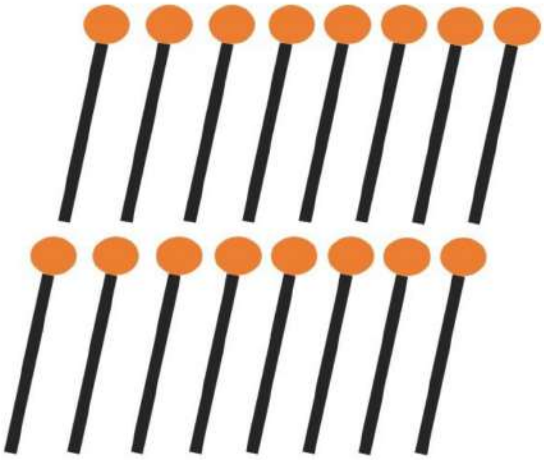
Chapter 3

Playing With Numbers

Introduction to Factors and Multiples

Introduction

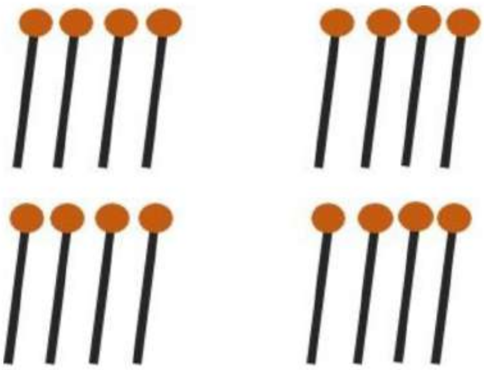
Lucy has a packet consisting of 16 candies, which she wishes to distribute among her 2 friends.



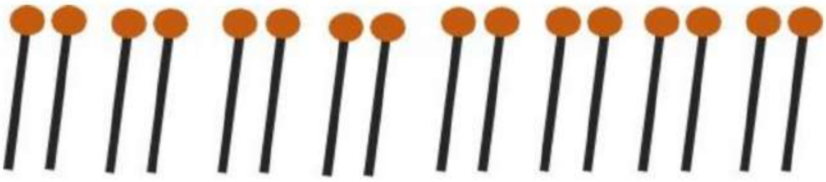
Now it would only be fair if each one gets the same number of candies.

$16 \div 2 = 8$
$2 \times 8 = 16$

Therefore, each friend will get 8 candies. Now, 2 more friends joined the group. Lucy has to distribute these candies equally among her 4 friends.
$16 \div (2 + 2) \Rightarrow 16 \div 4 = 4$
$4 \times 4 = 16$
What happens if 4 more friends come?



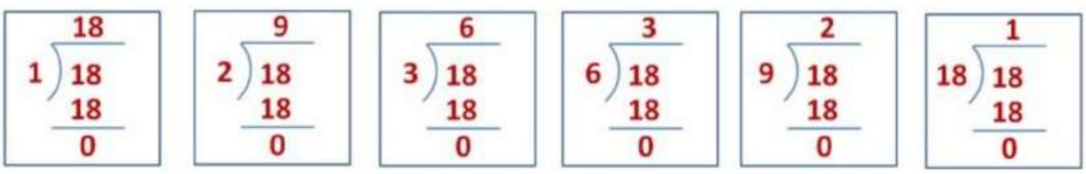
$16 \div (4 + 4) \Rightarrow 16 \div 8 = 2$



$8 \times 2 = 16$
So, we see that 16 can be written as a product of two numbers in different ways.
$16 = 2 \times 8$; $16 = 4 \times 4$; $16 = 8 \times 2$
2 divides 16 completely $\Rightarrow$ 2 is a factor of 16
4 divides 16 completely $\Rightarrow$ 4 is a factor of 16
8 divides 16 completely $\Rightarrow$ 8 is a factor of 16 and so on.
So, we see that 2, 4, 8 are exact divisors of 16. They are called the factors of 16.

Factors and Multiples

A factor of a number is an exact divisor of that number.



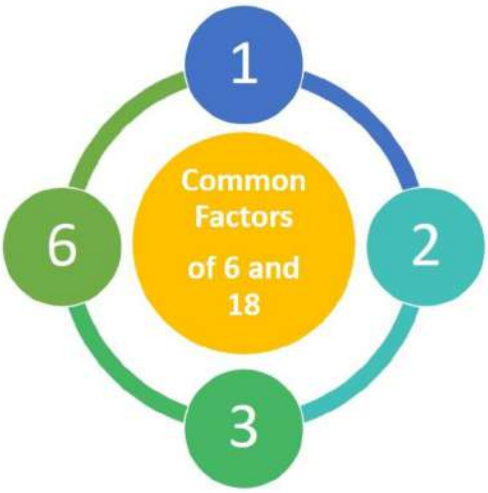
1, 2, 3, 6, 9 and 18 are exact divisors of 18.

These numbers are called factors of 18.

Factors and Common Factors

When two (or more) numbers have the same factor, that factor is known as a common factor.

The factors of 6 are 1, 2, 3 and 6
The factors of 18 are 1, 2, 3, 6, 9 and 18.



Example: Find the common factors of:

i) 20 and 32

ii) 15 and 35
Factors of 20 are 1, 2, 4, 5, 10 and 20.

Factors of 32 are 1, 2, 4, 8, 16, and 32.
Thus, common factors of 20 and 32 are 1, 2 and 4
ii) 15 and 35
Factors of 15 are 1, 3, 5 and 15
Factors of 35 are 1, 5, 7 and 35

Thus, common factors of 15 and 35 are 1 and 5.

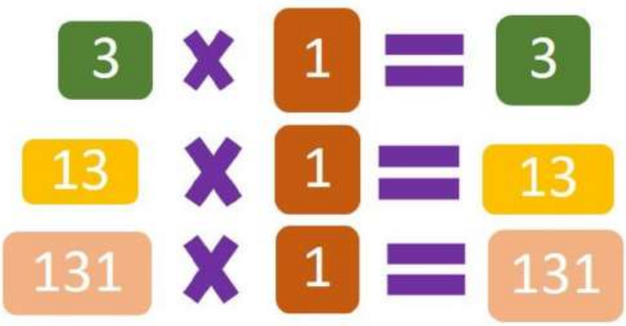
Example: Find the common factors of:
i) 4, 8 and 16   ii) 5, 10 and 25

i) 4, 8 and 16
Factors of 4 are 1, 2 and 4.
Factors of 8 are 1, 2, 4 and 8.
Factors of 16 are 1, 2, 4, 8 and 16.
Thus, common factors of 4, 8 and 16 are 1, 2 and 4.
ii) 5, 10 and 25
Factors of 5 are 1 and 5.
Factors of 10 are 1, 2, 5 and 10.
Factors of 25 are 1, 5 and 25.
Thus, common factors of 5, 10 and 25 are 1 and 5.

Properties of Factors

i) 1 is a factor of every number

1 is a number which occurs as a factor of every number.



ii) Every number is a factor of itself.

3 × 1 = 3

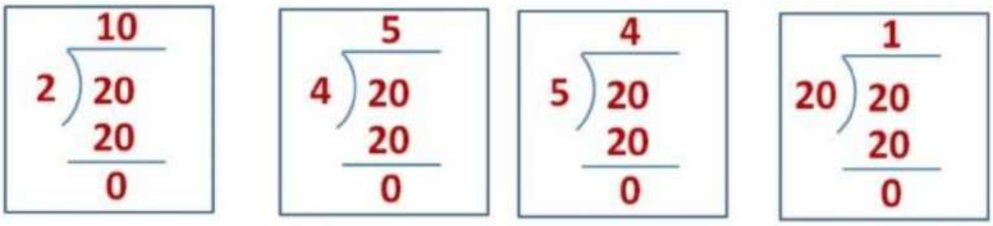
13 × 1 = 13

131 × 1 = 131

We can express every number in this way.

iii) Every factor of a number divides the given number exactly.

The factors of 20 are 1, 2, 4 and 5.



We see that every factor of 20 i.e. 1, 2, 4 and 5 and 20 divides the number 20 exactly.

iv) Every factor is less than or equal to the given number.

Factors of 64 are 1, 2, 4, 8, 16, 32 and 64.

Out of all these factors, the greatest factor is 64 and the other factors 1, 2, 4, 8, 16 and 32 are less than 64.

So, we can say that every factor is less than or equal to the given number.

v) Number of factors of a given number is finite.

Factors of 96 are 1, 2, 3, 4, 6, 8, 16, 24, 32, 48 and 96.

So, the number 96 has 11 factors, that is, we are able to count the number of factors.

Even for a large number like 137256, the number of factors is finite though the factorization of such large numbers is difficult.

Perfect Number

A perfect number is a number for which sum of all its factors is equal to twice the number. The numbers 6 and 28 are perfect numbers.

Now, the factors of 6 are 1, 2, 3 and 6.

$$1 + 2 + 3 + 6 = 12$$
$$12 = 2 \times 6$$

The factors of 28 are 1, 2, 4, 7, 14 and 28.

$$1 + 2 + 4 + 7 + 14 + 28 = 56$$
$$56 = 2 \times 28$$

Let us take one more example of number 10.

The factors of 10 are 1, 2, 5 and 10.

$$1 + 2 + 5 + 10 = 18$$
$$2 \times 10 = 20$$
$$18 \neq 20$$

Therefore, 10 is not a perfect number.

Example: Find the factors of the following numbers.
i) 15 ii) 27 iii) 36 iv) 23 v) 12

i) 15
$15 = 1 \times 15,$
$15 = 3 \times 5$
The factors of 15 are 1, 3, 5 and 15.
ii) 27
$27 = 1 \times 27,$
$27 = 3 \times 9$
The factors of 27 are 1, 3, 9 and 27.
iii) 36

$36 = 1 \times 36,$

$36 = 2 \times 18,$

$36 = 3 \times 12,$

$36 = 4 \times 9,$

$36 = 6 \times 6,$

The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18 and 36.iv) 23

$23 = 1 \times 23,$

The factors of 23 are 1 and 23.

v) 12

$12 = 1 \times 12,$

$12 = 2 \times 6,$

$12 = 4 \times 3,$

The factors of 12 are 1, 2, 3, 4, 6 and 12.

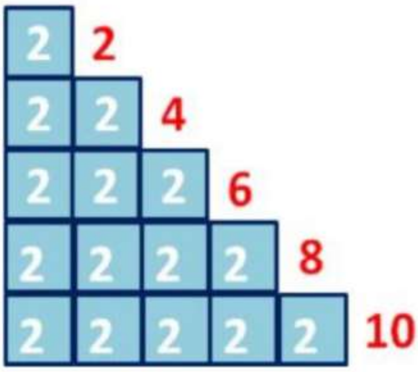
Multiples and Properties of Multiple

A multiple of any natural number is the product of that number and any non – zero whole number.

A number is a multiple of its factors.

Let us learn more about multiples with the help of an interesting pattern.

Collect paper strips of 2 units each and join them end to end as shown in the figure.



Length of the strip at the top = 2 units
Length of the strip below it is = 2 + 2 = 4 units
Or $2 \times 2 = 4$ units
Length of the next strip = 2 + 2 + 2 = 6
Or $2 \times 3 = 6$ units and so on.
We can say that the numbers 2, 4, 6, 8 and 10 are multiples of 2.

Common Multiples

A common multiple is a multiple that two or more numbers have in common.
Let us find common multiples of 4 and 6.
Multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 36, 40,............
Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, 54, ...
We see that 12, 24, 36...... are common multiples of 4 and 6.
Example: Find first three common multiples of:
i) 6 and 8 ii) 12 and 18
i) 6 and 8

Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 72, 78 ...
Multiples of 8 are 8, 16, 24, 32, 40, 48, 56, 64, 72,80 ...
First 3 common multiples of 6 and 8 are 24, 48 and 72

Properties of Multiples

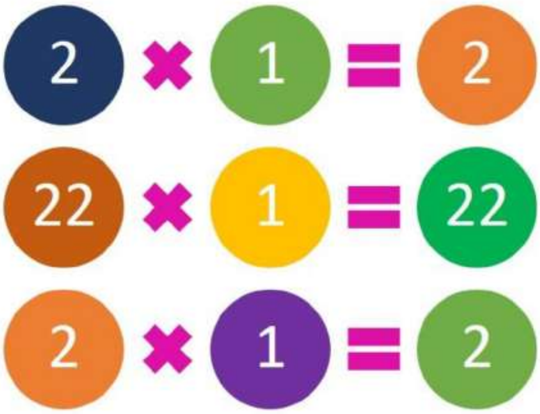
i) Every multiple of a number is greater than or equal to that number.

We know that the multiples of 2 are 2, 4, 6, 8, 10, ...
Here, each of these multiples is greater than or equal to 2.

ii) The number of multiples of a given number is infinite.

The multiples of 7 are 7, 14, 21, 28, 35, 42, ...
This list does not end here. It is endless. Therefore, there are infinite multiples of a given number.

iii) Every number is a multiple of itself.



Here, we see that every number is a multiple of itself.

Example: Write first five multiples of:
i) 5 ii) 6 iii) 8 iv) 9
i) 5
$5 \times 1 = 5$,

$5 \times 2 = 10$,
$5 \times 3 = 15$,
$5 \times 4 = 20$,
$5 \times 5 = 25$. The required multiples of 5 are 5, 10, 15, 20 and 25.
ii) 6
$6 \times 1 = 6$,
$6 \times 2 = 12$,
$6 \times 3 = 18$,
$6 \times 4 = 24$,
$6 \times 5 = 30$. The required multiples of 6 are 6, 12, 18, 24 and 30.
iii) 8
$8 \times 1 = 8$,
$8 \times 2 = 16$,
$8 \times 3 = 24$,
$8 \times 4 = 32$,
$8 \times 5 = 40$. The required multiples of 8 are 8, 16, 24, 32 and 40.

iv) 9
$9 \times 1 = 9,$
$9 \times 2 = 18,$
$9 \times 3 = 27,$
$9 \times 4 = 36,$
$9 \times 5 = 45.$ The required multiples of 9 are 9, 18, 27, 36 and 45

Prime and Composite Numbers

The numbers other than 1 whose only factors are 1 and the number itself are called prime numbers.

Number	Factors	Number of Factors
2 = 2 x 1	1, 2	2
3 = 3 x 1	1, 3	2
5 = 5 x 1	1, 5	2
11 = 11 x 1	1, 11	2

These are prime numbers as they have exactly two factors, 1 and the number itself.

The number 1 has only one factor.

The numbers having two or more than two factors are called Composite Numbers.

Number	Factors	Number of Factors
15	1, 3, 5, 15	4
18	1, 2, 3, 6, 18	5
25	1, 5, 25	3

These are composite numbers as they have more than two factors.

1 is neither a prime number nor a composite number.

Example: The numbers 13 and 31 are prime numbers. Both these numbers have same digits 1 and 3. Find such pairs of prime numbers up to 100.

Pairs of prime numbers having same digits up to 100 are 17 and 71; 13 and 31, 37 and 73; 79 and 97.

Example: Write down separately the prime and composite numbers less than 20.

Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17 and 19.
Composite Numbers less than 20 are 4, 6, 8, 9, 10, 12, 14, 15, 16 and 18.

Example: What is the greatest prime number between 1 and 10?

The greatest prime number between 1 and 10 is 7.

Example: Express the following as the sum of two odd primes.
i) 44 ii) 36 iii) 24 iv) 18

i) 44
$44 = 7 + 37$
44 can be expressed as a sum of odd primes, 7 and 37.
ii) 36
$36 = 7 + 29$
36 can be expressed as a sum of odd primes, 7 and 29.
iii) 24
$24 = 11 + 13$
24 can be expressed as a sum of odd primes, 11 and 13.
iv) 18
$18 = 5 + 13$
18 can be expressed as a sum of odd primes, 5 and 13.

Example: Write seven consecutive composite numbers less than 100 so that there is no prime number between them.

Composite Numbers from 90 to 96 do not have any prime number between them. Therefore, seven consecutive composite numbers less than 100 so that there is no prime number between them are 90, 91, 92, 93, 94, 95 and 96.

Example: Express each of the following numbers as the sum of three odd primes:
i) 21 ii) 31 iii) 53 iv) 61

i) 21

$21 = 3 + 7 + 11$

So, 21 can be expressed as a sum of three odd primes, 3, 7 and 11.

ii) 31

$31 = 3 + 11 + 17$

So, 31 can be expressed as a sum of three odd primes, 3, 11 and 17.

iii) 53

$53 = 13 + 17 + 23$

So, 53 can be expressed as a sum of three odd primes, 13, 17 and 23.

iv) 61

$61 = 13 + 19 + 29$

So, 61 can be expressed as a sum of three odd primes, 13, 19 and 29.

Example: Write five pairs of prime numbers less than 20 whose sum is divisible by 5.

Pairs of prime numbers less than 20 whose sum is divisible by 5 are

a) (2, 3)

$2 + 3 = 5$ (Sum is divisible by 5)

b) (2, 13)

$2 + 13 = 15$ (Sum is divisible by 5)

c) (3, 7)

$3 + 7 = 10$ (Sum is divisible by 5)

d) (3, 17)

$3 + 17 = 20$ (Sum is divisible by 5)

e) (5, 5)

5 + 5 = 10 (Sum is divisible by 5)

Eratosthenes Sieve to Find Prime Numbers

Greek Mathematician Eratosthenes, in the third century B.C., found a very simple method called 'Sieve of Eratosthenes' to find the prime and composite numbers.
Step 1: Place all the numbers from 1 to 100 in a table.

1 ✗	(2)	(3)	4 ✗	(5)	6 ✗	(7)	8 ✗	9 ✗	10 ✗
(11)	12 ✗	(13)	14 ✗	15 ✗	16 ✗	(17)	18 ✗	(19)	20 ✗
21 ✗	22 ✗	(23)	24 ✗	25 ✗	26 ✗	27 ✗	28 ✗	(29)	30 ✗
(31)	32 ✗	33 ✗	34 ✗	35 ✗	36 ✗	(37)	38 ✗	39 ✗	40 ✗
(41)	42 ✗	(43)	44 ✗	45 ✗	46 ✗	(47)	48 ✗	49 ✗	50 ✗
51 ✗	52 ✗	(53)	54 ✗	55 ✗	56 ✗	57 ✗	58 ✗	(59)	60 ✗
(61)	62 ✗	63 ✗	64 ✗	65 ✗	66 ✗	(67)	68 ✗	69 ✗	70 ✗
(71)	72 ✗	(73)	74 ✗	75 ✗	76 ✗	77 ✗	78 ✗	(79)	80 ✗
81 ✗	82 ✗	(83)	84 ✗	85 ✗	86 ✗	87 ✗	88 ✗	(89)	90 ✗
91 ✗	92 ✗	93 ✗	94 ✗	95 ✗	96 ✗	(97)	98 ✗	99 ✗	100 ✗

Step 2: Cross out 1 as it is not a prime number or a composite number.

Step 3: Encircle 2. All the numbers divisible by 2 are even numbers.
So cross out all the multiples of 2.

Step 3: Encircle 3; cross out multiples of 3.

Step 4: Encircle 5; cross out multiples of 5.

Step 5: Encircle 7; cross out multiples of 7.

Step 6: Encircle 11; cross out multiples of 11.

Step 7: Continue this process till all the numbers in the list are either encircled or crossed out.

All the encircled numbers are prime numbers. Rest all the crossed out numbers except 1 are composite numbers.

Prime Numbers from 1 to 100 are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89and 97.

This method is called the Sieve of Eratosthenes.

- **2 is the smallest prime number which is even**
- **Every prime number expect 2 is odd**

Therefore, $10000 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$

Co-Prime Numbers

If two numbers do not have a common factor other than 1, then the two numbers are said to be co – primes.
i) Factors of $2 = 1, 2$
Factors of $3 = 1, 3$
We see that the common factor of 2 and 3 is only 1, that is, there is no other common factor other than 1. Therefore, 2 and 3 are co – prime numbers.

Properties of Co-prime Numbers

i) Two prime numbers are co-prime to each other.

Factors of $7 = 1, 7$

Factors of $23 = 1, 23$

The only common factor of 7 and 23 is 1. So, 7 and 23 are co – prime numbers. Two prime numbers are always co – primes.

ii) Co-prime numbers need not to be prime numbers.

Factors of $7 = 1, 7$
Factors of $8 = 1, 2, 4, 8$

The only common factor of 7 and 8 is 1. So, they are co – prime numbers while 8 is not a prime number.

Factors of $9 = 1, 3, 9$

Factors of $10 = 1, 2, 5, 10$

The only common factor of 9 and 10 is 1. So, they are co – prime numbers while 9 and 10 are not prime numbers.

Twin Primes

Twin primes are a pair of primes which differ by 2.

Pairs of twin numbers between 1 and 100 are:
$(3, 5)$; $(5, 7)$; $(11, 13)$; $(17, 19)$; $(29, 31)$;
$(41, 43)$; $(59, 61)$; $(71, 73)$

Prime Triplet
A set of three consecutive prime numbers which differ by 2 is called a prime triplet.

The only prime triplet is $(3, 5, 7)$

Example: Write all the pairs of twin primes between 20 and 100.

The pairs of twin primes between 20 and 100 are $(29, 31)$;
$(41, 43)$; $(59, 61)$; $(71, 73)$

Example: Express each of the following numbers as the sum of twin primes:
i) 84 ii) 120

i) $84 = 41 + 43$
ii) $120 = 59 + 61$

Test of Divisibility

i) Divisibility by 2:

If the number ends with 2, 4, 6, 8 or 0, it is divisible by 2.

18, 122, 294, 4306, 93250, are divisible by 2.

Here 22, 94, 306, 3250 ends with 2, 4, 6 and 0 respectively.
Therefore, they are divisible by 2.

ii) Divisibility by 3:

If the sum of the digits of any number is divisible by 3 then that number is divisible by 3.

Let us consider the number 3246

Sum of the digits $= 3 + 2 + 4 + 6 = 15$, which is divisible by 3.

Therefore, 3246 is divisible by 3.

Consider the number 3654

Sum of the digits $= 3 + 6 + 5 + 4 = 18$, which is divisible by 3.

Therefore, 3654 is divisible by 3.

iii) Divisibility by 4:

If the last 2 digits of any number are divisible by 4, then that number is divisible by 4.

Consider the number 1548

The number formed by the last two digits $= 48$

We know, $48 \div 4 = 12$

Therefore, 1548 is divisible by 4.

Consider the number 24612

The number formed by the last two digits $= 12$

We know, $12 \div 4 = 3$

Therefore, 24612 is divisible by 4.

iii) Divisibility by 5:

If the digit in the ones place of a number is 5 or 0, then it is divisible by 5.

Consider the number 510

The number 510 ends in 0. Hence the number is divisible by 5.

Consider the number 402765

The number 402765 ends in 5. Hence the number is divisible by 5.

iv) Divisibility by 6:

If a number is divisible by 2 and 3, then that number is divisible by 6.

Consider the number 1356

1356 is divisible by 2 as it ends with 6.

Sum of digits $= 1 + 3 + 5 + 6 = 15$, which is divisible by 3.

So, 1356 is divisible by 3.

Therefore, 1356 is divisible by 6

v) Divisibility by 7:

Consider the number 6804.

- Take the last digit of the number 6804 and double it.

  $2 \times 4 = 8$

- Then subtract the result from the rest of the number.

  $680 - 8 = 672$

If the resulting number is evenly divisible by 7, so is the original number.

Now, $672 \div 7 = 96$ .Therefore the original number, 6804 is also divisible by 7.

vi) Divisibility by 8:

A number is divisible by 8 if the number formed by its last three digits is divisible by 8.

Consider the number 73512.

The number formed by the last three digits $= 512$

We know, $512 \div 8 = 64$

Therefore, 73512 is divisible by 8.

vii) Divisibility by 9:

A number is divisible by 9 if the sum of its digits is divisible by 9.

Consider the number 4608.

Sum of the digits $= 4 + 6 + 0 + 8 = 18$

We know, $18 \div 9 = 2$

Therefore, 4608 is divisible by 9.

viii) Divisibility by 10:

A number is divisible by 10 if it ends with a 0 (zero).

Consider the number 1070.

Here, the number ends with 0. Therefore, the number 1070 is divisible by 10.

ix) Divisibility by 11:

A number is divisible by 11 if the difference of the sum of its digits in odd places and the sum of its digits in the even places is either 0 or a multiple of 11.

Consider the number 863423.

Sum of digits in odd places $= 8 + 3 + 2 = 13$

Sum of digits in even places $= 6 + 4 + 3 = 13$

Difference of the two sums $= 13 - 13 = 0$

Therefore, the number 863423 is divisible by 11.

Example: Using divisibility tests, determine which of the following numbers are divisible by 4; by 8:

i) 572 ii) 5500 iii) 21084 iv) 31795072

i) 572

If the last 2 digits of any number are divisible by 4, then that number is divisible by 4.

Here, the number formed by the last two digits = 72

We know, $72 \div 4 = 12$

Therefore, 572 is divisible by 4.

A number is divisible by 8 if the number formed by its last three digits is divisible by 8.

The number formed by the last three digits = 572

572 is not divisible by 8.

ii) 5500

If the last 2 digits of any number are divisible by 4, then that number is divisible by 4.

Here, the number formed by the last two digits is 00, which is divisible by 4.

Therefore, 5500 is divisible by 4.

A number is divisible by 8 if the number formed by its last three digits is divisible by 8.

The number formed by the last three digits = 500, which is not divisible by 8.

Therefore, 5500 is not divisible by 8.

iii) 21084

If the last 2 digits of any number are divisible by 4, then that number is divisible by 4.

Here, the number formed by the last two digits is 84.

Now, $84 \div 4 = 21$

Therefore, 21084 is divisible by 4.

A number is divisible by 8 if the number formed by its last three digits is divisible by 8.

The number formed by the last three digits $= 084$, which is not divisible by 8.

Therefore, 21084 is not divisible by 8.

iv) 31795072

If the last 2 digits of any number are divisible by 4, then that number is divisible by 4.

Here, the number formed by the last two digits is 72,

Now, $72 \div 4 = 18$

Therefore, 31795072 is divisible by 4.

A number is divisible by 8 if the number formed by its last three digits is divisible by 8.

The number formed by the last three digits is072

$072 \div 8 = 9$

Therefore, 31795072 is divisible by 8.

Example: Using divisibility tests, determine which of following numbers are divisible by 6:
i) 1258 ii) 61233 iii) 438750 iv) 125830

i) 1258

If a number is divisible by 2 and 3, then that number is divisible by 6.

Now, 1258 is divisible by 2 as it ends with 8 (an even number).

Sum of digits $= 1 + 2 + 5 + 8 = 16$, which is not divisible by 3.

So, 1258 is not divisible by 3.

Since, 1258 is not divisible by both 2 and 3. Therefore, 1258 is not divisible by 6.

ii) 61233

If a number is divisible by 2 and 3, then that number is divisible by 6.
Now, 61233 is not divisible by 2 as it ends with 3 (odd number).

Sum of digits $= 6 + 1 + 2 + 3 + 3 = 15$, which is divisible by 3.

Since, 61233 is not divisible by both 2 and 3. Therefore, 61233 is not divisible by 6.

iii) 438750

If a number is divisible by 2 and 3, then that number is divisible by 6.

Now, 438750 is divisible by 2 as it ends with 0 (even number).

Sum of digits $= 4 + 3 + 8 + 7 + 5 + 0 = 27$, which is divisible by 3.

So, 43870 is divisible by 3.

Since, 438750 is divisible by both 2 and 3. Therefore, 438750 is divisible by 6.

iv) 125830

If a number is divisible by 2 and 3, then that number is divisible by 6.

Now, 125830 is divisible by 2 as it ends with 0 (even number).

Sum of digits $= 1 + 2 + 5 + 8 + 3 + 0 = 19$, which is not divisible by 3.

So, 125830 is not divisible by 3.

Since, 125830 is not divisible by both 2 and 3. Therefore, 125830 is not divisible by 6.

Example: Using divisibility tests, determine which of the following numbers are divisible by 11:

i) 10824 ii) 70169308 iii) 901153 iv) 5445

i) 10824

A number is divisible by 11 if the difference of the sum of its digits in odd places and the sum of its digits in the even places is either 0 or a multiple of 11.

Sum of digits in odd places = 1 + 8 + 4 = 13

Sum of digits in even places = 0 + 2 = 2

Difference of the two sums = 13 – 2 = 11

Therefore, the number 10824 is divisible by 11.

ii) 70169308

A number is divisible by 11 if the difference of the sum of its digits in odd places and the sum of its digits in the even places is either 0 or a multiple of 11.

Sum of digits in odd places = 7 + 1 + 9 + 0 = 17

Sum of digits in even places = 0 + 6 + 3 + 8 = 17

Difference of the two sums = 17 – 17 = 0

Therefore, the number 70169308 is divisible by 11.

iii) 901153

A number is divisible by 11 if the difference of the sum of its digits in odd places and the sum of its digits in the even places is either 0 or a multiple of 11.

Sum of digits in odd places = 9 + 1 + 5 = 15

Sum of digits in even places = 0 + 1 + 3 = 4

Difference of the two sums = 15 – 4 = 11

Therefore, the number 901153 is divisible by 11.

iv) 5445

A number is divisible by 11 if the difference of the sum of its digits in odd places and the sum of its digits in the even places is either 0 or a multiple of 11.

Sum of digits in odd places = 5 + 4 = 9

Sum of digits in even places = 4 + 5 = 9

Difference of the two sums = 9 – 9 = 0

Therefore, the number 5445 is divisible by 11.

Example: Write the smallest digit and the greatest digit in the blank space of each of the following numbers so that the number formed is divisible by 3:

i) _ 6724 ii) 4765 _ 2

i) _ 6724

If the sum of the digits of any number is divisible by 3 then that number is divisible by 3.

Here, sum of the remaining digits $= 6 + 7 + 2 + 4 = 19$

The smallest digit which can be added to 19 is 0. But 19 $(19 + 0 = 19)$, is not a multiple of 3.

$19 + 1 = 20$, not a multiple of 3.

$19 + 2 = 21$, is a multiple of 3.

Hence, the smallest digit that should be added to 19 is 2.

The greatest digit that could be added is 9.

But, $19 + 9 = 28$, which is not a multiple of 3.

$19 + 8 = 27$, which is a multiple of 3.

The greatest digit that should be added to 19 is 8.

ii) 4765_ 2

If the sum of the digits of any number is divisible by 3 then that number is divisible by 3.

Here, sum of the remaining digits $= 4 + 7 + 6 + 5 + 2 = 24$

Now, $24 + 0 = 24$, which is a multiple of 3.

Hence, the smallest digit that should be added to 24 is 0.

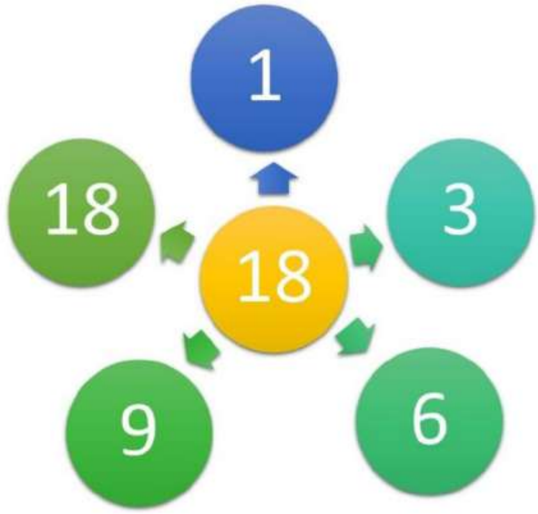
The greatest digit that could be added is 9.

Now, $24 + 9 = 33$, which is a multiple of 3.Therefore, the greatest digit that should be added to 24 is 9.

Some More Divisibility Rules

1) If a number is divisible by another number then it is divisible by each of the factors of that number.

The factors of 18 are 1, 3, 6, 9 and 18.



$36 \div 18 = 2$

So, 36 is divisible by 18.

If 36 is divisible by 18 then it is also divisible by each factor of 18, that is, 1, 3, 6, 9 and 18.

$36 \div 1 = 36, 36 \div 3 = 12, 36 \div 6 = 6, 36 \div 9 = 4, 36 \div 18 = 2$

Therefore, 36 is divisible by each of the factors of 18.

2) If a number is divisible by two co-prime numbers then it is divisible by their product also.

$40 \div 4 = 10$ and $40 \div 5 = 8$

So, 40 is divisible by both 4 and 5 and we know that, 4 and 5 are co – prime numbers.

Therefore, 40 will also be divisible by the product of 4 and 5, i.e.

$(4 \times 5 = 20)$

$40 \div 20 = 2$

Thus, 40 is divisible by product of the co-primes i.e. 4 and 5.

3) If two given numbers are divisible by a number, then their sum is also divisible by that number.

21 and 39 are divisible by 3

$21 \div 3 = 7$

$39 \div 3 = 13$

Sum of the two numbers $= 21 + 39 = 60$

$60 \div 3 = 20$

Therefore, if 21 and 39 are divisible by 3, then their sum, that is, 60 is also divisible by 3.

4) If two given numbers are divisible by a number, then their difference is also divisible by that number.

21 and 39 are divisible by 3

$21 \div 3 = 7$

$39 \div 3 = 13$

Difference of the two numbers $= 39 - 21 = 18$

$18 \div 3 = 6$

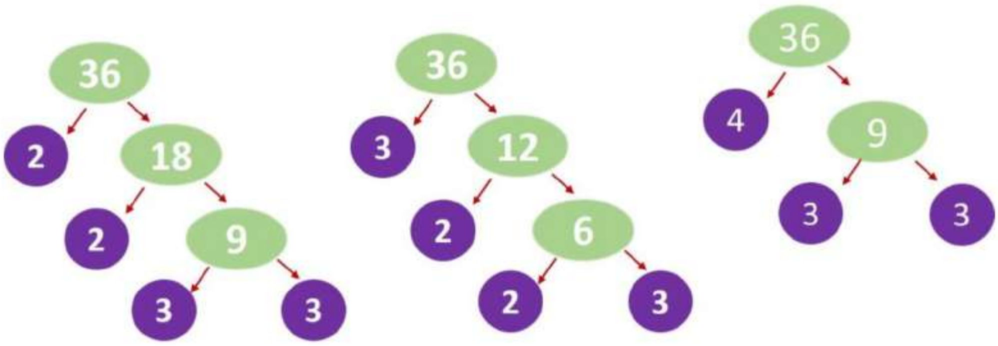
Therefore, if 39 and 21 are divisible by 3, then their difference, that is, 18 is also divisible by 3.

HCF and LCM

When a number is expressed as a product of its factors we say that the number has been factorised.

Now, we will factorise 36 in three different ways by using tree diagram method.

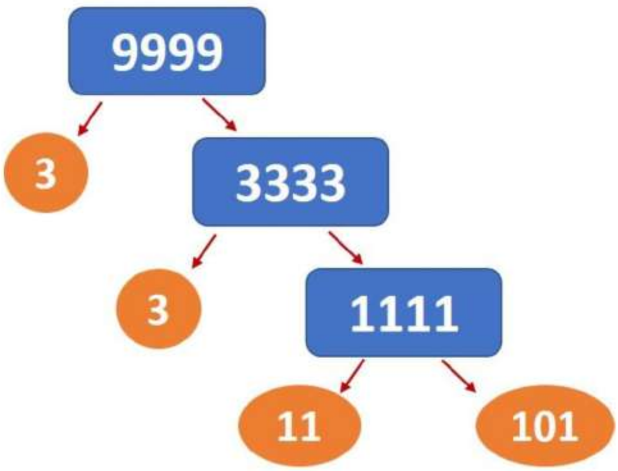
> **A tree diagram is a diagram that is used to break down a number into its factors until all the numbers left are prime.**



In all the above factorisations of 36, we ultimately arrive at only one factorisation $2 \times 2 \times 3 \times 3$.

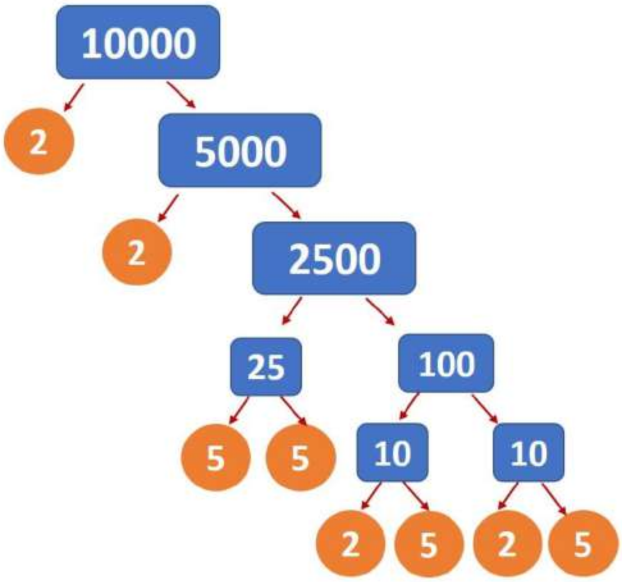
Example: Write the greatest 4-digit number and express it in terms of its prime factors.

The greatest 4 – digit number is 9999.



Therefore, $9999 = 3 \times 3 \times 11 \times 101$

Example: Write the smallest 5-digit number and express it in the form of its prime factors.
The smallest 5 – digit number is 10000.



Therefore, $10000 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$

Highest Common Factor (HCF)
The Highest Common Factor (HCF) of two or more given numbers is the highest (or greatest) of their common factors. It is also known as the Greatest Common Divisor (GCD).
To find the HCF of two or more numbers, we can use any of the following method.

- Common factor method
- Prime factorization method

HCF by Common Factor Method Let's find the HCF of 18, 24 and 42.

First, find all the factors of the given numbers individually.

Factors of 18 = 1, 2, 3, 6, 9, 18

Factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24

Factors of 42 = 1, 2, 3, 6, 7, 14, 21, 42

We see that the common factors of 18, 24 and 42 are 1, 2, 3 and 6.

Since 6 is the highest of these common factors.

Therefore, HCF of 18, 24 and 42 is 6.

HCF by Prime Factorization Method

Now we will find the HCF of 27 and 45 by Prime Factorization Method.

Prime factors of $27 = 3 \times 3 \times 3$
Prime factors of $45 = 3 \times 3 \times 5$
The common prime factors of the given numbers is 3 (occurring twice)

HCF of 27 and $45 = 3 \text{ x } 3 = 9$

Example: Find the HCF of the following numbers by Common Factor Method
i) 18, 48 ii) 18, 60 iii) 36, 84 iv) 70, 105, 175

i) 18, 48

Factors of 18 = 1, 2, 3, 6, 9, 18

Factors of 48 = 1, 2, 3, 4, 6, 8, 12, 24, 48

We see that the common factors of 18 and 48 are 1, 2, 3 and 6.

Since, 6isthe highest of these common factors. Therefore, HCF of 18 and 48 is 6.

ii) 18, 60

Factors of 18 = 1, 2, 3, 6, 9, 18

Factors of 60 = 1, 2, 3, 4, 5, 6, 12, 15, 20, 30, 60

We see that the common factors of 18 and 48 are 1, 2, 3 and 6.

Since, 6 is the highest of these common factors. Therefore, HCF of 18 and 48 is 6.

iii) 36, 84

Factors of 36 = 1, 2, 3, 4, 6, 9, 12, 18, 36

Factors of 60 = 1, 2, 3, 4, 5, 6, 12, 15, 20, 30, 60

We see that the common factors of 36 and 84 are 1, 2, 3, 4, 6 and 12.

Since 12 is the highest of these common factors. Therefore, HCF of 36 and 84 is 12.

iv) 70, 105 and 175

Factors of 70 = 1, 2, 5, 7, 10, 14, 35, 70

Factors of 105 = 1, 3, 5, 7, 15, 21, 35, 105

We see that the common factors of 36 and 84 are 1, 5, 7 and 35.

Since 35 is the highest of these common factors. Therefore, HCF of 70, 105 and 175 is 35.

Example: Find the HCF of the following numbers by Prime Factorization Method
i) 30, 42 ii) 27, 63 iii) 34, 102 iv) 91, 112, 49

i) 30, 42

Prime factors of 30 = $2 \times 3 \times 5$

Prime factors of 42 = $2 \times 3 \times 7$

The common prime factors of the given numbers are 2 and 3

HCF of 30 and 42 = 2 x 3 = 6

ii) 27, 63

Prime factors of 27 = $3 \times 3 \times 3$

Prime factors of 63 = $3 \times 3 \times 7$

The common prime factors of the given numbers is 3 (occurring twice)

HCF of 27 and 63 = 3 x 3 = 9

iii) 34 and 102

Prime factors of 34 = 2×17

Prime factors of 102 = $2 \times 3 \times 17$

The common prime factors of the given numbers are 2 and 17.
Therefore, HCF of 34 and 102 = 2 x 17 = 34

iv) 91, 112, 49

Prime factors of $91 = 7 \times 13$

Prime factors of $112 = 2 \times 2 \times 2 \times 2 \times 7$

Prime factors of $49 = 7 \times 7$

The common prime factors of the given numbers is 7. Therefore, HCF of 91, 112 and $49 = 7$

Lowest Common Multiple
The Lowest Common Multiple (LCM) of two or more given numbers is the lowest (or smallest or least) of their common multiples.

If two numbers are co-prime then the LCM is the product of the two numbers.

Consider the numbers 7 and 13

$7 = 7 \times 1$ and $13 = 1 \times 13$

So, LCM is $7 \times 13 = 91$

LCM by Common Multiple Method

The lowest common multiple of two or more numbers is the smallest number which is a multiple of each of the numbers.

Consider the numbers 8, 12 and 18.

Multiples of 8 are 8, 16, 24, 32, 40, 48, 56, 64, 72, 80.......

Multiples of 12 are 12, 24, 36, 48, 60, 72, 84, 96, 108, 120.......

Multiples of 18 are 18, 36, 54, 72, 90, 108.....

The lowest common multiple of 8, 12 and 18 is 72

L.C.M. of 8, 12 and 18 is 72.

LCM by Prime Factorization Method

To find the LCM by prime factorisation method we write the prime factorisations of the given numbers. Then the required LCM of these numbers is the product of all different prime factors of the numbers, which occur maximum number of times.

Consider the numbers 40, 48 and 45.

Prime factors of $40 = 2 \times 2 \times 2 \times 5$

Prime factors of $48 = 2 \times 2 \times 2 \times 2 \times 3$

Prime factors of $45 = 5 \times 3 \times 3$

The prime factor 2 appears maximum number of four times in the prime factorisation of 48, the prime factor 3 occurs maximum number of two times in the prime factorisation of 45.

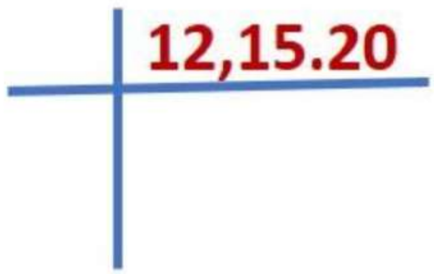
The prime factor 5 appears one time in the prime factorisations of 40 and 45, we take it only once.

Therefore, required $\text{LCM} = (2 \times 2 \times 2 \times 2) \times (3 \times 3) \times 5 = 720$

LCM by Division Method

Consider the numbers 12, 15 and 20.

Step 1: We write the given numbers in a row:



Step 2: Divide by the least prime number which divides any one of the given numbers and carry forward the numbers which are not divisible. In this case the least prime number is 2.

2	12, 15, 20
	6, 15, 10

Step 3: Again divide by 2. Continue this till we have no multiples of 2.

2	12,15.20
2	6, 15, 10
	3, 15, 5

Step 4: Divide by next prime number which is 3.

2	12, 15, 20
2	6, 15, 10
3	3, 15, 5
	1, 5, 5

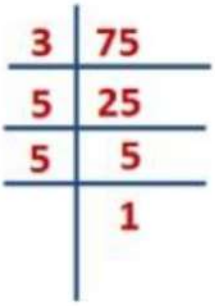
Step 5: Divide by next prime number which is 5.

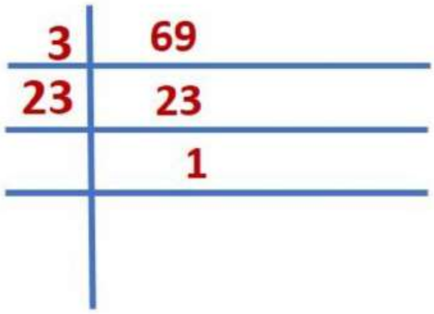
2	12, 15, 20
2	6, 15, 10
3	3, 15, 5
5	1, 5, 5
	1, 1, 1

So, $\text{LCM} = 2 \times 2 \times 3 \times 5 = 60$

Example: Rupa purchases two bags of fertilizer of weights 75 kg and 69 kg. Find the maximum value of weight which can measure the weight of the fertilizer exact number of times.

To find the maximum weight we have to find the HCF of 75 and 69.





Hence,

$75 = 3 \times 5 \times 5$

$69 = 3 \times 23$

So, HCF of 75 and 69 is 3.

Therefore, the required weight = 3 kg

Example: The traffic lights at three different road crossings change after every 48 seconds, 72seconds and 108 seconds respectively.

If they change simultaneously at 10 a.m., at what time will they change simultaneously again?

We have to find the LCM of 48, 72 and 108.

2	48, 72, 108
2	24, 36, 54
2	12, 18, 27
2	6, 9, 27
3	3, 9, 27
3	1, 3, 9
3	1, 1, 3
	1, 1, 1

LCM of 48, 72 and 108 $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 432$

432 seconds = 7 minutes 12 seconds

$$
\begin{array}{r|l}
 & 7 \\
60 & 432 \\
 & -420 \\
\hline
 & 12
\end{array}
$$

Therefore, the traffic lights change simultaneously after 7 minutes 12 seconds

The required time = 10 a.m. + 7 minutes 12 seconds or 7 minutes 12 seconds past 10 a.m.

Relationship between HCF and LCM

Let the two numbers be and b, then the product of these numbers is equal to the product of their HCF and LCM.

$a \times b =$ HCF x LCM

Example: The HCF of two numbers is 15 and their product is 2895.

Find their LCM.

Let the two numbers be a and b

Now, $a \times b = 2895$

$HCF = 15$

We know, $a \times b = HCF \times LCM$

$2895 = 15 \times LCM$

$$LCM = \frac{2895}{15} = 193$$