Characteristics of Instruments and Measurement Systems

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Measurements

Measurement is a process by which one can convert physical parameters to meaningful numbers. The measuring process is one in which the property of an object or system under consideration is compared to an accepted standard unit, a standard defined for that particular property.

Static Characteristics

1. Accuracy

It is the closeness with which an instrument reading approaches the true value of the quantity being measured.

2. Precision

It is a measure of the reproducibility of the measurements. It is a measure of degree of agreement within a group of measurements.

Remember:

- Precision is not the guarantee of accuracy.
- An instrument with more significant figure has more precision.

3. Sensitivity

It is the ratio of the magnitude of output signal to the magnitude of input signal applied to the instrument.

$$\text{Sensitivity} = \frac{\text{Output}}{\text{Input}}$$

Note:

- An instrument requires high degree of sensitivity.
- $\text{Sensitivity} \propto \dfrac{1}{\text{Deflection factor}}$

4. Resolution

The smallest change in input which can be detected with certainity by an instrument is its resolution.

5. Linearity

The output is linearly proportional to the input. For a linear instrument the sensitivity is constant for the entire range of instrument. Linearity is the most important parameter compared to all other parameters.

Remember:

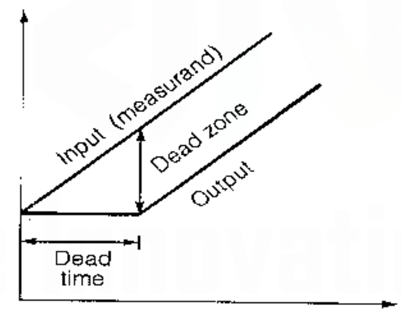
- Linearity is more important than the sensitivity.
- Accuracy is more important than resolution.

6. Dead Zone

It is the largest change of input quantity for which there is no output of the instrument.

7. Dead time

Time required by an instrument to begin to respond to the change in a measurand.



8. Range and Span

The difference between the maximum and minimum values of the scale is called range. The maximum value of the scale is called span.

Errors

$$\text{Error} = \text{Measured value} - \text{True value}$$

$$\text{Error} = -\text{Accuracy}$$

- Static Error

$$\delta A = A_m - A_t$$

where, A_m = Measured value of quantity or Actual value
A_t = True value of quantity or Nominal value

- Relative static error

$$\epsilon_r = \frac{\delta A}{A_t}$$

- Static correction

$$\delta C = A_t - A_m = -\delta A$$

- Static sensitivity

$$\text{Static sensitivity} = \frac{\Delta q_0}{\Delta q_i}$$

where, Δq_0 = Infinitesimal change in output
Δq_i = Infinitesimal change in input

- Non-linearity (N.L.)

$$\text{N.L.} = \frac{(\text{Max. deviation of output from the idealized straight line})}{\text{Full scale deflection}} \times 100$$

$$\text{Error at desired value} = \frac{\text{Full scale value} \times \text{Error at full scale}}{\text{Desired value}}$$

Combination of Quantities with Limiting Errors

Sum or Difference of Two or More than Two Quantities

Let $X = \pm x_1 \pm x_2 \pm x_3 \pm x_4$

$$\frac{\delta X}{X} = \pm\left(\frac{x_1}{X}\frac{\delta x_1}{x_1} + \frac{x_2}{X}\frac{\delta x_2}{x_2} + \frac{x_3}{X}\frac{\delta x_3}{x_3} + \frac{x_4}{X}\frac{\delta x_4}{x_4}\right)$$

where $\pm\delta x_1$ = Relative increment in quantity x_1
$\pm\delta x_2$ = Relative increment in quantity x_2
$\pm\delta X$ = Relative increment in X
$\frac{\delta x_1}{x_1}$ = Relative limiting error in quantity x_1
$\frac{\delta x_2}{x_2}$ = Relative limiting error in quantity x_2
$\frac{\delta X}{X}$ = Relative limiting error in X

Product or Quotient of Two or More than Two Quantities

Let $X = x_1 x_2 x_3$ or $X = \frac{x_1}{x_2 x_3}$ or $X = \frac{1}{x_1 x_2 x_3}$

$$\frac{\delta X}{X} = \pm\left(\frac{\delta x_1}{x_1} + \frac{\delta x_2}{x_2} + \frac{\delta x_3}{x_3}\right)$$

Composite Factors

Let $X = x_1^n \cdot x_2^m$

$$\frac{\delta X}{X} = \pm\left(n\frac{\delta x_1}{x_1} + m\frac{\delta x_2}{x_2}\right)$$

- Arithmetic Mean

$$\bar{X} = \frac{\Sigma x}{n} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

where, $x_1, x_2, \dots\dots, x_n$ = Readings or samples
n = Number of readings

Deviation

$$d_n = x_n - \bar{X}$$

Note:

Algebraic sum of deviation is zero.

- Average deviation

$$\bar{D} = \frac{\Sigma |d|}{n} = \frac{|d_1| + |d_2| + \dots |d_n|}{n}$$

- Standard deviation

For n > 20

$$S.D. = \sigma = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\frac{d_1^2 + d_2^2 + \cdots + d_n^2}{n}}$$

For n < 20

$$S.D. = s = \sqrt{\frac{\Sigma d^2}{n-1}}$$

- Variance

For n > 20

$$V = \sigma^2 = \frac{\Sigma d^2}{n}$$

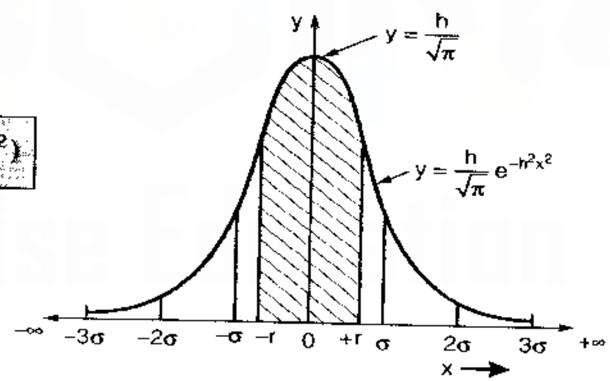
For n < 20

$$V = s^2 = \frac{\Sigma d^2}{n-1}$$

Normal or Gausian Curve of Errors

1. For Infinite Numbers of Reading

$$y = \frac{1}{\sigma\sqrt{2\pi}} \exp(-x^2/2\sigma^2)$$



where, x = magnitude of deviation from mean,
y = number of readings at any deviation x, (the probability of occurance of deviation x)
σ = standard deviation

- Precision Index

$$h = \frac{1}{\sigma\sqrt{2}}$$

- Probable error (P.E.)

$$r = \frac{0.4769}{h}$$

- Average deviation

$$\bar{D} = \frac{r}{0.8453} = \frac{1}{\pi h^2}$$

- Standard deviation

$$\sigma = \frac{r}{0.6745} = \frac{1}{h\sqrt{2}}$$

$$P.E. = r = 0.8453\,\bar{D} = 0.6745\,\sigma$$

2. For Finite Numbers of Reading

- For n > 20

$$P.E. = r = 0.6745\sqrt{\frac{\sum |d|^2}{n}}$$

- For n < 20

$$P.E. = r = 0.6745\sqrt{\frac{\sum |d|^2}{n-1}}$$

- Standard deviation of mean

$$\sigma_m = \frac{\sigma}{\sqrt{n}}$$

- Standard deviation of standard deviation

$$\sigma_\sigma = \frac{\sigma_m}{\sqrt{2}}$$

❑ Variance of combination of components

Let $X = f(x_1, x_2, \cdots, n)$

$$V_x = \left(\frac{\partial X}{\partial x_1}\right)^2 V_{x_1} + \left(\frac{\partial X}{\partial x_2}\right)^2 V_{x_2} + \cdots + \left(\frac{\partial X}{\partial x_n}\right)^2 V_{x_n}$$

where, $V_{x_1}, V_{x_2}, \cdots, V_{x_n}$ = Variance of $x_1, x_2, \cdots x_n$

❑ Standard Deviation of Combination of Components

Let $X = f(x_1, x_2, , \ldots\ldots, x_n)$

$$\sigma_x = \sqrt{\left(\frac{\partial X}{\partial x_1}\right)^2 \sigma_{x_1}^2 + \left(\frac{\partial X}{\partial x_2}\right)^2 \sigma_{x_2}^2 + \cdots + \left(\frac{\partial X}{\partial x_n}\right)^2 \sigma_{x_n}^2}$$

where, $\sigma_{x_1}, \sigma_{x_2}, \cdots, \sigma_{x_n}$ = Standard deviation of $x_1, x_2, \cdots, x_n$

❑ Probable Error of Combination of Components

Let $X = f(x_1, x_2, \ldots\ldots, x_n)$

$$r_x = \sqrt{\left(\frac{\partial X}{\partial x_1}\right)^2 r_{x_1}^2 + \left(\frac{\partial X}{\partial x_2}\right)^2 r_{x_2}^2 + \cdots + \left(\frac{\partial X}{\partial x_n}\right)^2 r_{x_n}^2}$$

where, $r_{x_1}, r_{x_2}, \cdots, r_{x_n}$ = Probable error of $x_1, x_2, \cdots, x_n$

❑ Uncertainty of Combination of Components

Let $X = f(x_1, x_2, \ldots\ldots, x_n)$

$$w_x = \sqrt{\left(\frac{\partial X}{\partial x_1}\right)^2 w_{x_1}^2 + \left(\frac{\partial X}{\partial x_2}\right)^2 w_{x_2}^2 + \cdots + \left(\frac{\partial X}{\partial x_n}\right)^2 w_{x_n}^2}$$

where, $w_{x_1}, w_{x_2}, \cdots, w_{x_n}$ = Uncertainties of $x_1, x_2, \cdots, x_n$

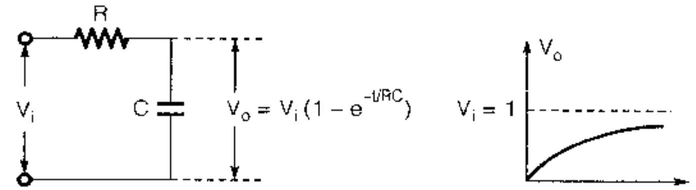
Order of Instrument

1. Zero Order System

As input changes, output also changes immediately called zero order system. Example : Resistor.

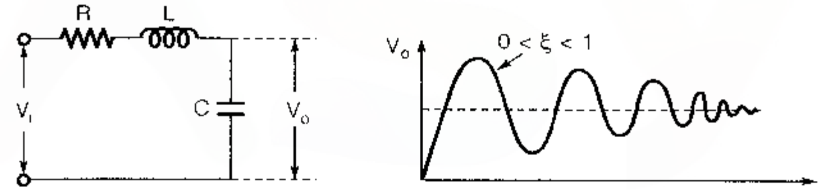
2. First Order System

As input changes, output also changes but not immediately, it takes some delay but without oscillation. Example : heater.



3. Second Order System

As input changes, output also changes, with some delay and oscillation.



Remember:

- The analog instruments are of second order instrument which has damping factor (ξ) between 0.6 to 0.8. It is an underdamped system.

Standards

Quantity	Unit	Definition
Length	Metre	The length of path travelled by light in an interval of $\frac{1}{299792458}$ sec.
Time	second	9.192631770×10^9 cycles of radiation from vapourised cesium-133 atom.
Temp.	Kelvin	The temperature difference between the absolute and the triple point of water is defined as 273.16°K.
Voltage	Volt	Standard cell voltage of weston cell i.e. 1.0186 V.
Current	Ampere	One ampere is the current flowing through two infinite long parallel conductor of negligible cross section placed 1 meter apart produced a force of 2×10^{-7} N/m.

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