** flows along a round uniform straight wire with cross-section radius R. Find the magnetic induction vector of this current at the point whose position relative to the axis of the wire is defined by a radius vector **r**. The magnetic permeability is assumed to be equal to unity throughout all the space.

3.234. Inside a long straight uniform wire of round cross-section there is a long round cylindrical cavity whose axis is parallel to the axis of the wire and displaced from the latter by a distance **l**. A direct current of density **j** flows along the wire. Find the magnetic induction inside the cavity. Consider, in particular, the case $\mathbf{l} = 0$.

3.235. Find the current density as a function of distance r from the axis of a radially symmetrical parallel stream of electrons if the magnetic induction inside the stream varies as $B = br^{\alpha}$, where b and α are positive constants.

3.236. A single-layer coil (solenoid) has length l and cross-section radius R. A number of turns per unit length is equal to n. Find the magnetic induction at the centre of the coil when a current I flows through it.

3.237. A very long straight solenoid has a cross-section radius R and n turns per unit length. A direct current I flows through the solenoid. Suppose that x is the distance from the end of the solenoid, measured along its axis. Find:

(a) the magnetic induction B on the axis as a function of x; draw an approximate plot of B vs the ratio x/R;

(b) the distance x_0 to the point on the axis at which the value of B differs by $\eta = 1\%$ from that in the middle section of the solenoid.

3.238. A thin conducting strip of width $h = 2.0$ cm is tightly wound in the shape of a very long coil with cross-section radius $R = 2.5$ cm to make a single-layer straight solenoid. A direct current $I = 5.0$ A flows through the strip. Find the magnetic induction inside and outside the solenoid as a function of the distance r from its axis.

3.239. $N = 2.5 \cdot 10^3$ wire turns are uniformly wound on a wooden toroidal core of very small cross-section. A current I flows through the wire. Find the ratio η of the magnetic induction inside the core to that at the centre of the toroid.

3.240. A direct current $I = 10$ A flows in a long straight round conductor. Find the magnetic flux through a half of wire's cross-section per one metre of its length.

3.241. A very long straight solenoid carries a current I. The cross-sectional area of the solenoid is equal to S, the number of turns per unit length is equal to n. Find the flux of the vector **B** through the end plane of the solenoid.

3.242. Fig. 3.65 shows a toroidal solenoid whose cross-section is rectangular. Find the magnetic flux through this cross-section if the current through the winding equals $I = 1.7$ A, the total number of turns is $N = 1000$, the ratio of the outside diameter to the inside one is $\eta = 1.6$, and the height is equal to $h = 5.0$ cm.

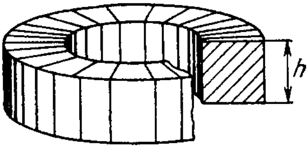

Fig. 3.65.

3.243. Find the magnetic moment of a thin round loop with current if the radius of the loop is equal to $R = 100$ mm and the magnetic induction at its centre is equal to $B = 6.0$ μT.

3.244. Calculate the magnetic moment of a thin wire with a current $I = 0.8$ A, wound tightly on half a tore (Fig. 3.66). The diameter of the cross-section of the tore is equal to $d = 5.0$ cm, the number of turns is $N = 500$.

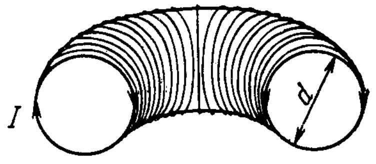

Fig. 3.66.

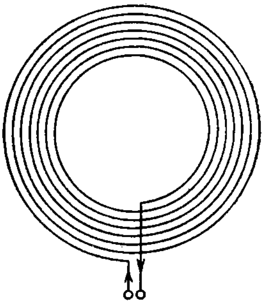
Fig. 3.67.

3.245. A thin insulated wire forms a plane spiral of $N = 100$ tight turns carrying a current $I = 8$ mA. The radii of inside and outside turns (Fig. 3.67) are equal to $a = 50$ mm and $b = 100$ mm. Find:

(a) the magnetic induction at the centre of the spiral;

(b) the magnetic moment of the spiral with a given current.

3.246. A non-conducting thin disc of radius R charged uniformly over one side with surface density σ rotates about its axis with an angular velocity ω. Find:

(a) the magnetic induction at the centre of the disc;

(b) the magnetic moment of the disc.

3.247. A non-conducting sphere of radius $R = 50$ mm charged uniformly with surface density $\sigma = 10.0$ μC/m² rotates with an angular velocity $\omega = 70$ rad/s about the axis passing through its centre. Find the magnetic induction at the centre of the sphere.

3.248. A charge q is uniformly distributed over the volume of a uniform ball of mass m and radius R which rotates with an angular velocity ω about the axis passing through its centre. Find the respective magnetic moment and its ratio to the mechanical moment.

3.249. A long dielectric cylinder of radius R is statically polarized so that at all its points the polarization is equal to $\mathbf{P} = \alpha\mathbf{r}$, where α is a positive constant, and $\mathbf{r}$ is the distance from the axis. The cylinder is set into rotation about its axis with an angular velocity $\boldsymbol{\omega}$. Find the magnetic induction $\mathbf{B}$ at the centre of the cylinder.

3.250. Two protons move parallel to each other with an equal velocity $v = 300$ km/s. Find the ratio of forces of magnetic and electrical interaction of the protons.

3.251. Find the magnitude and direction of a force vector acting on a unit length of a thin wire, carrying a current $I = 8.0$ A, at a point O, if the wire is bent as shown in

(a) Fig. 3.68*a*, with curvature radius $R = 10$ cm;

(b) Fig. 3.68*b*, the distance between the long parallel segments of the wire being equal to $l = 20$ cm.

3.252. A coil carrying a current $I = 10$ mA is placed in a uniform magnetic field so that its axis coincides with the field direction. The single-layer winding of the coil is made of copper wire with

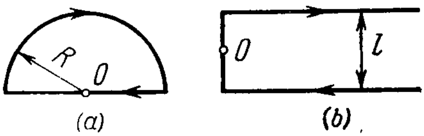

Fig. 3.68.

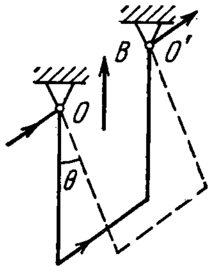

Fig. 3.69.

diameter $d = 0.10$ mm, radius of turns is equal to $R = 30$ mm. At what value of the induction of the external magnetic field can the coil winding be ruptured?

3.253. A copper wire with cross-sectional area $S = 2.5$ mm² bent to make three sides of a square can turn about a horizontal axis OO' (Fig. 3.69). The wire is located in uniform vertical magnetic field. Find the magnetic induction if on passing a current $I = 16$ A through the wire the latter deflects by an angle $\theta = 20°$.

3.254. A small coil C with $N = 200$ turns is mounted on one end of a balance beam and introduced between the poles of an electromagnet as shown in Fig. 3.70. The cross-sectional area of the coil

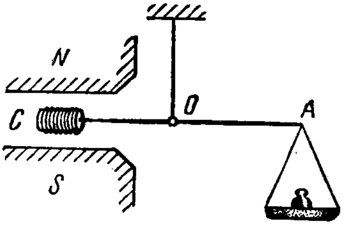

Fig. 3.70.

is $S = 1.0$ cm², the length of the arm OA of the balance beam is $l = 30$ cm. When there is no current in the coil the balance is in equilibrium. On passing a current $I = 22$ mA through the coil the equilibrium is restored by putting the additional counterweight of mass $\Delta m = 60$ mg on the balance pan. Find the magnetic induction at the spot where the coil is located.

3.255. A square frame carrying a current $I = 0.90$ A is located in the same plane as a long straight wire carrying a current $I_0 = 5.0$ A. The frame side has a length $a = 8.0$ cm. The axis of the frame passing through the midpoints of opposite sides is parallel to the wire and is separated from it by the distance which is $\eta = 1.5$ times greater than the side of the frame. Find:

(a) Ampere force acting on the frame;

(b) the mechanical work to be performed in order to turn the frame through 180° about its axis, with the currents maintained constant.

3.256. Two long parallel wires of negligible resistance are connected at one end to a resistance R and at the other end to a dc voltage source. The distance between the axes of the wires is $\eta = 20$ times greater than the cross-sectional radius of each wire. At what value of resistance R does the resultant force of interaction between the wires turn into zero?

3.257. A direct current I flows in a long straight conductor whose cross-section has the form of a thin half-ring of radius R. The same current flows in the opposite direction along a thin conductor located on the "axis" of the first conductor (point O in Fig. 3.61). Find the magnetic interaction force between the given conductors reduced to a unit of their length.

3.258. Two long thin parallel conductors of the shape shown in Fig. 3.71 carry direct currents I_1 and I_2. The separation between the conductors is a, the width of the right-hand conductor is equal to b. With both conductors lying in one plane, find the magnetic interaction force between them reduced to a unit of their length.

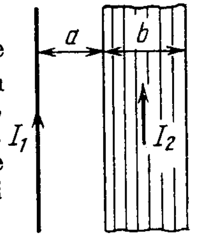

Fig. 3.71.

3.259. A system consists of two parallel planes carrying currents producing a uniform magnetic field of induction B between the planes. Outside this space there is no magnetic field. Find the magnetic force acting per unit area of each plane.

3.260. A conducting current-carrying plane is placed in an external uniform magnetic field. As a result, the magnetic induction becomes

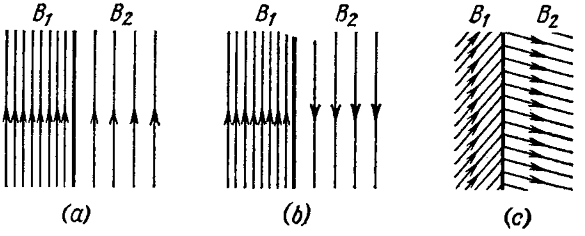

Fig. 3.72.

equal to B_1 on one side of the plane and to B_2, on the other. Find the magnetic force acting per unit area of the plane in the cases illustrated in Fig. 3.72. Determine the direction of the current in the plane in each case.

3.261. In an electromagnetic pump designed for transferring molten metals a pipe section with metal is located in a uniform magnetic field of induction B (Fig. 3.73). A current I is made to flow across this pipe section in the direction perpendicular both to the vector **B** and to the axis of the pipe. Find the gauge pressure produced by the pump if $B = 0.10$ T, $I = 100$ A, and $a = 2.0$ cm.

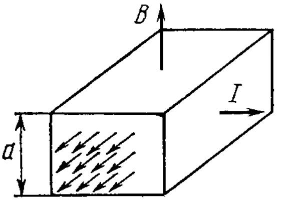

Fig. 3.73.

3.262. A current I flows in a long thin-walled cylinder of radius R. What pressure do the walls of the cylinder experience?

3.263. What pressure does the lateral surface of a long straight solenoid with n turns per unit length experience when a current I flows through it?

3.264. A current I flows in a long single-layer solenoid with cross-sectional radius R. The number of turns per unit length of the solenoid equals n. Find the limiting current at which the winding may rupture if the tensile strength of the wire is equal to F_{lim}.

3.265. A parallel-plate capacitor with area of each plate equal to S and the separation between them to d is put into a stream of conducting liquid with resistivity ρ. The liquid moves parallel to the plates with a constant velocity v. The whole system is located in a uniform magnetic field of induction B, vector **B** being parallel to the plates and perpendicular to the stream direction. The capacitor plates are interconnected by means of an external resistance R. What amount of power is generated in that resistance? At what value of R is the generated power the highest? What is this highest power equal to?

3.266. A straight round copper conductor of radius $R = 5.0$ mm carries a current $I = 50$ A. Find the potential difference between the axis of the conductor and its surface. The concentration of the conduction electrons in copper is equal to $n = 0.9 \cdot 10^{23}$ cm^{-3}.

3.267. In Hall effect measurements in a sodium conductor the strength of a transverse field was found to be equal to $E = 5.0$ μV/cm with a current density $j = 200$ A/cm^2 and magnetic induction $B = 1.00$ T. Find the concentration of the conduction electrons and its ratio to the total number of atoms in the given conductor.

3.268. Find the mobility of the conduction electrons in a copper conductor if in Hall effect measurements performed in the magnetic field of induction $B = 100$ mT the transverse electric field strength of the given conductor turned out to be $\eta = 3.1 \cdot 10^3$ times less than that of the longitudinal electric field.

3.269. A small current-carrying loop is located at a distance r from a long straight conductor with current I. The magnetic moment of the loop is equal to $\mathbf{p}_m$. Find the magnitude and direction of the force vector applied to the loop if the vector $\mathbf{p}_m$

(a) is parallel to the straight conductor;

(b) is oriented along the radius vector $\mathbf{r}$;

(c) coincides in direction with the magnetic field produced by the current I at the point where the loop is located.

3.270. A small current-carrying coil having a magnetic moment $\mathbf{p}_m$ is located at the axis of a round loop of radius R with current I flowing through it. Find the magnitude of the vector force applied to the coil if its distance from the centre of the loop is equal to x and the vector $\mathbf{p}_m$ coincides in direction with the axis of the loop.

3.271. Find the interaction force of two coils with magnetic moments $p_{1m} = 4.0$ mA·m² and $p_{2m} = 6.0$ mA·m² and collinear axes if the separation between the coils is equal to $l = 20$ cm which exceeds considerably their linear dimensions.

3.272. A permanent magnet has the shape of a sufficiently thin disc magnetized along its axis. The radius of the disc is $R = 1.0$ cm. Evaluate the magnitude of a molecular current I' flowing along the rim of the disc if the magnetic induction at the point on the axis of the disc, lying at a distance $x = 10$ cm from its centre, is equal to $B = 30$ μT.

3.273. The magnetic induction in vacuum at a plane surface of a uniform isotropic magnetic is equal to B, the vector $\mathbf{B}$ forming an angle α with the normal of the surface. The permeability of the magnetic is equal to μ. Find the magnitude of the magnetic induction B' in the magnetic in the vicinity of its surface.

3.274. The magnetic induction in vacuum at a plane surface of a magnetic is equal to B and the vector $\mathbf{B}$ forms an angle θ with the

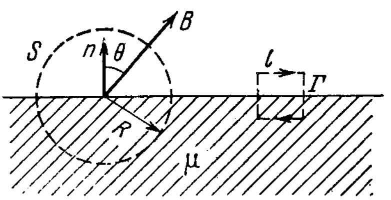

Fig. 3.74.

normal $\mathbf{n}$ of the surface (Fig. 3.74). The permeability of the magnetic is equal to μ. Find:

(a) the flux of the vector $\mathbf{H}$ through the spherical surface S of radius R, whose centre lies on the surface of the magnetic;

(b) the circulation of the vector $\mathbf{B}$ around the square path Γ with side l located as shown in the figure.

3.275. A direct current I flows in a long round uniform cylindrical wire made of paramagnetic with susceptibility χ. Find:

(a) the surface molecular current I'_s;

(b) the volume molecular current I'_v.
How are these currents directed toward each other?

3.276. Half of an infinitely long straight current-carrying solenoid is filled with magnetic substance as shown in Fig. 3.75. Draw the

Fig. 3.75.

approximate plots of magnetic induction B, strength H, and magnetization J on the axis as functions of x.

3.277. An infinitely long wire with a current I flowing in it is located in the boundary plane between two non-conducting media with permeabilities μ_1 and μ_2. Find the modulus of the magnetic induction vector throughout the space as a function of the distance r from the wire. It should be borne in mind that the lines of the vector **B** are circles whose centres lie on the axis of the wire.

3.278. A round current-carrying loop lies in the plane boundary between magnetic and vacuum. The permeability of the magnetic is equal to μ. Find the magnetic induction **B** at an arbitrary point on the axis of the loop if in the absence of the magnetic the magnetic induction at the same point becomes equal to $\mathbf{B}_0$. Generalize the obtained result to all points of the field.

3.279. When a ball made of uniform magnetic is introduced into an external uniform magnetic field with induction $\mathbf{B}_0$, it gets uniformly magnetized. Find the magnetic induction **B** inside the ball with permeability μ; recall that the magnetic field inside a uniformly magnetized ball is uniform and its strength is equal to $\mathbf{H}' = -\mathbf{J}/3$, where **J** is the magnetization.

3.280. $N = 300$ turns of thin wire are uniformly wound on a permanent magnet shaped as a cylinder whose length is equal to $l = 15$ cm. When a current $I = 3.0$ A was passed through the wiring the field outside the magnet disappeared. Find the coercive force H_c of the material from which the magnet was manufactured.

3.281. A permanent magnet is shaped as a ring with a narrow gap between the poles. The mean diameter of the ring equals $d = 20$ cm. The width of the gap is equal to $b = 2.0$ mm and the magnetic induction in the gap is equal to $B = 40$ mT. Assuming that the scattering of the magnetic flux at the gap edges is negligible, find the modulus of the magnetic field strength vector inside the magnet.

3.282. An iron core shaped as a tore with mean radius $R = 250$ mm supports a winding with the total number of turns $N = 1000$. The core has a cross-cut of width $b = 1.00$ mm. With a current $I = 0.85$ A flowing through the winding, the magnetic induction in the gap is equal to $B = 0.75$ T. Assuming the scattering of the magnetic flux at the gap edges to be negligible, find the permeability of iron under these conditions.

3.283. Fig. 3.76 illustrates a basic magnetization curve of iron (commercial purity grade). Using this plot, draw the permeability

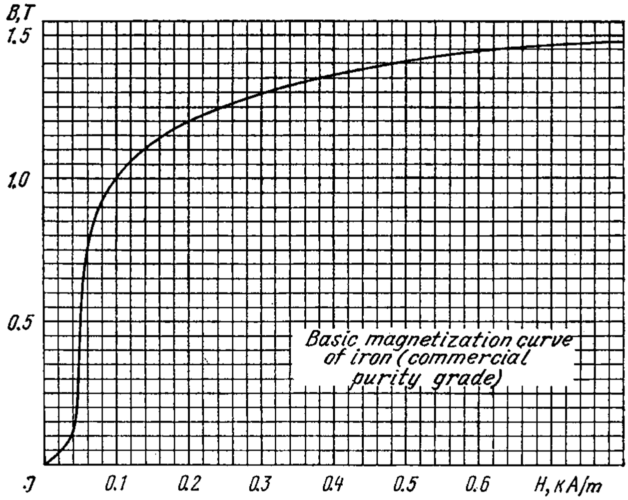

Fig. 3.76.

μ as a function of the magnetic field strength H. At what value of H is the permeability the greatest? What is μ_{max} equal to?

3.284. A thin iron ring with mean diameter $d = 50$ cm supports a winding consisting of $N = 800$ turns carrying current $I = 3.0$ A. The ring has a cross-cut of width $b = 2.0$ mm. Neglecting the scattering of the magnetic flux at the gap edges, and using the plot shown in Fig. 3.76, find the permeability of iron under these conditions.

3.285. A long thin cylindrical rod made of paramagnetic with magnetic susceptibility χ and having a cross-sectional area S is located along the axis of a current-carrying coil. One end of the rod is located at the coil centre where the magnetic induction is equal to B whereas the other end is located in the region where the magnetic field is practically absent. What is the force that the coil exerts on the rod?

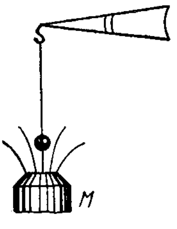

Fig. 3.77.

3.286. In the arrangement shown in Fig. 3.77 it is possible to measure (by means of a balance) the force with which a paramagnetic ball of volume $V = 41$ mm³ is attrabted to a pole of the electromagnet M. The magnetic induction at the axis of the poleshoe depends on the height x as $B = B_0 \exp(-ax^2)$, where $B_0 = 1.50$ T, $a = 100$ m^{-2}. Find:

(a) at what height x_m the ball experiences the maximum attraction;

(b) the magnetic susceptibility of the paramagnetic if the maximum attraction force equals $F_{max} = 160$ μN.

3.287. A small ball of volume V made of paramagnetic with susceptibility χ was slowly displaced along the axis of a current-carrying coil from the point where the magnetic induction equals B out to the region where the magnetic field is practically absent. What amount of work was performed during this process?

3.6. ELECTROMAGNETIC INDUCTION. MAXWELL'S EQUATIONS

• Faraday's law of electromagnetic induction:

$$\mathscr{E}_i = -\frac{d\Phi}{dt} \tag{3.6a}$$

• In the case of a solenoid and doughnut coil:

$$\Phi = N\Phi_1, \tag{3.6b}$$

where N is the number of turns, Φ_1 is the magnetic flux through each turn.

• Inductance of a solenoid:

$$L = \mu\mu_0 n^2 V. \tag{3.6c}$$

• Intrinsic energy of a current and interaction energy of two currents:

$$W = \frac{LI^2}{2}, \qquad W_{12} = L_{12} I_1 I_2. \tag{3.6d}$$

• Volume density of magnetic field energy:

$$w = \frac{B^2}{2\mu\mu_0} = \frac{\mathbf{BH}}{2}. \tag{3.6e}$$

• Displacement current density:

$$\mathbf{j}_{dis} = \frac{\partial \mathbf{B}}{\partial t}. \tag{3.6f}$$

• Maxwell's equations in differential form:

$$\begin{aligned} \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, \qquad \nabla \cdot \mathbf{B} = 0, \\ \nabla \times \mathbf{H} &= \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}, \qquad \nabla \cdot \mathbf{D} = \rho, \end{aligned} \tag{3.6g}$$

where $\nabla \times \equiv$ rot (the rotor) and $\nabla \cdot \equiv$ div (the divergence).

• Field transformation formulas for transition from a reference frame K to a reference frame K' moving with the velocity $\mathbf{v}_0$ relative to it.

In the case $v_0 \ll c$

$$\mathbf{E}' = \mathbf{E} + [\mathbf{v}_0\mathbf{B}], \quad \mathbf{B}' = \mathbf{B} - [\mathbf{v}_0\mathbf{E}]/c^2 \tag{3.6h}$$

In the general case

$$\begin{aligned} \mathbf{E}'_{\parallel} &= \mathbf{E}_{\parallel}, \qquad & \mathbf{B}'_{\parallel} &= \mathbf{B}_{\parallel}, \\ \mathbf{E}'_{\perp} &= \frac{\mathbf{E}_{\perp} + [\mathbf{v}_0\mathbf{B}]}{\sqrt{1-(v_0/c)^2}}, \qquad & B'_{\perp} &= \frac{\mathbf{B}_{\perp} - [\mathbf{v}_0\mathbf{E}]/c^2}{\sqrt{1-(v_0/c)^2}}, \end{aligned} \tag{3.6i}$$

where the symbols $\parallel$ and $\perp$ denote the field components, respectively parallel and perpendicular to the vector $\mathbf{v}_0$.

3.288. A wire bent as a parabola $y = ax^2$ is located in a uniform magnetic field of induction B, the vector **B** being perpendicular to the plane x, y. At the moment $t = 0$ a connector starts sliding translationwise from the parabola apex with a constant acceleration w (Fig. 3.78). Find the emf of electromagnetic induction in the loop thus formed as a function of y.

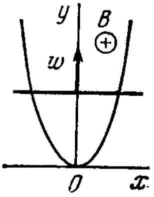

Fig. 3.78.

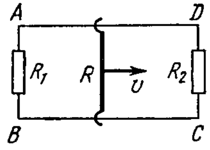

Fig. 3.79.

3.289. A rectangular loop with a sliding connector of length l is located in a uniform magnetic field perpendicular to the loop plane (Fig. 3.79). The magnetic induction is equal to B. The connector has an electric resistance R, the sides AB and CD have resistances R_1 and R_2 respectively. Neglecting the self-inductance of the loop, find the current flowing in the connector during its motion with a constant velocity v.

3.290. A metal disc of radius $a = 25$ cm rotates with a constant angular velocity $\omega = 130$ rad/s about its axis. Find the potential difference between the centre and the rim of the disc if

(a) the external magnetic field is absent;

(b) the external uniform magnetic field of induction $B = 5.0$ mT is directed perpendicular to the disc.

3.291. A thin wire AC shaped as a semi-circle of diameter $d = 20$ cm rotates with a constant angular velocity $\omega = 100$ rad/s in a uniform magnetic field of induction $B = 5.0$ mT, with $\boldsymbol{\omega} \uparrow\uparrow \mathbf{B}$. The rotation axis passes through the end A of the wire and is perpendicular to the diameter AC. Find the value of a line integral $\int \mathbf{E}\, d\mathbf{r}$ along the wire from point A to point C. Generalize the obtained result.

3.292. A wire loop enclosing a semi-circle of radius a is located on the boundary of a uniform magnetic field of induction B (Fig. 3.80). At the moment $t = 0$ the loop is set into rotation with a constant angular acceleration β about an axis O coinciding with a line of vector **B** on the boundary. Find the emf induced in the loop as a function of time t. Draw the approximate plot of this function. The arrow in the figure shows the emf direction taken to be positive.

3.293. A long straight wire carrying a current I and a П-shaped conductor with sliding connector are located in the same plane as shown in Fig. 3.81. The connector of length l and resistance R slides to the right with a constant velocity v. Find the current induced in

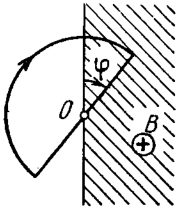

Fig. 3.80.

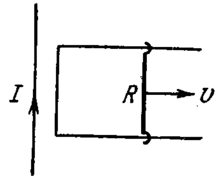

Fig. 3.81.

the loop as a function of separation r between the connector and the straight wire. The resistance of the Π-shaped conductor and the self-inductance of the loop are assumed to be negligible.

3.294. A square frame with side a and a long straight wire carrying a current I are located in the same plane as shown in Fig. 3.82. The frame translates to the right with a constant velocity v. Find the emf induced in the frame as a function of distance x.

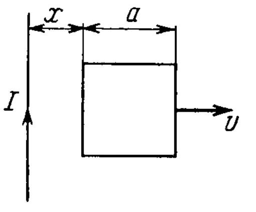

Fig. 3.82.

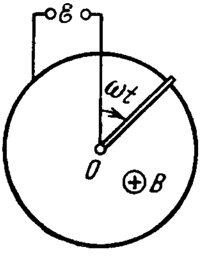

Fig. 3.83.

3.295. A metal rod of mass m can rotate about a horizontal axis O, sliding along a circular conductor of radius a (Fig. 3.83). The arrangement is located in a uniform magnetic field of induction B directed perpendicular to the ring plane. The axis and the ring are connected to an emf source to form a circuit of resistance R. Neglecting the friction, circuit inductance, and ring resistance, find the law according to which the source emf must vary to make the rod rotate with a constant angular velocity ω.

3.296. A copper connector of mass m slides down two smooth copper bars, set at an angle α to the horizontal, due to gravity (Fig. 3.84). At the top the bars are interconnected through a resistance R. The separation between the bars is equal to l. The system is located in a uniform magnetic field of induction B, perpendicular to the plane in which the connector slides. The resistances of the bars, the connector and the sliding contacts, as well as the self-inductance of the loop, are assumed to be negligible. Find the steady-state velocity of the connector.

3.297. The system differs from the one examined in the foregoing problem (Fig. 3.84) by a capacitor of capacitance C replacing the resistance R. Find the acceleration of the connector.

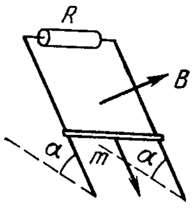

Fig. 3.84.

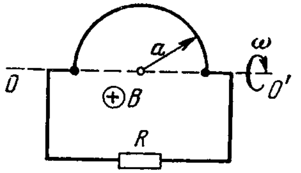

Fig. 3.85.

3.298. A wire shaped as a semi-circle of radius a rotates about an axis OO' with an angular velocity ω in a uniform magnetic field of induction B (Fig. 3.85). The rotation axis is perpendicular to the field direction. The total resistance of the circuit is equal to R. Neglecting the magnetic field of the induced current, find the mean amount of thermal power being generated in the loop during a rotation period.

3.299. A small coil is introduced between the poles of an electromagnet so that its axis coincides with the magnetic field direction. The cross-sectional area of the coil is equal to $S = 3.0$ mm^2, the number of turns is $N = 60$. When the coil turns through 180° about its diameter, a ballistic galvanometer connected to the coil indicates a charge $q = 4.5$ μC flowing through it. Find the magnetic induction magnitude between the poles provided the total resistance of the electric circuit equals $R = 40$ Ω.

3.300. A square wire frame with side a and a straight conductor carrying a constant current I are located in the same plane (Fig. 3.86).

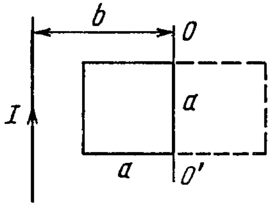

Fig. 3.86.

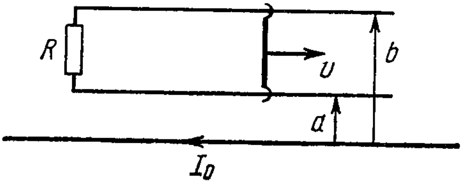

Fig. 3.87.

The inductance and the resistance of the frame are equal to L and R respectively. The frame was turned through 180° about the axis OO' separated from the current-carrying conductor by a distance b. Find the electric charge having flown through the frame.

3.301. A long straight wire carries a current I_0. At distances a and b from it there are two other wires, parallel to the former one, which are interconnected by a resistance R (Fig. 3.87). A connector slides without friction along the wires with a constant velocity v. Assuming the resistances of the wires, the connector, the sliding contacts, and the self-inductance of the freme to be negligible, find:

(a) the magnitude and the direction of the current induced in the connector;

(b) the force required to maintain the connector's velocity constant.

3.302. A conducting rod AB of mass m slides without friction over two long conducting rails separated by a distance l (Fig. 3.88). At the left end the rails are interconnected by a resistance R. The system is located in a uniform magnetic field perpendicular to the plane of the loop. At the moment $t = 0$ the rod AB starts moving to the right with an initial velocity v_0. Neglecting the resistances of the rails and the rod AB, as well as the self-inductance, find:

(a) the distance covered by the rod until it comes to a standstill;

(b) the amount of heat generated in the resistance R during this process.

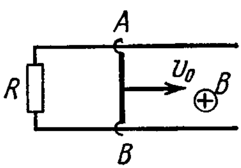

Fig. 3.88.

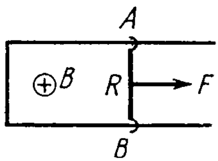

Fig. 3.89.

3.303. A connector AB can slide without friction along a П-shaped conductor located in a horizontal plane (Fig. 3.89). The connector has a length l, mass m, and resistance R. The whole system is located in a uniform magnetic field of induction B directed vertically. At the moment $t = 0$ a constant horizontal force F starts acting on the connector shifting it translationwise to the right. Find how the velocity of the connector varies with time t. The inductance of the loop and the resistance of the П-shaped conductor are assumed to be negligible.

3.304. Fig. 3.90 illustrates plane figures made of thin conductors which are located in a uniform magnetic field directed away from a

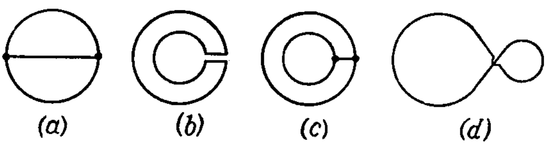

Fig. 3.90.

reader beyond the plane of the drawing. The magnetic induction starts diminishing. Find how the currents induced in these loops are directed.

3.305. A plane loop shown in Fig. 3.91 is shaped as two squares with sides $a = 20$ cm and $b = 10$ cm and is introduced into a uniform magnetic field at right angles to the loop's plane. The magnetic induction varies with time as $B = B_0 \sin \omega t$, where $B_0 = 10$ mT and $\omega = 100$ s^{-1}. Find the amplitude of the current induced in the loop if its resistance per unit length is equal to $\rho = 50$ mΩ/m. The inductance of the loop is to be neglected.

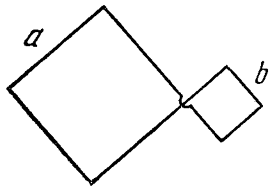

Fig. 3.91.

3.306. A plane spiral with a great number N of turns wound tightly to one another is located in a uniform magnetic field perpendicular to the spiral's plane. The outside radius of the spiral's turns is equal to a. The magnetic induction varies with time as $B = B_0 \sin \omega t$, where B_0 and ω are constants. Find the amplitude of emf induced in the spiral.

3.307. A Π-shaped conductor is located in a uniform magnetic field perpendicular to the plane of the conductor and varying with time at the rate $\dot{B} = 0.10$ T/s. A conducting connector starts moving with an acceleration $w = 10$ cm/s^2 along the parallel bars of the conductor. The length of the connector is equal to $l = 20$ cm. Find the emf induced in the loop $t = 2.0$ s after the beginning of the motion, if at the moment $t = 0$ the loop area and the magnetic induction are equal to zero. The inductance of the loop is to be neglected.

3.308. In a long straight solenoid with cross-sectional radius a and number of turns per unit length n a current varies with a constant velocity $\dot{I}$ A/s. Find the magnitude of the eddy current field strength as a function of the distance r from the solenoid axis. Draw the approximate plot of this function.

3.309. A long straight solenoid of cross-sectional diameter $d = 5$ cm and with $n = 20$ turns per one cm of its length has a round turn of copper wire of cross-sectional area $S = 1.0$ mm^2 tightly put on its winding. Find the current flowing in the turn if the current in the solenoid winding is increased with a constant velocity $\dot{I} = 100$ A/s. The inductance of the turn is to be neglected.

3.310. A long solenoid of cross-sectional radius a has a thin insulated wire ring tightly put on its winding; one half of the ring has the resistance η times that of the other half. The magnetic induction produced by the solenoid varies with time as $B = bt$, where b is a constant. Find the magnitude of the electric field strength in the ring.

3.311. A thin non-conducting ring of mass m carrying a charge q can freely rotate about its axis. At the initial moment the ring was at rest and no magnetic field was present. Then a practically uniform magnetic field was switched on, which was perpendicular to the plane of the ring and increased with time according to a certain law **B** (t). Find the angular velocity ω of the ring as a function of the induction **B** (t).

3.312. A thin wire ring of radius a and resistance r is located inside a long solenoid so that their axes coincide. The length of the solenoid is equal to l, its cross-sectional radius, to b. At a certain moment the solenoid was connected to a source of a constant voltage V. The total resistance of the circuit is equal to R. Assuming the inductance of the ring to be negligible, find the maximum value of the radial force acting per unit length of the ring.

3.313. A magnetic flux through a stationary loop with a resistance R varies during the time interval τ as $\Phi = at(\tau - t)$. Find the amount of heat generated in the loop during that time. The inductance of the loop is to be neglected.

3.314. In the middle of a long solenoid there is a coaxial ring of square cross-section, made of conducting material with resistivity ρ. The thickness of the ring is equal to h, its inside and outside radii are equal to a and b respectively. Find the current induced in the ring if the magnetic induction produced by the solenoid varies with time as $B = \beta t$, where β is a constant. The inductance of the ring is to be neglected.

3.315. How many metres of a thin wire are required to manufacture a solenoid of length $l_0 = 100$ cm and inductance $L = 1.0$ mH if the solenoid's cross-sectional diameter is considerably less than its length?

3.316. Find the inductance of a solenoid of length l whose winding is made of copper wire of mass m. The winding resistance is equal to R. The solenoid diameter is considerably less than its length.

3.317. A coil of inductance $L = 300$ mH and resistance $R = 140$ mΩ is connected to a constant voltage source. How soon will the coil current reach $\eta = 50\%$ of the steady-state value?

3.318. Calculate the time constant τ of a straight solenoid of length $l = 1.0$ m having a single-layer winding of copper wire whose total mass is equal to $m = 1.0$ kg. The cross-sectional diameter of the solenoid is assumed to be considerably less than its length.

Note. The time constant τ is the ratio L/R, where L is inductance and R is active resistance.

3.319. Find the inductance of a unit length of a cable consisting of two thin-walled coaxial metallic cylinders if the radius of the outside cylinder is $\eta = 3.6$ times that of the inside one. The permeability of a medium between the cylinders is assumed to be equal to unity.

3.320. Calculate the inductance of a doughnut solenoid whose inside radius is equal to b and cross-section has the form of a square with side a. The solenoid winding consists of N turns. The space inside the solenoid is filled up with uniform paramagnetic having permeability μ.

3.321. Calculate the inductance of a unit length of a double tape line (Fig. 3.92) if the tapes are separated by a distance h which is considerably less than their width b, namely, $b/h = 50$.

3.322. Find the inductance of a unit length of a double line if the radius of each wire is η times less than the distance between the axes of the wires. The field inside the wires is to be neglected, the permeability is assumed to be equal to unity throughout, and $\eta \gg 1$.

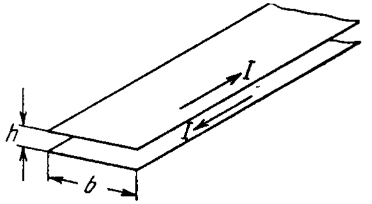

Fig. 3.92.

3.323. A superconducting round ring of radius a and inductance L was located in a uniform magnetic field of induction B. The ring plane was parallel to the vector **B**, and the current in the ring was equal to zero. Then the ring was turned through 90° so that its plane became perpendicular to the field. Find:

(a) the current induced in the ring after the turn;

(b) the work performed during the turn.

3.324. A current $I_0 = 1.9$ A flows in a long closed solenoid. The wire it is wound of is in a superconducting state. Find the current flowing in the solenoid when the length of the solenoid is increased by $\eta = 5\%$.

3.325. A ring of radius $a = 50$ mm made of thin wire of radius $b = 1.0$ mm was located in a uniform magnetic field with induction $B = 0.50$ mT so that the ring plane was perpendicular to the vector **B**. Then the ring was cooled down to a superconducting state, and the magnetic field was switched off. Find the ring current after that. Note that the inductance of a thin ring along which the surface current flows is equal to $L = \mu_0 a \left(\ln \frac{8a}{b} - 2\right)$.

3.326. A closed circuit consists of a source of constant emf $\mathscr{E}$ and a choke coil of inductance L connected in series. The active resistance of the whole circuit is equal to R. At the moment $t = 0$ the choke coil inductance was decreased abruptly η times. Find the current in the circuit as a function of time t.

Instruction. During a stepwise change of inductance the total magnetic flux (flux linkage) remains constant.

3.327. Find the time dependence of the current flowing through the inductance L of the circuit shown in Fig. 3.93 after the switch Sw is shorted at the moment $t = 0$.

3.328. In the circuit shown in Fig. 3.94 an emf $\mathscr{E}$, a resistance R, and coil inductances L_1 and L_2 are known. The internal resistance of the source and the coil resistances are negligible. Find the steady-state currents in the coils after the switch Sw was shorted.

3.329. Calculate the mutual inductance of a long straight wire and a rectangular frame with sides a and b. The frame and the wire lie in the same plane, with the side b being closest to the wire, separated by a distance l from it and oriented parallel to it.

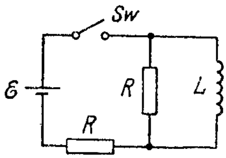

Fig. 3.93.

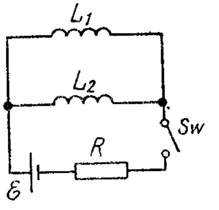

Fig. 3.94.

3.330. Determine the mutual inductance of a doughnut coil and an infinite straight wire passing along its axis. The coil has a rectangular cross-section, its inside radius is equal to a and the outside one, to b. The length of the doughnut's cross-sectional side parallel to the wire is equal to h. The coil has N turns. The system is located in a uniform magnetic with permeability μ.

3.331. Two thin concentric wires shaped as circles with radii a and b lie in the same plane. Allowing for $a \ll b$, find:

(a) their mutual inductance;

(b) the magnetic flux through the surface enclosed by the outside wire, when the inside wire carries a current I.

3.332. A small cylindrical magnet M (Fig. 3.95) is placed in the centre of a thin coil of radius a consisting of N turns. The coil is connected to a ballistic galvanometer. The active resistance of the whole circuit is equal to R. Find the magnetic moment of the magnet if its removal from the coil results in a charge q flowing through the galvanometer.

3.333. Find the approximate formula expressing the mutual inductance of two thin coaxial loops of the same radius a if their centres are separated by a distance l, with $l \gg a$.

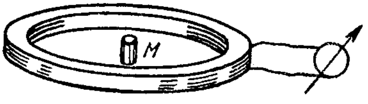

Fig. 3.95.

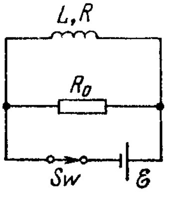

Fig. 3.96.

3.334. There are two stationary loops with mutual inductance L_{12}. The current in one of the loops starts to be varied as $I_1 = \alpha t$, where α is a constant, t is time. Find the time dependence $I_2(t)$ of the current in the other loop whose inductance is L_2 and resistance R.

3.335. A coil of inductance $L = 2.0\ \mu H$ and resistance $R = 1.0\ \Omega$ is connected to a source of constant emf $\mathscr{E} = 3.0$ V (Fig. 3.96). A resistance $R_0 = 2.0\ \Omega$ is connected in parallel with the coil. Find the amount of heat generated in the coil after the switch *Sw* is disconnected. The internal resistance of the source is negligible.

3.336. An iron tore supports $N = 500$ turns. Find the magnetic field energy if a current $I = 2.0$ A produces a magnetic flux across the tore's cross-section equal to $\Phi = 1.0$ mWb.

3.337. An iron core shaped as a doughnut with round cross-section of radius $a = 3.0$ cm carries a winding of $N = 1000$ turns through which a current $I = 1.0$ A flows. The mean radius of the doughnut is $b = 32$ cm. Using the plot in Fig. 3.76, find the magnetic energy stored up in the core. A field strength H is supposed to be the same throughout the cross-section and equal to its magnitude in the centre of the cross-section.

3.338. A thin ring made of a magnetic has a mean diameter $d = 30$ cm and supports a winding of $N = 800$ turns. The cross-sectional area of the ring is equal to $S = 5.0\ \text{cm}^2$. The ring has a cross-cut of width $b = 2.0$ mm. When the winding carries a certain current, the permeability of the magnetic equals $\mu = 1400$. Neglecting the dissipation of magnetic flux at the gap edges, find:

(a) the ratio of magnetic energies in the gap and in the magnetic;

(b) the inductance of the system; do it in two ways: using the flux and using the energy of the field.

3.339. A long cylinder of radius a carrying a uniform surface charge rotates about its axis with an angular velocity ω. Find the magnetic field energy per unit length of the cylinder if the linear charge density equals λ and $\mu = 1$.

3.340. At what magnitude of the electric field strength in vacuum the volume energy density of this field is the same as that of the magnetic field with induction $B = 1.0$ T (also in vacuum).

3.341. A thin uniformly charged ring of radius $a = 10$ cm rotates about its axis with an angular velocity $\omega = 100$ rad/s. Find the ratio of volume energy densities of magnetic and electric fields on the axis of the ring at a point removed from its centre by a distance $l = a$.

3.342. Using the expression for volume density of magnetic energy, demonstrate that the amount of work contributed to magnetization of a unit volume of para- or diamagnetic, is equal to $A = -\mathbf{J}\mathbf{B}/2$.

3.343. Two identical coils, each of inductance L, are interconnected (a) in series, (b) in parallel. Assuming the mutual inductance of the coils to be negligible, find the inductance of the system in both cases.

3.344. Two solenoids of equal length and almost equal cross-sectional area are fully inserted into one another. Find their mutual inductance if their inductances are equal to L_1 and L_2.

3.345. Demonstrate that the magnetic energy of interaction of two current-carrying loops located in vacuum can be represented as $W_{ia} = (1/\mu_0) \int \mathbf{B}_1\mathbf{B}_2\, dV$, where $\mathbf{B}_1$ and $\mathbf{B}_2$ are the magnetic inductions within a volume element dV, produced individually by the currents of the first and the second loop respectively.

3.346. Find the interaction energy of two loops carrying currents I_1 and I_2 if both loops are shaped as circles of radii a and b, with $a \ll b$. The loops' centres are located at the same point and their planes form an angle θ between them.

3.347. The space between two concentric metallic spheres is filled up with a uniform poorly conducting medium of resistivity ρ and permittivity ε. At the moment $t = 0$ the inside sphere obtains a certain charge. Find:

(a) the relation between the vectors of displacement current density and conduction current density at an arbitrary point of the medium at the same moment of time;

(b) the displacement current across an arbitrary closed surface wholly located in the medium and enclosing the internal sphere, if at the given moment of time the charge of that sphere is equal to q.

3.348. A parallel-plate capacitor is formed by two discs with a uniform poorly conducting medium between them. The capacitor was initially charged and then disconnected from a voltage source. Neglecting the edge effects, show that there is no magnetic field between capacitor plates.

3.349. A parallel-plate air condenser whose each plate has an area $S = 100\ \text{cm}^2$ is connected in series to an ac circuit. Find the electric field strength amplitude in the capacitor if the sinusoidal current amplitude in lead wires is equal to $I_m = 1.0$ mA and the current frequency equals $\omega = 1.6 \cdot 10^7\ \text{s}^{-1}$.

3.350. The space between the electrodes of a parallel-plate capacitor is filled with a uniform poorly conducting medium of conductivity σ and permittivity ε. The capacitor plates shaped as round discs are separated by a distance d. Neglecting the edge effects, find the magnetic field strength between the plates at a distance r from their axis if an ac voltage $V = V_m \cos \omega t$ is applied to the capacitor.

3.351. A long straight solenoid has n turns per unit length. An alternating current $I = I_m \sin \omega t$ flows through it. Find the displacement current density as a function of the distance r from the solenoid axis. The cross-sectional radius of the solenoid equals R.

3.352. A point charge q moves with a non-relativistic velocity $\mathbf{v} = \text{const}$. Find the displacement current density $\mathbf{j}_d$ at a point located at a distance r from the charge on a straight line

(a) coinciding with the charge path;

(b) perpendicular to the path and passing through the charge.

3.353. A thin wire ring of radius a carrying a charge q approaches the observation point P so that its centre moves rectilinearly with a constant velocity v. The plane of the ring remains perpendicular to the motion direction. At what distance x_m from the point P will the ring be located at the moment when the displacement current density at the point P becomes maximum? What is the magnitude of this maximum density?

3.354. A point charge q moves with a non-relativistic velocity $\mathbf{v} = \text{const}$. Applying the theorem for the circulation of the vector $\mathbf{H}$ around the dotted circle shown in Fig. 3.97, find $\mathbf{H}$ at the point A as a function of a radius vector $\mathbf{r}$ and velocity $\mathbf{v}$ of the charge.

3.355. Using Maxwell's equations, show that

(a) a time-dependent magnetic field cannot exist without an electric field;

(b) a uniform electric field cannot exist in the presence of a time-dependent magnetic field;

(c) inside an empty cavity a uniform electric (or magnetic) field can be time-dependent.

3.356. Demonstrate that the law of electric charge conservation, i.e. $\nabla \cdot \mathbf{j} = -\partial \rho / \partial t$, follows from Maxwell's equations.

3.357. Demonstrate that Maxwell's equations $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$ and $\nabla \cdot \mathbf{B} = 0$ are compatible, i.e. the first one does not contradict the second one.

3.358. In a certain region of the inertial reference frame there is magnetic field with induction B rotating with angular velocity $\boldsymbol{\omega}$. Find $\nabla \times \mathbf{E}$ in this region as a function of vectors $\boldsymbol{\omega}$ and $\mathbf{B}$.

3.359. In the inertial reference frame K there is a uniform magnetic field with induction $\mathbf{B}$. Find the electric field strength in the frame K' which moves relative to the frame K with a non-relativistic velocity $\mathbf{v}$, with $\mathbf{v} \perp \mathbf{B}$. To solve this problem, consider the forces acting on an imaginary charge in both reference frames at the moment when the velocity of the charge in the frame K' is equal to zero.

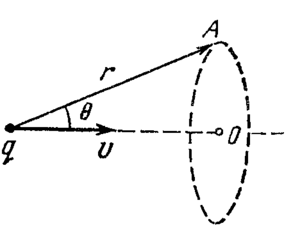

Fig. 3.97.

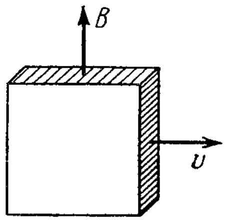

Fig. 3.98.

3.360. A large plate of non-ferromagnetic material moves with a constant velocity $v = 90$ cm/s in a uniform magnetic field with induction $B = 50$ mT as shown in Fig. 3.98. Find the surface density of electric charges appearing on the plate as a result of its motion.

3.361. A long solid aluminum cylinder of radius $a = 5.0$ cm rotates about its axis in a uniform magnetic field with induction $B = 10$ mT. The angular velocity of rotation equals $\omega = 45$ rad/s, with $\boldsymbol{\omega} \uparrow\uparrow \mathbf{B}$. Neglecting the magnetic field of appearing charges, find their space and surface densities.

3.362. A non-relativistic point charge q moves with a constant velocity $\mathbf{v}$. Using the field transformation formulas, find the magnetic induction $\mathbf{B}$ produced by this charge at the point whose position relative to the charge is determined by the radius vector $\mathbf{r}$.

3.363. Using Eqs. (3.6h), demonstrate that if in the inertial reference frame K there is only electric or only magnetic field, in any other inertial frame K' both electric and magnetic fields will coexist simultaneously, with $\mathbf{E}' \perp \mathbf{B}'$.

3.364. In an inertial reference frame K there is only magnetic field with induction $\mathbf{B} = b\,(y\mathbf{i} - x\mathbf{j})/\,(x^2 + y^2)$, where b is a constant, $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors of the x and y axes. Find the electric field strength $\mathbf{E}'$ in the frame K' moving relative to the frame K with a constant non-relativistic velocity $\mathbf{v} = v\mathbf{k}$; $\mathbf{k}$ is the unit vector of the z axis. The z' axis is assumed to coincide with the z axis. What is the shape of the field $\mathbf{E}'$?

3.365. In an inertial reference frame K there is only electric field of strength $\mathbf{E} = a\,(x\mathbf{i} + y\mathbf{j})/(x^2 + y^2)$, where a is a constant, $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors of the x and y axes. Find the magnetic induction $\mathbf{B}'$ in the frame K' moving relative to the frame K with a constant non-relativistic velocity $\mathbf{v} = v\mathbf{k}$; $\mathbf{k}$ is the unit vector of the z axis. The z' axis is assumed to coincide with the z axis. What is the shape of the magnetic induction $\mathbf{B}'$?

3.366. Demonstrate that the transformation formulas (3.6h) follow from the formulas (3.6i) at $v_0 \ll c$.

3.367. In an inertial reference frame K there is only a uniform electric field $E = 8$ kV/m in strength. Find the modulus and direction

(a) of the vector $\mathbf{E}'$, (b) of the vector $\mathbf{B}'$ in the inertial reference frame K' moving with a constant velocity $\mathbf{v}$ relative to the frame K at an angle $\alpha = 45°$ to the vector $\mathbf{E}$. The velocity of the frame K' is equal to a $\beta = 0.60$ fraction of the velocity of light.

3.368. Solve a problem differing from the foregoing one by a magnetic field with induction $B = 0.8$ T replacing the electric field.

3.369. Electromagnetic field has two invariant quantities. Using the transformation formulas (3.6i), demonstrate that these quantities are

(a) $\mathbf{EB}$; (b) $E^2 - c^2B^2$.

3.370. In an inertial reference frame K there are two uniform mutually perpendicular fields: an electric field of strength $E = 40$ kV/m and a magnetic field induction $B = 0.20$ mT. Find the electric strength E' (or the magnetic induction B') in the reference frame K' where only one field, electric or magnetic, is observed.

Instruction. Make use of the field invariants cited in the foregoing problem.

3.371. A point charge q moves uniformly and rectilinearly with a relativistic velocity equal to a β fraction of the velocity of light ($\beta = v/c$). Find the electric field strength $\mathbf{E}$ produced by the charge at the point whose radius vector relative to the charge is equal to $\mathbf{r}$ and forms an angle θ with its velocity vector.

3.7. MOTION OF CHARGED PARTICLES IN ELECTRIC AND MAGNETIC FIELDS

- Lorentz force:

$$\mathbf{F} = q\mathbf{E} + q\,[\mathbf{vB}]. \tag{3.7a}$$

- Motion equation of a relativistic particle:

$$\frac{d}{dt}\frac{m_0\mathbf{v}}{\sqrt{1-(v/c)^2}} = \mathbf{F}. \tag{3.7b}$$

- Period of revolution of a charged particle in a uniform magnetic field:

$$T = \frac{2\pi m}{qB}, \tag{3.7c}$$

where m is the relativistic mass of the particle, $m = m_0/\sqrt{1-(v/c)^2}$.

- Betatron condition, that is the condition for an electron to move along a circular orbit in a betatron:

$$B_0 = \frac{1}{2}\langle B\rangle, \tag{3.7d}$$

where B_0 is the magnetic induction at an orbit's point, $\langle B\rangle$ is the mean value of the induction inside the orbit.

3.372. At the moment $t = 0$ an electron leaves one plate of a parallel-plate capacitor with a negligible velocity. An accelerating voltage, varying as $V = at$, where $a = 100$ V/s, is applied between the plates. The separation between the plates is $l = 5.0$ cm. What is the velocity of the electron at the moment it reaches the opposite plate?

3.373. A proton accelerated by a potential difference V gets into the uniform electric field of a parallel-plate capacitor whose plates extend over a length l in the motion direction. The field strength varies with time as $E = at$, where a is a constant. Assuming the proton to be non-relativistic, find the angle between the motion directions of the proton before and after its flight through the capacitor; the proton gets in the field at the moment $t = 0$. The edge effects are to be neglected.

3.374. A particle with specific charge q/m moves rectilinearly due to an electric field $E = E_0 - ax$, where a is a positive constant, x is the distance from the point where the particle was initially at rest. Find:

(a) the distance covered by the particle till the moment it came to a standstill;

(b) the acceleration of the particle at that moment.

3.375. An electron starts moving in a uniform electric field of strength $E = 10$ kV/cm. How soon after the start will the kinetic energy of the electron become equal to its rest energy?

3.376. Determine the acceleration of a relativistic electron moving along a uniform electric field of strength E at the moment when its kinetic energy becomes equal to T.

3.377. At the moment $t = 0$ a relativistic proton flies with a velocity $\mathbf{v}_0$ into the region where there is a uniform transverse electric field of strength $\mathbf{E}$, with $\mathbf{v}_0 \perp \mathbf{E}$. Find the time dependence of

(a) the angle θ between the proton's velocity vector $\mathbf{v}$ and the initial direction of its motion;

(b) the projection v_x of the vector $\mathbf{v}$ on the initial direction of motion.

3.378. A proton accelerated by a potential difference $V = 500$ kV flies through a uniform transverse magnetic field with induction $B = 0.51$ T. The field occupies a region of space $d = 10$ cm in thickness (Fig. 3.99). Find the angle α through which the proton deviates from the initial direction of its motion.

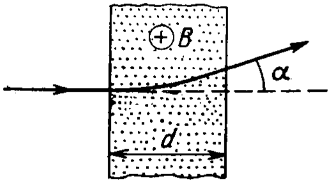

Fig. 3.99.

3.379. A charged particle moves along a circle of radius $r = 100$ mm in a uniform magnetic field with induction $B = 10.0$ mT. Find its velocity and period of revolution if that particle is

(a) a non-relativistic proton;

(b) a relativistic electron.

3.380. A relativistic particle with charge q and rest mass m_0 moves along a circle of radius r in a uniform magnetic field of induction B. Find:

(a) the modulus of the particle's momentum vector;

(b) the kinetic energy of the particle;

(c) the acceleration of the particle.

3.381. Up to what values of kinetic energy does the period of revolution of an electron and a proton in a uniform magnetic field exceed that at non-relativistic velocities by $\eta = 1.0\%$?

3.382. An electron accelerated by a potential difference $V = 1.0$ kV moves in a uniform magnetic field at an angle $\alpha = 30°$ to the vector $\mathbf{B}$ whose modulus is $B = 29$ mT. Find the pitch of the helical trajectory of the electron.

3.383. A slightly divergent beam of non-relativistic charged particles accelerated by a potential difference V propagates from a point A along the axis of a straight solenoid. The beam is brought into focus at a distance l from the point A at two successive values of magnetic induction B_1 and B_2. Find the specific charge q/m of the particles.

3.384. A non-relativistic electron originates at a point A lying on the axis of a straight solenoid and moves with velocity v at an angle α to the axis. The magnetic induction of the field is equal to B. Find the distance r from the axis to the point on the screen into which the electron strikes. The screen is oriented at right angles to the axis and is located at a distance l from the point A.

3.385. From the surface of a round wire of radius a carrying a direct current I an electron escapes with a velocity v_0 perpendicular to the surface. Find what will be the maximum distance of the electron from the axis of the wire before it turns back due to the action of the magnetic field generated by the current.

3.386. A non-relativistic charged particle flies through the electric field of a cylindrical capacitor and gets into a uniform transverse magnetic field with induction B (Fig. 3.100). In the capacitor the particle moves along the arc of a circle, in the magnetic field, along a semi-circle of radius r. The potential difference applied to the capacitor is equal to V, the radii of the electrodes are equal to a and b, with $a < b$. Find the velocity of the particle and its specific charge q/m.

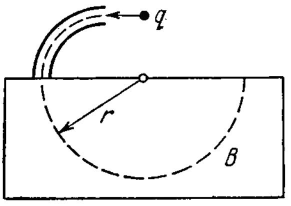

Fig. 3.100.

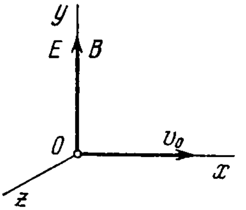

Fig. 3.101.

3.387. Uniform electric and magnetic fields with strength E and induction B respectively are directed along the y axis (Fig. 3.101). A particle with specific charge q/m leaves the origin O in the direction of the x axis with an initial non-relativistic velocity v_0. Find:

(a) the coordinate y_n of the particle when it crosses the y axis for the nth time;

(b) the angle α between the particle's velocity vector and the y axis at that moment.

3.388. A narrow beam of identical ions with specific charge q/m, possessing different velocities, enters the region of space, where there are uniform parallel electric and magnetic fields with strength E and induction B, at the point O (see Fig. 3.101). The beam direction coincides with the x axis at the point O. A plane screen oriented at right angles to the x axis is located at a distance l from the point O. Find the equation of the trace that the ions leave on the screen. Demonstrate that at $z \ll l$ it is the equation of a parabola.

3.389. A non-relativistic proton beam passes without deviation through the region of space where there are uniform transverse mutually perpendicular electric and magnetic fields with $E = 120$ kV/m and $B = 50$ mT. Then the beam strikes a grounded target. Find the force with which the beam acts on the target if the beam current is equal to $I = 0.80$ mA.

3.390. Non-relativistic protons move rectilinearly in the region of space where there are uniform mutually perpendicular electric and magnetic fields with $E = 4.0$ kV/m and $B = 50$ mT. The trajectory of the protons lies in the plane xz (Fig. 3.102) and forms an angle $\varphi = 30°$ with the x axis. Find the pitch of the helical trajectory along which the protons will move after the electric field is switched off.

3.391. A beam of non-relativistic charged particles moves without deviation through the region of space A (Fig. 3.103) where there are transverse mutually perpendicular electric and magnetic fields with

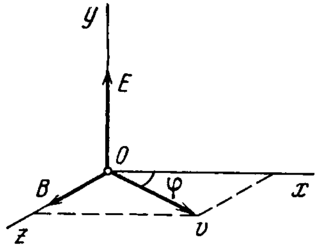

Fig. 3.102.

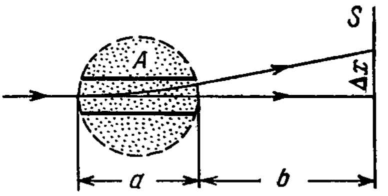

Fig. 3.103.

strength E and induction B. When the magnetic field is switched off, the trace of the beam on the screen S shifts by Δx. Knowing the distances a and b, find the specific charge q/m of the particles.

3.392. A particle with specific charge q/m moves in the region of space where there are uniform mutually perpendicular electric and magnetic fields with strength **E** and induction **B** (Fig. 3.104). At the moment $t = 0$ the particle was located at the point O and had zero velocity. For the non-relativistic case find:

(a) the law of motion $x(t)$ and $y(t)$ of the particle; the shape of the trajectory;

(b) the length of the segment of the trajectory between two nearest points at which the velocity of the particle turns into zero;

(c) the mean value of the particle's velocity vector projection on the x axis (the drift velocity).

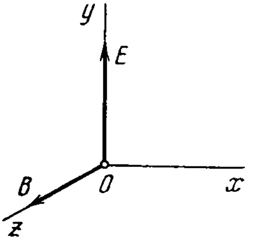

Fig. 3.104.

3.393. A system consists of a long cylindrical anode of radius a and a coaxial cylindrical cathode of radius b ($b < a$). A filament located along the axis of the system carries a heating current I producing a magnetic field in the surrounding space. Find the least potential difference between the cathode and anode at which the thermal electrons leaving the cathode without initial velocity start reaching the anode.

3.394. Magnetron is a device consisting of a filament of radius a and a coaxial cylindrical anode of radius b which are located in a uniform magnetic field parallel to the filament. An accelerating potential difference V is applied between the filament and the anode. Find the value of magnetic induction at which the electrons leaving the filament with zero velocity reach the anode.

3.395. A charged particle with specific charge q/m starts moving in the region of space where there are uniform mutually perpendicular electric and magnetic fields. The magnetic field is constant and has an induction B while the strength of the electric field varies with time as $E = E_m \cos \omega t$, where $\omega = qB/m$. For the non-relativistic case find the law of motion $x(t)$ and $y(t)$ of the particle if at the moment $t = 0$ it was located at the point O (see Fig. 3.104). What is the approximate shape of the trajectory of the particle?

3.396. The cyclotron's oscillator frequency is equal to $\nu = 10$ MHz. Find the effective accelerating voltage applied across the dees of that cyclotron if the distance between the neighbouring trajectories of protons is not less than $\Delta r = 1.0$ cm, with the trajectory radius being equal to $r = 0.5$ m.

3.397. Protons are accelerated in a cyclotron so that the maximum curvature radius of their trajectory is equal to $r = 50$ cm. Find:

(a) the kinetic energy of the protons when the acceleration is completed if the magnetic induction in the cyclotron is $B = 1.0$ T;

(b) the minimum frequency of the cyclotron's oscillator at which the kinetic energy of the protons amounts to $T = 20$ MeV by the end of acceleration.

3.398. Singly charged ions He^+ are accelerated in a cyclotron so that their maximum orbital radius is $r = 60$ cm. The frequency of a cyclotron's oscillator is equal to $\nu = 10.0$ MHz, the effective accelerating voltage across the dees is $V = 50$ kV. Neglecting the gap between the dees, find:

(a) the total time of acceleration of the ion;

(b) the approximate distance covered by the ion in the process of its acceleration.

3.399. Since the period of revolution of electrons in a uniform magnetic field rapidly increases with the growth of energy, a cyclotron is unsuitable for their acceleration. This drawback is rectified in a *microtron* (Fig. 3.105) in which a change ΔT in the period of revolution of an electron is made multiple with the period of accelerating field T_0. How many times has an electron to cross the accelerating gap of a microtron to acquire an energy $W = 4.6$ MeV if $\Delta T = T_0$, the magnetic induction is equal to $B = 107$ mT, and the frequency of accelerating field to $\nu = 3000$ MHz?

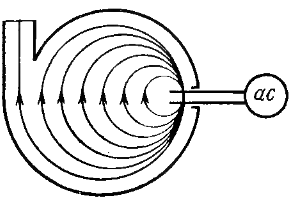

Fig. 3.105.

3.400. The ill effects associated with the variation of the period of revolution of the particle in a cyclotron due to the increase of its energy are eliminated by slow monitoring (modulating) the frequency of accelerating field. According to what law $\omega(t)$ should this frequency be monitored if the magnetic induction is equal to B and the particle acquires an energy ΔW per revolution? The charge of the particle is q and its mass is m.

3.401. A particle with specific charge q/m is located inside a round solenoid at a distance r from its axis. With the current switched into the winding, the magnetic induction of the field generated by the solenoid amounts to B. Find the velocity of the particle and the curvature radius of its trajectory, assuming that during the increase of current flowing in the solenoid the particle shifts by a negligible distance.

3.402. In a betatron the magnetic flux across an equilibrium orbit of radius $r = 25$ cm grows during the acceleration time at practically constant rate $\dot{\Phi} = 5.0$ Wb/s. In the process, the electrons acquire an energy $W = 25$ MeV. Find the number of revolutions made by the electron during the acceleration time and the corresponding distance covered by it.

3.403. Demonstrate that electrons move in a betatron along a round orbit of constant radius provided the magnetic induction on the orbit is equal to half the mean value of that inside the orbit (the betatron condition).

3.404. Using the betatron condition, find the radius of a round orbit of an electron if the magnetic induction is known as a function of distance r from the axis of the field. Examine this problem for the specific case $B = B_0 - ar^2$, where B_0 and a are positive constants.

3.405. Using the betatron condition, demonstrate that the strength of the eddy-current field has the extremum magnitude on an equilibrium orbit.

3.406. In a betatron the magnetic induction on an equilibrium orbit with radius $r = 20$ cm varies during a time interval $\Delta t = 1.0$ ms at practically constant rate from zero to $B = 0.40$ T. Find the energy acquired by the electron per revolution.

3.407. The magnetic induction in a betatron on an equilibrium orbit of radius r varies during the acceleration time at practically constant rate from zero to B. Assuming the initial velocity of the electron to be equal to zero, find:

(a) the energy acquired by the electron during the acceleration time;

(b) the corresponding distance covered by the electron if the acceleration time is equal to Δt.