Class XI Session 2024-25
Subject - Mathematics
Sample Question Paper - 3

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. cosec 150° = ? **[1]**

a) -2 b) $-\sqrt{2}$

c) 2 d) $\sqrt{2}$

2. Let A and B be finite sets containing m and n elements respectively. The number of relations that can be defined from A to B is **[1]**

a) 2^{m+n} b) 0

c) 2^{mn} d) mn

3. Three digits are chosen at random from 1, 2, 3, 4, 5, 6, 7, 8 and 9 without repeating any digit. What is the probability that the product is odd? **[1]**

a) $\frac{5}{108}$ b) $\frac{5}{42}$

c) $\frac{2}{3}$ d) $\frac{7}{48}$

4. $\lim\limits_{\theta \to 0} \frac{1-\cos 4\theta}{1-\cos 6\theta}$ is equal to **[1]**

a) 4/9 b) 1/2

c) -1 d) -1/2

5. The coordinates of the foot of perpendiculars from the point (2, 3) on the line y = 3x + 4 is given by **[1]**

a) $\frac{2}{3}, -\frac{1}{3}$ b) $\frac{10}{37}, -10$

c) $\frac{37}{10}, \frac{-1}{10}$ d) $\frac{-1}{10}, \frac{37}{10}$

6. Which set is the subset of all given sets? **[1]**

a) {1} b) {0}

c) {1, 2, 3, 4} d) { }

7. Let $x, y \in R$, then x + iy is a non real complex number if **[1]**

a) y = 0 b) x $\neq$ 0

c) x = 0 d) y $\neq$ 0

8. R is a relation from {11, 12, 13} to {8, 10, 12} defined by y = x - 3. Then, R^{-1} is **[1]**

a) {(10,13),(12,10)} b) {(10,13), (8,11), (12,10)}

c) {(11,8), (13,10)} d) {(8,11), (10,13)}

9. A man wants to cut three lengths from a single piece of board of length 91 cm. The second length is to be 3 cm longer than the shortest and third length is to be twice as long as the shortest. What are the possible lengths for the shortest board if the third piece is to be at least 5 cm longer than the second? **[1]**

a) 3 $\leq$ x $\leq$ 91 b) 3 $\leq$ x $\leq$ 5

c) 5 $\leq$ x $\leq$ 91 d) 8 $\leq$ x $\leq$ 22

10. $2 \sin 22\frac{1}{2}^\circ \cos 22\frac{1}{2}^\circ$ = ? **[1]**

a) $\sqrt{2}$ b) $\frac{1}{\sqrt{2}}$

c) $\frac{1}{2}$ d) 1

11. If A $\subset$ B, then **[1]**

a) $A^c \subset B^c$ b) $B^c \not\subset A^c$

c) $A^c = B^c$ d) $B^c \subset A^c$

12. If S be the sum, P the product and R be the sum of the reciprocals of n terms of a GP, then P^2 is equal to **[1]**

a) $(\frac{R}{S})^n$ b) $\frac{S}{R}$

c) $\frac{R}{S}$ d) $(\frac{S}{R})^n$

13. In Pascal's triangle, each row begins with 1 and ends in **[1]**

a) -1 b) 0

c) 2 d) 1

14. The solution set for |3x - 2| $\leq \frac{1}{2}$ **[1]**

a) $[\frac{5}{6}, \frac{2}{3}]$ b) $[\frac{2}{3}, \frac{2}{3}]$

c) $[\frac{1}{2}, \frac{5}{6}]$ d) $[\frac{5}{6}, \frac{1}{2}]$

15. For two sets A $\cup$ B = A if **[1]**

a) A = B b) A $\neq$ B

c) B $\subseteq$ A d) A $\subseteq$ B

16. If α and β are acute angles satisfying cos 2α = $\frac{3\cos 2\beta - 1}{3 - \cos 2\beta}$, then α is **[1]**

a) $\sqrt{2} \cot \beta$ b) $\sqrt{2} \tan \beta$

c) $\frac{1}{\sqrt{2}} \cot \beta$ d) $\frac{1}{\sqrt{2}} \tan \beta$

17. If $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is a real number and $0 < \theta < 2\pi$, then θ = **[1]**

a) $\frac{\pi}{3}$
b) $\frac{\pi}{2}$
c) π
d) $\frac{\pi}{6}$

18. How many even numbers can be formed by using all the digits 2, 3, 4, 5, 6? **[1]**

a) 72
b) 36
c) 120
d) 24

19. **Assertion (A):** The expansion of $(1 + x)^n$ = $n_{c_0} + n_{c_1}x + n_{c_2}x^2 \ldots + n_{c_n}x^n$. **[1]**

Reason (R): If x = -1, then the above expansion is zero.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** If each of the observations x_1, x_2, ..., x_n is increased by a, where a is a negative or positive number, then the variance remains unchanged. **[1]**

Reason (R): Adding or subtracting a positive or negative number to (or from) each observation of a group does not affect the variance.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. If A = {1, 2, 3), B = {3, 4} and C = {4 ,5 , 6}, find **[2]**

i. $A \times (B \cap C)$
ii. $(A \times B) \cap (A \times C)$

OR

Find the domain and range of the real valued function f(x) = $\frac{1}{\sqrt{16-x^2}}$.

22. Find the of derivative of the function from the first principle: sin x^2. **[2]**

23. Find the equation of the parabola whose: focus is (2, 3) and the directrix x - 4y + 3 = 0. **[2]**

OR

Find the equation of the circle which touches the lines 4x - 3y +10 = 0 and 4x - 3y - 30 = 0 and whose centre lies on the line 2x + y = 0.

24. Let A = {x : x ∈ N), B = (x : x = 2n, n ∈ N}, C = {x : x = 2n - 1, n ∈ N} and, D = {x : x is a prime natural number}. Find: A ∩ B. **[2]**

25. Find the equation of the perpendicular bisector of the line joining the points (1, 3) and (3, 1). **[2]**

Section C

26. Draw the graph of the Greatest Integer Function. **[3]**

27. Solve inequation and represent the solution set on the number line: $\frac{2x-1}{12} - \frac{x-1}{3} < \frac{3x+1}{4}$ where $x \in R$ **[3]**

28. Find the point in yz-plane which is equidistant from the points A(3, 2, -1), B(1, -1, 0) and C(2, 1, 2). **[3]**

OR

Show that the points (a, b, c), (b, c, a) and (c, a, b) are the vertices of an equilateral triangle.

29. Using binomial theorem, prove that (2^{3n} - 7n - 1) is divisible by 49, where $n \in N$ **[3]**

OR

Expand the given expression $\left(x+\frac{1}{x}\right)^6$

30. If $(a + ib) = \frac{c+i}{c-i}$, where c is real, prove that $a^2 + b^2 = 1$ and $\frac{b}{a} = \frac{2c}{c^2-1}$.. **[3]**

OR

Evaluate $\left[\frac{1}{1-4i} - \frac{2}{1+i}\right]\left[\frac{3-4i}{5+i}\right]$ to the standard form.

31. Let A = {a, e, i, o, u}, B = {a, d, e, o, v) and C = {e, o, t, m] . Using Venn diagrams, verify that: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ **[3]**

Section D

32. A bag contains 6 red, 4 white and 8 blue balls. If three balls are drawn at random, find the probability that: **[5]**

i. one is red and two are white
ii. two are blue and one is red
iii. one is red.

33. Find the derivative of x sinx from first principle. **[5]**

OR

Evaluate : $\lim\limits_{x \to \sqrt{10}} \frac{\sqrt{7-2x}-(\sqrt{5}-\sqrt{2})}{x^2-10}$

34. Find the three numbers in GP, whose sum is 52 and sum of whose product in pairs is 624. **[5]**

35. If cos x = $-\frac{3}{5}$ and x lies in the IIIrd quadrant, find the values of $\cos\frac{x}{2}$, $\sin\frac{x}{2}$ and sin 2x. **[5]**

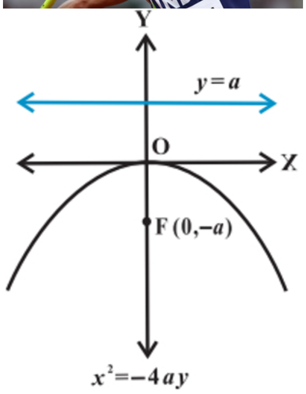
OR

Prove that: tan 20° tan 30° tan 40° tan 80° = 1

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Indian track and field athlete Neeraj Chopra, who competes in the Javelin throw, won a gold medal at Tokyo Olympics. He is the first track and field athlete to win a gold medal for India at the Olympics.



i. Name the shape of path followed by a javelin. If equation of such a curve is given by x^2 = -16y, then find the coordinates of foci. (1)
ii. Find the equation of directrix and length of latus rectum of parabola x^2 = -16y. (1)
iii. Find the equation of parabola with Vertex (0,0), passing through (5,2) and symmetric with respect to y-axis and also find equation of directrix. (2)

OR

Find the equation of the parabola with focus (2, 0) and directrix x = -2 and also length of latus rectum. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

For a group of 200 candidates, the mean and the standard deviation of scores were found to be 40 and 15 , respectively. Later on it was discovered that the scores of 43 and 35 were misread as 34 and 53, respectively.

Student	Eng	Hindi	S.St	Science	Maths
Ramu	39	59	84	80	41
Rajitha	79	92	68	38	75
Komala	41	60	38	71	82
Patil	77	77	87	75	42
Pursi	72	65	69	83	67
Gayathri	46	96	53	71	39

i. Find the correct variance. (1)

ii. What is the formula of variance. (1)

iii. Find the correct mean. (2)

OR

Find the sum of correct scores. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Ashish is writing examination. He is reading question paper during reading time. He reads instructions carefully. While reading instructions, he observed that the question paper consists of 15 questions divided in to two parts - part I containing 8 questions and part II containing 7 questions.

i. If Ashish is required to attempt 8 questions in all selecting at least 3 from each part, then in how many ways can he select these questions (1)

ii. If Ashish is required to attempt 8 questions in all selecting 3 from I part, then in how many ways can he select these questions (1)

iii. If Ashish is required to attempt 8 questions in all selecting 4 from part I and 4 from part II, then in how many ways can he select these questions (2)

OR

If Ashish is required to attempt 8 questions in all selecting 6 from one section and remaining from another section, then in how many ways can he select these questions (2)

Solution

Section A

1.
   (c) 2

   Explanation: cosec 150° = cosec (180° - 30°) = cosec 30° = 2.

2.
   (c) 2^{mn}

   Explanation: We have n(A) = m, n(B) = n.

   $\therefore$ Number of relations defined from A to B

   = number of possible subsets of A $\times$ B = $2^{n(A \times B)} = 2^{mn}$

3.
   (b) $\frac{5}{42}$

   Explanation: Here, n(S) = ${}^9C_3$, Let favourable event = E

   $\therefore$ n(E) = ${}^5C_3$,

   Now, P(E) = $\frac{n(E)}{n(S)} = \frac{{}^5C_3}{{}^9C_3} = \frac{5}{42}$

4. **(a)** 4/9

   Explanation: Given, $\lim\limits_{\theta \to 0} \frac{1-\cos 4\theta}{1-\cos 6\theta} = \lim\limits_{\theta \to 0} \frac{2\sin^2 2\theta}{2\sin^2 3\theta}$ $\left[\because 1-\cos\theta = 2\sin^2\frac{\theta}{2}\right]$

   $= \lim\limits_{\theta \to 0} \frac{\sin^2 2\theta}{\sin^2 3\theta} = \lim\limits_{\theta \to 0}\left[\frac{\sin 2\theta}{\sin 3\theta}\right]^2$

   $= \lim\limits_{\substack{\theta \to 0 \\ 2\theta \to 0 \\ 3\theta \to 0}} \left[\frac{\frac{\sin 2\theta}{2\theta} \times 2\theta}{\frac{\sin 3\theta}{3\theta} \times 3\theta}\right]^2 = \left[\frac{2\theta}{3\theta}\right]^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

5.
   (d) $\frac{-1}{10}, \frac{37}{10}$

   Explanation: Given equation is y = 3x + 4 …(i)

   Since, this equation is in y = mx + b form.

   Thus, the slope (m_1) of the given equation is 3

   Suppose equation of any line passing through the point (2, 3) is

   $y – y_1 = m(x – x_1)$

   $\Rightarrow$ y – 3 = m(x – 2) …(ii)

   Given that eq. (i) is perpendicular to eq. (ii)

   And we know that, if two lines are perpendicular then,

   $m_1 m_2 = -1$

   $\Rightarrow 3 \times m_2 = -1$

   $\Rightarrow m_2 = -\frac{1}{3}$

   $\therefore$ the slope of the required line $= -\frac{1}{3}$

   Substituting the value of slope in eq. (ii), we obtain

   $y - 3 = -\frac{1}{3}(x - 2)$

   $\Rightarrow$ 3y – 9 = -x + 2

   $\Rightarrow$ x + 3y – 9 – 2 = 0

   $\Rightarrow$ x + 3y – 11 = 0 …(iii)

   Now, we have to find the coordinates of foot of the perpendicular.

   Solving eq. (i) and (iii), we obtain

   x + 3(3x + 4) – 11 = 0 [from(i)]

   $\Rightarrow$ x + 9x + 12 – 11 = 0

$\Rightarrow$ 10x + 1 = 0

$\Rightarrow \mathrm{x}=-\frac{1}{10}$

Substituting the value of x in Eq (i), we obtain

$y=3\left(-\frac{1}{10}\right)+4$

$\Rightarrow \mathrm{y}=-\frac{3}{10}+4$

$\Rightarrow \mathrm{y}=\frac{-3+40}{10}$

$\Rightarrow \mathrm{y}=\frac{37}{10}$

So, the required coordinates are $\left(-\frac{1}{10}, \frac{37}{10}\right)$

6.

(d) { }

Explanation: { } denoted as null set and Null set is subset of all sets.

7.

(d) y $\neq$ 0

Explanation: If a complex number has to be a non real complex number then its imaginary part should not be zero

$\Rightarrow i y \neq 0 \Rightarrow y \neq 0$

8.

(d) {(8,11), (10,13)}

Explanation: Since, y = x – 3;

Therefore, for x = 11, y = 8.

For x = 12, y = 9. [But the value y = 9 does not exist in the given set.]

For x = 13, y =10.

So, we have R = {(11, 8), (13, 10)}

Now, R^{-1} = {(8, 11), (10, 13)}.

9.

(d) 8 $\leq$ x $\leq$ 22

Explanation: Let the length of the shortest piece be x cm. Then we have the length of the second and third pieces are x + 3 and 2x centimeters respectively.

According to the question,

x + (x + 3) + 2x $\leq$ 91

$\Rightarrow$ 4x + 3 $\leq$ 91

$\Rightarrow$ 4x $\leq$ 88

$\Rightarrow$ x $\leq$ 22

Also 2x $\geq$ (x + 3) + 5

$\Rightarrow$ 2x $\geq$ x + 8

$\Rightarrow$ x $\geq$ 8

$\Rightarrow$ 8 $\leq$ x $\leq$ 22

Hence the shortest piece may be atleast 8 cm long but it cannot be more than 22cm in length.

10.

(b) $\frac{1}{\sqrt{2}}$

Explanation: Using 2 sin A cos A = sin 2A, we get

$2 \sin 22 \frac{1}{2}^{\circ} \cos 22 \frac{1}{2}^{\circ}=\sin \left(2 \times \frac{45}{2}\right)^{\circ}=\sin 45^{\circ}=\frac{1}{\sqrt{2}}$

11.

(d) $B^{c} \subset A^{c}$

Explanation: Let A $\subset$ B

To prove $B^c \subset A^c$, it is enough to show that $x \in B^c \Rightarrow x \in A^c$

Let $x \in B^c$

$\Rightarrow x \notin B$

$\Rightarrow x \notin A$ since $A \subset B$

$\Rightarrow x \in A^c$

Hence $B^c \subset A^c$

12.

(d) $\left(\frac{S}{R}\right)^n$

Explanation: According to the question,

Sum of n terms of the G.P., S = $\frac{a(r^n-1)}{(r-1)}$

Product of n terms of the G.P., P = $a^n r\left[\frac{n(n-1)}{2}\right]$

Sum of the reciprocals of n terms of the G.P., R = $\frac{\left[\frac{1}{r^n}-1\right]}{a\left(\frac{1}{r}-1\right)}=\frac{(r^n-1)}{ar^{(n-1)}(r-1)}$

$\therefore P^2=\left\{a^2 r^{\frac{2(n-1)}{2}}\right\}^n$

$\Rightarrow P^2=\left\{\frac{\frac{a(r^n-1)}{(r-1)}}{\frac{\left(r^{n-1}\right)}{ar^{(n-1)}(r-1)}}\right\}^n$

$\Rightarrow P^2=\left\{\frac{S}{R}\right\}^n$

Let the first term of the G.P. be a and the common ratio be r.

Sum of n terms, S = $\frac{a(r^n-1)}{r-1}$

Product of the G.P., P = $a^n r^{\frac{n(n+1)}{2}}$

Sum of the reciprocals of n terms, R = $\frac{\left(\frac{1}{r^{\infty}-1}\right)}{a\left(\frac{1}{r-1}\right)}=\frac{\left(\frac{1-r^n}{r^n}\right)}{a\left(\frac{1-r}{r}\right)}$

$p^2 = \left\{a^2 r^{\frac{(n+1)}{2}}\right\}^n$

$p^2 = \left\{\frac{\frac{a\left(r^{n-1}-1\right)}{r-1}}{\frac{\left(\frac{1+n}{r^n}\right)}{a\left(\frac{1+}{r}\right)}}\right\}=\left\{\frac{S}{R}\right\}^n$

13.

(d) 1

Explanation:

The pascal's triangle is given by

1 1

1 2 1

1 3 3 1

14.

(c) $\left[\frac{1}{2}, \frac{5}{6}\right]$

Explanation: $|3x - 2| \leq \frac{1}{2}$

$\Rightarrow \frac{-1}{2} \leq 3x-2 \leq \frac{1}{2}$

$\Rightarrow \frac{-1}{2}+2 \leq 3x-2+2 \leq \frac{1}{2}+2$

$\Rightarrow \frac{3}{2} \leq 3x \leq \frac{5}{2} \quad [\because |x| \leq a \Leftrightarrow -a \leq x \leq a\ ]$

$\Rightarrow \frac{3}{2} \cdot \frac{1}{3} \leq 3x \cdot \frac{1}{3} \leq \frac{5}{2} \cdot \frac{1}{3}$

$\Rightarrow \frac{1}{2} \leq x \leq \frac{5}{6}$

$\Rightarrow x \in \left[\frac{1}{2}, \frac{5}{6}\right]$

15.

(c) $B \subseteq A$

Explanation: The union of two sets is a set of all those elements that belong to A or to B or to both A and B.

If $A \cup B = A$, then $B \subseteq A$

16.

(b) $\sqrt{2}\tan\beta$

Explanation: $\cos 2\alpha = \frac{3\cos 2\beta - 1}{3 - \cos 2\beta}$

$\Rightarrow \frac{\cos 2\alpha - 1}{\cos 2\alpha + 1} = \frac{(3\cos 2\beta - 1) - (3 - \cos 2\beta)}{(3\cos 2\beta - 1) + (3 - \cos 2\beta)}$ [Using compounds and dividendo]

$\Rightarrow \frac{\cos 2\alpha - 1}{\cos 2\alpha + 1} = \frac{4\cos 2\beta - 4}{2\cos 2\beta + 2}$

$\Rightarrow -\frac{1 - \cos 2\alpha}{1 + \cos 2\alpha} = \frac{-4(1 - \cos 2\beta)}{2(1 + \cos 2\beta)}$

$\Rightarrow \frac{1 - \cos 2\alpha}{1 + \cos 2\alpha} = \frac{2(1 - \cos 2\beta)}{(1 + \cos 2\beta)}$

$\Rightarrow \frac{2\sin^2\alpha}{2\cos^2\alpha} = \frac{2(2\sin^2\beta)}{(2\cos^2\beta)}$

$\Rightarrow \tan^2\alpha = 2\tan^2\beta$

$\therefore \tan\alpha = \sqrt{2}\tan\beta$

17.

(c) π

Explanation: π

Given:

$\frac{3 + 2i\sin\theta}{1 - 2i\sin\theta}$ is a real number

On rationalising, we get,

$\frac{3 + 2i\sin\theta}{1 - 2i\sin\theta} \times \frac{1 + 2i\sin\theta}{1 + 2i\sin\theta}$

$= \frac{(3 + 2i\sin\theta)(1 + 2i\sin\theta)}{(1)^2 - (2i\sin\theta)^2}$

$= \frac{3 + 2i\sin\theta + 6i\sin\theta + 4i^2\sin^2\theta}{1 + 4\sin^2\theta}$

$= \frac{3 - 4\sin^2\theta + 8i\sin\theta}{1 + 4\sin^2\theta}$ [$\because$ i^2 = -1]

$= \frac{3 - 4\sin^2\theta}{1 + 4\sin^2\theta} + i\frac{8\sin\theta}{1 + 4\sin^2\theta}$ For the above term to be real, the imaginary part has to be zero.

$\therefore \frac{8\sin\theta}{1 + 4\sin^2\theta} = 0$

$\Rightarrow$ 8 sin θ = 0

For this to be zero,

sin θ = 0

$\Rightarrow$ θ = 0,

$\pi, 2\pi, 3\pi$ ...

But

$0 < \theta < 2\pi$

Hence,

$\theta = \pi$

18. **(a)** 72

Explanation: To form an even number the last number can only be an even digit, therefore the number of impossibility for the last digit of number = 3

Now the ten's place can be filled by any of the remaining 4 digits, and hence the no. of ways for ten's place = 4

Then there remain three digits, so no. of ways of filling hundred's place = 3

Similarly no. of ways of filling thousand's place = 2 and of ten thousand = 1

Therefore, the total possibilities are = 3 × 4 × 3 × 2 × 1 = 72

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion:

$(1 + x)^n = n_{c_0} + n_{c_1}x + n_{c_2}x^2 \ldots + n_{c_n}x^n$

Reason:

$(1 + (-1))^n = n_{c_0}1^n + n_{c_1}(1)^{n-1}(-1)^1 + n_{c_2}(1)^{n-2}(-1)^2$ +... + ${}^nc_n(1)^{n-n}(-1)^n$

$= n_{c_8} - n_{c_1} + n_{c_2} - n_{c_3}$ + ... $(-1)^n n_{c_n}$

Each term will cancel each other

$\therefore (1 + (-1))^n = 0$

Reason is also the but not the correct explanation of Assertion.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Assertion: Let $\bar{x}$ be the mean of $x_1, x_2 ..., x_n$. Then, variance is given by

$\sigma_1^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

If a is added to each observation, the new observations will be

$y_i = x_i + a$

Let the mean of the new observations be $\bar{y}$.

Then,

$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{1}{n}\sum_{i=1}^{n}(x_i + a)$

$= \frac{1}{n}\left[\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} a\right]$

$= \frac{1}{n}\sum_{i=1}^{n} x_i + \frac{na}{n} = \bar{x} + a$

i.e. $\bar{y} = \bar{x} + a$ ...(ii)

Thus, the variance of the new observations is $\sigma_2^2 = \frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i + a - \bar{x} - a)^2$ [using Eqs. (i) and (ii)]

$= \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \sigma_1^2$

Thus, the variance of the new observations is same as that of the original observations.

Reason: We may note that adding (or subtracting) a positive number to (or from) each observation of a group does not affect the variance.

Section B

21. We have,

A = {1, 2, 3), B = {3, 4} and C = {4 ,5 , 6}

i. $\therefore \quad B \cap C = \{3, 4\} \cap \{4, 5, 6\}$ = {4}

$\therefore \quad A \times (B \cap C) = \{1, 2, 3, \} \times \{4\}$

= {(1, 4), (2, 4), (3, 4)}

$\Rightarrow \quad A \times (B \cap C)$ = {(1, 4), (2, 4), (3, 4)}

ii. $\therefore \quad A \times B = \{1, 2, 3, \} \times \{3, 4\}$

= {(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)}

and,

$A \times C = \{1, 2, 3\} \times \{4, 5, 6\}$

= {(1, 4), (1, 5), (1, 6), (2, 4),(2, 5), (2, 6), (3, 4), (3, 5), (3, 6)}

$(A \times B) \cap (A \times C) = \{(1, 4), (2, 4), (3, 4)\}$

OR

We know the square of a real number is never negative.

Clearly, f(x) takes real values only when $16 – x^2 \geq 0$

$= 16 \geq x^2$

$= x^2 \leq 16$

$= x^2 – 16 \leq 0$

$= x^2 – 4^2 \leq 0$

$= (x + 4)(x – 4) \leq 0$

$= x \geq –4$ and $x \leq 4$

$\therefore x \in [–4, 4]$

In addition, f(x) is also undefined when $16 – x^2 = 0$ because denominator will be zero and the result will be indeterminate.

$16 – x^2 = 0 \Rightarrow x = \pm 4$

Hence, $x \in [–4, 4] – \{–4, 4\}$

$\therefore x \in (–4, 4)$

Thus, domain of f = (–4, 4)

Let f(x) = y

$\Rightarrow \frac{1}{\sqrt{16-x^2}} = y$

$\Rightarrow \left(\frac{1}{\sqrt{16-x^2}}\right)^2 = y^2$

$\Rightarrow \frac{1}{16-x^2} = y^2$

= 1 = $(16 – x^2)y^2$

= 1 = $16y^2 – x^2y^2$

=$x^2y^2 + 1 – 16y^2 = 0$

= $(y^2)x^2 + (0)x + (1 – 16y^2) = 0$

As x ∈ R, the discriminant of this quadratic equation in x must be non-negative.

= $0^2 – 4(y^2)(1 – 16y^2) \geq 0$

= $–4y^2(1 – 16y^2) \geq 0$

= $4y^2(1 – 16y^2) \leq 0$

= $1 – 16y^2 \leq 0$ [$\because y^2 \geq 0$]

= $16y^2 – 1 \geq 0$

⇒ $(4y)^2 – 1^2 \geq 0$

= $(4y + 1)(4y – 1) \geq 0$

= 4y ≤ –1 and 4y ≥

$\Rightarrow$ y $\leq -\frac{1}{4}$ and y $\geq \frac{1}{4}$

$\Rightarrow$ y $\in \left(-\infty, -\frac{1}{4}\right] \cup \left[\frac{1}{4}, \infty\right)$

$\Rightarrow$ f(x) $\in \left(-\infty, -\frac{1}{4}\right] \cup \left[\frac{1}{4}, \infty\right)$

However, y is always positive because it is the reciprocal of a non-zero square root.

$\therefore$ f(x) $\in \left[\frac{1}{4}, \infty\right)$

Thus, range of f $= \left[\frac{1}{4}, \infty\right)$

Thus , is the required domain and range of the function.

22. Let $y = sin\, x^2$

Then, $y + \delta y = \sin(x + \delta x)^2$

$\Rightarrow \delta y = \sin(x + \delta x)^2 - \sin x^2$

Using first principle,

$\Rightarrow \frac{\delta y}{\delta x} = \frac{\sin(x+\delta x)^2 - \sin x^2}{\delta x}$

$\Rightarrow \frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$

$= \lim_{\delta x \to 0} \frac{\sin(x+\delta x)^2 - \sin x^2}{\delta x}$

$= \lim_{\delta x \to 0} \frac{2\cos\left[\frac{(x+\delta x)^2 + x^2}{2}\right] \sin\left[\frac{(x+\delta x)^2 - x^2}{2}\right]}{\delta x}$

$\left[\text{ using } (\sin C - \sin D) = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)\right]$

$= \lim_{\delta x \to 0} 2\cos\left[\frac{(x+\delta x)^2 + x^2}{2}\right] \frac{\sin\left[\left(x+\frac{\delta x}{2}\right)\cdot \delta x\right]}{\left(x+\frac{\delta x}{2}\right)\cdot \delta x}\left(x + \frac{\delta x}{2}\right)$

$= 2 \cdot \lim_{\delta x \to 0} \cos\left[\frac{(x+\delta x)^2 + x^2}{2}\right] \cdot \lim_{\delta x \to 0} \frac{\sin\left[\left(x+\frac{\delta x}{2}\right)\cdot \delta x\right]}{\left(x+\frac{\delta x}{2}\right)\cdot \delta x}$

$. \lim_{\delta x \to 0}\left(x + \frac{\delta x}{2}\right)$

$= \left[2 \times \cos x^2 \times 1 \times x\right] = 2x\cos x^2$

Hence, $\frac{d}{dx}\left(\sin x^2\right) = 2x\cos x^2$

23. Let P(x, y) be any point on the parabola whose focus is S(2, 3) and the directrix is x - 4y + 3 = 0

Draw PM perpendicular to x - 4y + 3 = 0

Thus, we have:

SP = PM

$\Rightarrow$ SP^2 = PM^2

$\Rightarrow$ (x - 2)^2 + (y - 3)^2 = $\left|\frac{x-4y+3}{\sqrt{1+16}}\right|^2$

$\Rightarrow$ (x - 2)^2 + (y - 3)^2 = $\left(\frac{x-4y+3}{\sqrt{17}}\right)^2$

$\Rightarrow$ 17 (x^2 + 4 - 4x + y^2 - 6y + 9) = x^2 + 16y^2 + 9 - 8xy - 24y + 6x

$\Rightarrow$ (17x^2 - 68x - 102y + 17y^2 + 13 $\times$ 17) = x^2 + 16y^2 + 9 - 8xy - 24y + 6x

$\Rightarrow$ 16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0

Which is the required equation of parabola.

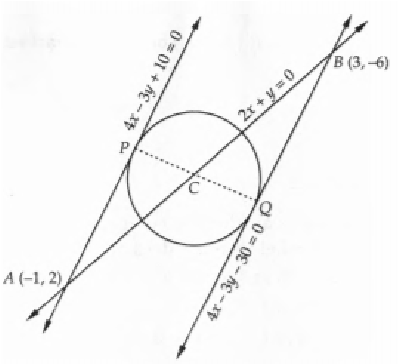
OR

Clearly, the lines 4x - 3y + 10 = 0 and 4x - 3y - 30 = 0 are parallel and are touching the circle.

It is given that the centre of the circle lies on the line 2x + y = 0 which intersects the lines 4x - 3y + 10 = 0 and 4x - 3y - 30 = 0 at A (-1, 2) and B (3, - 6) respectively.

Therefore, the centre of the circle is the mid-point of AB.

So, the coordinates of the centre C are (1, - 2)



Let d be the distance between parallel lines 4x -3y + 10 =0 and 4x -3y - 30 = 0 Then

PQ = $d=\left|\frac{10-(-30)}{\sqrt{4^2+(-3)^2}}\right|=8$

$\Rightarrow$ PQ = d = 8

Radius = $\frac{1}{2}(PQ)=\frac{1}{2}\times$ 8 = 4

$\Rightarrow$Radius = 4

Thus, the required circle has its centre at C (1, - 2) and radius = 4

Hence, its equation is (x - 1)^2 + (y + 2)^2 = 4^2

24. According to the question , we can state,

A = All natural numbers i.e. {1, 2, 3…..}

B = All even natural numbers i.e. {2, 4, 6, 8…}

C = All odd natural numbers i.e. {1, 3, 5, 7……}

D = All prime natural numbers i.e. {1, 2, 3, 5, 7, 11, …}

A $\cap$ B

A contains all elements of B

$\therefore$ B $\subset$ A

$\therefore$ A $\cap$ B = B

25. Given points are, A(1, 3) and B(3, 1).

Let C be the mid point of AB.

$\therefore$ Coordinates of $C=\left(\frac{1+3}{2},\frac{3+1}{2}\right)$ = (2, 2)

Slope of AB $=\frac{1-3}{3-1}=-1$

$\therefore$ Slope of the perpendicular bisector of AB = 1

Hence, the equation of the perpendicular bisector of AB is

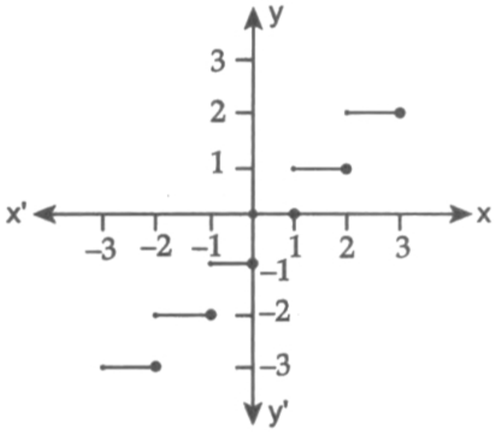
y - 2 = 1(x - 2)

$\Rightarrow$ x - y = 0

or, y = x

Section C

26. The greatest integer function is denoted by y = [x], For all real number, x, the greatest integer function returns the largest integer less than or equal to X.



Value of x	f(x) = [x]
⋰	⋰
$-3 \leq x < -2$	-3
$-2 \leq x < -1$	-2
$-1 \leq x < 0$	-1
$0 \leq x < 1$	0
$1 \leq x < 2$	1
$2 \leq x < 3$	2
$3 \leq x < 4$	3
⋰	⋰

27. Given:

$\frac{2x-1}{12} - \frac{x-1}{3} < \frac{3x+1}{4}$, where $x \in R$.

Multiply by 12 on both sides in the above equation

$\Rightarrow 12\left(\frac{2x-1}{12}\right) - 12\left(\frac{x-1}{3}\right) < 12\left(\frac{3x+1}{4}\right)$

$\Rightarrow (2x - 1) – 4(x - 1) < 3(3x + 1)$

$\Rightarrow 2x – 1 – 4x + 4 < 9x + 3$

$\Rightarrow 3 – 2x < 9x + 3$

Now, subtracting 3 on both sides in the above equation

$\Rightarrow 3 – 2x – 3 < 9x + 3 - 3$

$\Rightarrow -2x < 9x$

Now, subtracting 9x from both the sides in the above equation

$\Rightarrow -2x – 9x < 9x – 9x$

$\Rightarrow - 11x < 0$

Multiplying -1 on both the sides in above equation

$\Rightarrow (-11x)(-1) > (0)(-1)$

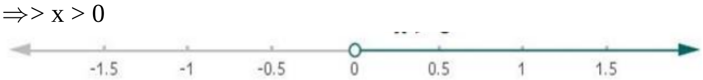
$\Rightarrow 11x > 0$

Dividing both sides by 11 in above equation

$\Rightarrow \frac{11x}{11} > \frac{0}{11}$

Therefore,

$\Rightarrow> x > 0$



28. The general point on yz plane is D(0, y, z).

Consider this point is equidistant to the points A(3, 2, -1), B(1, -1, 0) and C(2, 1, 2).

$\therefore$ AD = BD

$\sqrt{(0-3)^2+(y-2)^2+(z+1)^2}=\sqrt{(0-1)^2+(y+1)^2+(z-0)^2}$

Squaring both sides,

$(0 - 3)^2 + (y - 2)^2 + (z + 1)^2 = (0 - 1)^2 + (y + 1)^2 + (z - 0)^2$

$9 + y^2 - 4y + 4 + z^2 + 2z + 1 = 1 + y^2 + 2y + 1 + z^2$

$-6y + 2z + 12 = 0$ ….(1)

Also, AD = CD

$\sqrt{(0-3)^2+(y-2)^2+(z+1)^2}=\sqrt{(0-2)^2+(y-1)^2+(z-2)^2}$

Squaring both sides,

$(0 - 3)^2 + (y - 2)^2 + (z + 1)^2 = (0 - 2)^2 + (y - 1)^2 + (z - 2)^2$

$9 + y^2 - 4y + 4 + z^2 + 2z + 1 = 4 + y^2 - 2y + 1 + z^2 - 4z + 4$

$-2y + 6z + 5 = 0$ ….(2)

By solving equation (1) and (2) we get

$y= \frac{31}{16}$ $z = \frac{-3}{16}$

The point which is equidistant to the points A(3, 2, -1), B(1, -1, 0) and C(2, 1, 2) is $(\frac{31}{16}, \frac{-3}{16})$.

OR

Let A (a, b, c), B (b, c, a), and C (c, a, b) be the vertices of $\triangle$ABC. Then,

AB = $\sqrt{(b-a)^2+(c-b)^2+(a-c)^2}$

$=\sqrt{b^2-2ab+a^2+c^2-2bc+b^2+a^2-2ca+c^2}$

$=\sqrt{2a^2+2b^2+2c^2-2ab-2bc-2ca}$

$\mathrm{AB}=\sqrt{2\left(a^2+b^2+c^2-ab-bc-ca\right)}$

BC = $\sqrt{(c-b)^2+(a-c)^2+(b-a)^2}$

$=\sqrt{c^2-2bc+b^2+a^2-2ca+c^2+b^2-2ab+a^2}$

$=\sqrt{2a^2+2b^2+2c^2-2ab-2bc-2ca}$

$\mathrm{BC}=\sqrt{2\left(a^2+b^2+c^2-ab-bc-ca\right)}$

CA = $\sqrt{(a-c)^2+(b-a)^2+(c-b)^2}$

$=\sqrt{a^2-2ca+c^2+b^2-2ab+a^2+c^2-2bc+b^2}$

$=\sqrt{2a^2+2b^2+2c^2-2ab-2bc-2ca}$

$\mathrm{CA}=\sqrt{2\left(a^2+b^2+c^2-ab-bc-ca\right)}$

$\therefore$ AB = BC = CA

Therefore, $\triangle$ABC is an equilateral triangle.

29. To prove: $(2^{3n} - 7n - 1)$ is divisible by 49, where $n \in N$

$(2^{3n} - 7n - 1) = (2^3)n – 7n - 1$

$= 8^n - 7n - 1$

$= (1 + 7)^n - 7n - 1$

Now using binomial theorem..

$\Rightarrow {}^nC_0 1^n + {}^nC_1 1^{n-1} 7 + {}^nC_2 1^{n-2} 7^2 + \ldots\ldots + {}^nC_{n-1} 7^{n-1} + {}^nC_n 7^n - 7n - 1$

$= {}^nC_0 + {}^nC_1 7 + {}^nC_2 7^2 + \ldots\ldots + {}^nC_{n-1} 7^{n-1} + {}^nC_n 7^n - 7n - 1$

$= 1 + 7n + 7^2[{}^nC_2 + {}^nC_3 7 + \ldots + {}^nC_{n-1}\, 7^{n-3} + {}^nC_n\, 7^{n-2}] - 7n - 1$

$= 7^2[{}^nC_2 + {}^nC_3 7 + \ldots + {}^nC_{n-1}\, 7^{n-3} + {}^nC_n\, 7^{n-2}]$

$= 49[{}^nC_2 + {}^nC_3 7 + \ldots + {}^nC_{n-1}\, 7^{n-3} + {}^nC_n\, 7^{n-2}]$

$= 49K$, where $K = ({}^nC_2 + {}^nC_3 7 + \ldots + {}^nC_{n-1}\, 7^{n-3} + {}^nC_n 7^{n-2})$

Now, $(2^{3n} - 7n - 1) = 49K$

Therefore $(2^{3n} - 7n - 1)$ is divisible by 49.

OR

Using binomial theorem for the expansion of $\left(x+\frac{1}{x}\right)^6$ we have

$\left(x+\frac{1}{x}\right)^6 = ={}^6C_0(x)^6+{}^6C_1(x)^5\left(\frac{1}{x}\right)+{}^6C_2(x)^4\left(\frac{1}{x}\right)^2+{}^6C_3(x)^3\left(\frac{1}{x}\right)^3$

$+{}^6C_4(x)^2\left(\frac{1}{x}\right)^4+{}^6C_5(x)\left(\frac{1}{x}\right)^5+{}^6C_6\left(\frac{1}{6}\right)^6$

$= x^6 + 6 \cdot x^5 \cdot \frac{1}{x} + 15 \cdot 4x^4 \cdot \frac{1}{x^2} + 20 \cdot x^3 \cdot \frac{1}{x^3} + 15 \cdot x^2 \cdot \frac{1}{x^4} + 6 \cdot x \cdot \frac{1}{x^5} + \frac{1}{x^6}$

$= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$

30. Here $a + ib = \frac{c+i}{c-i}$

$= \frac{c+i}{c-i} \times \frac{c+i}{c+i} = \frac{(c+i)^2}{c^2-i^2}$

$= \frac{c^2+2ci+i^2}{c^2+1}$

$= \frac{c^2-1}{c^2+1} + \frac{2c}{c^2+1}i$

Comparing real and imaginary parts on both sides, we have

$a = \frac{c^2-1}{c^2+1}$ and $b = \frac{2c}{c^2+1}$

Now $a^2 + b^2 = \left(\frac{c^2-1}{c^2+1}\right)^2 + \left(\frac{2c}{c^2+1}\right)^2$

$= \frac{(c^2-1)^2+4c^2}{(c^2+1)^2} = \frac{(c^2+1)^2}{(c^2+1)^2} = 1$

Also $\frac{b}{a} = \frac{\frac{2c}{c^2+1}}{\frac{c^2-1}{c^2+1}} = \frac{2c}{c^2-1}$

OR

$\left[\frac{1}{1-4i} - \frac{2}{1+i}\right]\left[\frac{3-4i}{5+i}\right] = \left[\frac{1+i-2+8i}{(1-4i)(1+i)}\right]\left[\frac{3-4i}{5+i}\right]$

$= \left[\frac{-1+9i}{1+i-4i-4i^2}\right]\left[\frac{3-4i}{5+i}\right] = \left[\frac{-1+9i}{5-3i}\right]\left[\frac{3-4i}{5+i}\right]$

$= \frac{-3+4i+27i-36i^2}{25+5i-15i-3i^2} = \frac{33+31i}{28-10i} \times \frac{28+10i}{28+10i}$

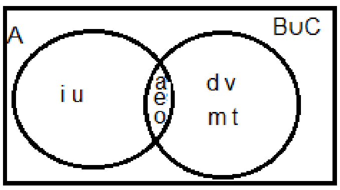
$= \frac{924+330i+868i+310i^2}{(28)^2-(10i)^2} = \frac{614+1198i}{784+100} (\because i^2 = -1)$

$= \frac{2(307+599i)}{884} = \frac{307+599i}{442}$

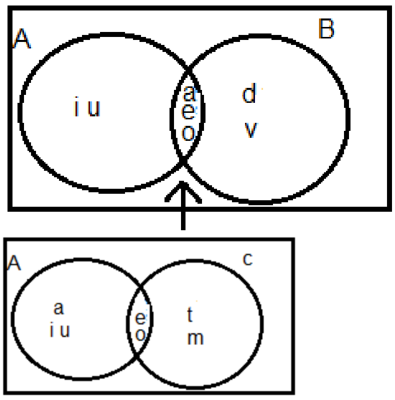
31. Here, it is given: A = {a, e, i, o, u}, B = {a, d, e, o, v} and C = {e, o, t, m}.

B ∪ C = {a, d, v, e, o, t, m} and A ∩ (B ∪ C) = {a, e, o}

LHS



R.H.S: A ∩ B = {a, e, o} and A ∩ C = {e, o}



$(A \cap B) \cup (A \cap C) =$ a e o

(A ∩ B) ∪ (A ∩ C) = {a, e, o}

L.H.S = R.H.S. [Verified]

Section D

32. Bag contains:

6 -Red balls

4 -White balls

8 -Blue balls

Since three ball are drawn,

$\therefore n(S) = {}^{18}C_3$

i. Let E be the event that one red and two white balls are drawn.

$\therefore n(E) = {}^6C_1 \times {}^4C_2$

$\therefore P(E) = \frac{{}^6C_1 \times {}^4C_2}{{}^{18}C_3} = \frac{6\times4\times3}{2} \times \frac{3\times2}{18\times17\times16}$

$P(E) = \frac{3}{68}$

ii. Let E be the event that two blue balls and one red ball was drawn.

$\therefore n(E) = {}^8C_2 \times {}^6C_1$

$\therefore P(E) = \frac{{}^8C_2 \times {}^6C_1}{{}^{18}C_3} = \frac{8\times7}{2} \times 6 \times \frac{3\times2\times1}{18\times17\times16} = \frac{7}{34}$

$P(E) = \frac{7}{34}$

iii. Let E be the event that one of the ball must be red.

$\therefore$ E = {(R,W,B) or (R,W,W) or (R,B,B)}

$\therefore n(E) = {}^6C_1 \times {}^4C_1 \times {}^8C_1 + {}^6C_1 \times {}^4C_2 + {}^6C_1 \times {}^8C_2$

$$\therefore P(E) = \frac{{}^6C_1 \times {}^4C_1 \times {}^8C_1 + {}^6C_1 \times {}^4C_2 + {}^6C_1 \times {}^8C_2}{{}^{18}C_3} = \frac{6\times4\times8 + \frac{6\times4\times3}{2\times1} + \frac{6\times8\times7}{2\times1}}{\frac{18\times17\times16}{3\times2\times1}}$$

$= \frac{396}{816} = \frac{33}{68}$

33. We have, f(x) = x sinx

By using first principle of derivative,

$f'(x) = \lim\limits_{h\to0} \frac{f(x+h)-f(x)}{h}$

$= \lim\limits_{h\to0} \frac{(x+h)\sin(x+h)-x\sin x}{h}$

$= \lim\limits_{h\to0} \frac{(x+h)[\sin x\cdot\cos h+\cos x\cdot\sin h]-x\sin x}{h}$ [$\because$ sin(x + y) = sinx cosy + cosx siny]

$= \lim\limits_{h\to0} \frac{[x\sin x\cdot\cos h+x\cdot\cos x\cdot\sin h+h\sin x\cdot\cos h+h\cos x\cdot\sin h-x\sin x]}{h}$

$= \lim\limits_{h\to0} \frac{[x\sin x(\cos h-1)+x\cdot\cos x\cdot\sin h+h(sinx\cdot cosh+cosx\cdot\sin h)]}{h}$

$= \lim\limits_{h\to0} \frac{x\sin x(\cos h-1)}{h} + \lim\limits_{h\to0} x\cdot\cos x\cdot\frac{\sin h}{h} + \lim\limits_{h\to0} \frac{h(\sin x\cdot\cos h+\cos x\cdot\sin h)}{h}$

$= x\sin x \lim\limits_{h\to0}\left[\frac{-(1-\cos h)}{h}\right]$ + x cos x + sin x

$= -2x\sin x\cdot \lim\limits_{\frac{h}{2}\to0} \frac{\sin^2\frac{h}{2}}{h\times\frac{h}{4}} \times \frac{h}{4}$ + xcos x + sin x

$= -x\cdot\sin x\cdot\frac{2}{4}\lim\limits_{\frac{h}{2}\to0}\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)^2 \times$ h + xcos x + sin x

$= -x\sin x\cdot\frac{1}{2}(1)\times$ 0 + x cos x + sin x $\left[\because \lim\limits_{x\to0}\frac{\sin x}{x} = 1\right]$

= xcos x + sin x

OR

We have,

$\lim\limits_{x\to\sqrt{10}} \frac{\sqrt{7-2x}-(\sqrt{5}-\sqrt{2})}{x^2-10}$

= $\lim\limits_{x\to\sqrt{10}} \frac{\sqrt{7-2x}-\sqrt{(\sqrt{5}-\sqrt{2})^2}}{x^2-10}$ $\left(\text{form } \frac{0}{0}\right)$

= $\lim\limits_{x\to\sqrt{10}} \frac{\sqrt{7-2x}-\sqrt{7-2\sqrt{10}}}{x^2-10}$ $\left(\text{form } \frac{0}{0}\right)$

= $\lim\limits_{x\to\sqrt{10}} \frac{\sqrt{7-2x}-\sqrt{7-2\sqrt{10}}}{x^2-10} \times \frac{\sqrt{7-2x}+\sqrt{7-2\sqrt{10}}}{\sqrt{7-2x}+\sqrt{7-2\sqrt{10}}}$

= $\lim\limits_{x\to\sqrt{10}} \frac{(7-2x)-(7-2\sqrt{10})}{(x-\sqrt{10})(x+\sqrt{10})\left\{\sqrt{7-2x}+\sqrt{7-2}\sqrt{10}\right\}}$

= $\lim\limits_{x\to\sqrt{10}} \frac{-2x+2\sqrt{10}}{(x-\sqrt{10})(x+\sqrt{10})\left\{\sqrt{7-2x}+\sqrt{7-2}\sqrt{10}\right\}}$

$= \lim\limits_{x \to \sqrt{10}} \frac{-2(x-\sqrt{10})}{(x-\sqrt{10})(x+\sqrt{10})\{\sqrt{7-2x}+\sqrt{7-2\sqrt{10}}\}}$

$= \lim\limits_{x \to \sqrt{10}} \frac{-2}{(x+\sqrt{10})\left\{\sqrt{7-2x}+\sqrt{7-2\sqrt{10}}\right\}}$

$= \lim\limits_{x \to \sqrt{10}} \frac{-2}{2\sqrt{10}\left\{\sqrt{7-2\sqrt{10}}+\sqrt{7-2\sqrt{10}}\right\}}$

$= \frac{-1}{\sqrt{10}\times 2\times\sqrt{7-2\sqrt{10}}}$ = $\frac{-1}{2\sqrt{10}(\sqrt{5}-\sqrt{2})}$ $[\because (\sqrt{5}-\sqrt{2})^2 = 7-2\sqrt{10}]$

$= \frac{-1}{2\sqrt{10}} \times \frac{(\sqrt{5}+\sqrt{2})}{3}$ = -$\frac{(\sqrt{5}+\sqrt{2})}{6\sqrt{10}}$

34. Let the three numbers in GP be $\frac{a}{r}$, a, ar.

Sum of three numbers = 52 [given]

$\Rightarrow \frac{a}{r}$ + a + ar = 52

$\Rightarrow$ a $\left(\frac{1}{r}+1+r\right)$ = 52 ...(i)

And sum of product in pair = 624

$\Rightarrow \frac{a}{r} \times$ a + a $\times$ ar + $\frac{a}{r} \times$ ar = 624

$\Rightarrow$ a^2 $\left(\frac{1}{r}+r+1\right)$= 624 ...(ii)

On dividing Eqs. (ii) by (i), we get

a = $\frac{624}{52} \Rightarrow$ a = 12

On putting a = 12 in Eq. (i), we get

12 ($\frac{1}{r}$ + r + 1) = 52

$\Rightarrow \frac{r^2+r+1}{r} = \frac{52}{12} \Rightarrow \frac{r^2+r+1}{r} = \frac{13}{3}$

$\Rightarrow$ 3r^2 +3r + 3 = 13r

$\Rightarrow$ 3r^2 - 10r + 3 = 0

$\Rightarrow$ (3r - 1) (r - 3) = 0

$\Rightarrow$ r = $\frac{1}{3}$ or r = 3

When r = $\frac{1}{3}$, then numers are $\frac{12}{\frac{1}{3}}$, 12, 12 $\times \frac{1}{3}$ i.e., 36, 12, 4.

When r = 3, then numbers are $\frac{12}{3}$, 12, 12 $\times \frac{1}{3}$ .i.e, 4, 12, 36.

35. We have to find the values of cos$\frac{x}{2}$, sin$\frac{x}{2}$ and sin 2x.

It is given that cos x = $-\frac{3}{5}$ and x lies in the IIIrd quadrant

We know,

cos 2x = 2 cos^2 x - 1

cos x = 2 cos$^2\frac{x}{2}$ - 1

$-\frac{3}{5}$ = 2 cos$^2\frac{x}{2}$ - 1 ... [$\because$ cos x = $-\frac{3}{5}$]

2 cos$^2\frac{x}{2}$ = $-\frac{3}{5}$+ 1

2 cos$^2\frac{x}{2}$ = $\frac{2}{5}$

cos$^2\frac{x}{2}$ = $\frac{1}{5}$

cos$\frac{x}{2}$ = $\pm\frac{1}{\sqrt{5}}$

Since,

x $\in \left(\pi, \frac{3\pi}{2}\right) \Rightarrow \frac{x}{2} \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$

cos$\frac{x}{2}$ will be negative in 3rd quadrant

So,

cos x = $-\frac{1}{\sqrt{5}}$

We know,

cos 2x = 1 - 2 sin^2 x

cos x = 1 - 2 sin$^2\frac{x}{2}$ ... [$\because$ cos x = $-\frac{3}{5}$]

$-\frac{3}{5}$ = 1 - 2 sin$^2\frac{x}{2}$

2 sin$^2\frac{x}{2}$ = $\frac{3}{5}$+ 1

2 sin$^2\frac{x}{2}$ = $\frac{8}{5}$

$\sin^2\frac{x}{2} = \frac{4}{5}$

$\sin\frac{x}{2} = \pm\frac{2}{\sqrt{5}}$

Since,

$x \in \left(\pi, \frac{3\pi}{2}\right) \Rightarrow \frac{x}{2} \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$

$\sin\frac{x}{2}$ will be positive in 2nd quadrant

So,

$\sin\frac{x}{2} = \frac{2}{\sqrt{5}}$

We know,

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$\sin^2 x = 1 - \left(-\frac{3}{5}\right)^2$ ... $[\because \cos x = -\frac{3}{5}]$

$\sin^2 x = 1 - \frac{9}{25}$

$\sin^2 x = \frac{25-9}{25}$

$\sin^2 x = \frac{16}{25}$

$\sin x = \pm\frac{4}{5}$

Since,

$x \in \left(\pi, \frac{3\pi}{2}\right)$

sinx will be negative in 3rd quadrant

So,

$\sin x = -\frac{4}{5}$

Now,

$\sin 2x = 2(\sin x)(\cos x)$ ... $[\because \cos x = -\frac{3}{5}$ and $\sin x = -\frac{4}{5}]$

$\sin 2x = 2 \times -\frac{4}{5} \times -\frac{3}{5}$

$\sin 2x = \frac{24}{25}$

Hence, values of $\cos\frac{x}{2}$, $\sin\frac{x}{2}$, sin 2x are $-\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}$ and $\frac{24}{25}$

OR

LHS = tan 20° tan 30° tan 40° tan 80°

$= \frac{1}{\sqrt{3}}(\tan 20^\circ \tan 40^\circ \tan 80^\circ)$ $\left[\because \tan 30^\circ = \frac{1}{\sqrt{3}}\right]$

$= \frac{(\sin 20^\circ \sin 40^\circ \sin 80^\circ)}{(\cos 20^\circ \cos 40^\circ \cos 80^\circ)\sqrt{3}}$

$= \frac{(2\sin 20^\circ \sin 40^\circ)\sin 80^\circ}{\sqrt{3}(2\cos 20^\circ \cos 40^\circ)\cos 80^\circ}$

Applying

$\Rightarrow$ 2 sin A sin B = cos (A - B) - cos (A + B) and 2 cos A cos B = cos(A + B) + cos (A - B), we get

$= \frac{[\cos(40^\circ-20^\circ)-\cos(20^\circ+40^\circ)]\sin 80^\circ}{[\cos(20^\circ+40^\circ)+\cos(40^\circ-20^\circ)]\cos 80^\circ\sqrt{3}}$

$= \frac{(\cos 20^\circ-\cos 60^\circ)\sin 80^\circ}{\sqrt{3}(\cos 60^\circ+\cos 20^\circ)\cos 80^\circ}$

$= \frac{\left(\cos 20^\circ-\frac{1}{2}\right)\sin 80^\circ}{\sqrt{3}\left(\frac{1}{2}+\cos 20^\circ\right)\cos 80^\circ}$

$= \frac{2\cos 20^\circ \sin 80^\circ-\sin 80^\circ}{\sqrt{3}(\cos 80^\circ+2\cos 20^\circ \cos 80^\circ)}$

Now,

$\Rightarrow$ 2 sin A cos B = sin (A + B) + sin (A - B)

$= \frac{\sin(80^\circ+20^\circ)+\sin(80^\circ-20^\circ)-\sin 80^\circ}{\sqrt{3}[\cos 80^\circ+\cos(20^\circ+80^\circ)+\cos(80^\circ-20^\circ)]}$

$= \frac{\sin 100^\circ+\sin 60^\circ-\sin 80^\circ}{\sqrt{3}(\cos 80^\circ+\cos 100^\circ+\cos 60^\circ)}$

$= \frac{\sin 100^\circ+\sin 60^\circ-\sin(180^\circ-100^\circ)}{\sqrt{3}(\cos 80^\circ+\cos(180^\circ-80^\circ)+\cos 60^\circ)}$

$= \frac{\sin 100^\circ + \frac{\sqrt{3}}{2} - \sin 100^\circ}{\sqrt{3}(\cos 80^\circ - \cos 80^\circ + \cos 60^\circ)}$

$= \frac{\frac{\sqrt{3}}{2}}{\sqrt{3}\left(\frac{1}{2}\right)}$ = 1 = RHS

Section E

36. i. The path traced by Javelin is parabola. A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point (not on the line) in the plane.

compare x^2 = -16y with x^2 = -4ay

$\Rightarrow$ - 4a = -16

$\Rightarrow$ a = 4

coordinates of focus for parabola x^2 = -4ay is (0, -a)

$\Rightarrow$ coordinates of focus for given parabola is (0, -4)

ii. compare x^2 = -16y with x^2 = -4ay

$\Rightarrow$ -4a = -16

$\Rightarrow$ a = 4

Equation of directrix for parabola x^2 = -4ay is y = a

$\Rightarrow$ Equation of directrix for parabola x^2 = -16y is y = 4

Length of latus rectum is 4a = 4 × 4 = 16

iii. Equation of parabola with axis along y - axis

x^2 = 4ay

which passes through (5, 2)

$\Rightarrow$ 25 = 4a $\times$ 2

$\Rightarrow$ 4a = $\frac{25}{2}$

hence required equation of parabola is

$x^2 = \frac{25}{2}y$

$\Rightarrow$ $2x^2$ = 25y

Equation of directrix is y= -a

Hence required equation of directrix is 8y + 25 = 0.

OR

Since the focus (2,0) lies on the x-axis, the x-axis itself is the axis of the parabola.

Hence the equation of the parabola is of the form either y^2 = 4ax or y^2 = -4ax.

Since the directrix is x = -2 and the focus is (2,0), the parabola is to be of the form y^2 = 4ax with a = 2.

Hence the required equation is y^2 = 4(2)x = 8x

length of latus rectum = 4a = 8

37. i. SD = σ = 15

$\Rightarrow$ Variance = 15^2 = 225

According to the formula,

Variance = $\left(\frac{1}{n}\sum {x_\mathrm{i}}^2\right) - \left(\frac{1}{n}\sum \mathrm{x_i}\right)^2$

$\therefore \frac{1}{200}\sum x_\mathrm{i}^2 - (40)^2$ = 225

$\Rightarrow \frac{1}{200}\sum (x_\mathrm{i})^2 - 1600$ = 225

$\Rightarrow \sum (x_\mathrm{i})^2$ = 200 $\times$ 1825 = 365000

This is an incorrect reading.

$\therefore$ Corrected $\sum (x_i)^2$ = 365000 - 34^2 - 53^2 + 43^2 + 35^2

= 365000 - 1156 - 2809 + 1849 + 1225

= 364109

Corrected variance = $\left(\frac{1}{n} \times \text{ Corrected } \sum x_i\right)$ - (Corrected mean)^2

= $\left(\frac{1}{200} \times 364109\right) - (39.955)^2$

= 1820.545 - 1596.402

= 224.14

ii. The formula of variance is $\frac{\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2}{n}$.

iii. Corrected mean = $\frac{\text{Corrected}\sum x_1}{200}$

= $\frac{7993}{200}$

= 39.955

OR

We have:

n = 200, $\bar{X}$ = 40, σ = 15

$\frac{1}{n}\sum x_{\text{i}}=\bar{X}$

$\therefore \frac{1}{200}\sum x_i$ = 40

$\Rightarrow \sum x_i$ = 40 $\times$ 200 = 8000

Since the score was misread, this sum is incorrect.

$\Rightarrow$ Corrected $\sum x_i$ = 8000 - 34 - 53 + 43 + 35

= 8000 - 7

= 7993

38. i. Since, at least 3 questions from each part have to be selected

Part I	Part II
3	5
4	4
3	5

So number of ways are

3 questions from part I and 5 questions from part II can be selected in $n^8C_3 \times {}^7C_5$ ways

4 questions from part I and 4 questions from part II can be selected in ${}^8C_4 \times {}^7C_4$ ways

5 questions from part I and 3 questions from part II can be selected in ${}^8C_5 \times {}^7C_3$ ways

So required number of ways are

${}^8C_3 \times {}^7C_5 + {}^8C_4 \times {}^7C_4 + {}^8C_5 \times {}^7C_3$

$\Rightarrow \frac{8!}{5!\times 3!} \times \frac{7!}{5!\times 2!} + \frac{8!}{4!\times 4!} \times \frac{7!}{4!\times 3!} + \frac{8!}{5!\times 3!} \times \frac{7!}{4!\times 3!}$

$\Rightarrow \frac{8\times 7\times 6}{3\times 2\times 1} \times \frac{7\times 6}{2\times 1} + \frac{8\times 7\times 6\times 5}{4\times 3\times 2\times 1} \times \frac{7\times 6\times 5}{3\times 2\times 1} + \frac{8\times 7\times 6}{3\times 2\times 1} \times \frac{7\times 6\times 5\times 4}{4\times 3\times 2\times 1}$

$\Rightarrow$ 56 $\times$ 21 + 70 $\times$ 35 + 56 $\times$ 35

$\Rightarrow$ 1176 + 2450 + 1960

$\Rightarrow$ 5586

ii. Ashish is selecting 3 questions from part I so he has to select remaining 5 questions from part II

The number of ways of selection is

3 questions from part I and 5 questions from part II can be selected in ${}^8C_3 \times {}^7C_5$ ways

$\Rightarrow {}^8C_3 \times {}^7C_5$

$\Rightarrow \frac{8!}{5!\times 3!} \times \frac{7!}{5!\times 2!}$

$\Rightarrow \frac{8\times 7\times 6}{3\times 2\times 1} \times \frac{7\times 6}{2\times 1}$

$\Rightarrow$ 56 $\times$ 21

$\Rightarrow$ 1176

iii. 4 questions from part I and 4 questions from part II can be selected

${}^8C_4 \times {}^7C_4$

$\Rightarrow \frac{8!}{4!\times 4!} \times \frac{7!}{4!\times 3!}$

$\Rightarrow \frac{8\times 7\times 6\times 5}{4\times 3\times 2\times 1} \times \frac{7\times 6\times 5}{3\times 2\times 1}$

$\Rightarrow$ 70 $\times$ 35

$\Rightarrow$ 2450

OR

6 questions from part I and 2 questions from part II can be selected or

2 questions from part I and 6 questions from part II can be selected

$\Rightarrow {}^8C_6 \times {}^7C_2 + {}^8C_2 \times {}^7C_6$

$\Rightarrow \frac{8!}{6!\times 2!} \times \frac{7!}{2!\times 5!} + \frac{8!}{6!\times 2!} \times \frac{7!}{1!\times 6!}$

$\Rightarrow \frac{8\times7}{2\times1} \times \frac{7\times6}{2\times1} + \frac{8\times7}{2\times1} \times 7$

$\Rightarrow 28 \times 21 + 28 \times 7$

$\Rightarrow 588 + 196 = 784$