Previous Years Paper

30 May 2023 - (Shift 2)

Q1. The value of $\int_{-\frac{\pi}{2}}^{\pi/2}(x^5+x^3+x+2)\,dx$ is:
(a) 0
(b) 2
(c) 2π
(d) π

Q2. Area bounded by the curves $y^2=9x$ and $y=3x$ is:
(a) $\frac{2}{3}$ sq units
(b) $\frac{1}{3}$ sq units
(c) $\frac{1}{2}$ sq units
(d) 2sq units

Q3. Let A and B be two events in a random experiment with sample space such that P(B) $\neq$ 0 and P(A|B) = 1. Which of the following is true?
(a) A C B
(b) B C A
(c) A = Ø
(d) B = Ø

Q4. A random variable x has the following probability distribution.

x	1	2	3	4	5	6
P(x)	α	0.1	0.3	β	0.4	0.1

In the above table, P[x = 1 or x = 4] is equal to:
(a) 0.05
(b) 0.01
(c) 0.1
(d) 0.5

Q5. The function $f(x)=\begin{vmatrix}x^2 & x\\ 3 & 1\end{vmatrix}, x \in \text{IR}$ has a:
(a) local maximum at x = 3
(b) local minimum at $x=\frac{3}{2}$
(c) local maximum at $x=\frac{3}{2}$
(d) local minimum at x = 0

Q6. $\begin{vmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{vmatrix}=\begin{vmatrix}2 & 3\\ 8 & 9\end{vmatrix}+y\begin{vmatrix}1 & 3\\ 7 & 9\end{vmatrix}+z\begin{vmatrix}1 & 2\\ 7 & 8\end{vmatrix}$ Then x + y + z is:
(a) 15
(b) 5
(c) 5
(d) 0

Q7. Given that the function $f(x)$ is continuous on R. Match List – I with List – II.

	List – I		List – II
(A)	$f(x)=\begin{cases}kx^2, & if\ x<3\\ 3, & if\ x\geq 3\end{cases}$	(I)	$k=\frac{5}{4}$
(B)	$f(x)=\begin{cases}kx+1 & if\ x\geq 4\\ x+2 & if\ x<4\end{cases}$	(II)	k = 1
(C)	$f(x)=\begin{cases}\frac{5+x}{2}, & if\ x\neq 0\\ 3k, & if\ x=0\end{cases}$	(II)	$k=\frac{1}{3}$
(D)	$f(x)=\begin{cases}x+k, & if\ x>-2\\ -3, & if\ x\leq -2\end{cases}$	(IV)	$k=\frac{5}{6}$

Choose the correct answer from the options given below:
(a) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)
(b) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)
(c) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)
(d) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

Q8. If A is a square matrix and |A| = 4, then the value of |AA'| where A' is transpose of A, is:
(a) 16
(b) 4
(c) 2
(d) 8

Q9. If $x=\log t^2$ and $y=(\log t)^2$ then $\frac{d^2y}{dx^2}$ is:
(a) 2
(b) $\frac{1}{2}$
(c) $\frac{\log t}{t}$
(d) $\frac{4\log 5-4}{t^6}$

Q10. If A, B and C are all the corner points of feasible region (bounded) of an LPP with objective function Z that needs to be maximized such that $Z_A>Z_B$ and $Z_C<Z_A$(Here Z_A denotes value of Z at A), then which of the following statements is TRUE?
(a) There are infinitely many optimal solutions
(b) There are exactly two optimal solutions
(c) There is a unique optimal solution
(d) Optimal solution does not exist

Q11. A square matrix $B=[bij]_{n\times n}$ where
bij = 0 when i#j
bij = k when i = j for some constant k is called:
(a) Diagonal matrix
(b) Identify matrix
(c) Scalar matrix
(d) Null matrix

Q12. A car starts from a point P at time t = 0 seconds and stops at Q. the distance x, in meters, covered by it in t seconds is given by $x(t)=t^3\left(3-\frac{t^2}{5}\right)$ the distance between P and Q is:
(a) $\frac{162}{3}m$
(b) $\frac{162}{5}m$
(c) 162 m
(d) $\frac{162}{9}m$

Q13.

Resource	Found 1 (x kg)	II (y kg)	Requirement units
Calcium (units/kg)	3	2	9
Vitamin (units /kg)	4	1	5
Cost (Rs/kg)	40	60	

Formulate the L.P.P. to minimize the cost

(a) Min z = 40x + 60y
$3x + 2y \geq 9$
$4x + y \geq 5$
$x, y \geq 0$

(b) $Min\ z = 9x + 5y$
$3x + 2y \geq 40$
$4x + y \geq 60$
$x, y \geq 0$

(c) Min z = 40x + 60 y
$3x + 2y \leq 9$
$4x + y \leq 10$
$x, y \leq 0$

(d) $Min\ z = 9x + 5y$
$3x + 2y \leq 40$
$4x + y \geq 60$
$x, y \geq 0$

Q14. Match List – I with List – II

	List – I		List – II
	General solution		Differential Equation
(A)	$y = cx$	(B)	$y' = \frac{1}{2\sqrt{x}} - 1$
(B)	$y = e^{-3x} + c$	(II)	$y' + 3y = 0$
(C)	$x + y = \sqrt{x}$+c	(III)	$y' = \frac{y^2}{1 - xy}$ $(xy \neq 1)$
(D)	$xy = \log y + c$	(IV)	xy' = y

(c is an arbitrary constant and $y' = \frac{dy}{dx}$)

(a) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)
(b) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)
(c) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)
(d) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

Q15. $\int_0^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{2-x}}$ dx is equal to:
(a) 0
(b) 2
(c) $\frac{3}{2}$
(d) 1

Q16. Solution of differential equation $\frac{dy}{dx} = \cos(x + y + 3)$ is:
(a) $y = -\sin(x + y + 3) + c$
(b) $y = \sin(x + y + 3) + c$
(c) $y = 2\tan^{-1}(x + c) - x - 3$
(d) $y = \frac{1}{2}\tan^{-1}(x + c) + x + 3$

Q17. If $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$ then value of $\alpha^2\beta^2\gamma^2 + 3(\alpha + \beta + \gamma) =$
(a) 8
(b) 0
(c) 10
(d) 10

Q18. The domain of the function $\sin^{-1}(4x - 1)$ is:
(a) $[-1, 1]$
(b) $[0, 1]$
(c) $\left[0, \frac{1}{2}\right]$
(d) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

Q19. The number of solutions of the system of equations
$\alpha^2 x + \alpha y = -1$
$\alpha x + \alpha^2 y = 1$
is infinite. Then α is:
(a) 0
(b) 1
(c) - 1
(d) 2

Q20. Maximize z = 2 x - y
Subject to constraints
$x + y \leq 4, x + 3y \leq 6, x \geq 0, y \geq 0.$
The corner points of the feasible region are:
(a) (0, 4), (4, 0), (0, 0), (2, 3)
(b) (0, 2), (6, 0), (0, 0), (1, 2)
(c) (0, 2), (4, 0), (0, 0), (3, 1)
(d) (0, 2), (4, 0), (0, 0), (1, 3)

Q21. Match List – I with List – II

	List – I		List – II
(A)	$\frac{d}{dx}(\sin x^2)$	(I)	$\frac{1}{5}$
(B)	$\frac{d}{dx}(e^{\sin x})$	(II)	0
(C)	$f(x) = \tan^{-1}x$ then f'(2)	(III)	$2x\cos x^2$
(D)	If $y = 3\cos x - 2\sin x$, then $\frac{d^2y}{dx^2} + y$	(IV)	$e^{\sin x}.\cos x$

Choose the correct answer from the options given below:
(a) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)
(b) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)
(c) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)
(d) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Q22. If $P(A \cap B) = 0.4, P(\bar{A} \cap \bar{B}) = 0.3$ then the value of $P(\bar{A})$+ $P(\bar{B})$is:
(a) 0.3
(b) 0.5
(c) 0.7
(d) 0.9

Q23. Equation of a line is $\frac{x-3}{2} = \frac{4-y}{3} = \frac{2-z}{4}$ then its direction cosines are:
(a) 2, 3, 4
(b) 2, −3, −4
(c) $\frac{2}{29}, \frac{-3}{29}, \frac{-4}{29}$,
(d) $\frac{2}{\sqrt{29}}, \frac{-3}{\sqrt{29}}, \frac{-4}{\sqrt{29}}$

Q24. Slope of the tangent to the parabola $y^2 = x + 2$ at a point in 1^{st} quadrant and lying on the line $y = x$ is:
(a) $\frac{1}{2}$

(b) $\frac{1}{4}$
(c) $\frac{1}{3}$
(d) 2

Q25. If $\vec{a} = 5\hat{\imath} + \hat{\jmath} + 3\hat{k}$ then the sum of projections $\vec{a}$ on x axis, y axis and z axis is :
(a) $\frac{9}{\sqrt{35}}$
(b) 9
(c) $\frac{5}{\sqrt{35}}$
(d) 5

Q26. If $\begin{bmatrix} x+y+z+w \\ x+y \\ x+z \\ x+w \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \\ 5 \\ 3 \end{bmatrix}$, find the value of $x^2 + y^2 - (w+z)^2$
(a) 16
(b) 20
(c) 4
(d) 8

Q27. If θ ∈ [0, π] is the angle between any two non zero vectors $\vec{a}$ and $\vec{b}$, such that $\left|\vec{a} \cdot \vec{b}\right| = \left|\vec{a} \times \vec{b}\right|$ then θ=
(a) $\frac{\pi}{2}$
(b) $\frac{\pi}{4}$
(c) 0
(d) π

Q28. The value of $\frac{3}{2}(\hat{\imath}\cdot\hat{\imath}) + \frac{1}{2}(\hat{\jmath}\cdot\hat{\jmath}) + \frac{|\hat{\imath}\times\hat{\imath}|}{(\hat{k}\cdot\hat{k})} + \frac{(\hat{k}\cdot\hat{\imath})}{|\hat{\imath}\times\hat{\jmath}|}$ is:
(a) $\frac{5}{2}$
(b) 0
(c) 2
(d) −2

Q29. If a line makes angles $\frac{2\pi}{3}$ and $\frac{\pi}{3}$ with positive direction of x-axis and y-axis respectively, then the acute angle made by the line with positive z-axis is
(a) $\frac{\pi}{3}$
(b) $\frac{\pi}{4}$
(c) $\frac{\pi}{6}$
(d) $\frac{\pi}{2}$

Q30. A is a 2 × 2 non - singular matrix such that adj A = $\begin{bmatrix} \sin\theta - \cos\theta & -\sin\theta\sec\theta \\ 2\cos^2\theta & \sin\theta - \cos\theta \end{bmatrix}$ then |A| is:
(a) 0
(b) 1
(c) −1
(d) −1 or 1

Q31. Let * be a binary operation set Q of rational numbers given by a*b = a + b + ab. Then identity element is:
(a) −1
(b) 0
(c) $\frac{1}{2}$
(d) 1

Q32. For a 3 × 3 matrix A, if A(adj A) = $\begin{bmatrix} 99 & 0 & 0 \\ 0 & 99 & 0 \\ 0 & 0 & 99 \end{bmatrix}$ then det (A) is equal to:
(a) 3×99
(b) $(99)^3$
(c) $(99)^2$
(d) 99

Q33. Let A= {1, 2, 3} and R = {(1, 1), (1, 3), (3, 1), (2, 2), (2, 1), (3, 3)} be a relation on A. Then, R is:
(a) Reflexive
(b) Both Reflexive and Symmetric
(c) Symmetric but not Reflexive
(d) Both Reflexive and Transitive

Q34. Number of solutions of the equation $\begin{vmatrix} -1 & 0 & \sin\theta \\ \sin\theta & -1 & 0 \\ 0 & sin\theta & -1 \end{vmatrix} = 0 \; in \; (0, \; \pi)$ is:
(a) exactly one
(b) exactly zero
(c) exactly two
(d) infinitely many

Q35. If the function $f(x) = \begin{cases} \frac{\sin 5x}{3x}, & x \neq 0 \\ \frac{k}{3}, & x = 0 \end{cases}$ is continuous at x = 0, then $k^2 - 2k + 10$ is equal to:
(a) 35
(b) 25
(c) 40
(d) 15

Q36. Area bounded between the parabola $y = x^2 - 2$ and the line y = x in square units is:
(a) $\frac{9}{2}$
(b) $\frac{11}{2}$
(c) $\frac{7}{2}$
(d) 5

Q37. If $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$, then matrix A is equal to:
(a) $\begin{bmatrix} 1 & 4 & -5 \\ 3 & -4 & 0 \end{bmatrix}$
(b) $\begin{bmatrix} 1 & -2 \\ -5 & 3 \\ 4 & 0 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$
(d) $\begin{bmatrix} 1 & -3 \\ 2 & -4 \\ 5 & 0 \end{bmatrix}$

Q38. The function $f(x) = x^3 + 1$ is:
(a) increasing in [2, 3]
(b) increasing at x = 1 only
(c) decreasing in [2, 3]
(d) neither increasing nor decreasing

Q39. Area bounded by the curve y = logx, x-axis, x =1 and x = 2 in square units is:
(a) log 4

(b) $\log\left(\frac{4}{e}\right)$
(c) 2log 2 +1
(d) log 4 –2

Q40. Let f :[–2, 2]→ $[-2,2]$ be a function defined by $f(x) = x|x|$, then f is:
(a) One-one but not onto
(b) Onto but not one-one
(c) Neither one-one nor onto
(d) Bijective

Q41. Number of defective bulbs in a lot of 500 bulbs follows a binomial distribution with probability of a randomly selected bulb to be defective equal to 0.3. A sample of 50 bulbs is drawn. Probability of 2 defective bulbs in the sample is:
(a) $1225\ (0.7)^{50}$
(b) $2450\ (0.3)^2\ (0.7)^{48}$
(c) $1225\ (0.3)^2\ (0.7)^{48}$
(d) $1225\ (0.3)^{50}$

Q42. The maximum number of equivalence relations on the set A = {a, b, c} is:
(a) 1
(b) 2
(c) 5
(d) 3

Q43. If $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $v = \tan^{-1} x, x \in (-1,1)$ then, $\frac{du}{dv}$ is equal to:
(a) 3
(b) 0
(c) 1
(d) 2

Q44. The function $z = \alpha x + \beta y\ (\alpha, \beta > 0)$ corresponds to the objective function of an LPP that needs to be maximized subject to $x + y \leq 1, x, y \geq 0$. Then the set of optimal solutions is:
(a) Empty set
(b) {(1, 0)} if α < β
(c) { (0, 1) } if α > β
(d) {(t, 1 –t) : t∈[0, 1]} if α = β

Q45. Order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^3\right]^{5/4} = \frac{d^2y}{dx^2}$ are:
(a) Order 1, degree 3
(b) Order 2, degree 4
(c) Order 1, degree 4
(d) Order 2, degree 5

Q46. $\lim_{x\to\infty}\left(\frac{x+7}{x+1}\right)^x$ is equal to:
(a) e
(b) e^3
(c) e^6
(d) e^7

Q47. The integral $\int \frac{dx}{\sqrt{\frac{1}{2}-5x-x^2}}$ is equal to:
(a) $\sin^{-1}\frac{2x+5}{3\sqrt{2}} + C$
(b) $\sin^{-1}\frac{2x+5}{3\sqrt{3}} + C$
(c) $\sin^{-1}\frac{2x-5}{3\sqrt{2}} + C$
(d) $\sin^{-1}\frac{2x-5}{3\sqrt{3}} + C$

Q48. The value of the integral $\int_0^{\pi} 2x\sin^3 x\, dx$ is:
(a) $\frac{2\pi}{3}$
(b) π
(c) $\frac{4\pi}{3}$
(d) $\frac{5\pi}{3}$

Q49. The distance between the line $\vec{r} = 2\hat{\imath} + 2\hat{\jmath} - 3\hat{k} + \lambda(\hat{\imath} + \hat{\jmath} + 4\hat{k})$ and the plane $\vec{r}\cdot(\hat{\imath} - 5\hat{\jmath} + \hat{k}) - 4 = 0$ is:
(a) 0
(b) $2\sqrt{3}$
(c) $\frac{4\sqrt{3}}{3}$
(d) $\frac{5\sqrt{3}}{3}$

Q50. Let X denote the number of tails in two tosses of a coin. If the mean and the variance of X are μ and σ^2 respectively, then $\mu + \sigma^2$ is equal to:
(a) $\frac{3}{2}$
(b) 1
(c) 2
(d) $\frac{5}{2}$

SOLUTIONS

S1. Ans. (c)

Sol. Given

$$\int_{\frac{-\pi}{2}}^{\pi/2} (x^5 + x^3 + x + 2)\, dx$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^5 + x^3 + x)\, dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2dx$$

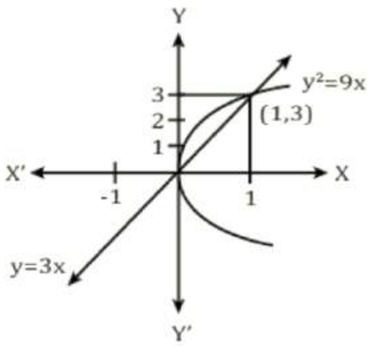
$= 0 + \{2x\}_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$ {Since $f(x) = x^5 + x^3 + x$ is an odd function}

$$= 2\left\{\frac{\pi}{2} + \frac{\pi}{2}\right\} = 2\pi$$

S2. Ans. (c)

Sol. Given

$y^2 = 9x$ and $y = 3x$



Solving above equations

$(3x)^2 = 9x$

$9x^2 - 9x = 0$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0, 1$

$x = 0 \Rightarrow y = 0$

$x = 1 \Rightarrow y = 3$

Area of bounded region $= \int_0^1 3\sqrt{x}\, dx - \int_0^1 3x dx$

$$= 3 \times \frac{2}{3}\left\{x^{\frac{3}{2}}\right\}_0^1 - 3\left\{\frac{x^2}{2}\right\}_0^1$$

$= 2 - \frac{3}{2} = \frac{1}{2}$ sq. unit

S3. Ans. (b)

Sol. Let A and B be two events in a random experiment with sample space such that P(B) ≠ 0 and P(A|B) = 1

Then $\frac{P(A \cap B)}{P(B)} = 1$

$P(A \cap B) = P(B)$

$\Rightarrow B \subset A$

S4. Ans. (c)

Sol. We have

$P(1) = \alpha, P(2) = 0.1, P(3) = 0.3, P(4) = \beta, P(5) = 0.4, P(6) = 0.1$

$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$

$\alpha + 0.1 + 0.3 + \beta + 0.4 + 0.1 = 1$

$\alpha + \beta + 0.9 = 1$

$\alpha + \beta = 0.1$

S5. Ans. (b)

Sol. Given

$f(x) = \begin{vmatrix} x^2 & x \\ 3 & 1 \end{vmatrix} = x^2 - 3x$

$f'(x) = 2x - 3$

$f'(x) = 0$

$2x - 3 = 0$

$x = \frac{3}{2}$

$f''(x) = 2 > 0$ (minima)

Local minimum at $x = \frac{3}{2}$

S6. Ans. (c)

Sol. Given

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = x\begin{vmatrix} 2 & 3 \\ 8 & 9 \end{vmatrix} + y\begin{vmatrix} 1 & 3 \\ 7 & 9 \end{vmatrix} + z\begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix}$$

$x = -4, y = 5, z = 6$

$x + y + z = -4 + 5 - 6 = -5$

S7. Ans. (d)

Sol. (A) Given

$$f(x) = \begin{cases} kx^2, & if\ x < 3 \\ 3, & if\ x \geq 3 \end{cases}$$

L.H.L. = R.H.L.

$\lim_{x \to 3^-} kx^2 = 3$

$\lim_{h \to 0} k(3-h)^2 = 3$

$9k = 3 \Rightarrow k = \frac{1}{3}$

(B) Given

$$f(x) = \begin{cases} kx + 1 & if\ x \geq 4 \\ x + 2 & if\ x < 4 \end{cases}$$

$\lim_{x \to 4^-} x + 2 = \lim_{x \to 4^+} kx + 1$

$\lim_{h \to 0}(4 - h) + 2 = \lim_{h \to 0} k(4 + h) + 1$

$6 = 4k + 1$

$k = \frac{5}{4}$

(C) Given

$$f(x) = \begin{cases} \frac{5 + x}{2}, & if\ x \neq 0 \\ 3k, & if\ x = 0 \end{cases}$$

L.H.L. = R.H.L.

$\lim_{x \to 0^-} \frac{5 + x}{2} = 3k$

$\lim_{h \to 0} \frac{5 + (0 - h)}{2} = 3k$

$k = \frac{5}{6}$

(D) Given

$$f(x) = \begin{cases} x + k, & if\ x > -2 \\ -3, & if\ x \leq -2 \end{cases}$$

L.H.L. = R.H.L.

$-3 = \lim_{x \to -2^+} x + k$

$\lim_{h \to 0}(-2 + h + k) = -3$

$-2 + k = -3$

$k = -1$

S8. Ans. (a)

Sol. Given

A is a square matrix and |A| = 4, then

$|AA'| = |A||A'| = |A||A| = |A|^2$ {since $|A'| = |A|$}
$= (4)^2 = 16$

S9. Ans. (b)

Sol. Given

$x = \log t^2$ and $y = (\log t)^2$

$\frac{dx}{dt} = \frac{1}{t^2} \times 2t = \frac{2}{t}$

$\frac{dy}{dt} = 2\log t \times \frac{1}{t} = \frac{2\log t}{t}$

$\frac{dy}{dx} = \frac{\frac{2\log t}{t}}{\frac{2}{t}} = \log t$

$\frac{d^2y}{dx^2} = \frac{1}{t} \times \frac{dt}{dx} = \frac{1}{t} \times \frac{t}{2} = \frac{1}{2}$

S10. Ans. (c)

Sol. If A, B and C are all the corner points of feasible region (bounded) of an LPP with objective function Z that needs to be maximized such that $Z_A > Z_B$ and $Z_C < Z_A$(Here Z_A denotes value of Z at A), then There is a unique optimal solution.

S11. Ans. (c)

Sol. A square matrix $B = [bij]_{n\times n}$where
bij = 0 when i#j
bij = k when i = j for some constant k is called scalar matrix.

S12. Ans. (b)

Sol. Given

$x(t) = t^3\left(3 - \frac{t^2}{5}\right) = 3t^3 - \frac{t^5}{5}$

At P and Q, the velocity of car is 0.
Let v be the velocity of car, then

$v = \frac{dx}{dt} = 9t^2 - t^4$

Putting $v = 0$

$9t^2 - t^4 = 0$

$t^2(9 - t^2) = 0$

$t = 0, 3$

It takes 3 seconds to reach from P to Q
Distance PQ = Distance travelled in 3 second

$x = t^3\left(3 - \frac{t^2}{5}\right) = (3)^3\left(3 - \frac{9}{5}\right) = 27 \times \frac{6}{5} = \frac{162}{5}$ m

S13. Ans. (a)

Sol. Min z = 40x + 60y

$3x + 2y \geq 9$

$4x + y \geq 5$

$x, y \geq 0$

S14. Ans. (d)

Sol. (A) Given

$y = cx$

$y' = c$

Multiplying by x,

$xy' = cx$

$xy' = y$

(B) We have

$y = e^{-3x} + c$

$y' = -3e^{-3x}$

$y' + 3y = 0$

(C) We have

$x + y = \sqrt{x}$+c

$1 + y' = \frac{1}{2\sqrt{x}}$

$y' = \frac{1}{2\sqrt{x}} - 1$

(D) We have

$xy = \log y + c$

$xy' + y = \frac{1}{y}y'$

$xyy' + y^2 = y'$

$y^2 = y'(1 - xy)$

$y' = \frac{y^2}{1-xy}$

S15. Ans. (d)

Sol. Let

$I = \int_0^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{2-x}}\,dx$ .... (i)

$I = \int_0^2 \frac{\sqrt{2-x}}{\sqrt{2-x}+\sqrt{x}}\,dx$ .... (ii)

Adding eq. (i) & (ii), we get

$I + I = \int_0^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{2-x}}\,dx + \int_0^2 \frac{\sqrt{2-x}}{\sqrt{2-x}+\sqrt{x}}\,dx$

$2I = \int_0^2 1\,dx = (2 - 0)$

$I = 1$

S16. Ans. (c)

Sol. Given

$\frac{dy}{dx} = \cos(x + y + 3)$

Put $x + y = v \Rightarrow 1 + \frac{dy}{dx} = \frac{dv}{dx} \Rightarrow \frac{dy}{dx} = \frac{dv}{dx} - 1$

$\frac{dv}{dx} - 1 = \cos(v + 3)$

$\frac{dv}{dx} = 1 + \cos(v + 3)$

$\frac{dv}{1+\cos(v+3)} = dx$

$\frac{dv}{1+2\cos^2\left(\frac{v+3}{2}\right)-1} = dx$

$\frac{dv}{2\cos^2\left(\frac{v+3}{2}\right)} = dx$

$\frac{1}{2}\int \sec^2\left(\frac{v+3}{2}\right)dv = \int dx$

$\frac{1}{2}\tan\left(\frac{v+3}{2}\right) \times 2 = x + c$

$\tan\left(\frac{v+3}{2}\right) = x + c$

$\tan\left(\frac{x+y+3}{2}\right) = x + c$

$\frac{x+y+3}{2} = \tan^{-1}(x + c)$

$x + y + 3 = 2\tan^{-1}(x + c)$

$y = 2\tan^{-1}(x + c) - x - 3$

S17. Ans. (a)

Sol. Given

$\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$

We have

$0 \leq \cos^{-1}\alpha \leq \pi$

Maximum value of $\cos^{-1}\alpha$ satisfies the given equation.

i.e. $\cos^{-1}\alpha = \pi \Rightarrow \alpha = \cos\pi \Rightarrow \alpha = -1$

Similarly, we can prove $\beta = \gamma = -1$

Now

$\alpha^2\beta^2\gamma^2 + 3(\alpha + \beta + \gamma) = (-1)^2(-1)^2(-1)^2 + 3(-1 - 1 - 1)$

$= 1 + 3(-3) = 1 - 9 = -8$

S18. Ans. (c)

Sol. Given function is

$\sin^{-1}(4x - 1)$

We have

$-1 \le (4x-1) \le 1$
$0 \le 4x \le 2$
$0 \le x \le \frac{1}{2}$

S19. Ans. (c)

Sol. Given system of equations
$\alpha^2 x + \alpha y = -1$
$\alpha x + \alpha^2 y = 1$
Here
$A = \begin{bmatrix} \alpha^2 & \alpha \\ \alpha & \alpha^2 \end{bmatrix}$
ATQ.
$|A| = \alpha^4 - \alpha^2 = 0$
$\alpha^2(\alpha^2 - 1) = 0$
$\alpha = 0, 1, -1$
$\alpha = -1$ satisfy the given equations.

S20. Ans. (c)

Sol. Given
Maximize z = 2 x - y
Subject to constraints
$x + y \le 4, x + 3y \le 6, x \ge 0, y \ge 0.$
We have
$x + y = 4$ .... (i)
$x + 3y = 6$ .... (ii)
From eq. (i), we get
(4, 0) & (0, 4)
From eq. (ii), we get
(6, 0) & (0, 2)
From eq. (i) & (ii)
$2y = 2 \Rightarrow y = 1$
$x + 1 = 4 \Rightarrow x = 3$
So, intersection points are (3, 1)
Corner points of feasible region are (0, 2), (4, 0), (0, 0), (3, 1)

S21. Ans. (c)

Sol. (A) We have
$\frac{d}{dx}(\sin x^2) = \cos x^2 \times 2x = 2x \cos x^2$

(B) We have
$\frac{d}{dx}(e^{\sin x}) = e^{\sin x} \cos x = \cos x\, e^{\sin x}$

(C) $f(x) = \tan^{-1} x$
$f'(x) = \frac{1}{1+x^2}$
$f'(2) = \frac{1}{5}$

(D) $y = 3\cos x - 2\sin x$
$\frac{dy}{dx} = -3\sin x - 2\cos x$
$\frac{d^2y}{dx^2} = -3\cos x + 2\sin x$
$\frac{d^2y}{dx^2} + y = -3\cos x + 2\sin x + 3\cos x - 2\sin x = 0$

S22. Ans. (d)

Sol. Given
$P(A \cap B) = 0.4, P(\bar{A} \cap \bar{B}) = 0.3$
We have
$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B) \Rightarrow P(A \cup B) = 1 - 0.3 = 0.7$
We know
$P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow P(A) + P(B) = P(A \cup B) + P(A \cap B)$
Now
$P(\bar{A}) + P(\bar{B}) = [1 - P(A)] + [1 - P(B)] = 2 - [P(A) + P(B)]$
$= 2 - [P(A \cup B) + P(A \cap B)] = 2 - (0.7 + 0.4) = 2 - (1.1) = 0.9$

S23. Ans. (d)

Sol. Equation of a line is
$\frac{x-3}{2} = \frac{4-y}{3} = \frac{2-z}{4}$
Or $\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-2}{-4}$
Direction ratios of line are $2, -3, -4$
So, direction cosines are
$\frac{2}{\sqrt{(2)^2+(-3)^2+(-4)^2}}, \frac{-3}{\sqrt{(2)^2+(-3)^2+(-4)^2}}, \frac{-4}{\sqrt{(2)^2+(-3)^2+(-4)^2}}$
i.e.
$\frac{2}{\sqrt{29}}, \frac{-3}{\sqrt{29}}, \frac{-4}{\sqrt{29}}$

S24. Ans. (b)

Sol. Given curve
$y^2 = x + 2$
$2y\frac{dy}{dx} = 1$
$\frac{dy}{dx} = \frac{1}{2y}$
Given line $y = x$
$\Rightarrow y^2 - y - 2 = 0$
$y^2 - 2y + y - 2 = 0$
$(y-2)(y+1) = 0$
$y = 2, -1$
Only $y = 2$ lie in first quadrant.
So, slope $= \frac{1}{4}$

S25. Ans. (b)

Sol. Given
$\vec{a} = 5\hat{i} + \hat{j} + 3\hat{k}$
Projection of $\vec{a} = 5\hat{i} + \hat{j} + 3\hat{k}$ on x – axis is 5
Projection of $\vec{a} = 5\hat{i} + \hat{j} + 3\hat{k}$ on y – axis is 1
Projection of $\vec{a} = 5\hat{i} + \hat{j} + 3\hat{k}$ on z – axis is 3
Now sum of projection is 9.

S26. Ans. (c)

Sol. Given
$\begin{bmatrix} x+y+z+w \\ x+y \\ x+z \\ x+w \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \\ 5 \\ 3 \end{bmatrix}$
$x + y + z + w = 10$ .... (i)
$x + y = 6$ ....(ii)
$x + z = 5$ .... (iii)
$x + w = 3$ .... (iv)
$6 + z + w = 10 \Rightarrow z + w = 4$
Eq. (iii) – (iv)
$z - w = 2$
Solving above equations we get
$z = 3$ & $w = 1$
$\Rightarrow x = 2$ & $y = 4$
$x^2 + y^2 - (w+z)^2 = 4 + 16 - 16 = 4$

S27. Ans. (b)

Sol. Given
$|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$
$|\vec{a}||\vec{b}| \cos\theta = |\vec{a}||\vec{b}| \sin\theta$

$\tan\theta = 1$
$\theta = \frac{\pi}{4}$

S28. Ans. (c)
Sol. We have
$\frac{3}{2}(\hat{\imath}\cdot\hat{\imath}) + \frac{1}{2}(\hat{\jmath}\cdot\hat{\jmath}) + \frac{|\hat{\imath}\times\hat{\imath}|}{(\hat{k}\cdot\hat{k})} + \frac{(\hat{k}\cdot\hat{\imath})}{|\hat{\imath}\times\hat{\jmath}\,|} = \frac{3}{2} + \frac{1}{2} + 0 + 0 = 2$

S29. Ans. (b)
Sol. Given
$\alpha = \frac{2\pi}{3}$ and $\beta = \frac{\pi}{3}$
We have
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
$\cos^2\frac{2\pi}{2} + \cos^2\frac{\pi}{3} + \cos^2\gamma = 1$
$\frac{1}{4} + \frac{1}{4} + \cos^2\gamma = 1$
$\cos^2\gamma = \frac{1}{2}$
$\gamma = \frac{\pi}{4}$

S30. Ans. (b)
Sol. Given
adj A $= \begin{bmatrix} \sin\theta - \cos\theta & -\sin\theta\sec\theta \\ 2\cos^2\theta & \sin\theta - \cos\theta \end{bmatrix}$
$|adj(A)| = |A|^{2-1} = |A|$
$|A| = (\sin\theta - cos\theta)^2 + 2\sin\theta\cos\theta = 1$

S31. Ans. (b)
Sol. Given
a*b = a + b + ab
Let 'e' be the identity element, then
$a * e = a$
$a + e + ae = a$
$e + ae = 0 \Rightarrow e(1 + a) = 0 \Rightarrow e = 0$

S32. Ans. (d)
Sol. We have
A(adj A) $= \begin{bmatrix} 99 & 0 & 0 \\ 0 & 99 & 0 \\ 0 & 0 & 99 \end{bmatrix}$
We know
$A(adjA) = |A|I$
$\Rightarrow$A(adj A) $= \begin{bmatrix} 99 & 0 & 0 \\ 0 & 99 & 0 \\ 0 & 0 & 99 \end{bmatrix} = 99I$
$|A| = 99$

S33. Ans. (a)
Sol. We have
R = {(1, 1), (1, 3), (3, 1), (2, 2), (2, 1), (3, 3)} is Reflexive.

S34. Ans. (a)
Sol. Given
$\begin{vmatrix} -1 & 0 & \sin\theta \\ \sin\theta & -1 & 0 \\ 0 & sin\theta & -1 \end{vmatrix} = 0$
$-1(1-0) + 0 + \sin\theta\,(\sin^2\theta - 0) = 0$
$\sin^3\theta - 1 = 0$
$\sin\theta = 1$
$\theta = \frac{\pi}{2}$
Only one solution.

S35. Ans. (b)
Sol. Given function
$f(x) = \begin{cases} \frac{\sin 5x}{3x}, & x \neq 0 \\ \frac{k}{3}, & x = 0 \end{cases}$ is continuous at $x = 0$,
then
$\lim_{x\to 0}\frac{\sin 5x}{3x} = \frac{k}{3}$
$\frac{5}{3}\lim_{x\to 0}\frac{\sin 5x}{5x} = \frac{k}{3}$
$\frac{5}{3}\times 1 = \frac{k}{3}$
$k = 5$
Now
$k^2 - 2k + 10 = 25 - 10 + 10 = 25$

S36. Ans. (a)
Sol. Given
parabola $y = x^2 - 2$ and the line y = x
$x = x^2 - 2 \Rightarrow x^2 - x - 2 = 0$
$\Rightarrow x = 2, -1$
$x = 2 \Rightarrow y = 2$
$x = -1 \Rightarrow= y = -1$
Area of bounded region $= \int_{-1}^{2} x dx - \int_{-1}^{2}(x^2 - 2)dx$
$= \left\{\frac{x^2}{2}\right\}_{-1}^{2} - \left\{\frac{x^3}{3} - 2x\right\}_{-1}^{2} = \left(2 - \frac{1}{2}\right) - \left(\frac{8}{3} - 4 + \frac{1}{3} - 2\right) = \frac{9}{2}$

S37. Ans. (c)
Sol. Given
$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$
Let $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$
$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$
$\begin{bmatrix} 2a - d & 2b - e & 2c - f \\ a & b & c \\ -3a + 4d & -3b + 4e & -3c + 4f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$
We get
$a = 1, b = -2, c = -5, d = 3, e = 4, f = 0$
$A = \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$

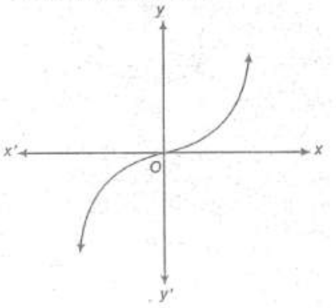
S38. Ans. (a)
Sol. Given function $f(x) = x^3 + 1$
$f'(x) = 3x^2 \geq 0$
Increasing in R.
So, increasing in [2, 3]

S39. Ans. (b)
Sol. Given curve
$y = \log x$ & x-axis, x =1 and x =2
Now area of bounded region $= \int_1^2 y dx =$
$\int_1^2 \log x\, dx = \{x\log x - x\}_1^2 = 2\log 2 - 2 + 1$
$= 2\log 2 - 1 = \log 2^2 - \log e = \log\left(\frac{4}{e}\right)$

S40. Ans. (d)
Sol. Given $f(x) = x|x|$
$f(x) = \begin{cases} -x^2, & x < 0 \\ 0, & x = 0 \\ x^2, & x > 0 \end{cases}$

$f(x)$ is one – one and onto.

$\Rightarrow f(x)$ is bijective.



S41. Ans. (c)

Sol. Here

$n = 50, r = 2, p = 0.3, q = 1 - 0.3 = 0.7$

Required probability $= 50_{C_2}(0.3)^2(0.7)^{50-2} = 1225(0.3)^2(0.7)^{48}$

S42. Ans. (c)

Sol. Given set

$\{1, 2, 3\}$

Equivalence relations on the set are

$\{(1,1),(2,2),(3,3)\}$

$\{(1,1),(2,2),(3,3),(1,2),(2,1)\}$

$\{(1,1),(2,2),(3,3),(1,3),(3,1)\}$

$\{(1,1),(2,2),(3,3),(2,3),(3,2)\}$

$\{(1,1),(2,2),(3,3),(1,2),(2,1),$
$(1,3),(3,1),(2,3),(3,2)\}$

S43. Ans. (d)

Sol. Given

$u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$

$u = \sin^{-1}\left(\frac{2tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta = 2\tan^{-1} x$

$\frac{du}{dx} = \frac{2}{1+x^2}$

Also

$v = \tan^{-1} x \Rightarrow \frac{dv}{dx} = \frac{1}{1+x^2}$

$\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = 2$

S44. Ans. (d)

Sol. Given

$z = \alpha x + \beta y \ (\alpha, \beta > 0)$

$x + y = 1$

Corner Points are $(1, 0)$ & $(0, 1)$

Then the set of optimal solutions is:

$\{(t, 1 - t) : t \in [0, 1]\}$ if $\alpha = \beta$

S45. Ans. (b)

Sol. Given

$$\left[1 + \left(\frac{dy}{dx}\right)^3\right]^{5/4} = \frac{d^2y}{dx^2}$$

Taking power 4 both sides

$$\left[1 + \left(\frac{dy}{dx}\right)^3\right]^5 = \left\{\frac{d^2y}{dx^2}\right\}^4$$

Order = 2, degree = 4

S46. Ans. (c)

Sol. We have

$$\lim_{x\to\infty}\left(\frac{x+7}{x+1}\right)^x = \lim_{x\to\infty}\left(\frac{x+1}{x+1} + \frac{6}{x+1}\right)^x = \lim_{x\to\infty}\left(1 + \frac{6}{x+1}\right)^x = e^6$$

S47. Ans. (b)

Sol. Given

$$\int \frac{dx}{\sqrt{\frac{1}{2} - 5x - x^2}} = \int \frac{dx}{\sqrt{-\left[x^2 + 2.x.\frac{5}{2} + \frac{25}{4} - \frac{25}{4} - \frac{1}{2}\right]}}$$

$$= \int \frac{dx}{\sqrt{-\left[\left(x+\frac{5}{2}\right)^2 - \frac{27}{4}\right]}}$$

$$\int \frac{dx}{\sqrt{\frac{27}{4} - \left(x+\frac{5}{2}\right)}} = \sin^{-1}\left(\frac{x+\frac{5}{2}}{\frac{3\sqrt{3}}{2}}\right) = \sin^{-1}\left(\frac{2x+5}{3\sqrt{3}}\right) + C$$

S48. Ans. (c)

Sol. We have

$$sin3x = 3\sin x - 4\sin^3 x \Rightarrow \sin^3 x = \frac{3\sin x - \sin 3x}{4}$$

Now

$$\int_0^{\pi} 2x\sin^3 x\, dx = \frac{3}{2}\int_0^{\pi} xsinxdx - \frac{1}{2}\int_0^{\pi} x\sin 3x\, dx$$

$$= \frac{3}{2} \times \pi - \frac{1}{2} \times \frac{\pi}{3} = \frac{4\pi}{3}$$

S49. Ans. (d)

Sol. Given equation of line is

$\vec{r} = 2\hat{\imath} + 2\hat{\jmath} - 3\hat{k} + \lambda(\hat{\imath} + \hat{\jmath} + 4\hat{k})$ and the plane $\vec{r} \cdot (\hat{\imath} - 5\hat{\jmath} + \hat{k}) - 4 = 0$

Distance between line and plane is

$$\left|\frac{1(2) - 5(2) + 1(-3) - 4}{\sqrt{1^2 + (-5)^2 + (1)^2}}\right| = \left|\frac{-15}{\sqrt{27}}\right| = \frac{15}{3\sqrt{3}} = \frac{5\sqrt{3}}{3}$$

S50. Ans. (a)

Sol. Let X denote the number of tails in two tosses of a coin.

If the mean and the variance of X are μ and σ^2 respectively, then we have

$\mu = 1$ & $\sigma^2 = \frac{1}{2}$

Now

$\mu + \sigma^2 = 1 + \frac{1}{2} = \frac{3}{2}$