

Statistics

Exercise 14.1

Q. 1. A survey was conducted by a group of students as a part of their environment awareness programmed, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

No. of plants	0-2	2-4	4-6	6-8	8-10	10-12	12-14
No. of houses	1	2	1	5	6	2	3

Answer : Let's find mean of the data using direct method.

First, construct a table for ease of calculation.

No. of plants	Midpoints (x_i)	No. of houses (frequency) (f_i)	$x_i f_i$
0-2	1	1	$1 \times 1 = 1$
2-4	3	2	$3 \times 2 = 6$
4-6	5	1	$5 \times 1 = 5$
6-8	7	5	$7 \times 5 = 35$
8-10	9	6	$9 \times 6 = 54$
10-12	11	2	$11 \times 2 = 22$
12-14	13	3	$13 \times 3 = 39$
TOTAL		$\Sigma f_i = 20$	$\Sigma x_i f_i = 162$

Mean is given by

$$\text{Mean} = \frac{\Sigma x_i f_i}{\Sigma f_i}$$

$$\Rightarrow \text{Mean} = 162/20$$

$$\Rightarrow \text{Mean} = 8.1$$

Thus, mean number of plants per house is 8.1.

Q. 2. Consider the following distribution of daily wages of 50 workers of a factory.

Daily wages in Rupees	200-250	250-300	300-350	350-400	400-450
Number of workers	12	14	8	6	10

Find the mean daily wages of the workers of the factory by using an appropriate method.

Answer : Let's find mean of the data using assumed mean method.

We are using assumed mean method to avoid miscalculation and false answer, as x_i 's are large in this question.

First, construct a table for ease of calculation.

Daily wages (Rs.)	Midpoints (x_i)	$d_i = x_i - 325$	No. of workers (frequency) (f_i)	$f_i d_i$
200-250	225	-100	12	-1200
250-300	275	-50	14	-700
300-350	325 = A	0	8	0
350-400	375	50	6	300
400-450	425	100	10	1000
TOTAL			$\Sigma f_i = 50$	$\Sigma f_i d_i = -600$

Mean is given by

$$\text{Mean} = A + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

Where A = assumed mean

$$\Rightarrow \text{Mean} = 325 + \frac{-600}{50}$$

$$\Rightarrow \text{Mean} = 325 - 12 = 313$$

Thus, mean daily wages of the workers is Rs. 313.

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200-250	225	-100	12	-1200
250-300	275	-50	14	-700
300-350	325 = A	0	8	0
350-400	375	50	6	300
400-450	425	100	10	1000
TOTAL			$\Sigma f_i = 50$	$\Sigma f_i d_i = -600$

Mean is given by

$$\text{Mean} = A + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

Where A = assumed mean

$$\Rightarrow \text{Mean} = 325 + \frac{-600}{50}$$

$$\Rightarrow \text{Mean} = 325 - 12 = 313$$

Thus, mean daily wages of the workers is Rs. 313.

Q. 3. The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is ` 18. Find the missing frequency f.

Daily pocket allowance (in Rupees)	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Number of children	7	6	9	13	f	5	4

Answer : To find f , we'll need to find mean pocket allowance by direct method and equate it to the given mean pocket allowance, 18.

So, let's construct a table finding midpoints and stating frequencies.

Daily pocket allowance (Rs.)	Midpoints (x_i)	No. of children (frequency) (f_i)	$x_i f_i$
11-13	12	7	$12 \times 7 = 84$
13-15	14	6	$14 \times 6 = 84$
15-17	16	9	$16 \times 9 = 144$
17-19	18	13	$18 \times 13 = 234$
19-21	20	f	$20 \times f = 20f$
21-23	22	5	$22 \times 5 = 110$
23-25	24	4	$24 \times 4 = 96$
TOTAL		$\Sigma f_i = 44 + f$	$\Sigma x_i f_i = 752 + 20f$

Mean is given by

$$\text{Mean} = \frac{\Sigma x_i f_i}{\Sigma f_i}$$

$$\Rightarrow 18 = \frac{752+20f}{44+f} \quad [\text{given, mean} = 18]$$

$$\Rightarrow 18(44 + f) = 752 + 20f$$

$$\Rightarrow 792 + 18f = 752 + 20f$$

$$\Rightarrow 20f - 18f = 792 - 752$$

$$\Rightarrow 2f = 40$$

$$\Rightarrow f = 40/2$$

$$\Rightarrow f = 20$$

Thus, the missing frequency f is 20.

Q. 4. Thirty women were examined in a hospital by a doctor and their of heart beats per minute were recorded and summarized as shown. Find the mean heart beats per minute for these women, choosing a suitable method.

Number of heart beats/minute	65-68	68-71	71-74	74-77	77-80	80-83	83-86
Number of women	2	4	3	8	7	4	2

Answer : We shall use assumed mean method to find mean heart beats per minute, as this data has large values of x_i 's and has a risk of miscalculation. But by using assumed mean method, we'll be able to solve it with ease and more accuracy.

No. of heart beats per minute	Midpoints (x_i)	$d_i = x_i - 75.5$	No. of women (frequency) (f_i)	$f_i d_i$
65-68	66.5	$66.5 - 75.5 = -9$	2	$-9 \times 2 = -18$
68-71	69.5	$69.5 - 75.5 = -6$	4	$-6 \times 4 = -24$
71-74	72.5	$72.5 - 75.5 = -3$	3	$-3 \times 3 = -9$
74-77	$75.5 = A$	$75.5 - 75.5 = 0$	8	$0 \times 8 = 0$
77-80	78.5	$78.5 - 75.5 = 3$	7	$3 \times 7 = 21$
80-83	81.5	$81.5 - 75.5 = 6$	4	$6 \times 4 = 24$
83-86	84.5	$84.5 - 75.5 = 9$	2	$9 \times 2 = 18$
TOTAL			$\Sigma f_i = 30$	$\Sigma f_i d_i = 12$

Mean is given by

$$\text{Mean} = A + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$\Rightarrow \text{Mean} = 75.5 + \frac{12}{30}$$

$$\Rightarrow \text{Mean} = \frac{755}{10} + \frac{4}{10}$$

$$\Rightarrow \text{Mean} = \frac{755+4}{10} = \frac{759}{10}$$

$$\Rightarrow \text{Mean} = 75.9$$

Thus, mean heart beats per minute of these women is 75.9.

Q. 5. In a retail market, fruit vendors were selling oranges kept in packing basket. These baskets contained varying number of oranges. The following was distribution of oranges.

Number of oranges	10-14	15-19	20-24	25-29	30-34
Number of baskets	15	110	135	115	25

Find the mean number of oranges kept in each basket. Which method of finding the mean did you choose?

Answer : We shall use assumed mean method to find mean number of oranges.

So, while converting data into exclusive type won't change any calculation in finding mean, we are however converting the inclusive data type to exclusive data type for better representation.

No. of oranges	Midpoints (x_i)	$d_i = x_i - 22$	No. of baskets (frequency) (f_i)	$f_i d_i$
9.5-14.5	12	$12 - 22 = -10$	15	$-10 \times 15 = -150$
14.5-19.5	17	$17 - 22 = -5$	110	$-5 \times 110 = -550$
19.5-24.5	22 = A	$22 - 22 = 0$	135	$0 \times 135 = 0$
24.5-29.5	27	$27 - 22 = 5$	115	$5 \times 115 = 575$
29.5-34.5	32	$32 - 22 = 10$	25	$10 \times 25 = 250$
TOTAL			$\Sigma f_i = 400$	$\Sigma f_i d_i = 125$

Mean is given by

$$\text{Mean} = A + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$\Rightarrow \text{Mean} = 22 + \frac{125}{400}$$

$$\Rightarrow \text{Mean} = 22 + 0.3125$$

$$\Rightarrow \text{Mean} = 22.3125$$

Thus, mean number of oranges is 22.31.

We have used assumed mean method here, because of larger data values of f_i 's. Assumed mean method gives result to accuracy and ease when data is large.

Q. 6. The table below shows the daily expenditure on food of 25 households in a locality.

Daily expenditure (in Rupees)	100-150	150-200	200-250	250-300	300-350
Number of house holds	4	5	12	2	2

Find the mean daily expenditure on food by a suitable method.

Answer :

Let's find mean of the data using assumed mean method for ease of calculation.

Daily expenditure (Rs.)	Midpoints (x_i)	$d_i = x_i - 225$	No. of house holds (frequency) (f_i)	$f_i d_i$
100-150	125	$125 - 225 = -100$	4	$-100 \times 4 = -400$
150-200	175	$175 - 225 = -50$	5	$-50 \times 5 = -250$
200-250	225 = A	$225 - 225 = 0$	12	$0 \times 12 = 0$
250-300	275	$275 - 225 = 50$	2	$50 \times 2 = 100$
300-350	325	$325 - 225 = 100$	2	$100 \times 2 = 200$
TOTAL			$\Sigma f_i = 25$	$\Sigma x_i f_i = -350$

Mean is given by

$$\text{Mean} = A + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$\Rightarrow \text{Mean} = 225 + \frac{-350}{25}$$

$$\Rightarrow \text{Mean} = 225 - 14$$

⇒ Mean = 211

Thus, mean daily expenditure on food is 211.

Q. 7. To find out the concentration of SO_2 in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below:

Concentration of SO_2 in ppm	0.00-0.04	0.04-0.08	0.08-0.12	0.12-0.16	0.16-0.20	0.20-0.24
Frequency	4	9	9	2	4	2

Find the mean concentration of SO_2 in the air.

Answer : Let's find mean of the data using direct method as data given have smaller values.

First, construct a table for ease of calculation.

Concentration of SO_2 (ppm)	Midpoints (x_i)	Frequency (f_i)	$x_i f_i$
0.00-0.04	0.02	4	$0.02 \times 4 = 0.08$
0.04-0.08	0.06	9	$0.06 \times 9 = 0.54$
0.08-0.12	0.10	9	$0.10 \times 9 = 0.90$
0.12-0.16	0.14	2	$0.14 \times 2 = 0.28$
0.16-0.20	0.18	4	$0.18 \times 4 = 0.72$
0.20-0.24	0.22	2	$0.22 \times 2 = 0.44$
TOTAL		$\Sigma f_i = 30$	$\Sigma x_i f_i = 2.96$

Mean is given by

$$\text{Mean} = \frac{\Sigma x_i f_i}{\Sigma f_i}$$

$$\Rightarrow \text{Mean} = 2.96/30$$

$$\Rightarrow \text{Mean} = 0.0987$$

Or approximately, mean = 0.099

Thus, mean concentration of SO_2 in the air is 0.099 ppm.

Q. 8. A class teacher has the following attendance record of 40 students of a class for the whole term. Find the mean number of days a student was present out of 56 days in the term

Number of days	35-38	38-41	41-44	44-47	47-50	50-53	53-56
Number of students	1	3	4	4	7	10	11

Answer : Here, we can find mean by either direct or assumed mean method. Let's find mean of the data using direct method as data given have smaller values.

First, construct a table for ease of calculation.

No. of days	Midpoints (x_i)	No. of student (frequency) (f_i)	$x_i f_i$
35-38	36.5	1	$36.5 \times 1 = 36.5$
38-41	39.5	3	$39.5 \times 3 = 118.5$
41-44	42.5	4	$42.5 \times 4 = 170$
44-47	45.5	4	$45.5 \times 4 = 182$
47-50	48.5	7	$48.5 \times 7 = 339.5$
50-53	51.5	10	$51.5 \times 10 = 515$
53-56	54.5	11	$54.5 \times 11 = 599.5$
TOTAL		$\Sigma f_i = 40$	$\Sigma x_i f_i = 1961$

Mean is given by

$$\text{Mean} = \frac{\Sigma x_i f_i}{\Sigma f_i}$$

$$\Rightarrow \text{Mean} = 1961/40$$

$$\Rightarrow \text{Mean} = 49.025$$

Or approximately, mean = 49

Thus, mean number of days a student was present is 49.

Q. 9. The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate in %	45-55	55-65	65-75	75-85	85-95
Number of cities	3	10	11	8	3

Answer :Let's find mean of the data using direct method as data given have smaller values.

First, construct a table for ease of calculation.

Literacy rate (%)	Midpoints (x_i)	No. of cities (frequency) (f_i)	$x_i f_i$
45-55	50	3	$50 \times 3 = 150$
55-65	60	10	$60 \times 10 = 600$
65-75	70	11	$70 \times 11 = 770$
75-85	80	8	$80 \times 8 = 640$
85-95	90	3	$90 \times 3 = 270$
TOTAL		$\Sigma f_i = 35$	$\Sigma x_i f_i = 2430$

Mean is given by

$$\text{Mean} = \frac{\Sigma x_i f_i}{\Sigma f_i}$$

$$\Rightarrow \text{Mean} = 2430/35$$

$$\Rightarrow \text{Mean} = 69.429$$

Or approximately, mean = 69.43

Thus, mean literacy rate is 69.43%.

Exercise 14.2

Q. 1. The following table shows the ages of the patients admitted in a hospital during a year :

Age (in years)	5-15	15-25	25-35	35-45	45-55	55-65
Number of patients	6	11	21	23	14	5

Find the made and the mean of the data given above. Compare and interpret the two measures of central tendency.

Answer :

For mode:

Here, highest frequency is 23.

So, the modal class = 35-45

Mode is given by

$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

Where,

L = Lower class limit of the modal class = 35

h = class interval of the modal class = 10

f_1 = frequency of the modal class = 23

f_0 = frequency of the class preceding the modal class = 21

f_2 = frequency of the class succeeding the modal class = 14

Substituting values in the formula of mode,

$$\text{Mode} = 35 + \frac{23 - 21}{2 \times 23 - 21 - 14} \times 10$$

$$\Rightarrow \text{Mode} = 35 + \frac{2}{11} \times 10$$

$$\Rightarrow \text{Mode} = 35 + 20/11$$

⇒ Mode = 36.8

For mean:

Let's draw a table showing midpoints and frequencies.

Age (in yrs)	Midpoints (x_i)	No. of patients (frequency) (f_i)	$x_i f_i$
5-15	10	6	$10 \times 6 = 60$
15-25	20	11	$20 \times 11 = 220$
25-35	30	21	$30 \times 21 = 630$
35-45	40	23	$40 \times 23 = 920$
45-55	50	14	$50 \times 14 = 700$
55-65	60	5	$60 \times 5 = 300$
TOTAL		$\Sigma f_i = 80$	$\Sigma x_i f_i = 2830$

Mean is given by

$$\text{Mean} = \frac{\Sigma x_i f_i}{\Sigma f_i}$$

⇒ Mean = 2830/80

⇒ Mean = 35.37

Thus, mode is 36.8 and mean is 35.37.

The mode shows maximum number of patients admitted in the hospital is of age 36.8 years approximately and mean shows that on an average, number of patients admitted is of age 35.37 years.

Q. 2. The following data gives the information on the observed life times (in hours) of 225 electrical components :

Determine the modal lifetimes of the components.

Lifetimes (in hours)	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	10	35	52	61	38	29

Answer : For mode:

Here, highest frequency is 61.

So, the modal class = 60-80

Mode is given by

$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

Where,

L = Lower class limit of the modal class = 60

h = class interval of the modal class = 20

f_1 = frequency of the modal class = 61

f_0 = frequency of the class preceding the modal class = 52

f_2 = frequency of the class succeeding the modal class = 38

Substituting values in the formula of mode,

$$\text{Mode} = 60 + \frac{61 - 52}{2 \times 61 - 52 - 38} \times 20$$

$$\Rightarrow \text{Mode} = 60 + \frac{9}{32} \times 20$$

$$\Rightarrow \text{Mode} = 60 + 180/32$$

$$\Rightarrow \text{Mode} = 60 + 5.625$$

$$\Rightarrow \text{Mode} = 65.625$$

Thus, modal lifetime of the components is 65.625 years.

Q. 3. The following data gives the distribution of total monthly household expenditure of 200 families of Gummandidala village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure.

Expenditure (in rupees)	1000- 1500	1500- 2000	2000- 2500	2500- 3000	3000- 3500	3500- 4000	4000- 4500	4500- 5000
Number of families	24	40	33	28	30	22	16	7

Answer : For mode:

Here, highest frequency is 40.

So, the modal class = 1500-2000

Mode is given by

$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

Where,

L = Lower class limit of the modal class = 1500

h = class interval of the modal class = 500

f_1 = frequency of the modal class = 40

f_0 = frequency of the class preceding the modal class = 24

f_2 = frequency of the class succeeding the modal class = 33

Substituting values in the formula of mode,

$$\text{Mode} = 1500 + \frac{40 - 24}{2 \times 40 - 24 - 33} \times 500$$

$$\Rightarrow \text{Mode} = 1500 + \frac{16}{23} \times 500$$

$$\Rightarrow \text{Mode} = 1500 + 347.8$$

$$\Rightarrow \text{Mode} = 1847.8$$

For mean:

Using assumed-mean method, as values of the data are large.

Let's draw a table showing midpoints and frequencies.

Expenditure (Rs.)	Midpoints (x_i)	$d_i = x_i - 2750$	No. of families (frequency) (f_i)	$f_i d_i$
1000-1500	1250	$1250 - 2750 = -1500$	24	$-1500 \times 24 = -36000$
1500-2000	1750	$1750 - 2750 = -1000$	40	$-1000 \times 40 = -40000$
2000-2500	2250	$2250 - 2750 = -500$	33	$-500 \times 33 = -16500$
2500-3000	2750 = A	$2750 - 2750 = 0$	28	$0 \times 28 = 0$
3000-3500	3250	$3250 - 2750 = 500$	30	$500 \times 30 = 15000$
3500-4000	3750	$3750 - 2750 = 1000$	22	$1000 \times 22 = 22000$
4000-4500	4250	$4250 - 2750 = 1500$	16	$1500 \times 16 = 24000$
4500-5000	4750	$4750 - 2750 = 2000$	7	$2000 \times 7 = 14000$
TOTAL			$\Sigma f_i = 200$	$\Sigma f_i d_i = -17500$

Mean is given by

$$\text{Mean} = A + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$\Rightarrow \text{Mean} = 2750 + \frac{-17500}{200}$$

$$\Rightarrow \text{Mean} = 2750 - 87.5$$

$$\Rightarrow \text{Mean} = 2662.5$$

Thus, mode is 1847.8 and mean is 2662.5.

Q. 4. The following distribution gives the state-wise, teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures.

Number of students	15-20	20-25	25-30	30-35	35-40	40-45	45-50	50-55
Number of States	3	8	9	10	3	0	0	2

Answer : For mode:

Here, highest frequency is 10.

So, the modal class = 30-35

Mode is given by

$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

Where,

L = Lower class limit of the modal class = 30

h = class interval of the modal class = 5

f_1 = frequency of the modal class = 10

f_0 = frequency of the class preceding the modal class = 9

f_2 = frequency of the class succeeding the modal class = 3

Substituting values in the formula of mode,

$$\text{Mode} = 30 + \frac{10 - 9}{2 \times 10 - 9 - 3} \times 5$$

$$\Rightarrow \text{Mode} = 30 + \frac{1}{8} \times 5$$

$$\Rightarrow \text{Mode} = 30 + 0.625$$

$$\Rightarrow \text{Mode} = 30.625$$

Or approximately, mode = 30.6

For mean:

Let's solve by using direct method as values are small.

Let's draw a table showing midpoints and frequencies.

No. of students	Midpoints (x_i)	No. of states (frequency) (f_i)	$x_i f_i$
15-20	17.5	3	$17.5 \times 3 = 52.5$
20-25	22.5	8	$22.5 \times 8 = 180$
25-30	27.5	9	$27.5 \times 9 = 247.5$
30-35	32.5	10	$32.5 \times 10 = 325$
35-40	37.5	3	$37.5 \times 3 = 112.5$
40-45	42.5	0	$42.5 \times 0 = 0$
45-50	47.5	0	$47.5 \times 0 = 0$
50-55	52.5	2	$52.5 \times 2 = 105$
TOTAL		$\Sigma f_i = 35$	$\Sigma x_i f_i = 1022.5$

Mean is given by

$$\text{Mean} = \frac{\Sigma x_i f_i}{\Sigma f_i}$$

$$\Rightarrow \text{Mean} = 1022.5/35$$

$$\Rightarrow \text{Mean} = 29.2$$

Thus, mode is 30.6 and mean is 29.2.

Mode implies that most states have student teacher ratio of 30.6 and mean implies that on an average, student teacher ratio in most states is found to be 29.2.

Q. 5. The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches.

Runs	3000-4000	4000-5000	5000-6000	6000-7000	70000-8000	8000-9000	9000-10000	10000-11000
Number of batsman	4	18	9	7	6	3	1	1

Find the mode of the data.

Answer :

For mode:

Here, highest frequency is 18.

So, the modal class = 4000-5000

Mode is given by

$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

Where,

L = Lower class limit of the modal class = 4000

h = class interval of the modal class = 1000

f_1 = frequency of the modal class = 18

f_0 = frequency of the class preceding the modal class = 4

f_2 = frequency of the class succeeding the modal class = 9

Substituting values in the formula of mode,

$$\text{Mode} = 4000 + \frac{18 - 4}{2 \times 18 - 4 - 9} \times 1000$$

$$\Rightarrow \text{Mode} = 4000 + \frac{14}{23} \times 1000$$

$$\Rightarrow \text{Mode} = 4000 + 608.7$$

$$\Rightarrow \text{Mode} = 4608.7$$

Thus, the mode of the data is 4608.7.

Q. 6. A student noted the number of cars passing through a spot on a road for 100 periods, each of 3 minutes, and summarized this in the table given below.

Number of cars	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
frequency	7	14	13	12	20	11	15	8

Find the mode of the data.

Answer :

For mode: Here, highest frequency is 20.

So, the modal class = 40-50

Mode is given by

$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

Where,

L = Lower class limit of the modal class = 40

h = class interval of the modal class = 10

f_1 = frequency of the modal class = 20

f_0 = frequency of the class preceding the modal class = 12

f_2 = frequency of the class succeeding the modal class = 11

Substituting values in the formula of mode,

$$\text{Mode} = 40 + \frac{20 - 12}{2 \times 20 - 12 - 11} \times 10$$

$$\Rightarrow \text{Mode} = 40 + \frac{8}{17} \times 10$$

$$\Rightarrow \text{Mode} = 40 + 4.7$$

$$\Rightarrow \text{Mode} = 44.7$$

Thus, the mode of the data is 44.7.

Exercise 14.3

Q. 1. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption	65-85	85-105	105-125	125-145	145-165	165-185	185-205
Number of consumers	4	5	13	20	14	8	4

Answer : Let's draw a table showing midpoints, frequencies and cumulative frequency.

Monthly consumption	Midpoint (x_i)	$d_i = x_i - 135$	No. of consumers (frequency) (f_i)	$f_i d_i$	Cumulative frequency <u>cf</u>
65-85	75	$75 - 135 = -60$	4	$4 \times -60 = -240$	4
85-105	95	$95 - 135 = -40$	5	$5 \times -40 = -200$	$4 + 5 = 9$
105-125	115	$115 - 135 = -20$	13	$13 \times -20 = -260$	$9 + 13 = 22$
125-145	135 = A	$135 - 135 = 0$	20	$20 \times 0 = 0$	$22 + 20 = 42$
145-165	155	$155 - 135 = 20$	14	$14 \times 20 = 280$	$42 + 14 = 56$
165-185	175	$175 - 135 = 40$	8	$8 \times 40 = 320$	$56 + 8 = 64$
185-205	195	$195 - 135 = 60$	4	$4 \times 60 = 240$	$64 + 4 = 68$
TOTAL			$\Sigma f_i = 68$	$\Sigma f_i d_i = 140$	$N = 68$

For median:

We have, total frequency, $N = 68$

$$N/2 = 68/2 = 34$$

Observe, $cf = 42$ is just greater than 34.

Thus, median class = 125-145

Median is given by

$$\text{Median} = L + \left[\frac{\frac{N}{2} - cf}{f} \right] \times h$$

Where,

L = Lower class limit of median class = 125

$$N/2 = 34$$

cf = cumulative frequency of the class preceding median class = 22

f = frequency of the median class = 20

h = class interval of the median class = 20

Substituting these values in the formula of median, we get

$$\text{Median} = 125 + \left[\frac{34 - 22}{20} \right] \times 20$$

$$\Rightarrow \text{Median} = 125 + 12$$

$$\Rightarrow \text{Median} = 137$$

For mean:

Mean is given by

$$\text{Mean} = A + \frac{\sum f_i d_i}{\sum f_i}$$

$$\Rightarrow \text{Mean} = 135 + \frac{140}{68}$$

$$\Rightarrow \text{Mean} = 135 + 2.06$$

$$\Rightarrow \text{Mean} = 137.06$$

For mode:

Here, highest frequency is 20.

So, the modal class = 125-145

Mode is given by

$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

Where,

L = Lower class limit of the modal class = 125

h = class interval of the modal class = 20

f_1 = frequency of the modal class = 20

f_0 = frequency of the class preceding the modal class = 13

f_2 = frequency of the class succeeding the modal class = 14

Substituting values in the formula of mode,

$$\text{Mode} = 125 + \frac{20 - 13}{2 \times 20 - 13 - 14} \times 20$$

$$\Rightarrow \text{Mode} = 125 + \frac{7}{13} \times 20$$

$$\Rightarrow \text{Mode} = 125 + 10.76$$

$$\Rightarrow \text{Mode} = 135.76$$

Thus, median is 137, mean is 137.06 and mode is 135.76.

Median and mean, both gives us the average value of a data, with just the difference that mean can give us quantitative measure only but median can give us both quantitative as well as qualitative measure of a data. And that is why, mean and median have come out to be very close in this question.

Mode gives the value that appears most in a given data. Thus, the maximum monthly consumption of electricity is 135.76.

Q. 2. If the median of 60 observations, given below is 28.5, find the values of x and y.

Class interval	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	5	x	20	15	y	5

Answer :

Class interval	Frequency (fi)	Cumulative frequency cf
0-10	5	5
10-20	x	5 + x
20-30	20	5 + x + 20 = 25 + x
30-40	15	25 + x + 15 = 40 + x
40-50	y	40 + x + y
50-60	5	40 + x + y + 5 = 45 + x + y
TOTAL	$\Sigma f_i = 45 + x + y$	$N = 45 + x + y$

For median:

Since, given that: Median = 28.5

We know, median class = 20-30

Median is given by

$$\text{Median} = L + \left[\frac{\frac{N}{2} - cf}{f} \right] \times h$$

Where,

L = Lower class limit of median class = 20

$N/2 = 60/2 = 30$ [$\because$, given: total observation = 60]

cf = cumulative frequency of the class preceding median class = 5 + x

f = frequency of the median class = 20

h = class interval of the median class = 10

Substituting these values in the formula of median, we get

$$\text{Median} = 20 + \left[\frac{30 - 5 - x}{20} \right] \times 10$$

$$\Rightarrow \text{Median} = 20 + (25 - x)/2$$

$$\Rightarrow 28.5 = (40 + 25 - x)/2$$

$$\Rightarrow 57 = 65 - x$$

$$\Rightarrow x = 65 - 57 = 8$$

Recall, $N = 45 + x + y = 60$ [given]

And $x = 8$

$$\Rightarrow 45 + 8 + y = 60$$

$$\Rightarrow 53 + y = 60$$

$$\Rightarrow y = 60 - 53$$

$$\Rightarrow y = 7$$

Thus, $x = 8$ and $y = 7$.

Q. 3. A life insurance agent found the following data about distribution of ages of 100 policy holders. Calculate the median age. [Policies are given only to persons having age 18 years onwards but less than 60 years.]

Age (in years)	Below 20	Below 25	Below 30	Below 35	Below 40	Below 45	Below 50	Below 55	Below 60
Number of policy holders	2	6	24	45	78	89	92	98	100

Answer :

Age in years	Class	No. of consumers (frequency) (f_i)	Cumulative frequency cf
Below 20	15-20	2	2
Below 25	20-25	6	$2 + 6 = 8$
Below 30	25-30	24	$8 + 24 = 32$
Below 35	30-35	45	$32 + 45 = 77$
Below 40	35-40	78	$77 + 78 = 155$
Below 45	40-45	89	$155 + 89 = 244$
Below 50	45-50	92	$244 + 92 = 336$
Below 55	50-55	98	$336 + 98 = 434$
Below 60	55-60	100	$434 + 100 = 534$
TOTAL			$N = 534$

For median:

We have, total frequency, $N = 534$

$$N/2 = 534/2 = 267$$

Observe, $cf = 336$ is just greater than 267.

Thus, median class = 45-50

Median is given by

$$\text{Median} = L + \left[\frac{\frac{N}{2} - cf}{f} \right] \times h$$

Where,

L = Lower class limit of median class = 45

N/2 = 267

cf = cumulative frequency of the class preceding median class = 244

f = frequency of the median class = 92

h = class interval of the median class = 5

Substituting these values in the formula of median, we get

$$\text{Median} = 45 + \left[\frac{267 - 244}{92} \right] \times 5$$

$$\Rightarrow \text{Median} = 45 + \left[\frac{23}{92} \right] \times 5$$

$$\Rightarrow \text{Median} = 45 + 115/92$$

$$\Rightarrow \text{Median} = 45 + 1.25$$

$$\Rightarrow \text{Median} = 46.25$$

Thus, the median is 46.25.

Q. 4. The lengths of 40 leaves of a plant are measured correct to the nearest millimeter, and the data obtained is represented in the following table :

Length (in mm)	118- 126	127- 135	136- 144	145- 153	154- 162	163- 171	172- 180
Number of leaves	3	5	9	12	5	4	2

Find the median length of the leaves. (Hint : The data needs to be converted to continuous classes for finding the median, since the formula assumes

continuous classes. The classes then change to 117.5 – 126.5, 126.5 – 135.5,, 171.5 – 180.5.)

Answer : We'll convert this inclusive data into exclusive data type.

Length (mm)	No. of leaves (frequency) (f_i)	Cumulative frequency cf
117.5-126.5	3	3
126.5-136.5	5	$3 + 5 = 8$
136.5-144.5	9	$8 + 9 = 17$
144.5-153.5	12	$17 + 12 = 29$
153.5-162.5	5	$29 + 5 = 34$
162.5-171.5	4	$34 + 4 = 38$
171.5-180.5	2	$38 + 2 = 40$
		$N = 40$

For median:

We have, total frequency, $N = 40$

$$N/2 = 40/2 = 20$$

Observe, $cf = 29$ is just greater than 20.

Thus, median class = 144.5-153.5

Median is given by

$$\text{Median} = L + \left[\frac{\frac{N}{2} - cf}{f} \right] \times h$$

Where,

L = Lower class limit of median class = 144.5

$$N/2 = 20$$

cf = cumulative frequency of the class preceding median class = 17

f = frequency of the median class = 12

h = class interval of the median class = 9

Substituting these values in the formula of median, we get

$$\text{Median} = 144.5 + \left[\frac{20 - 17}{12} \right] \times 9$$

$$\Rightarrow \text{Median} = 144.5 + \left[\frac{3}{12} \right] \times 9$$

$$\Rightarrow \text{Median} = 144.5 + 9/4$$

$$\Rightarrow \text{Median} = 144.5 + 2.25$$

$$\Rightarrow \text{Median} = 146.75$$

Thus, the median is 146.75 mm.

Q. 5. The following table gives the distribution of the life-time of 400 neon lamps

Life time (in hours)	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000
Number of lamps	14	56	60	86	74	62	48

Find the median life time of a lamp.

Answer :

Life time (hours)	No. of lamps (frequency) (f_i)	Cumulative frequency cf
1500-2000	14	14
2000-2500	56	$14 + 56 = 70$
2500-3000	60	$70 + 60 = 130$
3000-3500	86	$130 + 86 = 216$
3500-4000	74	$216 + 74 = 290$
4000-4500	62	$290 + 62 = 352$
4500-5000	48	$352 + 48 = 400$
		$N = 400$

For median:

We have, total frequency, $N = 400$

$$N/2 = 400/2 = 200$$

Observe, cf = 216 is just greater than 200.

Thus, median class = 3000-3500

Median is given by

$$\text{Median} = L + \left[\frac{\frac{N}{2} - cf}{f} \right] \times h$$

Where,

L = Lower class limit of median class = 3000

$$N/2 = 200$$

cf = cumulative frequency of the class preceding median class = 130

f = frequency of the median class = 86

h = class interval of the median class = 500

Substituting these values in the formula of median, we get

$$\text{Median} = 3000 + \left[\frac{200 - 130}{86} \right] \times 500$$

$$\Rightarrow \text{Median} = 3000 + \left[\frac{70}{86} \right] \times 500$$

$$\Rightarrow \text{Median} = 3000 + 406.98$$

$$\Rightarrow \text{Median} = 3406.98$$

Thus, the median is 3406.98 hours.

Q. 6. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabet in the surnames was obtained as follows

Number of letters	1-4	4-7	7-10	10-13	13-16	16-19
Number of lamps	6	30	40	16	4	4

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames? Also, find the modal size of the surnames.

Answer :

No. of letters	Midpoint (x_i)	No. of lamps (frequency) (f_i)	$f_i x_i$	Cumulative frequency cf
1-4	2.5	6	$2.5 \times 6 = 15$	6
4-7	5.5	30	$5.5 \times 30 = 165$	$6 + 30 = 36$
7-10	8.5	40	$8.5 \times 40 = 340$	$36 + 40 = 76$
10-13	11.5	16	$11.5 \times 16 = 184$	$76 + 16 = 92$
13-16	14.5	4	$14.5 \times 4 = 58$	$92 + 4 = 96$
16-19	17.5	4	$17.5 \times 4 = 70$	$96 + 4 = 100$
TOTAL		$\Sigma f_i = 100$	$\Sigma f_i x_i = 832$	$N = 100$

For median:

We have, total frequency, $N = 100$

$$N/2 = 100/2 = 50$$

Observe, cf = 76 is just greater than 50.

Thus, median class = 7-10

Median is given by

$$\text{Median} = L + \left[\frac{\frac{N}{2} - cf}{f} \right] \times h$$

Where,

L = Lower class limit of median class = 7

N/2 = 50

cf = cumulative frequency of the class preceding median class = 36

f = frequency of the median class = 40

h = class interval of the median class = 3

Substituting these values in the formula of median, we get

$$\text{Median} = 7 + \left[\frac{50 - 36}{40} \right] \times 3$$

$$\Rightarrow \text{Median} = 7 + \left[\frac{14}{40} \right] \times 3$$

$$\Rightarrow \text{Median} = 7 + 1.05$$

$$\Rightarrow \text{Median} = 8.05$$

For mean:

Mean is given by

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$$

$$\Rightarrow \text{Mean} = \frac{832}{100} \text{ [Refer the values from the table given above]}$$

$$\Rightarrow \text{Mean} = 8.32$$

For mode:

Here, highest frequency is 40.

So, the modal class = 7-10

Mode is given by

$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

Where,

L = Lower class limit of the modal class = 7

h = class interval of the modal class = 3

f_1 = frequency of the modal class = 40

f_0 = frequency of the class preceding the modal class = 30

f_2 = frequency of the class succeeding the modal class = 16

Substituting values in the formula of mode,

$$\text{Mode} = 7 + \frac{40 - 30}{2 \times 40 - 30 - 16} \times 3$$

$$\Rightarrow \text{Mode} = 7 + \frac{10}{34} \times 3$$

$$\Rightarrow \text{Mode} = 7 + 0.88$$

$$\Rightarrow \text{Mode} = 7.88$$

Thus, median is 8.05, mean is 8.32 and mode is 7.88.

Q. 7. The distribution below gives the weights of 30 students of a class. Find the median weight of the students.

Weight (in kg)	40- 45	45- 50	50- 55	55- 60	60- 65	65- 70	70- 75
Number of students	2	3	8	6	6	3	2

Answer :

Weight (kg)	No. of students (frequency) (f_i)	Cumulative frequency cf
40-45	2	2
45-50	3	$2 + 3 = 5$
50-55	8	$5 + 8 = 13$
55-60	6	$13 + 6 = 19$
60-65	6	$19 + 6 = 25$
65-70	3	$25 + 3 = 28$
70-75	2	$28 + 2 = 30$
TOTAL		$N = 30$

For median:

We have, total frequency, $N = 30$

$$N/2 = 30/2 = 15$$

Observe, cf = 19 is just greater than 15.

Thus, median class = 55-60

Median is given by

$$\text{Median} = L + \left[\frac{\frac{N}{2} - cf}{f} \right] \times h$$

Where,

L = Lower class limit of median class = 55

$$N/2 = 15$$

cf = cumulative frequency of the class preceding median class = 13

f = frequency of the median class = 6

h = class interval of the median class = 5

Substituting these values in the formula of median, we get

$$\text{Median} = 55 + \left[\frac{15 - 13}{6} \right] \times 5$$

$$\Rightarrow \text{Median} = 55 + \left[\frac{2}{6} \right] \times 5$$

$$\Rightarrow \text{Median} = 55 + 1.67$$

$$\Rightarrow \text{Median} = 56.67$$

Thus, the median is 56.67 kg.

Q. 1. The following distribution gives the daily income of 50 workers of a factory.

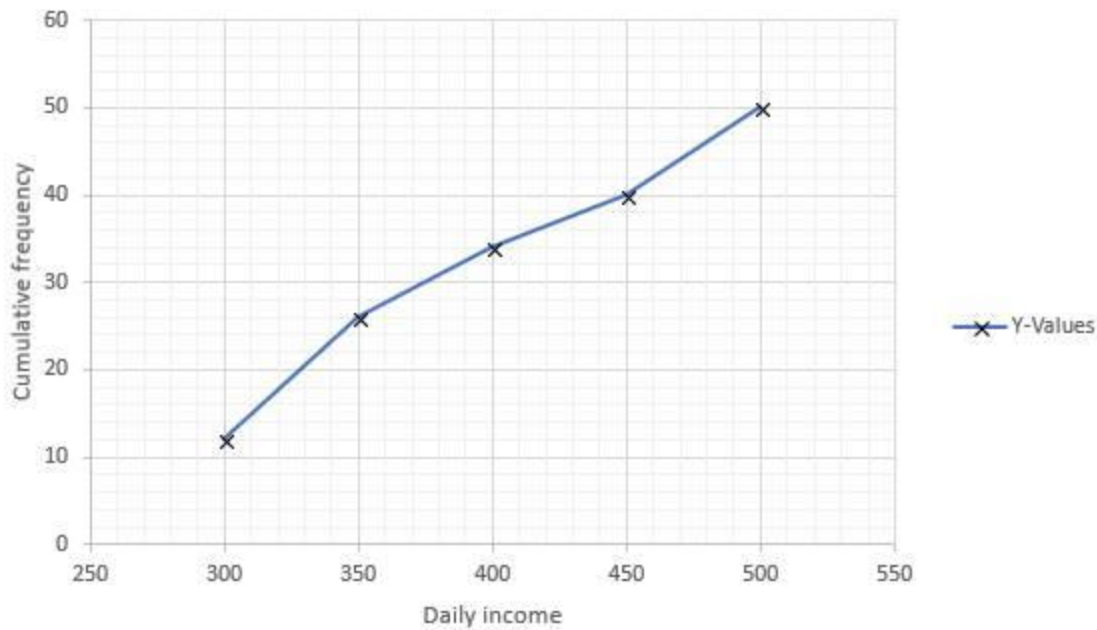
Daily income (in Rupees)	250-300	300-350	350-400	400-450	450-500
Number of workers	12	14	8	6	10

Convert the distribution above to a less than type cumulative frequency distribution, and draw its ogive.

Answer : Let's make less than type cumulative frequency table:

Daily income (less than type)	Cumulative frequency cf
Less than 300	12
Less than 350	12 + 14 = 26
Less than 400	26 + 8 = 34
Less than 450	34 + 6 = 40
Less than 500	40 + 10 = 50

Plot points (300,12), (350,26), (400, 34), (450, 40) and (500,50) on a graph.

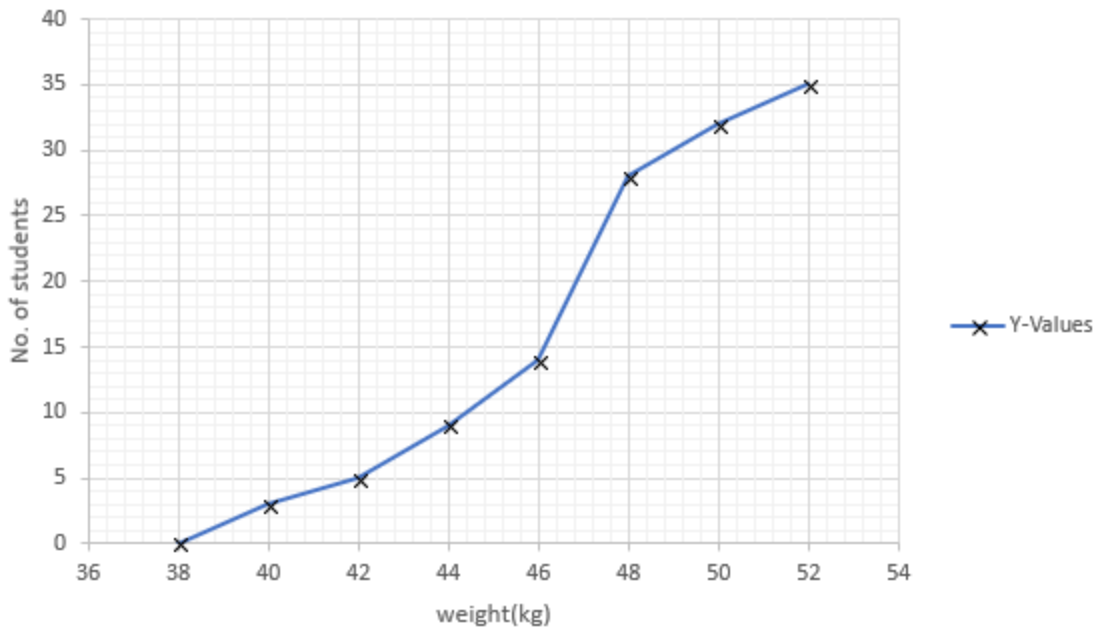


Q. 2. During the medical check-up of 35 students of a class, their weights were recorded as follows :

Weight (in kg)	Number of students
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

Draw a less than type ogive for the given data. Hence obtain the median weight from the graph and verify the result by using the formula.

Answer : Using points (38,0), (40,3), (42,5), (44,9), (46,14), (48,28), (50, 32) and (52,35), plot a graph.

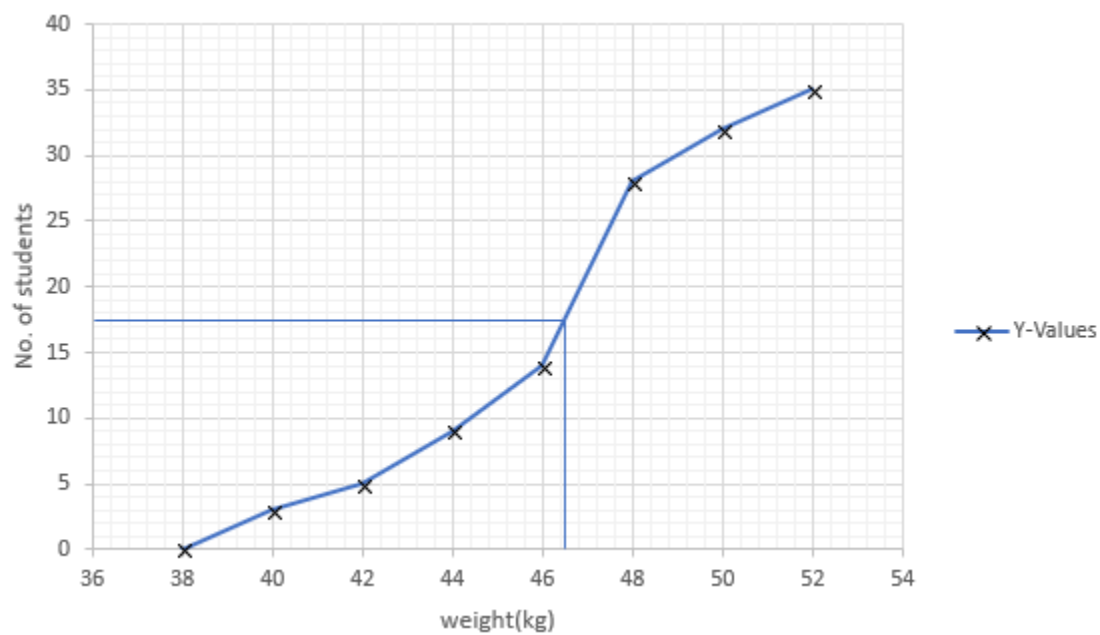


To find median graphically,

We have total frequency, $N = 35$

$$\Rightarrow N/2 = 35/2 = 17.5$$

Plot 17.5 on the y-axis and draw a horizontal line intersecting the ogive parallel to x-axis.



Observe, from the graph the vertical line parallel to y-axis touches y-axis at 46.5 (approx.).

Hence, median is 46.5.

Finding median by formula:

Weight (kg)	Cumulative frequency cf	Class	Frequency (f _i)
Less than 38	0	36-38	0
Less than 40	3	38-40	3 - 0 = 3
Less than 42	5	40-42	5 - 3 = 2
Less than 44	9	42-44	9 - 5 = 4
Less than 46	14	44-46	14 - 9 = 5
Less than 48	28	46-48	28 - 14 = 14
Less than 50	32	48-50	32 - 28 = 4
Less than 52	35	50-52	35 - 32 = 3
	N = 35		

For median:

We have, total frequency, N = 35

$$N/2 = 35/2 = 17.5$$

Observe, cf = 28 is just greater than 17.5.

Thus, median class = 46-48

Median is given by

$$\text{Median} = L + \left[\frac{\frac{N}{2} - cf}{f} \right] \times h$$

Where,

L = Lower class limit of median class = 46

$$N/2 = 17.5$$

cf = cumulative frequency of the class preceding median class = 14

f = frequency of the median class = 14

h = class interval of the median class = 2

Substituting these values in the formula of median, we get

$$\text{Median} = 46 + \left[\frac{17.5 - 14}{14} \right] \times 2$$

$$\Rightarrow \text{Median} = 46 + \left[\frac{3.5}{14} \right] \times 2$$

$$\Rightarrow \text{Median} = 46 + 0.5$$

$$\Rightarrow \text{Median} = 46.5$$

Thus, median is 46.5.

Hence, verified.

Q. 3. the following table gives production yield per hectare of wheat of 100 farms of a village.

Production yield (Qui/Hec)	50-55	55-60	60-65	65-70	70-75	75-80
Number of farmers	2	8	12	24	38	16

Change the distribution to a more than type distribution and draw its ogive.

Answer : Let's make more than type cumulative frequency table:

Production yield (more than type)	Cumulative frequency cf
More than 50	$2 + 8 + 12 + 24 + 38 + 16 = 100$
More than 55	$100 - 2 = 98$
More than 60	$98 - 8 = 90$
More than 65	$90 - 12 = 78$
More than 70	$78 - 24 = 54$
More than 75	$54 - 38 = 16$

Plot points (50,100), (55,98), (60, 90), (65, 78), (70, 54) and (75,16) on a graph.

