CHAPTER – 16

PLAYING WITH NUMBERS

EXERCISE – 16.1

Question – 1 Find the values of the letters in each of the following and give reasons for the steps involved

$$\begin{array}{r} 3\ A \\ +\ 2\ 5 \\ \hline B\ 2 \end{array}$$

Answer:

The addition of A and 5 is giving 2 which means that a number whose ones digit is 2.

But, this is only possible when digit A is 7

Well in that case, the addition of A =7 and 5 will give 12

And, 1 will be the carry for the next step.

Now,

In the next step,

1 + 3 + 2 = 6

Therefore, the addition is as follows:

$$\begin{array}{r} 3\ 7 \\ +\ 2\ 5 \\ \hline 6\ 2 \end{array}$$

Clearly, B is 6

Hence, A and B are 7 and 6 respectively.

Question – 2

Find the values of the letters in each of the following and give reasons for the steps involved

$$\begin{array}{r} 4\ A \\ +\ 9\ 8 \\ \hline C\ B\ 3 \end{array}$$

Answer:

While adding A and 8, we are getting 3 which shows that a number whose one's digit is 3.

But, this is only possible when digit A is 5.

In that case, the addition of A and 8 will give 13

And, 1 will be the carry for the next step.

Now,

In the next step,

1 + 4 + 9 = 14

Therefore, the addition is as follows:

$$\begin{array}{r} 4\ 5 \\ +\ 9\ 8 \\ \hline 1\ 4\ 3 \end{array}$$

Clearly, B and C are 4 and 1 respectively

Hence, A, B, and C are 5, 4, and 1 respectively

Question – 3 Find the values of the letters in each of the following and give reasons for the steps involved

$$\begin{array}{r} 1\ A \\ \times\ A \\ \hline 9\ A \end{array}$$

Answer:

While multiplying A with A itself, we are getting a number whose ones digit is A again.

But, this happens only when A = 1, 5, or 6

Now,

If A = 1, then the multiplication will be 11 × 1 = 11. But, here the tens digit is given as 9

Therefore, A = 1 is invalid.

Similarly,

If A = 5, then the multiplication will be 15 × 5 = 75. Thus, A = 5 is also invalid.

If we take A = 6, then 16 × 6 = 96

Therefore, A should be 6

The multiplication is as follows:

$$\begin{array}{r} 1\ 6 \\ \times\ 6 \\ \hline 9\ 6 \end{array}$$

Hence, the value of A is 6

Question – 4 Find the values of the letters in each of the following and give reasons for the steps involved

$$\begin{array}{r} A\ B \\ \times\ 3\ 7 \\ \hline 6\ A \end{array}$$

Answer:

While adding A and 3 we obtain 6. There can be two cases.

(i) First step is not producing a carry

In that case,

A comes to be 3 as 3 + 3 = 6

Taking the first step in which the addition of B and 7 is giving A (i.e., 3), B should be a number such that the unit's digit of this addition comes to be 3.

It is only possible only when B = 6

In that case, A = 6 + 7 = 13

But, A is a single digit number

Hence, it is invalid.

(ii) First step is producing a carry

In that case,

A comes to be 2 as 1 + 2 + 3 = 6

Taking the first step in which the addition of B and 7 is giving A (i.e., 2), B should be a number such that the unit's digit of this addition comes to be 2.

But, it is possible only when B = 5 and 5 + 7 = 12

$$\begin{array}{r} 2\ 5 \\ \times\ 3\ 7 \\ \hline 6\ 2 \end{array}$$

Hence, the values of A and B are 2 and 5 respectively

Question – 5 Find the values of the letters in each of the following and give reasons for the steps involved

$$\begin{array}{r} A\ B \\ \times\ 3 \\ \hline C\ A\ B \end{array}$$

Answer:

The multiplication of 3 and B gives a number whose ones digit is B again

Therefore, B must be 0 or 5

Let B is 5

Multiplication of first step = $3 \times 5 = 15$

Now,

1 will be a carry for the next step

$3 \times A + 1 = CA$

This is not valid for any value of A

Hence, B must be 0 only.

If B = 0, then there won't be any carry for the next step

We will get, $3 \times A = CA$

That is, the one's digit of $3 \times A$ should be A

This is possible when A = 5 or 0

But, A cannot be 0 as AB is a two-digit number

Therefore, A must be 5 only.

The multiplication is as follows:

$$\begin{array}{r} 5\ 0 \\ \times\ 3 \\ \hline 1\ 5\ 0 \\ \hline \end{array}$$

Hence, the values of A, B, and C are 5, 0, and 1 respectively

Question – 6 Find the values of the letters in each of the following and give reasons for the steps involved

$$\begin{array}{r} A\ B \\ \times\ 5 \\ \hline C\ A\ B \\ \hline \end{array}$$

Answer:

While multiplying B and 5 we are getting a number whose ones digit is B again.

This is only possible when B = 5 or B = 0

In that case,

The product will be, $B \times 5 = 5 \times 5 = 25$

2 will be a carry for the next step

Now,

We have, $5 \times A + 2 = CA$, which is possible for A = 2 or 7

The multiplication is as follows:

$$\begin{array}{r} 2\ 5 \\ \times\ 5 \\ \hline 1\ 2\ 5 \\ \hline \end{array} \qquad \begin{array}{r} 7\ 5 \\ \times\ 5 \\ \hline 3\ 7\ 5 \\ \hline \end{array}$$

If B = 0,

$B \times 5 = B$

$\Rightarrow 0 \times 5$

$= 0$

There will not be any carry in this step.

In the next step, $5 \times A = CA$

It can only happen when A = 5 or A = 0

But, A cannot be 0 as AB is a two-digit number

Hence, A can be 5 only.

The multiplication is as follows:

$$\begin{array}{r} 5\ 0 \\ \times\ 5 \\ \hline 2\ 5\ 0 \\ \hline \end{array}$$

Hence, there are 3 possible values of A, B, and C:

(i) 5, 0, and 2 respectively

(ii) 2, 5, and 1 respectively

(iii) 7, 5, and 3 respectively

Question – 7 Replace A, B by suitable numerals.

$$\begin{array}{r} A\ B \\ \times\ 6 \\ \hline B\ B\ B \\ \hline \end{array}$$

Answer:

while multiplying 6 and B we get a number whose one's digit is B again

It is only possible when B = 0, 2, 4, 6, or 8

If B = 0, then the product will be 0.

Therefore, this value of B is impossible.

In that case,

If B = 2, then B × 6 = 12 and 1 will be a carry for the next step

$6A + 1 = BB = 22$

$\Rightarrow 6A = 21$

Hence, any integer value of A is not possible

If B = 6, then $B \times 6 = 36$ and 3 will be a carry for the next step

$6A + 3 = BB = 66$

$\Rightarrow 6A = 63$

Hence, any integer value of A is not possible

If B = 8, then $B \times 6 = 48$ and 4 will be a carry for the next step.

$6A + 4 = BB = 88$

$\Rightarrow 6A = 84$

Therefore,

$A = 14$

However, A is a single digit number. Therefore, this value of A is not possible

If B = 4, then $B \times 6 = 24$ and 2 will be a carry for the next step.

$6A + 2 = BB = 44$

$\Rightarrow 6A = 42$

And hence, $A = 7$

The multiplication is as follows:

$$\begin{array}{r} 7\,4 \\ \times\, 6 \\ \hline 4\,4\,4 \end{array}$$

Hence, the values of A and B are 7 and 4 respectively

Question – 8 Find the values of the letters in each of the following and give reasons for the steps involved

$$\begin{array}{r} A\ 1 \\ \times\ 1\ B \\ \hline B\ 0 \end{array}$$

Answer:

While adding 1 and B we get 0 i.e., a number whose one's digits is 0.

This is only possible when digit B is 9.

In that case, the addition of 1 and B will give 10 and, 1 will be the carry for the next step.

In the next step,

$1 + A + 1 = B$

Clearly,

A is 7 as $1 + 7 + 1 = 9 = B$

Therefore, the addition is as follows:

$$\begin{array}{r} 7\ 1 \\ \times\ 1\ 9 \\ \hline 9\ 0 \end{array}$$

Hence, the values of A and B are 7 and 9 respectively

Question – 9 Find the values of the letters in each of the following and give reasons for the steps involved

$$\begin{array}{r} 2\ A\ B \\ \times\ A\ B\ 1 \\ \hline B\ 1\ 8 \end{array}$$

Answer:

While adding B and 1 we get 8 i.e., a number whose ones digits is 8.

This is only possible when digit B is 7.

In that case, the addition of B and 1 will give 8.

In the next step,

A + B = 1

Clearly, A is 4

4 + 7 = 11 and 1 will be a carry for the next step.

In the next step,

1 + 2 + A = B

1 + 2 + 4 = 7

Therefore, the addition is as follows:

$$\begin{array}{r} 2\,4\,7 \\ \times\, 4\,7\,1 \\ \hline 7\,1\,8 \end{array}$$

Hence, the values of A and B are 4 and 7 respectively

Question – 10 Find the values of the letters in each of the following and give reasons for the steps involved

$$\begin{array}{r} 1\,2\,A \\ \times\, 6\,A\,B \\ \hline A\,0\,9 \end{array}$$

Answer:

While adding A and B we get 9 i.e., a number whose ones digits is 9.

The sum can be 9 only as the sum of two single digit numbers cannot be 19.

Therefore, there will not be any carry in this step

In the next step, $2 + A = 0$

It is possible only when $A = 8$

$2 + 8 = 10$ and 1 will be the carry for the next step.

$1 + 1 + 6 = A$

Clearly, A is 8.

We know that the addition of A and B is giving 9. As A is 8, therefore, B is 1

Hence, the addition is as follows:

$$\begin{array}{r} 1\,2\,8 \\ \times\, 6\,8\,1 \\ \hline 8\,0\,9 \end{array}$$

Hence, the values of A and B are 8 and 1 respectively

EXERCISE – 16.2

Question – 1 If 21y5 is a multiple of 9, where y is a digit, what is the value of y?

Answer:

We know that. if a number is a multiple of 9, then the sum of its digits will be divisible by 9

Sum of digits of 21*y*5 = 2 + 1 + *y* + 5 = 8 + *y*

Hence, 8 + *y* should be a multiple of 9

This is possible when 8 + *y* is any one of these numbers 0, 9, 18, 27, and so on …

However, since *y* is a single digit number, this sum can be 9 only.

8 + y = 9
y = 9 - 8y = 1

Therefore, *y* should be 1 only

Question – 2 If 31z5 is a multiple of 9, where z is a digit, what is the value of z?
You will find that there are two answers for the last problem. Why is this so?

Answer:

We know that if a number is a multiple of 9, then the sum of its digits will be divisible by 9

Sum of digits of 31*z*5 = 3 + 1 + *z* + 5 = 9 + *z*

Hence, 9 + *z* should be a multiple of 9

This is possible when 9 + *z* is any one of these numbers 0, 9, 18, 27, and so on …

But, since z is a single digit number, this sum can be either 9 or 18

Therefore, z should be either 0 or 9

Question – 3 If 24x is a multiple of 3, where x is a digit, what is the value of x?
(Since 24x is a multiple of 3, its sum of digits 6 + x is a multiple of 3; so 6 + x is one of these numbers: 0, 3, 6, 9, 12, 15, 18, ... But since x is a digit, it can only be that6 + x = 6 or 9 or 12 or 15. Therefore, x = 0 or 3 or 6 or 9. Thus, x can have any of four different values)

Answer:

Since $24x$ is a multiple of 3, the sum of its digits is a multiple of 3

Sum of digits of $24x = 2 + 4 + x = 6 + x$

Hence, $6 + x$ is a multiple of 3

This is possible when $6 + x$ is any one of these numbers 0, 3, 6, 9, and so on …

Since x is a single digit number, the sum of the digits can be 6 or 9 or 12 or 15 and the value of x comes to 0 or 3 or 6 or 9 respectively

Thus, x can have its value as any of the four different values 0, 3, 6, or 9

Question – 4 If 31z5 is a multiple of 3, where z is a digit, what might be the values of z?

Answer:

Since $31z5$ is a multiple of 3, the sum of its digits will be a multiple of 3

That is, $3 + 1 + z + 5 = 9 + z$ is a multiple of 3

This is possible when $9 + z$ is any one of 0, 3, 6, 9, 12, 15, 18, and so on …

Since z is a single digit number, the value of $9 + z$ can only be 9 or 12 or 15 or 18 and thus, the value of x comes to 0 or 3 or 6 or 9 respectively

Thus, z can have its value as any one of the four different values 0, 3, 6, or 9