

# 10. Electrostatics

- **Charging by induction:** This is a process of charging an uncharged body with the help of a charged body without any actual contact.
- A charge of opposite sign develops on uncharged body.

## Basic Properties of Electric Charges

- The total electric charge on an object is equal to the algebraic sum of all the electric charges distributed on the different parts of the object
- The total charge of an isolated system remains constant with time.
- All observable charges are always some integral multiple of elementary charge,  $e$  ( $= \pm 1.6 \times 10^{-19} \text{ C}$ )

## Coulomb's Law

- Two point charges attract or repel each other with a force which is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them.
- $F = K \frac{q_1 q_2}{r^2}$ ,  $K = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}$

## Principle of Superposition

- It is based on the property that the forces with which two charges attract or repel each other are not affected by the presence of a third (or more) additional charge(s).
- The total force on a given charge due to number of charges is equal to the vector sum of the individual forces exerted on the given charge by all the other charges.

## Electric Field

- It is the space around a charge, in which any other charge experiences an electrostatic force.

## Electric Field Intensity

- The electric field intensity at a point due to a source charge is defined as the force experienced per unit positive test charge placed at that point without disturbing the source charge.
- Electric field due to a point charge at distance  $r$  from it is  $E = \frac{q}{4\pi\epsilon_0 r^2}$
- Electric field due to a number of charges is found by adding the individual electric fields vectorially.

## Electric Field Lines

- An electric line of force is the path along which a unit positive charge would move, if it is free to do so.
- Properties of electric field lines

- They are continuous curves without any breaks
- They cannot cross each other.
- They cannot form closed loops.

### Continuous Charge Distribution

- **Linear charge density** – When charge is distributed along a line then charge density is given by  $\lambda$ .

- $\lambda = \frac{q}{L}$

- **Surface charge density** – When charge is distributed along a surface, the charge density is given by  $\sigma$ .

- $\sigma = \frac{q}{A}$

- **Volume charge density** – When charge is distributed along a volume, the charge density is given by  $\delta$ .

$$\delta = \frac{q}{V}$$

### Electric Dipole

- System of two equal and opposite charges separated by a certain small distance.

### Electric Dipole Moment

- It is a vector quantity, with magnitude equal to the product of either of the charges and the length of the electric dipole and direction from the negative charge to the positive charge.

$$\vec{p} = q(2a)$$

### Electric Field on Axial Line of an Electric Dipole

$$E = \frac{1}{4\pi\epsilon_0} \frac{2p}{r^3}$$

### Electric Field for Points on the Equatorial Plane

$$E = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3}$$

### Dipole in a Uniform External Field

In a uniform electric field  $E$ , a dipole experiences a torque  $\tau$ , due to two equal and unlike parallel forces acting on dipole

$$\tau = \text{Force} \times \text{Perpendicular distance between the two forces}$$

$$\tau = pE \sin\theta$$

$$\therefore \vec{\tau} = \vec{p} \times \vec{E}$$

### Electrostatic Potential energy

- Work done by an external force in bringing a charge  $q$  from a point R to a point P in electric field of a certain charge configuration is UP-UR, which is the difference in potential energy of charge  $q$  between the final and initial points.
- Potential energy at a point is the work done by an external force in moving a charge from infinity to that point.

### Electrostatic Potential

- Electrostatic potential at any point in a region of electrostatic field is the minimum work done in carrying a unit positive charge (without acceleration) from infinity to that point.
- Electric potential due to a point charge of magnitude  $q$  at a distance  $r$  from the charge is given as  $V = \frac{q}{4\pi\epsilon_0 r}$
- Potential difference between two points P and R can be written as  $V_P - V_R = \frac{UP - UR}{q}$

### Equipotential Surfaces

An equipotential surface is that surface at every point of which, the electric potential is same.

No work is done in moving a test charge from one point of the equipotential surface to the other.

The equipotential surface for a point charge are concentric spherical surfaces centered at the charge.

### Potential due to a System of Charges

- For a system of point charges  $q_1, q_2, q_3, \dots, q_n$  at distances  $r_1, r_2, r_3, \dots, r_n$  respectively from the point P,
- The potential at a point P is given by the superposition principle

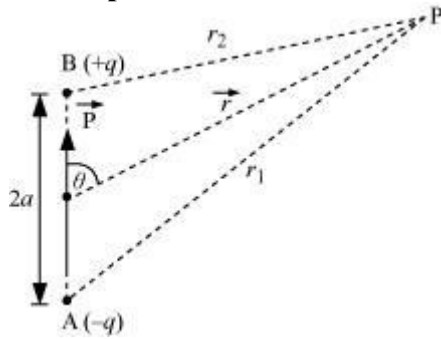
$$V = \frac{1}{4\pi\epsilon_0} \left( \frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \dots + \frac{q_n}{r_n} \right)$$

$$V = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i}$$

### Dipole

- A dipole is a system of two charges of equal magnitude  $q$  and opposite polarity separated by a distance  $2a$ . The dipole moment of the dipole is  $\vec{p} \rightarrow$  with magnitude  $p = q \times 2a$  and direction -  $q$  to  $+q$ .

## Potential due to dipole.



- Potential at point  $P$  due to charge at point  $A$  is given as:

$$V_1 = \frac{-q}{4\pi\epsilon_0 r_1}$$

- Potential at point  $P$  due to charge at point  $B$  is given as:

$$V_2 = \frac{q}{4\pi\epsilon_0 r_2}$$

- Total Potential at point  $P$  due to dipole

$$V = V_1 + V_2$$

$$V = \frac{q}{4\pi\epsilon_0} \left[ \frac{1}{r_2} - \frac{1}{r_1} \right]$$

$V$  at position vector  $\vec{r}$  can be expressed as

$$\boxed{V = \frac{\vec{p} \cdot \hat{r}}{4\pi\epsilon_0 r^2}} \quad \text{for } (r \gg a)$$

## Potential Energy of a Single Charge

It is the work done in bringing a charge  $q$  from infinity to a point  $P$  whose position vector is  $\vec{r}$  and  $V(\vec{r})$  is potential due to external field there.

The magnitude of work done  $= q \cdot V(\vec{r})$ .

## Potential Energy of a System of Two Charges in an External Field

- It is the sum of the work done in bringing  $q_1$  and  $q_2$  from infinity to  $\vec{r}_1$  and  $\vec{r}_2$  respectively and assembling the charges at their respective locations.

$$U = W_1 + W_2 + W_3$$

$$U = q_1 \cdot V(\vec{r}_1) + q_2 \cdot V(\vec{r}_2) + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$$

### Potential Energy of an Electric Dipole, When Placed in a Uniform Electric Field

- The potential energy of an electric dipole in a uniform electric field is given as  $U = - \vec{p} \cdot \vec{E}$

where  $\vec{p}$  = dipole moment of the dipole

$\vec{E}$  = strength of external electric field