

# CAT 2017 Question Paper Slot 2

## Quant

67. The numbers 1, 2, ..., 9 are arranged in a 3 X 3 square grid in such a way that each number occurs once and the entries along each column, each row, and each of the two diagonals add up to the same value. If the top left and the top right entries of the grid are 6 and 2, respectively, then the bottom middle entry is
68. In a 10 km race, A, B, and C, each running at uniform speed, get the gold, silver, and bronze medals, respectively. If A beats B by 1 km and B beats C by 1 km, then by how many metres does A beat C?
69. Bottle 1 contains a mixture of milk and water in 7: 2 ratio and Bottle 2 contains a mixture of milk and water in 9: 4 ratio. In what ratio of volumes should the liquids in Bottle 1 and Bottle 2 be combined to obtain a mixture of milk and water in 3:1 ratio?
- A 27:14
- B 27:13
- C 27:16
- D 27:18
70. Arun drove from home to his hostel at 60 miles per hour. While returning home he drove half way along the same route at a speed of 25 miles per hour and then took a bypass road which increased his driving distance by 5 miles, but allowed him to drive at 50 miles per hour along this bypass road. If his return journey took 30 minutes more than his onward journey, then the total distance traveled by him is
- A 55 miles
- B 60 miles
- C 65 miles
- D 70 miles
71. Out of the shirts produced in a factory, 15% are defective, while 20% of the rest are sold in the domestic market. If the remaining 8840 shirts are left for export, then the number of shirts produced in the factory is
- A 13600
- B 13000
- C 13400
- D 14000
72. The average height of 22 toddlers increases by 2 inches when two of them leave this group. If the average height of these two toddlers is one-third the average height of the original 22, then the average height, in inches, of the remaining 20 toddlers is

- A 30
- B 28
- C 32
- D 26

73. The manufacturer of a table sells it to a wholesale dealer at a profit of 10%. The wholesale dealer sells the table to a retailer at a profit of 30% Finally, the retailer sells it to a customer at a profit of 50%. If the customer pays Rs 4290 for the table, then its manufacturing cost (in Rs) is

- A 1500
- B 2000
- C 2500
- D 3000

74. A tank has an inlet pipe and an outlet pipe. If the outlet pipe is closed then the inlet pipe fills the empty tank in 8 hours. If the outlet pipe is open then the inlet pipe fills the empty tank in 10 hours. If only the outlet pipe is open then in how many hours the full tank becomes half-full?

- A 20
- B 30
- C 40
- D 45

75. Mayank buys some candies for Rs 15 a dozen and an equal number of different candies for Rs 12 a dozen. He sells all for Rs 16.50 a dozen and makes a profit of Rs 150. How many dozens of candies did he buy altogether?

- A 50
- B 30
- C 25
- D 45

76. In a village, the production of food grains increased by 40% and the per capita production of food grains increased by 27% during a certain period. The percentage by which the population of the village increased during the same period is nearest to
- A 16  
B 13  
C 10  
D 7
77. If  $a, b, c$  are three positive integers such that  $a$  and  $b$  are in the ratio 3 : 4 while  $b$  and  $c$  are in the ratio 2:1, then which one of the following is a possible value of  $(a + b + c)$ ?
- A 201  
B 205  
C 207  
D 210
78. A motorbike leaves point A at 1 pm and moves towards point B at a uniform speed. A car leaves point B at 2 pm and moves towards point A at a uniform speed which is double that of the motorbike. They meet at 3:40 pm at a point which is 168 km away from A. What is the distance, in km, between A and B?
- A 364  
B 378  
C 380  
D 388
79. Amal can complete a job in 10 days and Bimal can complete it in 8 days. Amal, Bimal and Kamal together complete the job in 4 days and are paid a total amount of Rs 1000 as remuneration. If this amount is shared by them in proportion to their work, then Kamal's share, in rupees, is
- A 100  
B 200  
C 300  
D 400
80. Consider three mixtures — the first having water and liquid A in the ratio 1:2, the second having water and liquid B in the ratio 1:3, and the third having water and liquid C in the ratio 1:4. These three mixtures of A, B, and C, respectively, are further mixed in the proportion 4: 3: 2. Then the resulting mixture has

- A The same amount of water and liquid B
- B The same amount of liquids B and C
- C More water than liquid B
- D More water than liquid A

81. Let ABCDEF be a regular hexagon with each side of length 1 cm. The area (in sq cm) of a square with AC as one side is
- A  $3\sqrt{2}$
  - B 3
  - C 4
  - D  $\sqrt{3}$
82. The base of a vertical pillar with uniform cross section is a trapezium whose parallel sides are of lengths 10 cm and 20 cm while the other two sides are of equal length. The perpendicular distance between the parallel sides of the trapezium is 12 cm. If the height of the pillar is 20 cm, then the total area, in sq cm, of all six surfaces of the pillar is
- A 1300
  - B 1340
  - C 1480
  - D 1520
83. The points (2, 5) and (6, 3) are two end points of a diagonal of a rectangle. If the other diagonal has the equation  $y = 3x + c$ , then c is
- A -5
  - B -6
  - C -7
  - D -8
84. ABCD is a quadrilateral inscribed in a circle with centre O such that O lies inside the quadrilateral. If  $\angle COD = 120$  degrees and  $\angle BAC = 30$  degrees, then the value of  $\angle BCD$  (in degrees) is
85. If three sides of a rectangular park have a total length 400 ft, then the area of the park is maximum when the length (in ft) of its longer side is
86. Let P be an interior point of a right-angled isosceles triangle ABC with hypotenuse AB. If the perpendicular distance of P from each of AB, BC, and CA is  $4(\sqrt{2} - 1)$  cm, then the area, in sq cm, of the triangle ABC is

87. If the product of three consecutive positive integers is 15600 then the sum of the squares of these integers is

- A 1777
- B 1785
- C 1875
- D 1877

88. If  $x$  is a real number such that  $\log_3 5 = \log_5(2 + x)$ , then which of the following is true?

- A  $0 < x < 3$
- B  $23 < x < 30$
- C  $x > 30$
- D  $3 < x < 23$

89. Let  $f(x) = x^2$  and  $g(x) = 2^x$ , for all real  $x$ . Then the value of  $f[f(g(x)) + g(f(x))]$  at  $x = 1$  is

- A 16
- B 18
- C 36
- D 40

90. The minimum possible value of the sum of the squares of the roots of the equation  $x^2 + (a + 3)x - (a + 5) = 0$  is

- A 1
- B 2
- C 3
- D 4

91. If  $9^{x-\frac{1}{2}} - 2^{2x-2} = 4^x - 3^{2x-3}$ , then  $x$  is

- A  $\frac{3}{2}$
- B  $\frac{2}{5}$
- C  $\frac{3}{4}$
- D  $\frac{4}{9}$

92. If  $\log(2^a \times 3^b \times 5^c)$  is the arithmetic mean of  $\log(2^2 \times 3^3 \times 5)$ ,  $\log(2^6 \times 3 \times 5^7)$ , and  $\log(2 \times 3^2 \times 5^4)$ , then  $a$  equals

93. Let  $a_1, a_2, a_3, a_4, a_5$  be a sequence of five consecutive odd numbers. Consider a new sequence of five consecutive even numbers ending with  $2a_3$ .  
If the sum of the numbers in the new sequence is 450, then  $a_5$  is

94. How many different pairs  $(a, b)$  of positive integers are there such that  $a \geq b$  and  $\frac{1}{a} + \frac{1}{b} = \frac{1}{9}$ ?

95. In how many ways can 8 identical pens be distributed among Amal, Bimal, and Kamal so that Amal gets at least 1 pen, Bimal gets at least 2 pens, and Kamal gets at least 3 pens?

96. How many four digit numbers, which are divisible by 6, can be formed using the digits 0, 2, 3, 4, 6, such that no digit is used more than once and 0 does not occur in the left-most position?

97. If  $f(ab) = f(a)f(b)$  for all positive integers  $a$  and  $b$ , then the largest possible value of  $f(1)$  is

98. Let  $f(x) = 2x - 5$  and  $g(x) = 7 - 2x$ . Then  $|f(x) + g(x)| = |f(x)| + |g(x)|$  if and only if

- A  $\frac{5}{2} < x < \frac{7}{2}$
- B  $x \leq \frac{5}{2}$  or  $x \geq \frac{7}{2}$
- C  $x < \frac{5}{2}$  or  $x \geq \frac{7}{2}$
- D  $\frac{5}{2} \leq x \leq \frac{7}{2}$

99. An infinite geometric progression  $a_1, a_2, \dots$  has the property that  $a_n = 3(a_{n+1} + a_{n+2} + \dots)$  for every  $n \geq 1$ . If the sum  $a_1 + a_2 + a_3 + \dots = 32$ , then  $a_5$  is

- A  $1/32$
- B  $2/32$
- C  $3/32$
- D  $4/32$

100. If  $a_1 = \frac{1}{2 \times 5}, a_2 = \frac{1}{5 \times 8}, a_3 = \frac{1}{8 \times 11}, \dots$ , then  $a_1 + a_2 + a_3 + \dots + a_{100}$  is

- A  $\frac{25}{151}$
- B  $\frac{1}{2}$
- C  $\frac{1}{4}$
- D  $\frac{111}{55}$

## Answers

### Quant

67. <b>3</b>	68. <b>1900</b>	69. <b>B</b>	70. <b>C</b>	71. <b>B</b>	72. <b>C</b>	73. <b>B</b>	74. <b>A</b>
75. <b>A</b>	76. <b>C</b>	77. <b>C</b>	78. <b>B</b>	79. <b>A</b>	80. <b>C</b>	81. <b>B</b>	82. <b>C</b>
83. <b>D</b>	84. <b>90</b>	85. <b>200</b>	86. <b>16</b>	87. <b>D</b>	88. <b>D</b>	89. <b>C</b>	90. <b>C</b>
91. <b>A</b>	92. <b>3</b>	93. <b>51</b>	94. <b>3</b>	95. <b>6</b>	96. <b>50</b>	97. <b>1</b>	98. <b>D</b>
99. <b>C</b>	100. <b>A</b>						

## Explanations

### Quant

67.3

According to the question each column, each row, and each of the two diagonals of the 3X3 matrix add up to the same value. This value must be 15.

Let us consider the matrix as shown below:

6		2

Now we'll try substituting values from 1 to 9 in the exact middle grid shown as 'x'.

If  $x = 1$  or  $3$ , then the value in the left bottom grid will be more than 9 which is not possible.

$x$  cannot be equal to 2.

If  $x = 4$ , value in the left bottom grid will be 9. But then addition of first column will come out to be more than 15. Hence, not possible.

If  $x=5$ , we get the grid as shown below:

6	7	2
1	5	9
8	3	4

Hence, for  $x = 5$  all conditions are satisfied. We see that the bottom middle entry is 3.

Hence, 3 is the correct answer.

68. 1900

By the time A traveled 10 KM, B traveled 9 KM

Hence  $Speed_A : Speed_B = 10 : 9$

Similarly  $Speed_B : Speed_C = 10 : 9$

Hence  $Speed_A : Speed_B : Speed_C = 100 : 90 : 81$

Hence by the time A traveled 10 KMs, C should have traveled 8.1 KMs

So A beat C by 1.9 KMs = 1900 Mts

69. B

The ratio of milk and water in Bottle 1 is 7:2 and the ratio of milk and water in Bottle 2 is 9:4

Therefore, the proportion of milk in Bottle 1 is  $\frac{7}{9}$  and the proportion of milk in Bottle 2 is  $\frac{9}{13}$

Let the ratio in which they should be mixed be equal to  $X:1$ .

Hence, the total volume of milk is  $\frac{7X}{9} + \frac{9}{13}$

The total volume of water is  $\frac{2X}{9} + \frac{4}{13}$

They are in the ratio 3:1

Hence,  $\frac{7X}{9} + \frac{9}{13} = 3 * (\frac{2X}{9} + \frac{4}{13})$

Therefore,  $91X + 81 = 78X + 108$

Therefore  $X = \frac{27}{13}$



70. **C**

Let the distance between the home and office be  $2x$  miles

Time taken for going in the morning =  $\frac{2x}{60}$  hrs

Time taken for going back in the evening =  $\frac{x}{25} + \frac{x+5}{50}$  hrs

It is given that he took 30 minutes (0.5 hrs) more in the evening

Hence  $\frac{2x}{60}$  hrs + 0.5 =  $\frac{x}{25} + \frac{x+5}{50}$

Solving for  $x$ , we get  $x = 15$  miles.

Total distance traveled =  $2x + x + x + 5 = 4x + 5 = 65$  Miles

71. **B**

Let the total number of shirts be  $x$ . Hence number of non defective shirts =  $x - 15\%$  of  $x = 0.85x$

Number of shirts left for export = No of non defective shirts - number of shirts sold in domestic market

= No of non defective shirts - 20% of No of non defective shirts

= 80% of No of non defective shirts

Hence  $8840 = 0.8 * (0.85x)$ . Solving for  $x$  we get,  $x = 13000$

72. **C**

Let the average height of 22 toddlers be  $3x$ .

Sum of the height of 22 toddlers =  $66x$

Hence average height of the two toddlers who left the group =  $x$

Sum of the height of the remaining 20 toddlers =  $66x - 2x = 64x$

Average height of the remaining 20 toddlers =  $64x/20 = 3.2x$

Difference =  $0.2x = 2$  inches  $\Rightarrow x = 10$  inches

Hence average height of the remaining 20 toddlers =  $3.2x = 32$  inches

73. **B**

Let the manufacturing price of the table =  $x$

Hence the price at which the wholesaler bought from the manufacturer =  $1.1 \times x$

The price at which the retailer bought from the wholesaler =  $1.3 \times 1.1 \times x$

The price at which the customer bought from the retailer =  $1.5 \times 1.3 \times 1.1 \times x$

$1.5 \times 1.3 \times 1.1 \times x = 4290$

$\Rightarrow x = 2000$

74. **A**

Let the time taken by the outlet pipe to empty =  $x$  hours

Then,  $\frac{1}{8} - \frac{1}{x} = \frac{1}{10}$

$\Rightarrow x = 40$

Hence time taken by the outlet pipe to make the tank half-full =  $40/2 = 20$  hour

75. **A**

Let the number of dozens of candies he bought of each variety be  $x$

Hence total cost =  $12x + 15x = 27x$

Total selling price =  $16.50 * 2x = 33x$

Profit =  $33x - 27x = 6x$

Given  $6x = 150 \Rightarrow x = 25$

Hence he bought 50 dozens of candies in total

76. **C**

Let initial population and production be  $x, y$  and final population be  $z$

Final production =  $1.4y$ , final percapita = 1.27 times initial percapita

$\Rightarrow \frac{1.4y}{z} = 1.27 \times \frac{y}{x}$

$\Rightarrow \frac{z}{x} = \frac{1.4}{1.27} \approx 1.10$

Hence the percentage increase in population = 10%

77. **C**

$a : b = 3:4$  and  $b : c = 2:1 \Rightarrow a:b:c = 3:4:2$

$\Rightarrow a = 3x, b = 4x, c = 2x$

$\Rightarrow a + b + c = 9x$

$\Rightarrow a + b + c$  is a multiple of 9.

From the given options only, option C is a multiple of 9

78. **B**

Let the distance traveled by the car be  $x$  KMs

Distance traveled by the bike = 168 KMs

Speed of car is double the speed of bike

$$\Rightarrow \frac{x}{3:40-2:00} = 2 \times \frac{168}{3:40-1:00}$$

$$\Rightarrow \frac{x}{100} = 2 \times \frac{168}{160}$$

$$\Rightarrow x = 210$$

Hence the distance between A and B is  $x + 168 = 378$  KMs

79. **A**

Let the time take by kamal to complete the task be  $x$  days.

$$\text{Hence we have } \frac{1}{10} + \frac{1}{8} + \frac{1}{x} = \frac{1}{4}$$

$$\Rightarrow x = 40 \text{ days.}$$

$$\text{Ratio of the work done by them} = \frac{1}{10} : \frac{1}{8} : \frac{1}{40} = 4 : 5 : 1$$

$$\text{Hence the wage earned by Kamal} = \frac{1}{10} * 1000 = 100$$

80. **C**

The proportion of water in the first mixture is  $\frac{1}{3}$

The proportion of Liquid A in the first mixture is  $\frac{2}{3}$

The proportion of water in the second mixture is  $\frac{1}{4}$

The proportion of Liquid B in the second mixture is  $\frac{3}{4}$

The proportion of water in the third mixture is  $\frac{1}{5}$

The proportion of Liquid C in the third mixture is  $\frac{4}{5}$

$$\text{As they are mixed in the ratio } 4:3:2, \text{ the final amount of water is } 4 \times \frac{1}{3} + 3 \times \frac{1}{4} + 2 \times \frac{1}{5} = \frac{149}{60}$$

$$\text{The final amount of Liquid A in the mixture is } 4 \times \frac{2}{3} = \frac{8}{3}$$

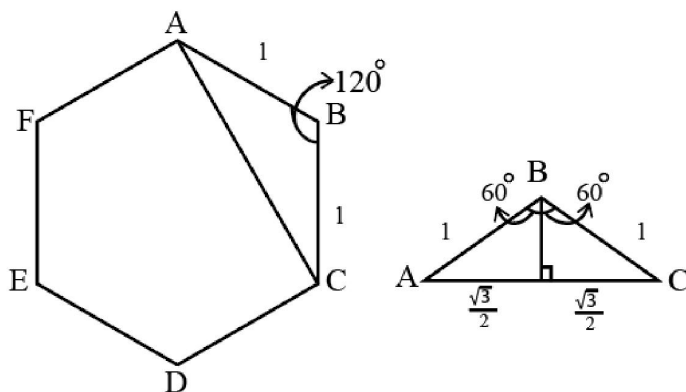
$$\text{The final amount of Liquid B in the mixture is } 3 \times \frac{3}{4} = \frac{9}{4}$$

$$\text{The final amount of Liquid C in the mixture is } 2 \times \frac{4}{5} = \frac{8}{5}$$

$$\text{Hence, the ratio of Water : A : B : C in the final mixture is } \frac{149}{60} : \frac{8}{3} : \frac{9}{4} : \frac{8}{5} = 149 : 160 : 135 : 96$$

From the given choices, only option C is correct.

81. **B**



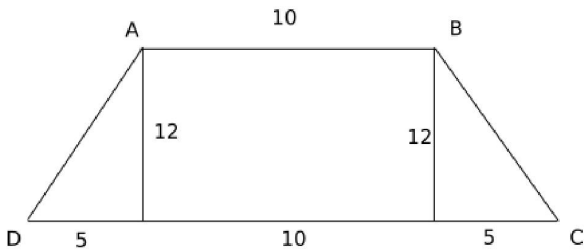
The length of the diagonals of a regular hexagon with side  $s$  are  $\sqrt{3}s$ .

Here length of  $AC = \sqrt{3}s = \sqrt{3}$  cms

Hence area of the square =  $\sqrt{3}^2 = 3$  sq cm

82. **C**

See the diagram below



Length of side  $AD = \sqrt{12^2 + 5^2} = 13$

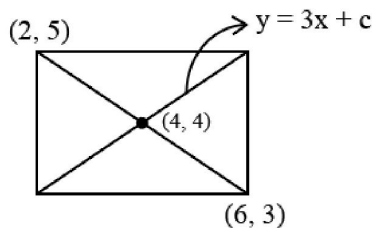
Area of the trapezium =  $12 * (10 + 20)/2 = 180$

Perimeter of the trapezium =  $10 + 20 + 13 + 13 = 56$

Area of the sides of the pillar =  $56 * \text{height} = 56 * 20 = 1120$ .

Total area of the pillar =  $1120 + \text{area of base} + \text{area of the top} = 1120 + 180 + 180 = 1480$

83. **D**



The midpoint of one diagonal lies on the other diagonal.

Midpoint is  $((2+6)/2, (5+3)/2) = (4, 4)$

Hence  $4 = 3 * 4 + c \Rightarrow c = -8$

84. **90**

$\angle COD = 120 \Rightarrow \angle CAD = 120/2 = 60$  (The angle subtended by the chord DC at the major arc is half the angle subtended at the centre of the circle.)

$\angle BAC = 30$

$\angle BAD = \angle BAC + \angle CAD = 30 + 60 = 90$ .

$\angle BCD = 180 - \angle BAD = 180 - 90 = 90$

85. **200**

Let the length and breadth of the park be  $l, b, l > b$

Case 1:  $2l + b = 400$

Area =  $lb$ . Area is maximum when  $2l * b$  is maximum, which is maximum when  $2l = b$  (using  $AM \geq GM$  inequality)  $\Rightarrow l = 100, b = 200$ . Which can't happen since  $l > b$

Case 2:  $l + 2b = 400$

Area =  $lb$ . Area is maximum when  $l * 2b$  is maximum, which is maximum when  $l = 2b$  (using  $AM \geq GM$  inequality)  $\Rightarrow l = 200, b = 100$ .

Hence length of the longer side is 200 ft

86. 16

Let the length of non-hypotenuse sides of the right angled triangle be  $a$ . Then the hypotenuse  $h = \sqrt{2}a$

P is equidistant from all the side of the triangle. Hence P is the incenter and the perpendicular distance is the inradius.

In a right angled triangle, inradius =  $\frac{a+b-h}{2}$

$$\Rightarrow \frac{a+a-\sqrt{2}a}{2} = 4(\sqrt{2} - 1)$$

$$\Rightarrow \sqrt{2}a(\sqrt{2} - 1) = 8(\sqrt{2} - 1)$$

$$\Rightarrow a = 4\sqrt{2}$$

$$\text{Area of the triangle} = \frac{1}{2}a^2 = 16 \text{ sq cm}$$

87. D

$$(x - 1)x(x + 1) = 15600$$

$$\Rightarrow x^3 - x = 15600$$

The nearest cube to 15600 is 15625 =  $25^3$

We can verify that  $x = 25$  satisfies the equation above.

Hence the three numbers are 24, 25, 26. Sum of their squares = 1877

88. D

$$1 < \log_3 5 < 2$$

$$\Rightarrow 1 < \log_5(2 + x) < 2$$

$$\Rightarrow 5 < 2 + x < 25$$

$$\Rightarrow 3 < x < 23$$

89. C

$$f[f(g(1)) + g(f(1))]$$

$$= f[f(2^1) + g(1^2)]$$

$$= f[f(2) + g(1)]$$

$$= f[2^2 + 2^1]$$

$$= f(6)$$

$$= 6^2 = 36$$

90. C

Let the roots of the equation  $x^2 + (a + 3)x - (a + 5) = 0$  be equal to  $p, q$

$$\text{Hence, } p + q = -(a + 3) \text{ and } p \times q = -(a + 5)$$

$$\text{Therefore, } p^2 + q^2 = a^2 + 6a + 9 + 2a + 10 = a^2 + 8a + 19 = (a + 4)^2 + 3$$

As  $(a + 4)^2$  is always non negative, the least value of the sum of squares is 3

91. A

$$\text{It is given that } 9^{x-\frac{1}{2}} - 2^{2x-2} = 4^x - 3^{2x-3}$$

Let us try to reduce them to powers of 3 and 2

$$\text{The given equation can be reduced to } 3^{2x-1} + 3^{2x-3} = 2^{2x} + 2^{2x-2}$$

$$\text{Hence, } 3^{2x-3} \times 10 = 2^{2x-2} \times 5$$

$$\text{Therefore, } 3^{2x-3} = 2^{2x-3}$$

This is possible only if  $2x - 3 = 0$  or  $x = 3/2$

92.3

$$\log(2^a \times 3^b \times 5^c) = \frac{\log(2^2 \times 3^3 \times 5) + \log(2^6 \times 3 \times 5^7) + \log(2 \times 3^2 \times 5^4)}{3}$$

$$\log(2^a \times 3^b \times 5^c) = \frac{\log(2^{2+6+1} \times 3^{3+1+2} \times 5^{1+7+4})}{3}$$

$$\log(2^a \times 3^b \times 5^c) = \frac{\log(2^9 \times 3^6 \times 5^{12})}{3}$$

$$3\log(2^a \times 3^b \times 5^c) = \log(2^9 \times 3^6 \times 5^{12})$$

Hence,  $3a = 9$  or  $a = 3$

93.51

Sum of the sequence of even numbers is  $2a_3 + (2a_3 - 2) + (2a_3 - 4) + (2a_3 - 6) + (2a_3 - 8) = 450$

$$\Rightarrow 10a_3 - 20 = 450$$

$$\Rightarrow a_3 = 47$$

$$\text{Hence } a_5 = 47 + 4 = 51$$

94.3

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{9}$$

$$\Rightarrow ab = 9(a + b)$$

$$\Rightarrow ab - 9(a + b) = 0$$

$$\Rightarrow ab - 9(a + b) + 81 = 81$$

$$\Rightarrow (a - 9)(b - 9) = 81, a > b$$

Hence we have the following cases,

$$a - 9 = 81, b - 9 = 1 \Rightarrow (a, b) = (90, 10)$$

$$a - 9 = 27, b - 9 = 3 \Rightarrow (a, b) = (36, 12)$$

$$a - 9 = 9, b - 9 = 9 \Rightarrow (a, b) = (18, 18)$$

Hence there are three possible positive integral values of (a,b)

95.6

After Amal, Bimal and Kamal are given their minimum required pens, the pens left are  $8 - (1 + 2 + 3) = 2$  pens

Now these two pens have to be divided between three persons so that each person can get zero pens =

$${}^{2+3-1}C_{3-1} = {}^4C_2 = 6$$

96.50

For the number to be divisible by 6, the sum of the digits should be divisible by 3 and the units digit should be even. Hence we have the digits as

Case I: 2, 3, 4, 6

Now the units place can be filled in three ways (2,4,6), and the remaining three places can be filled in  $3! = 6$  ways.

$$\text{Hence total number of ways} = 3 \times 6 = 18$$

Case II: 0, 2, 3, 4

case II a: 0 is in the units place  $\Rightarrow 3! = 6$  ways

case II b: 0 is not in the units place  $\Rightarrow$  units place can be filled in 2 ways (2,4), thousands place can be filled in 2 ways (remaining 3 - 0) and remaining can be filled in  $2! = 2$  ways. Hence total number of ways =  $2 \times 2 \times 2 = 8$

Total number of ways in this case =  $6 + 8 = 14$  ways.

Case III: 0, 2, 4, 6

case III a: 0 is in the units place  $\Rightarrow 3! = 6$  ways

case II b: 0 is not in the units place  $\Rightarrow$  units place can be filled in 3 ways (2,4,6), thousands place can be filled in 2 ways (remaining 3 - 0) and remaining can be filled in  $2! = 2$  ways. Hence total number of ways =  $3 \times 2 \times 2 = 12$

Total number of ways in this case =  $6 + 12 = 18$  ways.

Hence the total number of ways =  $18 + 14 + 18 = 50$  ways

97. **1**

$$f(1 * 1) = f(1)f(1)$$

$$\Rightarrow f(1) = f(1)f(1)$$

$$\Rightarrow f(1) = 0 \text{ or } f(1) = 1$$

Hence maximum value of  $f(1)$  is 1

98. **D**

$$|f(x) + g(x)| = |f(x)| + |g(x)| \text{ if and only if}$$

$$\text{case 1: } f(x) \geq 0 \text{ and } g(x) \geq 0$$

$$\Leftrightarrow 2x - 5 \geq 0 \text{ and } 7 - 2x \geq 0$$

$$\Leftrightarrow x \geq \frac{5}{2} \text{ and } \frac{7}{2} \geq x$$

$$\Leftrightarrow \frac{5}{2} \leq x \leq \frac{7}{2}$$

$$\text{case 2: } f(x) \leq 0 \text{ and } g(x) \leq 0$$

$$\Leftrightarrow 2x - 5 \leq 0 \text{ and } 7 - 2x \leq 0$$

$$\Leftrightarrow x \leq \frac{5}{2} \text{ and } \frac{7}{2} \leq x$$

So  $x \leq 5/2$  and  $x \geq 7/2$  which is not possible.

Hence, answer is

$$\Leftrightarrow \frac{5}{2} \leq x \leq \frac{7}{2}$$

99. **C**

Let the common ratio of the G.P. be  $r$ .

$$\text{Hence we have } a_n = 3(a_{n+1} + a_{n+2} + \dots)$$

The sum up to infinity of GP is given by  $\frac{a}{1-r}$  where  $a$  here is  $a_{n+1}$

$$\Rightarrow a_n = 3\left(\frac{a_{n+1}}{1-r}\right)$$

$$\Rightarrow a_n = 3\left(\frac{a_n \times r}{1-r}\right)$$

$$\Rightarrow r = \frac{1}{4}$$

$$\text{Now, } a_1 + a_2 + a_3 + \dots = 32$$

$$\Rightarrow \frac{a_1}{1-r} = 32$$

$$\Rightarrow \frac{a_1}{3/4} = 32$$

$$\Rightarrow a_1 = 24$$

$$a_5 = a_1 \times r^4$$

$$a_5 = 24 \times (1/4)^4 = \frac{3}{32}$$

100. **A**

$$a_{100} = \frac{1}{(3 \times 100 - 1) \times (3 \times 100 + 2)} = \frac{1}{299 \times 302}$$

$$\frac{1}{2 \times 5} = \frac{1}{3} \times \left(\frac{1}{2} - \frac{1}{5}\right)$$

$$\frac{1}{5 \times 8} = \frac{1}{3} \times \left(\frac{1}{5} - \frac{1}{8}\right)$$

$$\frac{1}{8 \times 11} = \frac{1}{3} \times \left(\frac{1}{8} - \frac{1}{11}\right)$$

....

$$\frac{1}{299 \times 302} = \frac{1}{3} \times \left(\frac{1}{299} - \frac{1}{302}\right)$$

$$\text{Hence } a_1 + a_2 + a_3 + \dots + a_{100} = \frac{1}{3} \times \left(\frac{1}{2} - \frac{1}{5}\right) + \frac{1}{3} \times \left(\frac{1}{5} - \frac{1}{8}\right) + \frac{1}{3} \times \left(\frac{1}{8} - \frac{1}{11}\right) + \dots + \frac{1}{3} \times \left(\frac{1}{299} - \frac{1}{302}\right)$$

$$= \frac{1}{3} \times \left(\frac{1}{2} - \frac{1}{302}\right)$$

$$= \frac{25}{151}$$