Centre of Mass, Linear Momentum, Collision

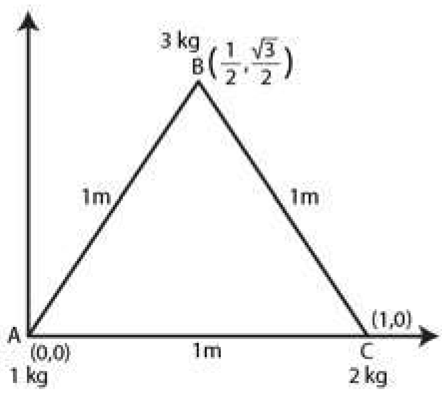
Exercise Solutions

Solution 1:

Here, $m_A = 1$ kg, $m_B = 3$ kg and $m_C = 2$ kg

$x_A = 0$, $x_B = 1/2$ and $x_C = 1$

$y_A = 0$, $y_B = \sqrt{3}/2$ and $y_C = 0$



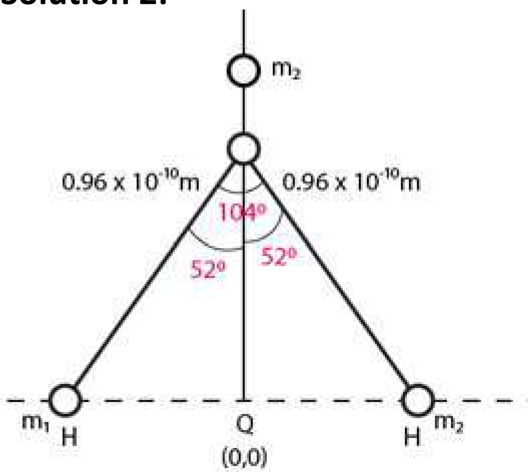
$$\text{Centre of mass} = \left(\frac{m_A x_A + m_B x_B + m_C x_C}{m_A + m_B + m_C}, \frac{m_A y_A + m_B y_B + m_C y_C}{m_A + m_B + m_C}\right)$$

$$= \left(\frac{1 \times 0 + 3 \times \frac{1}{2} + 2 \times 1}{1 + 2 + 3}, \frac{1 \times 0 + 3 \times \frac{\sqrt{3}}{2} + 2 \times 0}{1 + 2 + 3}\right)$$

$$= \left(\frac{7}{12}, \frac{\sqrt{3}}{4}\right)$$

This is the centre of mass from the point A.

Solution 2:



Let θ be the origin of the system
From the figure:
$m_1 = 1$ gm, $m_2 = 1$ gm

$x_1 = -(0.96 \times 10^{-10}) \sin 52^\circ$, $x_2 = -(0.96 \times 10^{-10}) \sin 52^\circ$ and $x_3 = 0$

$y_1 = 0$, $y_2 = 0$ and $y_3 = (0.96 \times 10^{-10}) \cos 52^\circ$

The position of the mass is:

$$\text{Centre of mass} = \left(\frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}, \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}\right)$$

$$= \left(\frac{-(0.96 \times 10^{-10}) \times \sin 52 + (0.96 \times 10^{-10}) \sin 52 + 16 \times 0}{1+1+16}, \frac{0+0+16y_3}{18}\right)$$

$= (0, (8/9) \times 0.96 \times 10^{-10} \cos 52^\circ)$

Solution 3:
Let the mass of each brick be 'm'. Centre of mass of each brick will be at its length L/2 . (Given)

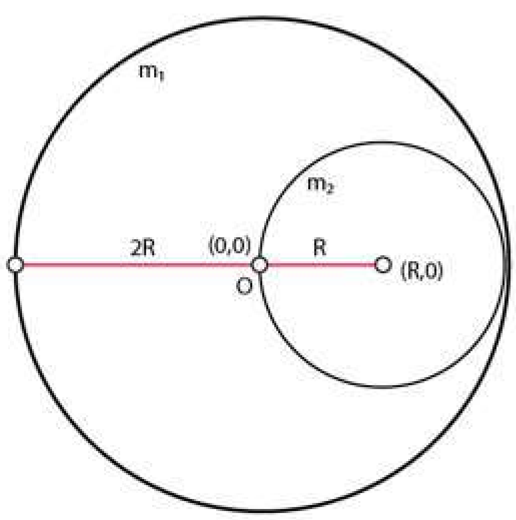
Let O be the origin, the x coordinate of center of mass of the system:

$$= \frac{m\left(\frac{L}{2}\right) + m\left(\frac{L}{2} + \frac{L}{10}\right) + m\left(\frac{L}{2} + 2\frac{L}{10}\right) m\left(\frac{L}{2} + 3\frac{L}{10}\right) + m\left(\frac{L}{2} + 2\frac{L}{10}\right) + m\left(\frac{L}{2} + \frac{L}{10}\right) + m\left(\frac{L}{2}\right)}{m + m + m + m + m + m + m}$$

$$= \frac{44\,L}{70}$$

$$= \frac{11L}{35}$$

Solution 4:



Let the center of the bigger disc be the origin.
And density (mass/volume) of the both the discs be 'ρ' and thickness 't'.

Mass of the bigger disc = $m_1 = \pi (2R)^2 t \rho$

And mass of the smaller disc = $m_2 = \pi R^2 t \rho$

Position of center of mass of the bigger disc $(x_1, y_1) = (0,0)$

Position of center of mass of the smaller disc $(x_2, y_2) = (R,0)$

The position of the center of mass (x, y)

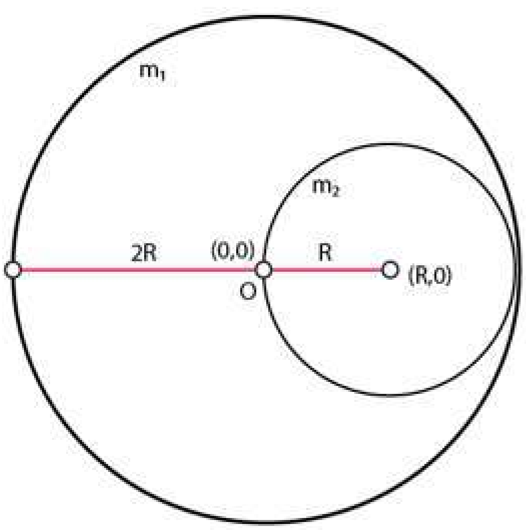
$$= \left(\frac{m_1x_1 + m_2x_2}{m_1 + m_2}, \frac{m_1y_1 + m_2y_2}{m_1 + m_2}\right)$$

$$= \left(\frac{[(\pi(2R)^2 t\,\rho) \times 0] + [(\pi R^2 t\,\rho) \times R]}{\pi(2R)^2 t\rho + \pi R^2 t\rho}, \frac{[(\pi(2R)^2 t\rho) \times 0] + [(\pi R^2 t\rho) \times 0]}{\pi(2R)^2 t\rho + \pi R t\rho}\right)$$

$$= \left(\frac{\pi R^3 t\rho}{5\pi R^2 t\rho}, 0\right)$$

$$= \left(\frac{R}{5}, 0\right)$$

Solution 5:
Let the center of the bigger disc be the origin.



Let the density of the both the discs be 'ρ' and thickness 't'.

Mass of the bigger disc = m1 = $\pi (2R)^2$ t ρ

Mass of the maller disc = $m_2 = \pi R^2$ t ρ

Position of center of mass of the bigger disc $(x_1, y_1) = (0,0)$

Position of center of mass of the smaller disc $(x_2, y_2) = (R,0)$

Now, position of the new center of mass (x, y)

$$= \left(\frac{[(\pi(2R)^2 t\,\rho) \times 0] - [(\pi R^2 t\,\rho) \times R]}{\pi(2R)^2 t\rho - \pi R^2 t\rho}, \frac{[(\pi(2R)^2 t\rho) \times 0] - [(\pi R^2 t\rho) \times 0]}{\pi(2R)^2 t\rho - \pi R t\rho}\right)$$

$$= \left(\frac{-\pi R^3 t\rho}{3\pi R^2 t\rho}, 0\right)$$

$$= \left(\frac{-R}{3}, 0\right)$$

Solution 6:

Given: Mass per unit area for the two plates is same.

Mass of the circular disc = $m_1 = m\pi (d/2)^2$

Mass of the square plate= $m_2 = md^2$

Let origin to be at centre of the circular disc.

Position of the centre of mass :

$$\text{Centre of mass of the system} = \left(\frac{m_1x_1 + m_2x_2}{m_1 + m_2}, \frac{m_1y_1 + m_2y_2}{m_1 + m_2}\right)$$

$$= \left(\frac{\left(m\pi\left(\frac{d}{2}\right)^2 \times 0\right) + (md^2 \times d)}{m\pi\left(\frac{d}{2}\right)^2 + md^2}, \frac{\left(m\pi\left(\frac{d}{2}\right)^2 \times 0\right) + (md^2 \times 0)}{m\pi\left(\frac{d}{2}\right)^2 + md^2}\right)$$

$$= \left(\frac{md^2}{md^2\left(\frac{\pi}{4} + 1\right)}, 0\right)$$

$$= \left(\frac{4d}{\pi + 4}, 0\right)$$

The new centre of mass (4d/(π+4) right of the centre of circular disc.

Solution 7:

Velocity of centre of mass of the given system, v

$$= \left(\frac{m_1v_1 + m_2v_2 + m_3v_3 + m_4v_4 + m_5v_5}{m_1 + m_2 + m_3 + m_4 + m_5}\right)$$

$$= \frac{\begin{bmatrix} 1(-1.5\cos37°\,\hat{i} - 1.5\sin37°\,\hat{j}) + 1.2(0\,\hat{i} + 0.4\,\hat{j}) + 1.5(-1\cos37°\,\hat{i} \\ + 1\sin37°\,\hat{j}) + 0.5(3\,\hat{i} + 0\,\hat{j}) + 1(2\cos37°\,\hat{i} - 2\sin37°\,\hat{j}) \end{bmatrix}}{1 + 1.2 + 1.5 + 0.5 + 1}$$

$$= \frac{0.7\,\hat{i} - 0.72\,\hat{j}}{5.2}$$

Solution 8:

Let two masses are placed on the x-axis.

Centre of mass of the system be its origin,

Initial position of 10 kg mass = $(-x_1, 0)$

Initial position of 20 kg mass = $(x_2, 0)$

Final position of 10kg mass = $(-x_1+2, 0)$

Final position of 20kg mass = $(x_2+x, 0)$

Now,

x-cordinate of centre of mass with respect to initial position = $(-10x_1 + 20x_2)/(10+20) = 0$

x-cordinate of centre of mass with respect to final position = $[10(-x_1 +2) + 20(x_2+x)]/(10+20) = 0$

Equating above expressions, we have

$(20+20x)/30 = 0$

=> $x = -1$

The 2nd mass must be moved 1 cm towards the centre of mass to keep the centre of mass of the system unchanged.

Solution 9:
Shift y-coordinate of the 10kg block = 7 cm
Shift in y-coordinate of the 20kg block = y cm
Shift in y-coordinate of centre of mass of the system = 1 cm

=> (10x7+30y)/(10=30) = 1

=> y = -1
The 30kg block should be displayed 1 cm downwards to raise the centre of mass through 1cm.

Solution 10:
As hall is gravity free and the block of ice is at rest. (Given)

=> No external forces are acting on the block.
Therefore, the centre of mass of the system will stay the same.

So, the centre of mass of the system would not move.

Solution 11:
Let the mass per unit of the plate be m.
Mass of the semi-circular plate with radius $R_2 = m_2 = \pi(R_2)^2$ m

Mass of the semi-circular plate having radius $R_1 = m_1 = \pi (R_1)^2$ m

Consider, The x coordinate of centre of mass with zero.

y-coordinate of centre of mass of semi-circular plate having radius $R_2 = y_2 = 4R_2/3\pi$

y-coordinate of centre of mass of semi-circular plate having radius $R_1 = y_1 = 4R_1/3\pi$

The centre of mass of the plate will be on the symmetrical axis is

$$= \frac{\pi R_2^{\,2} m \times \frac{4R_2}{3\pi} - \pi R_1^{\,2} m \times \frac{4R_1}{3\pi}}{\pi R_2^{\,2} m - \pi R_1^{\,2} m}$$

$$= \frac{4(R_2^{\,3} - R_1^{\,3})}{3(R_2^{\,2} - R_1^{\,2})}$$

$$= \frac{4}{3\pi} \frac{(R_2 - R_1)(R_2^{\,2} + R_1^{\,2} + R_1 R_2)}{(R_2 - R_1)(R_2 + R_1)}$$

$$= \frac{4(R_2^{\,2} + R_1^{\,2} + R_1 R_2)}{3\pi(R_1 + R_2)}$$

Solution 12:

Mass of Mr. Mathur = m_1 = 60 kg
Mass of Mr. Verma = m_2 = 50 kg
Mass of the boat = m_3 = 40 kg
Taking the origin of the 1-D coordinate system to be at one extreme end of the boat where Mr. Verma and Mr. Mathur are sitting.

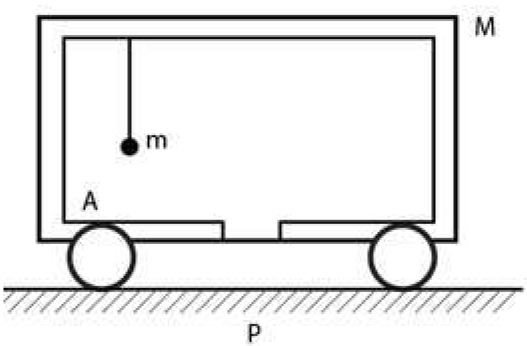
The centre of mass will be at a distance:

$$\frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{60 \times 0 + 50 \times 4 + 40 \times 2}{50 + 60 + 40} = \frac{280}{150} = 1.87 \text{ m}$$

Final centre of mass of the system when both of them are sitting at the centre of the boat = 2 m
Change in position of centre of mass of the boat = 2 – 1.87 = 0.13 m = 13 cm

Therefore, boat will move 13cm towards the side Mr. Verma was sitting.

Solution 13:



Let the bob fall at A.
Let the mass of the bob = m
And mass of the cart = M

Initially, position of centre of mass of the system

$$= \frac{-mL + M \times 0}{m + M} = \frac{-mL}{m + M}$$

Finally, position of centre of mass of the system = 0

Shift in centre of mass of the system

$$= 0 - \frac{-mL}{m + M} = \frac{mL}{m + M}$$

But there is no external force in horizontal direction. So the cart displaces a distance mL/(m+M) towards right.

Solution 14:
Initially, the balloon and monkey are at rest
And position of centre of mass of the system = -mL/m+M

Finally, when the monkey reaches the top, position of centre of mass of the system = 0

Shift in centre of mass of the system = mL/m+M

=> The balloon descends by a distance of mL/m+M

Solution 15:
Let m_1 and m_2 masses of the particles and v_1 and v_2 are the velocities of the two particles.
Here $m_1 = 1$ kg and $m_2 = 4$ kg

Equating the kinetic energies of the particles, we have
$(1/2)\ m_1 v_1^2 = (1/2)\ m_2 v_2^2$

On substituting the values of m_1 and m_2

=> $(v_1/v_2)^2 = 4/1$

Or $(v_1/v_2) = 2/1$
Thus, Ratio of their linear momenta = 1:2

Solution 16:
Mass of the uranium-238 nucleus, m = 238 (Given)
Initial speed of the nucleus = u = 0 m/s
Final speed of the nucleus = v
Final speed of the alpha particle = $v_2 = 1.4 \times 10^7$ m/s
Mass of the nucleus after emission, $m_1 = 234$
Mass of the alpha particle, $m_2 = 4$

Applying conservation of momentum , $mu = m_1v_1 + m_2v_2$

$238\times0 = 234\ v + 4 \times 1.4 \times 10^7$
$v = -2.39 \times 10^5$ m/s or -2.4×10^5 m/s (approx.)

Solution 17:

Let m1 and m2 be the mass of man and mass of earth respectively.

$m_1 = 50$ kg

$m_2 = 6 \times 10^{24}$ kg

Let v_1 and v_2 be the speed of man and speed of earth

$v_1 = 1.8$ m/s

By conservation of momentum, $m_1 v_1 = m_2 v_2$

$50 \times 1.8 = 6 \times 10^{24} \times v_2$

or $v_2 = 1.5 \times 10^{-23}$ m/s

Solution 18:

Given: Mass of the proton =

Momentum of electron = 1.4×10^{-26} kg-m/s

Momentum of antineutrino = 6.4×10^{-27} kg-m/s

Let the velocity of proton be 'v' m/s

(a) Applying conservation of momentum,

Initial momentum of neutron = momentum of electron + momentum of proton + momentum of antineutrino

$0 = 1.67 \times 10^{-27} \times v + 1.4 \times 10^{-27} + 6.4 \times 10^{-27}$

$\Rightarrow -1.67 \times 10^{-27} \times v = 20.4 \times 10^{-27}$

or $v = -12.2$ m/s

Which is velocity of proton in direction opposite to electron and antineutrino.

(b) Resultant momentum of electron and antineutrino = $\sqrt{(14^2 + 6.4^2)} \times 10^{-27}$

$= 15.4 \times 10^{-27}$ kg m/s

Again,

Initial momentum of neutron = momentum of electron + momentum of proton + momentum of antineutrino

$0 = 1.67 \times 10^{-27} \times v + 1.4 \times 10^{-27} + 6.4 \times 10^{-27}$

or

$v = -9.2$ m/s

Solution 19:

Mass of man = M and initial velocity = 0 and mass of bad = m

Let t be the total time man will take to reach the height h = time taken in falling through height H – time taken in falling through height (H-h)

$$\Rightarrow t = \frac{\sqrt{2H}}{g} - \frac{\sqrt{2(H-h)}}{g}$$

Horizontal distance covered in this time = x

Horizontal velocity of the man, $v_2 = x/t$

Now, $mv_1 = mv_2$

$\Rightarrow v_1 = Mv_2/m$

=> Distance the bag travels = $v_1t = Mx/m$

and

$$v_1 = \frac{M}{m}\frac{xg}{\sqrt{2H} - \sqrt{2(H-h)}}$$

Which is the minimum horizontal velocity imparted to the bag.

Solution 20:

Mass of the ball, m = 50 g or 0.05 kg

Initial velocity of the ball = u = -2 cos45° î + 2 sin45° ĵ

Final velocity of ball after reflection = v = 2 cos45° î + 2 sin45° ĵ

(a) Change in momentum = m(v-u)

= 0.05×[(2cos45° î + 2sin45° ĵ) – (-2cos45° î + 2sin45° ĵ)]

= 0.14 î

Magnitude of change in momentum = 0.2/√2 = 0.14 kg m/s

(b) Magnitude of initial momentum = m|u|

= 0.05×2

= 0.10 kg m/s

Magnitude of final momentum = $m|v|$

$= 0.05 \times 2$

= 0.10 kg m/s

Change in magnitude of momentum = $m|v| - m|u|$ = 0 kg m/s

Solution 21:

The angle of reflection will be the same as angle of incidence, i.e. θ

Linear momentum of incident light = $P_i = -(h/\lambda)\cos\theta\ \hat{i} + (h/\lambda)\sin\theta\ \hat{j}$

Linear momentum of reflected light = $P_r = (h/\lambda)\cos\theta\ \hat{i} + (h/\lambda)\sin\theta\ \hat{j}$

Change in linear momentum = $P_r - P_i$

$= [(h/\lambda)\cos\theta\ \hat{i} + (h/\lambda)\sin\theta\ \hat{j}] – [-(h/\lambda)\cos\theta\ \hat{i} + (h/\lambda)\sin\theta\ \hat{j}]$

$= -2(h/\lambda)\cos\theta\ \hat{i} + 0\ \hat{j}$

=> Change in linear momentum is in x-direction = $(2h\cos\theta)/\lambda$

Solution 22:

As the block is exploded only due to its internal energy. So, the net external force during this process is zero.

P_i = 0 kg m/s

Let the mass of each part be m kg and velocity of the third part be $v_x\hat{i} + v_y\hat{j}$

Final momentum = $P_f = 10m\ \hat{i} + 10m\ \hat{j} + m(v_x\ \hat{i} + v_y\ \hat{j})$

By conservation of momentum, we have

pi = pf

$0 = 10m\ \hat{i} + 10m\ \hat{j} + m(v_x\ \hat{i} + v_y\ \hat{j})$

$v_x\ \hat{i} + v_y\ \hat{j} = -10\ \hat{i} - 10\ \hat{j}$

or vx = -10 and vy = -10

Magnitude of velocity of third part = $\sqrt{((-10)^2+(-10)^2)}$ = $10\sqrt{2}$ m/s

Angle with the horizontal = $\tan^{-1}(-10/10)$ = 135° from positive x axis or 45° below the x axis.

Solution 23:
Two fat astronauts each of mass 120 kg are travelling in a closed spaceship moving at a speed of 15 km/s in the outer space far removed from all other material objects.
The spaceship is moving in vacuum and there is absolutely no external force acting on it. So, the total mass of the spaceship remain unaffecting and also its velocity.

Velocity of the spaceship = same as before = 15 km/s

Solution 24:
Here d = 1 cm, v = 20 m/s and u = 0 and density = 900 kg/m^3

Mass of a hailstone, m = Density×Volume
= $900 \times 4/3 \times \pi \times (\text{Radius})^3$

= $900\times4/3\times\pi\times(0.5\times10^{-2})^3$

= 0.471×10^{-3}kg

Mass of 2000 hailstones, M = 2000×m = 0.942 kg

Rate of change in momentum per unit area = Rate of change of momentum

= M× (20 – 0)

= 18.84 N or 19 N (approx.)

Total force exerted on the roof = 19 × area of the roof

= 19 × 10 × 10 = 1900 N

Solution 25:

Let a ball of mass m dropped onto a floor from a certain height "h".

Velocity of the ball before the collision = $\sqrt{(2gh)}$ and $-\sqrt{(2gh)}$ after the collision.

Considering, $v = \sqrt{(2gh)}$

Time taken by the ball to hit the ground, $t = v/g = \sqrt{(2h/g)}$

Total time = $2t = 2\sqrt{(2h/g)}$

Average force exerted by the ball on the floor = Rate of change of momentum of the ball

$$= \frac{m\sqrt{2gh} - (-\sqrt{2gh})}{\text{Total time}}$$

$$= \frac{2m\sqrt{2gh}}{2\sqrt{\frac{2h}{g}}}$$

$$= mg$$

Solution 26:

Mass of the car = M and Mass of man = m

Initial velocity of the car, $u_2 = 0$ and Initial velocity of man = u_1

Velocity of man in the car = Recoil velocity of the car = v

Now,

$mu_1 + Mu_2 = (m+M)v$

[conservation of momentum]

$mu_1 + 0 = (m+M)v$

$u_1 = (m+M)v/m$

$u_1 = v(1 + M/m)$; is the velocity with which man approaches the engine.

Solution 27:
By conservation of momentum,

=> $0 = 49m\times v + m\times 200$

=> $v = -200/49$

After first shot, velocity of the car = (-200/49) m/s

After another shot, velocity of the car = (-200/48) m/s

Total recoiling velocity of the car = [-200(1/49 + 1/48)] m/s

Solution 28:

Let v be the recoil velocity of the railroad car when the man on the right jumps to the left.

Applying conservation of momentum,

$\Rightarrow 0 = -mu + (m+M)v$
=> $v = mu/(m+M)$

Let v' be the recoil velocity of the railroad car when the man on the left jumps to the right

=> $0 = mu - Mv'$

=> $v' = mu/M$

Now,

Net recoil velocity the car = v+v'

$$= \frac{mu}{m+M} - \frac{mu}{M}$$

$$= \frac{mMu - m^2u - mMu}{M(m+M)}$$

$$= -\frac{m^2u}{M(m+M)}$$

Which is the velocity of the car after both people have jumped toward left.

Solution 29:

Using conservation of momentum $\Rightarrow mv + M \times 0 = (m+M)V$
Where V be the speed of the bigger block.

$mv = (m+M)V$
$\Rightarrow V = mv/(m+M)$, which is speed of the bigger box.

Solution 30:
Initial momentum of the school boy before sitting = $4 \times 25 = 100$ kg km/h

Initial momentum of the bugghi before the boy sat on it = $10 \times 200 = 2000$ kg km/h

Total initial momentum of the bugghi and the school boy = $2000+100 = 2100$ kg km/h

Final momentum of the bugghi after the school boy sat on it = $v \times (100+25) = 125v$ kg km/h

Where v km/h be the new velocity of the bugghi.

Now, total linear momentum,

$\Rightarrow 125v = 2100$
or $v = (28/3)$ km/h

Solution 31:
Initial momentum of the ball before collision = $0.50 \times 5 = 2.5$ kg m/s
Initial momentum of the other ball before collision = $1.0 \times v = v$ kg m/s
Where, v be the velocity of the other ball before collision.

Total initial momentum of the two balls = $(2.5+v)$ kg m/s and both the balls come to rest after collision.

Final momentum of the two balls after collision = 0 kg m/s

By conservation of total linear momentum

$\Rightarrow 2.5 + v = 0$ or $v = -2.5$ m/s

Solution 32:

Total linear momentum of both the skaters before the collision = 60×10 + 40× 0 = 600 kg m/s

Total linear momentum of both skaters after the collision = v×(60+40) = 100v kg m/s

Where v be the velocity of both the skaters after they cling to each other.

Find v:

using conservation of total linear momentum,

100v = 600 or v = 6 m/s

Now,

Total KE of both skaters before the collision = (1/2) (60 x 10^2) + (1/2)(40-0) = 3000 Joule

And, Total KE of both skaters after the collision = (1/2) (60 x 40)$(6)^2$ = 1800 Joule

Therefore, Total loss in kinetic energy during this collision = 3000 – 1800 = 1200 J

Solution 33:

The collision starts at t = 0 and the particles interact for a time interval Δt.

During the collision the speed of the two particles of masses m_1 and m_2 are u(t) and u' respectively.

By law of conservation of momentum

$m_1u_1 + m_2u_2 = m_1u(t) + m_2u'$

since,

$$u(t) = u_1 + \frac{t}{\Delta t}(u_2 - u_1)$$

Then,

$$m_1u_1 + m_2u_2 - m_1\left[u_1 + \frac{t}{\Delta t}(u_2 - u_1)\right] = m_2u'$$

$$m_2u_2 - m_1\left[\frac{t}{\Delta t}(u_2 - u_1)\right] = m_2u'$$

$$\frac{m_2u_2 - m_1\left[\frac{t}{\Delta t}(u_2 - u_1)\right]}{m_2} = u'$$

Which is the speed of the second particle as a function of time during the collision.

Solution 34:
From law of conservation of momentum

$mv + 0 = (m' + m)u' + (M - m')u_1$

or

$$u' = \frac{mv - (M - m')u_1}{(m' + m)}$$

Where u' is the velocity of the bullet with frictional mass.

Solution 35:
Here $mv_1 + mv_2 = mv + 0$

Where v_1 and v_2 are the final velocities of the first ball and second ball and m is the mass of the second ball and v is the velocity of the first ball.

$v_1 + v_2 = v$ and $v_1 - v_2 = ev$

Final KE = (3/4)(initial KE)

$$\frac{1}{2}(mv_1^2) + \frac{1}{2}(mv_2^2) = \frac{3}{4} \times \frac{1}{2}(mv^2)$$

$$v_1^2 + v_2^2 = \frac{3}{4} \times v^2$$

$$\frac{(v_1 + v_2)^2 + (v_1 - v_2)^2}{2} = \frac{3}{4} \times v^2$$

Equations $v_1 + v_2 = v$ and $v_1 - v_2 = ev$ implies

$$\frac{v^2 + e^2v^2}{2} = \frac{3}{4}v^2$$

$$(1 + e^2) = \frac{3}{2}$$

$$e = \frac{1}{\sqrt{2}}$$

Which is the coefficient of restitution.

Solution 36:

From law of conservation of momentum

mu + m' x 0 = (m' + m)v ...(1)

Where m = mass of 1st block = 2kg, u = Initial speed before collision = 2 m/s and m' = ass of the second block = 2kg

on putting values, we have

(1)=> v = 1 m/s

(a) Loss in kinetic energy in elastic collision = (1/2) mu^2 + 0 - (1/2)(m' + m)v^2

= 2 J

(b) Actual loss = (1/2) x (maximum loss) = 2/2 = 1 J

Let v_1 and v_2 are the final velocities of the blocks of masses 2kg

Where $v_1 + v_2 = u$ and $v_1 - v_2 = eu$.

Find Actual loss:

$$\frac{1}{2}mu^2 - \frac{1}{2}mv_1{}^2 - \frac{1}{2}m'(v_2)^2 = 1$$

$$\frac{1}{2} \times 2 \times 2 \times 2 - \frac{1}{2} \times 2 \times v_1^2 - \frac{1}{2} \times 2 \times v_2^2 = 1$$

$$4 - \frac{1}{2}(v_1^2 + v_2^2) = 1$$

since $v_1+v_2 = u$ and $v_1-v_2 = eu$

$$4 - \frac{u^2 + e^2u^2}{2} = 1$$

$$4 - \frac{(1 + e^2)2^2}{2} = 1$$

$$e = \frac{1}{\sqrt{2}}$$

Which is the coefficient of restitution.

Solution 37:

Here u is the initial speed and KE after collision is 0.2J

Now, Initial kinetic energy = $(1/2)\ mu^2$

$= (1/2) \times 0.1 \times u^2$

$= 0.05\ u^2$

We know, $mu = mv_1 = mv_2$

Where v_1 and v_2 are the final speed of first block and final speed of second block respectively.

$v_1 + v_2 = u$.....(1)

Again,
$(v_1 - v_2) + L\ (a_1\text{-}u_2) = 0$

As $u_1 = u$ and $u_2 = 0$

$L\ a_1 = (v_2 - v_1)$ ....(2)

Adding (1) and (2), we get
$v_1 = [u(1\text{-}L)/2]$

and $v_2 = [u(1+L)/2]$

Total kinetic energy after collision = $(1/2)mv_1^2 + (1/2)\ mv\ v_2^2$

=> $0.2 = v_1^2 + v_2^2 = 4$

on substituting the values of v_1 and v_2, we get

$u^2 = 8/(1+L^2)$

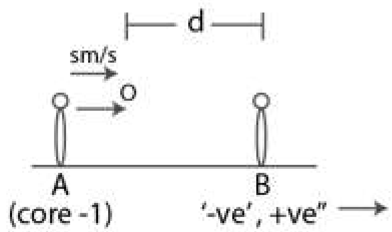
For minimum value of u denominator should be maximum:

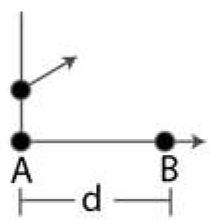
L=1 and $u^2 = 4$ or $u = 2m/s$

For maximum value of u denominator should be minimum

L = 0 and $u^2 = 8$ or $u = 2\sqrt{2}$ m/s

Solution 38:





Two friends A & B (each weighing 40kg) are sitting on a frictionless platform some distance d apart A rolls a ball of mass 4kg on the platform towards B, which B catches.

Then B rolls the ball towards A and A catches it. The ball keeps on moving back & forth between A and B.

(a) **Total momentum of man A and the ball will remain constant.**

$mu = (m+m)v_1$

$0 = 20 - 40v$

=> $v = 0.5$ m/s towards left

(b) **When the man at B catches the ball, momentum between B and the ball will remain constant.**

$20 = 44v$

=> $v = (20/44)$ m/s

when B throws ball, then applying LCLM (Law of Conservation of Linear Momentum)

=> $Mv_1 = -mu + Mv_2$
$44 \times (20/44) = -4 \times 5 + 40 \times v_2$

$V_2 = 1$ m/s towards right

When A catches the ball, then applying LCLM

$-mu + M(-v) = (M+m)v'$

$-4 \times 5 + (-0.5) \times 40 = -44 v'$

$v' = 10/11$ m/s towards left

(c) speeds of A and B after the ball has made 5 round trips and is held by A

When A throws the ball

$(M+m)v_3 = mu - Mv_4$

$44 \times (10/11) = 4 \times 5 - 40 \times v_4$

$v_4 = 3/2$ m/s towards left

When B receives the ball,

$Mv_2 + mu = (M+m)v_5$

$40 \times 1 + 4 \times 5 = 44 \times v_5$

$v_5 = 15/11$ m/s towards right

When B throws the ball,

$(M+m)v_5 = -mu + Mv_6$

$44 \times (66/44) = -4 \times 5 + 40 \times v_6$

$V_6 = 2$ m/s towards right

When A catches the ball,

$-mu - Mv_4 = -(M+m)v'$

$-4 \times 5 -40 \times (3/2) = -44v'$

$v'= 20/11$ m/s towards left

Similarly, After 5 round trips:

Velocity of A will be 50/11 and velocity of B will be 5 m/s

(d) **After 6th round trip**

Velocity of A = (60/11) m/s > 5 m/s, can't reach the ball. So only can roll the ball six.

(e) **Let the ball and body A at the initial position be at origin**

= [Mv+mu+Md]/[M+M+m]

= [40x0 + 4x0+40xd]/[40+40+4]

= 10d/11

Solution 39:
we know, Velocity = u = $\sqrt{(2gh)}$
Where, g is the acceleration due to gravity and h is the height.
=> v = $\sqrt{(2x9.8x1.5)}$

Let u is the velocity of the ball when it approaches the ground
[Here g = 9.8 and h = 2]
=> u = $\sqrt{(2gh)}$ = $\sqrt{(2x9.8x2)}$

Now, v + lu = 0
Where, l is the coefficient of restitution.

or l = -v/u

or l = $\sqrt{3}/2$

Solution 40:
kinetic energy of nucleus = (1/2) mv^2
Given that, internal energy of a nucleus of mass M decreases, and Linear momentum is E/c and the nucleus recoils.

=> KE = (1/2) m $(E/mc)^2$

KE = $E/2mc^2$

Gamma photon energy is E.

Decrease in internal energy = E + $E/2mc^2$

[As, decrease in internal energy will be equal to sum of energy limited by gamma photon and kinetic energy of nucleus]

Solution 41:

Let M_1 and M_2 masses of blocks which is 2 kg each.
Velocicty of 1st block = v_1 = 1 m/s
Velocicty of 2nd block = v_2 = 0 m/s

After collision velocity starts to decrease continuously and block 1 and block 2 starts moving together with common velocity.

using law of conservation energy

$$\frac{1}{2}M_1v_1^3 + \frac{1}{2}M_2v_2^2 = \frac{1}{2}M_1v^2 + \frac{1}{2}M_2v^2 + \frac{1}{2}kx^2$$

$$\frac{1}{2} \times 2 \times 1^2 + 0 = \frac{1}{2} \times 2 \times v^2 + \frac{1}{2} \times 2 \times v^2 + \frac{1}{2} \times x^2 \times 100$$

=> $2v^2 + 50\, x^2 = 1$ ....(1)

When no external force in horizontal direction, the momentum should be conserved.

$M_1v_1 + M_2v_2 = (M_1 + M_2)v$

$2x_1 + 0 = 4xv$

=> v = (1/2) m/s

(1)=> x = 10 cm, is the maximum compression of the spring.

Solution 42:
Here m = Mass of bullet = 20 g or 0.02 kg and M = Mass of block = 10 kg

Initial velocity of bullet = v_1 = 500 m/s and Initial velocity of block = v_2 = 0 m/s and Final velocity of bullet = v= 100 m/s

let v' be the final velocity of block when bullet comes out

=> $mv_1 + Mv_2 = mv + Mv'$

=> $0.02 \times 500 + 10 \times 0 = 0.02 \times 100 + 10 \times v'$

=> $v' = 0.8$ m/s

Now,
let μ friction coefficient between the block and the surface

Find μ:

As we are given, After moving a distance 0.2 it stops

$0 - (1/2) \times 10 \times (0.8)^2 = -\mu \times 10 \times 10 \times 0.2$

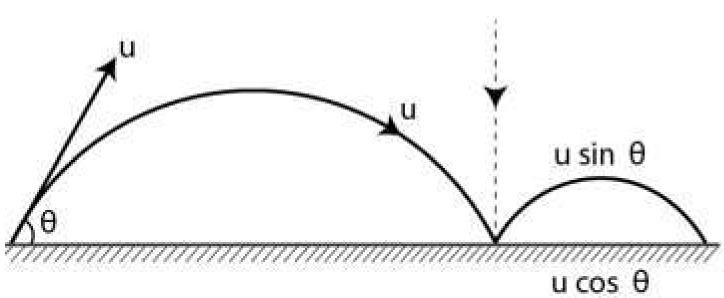
[By work energy principle]

=> $\mu = 0.16$

Solution 43: A projectile is fired with a speed u at an angle θ above a horizontal field. The coefficient of restitution of collision between the projectile and the field is e.

Let projection angle is Θ and projection velocity v.

g = 10 m/s



$e = (u \sin\theta)/v \times v = eu \sin\theta$

Velocity of projection for 2nd projectile motion and angle of projection,

$$U = \sqrt{(u\cos\theta)^2 + (eu\sin\theta)^2}$$

and

$$\emptyset = \tan^{-1}\left(\frac{eu\sin\theta}{u\cos\theta}\right)$$

$$\tan\emptyset = e\tan\theta \quad ....(1)$$

Since equation for path of projectile motion

$y = x\tan\phi - [gx^2\sec^2\phi]/2U^2$ ....(2)

Put $y = 0$, $\tan\phi = e\tan\theta$, $\sec^2\phi = 1+ e^2\tan^2\theta$ and

$U = (u\cos\theta)^2 + (eu\sin\theta)^2$

(2)=>

$$y = xe\tan\theta - \frac{gx^2(1 + e^2\tan^2\theta)}{2u^2(\cos^2\theta + e^2\sin^2\theta)}$$

$$x = \frac{\left(2u^2(\cos^2\theta + e^2\sin^2\theta)\right) \times (e\tan\theta)}{gx^2(1 + e^2\tan^2\theta)}$$

$$x = \frac{2eu^2\tan\theta - \cos^2\theta}{g}$$

$$x = \frac{eu^2\sin 2\theta}{g}$$

Now,
From starting point O the projectile makes its second collision with the field at a distance x'

$$x' = \frac{u^2\sin 2\theta}{g} + \frac{eu^2\sin 2\theta}{g}$$

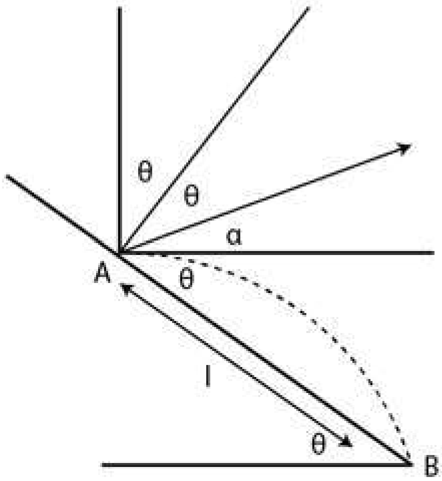
$$x' = \frac{u^2\sin 2\theta}{g}(1 + e)$$

Solution 44:

Let the striking velocity of the projectile with the inclined plane = v = √(2gh)

Now, the projectile makes an angle = α = (90° - 2θ)

velocity of the projectile = u = √(2gh)



Let AB = l

x = l cosθ and y = -l sinθ

using equation of trajectory

$$y = x\tan\alpha - \frac{gx^2\sec^2\alpha}{2u^2}$$

$$-l\sin\theta = l\cos\theta.\tan(90^\circ - 2\theta) - \frac{g(l\cos\theta)^2.\sec^2(90^\circ - 2\theta)}{2(\sqrt{2gh})^2}$$

$$-l\sin\theta = l\cos\theta.\cot 2\theta - \frac{gl^2\cos^2\theta.\mathrm{cosec}^2 2\theta}{4gh}$$

$$\frac{l\cos^2\theta.\mathrm{cosec}^2 2\theta}{4h} = \sin\theta + \cos\theta.\cot 2\theta$$

$$l = \frac{4h \times \sin^2 2\theta}{\cos^2\theta}\left(\sin\theta + \cos\theta.\frac{\cos 2\theta}{\sin 2\theta}\right)$$

$$l = 16h\sin^2\theta \times \frac{\cos\theta}{2\sin\theta\cos\theta}$$

$$l = 8h\sin\theta$$

Solution 45:

Let v be the striking velocity of the projectile with the inclined plane = $v = \sqrt{(2gh)}$
$= \sqrt{(2 \times 10 \times 5)} = 10$ m/s

After collision, let it make an angle θ with horizontal. Its horizontal component remain unchanged and velocity in perpendicular direction to the plane after collision is

$v_y = ev \cos\theta$

$= (3/4) \times 10 \times \cos 45^o$

$= (3.75)\sqrt{2}$ m/s

and $v_x = v \cos \theta = (5/\sqrt{2})$ m/s

So $u = \sqrt{(v_x^2 + v_y^2)}$

$= \sqrt{(50+28.125)}$

$= 8.83$ m/s

Again, β be the angle of reflection from the wall.

$\Rightarrow \beta = \tan^{-1} (3.75\sqrt{2}/5\sqrt{2})$

$= \tan^{-1}(\tan 37^o) = 37^o$

Find angle of projection:

Let α be the angle of projection

$\alpha = 90^o - (45^o + 37^o) = 8^o$

If l be the distance from it falls, coordinate are

$x = l \cos\theta$, $y = -l \sin\theta$

From equation of trajectory:

$$y = x\tan\alpha - \frac{gx^2\sec^2\alpha}{2u^2}$$

$$-l\sin\theta = l\cos\theta \,.\, \tan\alpha - \frac{gx^2\sec^2\alpha}{2u^2}$$

Substituting the values,

$$-l\sin 45^\circ = l\cos 45^\circ \,.\, \tan 8^\circ - \frac{10(l\cos 45^\circ)^2 \,.\, \sec^2 8^\circ}{2(8.83)^2}$$

Solving above, we have the value of l i.e.

l = 18.5 m

it will hit the plane again at 18.5 m along the incline.

Solution 46:

The string is stretched by a distance x =1.00 cm= 0.01m.
In the equilibrium condition
Mg = kx
Where M = mass of block = 0.2 kg and g = 10 m/s

=> k = 200 N/m

The velocity with which the particle m will strike M is

u = √(2gh)

= √(2x10x0.45

= 3 m/s

After the collision the velocity of the particle and the block:

v = mu/(m+M)

=> v = (0.12x3)/0.32 = (9/8) m/s

From law of conservation of energy

$$\frac{1}{2}mu^2 + \frac{1}{2}kx^2 = \frac{1}{2}(M+m)v^2 + \frac{1}{2}k(x+\delta)^2$$

Where, string be stretched through an extra deflection δ.

On substituting the values and solving, we have

$\delta = 6.1$ cm

Solution 47:

Let v be the final velocity and u is the strike pendulum.

Now, mu = (M+m)v

Here, m = 25 g = 0.025 kg, M = 5 kg

=> v = u/201

Using conservation of energy, we have
(1/2) (M+m)v^2 = (M+m)gh

$v^2 = 2gh$

Using value of v,

u = 280 m/s

Therefore, the bullet strikes pendulum with velocity u = 280 m/s.

Solution 48:
Let the velocity of block is v' and bullet emerges out with velocity is v.

Using law of conservation of momentum:

mu + Mu' = mv + Mv'

or mu = Mv' + mv ...(1)

Work energy principle for the block after the collision

$(1/2)Mu'^2 - (1/2)\ Mv'^2 = -Mgh$

[here $h = 0.2$ m and $u' = 0$ m/sec, $M = 500$gm $= 0.5$kg, and $m = 20$gm $= 0.02$kg and $u =$ 300 m/sec.]

substituting the values and solving , we have

$v' = 2$ m/s

(1)=> $v = 250$ m/s

Therefore, speed of the bullet as it emerges from the block is $v= 250$ m/s.

Solution 49:

For the blocks to come to rest again, let the distance travelled by two blocks of mass m_1 and m_2 is x_1 and x_2 respectively.

As zero external force acts in horizontal direction,
$m_1x_1 = m_2x_2$ ....(1)

By law of conservation of energy

$(1/2)\ kx^2 = (1/2)\ k\ (x_1 + x_2 - x_0)^2$

Thus, $x_0 = x_1 + x_2 - x_0$

or $x_1 = 2x_0 - x_2$

(1)=> $m_1(2x_0 - x_2) = m_2x_2$

$x_2 = 2m_1x_0/(m_1 + m_2)$

similarly, we can find the value of x_1

$x_1 = 2m_2x_0/(m_1 + m_2)$

Solution 50:

(a) Let v_c be the velocity of center of mass

$$v_c = \frac{m_2 \times v_0 + m_1 \times 0}{m_1 + m_2}$$

$$v_c = \frac{m_2 v_0}{m_1 + m_2}$$

(b)
when both the blocks will attain the velocity of centre of mass.

Let x be the maximum elongation of spring.

Change in kinetic energy = potential energy stored in spring

$$\frac{1}{2}m_2 v_0^2 - \frac{1}{2}(m_1 + m_2)v_c^2 = \frac{1}{2}Kx^2$$

Putting value of v_c

$$Kx^2 = m_2 v_0^2\left(1 - \frac{m_2}{m_1 + m_2}\right)$$

$$x = \left[\left(\frac{1}{k}\right)\left\{\frac{m_1 m_2}{m_1 + m_2}\right\}\right]^{\frac{1}{2}} v_0$$