ICSE 2025 EXAMINATION

Sample Question Paper - 12

Mathematics

Time: 2 ½ Hours. **Total Marks: 80**

General Instructions:

1. *Answers to this Paper must be written on the paper provided separately.*
2. *You will not be allowed to write during the first 15 minutes. This time is to be spent in reading the question paper.*
3. *The time given at the head of this Paper is the time allowed for writing the answers.*
4. *Attempt **all** questions from **Section A** and **any four** questions from **Section B**.*
5. *The intended marks for questions or parts of questions are given in brackets [ ]*

Section A

(Attempt all questions from this section.)

Question 1

Choose the correct answers to the questions from the given options. [15]

i) If $\begin{bmatrix} 4 & y \end{bmatrix}\begin{bmatrix} 1 \\ y \end{bmatrix} = \begin{bmatrix} 8 \end{bmatrix}$ then find y.

(a) ± 3
(b) -3
(c) ± 2
(d) -2

ii) The value of q such that $x = 2$ is a solution of $8x^2 + qx - 4 = 0$ is _____.

(a) -14
(b) 14
(c) 28
(d) -28

iii) Mr. Pankaj took health insurance policy for his family and paid Rs. 900 as SGST. Find the total annual premium paid by him for this policy, rate of GST being 18%.

(a) Rs. 5,000
(b) Rs. 10,000
(c) Rs. 15,000
(d) Rs. 20,000

iv) Solve for x: $\frac{4x-5}{5x-4}=\frac{x+2}{3x+4}$

(a) $x=-\frac{12}{7}$ or $x=-1$

(b) $x=\frac{12}{7}$ or $x=-1$

(c) $x=\frac{12}{7}$ or $x=1$

(d) $x=-\frac{12}{7}$ or $x=1$

v) In an A.P., the two consecutive terms are (2n + 3) and (2n + 5). Find the common difference of the A.P.

(a) 2n + 2
(b) 2
(c) 1
(d) 4

vi) If x, y, z are in continued proportion, then, $\frac{(x+y)^2}{(y+z)^2}=$

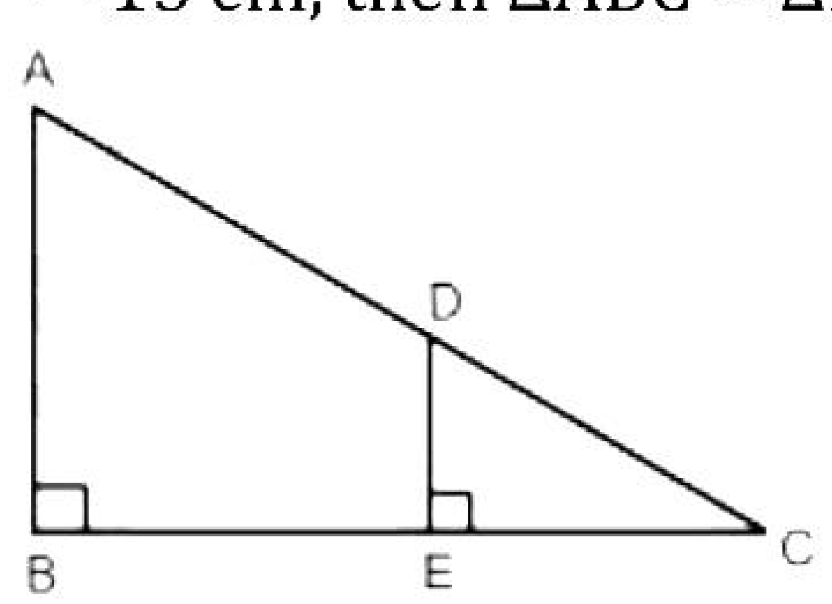
(a) $\frac{z}{x}$

(b) $\frac{x^2}{z^2}$

(c) $\frac{z^2}{x^2}$

(d) $\frac{x}{z}$

vii) In the given figure, AB and DE are perpendiculars to BC. If AB = 5 cm, DE = 4 cm and AC = 13 cm, then $\Delta ABC \sim \Delta DEC$ by______



(a) SSS test
(b) AA test
(c) SAS test
(d) RHS test

viii) The volume of the vessel, in the form of a right circular cylinder, is and its height is 7 cm. Find the radius of its base.

(a) 1 cm
(b) 2 cm
(c) 4 cm
(d) 8 cm

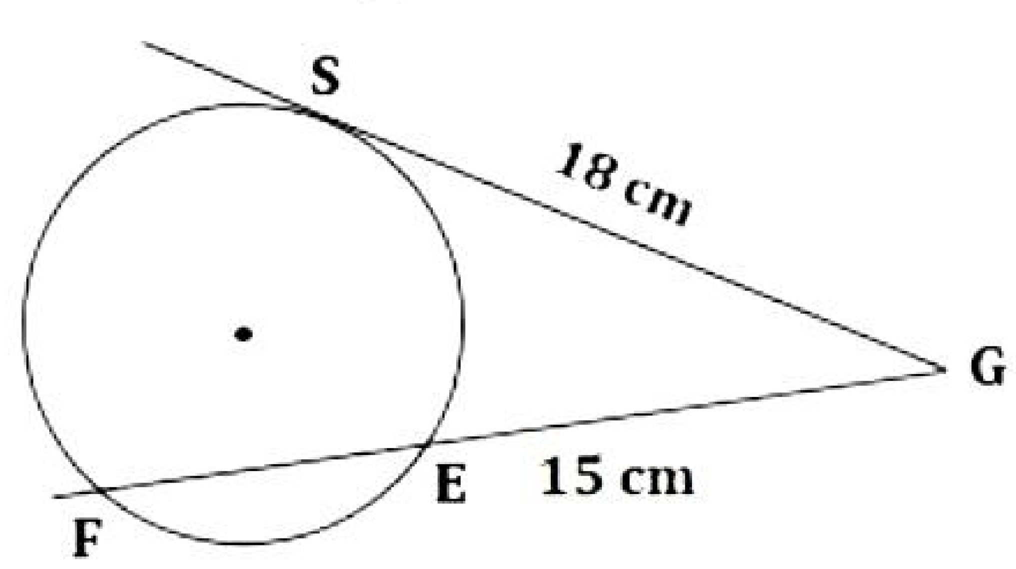
ix) A bag contains six identical black balls. A child withdraws one ball from the bag without looking into it. What is the probability that he takes out a white ball?

(a) 0
(b) 1
(c) $\frac{1}{2}$
(d) $\frac{2}{3}$

x) Less than ogive curve is ____

(a) an increasing curve
(b) a decreasing curve
(c) parallel to X-axis
(d) parallel to Y-axis

xi) Find the length of EF.



(a) 6.6 cm
(b) 6.5 cm
(c) 6.4 cm
(d) 6.2 cm

xii) **Statement 1:** A matrix which has an equal number of rows and columns is called a rectangular matrix.

Statement 2: If each element of a matrix is zero, it is called a zero matrix.

Which of the following is valid?

(a) Both the statements are true.
(b) Both the statements are false.
(c) Statement 1 is true, and Statement 2 is false.
(d) Statement 1 is false, and Statement 2 is true.

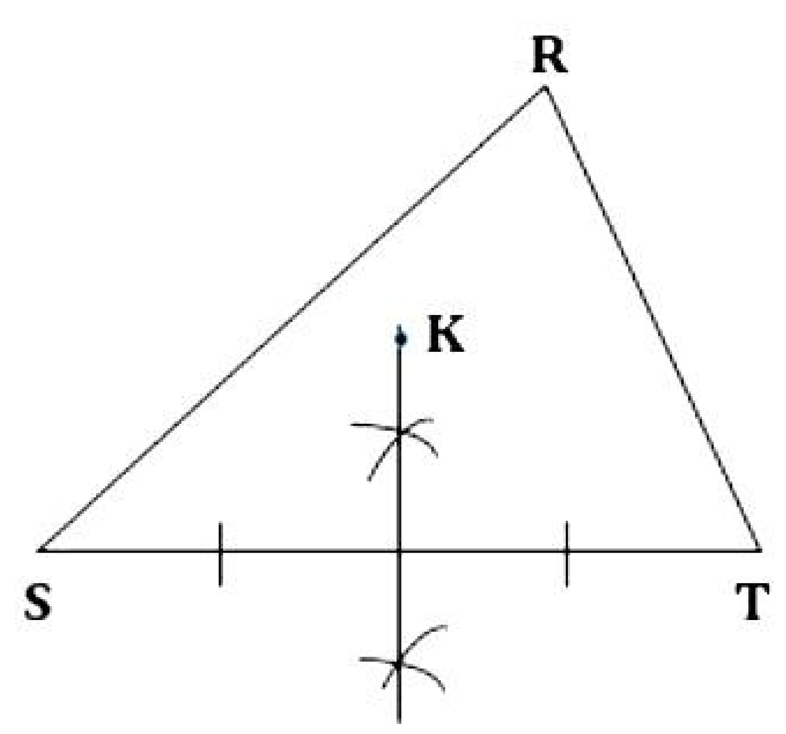
xiii) The money required to buy 220, Rs. 30 shares at a discount of Rs. 10 is

(a) Rs. 4000
(b) Rs. 4400
(c) Rs. 4800
(d) Rs. 5000

xiv) The ratio in which the join of (–4, 7) and (3, 0) is divided by the y-axis is

(a) 3 : 4
(b) 4 : 3
(c) 1 : 4
(d) 1 : 3

xv) **Assertion (A):** Point K is equidistant from R and T.



Reason (R): The locus of points equidistant from two given points is the perpendicular bisector of the line joining the two points.

(a) A is true, R is false
(b) A is false, R is true
(c) Both A and R are true, and R is the correct reason for A.
(d) Both A and R are true, and R is the incorrect reason for A.

Question 2

i) A solid sphere of radius 6 cm is melted and then recast into small spherical balls each of diameter 0.4 cm. Find the number of balls thus obtained. [4]

ii) Tanushree opens a recurring deposit account of Rs. 400 per month in a bank paying 6% p.a. How many instalments does she have to pay to get a maturity amount of Rs. 15732? [4]

iii) Prove that $\frac{\cos A}{1+\sin A}+\tan A=\sec A$. [4]

Question 3

i) If a, b, c and d are in proportion, prove that $\frac{ma^2+nb^2}{mc^2+nd^2}=\frac{ma^2-nb^2}{mc^2-nd^2}$ [4]

ii) In the given figure, O is the centre of the circle, $\angle BAD = 60^\circ$ and chord BC = chord CD. [4]

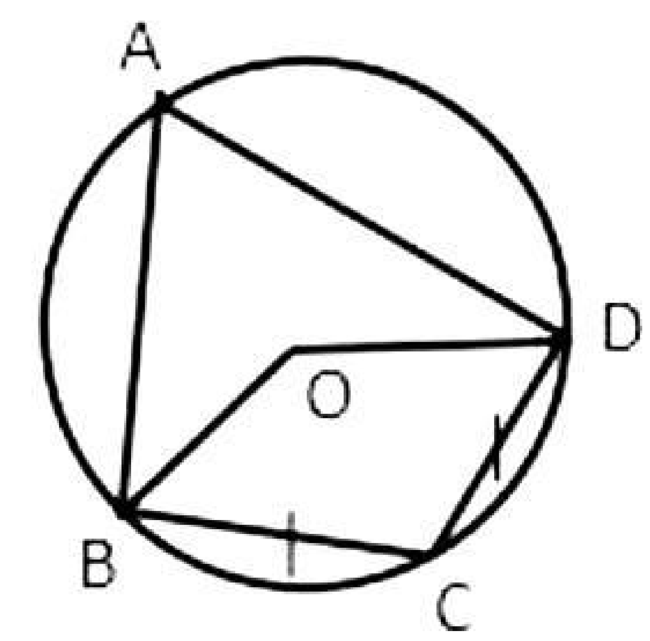
Find

a. $\angle CBD$

b. $\angle BOD$

c. $\angle OBC$

d. $\angle BOC$



iii) Draw a cumulative frequency curve for the following distribution: [5]

Class interval	10–20	20–30	30–40	40–50	50–60
Frequency	8	13	5	9	10

Section B

(Attempt any four questions from this Section.)

Question 4

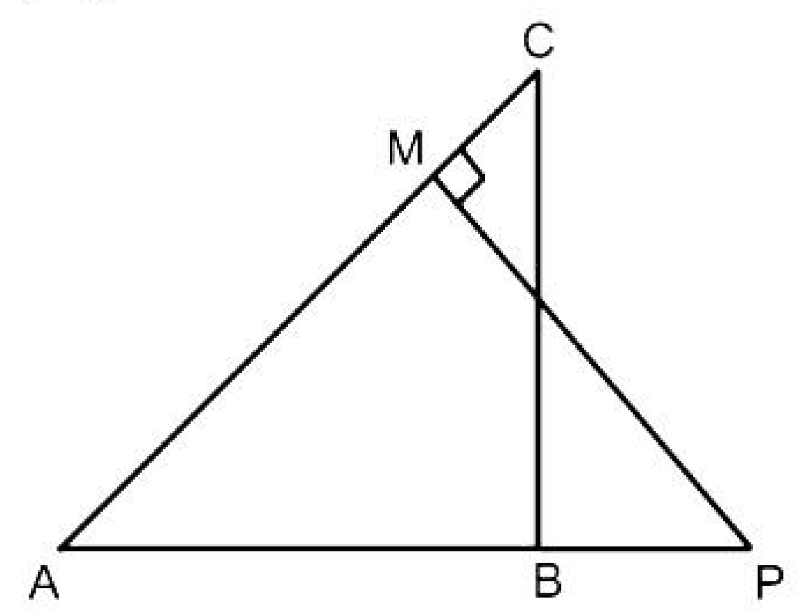
i) Find the matrix M, such that $-A + 3B + M = 0$, where $A = \begin{bmatrix} -2 & 6 \\ 5 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$ [3]

ii) Given that 2 is a root of the equation $3x^2 - p(x + 1) = 0$ and that the equation $px^2 - qx + 9 = 0$ has equal roots, find the values of p and q. [3]

iii) In the figure, ΔABC and ΔAMP are right-angled at B and M, respectively. AC = 10 cm, AP = 15 cm and PM = 12 cm. [4]

(a) Prove that $\Delta ABC \sim \Delta AMP$.

(b) Find AB and BC.



Question 5

i) 100 surnames were randomly picked up from a local telephone directory, and the distribution of the number of letters of the English alphabet in the surnames was obtained as follows: [3]

No. of letters	No. of surnames
1–4	6
4–7	30
7–10	40
10–13	16
13–16	4
16–19	4

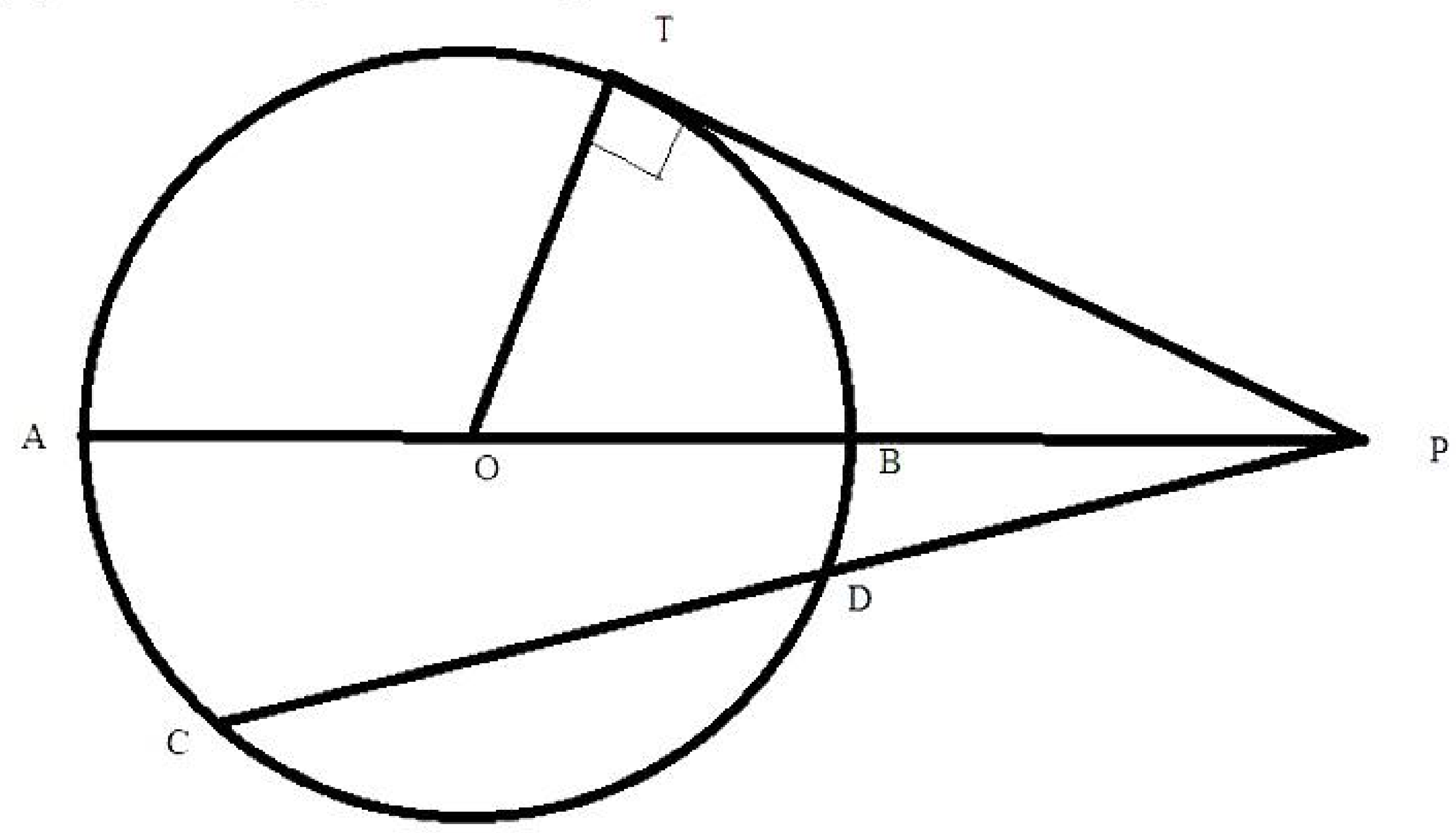
Determine the mean of the above data.

ii) A manufacturer in Banaras manufactures a machine and marks it at Rs. 65,000. He sells the machine to a wholesaler (in Gujarat) at a discount of 30%. The wholesaler sells the machine to a dealer (in Punjab) at a discount of 20% on the marked price. If the rate of GST is 26%, find the tax paid by the wholesaler to the central-government. [3]

iii) In the figure given below, diameter AB and chord CD of a circle meet at P. PT is a tangent to the circle at T. CD = 7.8 cm, PD = 5 cm, PB = 4 cm. Find [3]

(i) AB.

(ii) the length of tangent PT.



Question 6

i) Find the sum of first 8 terms of the G.P.: $\sqrt{2}, 2, 2\sqrt{2}, ...$ [3]

ii) Draw a histogram for the following distribution: [3]

Class Interval	29.5–39.5	39.5–49.5	49.5–59.5	59.5–69.5	69.5–79.5
Frequency	24	16	09	15	20

iii) $\left(\frac{3}{4}\right)^{th}$ part of a conical vessel of internal radius 5 cm and height 24 cm is full of water. The water is emptied into a cylindrical vessel with internal radius 10 cm. Find the height of water in the cylindrical vessel. [4]

Question 7

i) (a) Find the equation of the altitudes AD of the triangle ABC whose vertices are A(7, −1), B(−2, 8) and C(1, 2).

(b) Find the equation of the line perpendicular to $5x - 2y = 8$ and which passes through the midpoint of the line segment joining (2, 3) and (4, 5). [5]

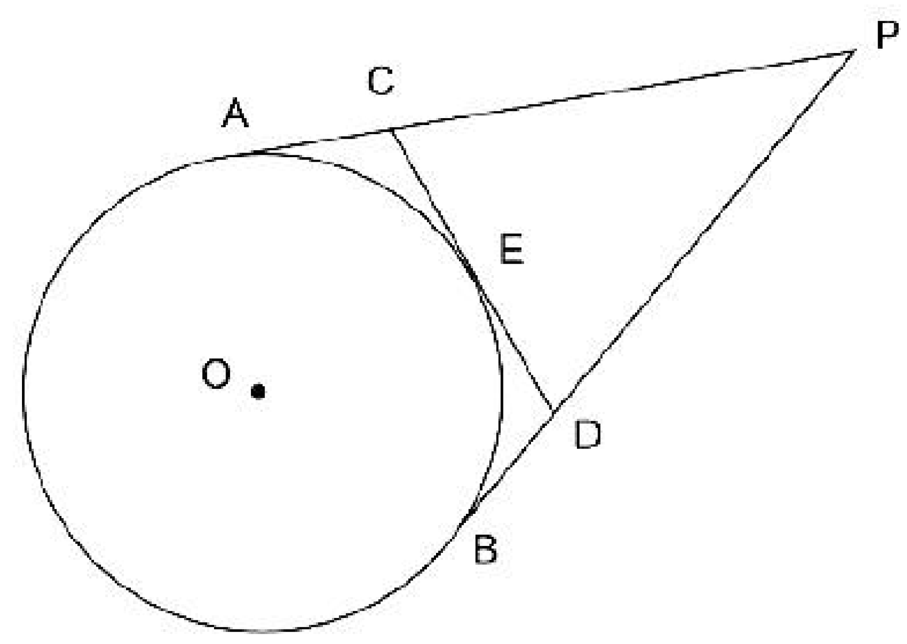
ii) An aeroplane at an altitude of 1500 metres, finds that two ships are sailing towards it in the same direction. The angles of depression as observed from the aeroplane are 45^o and 30^o respectively. Find the distance between the two ships. ($\sqrt{3} = 1.732$) [5]

Question 8

i) Solve the following inequation, write the solution set and represent it on the number line. [3]

$$-3(x-7) \geq 15-7x > \frac{x+1}{3}, x \in R.$$

ii) From an external point P, tangents PA and PB are drawn to a circle with centre O. If CD is the tangent to the circle at a point E and PA = 14 cm, find the perimeter of ΔPCD. [3]



iii) Points A, B, C and D divide the line segment joining the point (5, −10) and the origin in five equal parts. Find the co-ordinates of A, B, C and D. [4]

Question 9

i) Find the mean proportional between [3]

a) 17.5 and 0.007

b) $6 + 3\sqrt{3}$ and $8 - 4\sqrt{3}$

c) $a - b$ and $a^3 - a^2b$

ii) A bus covers a distance of 240 km at a uniform speed. Due to heavy rain its speed gets reduced by 10 km/hr and as such it takes two hours longer to cover the total distance. Assuming the uniform speed to be 'x' km/hr, form an equation and solve it to evaluate 'x'. [3]

iii) Construct a triangle ABC with AB = 5.5 cm, AC = 6 cm and $\angle BAC = 105°$. [4]

Hence:

a) Construct the locus of points equidistant from BA and BC.

b) Construct the locus of points equidistant from B and C.

c) Mark the point which satisfies the above two loci as P. Measure and write the length of PC.

Question 10

i) If (x – 2) is a factor of the expression $2x^3 + ax^2 + bx - 14$ and when the expression is divided by (x – 3), it leaves a remainder 52, find the values of a and b. [3]

ii) From a pack of 52 playing cards, the Jack, Queen and King of clubs are removed, and the pack is well shuffled. From the remaining cards, a card is drawn. Find the probability of getting [3]

a) a club
b) a red face card
c) a black face card

iii) The points A(3, 2), B(0, 4) and C(−4, −3) are the vertices of a triangle. [4]

a) Plot the points on a graph paper.
b) Draw the triangle formed by the reflection of these points at the x-axis.
c) Are the two triangles congruent?

Solution

Section A

Solution 1

i)

Correct Option: (c)

Explanation:

$$\begin{bmatrix} 4 & y \end{bmatrix}\begin{bmatrix} 1 \\ y \end{bmatrix} = [8]$$

$$\Rightarrow \left[4 + y^2\right] = [8]$$

$$\Rightarrow 4 + y^2 = 8$$

$$\Rightarrow y^2 = 4$$

$$\Rightarrow y = \pm 2$$

ii)

Correct Option: (a)

Explanation:

If $x = 2$ is a solution of $8x^2 + qx - 4 = 0$ then $x = 2$ must satisfy the equation

$8x^2 + qx - 4 = 0$

$\Rightarrow 8(2)^2 + q(2) - 4 = 0$

$\Rightarrow 32 + 2q - 4 = 0$

$\Rightarrow 28 + 2q = 0$

$\Rightarrow 2q = -28$

$\Rightarrow q = -14$

iii)

Correct Option: (b)

Explanation:

Let the total annual premium paid by Mr. Pankaj be Rs. x.

According to the question,

18% of x = SGST + CGST

$\Rightarrow$ 18% of x = 1800 ..... ($\because$ SGST = CGST)

$$\Rightarrow \frac{18}{100} \times x = 1800$$

$\Rightarrow$ x = Rs. 10,000

iv)

Correct option: (b)

Explanation:

$$\frac{4x-5}{5x-4}=\frac{x+2}{3x+4}$$

$$(4x-5)(3x+4)=(x+2)(5x-4)$$

$$12x^2+16x-15x-20=5x^2-4x+10x-8$$

$$7x^2-5x-12=0$$

$$7x^2-12x+7x-12=0$$

$$x(7x-12)+(7x-12)=0$$

$$(7x-12)(x+1)=0$$

$$x=\frac{12}{7}\text{ or }x=-1$$

v)

Correct option: (b)

Explanation:

Common difference is found out by taking the difference of the consecutive terms.

So, common difference = (2n + 5) – (2n + 3) = 2

vi)

Correct Option: (d)

Explanation:

x, y, z are in continued proportion,

$$\therefore\ \frac{x}{y}=\frac{y}{z}\Rightarrow y^2=zx \qquad \text{....(1)}$$

$$\frac{x+y}{y}=\frac{y+z}{z} \qquad \text{(By componendo)}$$

$$\Rightarrow\frac{x+y}{y+z}=\frac{y}{z} \qquad \text{(By alternendo)}$$

$$\Rightarrow\frac{(x+y)^2}{(y+z)^2}=\frac{y^2}{z^2}=\frac{zx}{z^2}=\frac{x}{z}$$

vii)

Correct option: (b)

Explanation:

In ΔABC and ΔDEC,

$\angle ABC = \angle DEC = 90^\circ$

$\angle C = \angle C$ (Common)

$\therefore \Delta ABC \sim \Delta DEC$ (By AA similarity) viii)

Correct Option: (d)

Explanation:

Volume of the vessel = $448\pi\ cm^3$

Height (h) = 7 cm

$\therefore \pi r^2 h = 448\pi\ cm^3$

$\Rightarrow r^2 h = 448\ cm^3$

$\Rightarrow r^2 \times 7 = 448\ cm^3$

$\Rightarrow r^2 = \frac{448}{7} cm^2$

$\Rightarrow r^2 = 64\ cm^2$

$\Rightarrow r = 8\ cm$

Radius of the base = 8 cm

ix)

Correct Option: (a)

Explanation:

Possible number of outcomes = 6 = number of balls in the bag

n(S) = 6

Let E = event of drawing a white ball

Number of white balls in the bag = 0

n(E) = 0

Probability of drawing a white ball = $P(S) = \frac{n(E)}{n(S)} = \frac{0}{6} = 0$

x)

Correct Option: (a)

Explanation:

Less than ogive curve is an increasing curve.

xi)

Correct option: (a)

Explanation:

If a chord and a tangent intersect externally, then the product of the lengths of the segments of the chord is equal to the square of the length of the tangent from the point of contact to the point of intersection.

$\Rightarrow GS^2 = GF \times GE$

$\Rightarrow 18^2 = (15 + EF) \times 15$

$\Rightarrow 324 = 225 + 15EF$

$\Rightarrow EF = 6.6$ cm

xii)

Correct option: (d)

Explanation:

A matrix which has an equal number of rows and columns is called a square matrix.

Hence, statement 1 is false.

The statement provided in statement 2 is correct.

Hence, statement 2 is true.

xiii)

Correct Option: (b)

Explanation:

Total shares = 220

Nominal value = Rs. 30

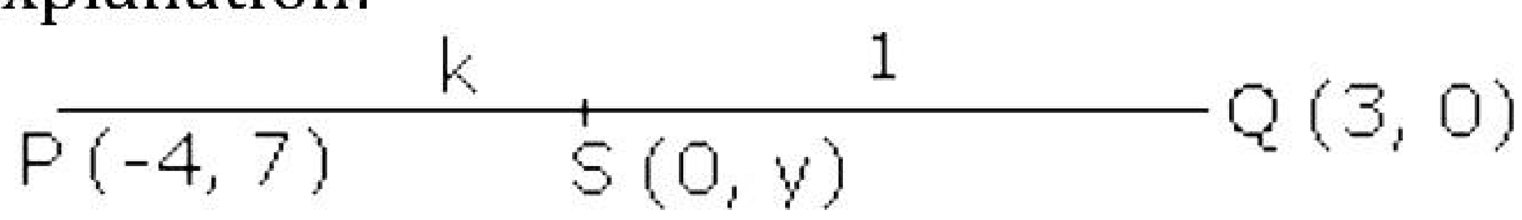
Market value = Rs. 30 – Rs. 10 = Rs. 20

Money required to buy 220 shares = Rs. 20 × 220 = Rs. 4400

xiv)

Correct Option: (b)

Explanation:



Let S (0, y) be the point on the y-axis which divides the line segment PQ in the ratio k : 1.

Using section formula, we have

$$0=\frac{3k-4}{k+1}$$

$$\Rightarrow 3k=4$$

$$\Rightarrow k=\frac{4}{3}$$

Hence, the required ratio is $4:3$.

xv)

Correct Option: (b)

Explanation:

The statement given in reason is correct and hence reason is true.

Since the point K lies on the perpendicular bisector of ST, it is equidistant from **S** and **T**.

Hence, assertion is false.

Solution 2

i)

Volume of the sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 6^3 = 288\pi\ cm^3$

$\Rightarrow$ Radius of the spherical ball = $\frac{0.4}{2} = 0.2\,cm$

$\Rightarrow$ Volume of the spherical ball = $\frac{4}{3}\pi \times 0.2^3 = \frac{32\pi}{3000}$ cm^3

$\Rightarrow$ Number of balls × volume of one spherical ball = Volume of the solid sphere

$\Rightarrow$ Number of balls × $\frac{32\pi}{3000} = 288\pi$

$\Rightarrow$ Number of balls = 27000

Hence, 27000 spherical balls are obtained.

ii)

Monthly instalment = P = Rs. 400, rate of interest = r = 6%, maturity value = Rs. 15732

Let the number of months be n.

Maturity Value = $P \times n + P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$

$15732 = 400n + 400 \times \frac{n(n+1)}{2 \times 12} \times \frac{6}{100}$

$15732 = 400n + n(n+1)$

$400n + n^2 + n - 15732 = 0$

$n^2 + 401n - 15732 = 0$

$n^2 + 437n - 36n - 15732 = 0$

$n(n + 437) - 36(n + 437) = 0$

$(n + 437)(n - 36) = 0$

$n \neq -437$; hence, $n = 36$

Hence, the number of instalments is 36.

iii)

$$\text{L.H.S.} = \frac{\cos A}{1 + \sin A} + \tan A$$

$$= \frac{\cos A(1 - \sin A)}{(1 + \sin A)(1 - \sin A)} + \frac{\sin A}{\cos A}$$

$$= \frac{\cos A - \sin A \cos A}{1 - \sin^2 A} + \frac{\sin A}{\cos A}$$

$$= \frac{\cos A - \sin A \cos A}{\cos^2 A} + \frac{\sin A}{\cos A}$$

$$= \frac{1}{\cos A} - \frac{\sin A}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \frac{1}{\cos A}$$

$$= \sec A$$

= R.H.S.

Solution 3

i)

a, b, c and d are in proportion.

$$\Rightarrow \frac{a}{b} = \frac{c}{d} = k$$

$$\Rightarrow a = bk \text{ and } c = dk$$

$$\text{L.H.S.} = \frac{ma^2 + nb^2}{mc^2 + nd^2}$$

$$= \frac{m(bk)^2 + nb^2}{m(dk)^2 + nd^2}$$

$$= \frac{mb^2k^2 + nb^2}{md^2k^2 + nd^2}$$

$$= \frac{b^2(mk^2 + n)}{d^2(mk^2 + n)}$$

$$= \frac{b^2}{d^2} \qquad ....(1)$$

$$\text{R.H.S.} = \frac{ma^2 - nb^2}{mc^2 - nd^2}$$

$$= \frac{m(bk)^2 - nb^2}{m(dk)^2 - nd^2}$$

$$= \frac{mb^2k^2 - nb^2}{md^2k^2 - nd^2}$$

$$= \frac{b^2(mk^2 - n)}{d^2(mk^2 - n)}$$

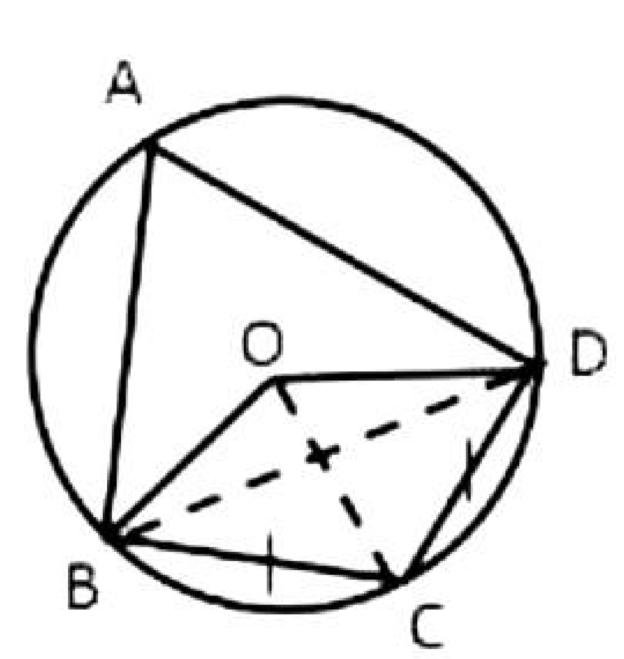
$$= \frac{b^2}{d^2} \qquad ....(2)$$

From (1) and (2),

$$\frac{ma^2 + nb^2}{mc^2 + nd^2} = \frac{ma^2 - nb^2}{mc^2 - nd^2}$$

ii)

Join BD and OC.



a. $\angle BAD + \angle BCD = 180°$ (opposite angles of a cyclic quadrilateral are supplementary)

$\Rightarrow 60° + \angle BCD = 180°$

$\Rightarrow \angle BCD = 120°$ ...(i)

$BC = CD$ (Given)

$\Rightarrow \angle CBD = \angle BDC$ (angles opposite to equal sides are equal)

Now,

$\angle CBD + \angle BDC + \angle BCD = 180°$ (sum of angles of a triangle)

$\Rightarrow \angle CBD = \frac{1}{2}(180° - \angle BCD) = \frac{1}{2}(180° - 120°)$

$\Rightarrow \angle CBD = 30°$ ...(ii)

b. $\angle BAD = 60°$

$\Rightarrow \angle BOD = 2\angle BAD$ (angle subtended by the same arc at the centre)

$= 2 \times 60°$

$= 120°$

c. In ΔOBD,

$OB = OD$ (radii of the same circle)

$\Rightarrow \angle ODB = \angle OBD$ (angles opposite to equal sides are equal)

Now,

$\angle OBD + \angle ODB + \angle BOD = 180^o$ (sum of angles of a triangle)

$\Rightarrow \angle OBD = \frac{1}{2}(180° - \angle BOD) = \frac{1}{2}(180° - 120°)$

$\Rightarrow \angle OBD = 30^o$ ...(iii)

Then, $\angle OBC = \angle OBD + \angle CBD = 30^o + 30^o = 60^o$

d. In ΔOBC,

$OB = OC$ (radii of the same circle)

$\angle OBC = \angle OCB = 60^o$ (angles opposite to equal sides are equal)

$\Rightarrow \angle OBC + \angle OCB + \angle BOC = 180^o$ (sum of angles of a triangle)

$\Rightarrow 60^o + 60^o + \angle BOC = 180^o$

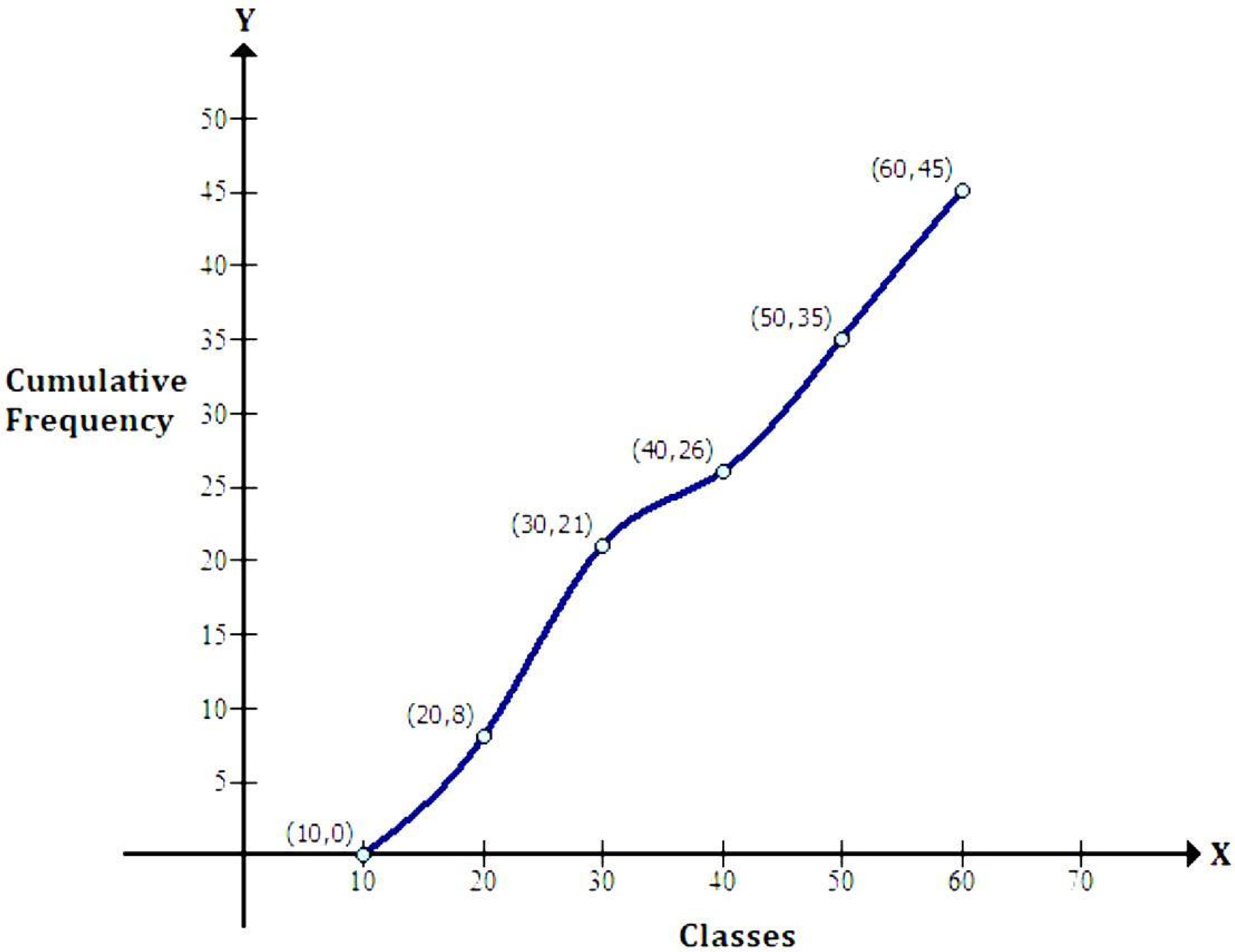
$\Rightarrow \angle BOC = 60^o$

iii)

Let us first prepare the cumulative frequency table:

Class Interval	Frequency	Cumulative Frequency
10–20	8	8
20–30	13	21
30–40	5	26
40–50	9	35
50–60	10	45

Taking the upper-class limits along the x-axis and corresponding cumulative frequencies along the y-axis, mark the points (10, 0), (20, 8), (30, 21), (40, 26), (50, 35) and (60, 45). Join the points marked by a free-hand curve (as shown below).



Section B

Solution 4

i) $-A + 3B + M = 0$

$\Rightarrow M = A - 3B$

$$= \begin{bmatrix} -2 & 6 \\ 5 & 8 \end{bmatrix} - 3\begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 6 \\ 5 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ -6 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} -2-3 & 6-6 \\ 5+6 & 8-9 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 0 \\ 11 & -1 \end{bmatrix}$$

ii)

Since 2 is a root of the equation $3x^2 - p(x+1) = 0$

$\Rightarrow 3(2)^2 - p(2+1) = 0$

$\Rightarrow 3 \times 4 - 3p = 0$

$\Rightarrow 12 - 3p = 0$

$\Rightarrow 3p = 12$

$\Rightarrow p = 4$

Now, the other equation becomes $4x^2 - qx + 9 = 0$

Here, $a = 4$, $b = -q$ and $c = 9$

Since the roots are equal, we have

$b^2 - 4ac = 0$

$\Rightarrow (-q)^2 - 4 \times 4 \times 9 = 0$

$\Rightarrow q^2 - 144 = 0$

$\Rightarrow q^2 = 144$

$\Rightarrow q = 12$

Hence, $p = 4$ and $q = 12$.

iii)

(a) In ΔABC and ΔAMP,

$\angle ABC = \angle AMP$ (Each 90°)

$\angle BAC = \angle MAP$ (common angle)

$\therefore \Delta ABC \sim \Delta AMP$ (AA similarity)

(b)

In ΔAMP,

$AM^2 + MP^2 = AP^2$

$\therefore AM \quad \sqrt{AP^2 \quad MP^2}$

$\therefore AM = \sqrt{15^2 - 12^2}$

$\therefore AM = 9$ cm

Since $\Delta ABC \sim \Delta AMP$

$\Rightarrow \frac{AB}{AM} = \frac{AC}{AP}$

$\Rightarrow \frac{AB}{9} = \frac{10}{15}$

$\Rightarrow AB = 6$ cm

Also,

$\frac{AC}{AP} = \frac{BC}{MP}$

$\Rightarrow \frac{10}{15} = \frac{BC}{12}$

$\Rightarrow BC = 8$ cm

Solution 5

i) Let the assumed mean A be 8.5. Class interval i = 3.

Letters	Frequency f	Mid-value x	$t = \frac{x - A}{i}$	$f \times t$	Cumulative frequency
1 – 4	6	2.5	–2	–12	6
4 – 7	30	5.5	–1	–30	36
7 – 10	40	8.5 = A	0	0	76
10 – 13	16	11.5	1	16	92
13 – 16	4	14.5	2	8	96
16 – 19	4	17.5	3	12	100
	N = 100			$\sum ft = -6$	

N = Total frequency = 100

$$\text{Mean} = A + \frac{\sum ft}{\sum f} \times i$$

$$= 8.5 + 3 \times \left(\frac{-6}{100}\right)$$

$$= 8.5 - \frac{18}{100}$$

$$= 8.5 - 0.18$$

$$= 8.32$$

ii) At each stage of sale of the machine, there is an inter-state transaction

$\therefore$ The rate of GST on each transaction of machine is 26%.

For the manufacturer,

Marked price = Rs. 65,000 and discount = 30% of Rs. 65,000 = Rs. 19,500

$\therefore$ S.P. = M.P. – Discount

= Rs. (65,000 – 19,500) = Rs. 45,500

For the wholesaler,

C.P. = Rs. 45,500 and discount = 20% of Rs. 65,000 = Rs. 13,000

And S.P. = M.P. – Discount

= Rs. (65,000 – 13,000)

= Rs. 52,000

Therefore, tax paid by the wholesaler to the central government

= Output tax – Input tax

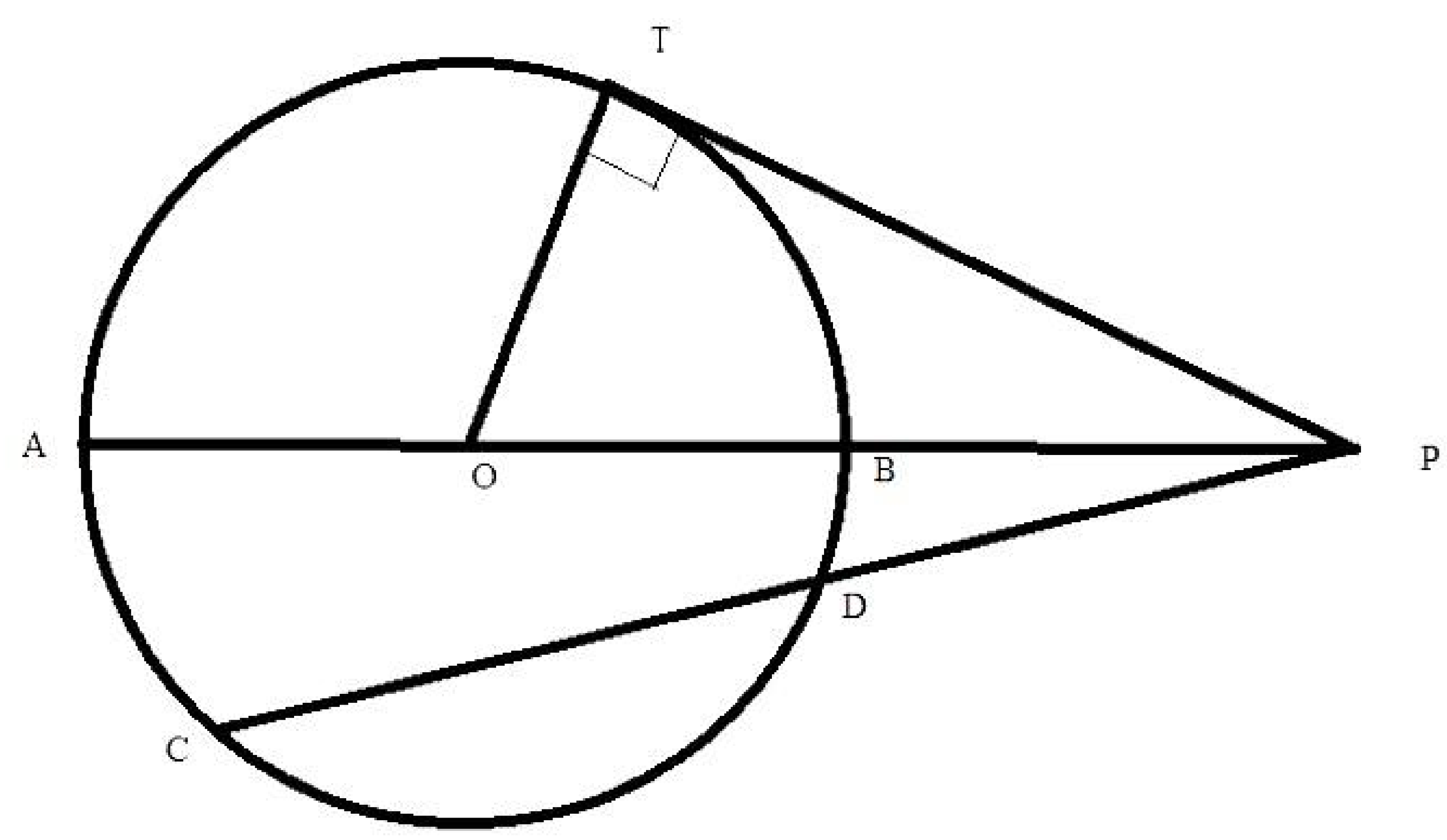
= Tax on S.P. – Tax on C.P.

= 26% of Rs. 52,000 – 26% of Rs. 45,500

= Rs. (13,520 – 11,830)

= Rs. 1,690

iii)



As chord CD and tangent at point T intersect each other at P,

$PC \times PD = PT^2$ ..... (i)

Diameter AB and tangent at point T intersect each other at P,

$PA \times PB = PT^2$ ....(ii)

From (i) and (ii), $PC \times PD = PA \times PB$ ... (iii)

Given: PD = 5 cm, CD = 7.8 cm

PA = PB + AB = 4 + AB,

And PC = PD + CD = 5 + 7.8 = 12.8 cm

Substituting these values in (iii), we get

$12.8 \times 5 = (4 + AB) \times 4$

$\Rightarrow 4 + AB = \frac{12.8 \times 5}{4}$

$\Rightarrow 4 + AB = 16$ cm

$\Rightarrow AB = 12$ cm

(ii)

$PC \times PD = PT^2$

$\Rightarrow PT^2 = 12.8 \times 5 = 64$

$\Rightarrow PT = 8$ cm

Thus, the length of the tangent PT is 8 cm.

Solution 6

i) Given G.P.: $\sqrt{2}, 2, 2\sqrt{2}, ...$

First term, a = $\sqrt{2}$ and number of terms to be added = 8

Common ratio, r = $\frac{2}{\sqrt{2}} = \sqrt{2} \Rightarrow |r| > 1$

Therefore,

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_8 = \frac{\sqrt{2}\left(\left(\sqrt{2}\right)^8 - 1\right)}{\sqrt{2} - 1}$$

$$= \frac{\sqrt{2}(16 - 1)}{\sqrt{2} - 1}$$

$$= \frac{15\sqrt{2}}{\sqrt{2} - 1}$$

$$= \frac{15\sqrt{2}}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1}$$

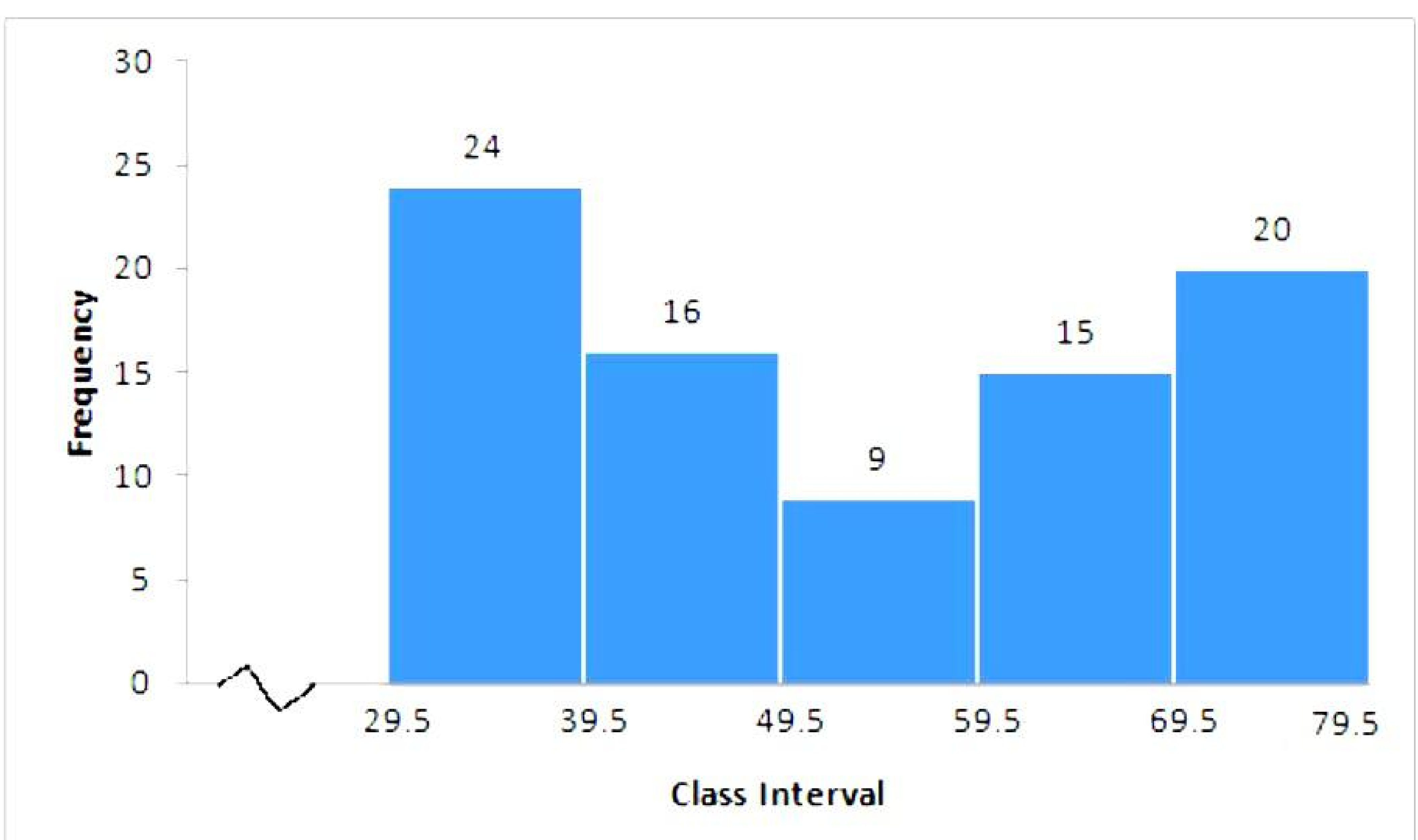
$$= 30 + 15\sqrt{2}$$

$$= 15\left(2 + \sqrt{2}\right)$$

ii) Steps of construction:

(a) Taking suitable scales, mark class intervals on the X-axis and frequencies on the Y-axis.

(b) Construct rectangles with class intervals as bases and corresponding frequencies as heights.



iii) Height of the conical vessel, h = 24 cm

Radius of the conical vessel, r = 5 cm

Then, volume filled with water $=\frac{3}{4}\times\frac{1}{3}\times\pi r^2 h$

$=\frac{3}{4}\times\frac{1}{3}\times\pi\times 25\times 24$

$=150\pi\ cm^2$

Let H be the height of the cylindrical vessel which is filled by water of the conical vessel.

Radius of the cylindrical vessel = 10 cm

Volume of the cylindrical vessel = volume of water

$\Rightarrow \pi(10)^2 H = 150\pi$

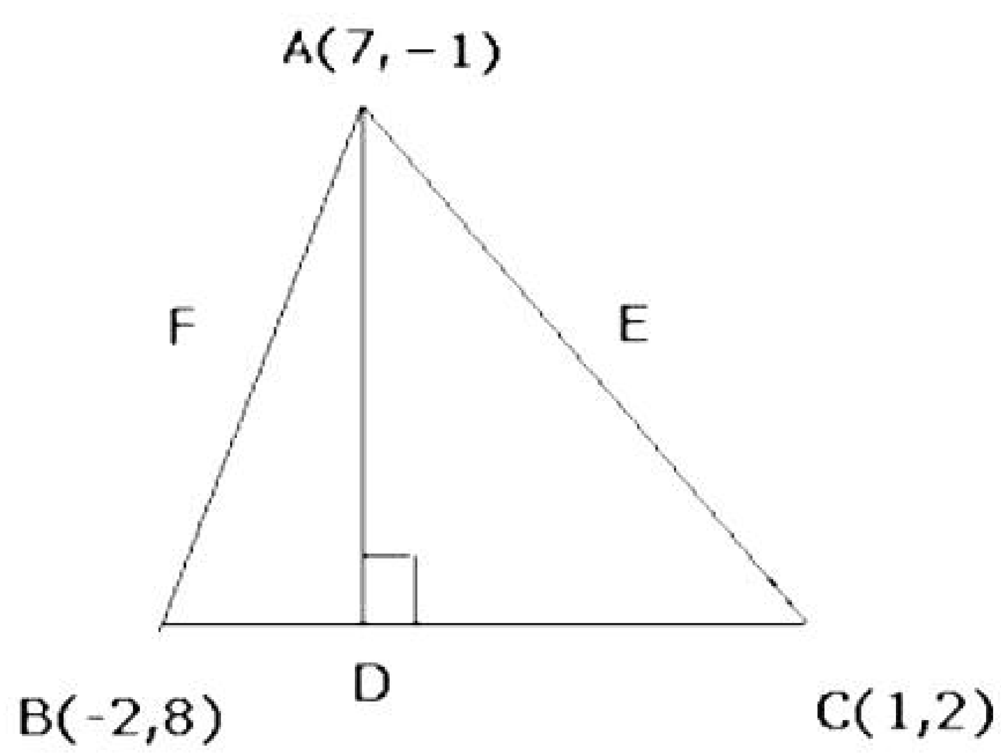
$\Rightarrow 100\times H = 150$

$\Rightarrow H = 1.5$ cm

Thus, the height of the cylindrical vessel is 1.5 cm.

Solution 7

i) Let ABC be the given triangle and AD be its altitude.



Slope of BC $=\frac{2-8}{1+2}=\frac{-6}{3}=-2$

AD $\perp$ BC.

$\therefore$ Slope of AD = $\frac{1}{2}$

Equation of AD, $y=\frac{1}{2}x+c$

Since the altitude AD passes through the point A(7, −1), we have

$-1=\frac{1}{2}\times 7+c$

$\Rightarrow\ c=-1-\frac{7}{2}=\frac{-9}{2}$

Equation of AD is given by

$\Rightarrow y=\frac{1}{2}x-\frac{9}{2}$

$\Rightarrow 2y=x-9$

$\Rightarrow x-2y-9=0$

(b) Let M be the mid-point of line segment joining (2, 3) and (4, 5)

$$\Rightarrow M \equiv \left(\frac{4+2}{2}, \frac{5+3}{2}\right) \Rightarrow M \equiv (3,\ 4)$$

Equation of the given line is 5x – 2y = 8.

$\Rightarrow$ 2y = 5x – 8

$$\Rightarrow y = \frac{5}{2}x - 4$$

$\therefore$ Slope of the given line is $\frac{5}{2}$

Therefore, slope of the line perpendicular to the line 5x – 2y = 8 is $-\frac{2}{5}$

Equation of the perpendicular line is y = mx + k

$$\Rightarrow y = -\frac{2}{5}x + k$$

$$\Rightarrow 2x + 5y - k = 0$$

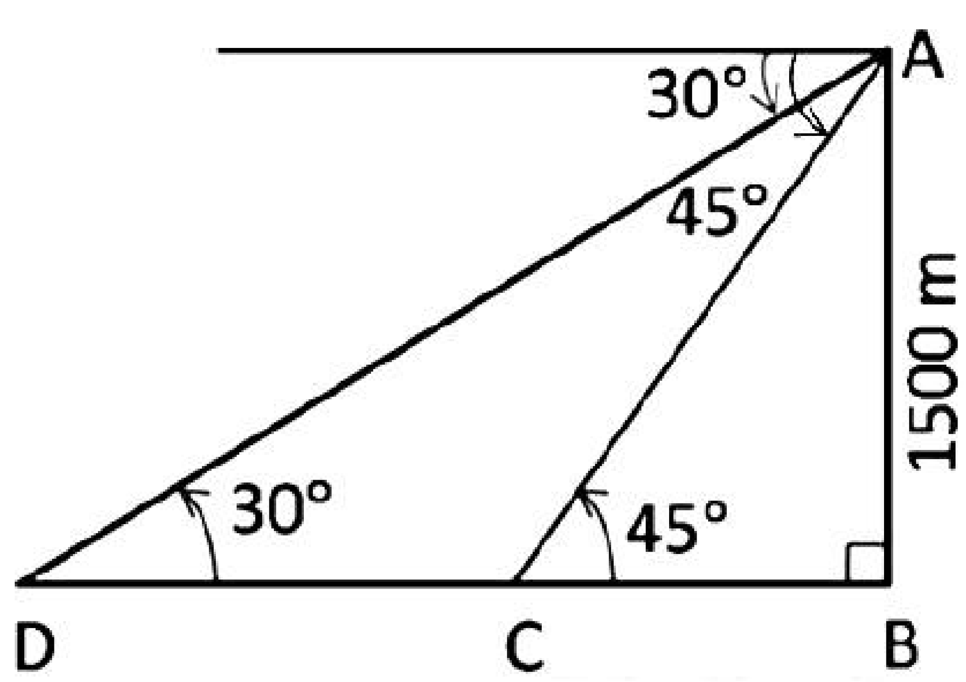
It passes through M(3, 4).

$\therefore$ $2 \times 3 + 5 \times 4 - k = 0$

$\Rightarrow$ k = 26

Hence, 2x + 5y – 26 = 0 is the required equation.

ii) A is the aeroplane. D and C are the ships sailing towards A.
Ships are sailing towards the aeroplane in the same direction.



In the figure, height AB = 1500 m

Distance between two ships D and C = CD

In the right-angled ΔABC,

$$\tan 45° = \frac{AB}{BC}$$

$$\Rightarrow 1 = \frac{1500}{BC}$$

$$\Rightarrow BC = 1500 \text{ m}$$

In the right-angled ΔABD,

$$\tan 30° = \frac{AB}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1500}{BD}$$

$$\Rightarrow BD = 1500\sqrt{3} \text{ m}$$

$$\Rightarrow BD = 1500(1.732) = 2598 \text{ m}$$

Therefore, distance between two ships D and C = CD

= BD – BC

= (2598 – 1500) m

= 1098 m

Solution 8

i)

$$-3(x-7) \geq 15 - 7x > \frac{x+1}{3}, \ x \in R$$

$$\Rightarrow -3(x-7) \geq 15 - 7x \ \text{ and } \ 15 - 7x > \frac{x+1}{3}$$

$$\Rightarrow -3x + 21 \geq 15 - 7x \ \text{ and } \ 45 - 21x > x + 1$$

$$\Rightarrow -3x + 7x \geq 15 - 21 \ \text{ and } \ 45 - 1 > x + 21x$$

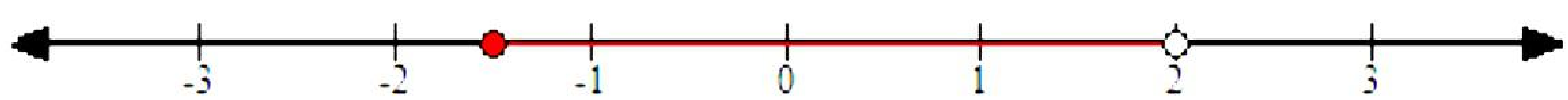
$$\Rightarrow 4x \geq -6 \ \text{ and } \ 44 > 22x$$

$$\Rightarrow x \geq \frac{-3}{2} \ \text{ and } \ 2 > x$$

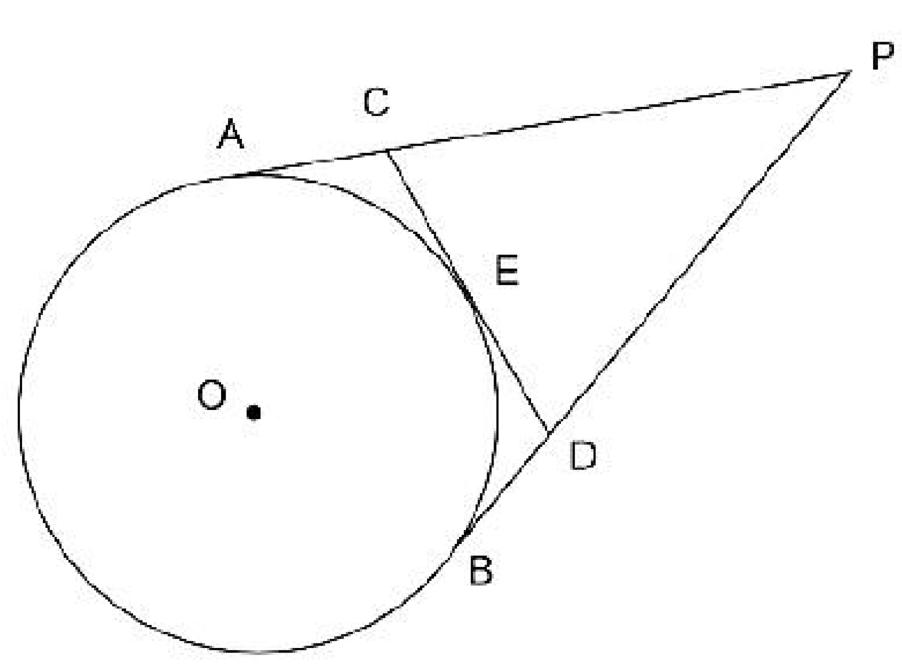
$$\Rightarrow x \geq -1.5 \ \text{ and } \ 2 > x$$

$\therefore$ The solution set is $\{x : x \in R, \ -1.5 \leq x < 2\}$.

The solution set is represented on the number line as follows:



ii) Two tangents PA and PB are drawn from an external point P to a circle with centre O.



Since the tangents from an external point to a circle are equal,

PA = PB ....(i)

Also, CA = CE and DB = DE

Perimeter of ΔPCD = PC + CD + PD

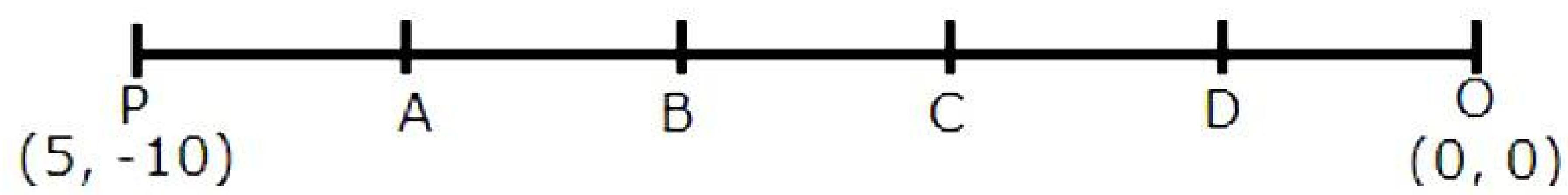
= (PA – CA) + (CE + DE) + (PB – DB)

= (PA – CE) + (CE + DE) + (PB – DE)

$= PA + PB$
$= 2PA$ [From (i)]
$= 2 \times 14$ cm
$= 28$ cm

Hence, the perimeter of ΔPCD is 28 cm.

iii)



Point A divides PO in the ratio 1:4.

$\therefore$ Co-ordinates of point A = $\left(\frac{1\times0+4\times5}{1+4}, \frac{1\times0+4\times(-10)}{1+4}\right) = \left(\frac{20}{5}, \frac{-40}{5}\right) = (4, -8)$

Point B divides PO in the ratio 2:3.

$\therefore$ Co-ordinates of point B = $\left(\frac{2\times0+3\times5}{2+3}, \frac{2\times0+3\times(-10)}{2+3}\right) = \left(\frac{15}{5}, \frac{-30}{5}\right) = (3, -6)$

Point C divides PO in the ratio 3:2.

$\therefore$ Co-ordinates of point C = $\left(\frac{3\times0+2\times5}{3+2}, \frac{3\times0+2\times(-10)}{3+2}\right) = \left(\frac{10}{5}, \frac{-20}{5}\right) = (2, -4)$

Point D divides PO in the ratio 4:1.

$\therefore$ Co-ordinates of point D = $\left(\frac{4\times0+1\times5}{4+1}, \frac{4\times0+1\times(-10)}{4+1}\right) = \left(\frac{5}{5}, \frac{-10}{5}\right) = (1, -2)$

Solution 9

i)

a) Let the mean proportional between 17.5 and 0.007 be x.

$\Rightarrow$ 17.5, x and 0.007 are in continued proportion.

$\Rightarrow 17.5 : x = x : 0.007$

$\Rightarrow x \times x = 17.5 \times 0.007$

$\Rightarrow x^2 = \frac{175}{10} \times \frac{7}{1000} = \frac{1225}{10000}$

$\Rightarrow x^2 = \left(\frac{35}{100}\right)^2$

$\Rightarrow x = \frac{35}{100} = 0.35$

b) Let the mean proportional between $6 + 3\sqrt{3}$ and $8 - 4\sqrt{3}$ be x.

$\Rightarrow 6 + 3\sqrt{3}$, x and $8 - 4\sqrt{3}$ are in continued proportion.

$\Rightarrow 6 + 3\sqrt{3} : x = x : 8 - 4\sqrt{3}$

$\Rightarrow x \times x = (6 + 3\sqrt{3})(8 - 4\sqrt{3})$

$\Rightarrow x^2 = 48 + 24\sqrt{3} - 24\sqrt{3} - 36$

$\Rightarrow x^2 = 12$

$\Rightarrow x = 2\sqrt{3}$

c) Let the mean proportional between $a - b$ and $a^3 - a^2b$ be x.

$\Rightarrow a - b, x, a^3 - a^2b$ are in continued proportion.

$\Rightarrow a - b : x = x : (a^3 - a^2b)$

$\Rightarrow x \times x = (a - b)(a^3 - a^2b)$

$\Rightarrow x^2 = (a - b) \times a^2(a - b) = [a(a - b)]^2$

$\Rightarrow x = a(a - b)$

ii) Time taken by bus to cover total distance with speed x km/hr $= \frac{240}{x}$

Time taken by bus to cover total distance with speed (x – 10) km/hr $= \frac{240}{x-10}$

According to the given condition,

$\frac{240}{x-10} - \frac{240}{x} = 2$

$\Rightarrow 240\left(\frac{1}{x-10} - \frac{1}{x}\right) = 2$

$\Rightarrow \frac{1}{x-10} - \frac{1}{x} = \frac{1}{120}$

$\Rightarrow \frac{x-x+10}{x(x-10)} = \frac{1}{120}$

$\Rightarrow \frac{10}{x^2-10x} = \frac{1}{120}$

$\Rightarrow x^2 - 10x = 1200$

$\Rightarrow x^2 - 10x - 1200 = 0$

$\Rightarrow (x-40)(x+30) = 0$

$\Rightarrow x - 40 = 0$ or $x + 30 = 0$

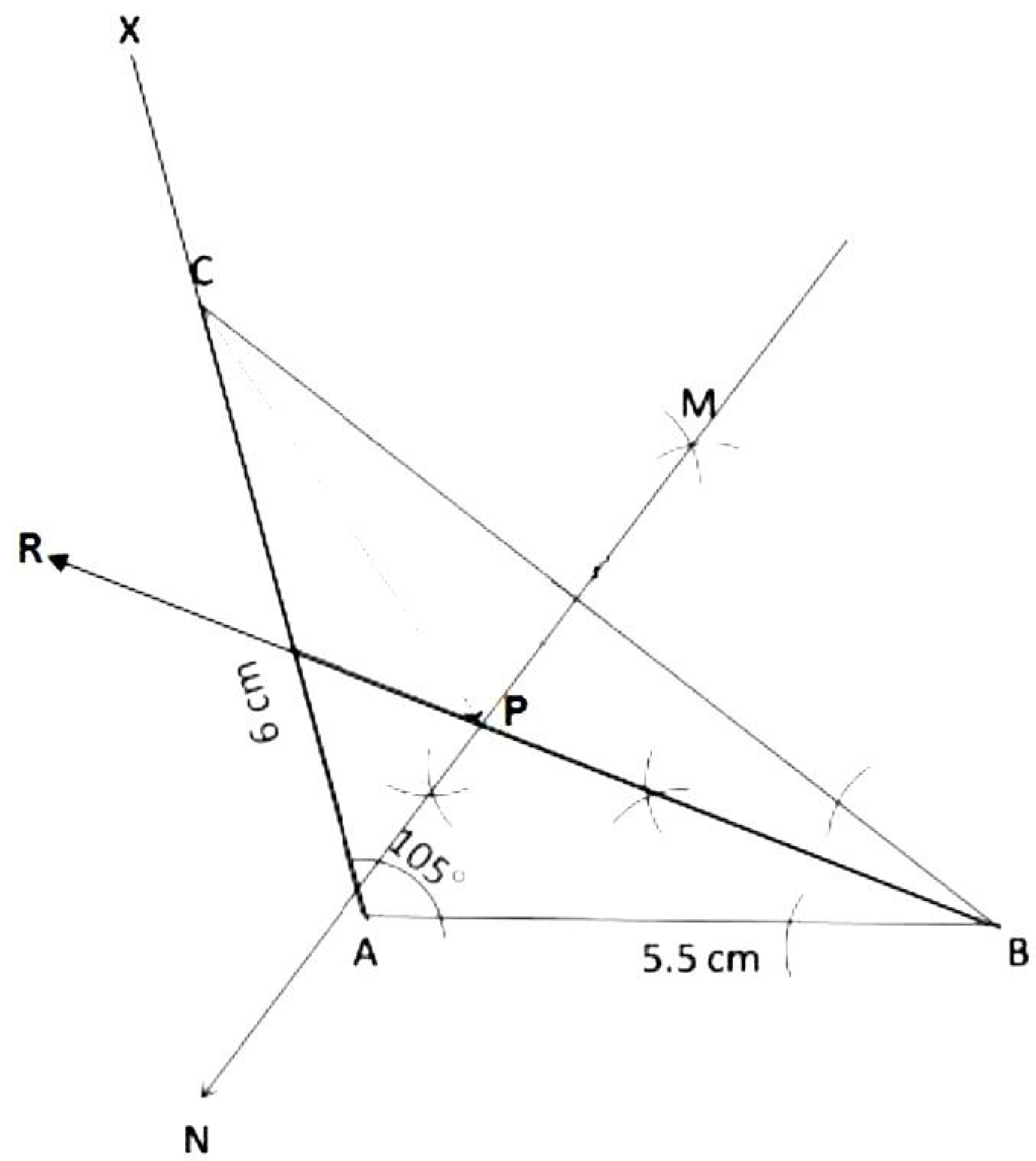
$\Rightarrow x = 40$ or $x = -30$

Since the speed cannot be negative, the uniform speed is 40 km/hr.

iii)

1) Draw a line segment AB of length 5.5 cm.
2) Construct $\angle BAX = 105°$.
3) Draw an arc AC with radius 6 cm on AX with centre at A.
4) Join BC. Thus, ΔABC is the required triangle.

a) Draw BR, the bisector of ∠ABC, which is the locus of points equidistant from BA and BC.
b) Draw MN, the perpendicular bisector of BC, which is the locus of points equidistant from B and C.
c) The angle bisector of ∠ABC and perpendicular bisector of BC meet at point P. Thus, P satisfies the above two loci. Length of PC = 4.8 cm



Solution 10

i)

Since $(x-2)$ is a factor of polynomial $2x^3+ax^2+bx-14$, we have

$2(2)^3+a(2)^2+b(2)-14=0$

$\Rightarrow 16+4a+2b-14=0$

$\Rightarrow 4a+2b+2=0$

$\Rightarrow 2a+b+1=0$

$\Rightarrow 2a+b=-1 \quad ....(i)$

On dividing by $(x-3)$, the polynomial $2x^3+ax^2+bx-14$ leaves remainder 52,

$\Rightarrow 2(3)^3+a(3)^2+b(3)-14=52$

$\Rightarrow 54+9a+3b-14=52$

$\Rightarrow 9a+3b+40=52$

$\Rightarrow 9a+3b=12$

$\Rightarrow 3a+b=4 \quad ....(ii)$

Subtracting (i) from (ii), we get $a = 5$

Substituting $a = 5$ in (i), we get

$2 \times 5 + b = -1$

$\Rightarrow 10 + b = -1$

$\Rightarrow b = -11$

Hence, $a = 5$ and $b = -11$.

ii) Number of cards removed = 3 (face cards of clubs)

Remaining number of cards = 52 – 3 = 49

a) Remaining number of club cards = 13 – 3 = 10

$\therefore$ P (a club card) = $\frac{10}{49}$

b) Number of red face cards = 6 (i.e. 3 of diamonds and 3 of hearts)

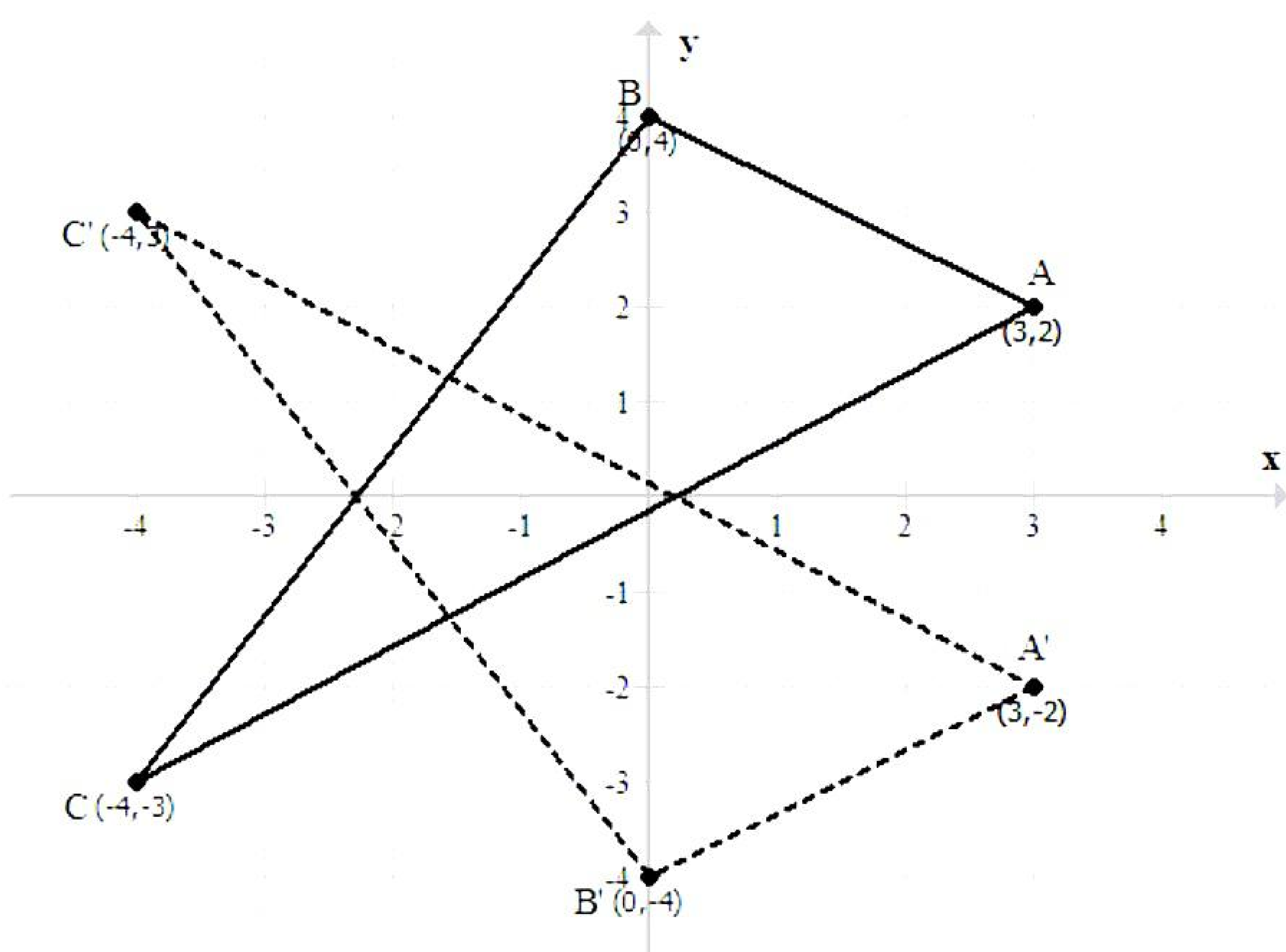
P (a red face card) = $\frac{6}{49}$

c) Number of black face cards = 3 (3 of spade)

P (a black face card) = $\frac{3}{49}$

iii)

a)



b) Reflection on the x-axis is represented by

$r : (x, y) \rightarrow (x, -y)$

$\therefore A(3, 2) \rightarrow A'(3, -2)$

$\therefore$ B(0, 4) $\rightarrow$ B'(0, -4)
$\therefore$ C(-4, -3) $\rightarrow$ C'(-4, 3)

c) By measuring the lengths of the sides of ΔABC and ΔA'B'C', we can say that
ΔABC $\cong$ ΔA'B'C [By SSS rule]