Code No. 041

Roll No. | | | | | | | |

- Please check that this question paper contains **4** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **33** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

MATHEMATICS–XII

Sample Paper (Solved)

Time allowed: 3 hours *Maximum Marks: 80*

General Instructions:

PART A

Section I

All questions are compulsory. In case of internal choices attempt any one.

1. State whether the function $f : N \to N$ given by $f(x) = 5x$ is injective, surjective or both.

Or

Let R be the equivalence relation in the set $A = \{0, 1, 2, 3, 4, 5\}$ given by
$R = \{(a, b) : 2 \text{ divides } (a - b)\}$. Write the equivalence class [0].

2. Differentiate $\sin^2(x^2)$ w.r.t. x^2.

3. Write total number of one-one functions from set A to set B where $A = \{1, 2, 3\}$, $B = \{a, b, c, d\}$.

Or

Write total number of one-one functions from set A to set B where $A = \{1, 2, 3, 4\}$, $B = \{a, b, c\}$.

4. For what value of a, $\begin{pmatrix} 2a & -1 \\ -8 & 3 \end{pmatrix}$ is a singular matrix?

5. A square matrix A, of order 3, has $|A| = 5$, find $|A.adj\ A|$.

Or

What is the value of $|3I_3|$, where I_3 is the identity matrix of order 3?

6. For what value of k, the matrix $\begin{bmatrix} 2-k & 3 \\ -5 & 1 \end{bmatrix}$ is not invertible?

7. Write the antiderivative of $\left(3\sqrt{x} + \frac{1}{\sqrt{x}}\right)$.

Or

Evaluate : $\int_0^3 \frac{dx}{9 + x^2}$

8. Evaluate : $\int_{1}^{\sqrt{3}} \frac{dx}{1+x^2}$

9. Evaluate : $\int \frac{x^3 - x^2 + x - 1}{x-1}\, dx$

Or

Evaluate : $\int_{0}^{1} \frac{2x}{1+x^2}\, dx$

10. Find the distance of the point (a, b, c) from x-axis.

11. In a triangle ABC, the sides AB and BC are represented by vectors $2\hat{i} - \hat{j} + 2\hat{k}$, $\hat{i} + 3\hat{j} + 5\hat{k}$ respectively. Find the vector representing CA.

12. Find the area of the parallelogram, whose diagonals are $\vec{d_1} = 5\hat{i}$ and $\vec{d_2} = 2\hat{j}$.

13. Find the direction cosines of a line, passing through origin and lying in the first octant, making equal angles with the three coordinate axes.

14. If the lines $\frac{x-1}{-2} = \frac{y-4}{3p} = \frac{z-3}{4}$ and $\frac{x-2}{4p} = \frac{y-5}{2} = \frac{z-1}{-7}$ are perpendicular to each other, then find the value of p.

15. If A and B are two events such that P(A) = 0.4, P(B) = 0.8 and P(B/A) = 0.6, then find P(A/B).

16. If A and B are two independent events, then prove that the probability of occurrence of atleast one of A and B is given by 1 – P(A′).P(B ′).

Section II

Both the case-study based questions are compulsory. Attempt any 4 sub-parts from each question (17–21) and (22 – 26). Each question carries 1 mark.

17. Case Study—The given figure shows the newspapers in two languages *i.e.*, Hindi and English. In a Hostel, 60% of the students read the Hindi Newspapers, 40% read the English newspapers and 20% read both Hindi and English newspaper. A student is selected at random:

(*i*) Find the probability that she reads Hindi or English newspaper.

(*a*) $\frac{3}{5}$ (*b*) $\frac{1}{5}$

(*c*) $\frac{4}{5}$ (*d*) $\frac{2}{5}$

(*ii*) Find the probability that she reads neither Hindi nor English newspaper.

(*a*) $\frac{2}{5}$ (*b*) 0 (*c*) $\frac{4}{5}$ (*d*) $\frac{1}{5}$

(*iii*) If she reads Hindi newspaper, find the probability that she reads English newspaper.

(*a*) $\frac{1}{3}$ (*b*) $\frac{2}{3}$ (*c*) $\frac{4}{10}$ (*d*) $\frac{1}{2}$

(*iv*) If she reads English newspaper, find the probability that she reads Hindi newspaper.

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{4}$ (*c*) $\frac{2}{10}$ (*c*) $\frac{3}{10}$

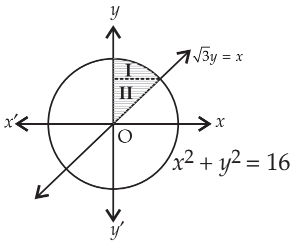
(*v*) Check that events Hindi and English independent or not.

(*a*) Yes (*b*) No

(*c*) Mutually exclusive (*d*) none of the above

18. Case Study— ***Archery School vs. School Competition***

This figure shows an archery board in which an arrow across the middle of the bow is placed with the bowstring in the arrow's nock.



Answer the following questions:

(*i*) Equation of the circle is:

(*a*) $x^2 + y^2 = r^2$ (*b*) $x^2 - y^2 = r^2$

(*c*) $(x-h)^2+(y-k)^2=r^2$ (*d*) None of the above

(*ii*) Equation of the circle in terms of *y* is:

(*a*) $x = \sqrt{16 - y^2}$ (*b*) $x = \sqrt{4 - y^2}$ (*c*) $x = \sqrt{4 + y^2}$ (*d*) $x = \sqrt{y^2 + 16}$

(*iii*) Intersection point on *y*-axis is:

(*a*) $y = 0$ (*b*) $y = 4$ (*c*) $y = 1$ (*d*) $y = 2$

(*iv*) Radius of given circle is:

(*a*) 0 (*b*) 2 (*c*) 4 (*d*) 3

(*v*) Area of part I is:

(*a*) $\sqrt{3}$ sq. units (*b*) $2\sqrt{3}$ sq. units (*c*) $\frac{4\pi}{\sqrt{3}}$ sq. units (*d*) $4\sqrt{3}$ sq. units

PART B

Section III

19. Prove that if $\frac{1}{2} \le x \le 1$ then $\cos^{-1} x + \cos^{-1}\left[\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right] = \frac{\pi}{3}$.

20. Let $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -6 \\ -2 & 4 \end{bmatrix}$. Then compute AB. Hence, solve the following system of equations : $2x + y = 4$; $3x + 2y = 1$.

Or

If $A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$, then using A^{-1}, solve the following system of equations : $x - 2y = -1$, $2x + y = 2$.

21. If $f(x) = \sin 2x - \cos 2x$, find $f'\left(\frac{\pi}{6}\right)$.

22. Find the equations of tangent and normal to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point where it cuts the *x*-axis.

23. Evaluate : $\int \frac{\sin x - \cos x}{\sin x . \cos x}\, dx$.

Or

Evaluate : $\int e^x \left(\frac{x^2+1}{(x+1)^2}\right) dx$.

24. Find the area bounded by the curves $y = \sqrt{x}$, $2y + 3 = x$ and *x*-axis.

25. Solve the following differential equation : $x \log x \frac{dy}{dx} + y = 2 \log x$.

26. Let $\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and satisfying $\vec{d} . \vec{c} = 21$.

27. Find the values of a so that the following lines are skew:

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-a}{4};\ \frac{x-4}{5}=\frac{y-1}{2}=z.$$

28. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of diamond cards drawn.

Or

P speaks truth in 70% of the cases and Q in 80% of the cases. In what percent of cases are they likely to agree in stating the same fact?

Section IV

All questions are compulsory. In case of internal choices attempt any one.

29. Let A = {1, 2, 3, …, 9} and R be the relation in A × A defined by (a, b) R (c, d) if $a + d = b + c$ for $a, b, c, d \in$ A. Prove that R is an equivalence relation. Also obtain the equivalence class [(2, 5)].

30. If $x = \sin t$, $y = \sin kt$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + k^2y = 0$.

31. If $y^x + x^y + x^x = a^b$, find $\frac{dy}{dx}$.

Or

Find all the points of discontinuity of the function $f(x) = [x^2]$ on [1, 2), where [.] denotes the greatest integer function.

32. Separate the interval $\left[0, \frac{\pi}{2}\right]$ into subintervals in which the function $f(x) = \sin^4 x + \cos^4 x$ is strictly increasing or strictly decreasing.

33. Evaluate : $\int_{-1}^{1/2} |x\cos(\pi x)|\, dx$.

34. Using integration, find the area of the region bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

Or

Using integration, find the area of the region : $\{(x, y) : 0 \le 2y \le x^2, 0 \le y \le x, 0 \le x \le 3\}$

35. Solve the following differential equation :

$$\sqrt{1 + x^2 + y^2 + x^2y^2} + xy\frac{dy}{dx} = 0.$$

Section V

All questions are compulsory. In case of internal choices attempt any one.

36. Using matrices, solve the following system of equations :

$x + 2y + z = 7$, $\quad x + 3z = 11$, $\quad 2x - 3y = 1$.

Or

Using matrices, solve the following system of equations :

$x + y + z = 6$, $\quad x + 2z = 7$, $\quad 3x + y + z = 12$

37. Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point (1, 3, 4) from the plane $2x - y + z + 3 = 0$. Find also, the image of the point in the plane.

Or

Show that the lines $\frac{x+3}{-3}=\frac{y-1}{1}=\frac{z-5}{5}$ and $\frac{x+1}{-1}=\frac{y-2}{2}=\frac{z-5}{3}$ are coplanar. Also find the equation of the plane.

38. Solve the following graphically and also find the maximum profit.

Maximum Profit, $Z = 1000x + 600y$

Subject to constraints: $x + y \le 200$; $x \ge 20$; $y > 4x$ and $x \ge 0, y \ge 0$.

Or

Solve the following graphically and also find the maximum profit.

Maximum Profit, $Z = x + y$

Subject to constraints: $2x + 3y \le 120$; $\frac{x}{50}+\frac{y}{80} \le 1$; $x \ge 0, y \ge 0$.

Answer Sheet

S	A	M	P	L	E							P	A	P	E	R		

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MATHEMATICS

1. The function $f(x) = 5x$ is injective (one-one)
for $f(x_1) = 5x_1$, $f(x_2) = 5x_2$
$f(x_1) = f(x_2) \Rightarrow 5x_1 = 5x_2$
$\Rightarrow x_1 = x_2$ $\therefore$ $f(x)$ is injective
Further f is not surjective (onto), as for $1 \in N$, there does not exist any x in N such that $f(x) = 5x = 1$
$\therefore$ **f is injective (one-one) but not surjective (onto).**

Or

$0 \sim 0$ ($\because$ 2 divides 0)
$2 \sim 0$ ($\because$ 2 divides 2)
$4 \sim 0$ ($\because$ 2 divides 4)
$\therefore$ **The equivalence class [0] = {0, 2, 4}**

2. We have $f(x^2) = \sin^2 (x^2)$
$f'(x^2) = 2 \sin(x^2) \cos(x^2)$ $\Rightarrow$ $\mathbf{f'(x^2) = \sin (2x^2)}$ **...[$\because$ 2 sin θ cos θ = sin 2θ**

3. Let $p = n(A) = 3$, and $q = n(B) = 4$, $p < q$

$\therefore$ **Number of one-one functions from A to B** = $\frac{q!}{(q-p)!} = \frac{4!}{(4-3)!}$ = **4! = 24**

Or

Let $p = n(A) = 4$, and $q = n(B) = 3$; Since $p > q$
$\therefore$ **Number of one-one function from A to B = 0.**

4. Since $\begin{pmatrix} 2a & -1 \\ -8 & 3 \end{pmatrix}$ is a singular matrix

$\therefore$ $\begin{vmatrix} 2a & -1 \\ -8 & 3 \end{vmatrix} = 0$

$\Rightarrow$ $6a - 8 = 0$ $\Rightarrow$ $6a = 8$ $\therefore$ $a = \frac{8}{6} = \mathbf{\frac{4}{3}}$

5. A.*adj* A = |A|I
A.*adj* A = 5I **...[Given |A| = 5**

$\therefore$ $|A.adj\, A| = |5I| \Rightarrow |A.\text{adj}\, A| = \begin{vmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{vmatrix} = 5(25 - 0) = \mathbf{125}$

Alternatively.

$$|A.adj\,A| = |A|\,|adj\,A|$$
$$= |A|\,|A|^{n-1} = |A|^n$$
$$= (5)^3 = \mathbf{125}$$

Or

$$|3I_3| = 3^3|I_3| \qquad [\because |kA| = k^n|A|]$$
$$= 27(1) = \mathbf{27} \qquad [\because |I_3| = 1]$$

6. $\begin{bmatrix} 2-k & 3 \\ -5 & 1 \end{bmatrix}$ is not invertible

$\therefore \quad \begin{vmatrix} 2-k & 3 \\ -5 & 1 \end{vmatrix} = 0 \qquad \Rightarrow \quad 2-k+15 = 0$

$\Rightarrow \quad 17-k = 0 \qquad \Rightarrow \quad \mathbf{k = 17}$

7. $$\int\left(3\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx = \int\left(3x^{1/2} + x^{-1/2}\right)dx$$
$$= 3.\frac{2}{3}x^{3/2} + \frac{2}{1}x^{1/2} + C$$
$$= 2x\sqrt{x} + 2\sqrt{x} + C = \mathbf{2\sqrt{x}\,(x+1) + C}$$

Or

$$\int_0^3 \frac{dx}{9+x^2} = \int_0^3 \frac{dx}{3^2+x^2}$$
$$= \frac{1}{3}\left[\tan^{-1}\left(\frac{x}{3}\right)\right]_0^3 \qquad \left[\because \int\frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C\right.$$
$$= \frac{1}{3}\left[\tan^{-1}\left(\frac{3}{3}\right) - \tan^{-1}\left(\frac{0}{3}\right)\right] = \frac{1}{3}[\tan^{-1}(1) - \tan^{-1}(0)]$$
$$= \frac{1}{3}\left[\frac{\pi}{4} - 0\right] = \mathbf{\frac{\pi}{12}} \qquad \left[\because \tan^{-1}(1) = \frac{\pi}{4},\ \tan^{-1}(0) = 0\right.$$

8. $$\int_1^{\sqrt{3}} \frac{dx}{1+x^2} = \left(\tan^{-1}x\right)_1^{\sqrt{3}}$$
$$= \tan^{-1}\sqrt{3} - \tan^{-1}1 = \frac{\pi}{3} - \frac{\pi}{4}$$
$$= \frac{4\pi - 3\pi}{12} = \mathbf{\frac{\pi}{12}}$$

9. $$\int \frac{x^3 - x^2 + x - 1}{x-1}\,dx = \int \frac{x^2(x-1) + 1(x-1)}{(x-1)}\,dx$$
$$= \int \frac{(x-1)(x^2+1)}{(x-1)}\,dx$$
$$= \int (x^2+1)\,dx = \mathbf{\frac{x^3}{3} + x + c}$$

Or

$$\int_0^1 \frac{2x}{1+x^2}\,dx = \int_1^2 \frac{dp}{p}$$

Let $p = 1 + x^2$
$dp = 2x\,dx$
When $x = 1$, $p = 1 + 1 = 2$
When $x = 0$, $p = 1 + 0 = 1$

$= [\log |p|]_1^2$

$= \log 2 - \log 1 = \mathbf{\log 2}$ **[$\because$ log 1 = 0]**

10. For each point A(a, b, c) on the x-axis is B(a, 0, 0)

$\therefore$ Distance of A from x-axis

$$\mathbf{AB} = \sqrt{(a-a)^2 + (0-b)^2 + (0-c)^2}$$

$$= \sqrt{0 + b^2 + c^2} = \sqrt{\mathbf{b^2 + c^2}} \textbf{ units}$$

11. $\vec{AB} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{BC} = \hat{i} + 3\hat{j} + 5\hat{k}$

$\therefore$ $\vec{AC} = \vec{AB} + \vec{BC} = (2\hat{i} - \hat{j} + 2\hat{k}) + (\hat{i} + 3\hat{j} + 5\hat{k}) = 3\hat{i} + 2\hat{j} + 7\hat{k}$

As we know, $\vec{\mathbf{CA}} = -\vec{AC}$

$= -(3\hat{i} + 2\hat{j} + 7\hat{k})$

$= \mathbf{-3\hat{i} - 2\hat{j} - 7\hat{k}}$ **or** $\mathbf{-(3\hat{i} + 2\hat{j} + 7\hat{k})}$

12. Area of the parallelogram $= \frac{1}{2}|\vec{d_1} \times \vec{d_2}| = \frac{1}{2}\sqrt{100}$

$= \frac{1}{2}(10) = \mathbf{5}$ **sq. units.**

$\ldots\left[\vec{d_1} \times \vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 0 & 0 \\ 0 & 2 & 0 \end{vmatrix} = \hat{i}(0) - \hat{j}(0) + \hat{k}(10) = 10\hat{k}\right.$

13. $\alpha = \beta = \gamma$ **...[given**

$\therefore$ $\cos\alpha = \cos\beta = \cos\gamma$

$l = m = n$ ...(i)

But $l^2 + m^2 + n^2 = 1$

$l^2 + l^2 + l^2 = 1$ **...[From (i)**

$\Rightarrow$ $3l^2 = 1$

$\Rightarrow$ $l^2 = \frac{1}{3}$ $\Rightarrow$ $l = \frac{1}{\sqrt{3}}$

$\therefore$ $l = m = n = \frac{1}{\sqrt{3}}$

$\therefore$ **Direction cosines of the line are** $< \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} >$

14. Direction ratios of given first line are -2, $3p$, 4

Direction ratios of given second line are $4p$, 2, -7

Using $a_1a_2 + b_1b_2 + c_1c_2 = 0$ **...[$\because$ They are $\perp$**

$-2(4p) + 3p(2) + 4(-7) = 0$ $\Rightarrow$ $-8p + 6p - 28 = 0$

$\Rightarrow$ $-2p = 28$ $\Rightarrow$ $\mathbf{p = -14}$

15. We have, P(A) = 0.4, P(B) = 0.8, P(B/A) = 0.6

We know that, $P(B/A) = \frac{P(A \cap B)}{P(A)}$

$\Rightarrow$ $P(A \cap B) = P(B/A)\,P(A)$

$\Rightarrow$ $P(A \cap B) = (0.6)(0.4)$

$\therefore$ $P(A \cap B) = 0.24$

Now, $\mathbf{P(A/B)} = \frac{P(A \cap B)}{P(B)} = \frac{0.24}{0.8} = \mathbf{\frac{3}{10}}$

16. A and B are Independent events **...[Given**

$\therefore$ $P(A \cap B) = P(A) \times P(B)$ ...(i)

P (at least one of A and B) $= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A) \times P(B)$ **[From (*i*)**

$= P(A) + P(B)[1 - P(A)]$

$= 1 - P(A') + P(B).P(A')$ $\left[\because P(A) + P(A') = 1 \Rightarrow P(A) = 1 - P(A')\right.$

$= 1 - P(A')[1 - P(B)]$ $\left[\because 1 - P(B) = P(B')\right.$

$= \mathbf{1 - P(A')P(B')}$ **[Hence proved]**

17. Let H and E be the events to reads Hindi and English newspaper respectively.

$\therefore$ $P(H) = 60\% = \frac{60}{100} = \frac{3}{5}$; $P(E) = 40\% = \frac{40}{100} = \frac{2}{5}$; $P(H \cap E) = 20\% = \frac{20}{100} = \frac{1}{5}$

(*i*) **(*c*)**; P (she reads Hindi or English newspaper), $P(H \cup E) = P(H) + P(E) - P(H \cap E)$

$= \frac{3}{5} + \frac{2}{5} - \frac{1}{5} = \mathbf{\frac{4}{5}}$

(*ii*) **(*d*)**; P (she reads neither Hindi nor English newspaper = $P(\overline{H} \cap \overline{E}) = P(\overline{H \cup E})$

$= 1 - P(H \cap E) = 1 - [P(H) + P(E) - P(H \cap E)]$

$= 1 - \left(\frac{3}{5} + \frac{2}{5} - \frac{1}{5}\right) = 1 - \frac{4}{5} = \mathbf{\frac{1}{5}}$

(*iii*) **(*a*)**; P(she read English newspaper when it is given that she reads Hindi newspaper)

$= P(E/H) = \frac{P(E \cap H)}{P(H)} = \frac{1/5}{3/5} = \frac{1}{5} \times \frac{5}{3} = \mathbf{\frac{1}{3}}$

(*iv*) **(*a*)**; P(she reads Hindi newspaper when it is given that she reads English newspaper)

$= P(H/E) = \frac{P(H \cap E)}{P(E)} = \frac{1/5}{2/5} = \frac{1}{5} \times \frac{5}{2} = \mathbf{\frac{1}{2}}$

(*v*) **(*b*)**; Here $P(H) = \frac{3}{5}$, $P(E) = \frac{2}{5}$ and $P(H \cap E) = \frac{1}{5}$

Now, $P(H) \cdot P(E) = \frac{3}{5} \times \frac{2}{5} = \mathbf{\frac{6}{25}}$

$\therefore$ $P(H \cap E) \neq P(H) \cdot P(E)$

Hence the events Hindi and English are not independent.

18. **(*i*)** **(*c*)**; Equation of the circle is $(x - h)^2 + (y - k)^2 = r^2$

(*ii*) **(*a*)**; Given equation of circle is $x^2 + y^2 = 16$

$\Rightarrow x^2 = 16 - y^2 \Rightarrow x = \sqrt{16 - y^2}$

(*iii*) **(*d*)**; We have, $x^2 + y^2 = 16$ …(*i*) and $x = \sqrt{3}\, y$ …(*ii*)

$\Rightarrow (\sqrt{3}y)^2 + y^2 = 16$ **[From (*ii*)**

$\Rightarrow 3y^2 + y^2 = 16$

$\Rightarrow 4y^2 = 16 \Rightarrow y^2 = \frac{\cancel{16}^{4}}{\cancel{4}}$

$\Rightarrow y^2 = 4$ $\therefore$ $\mathbf{y = 2}$

(*iv*) **(*c*)**; **Given:** $x^2 + y^2 = 16$

$(x - 0)^2 + (y - 0)^2 = (r)^2$

Comparing the above with $(x - h)^2 + (y - k)^2 = r^2$

$\therefore$ $\mathbf{r = 4}$

(*v*) **(*b*)**; Area Part I $= \int_0^2 \sqrt{3}y$

$= \frac{\sqrt{3}}{2}\left[y^2\right]_0^2 = \frac{\sqrt{3}}{2}[4 - 0] = \mathbf{2\sqrt{3}}$ **sq. units**.

19. **L.H.S.** $\cos^{-1} x + \cos^{-1}\left[\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right]$

Let $\cos^{-1} x = \theta$ for all $x \in \left[\frac{1}{2}, 1\right], \theta \in \left[0, \frac{\pi}{3}\right] \quad \Rightarrow \quad x = \cos\theta$

$$= \theta + \cos^{-1}\left[\frac{\cos\theta}{2} + \frac{\sqrt{3 - 3\cos^2\theta}}{2}\right]$$

$$= \theta + \cos^{-1}\left[\frac{\cos\theta}{2} + \frac{\sqrt{3(1-\cos^2\theta)}}{2}\right] = \theta + \cos^{-1}\left[\frac{\cos\theta}{2} + \frac{\sqrt{3}\sin\theta}{2}\right]$$

$$= \theta + \cos^{-1}\left[\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta\right] = \theta + \cos^{-1}\left[\cos\frac{\pi}{3}\cos\theta + \sin\frac{\pi}{3}\sin\theta\right]$$

$$= \theta + \cos^{-1}\left[\cos\left(\frac{\pi}{3} - \theta\right)\right]$$ **...[$\because \cos x \cos y + \sin x \sin y = \cos(x-y)$**

$= \theta + \frac{\pi}{3} - \theta = \frac{\pi}{3}$ = **R.H.S. (Hence proved)**

20. $AB = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 4 & -6 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 8-6 & -12+12 \\ 4-4 & -6+8 \end{bmatrix}$

$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$AB = 2I$

$A^{-1}(AB) = A^{-1}(2I)$ **...[Pre-multiplying both sides by A^{-1}; $|A| \neq 0$**

$IB = 2A^{-1} I$ **...[$A^{-1}(A) = I$**

$A^{-1} = \frac{1}{2}B$

$A^{-1} = \frac{1}{2}\begin{bmatrix} 4 & -6 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ ...(*i*)

The given system of equations can be written in the form

$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$A^t X = C$ where $X = \begin{bmatrix} x \\ y \end{bmatrix}, C = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$X = (A^t)^{-1} C$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \end{bmatrix}$ **[$\because (A^t)^{-1} = (A^{-1})^t$** ... [From (*i*)

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8-1 \\ -12+2 \end{bmatrix} = \begin{bmatrix} 7 \\ -10 \end{bmatrix}$ $\therefore$ ***x* = 7, *y* = 10**

Or

$A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$

$|A| = 1 + 4 = 5 \neq 0$ $\therefore$ A^{-1} does exist.

$adj\, A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}.adj\ A \quad \Rightarrow A^{-1} = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$ ...(*i*)

Given system of equations is AX = B, where $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

Writing in matrix form,

$$\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$AX = B$$

$$A^{-1}AX = A^{-1}B$$ ...[Pre multiplying both sides by A^{-1}

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -1+4 \\ 2+2 \end{bmatrix} \quad \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{5} \\ \frac{4}{5} \end{bmatrix}$$

$\therefore$ **$x = \frac{3}{5}$ and $y = \frac{4}{5}$**

21. $f(x) = \sin 2x - \cos 2x$

Differentiating both sides w.r.t. x, we get

$$f'(x) = 2\cos 2x + 2\sin 2x$$

$$f'\left(\frac{\pi}{6}\right) = 2\cos\frac{\pi}{3} + 2\sin\frac{\pi}{3}$$

$$= 2\left(\frac{1}{2}\right) + 2\left(\frac{\sqrt{3}}{2}\right) = \mathbf{(1 + \sqrt{3})}$$

22. **(*i*)** For x-axis intercept, put $y = 0$

$$y = \frac{(x-7)}{(x-2)(x-3)} \quad \Rightarrow \quad 0 = \frac{(x-7)}{(x-2)(x-3)}$$

$$x - 7 = 0 \quad \Rightarrow \quad x = 7$$

$\therefore$ The given curve cuts x-axis at point (7, 0),

$$y = \frac{x-7}{(x-2)(x-3)} = \frac{x-7}{x^2-5x+6}$$

$$\therefore \frac{dy}{dx} = \frac{(x^2-5x+6).1-(x-7)(2x-5)}{(x^2-5x+6)}$$

$\therefore$ Slope of tangent, $\frac{dy}{dx}$ at (7, 0) $= \frac{(49-35+6)-(7-7)(14-5)}{(49-35+6)} = \frac{20}{(20)^2} = \frac{1}{20}$

Equation of tangent to the curve at (7, 0) is $y - 0 = \frac{1}{20}(x-7)$

$\Rightarrow 20y = x - 7 \quad \Rightarrow$ **$20y - x + 7 = 0$**

(*ii*) $\therefore$ Slope of normal = –20

Equation of normal to the curve at (7, 0) is $y - 0 = -20(x - 7)$

$\Rightarrow$ $y = -20x + 140$

$\Rightarrow$ $\mathbf{20x + y - 140 = 0}$

23. Let $I = \int \frac{\sin x - \cos x}{\sqrt{\sin x \cos x}} dx = \int \frac{dp}{\sqrt{\frac{1-p^2}{2}}}$

$$\left[\begin{aligned} &\textbf{Let } p = \sin x - \cos x \\ &\therefore\ dp = (\cos x + \sin x)dx \\ &\therefore\ p^2 = \sin^2 x + \cos^2 x - 2\sin x \cos x \\ &\quad p^2 = 1 - 2\sin x \cos x \\ &\Rightarrow 2\sin x \cos x = 1 - p^2 \\ &\Rightarrow \sin x \cos x = \frac{1-p^2}{2} \end{aligned}\right.$$

$$= \sqrt{2} \int \frac{dp}{\sqrt{1^2 - p^2}}$$

$$= \sqrt{2} \sin^{-1}\left(\frac{p}{1}\right) + c \qquad \left[\because \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C\right.$$

$$= \sqrt{2}\, \mathbf{\sin^{-1}(\sin x - \cos x) + c}$$

Or

Let $I = \int \frac{e^x(x^2+1)}{(x+1)^2} dx = \int e^x \frac{(x^2 - 1 + 1 + 1)}{(x+1)^2} dx$

$$= \int e^x \left[\frac{(x-1)(x+1)}{(x+1)^2} + \frac{2}{(x+1)^2}\right] dx$$

$$= \int e^x \left[\frac{x-1}{x+1} + \frac{2}{(x+1)^2}\right] dx$$

$$= \int e^x [f(x) + f'(x)]\, dx$$

$$= e^x f(x) + c$$

$$= \mathbf{e^x.\left(\frac{x-1}{x+1}\right) + c}$$

$$\left[\begin{aligned} &\textbf{where } f(x) = \frac{x-1}{x+1} \\ &f(x) = \frac{(x+1)1 - (x-1)1}{(x+1)^2} \\ &\quad = \frac{x+1-x+1}{(x+1)^2} = \frac{2}{(x+1)^2} \end{aligned}\right.$$

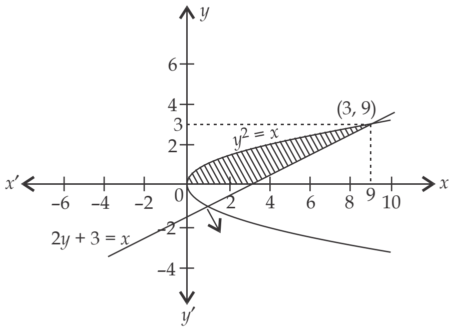
24. The given curves are, $y = \sqrt{x}$ and $2y + 3 = x$

$y = \sqrt{x}$ or $y^2 = x$ ...(*i*)

x	0	1	4
y	0	±1	±2

$2y + 3 = x$

x	0	1	3
y	–1.5	–1	0



For point of intersection,

$2y + 3 = x$

$2y + 3 = y^2$ ...**[From (*i*)**

$y^2 - 2y - 3 = 0$

$\Rightarrow$ $(y + 1)(y - 3) = 0$

$y = 3$ or $y = -1$

$\Rightarrow$ $y = 3$ (As $y > 0$, $y^2 = x$ lies in 1st quadrant)

$\therefore$ $\mathbf{x = 9}$

Required area of shaded region $= \int_0^3 [(2y+3) - y^2]\, dy$

$$= \left[\frac{2y^2}{2} + 3y - \frac{y^3}{3}\right]_0^3 = \left[(3)^2 + 3(3) - \frac{(3)^3}{3}\right]$$

$= 9 + 9 - 9 =$ **9 sq. units**

25. $x \log x \dfrac{dy}{dx} + y = 2 \log x$

Dividing both sides by $x \log x$, we get

$$\frac{dy}{dx} + \frac{y}{x \log x} = \frac{2 \log x}{x \log x}$$

On comparing with $\dfrac{dy}{dx} + \text{P}y = \text{Q}$

Here, 'P' = $\dfrac{1}{x \log x}$, 'Q' = $\dfrac{2}{x}$

$$\text{I.F.} = e^{\int \text{P}\, dx} = e^{\int \frac{dx}{x \log x}}$$

$$= e^{\int \frac{dt}{t}} = e^{\log |t|} = t = \log x \qquad \ldots \left[\begin{array}{l}\textbf{let } t = \log x \\ dt = \frac{1}{x} dx\end{array}\right.$$

Required solution is $y(\text{IF}) = \int \text{Q}\,(\text{I.F.})\; dx$

$$y.\log x = \int \frac{2}{x} \log x\, dx$$

$$= 2\int t\, dt = 2\frac{t^2}{2} + c \qquad \ldots \left[\begin{array}{l}\textbf{let } t = \log x \\ dt = \frac{1}{x} dx\end{array}\right.$$

$$y(\log x) = (\log x)^2 + c$$

$\therefore \quad \boldsymbol{y = \log x + \dfrac{c}{\log x}}$

26. Let $\vec{d} = x\hat{i} + y\hat{j} + z\hat{k}$ …(*i*)

As $\vec{d} \perp \vec{a}$

$\therefore \quad \vec{d} \cdot \vec{a} = 0$

$(x\hat{i} + y\hat{j} + z\hat{k})(4\hat{i} + 5\hat{j} - \hat{k}) = 0$

$4x + 5y - z = 0$ …(*ii*)

As $\vec{d} \perp \vec{b}$

$\therefore \quad \vec{d} \cdot \vec{b} = 0$

$(x\hat{i} + y\hat{j} + z\hat{k})(\hat{i} - 4\hat{j} + 5\hat{k}) = 0$

$x - 4y + 5z = 0$ …(*iii*)

$\vec{d} \cdot \vec{c} = 21$ **…[Given**

$(x\hat{i} + y\hat{j} + z\hat{k})(3\hat{i} + \hat{j} - \hat{k}) = 21$

$\Rightarrow 3x + y - z = 21$ …(*iv*)

Multiplying equation (*ii*) by 5 and (*iii*) by 1, we have

$$\begin{array}{r} 20x + 25y - 5z = 0 \\ x - \;\; 4y + 5z = 0 \\ \hline 21x + 21y \qquad = 0 \end{array}$$

$x + y = 0$ **[dividing by 21**

$\Rightarrow \quad x = -y$ …(*v*)

Solving (*ii*) and (*iv*), we get

$$\begin{array}{r} 4x + 5y - z = \;\; 0 \\ 3x + \;\; y - z = \;21 \\ - \quad - \quad + \quad - \\ \hline x + 4y \quad = -21 \end{array}$$

$\Rightarrow \quad -y + 4y = -21$ **…[From (*v*)**

$\Rightarrow \quad 3y = -21 \qquad \Rightarrow \quad y = -7$

Putting the value of y in (v), we get

$x = -(-7) = 7$

Putting the values of x and y in (ii), we get

$4(7) + 5(-7) = z \Rightarrow 28 - 35 = z \Rightarrow z = -7$

Putting the values of x, y and z in (i), we get $\vec{d} = 7\hat{i} - 7\hat{j} - 7\hat{k}$

27. Direction ratios of given 1st line are 2, 3, 4

Direction ratios of given 2nd line are 5, 2, 1

As $2 : 3 : 4 \neq 5 : 2 : 1$, the given lines are not parallel.

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-a}{4} = p \text{ (Let)}$$

Any point on the given first line is $(2p + 1, 3p + 2, 4p + a)$

$$\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1} = q \text{ (Let)}$$

Any point on the given second line is $(5q + 4, 2q + 1, q)$

As lines are skew **...[Given**

If apart from being non parallel, they do not intersect,

There must not exist a pair of values of p, q which satisfy the below three equations simultaneously,

$2p + 1 = 5q + 4,$

$p = \dfrac{5q+3}{2}$ ...(i)

$\therefore\ p = \dfrac{-5+3}{2}$ **...[From (ii)**

$p = -1$

$3p + 2 = 2q + 1,$

$3\left(\dfrac{5q+3}{2}\right) + 2 = 2q + 1$ **...[From (i)**

$15q + 9 + 4 = 4q + 2$

$15q - 4q = 2 - 13$

$11q = -11$

$q = -1$ ...(ii)

$4p + a = q$

Solving the first two equations, we have $p = -1$, $q = -1$.

These values will not satisfy the third equation.

$4p + a \neq q$

$4(-1) + a \neq -1$ **...[From (i) and (ii)**

$-4 + a \neq -1$

$\boldsymbol{a \neq 3}$

28. Let p = P(a diamond card) = $\dfrac{13}{52} = \dfrac{1}{4}$

$\therefore\ q = 1 - \dfrac{1}{4} = \dfrac{3}{4}, n = 2$

Here random variable X can take values 0, 1, 2.

Using $P(r) = {}^nC_r \cdot q^{n-r} \cdot p^r$

$$P(X = 0) = {}^2C_0\, q^{2-0} p^0 = q^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

$$P(X = 1) = {}^2C_1\, q^{2-1} p^1 = 2qp = 2\left(\frac{3}{4}\right)\left(\frac{1}{4}\right) = \frac{6}{16}$$

$$P(X = 2) = {}^2C_2\, q^{2-2} p^2 = p^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

Probability distribution is

X	0	1	2
P(X)	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

Or

$p_1 = \text{P(P speaks truth)} = 70\% = \frac{70}{100} = \frac{7}{10}$

$\therefore \quad q_1 = 1 - p_1 = 1 - \frac{7}{10} = \frac{3}{10}$

$p_2 = \text{P(Q speaks truth)} = 80\% = \frac{80}{100} = \frac{8}{10}$

$\therefore \quad q_2 = 1 - p_2 = 1 - \frac{8}{10} = \frac{2}{10}$

$$\begin{aligned}\text{P(They agree)} &= p_1p_2 + q_1q_2 \\ &= \frac{7}{10}\times\frac{8}{10} + \frac{3}{10}\times\frac{2}{10} \\ &= \frac{56}{100} + \frac{6}{100} = \frac{62}{100} = 0.62\end{aligned}$$

$\therefore$ **Required %** $= \frac{62}{100} \times 100 =$ **62%**

29. (*i*) For all $a, b \in$ A

(a, b) R (a, b)

$a + b = b + a,$ $\quad \therefore$ **R is reflexive**

(*ii*) For $a, b, c, d \in$ A

Let (a, b) R (c, d) $\quad \therefore \quad a + d = b + c$

$\Rightarrow c + b = d + a \quad \Rightarrow \quad (c, d)$ R (a, b) $\quad \therefore$ **R is symmetric**

(*iii*) For $a, b, c, d, e, f \in$ A

(a, b) R (c, d) and (c, d) R (e, f)

$\therefore a + d = b + c \quad \ldots(i)$ and $c + f = d + e \quad \ldots(ii)$

Adding (*i*) and (*ii*),

$a + d + c + f = b + c + d + e$

$\Rightarrow \quad a + f = b + e \quad \Rightarrow \quad (a, b)$ R (e, f) $\quad \therefore$ **R is transitive**

Hence R is an **equivalence relation** and equivalence class [(2, 5)] is

{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)}

30. $x = \sin t, \quad \ldots(i)$

Differentiating both sides w.r.t. x,

$$\frac{dx}{dt} = \cos t$$

$y = \sin kt \quad \ldots(ii)$

Differentiating both sides w.r.t. x,

$$\frac{dy}{dt} = k \cos kt$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = k \cos kt \times \frac{1}{\cos t}$$

$$\Rightarrow \quad \cos t \, . \, \frac{dy}{dx} = k \cos kt$$

Squaring both sides, we get

$$\cos^2 t \, . \left(\frac{dy}{dx}\right)^2 = k^2 \cos^2 kt$$

$$(1 - \sin^2 t)\left(\frac{dy}{dx}\right)^2 = k^2(1 - \sin^2 kt)$$

$$(1 - x^2)\left(\frac{dy}{dx}\right)^2 = k^2(1 - y^2)$$ **...[From (*i*) and (*ii*)**

Differentiating both sides w.r.t. x, we have

$$(1-x^2)\,.\;2\left(\frac{dy}{dx}\right).\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2.(-2x)=k^2\left(-2y\frac{dy}{dx}\right)$$

Dividing both sides by $2\frac{dy}{dx}$, we have

$$(1-x^2)\,\frac{d^2y}{dx^2}-x\frac{dy}{dx}=-k^2y$$

$$\Rightarrow\quad (1-x^2)\,\frac{d^2y}{dx^2}-x\frac{dy}{dx}+k^2y=0$$ **(Hence proved)**

31. Let $A = y^x$ ...(i)

Taking log on both sides,

$\log A = x\,.\,\log y$

Differentiating both sides w.r.t. x, we have

$$\frac{1}{A}.\frac{dA}{dx}=x.\frac{1}{y}\frac{dy}{dx}+\log y\,.\,1$$

$$\frac{dA}{dx}=A\left(\frac{x}{y}\frac{dy}{dx}+\log y\right)$$

$$\frac{dA}{dx}=y^x\left(\frac{x}{y}\frac{dy}{dx}+\log y\right)$$ ...(iii) **[From (i)**

Let $B = x^y$ ...(ii)

Taking log on both sides,

$\log B = y\,.\,\log x$

Differentiating both sides w.r.t. x, we have

$$\frac{1}{B}.\frac{dB}{dx}=y.\frac{1}{x}+\log x.\frac{dy}{dx}$$

$$\frac{dB}{dx}=B\left(\frac{y}{x}+\log x\frac{dy}{dx}\right)$$

$$\frac{dB}{dx}=x^y\left(\frac{y}{x}+\log x\frac{dy}{dx}\right)$$ ...(iv)

Let $C = x^x$ ...(v)

Taking log on both sides, we have

$\log C = x \log x$

Differentiating both sides w.r.t. x, we have

$$\frac{1}{C}.\frac{dC}{dx}=x.\frac{1}{x}+\log x\,.\,1$$

$$\frac{dC}{dx}=C(1+\log x)$$

$$\frac{dC}{dx}=x^x(1+\log x)$$ ...(vi) **[From (v)**

Now $y^x + x^y + x^x = a^b$

$A + B + C = a^b$ ...**[From (i), (ii) and (v)**

Differentiating both sides w.r.t. x, we have

$$\frac{dA}{dx}+\frac{dB}{dx}+\frac{dC}{dx}=0$$

$$y^x\left(\frac{x}{y}\frac{dy}{dx}+\log y\right)+x^y\left(\frac{y}{x}+\log x\frac{dy}{dx}\right)+x^x(1+\log x)=0$$ ...**[From (iii), (iv) and (vi)**

$$(xy^{x-1}+x^y\log x)\frac{dy}{dx}=-[y^x\,.\,\log y+y\,.\,x^{y-1}+x^x(1+\log x)]$$

$$\therefore\quad \boldsymbol{\frac{dy}{dx}=\frac{-[y^x\log y+y\,.\,x^{y-1}+x^x(1+\log x)]}{(x\,.y^{x-1}+x^y.\log x)}}$$

Or

$[x] = -1$, for $-1 \le x < 0$

$= 0$, for $0 \le x < 1$

$= 1$, for $1 \le x < 2$

$[x] = 2$, for $2 \le x < 3$

$[x] = 3$, for $3 \le x < 4$

... $\left[\begin{array}{l} \boldsymbol{x=\sqrt{2}-h, h>0} \\ \boldsymbol{x=\sqrt{2}+h, h>0} \\ \boldsymbol{\sqrt{2}=1.4} \end{array}\right.$

$$f(x) = \begin{cases} 1, & 1 \le x < \sqrt{2} \\ 2, & \sqrt{2} \le x < \sqrt{3} \\ 3, & \sqrt{3} \le x < 2 \end{cases}$$

At $x = \sqrt{2}$

L.H.L. $= \lim_{x \to \sqrt{2}^-} f(x)$

$= \lim_{x \to \sqrt{2}^-} 1$

$= 1$

R.H.L. $= \lim_{x \to \sqrt{2}^+} f(x)$

$= \lim_{x \to \sqrt{2}^+} 2$

$= 2$

L.H.L. $\neq$ R.H.L.

$\therefore$ $f(x)$ is discontinuous at $x = \sqrt{2}$

At $x = \sqrt{3}$

L.H.L. $= \lim_{x \to \sqrt{3}^-} f(x)$

$= \lim_{x \to \sqrt{3}^-} 2$

$= 2$

R.H.L. $= \lim_{x \to \sqrt{3}^+} f(x)$

$= \lim_{x \to \sqrt{3}^+} 3$

$= 3$

L.H.L. $\neq$ R.H.L.

$\therefore$ $f(x)$ is discontinuous at $x = \sqrt{3}$.

$\therefore$ **$x = \sqrt{2}$ and $\sqrt{3}$ are two discontinuities in [1, 2).**

32. $f(x) = \sin^4 x + \cos^4 x, \quad \left(0, \frac{\pi}{2}\right)$

Differentiating both sides w.r.t. x, we have

$$\begin{aligned} f'(x) &= 4 \sin^3 x \,.\, \cos x + 4 \cos^3 x \,.\, (-\sin x) \\ &= 4 \sin x \cos x (\sin^2 x - \cos^2 x) \\ &= -2 \,.\, 2 \sin x \cos x (\cos^2 x - \sin^2 x) \\ &= -2 \,.\, \sin 2x \,.\, \cos 2x = -\sin 4x \end{aligned}$$

When $f'(x) = 0$

$-\sin 4x = 0 \quad \Rightarrow \sin 4x = 0$

$\Rightarrow \quad 4x = 0, \pi, 2\pi, \ldots \quad \Rightarrow x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \ldots$

Intervals	*Checking point*	*Sign of $-\sin 4x$*	*Sign of $f'(x)$*	*Nature of $f(x)$*
$\left(0, \frac{\pi}{4}\right)$	$x = \frac{\pi}{6}$	$-$ve as $0 < 4x < \pi$	≤ 0	decreasing
$\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$	$x = \frac{\pi}{2}$	$+$ve as $\pi < 4x < 2\pi$	≥ 0	increasing

So, $f(x)$ is strictly decreasing on $\left[0, \frac{\pi}{4}\right]$ and strictly increasing on $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$.

33. Let $y = x \cos \pi x$

When $x = -1$, $y = -1 \cos(-\pi) = -1(-1) = 1$

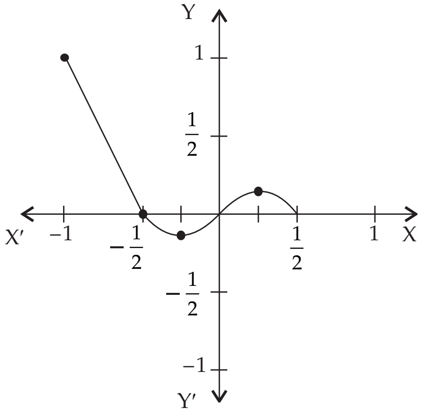
When $x = -\frac{1}{2}$, $y = -\frac{1}{2} \cos\left(\frac{-\pi}{2}\right) = 0$

When $x = -\frac{1}{4}, y = -\frac{1}{4}\cos\left(\frac{-\pi}{4}\right) = \frac{-1}{4}\left(\frac{1}{\sqrt{2}}\right) = \frac{-1}{4\sqrt{2}}$

When $x = 0$, $y = 0 \cos (0) = 0$

When $x = \frac{1}{4}, y = \frac{1}{4}\cos\frac{\pi}{4} = \frac{1}{4}\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{4\sqrt{2}}$

When $x = \frac{1}{2}, y = \frac{1}{2}\cos\frac{\pi}{2} = \frac{1}{2}(0) = 0$



x	-1	$-\frac{1}{2}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$
y	1	0	$\frac{-1}{4\sqrt{2}}$	0	$\frac{1}{4\sqrt{2}}$	0

$$|x\cos\pi x| = \begin{cases} +x\cos\pi x, & -1 < x < -\frac{1}{2} \\ -x\cos\pi x, & -\frac{1}{2} < x < 0 \\ +x\cos\pi x, & 0 < x < \frac{1}{2} \end{cases}$$

$$\int_{-1}^{1/2} |x\cos\pi x|\,dx = \int_{-1}^{-1/2} x\cos\pi x\,dx + \int_{-1/2}^{0} -x\cos\pi x + \int_{0}^{1/2} x\cos\pi x\,dx$$

$\left[\because \int x\cos\pi x\,dx = x\frac{\sin\pi x}{\pi} - \int 1.\frac{\sin\pi x}{\pi}dx = \frac{1}{\pi}x\sin\pi x - \frac{1}{\pi}\left(\frac{-\cos\pi x}{\pi}\right) \ldots(i)\right.$

$$= \left[\frac{x}{\pi}\sin\pi x + \frac{1}{\pi^2}\cos\pi x\right]_{-1}^{-1/2} - \left[\frac{x}{\pi}\sin\pi x + \frac{1}{\pi^2}\cos\pi x\right]_{-1/2}^{0} + \left[\frac{x}{\pi}\sin\pi x + \frac{1}{\pi^2}\cos\pi x\right]_{0}^{1/2}$$

$$= \left[\frac{1}{\pi}\left(\frac{-1}{2}\right)\sin\left(\frac{-\pi}{2}\right) + \frac{1}{\pi^2}\cos\left(\frac{-\pi}{2}\right)\right] - \left[\frac{1}{\pi}(-1)\sin(-\pi) + \frac{1}{\pi^2}\cos(-\pi)\right]$$

$$- \left[0 + \frac{1}{\pi^2}\cos(0)\right] + \left[\frac{-1}{2\pi}\sin\left(\frac{-\pi}{2}\right) + \frac{1}{\pi^2}\cos\left(\frac{-\pi}{2}\right)\right] + \left[\frac{1}{2\pi}\sin\frac{\pi}{2} + \frac{1}{\pi^2}\cos\frac{\pi}{2}\right] - \left[0 + \frac{1}{\pi^2}\cos 0\right]$$

$$= \left[\frac{-1}{2\pi}(-1) + \frac{1}{\pi^2}(0)\right] - \left[\frac{-1}{\pi}(0) + \frac{1}{\pi^2}(-1)\right] - \left[0 + \frac{1}{\pi^2}(1)\right] + \left[\frac{-1}{2\pi}(-1) + \frac{1}{\pi^2}(0)\right]$$

$$+ \left[\frac{1}{2\pi}(1) + \frac{1}{\pi^2}(0)\right] - \left[(0) + \frac{1}{\pi^2}(1)\right]$$

$$= \frac{1}{2\pi} + \frac{1}{\pi^2} - \frac{1}{\pi^2} + \frac{1}{2\pi} + \frac{1}{2\pi} - \frac{1}{\pi^2} = \mathbf{\frac{3}{2\pi} - \frac{1}{\pi^2}}$$

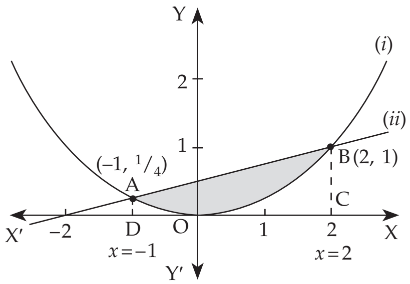
34. Given equations of curve and line are

$x^2 = 4y$ (parabola) …(*i*)

$x = 4y - 2$ (line) …(*ii*)

Table for line $x = 4y - 2$

x	0	-2
y	0.5	0



Solving (*i*) and (*ii*), we get

$x = x^2 - 2$ $\Rightarrow$ $x^2 - x - 2 = 0$

$\Rightarrow$ $x^2 - 2x + x - 2 = 0$

$\Rightarrow$ $x(x - 2) + 1(x - 2) = 0$

$\Rightarrow \quad (x-2)(x+1) = 0 \qquad \therefore \quad x = 2, \text{ or } x = -1$

When x = 2, $\qquad y = 1$ **…[using (*i*)**

When x = –1, $\qquad y = \frac{1}{4}$ **…[using (*ii*)**

The points of intersection are A $\left(-1, \frac{1}{4}\right)$ and B(2, 1).

From (*i*), $y = \frac{x^2}{4}$

From (*ii*), $4y = x + 2 \quad \Rightarrow y = \frac{x+2}{4}$

Required area of the bounded region ABOA = Area of the region ABCODA
– Area of the region ABOA

$$= \int_{-1}^{2} [(y \text{ line}) - y\,(\text{parabola})]\,dx = \int_{-1}^{2}\left[\frac{x+2}{4} - \frac{x^2}{4}\right]dx$$

$$= \frac{1}{4}\int_{-1}^{2}(x+2-x^2)\,dx = \frac{1}{4}\left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2} = \frac{1}{4}\left[\left(\frac{4}{2}+4-\frac{8}{3}\right) - \left(\frac{1}{2}-2+\frac{1}{3}\right)\right]$$

$$= \frac{1}{4}\left[\left(2+4-\frac{8}{3}\right) - \left(\frac{3-12+2}{6}\right)\right] = \frac{1}{4}\left[6 - \frac{8}{3} + \frac{7}{6}\right] = \frac{1}{4}\left[\frac{36-16+7}{6}\right]$$

$$= \mathbf{\frac{1}{4} \times \frac{27}{6} = \frac{9}{8}} \textbf{ sq. units}$$

Or

$0 \le 2y \le x^2$; $\qquad 0 \le y \le x$; $\qquad 0 \le x \le 3$

Let $2y = 0$, $2y = x^2$ $\qquad$ Let $y = x$ $\qquad$ Let $x = 0$

$\Rightarrow \quad y = 0$ $\qquad\qquad\qquad\qquad$ $x = 3$

For Point of intersection,

We have $x^2 = 2y$

$\therefore \quad x^2 = 2x$ **…[From (*i*)**

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0$ or $x = 2$

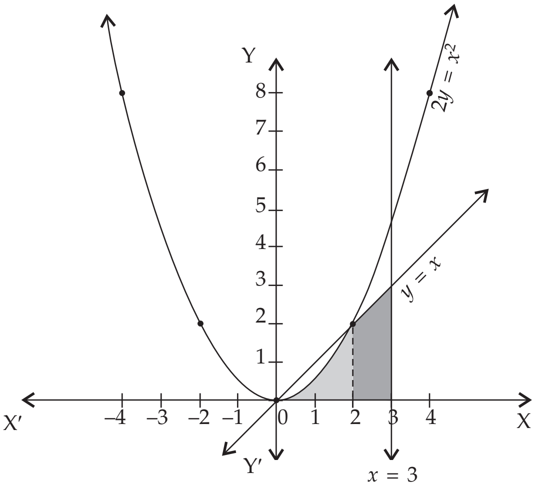
Now, $2y = x^2$

$y = \frac{1}{2}x^2$

x	0	±2	±4
y	0	2	8

$y = x$ $\qquad$ …(*i*)

x	0	3	2
y	0	3	2



Area of the shaded region

$$= \int_0^2 \frac{1}{2}x^2\,dx + \int_2^3 x\,dx = \frac{1}{2}\times\frac{1}{3}\left[x^3\right]_0^2 + \frac{1}{2}\left[x^2\right]_2^3$$

$$= \frac{1}{6}[(2)^3 - 0)] + \frac{1}{2}(3^2 - 2^2)$$

$$= \frac{8}{6} + \frac{1}{2}(9-4) = \frac{8+15}{6} = \mathbf{\frac{23}{6}} \textbf{ sq. units}$$

35. $\sqrt{1 + x^2 + y^2 + x^2y^2} + xy\frac{dy}{dx} = 0$

$$\Rightarrow \quad \sqrt{1(1+x^2) + y^2(1+x^2)} + xy\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \sqrt{(1+x^2)(1+y^2)} + xy\frac{dy}{dx} = 0$$

$$\Rightarrow \quad xy\frac{dy}{dx} = -\sqrt{(1+x^2)(1+y^2)}$$

$$\Rightarrow \quad \frac{y\,dy}{\sqrt{1+y^2}} = -\frac{\sqrt{1+x^2}}{x}\,dx$$

Integrating both sides, we get

$$\int \frac{y\,dy}{\sqrt{1+y^2}} = -\int \frac{x\sqrt{1+x^2}dx}{x^2}$$

Let $p = 1 + y^2$ | Let $z^2 = 1 + x^2$ or $z^2 - 1 = x^2$

$dp = 2y\,dy$ | $2z\,dz = 2x\,dx$

$\frac{dp}{2} = y\,dy$

$$\Rightarrow \quad \frac{1}{2}\int \frac{dp}{\sqrt{p}} = -\int \frac{z.z}{z^2-1}\,dz \qquad \Rightarrow \quad \frac{1}{2}\int \frac{dp}{\sqrt{p}} = -\int \frac{z^2}{z^2-1}\,dz$$

$$\Rightarrow \quad \frac{1}{2}\int p^{-\frac{1}{2}}dp = -\left[\frac{(z^2-1)+1}{z^2-1}\right]dz$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{2}{1}p^{\frac{1}{2}}\right) = -\int\left(1+\frac{1}{z^2-1^2}\right)dz$$

$$\Rightarrow \quad p^{\frac{1}{2}} = -\left[z + \frac{1}{2}\log\left|\frac{z-1}{z+1}\right|\right] + c \qquad \ldots\left[\because \int\frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c\right]$$

$$\Rightarrow \quad \sqrt{1+y^2} = -\sqrt{1+x^2} - \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right| + c$$

36. Given equations can be written as

$AX = B \qquad \Rightarrow X = A^{-1}B \qquad \ldots(i)$

where

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$$

$|A| = 1(9) - 2(-6) + 1(-3)$

$= 9 + 12 - 3 = 18 \neq 0$

$\therefore$ A^{-1} exists.

$A_{11} = 9$, $\quad A_{12} = -(-6) = 6$, $\quad A_{13} = -3$

$A_{21} = -(0+3) = -3$ $\quad A_{22} = 0 - 2 = -2$, $\quad A_{23} = -(-3-4) = 7$

$A_{31} = 6 - 0 = 6$, $\quad A_{32} = -(3-1) = -2$ $\quad A_{33} = 0 - 2 = -2$

$$adj\,A = \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}.adj\,A = \frac{1}{18}\begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$$

$X = A^{-1}B \qquad$ **...[From (*i*)**

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{18}\begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}\begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix} = \frac{1}{18}\begin{bmatrix} 63-33+6 \\ 42-22-2 \\ -21+77-2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{18}\begin{bmatrix} 36 \\ 18 \\ 54 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$\therefore$ $\quad$ $\boldsymbol{x = 2, y = 1, z = 3}$

Or

Given equations can be written as

$AX = B \qquad \Rightarrow X = A^{-1}B \qquad \ldots(i)$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$|A| = 1(0-2) - 1(1-6) + 1(1-0)$

$= -2 + 5 + 1 = 4 \neq 0$

$\therefore$ $\quad$ A^{-1} exists.

$A_{11} = -2, \qquad A_{12} = -(-5) = 5, \qquad A_{13} = 1$

$A_{21} = -(1-1) = 0, \qquad A_{22} = 1 - 3 = -2, \qquad A_{23} = -(1-3) = 2$

$A_{31} = 2 - 0 = 2, \qquad A_{32} = -(2-1) = -1, \qquad A_{33} = 0 - 1 = -1$

$$adj\ A = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}.\,adj\ A = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

From (i), $X = A^{-1}B = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -12+0+24 \\ 30-14-12 \\ 6+14-12 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 12 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

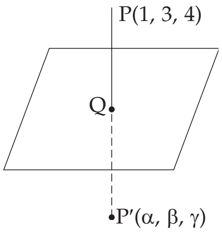
$\therefore$ $\quad$ $\boldsymbol{x = 3, y = 1, z = 2}$

37. **(*i*)** Let P (1, 3, 4) be the given point and Q be the foot of the perpendicular from P to the given plane.

Direction ratios of the normal to the given plane are 2, –1, 1.

$\therefore$ Direction ratios of the line PQ are 2, –1, 1

Equation of line PQ using $\dfrac{x - x_1}{a} = \dfrac{y - y_1}{b} = \dfrac{z - z_1}{c}$



$\therefore$ $\dfrac{x-1}{2} = \dfrac{y-3}{-1} = \dfrac{z-4}{1} = \lambda$ (let)

The coordinates of Q (for some value of λ) are

$(2\lambda + 1, -\lambda + 3, \lambda + 4) \qquad \ldots(i)$

Pt. Q lies on the given plane $2x - y + z + 3 = 0$

$2(2\lambda + 1) - (-\lambda + 3) + \lambda + 4 + 3 = 0$

$4\lambda + 2 + \lambda - 3 + \lambda + 4 + 3 = 0$

$6\lambda = -6 \quad \Rightarrow \lambda = -1$

Putting the value of $\lambda = -1$ in (i),

$\therefore$ Coordinates of foot of perpendicular, Q = $(-2 + 1, 1 + 3, -1 + 4)$ = **(–1, 4, 3)**

(*ii*) **Perpendicular Distance PQ** = $\sqrt{(-1-1)^2 + (4-3)^2 + (3-4)^2}$

$= \sqrt{4+1+1} = \sqrt{6}$ **units**

(*iii*) Let $P'(\alpha, \beta, \gamma)$ be the required image of point P

$\therefore$ Mid point of PP′ = Point Q

$$\left(\frac{\alpha+1}{2}, \frac{\beta+3}{2}, \frac{\gamma+4}{2}\right) = (-1, 4, 3)$$

$\frac{\alpha+1}{2} = -1,$	$\frac{\beta+3}{2} = 4,$	$\frac{\gamma+4}{2} = 3$
$\alpha + 1 = -2,$	$\beta + 3 = 8,$	$\gamma + 4 = 6$
$\alpha = -3$	$\beta = 5$	$\gamma = 2$

$\therefore$ **Image of P is P′(–3, 5, 2).**

Or

Lines $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and $\frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$ are coplanar if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Here $x_1 = -3,\quad y_1 = 1,\quad z_1 = 5$

$x_2 = -1,\quad y_2 = 2,\quad z_2 = 5$

$a_1 = -3,\quad b_1 = 1,\quad c_1 = 5$

$a_2 = -1,\quad b_2 = 2,\quad c_2 = 5$

Now $$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} -1-(-3) & 2-1 & 5-5 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 0 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix}$$

Expanding along $R_1 = 2(5 - 10) - 1(-15 + 5)$

$= 2(-5) - 1(-10) = -10 + 10 = 0$

$\therefore$ **Given lines are coplanar.**

Equation of plane containing given lines is

$$\begin{vmatrix} x-(-3) & y-1 & z-5 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x+3 & y-1 & z-5 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = 0$$

$\Rightarrow (x + 3)(5 - 10) - (y - 1)(-15 + 5) + (z - 5)(-6 + 1) = 0$

$\Rightarrow -5(x + 3) + 10(y - 1) - 5(z - 5) = 0$

$\Rightarrow -5x - 15 + 10y - 10 - 5z + 25 = 0$

$\Rightarrow -5x + 10y - 5z = 0$

$\therefore$ $\boldsymbol{x - 2y + z = 0}$ **is the equation of the plane.**

38. Maximise Profit $Z = 1000x + 600y$

Subject to the constraints,

$x + y \leq 200,\quad x \geq 20,\quad y \geq 4x$

$x \geq 0,\quad y \geq 0$

Let $x + y = 200,$

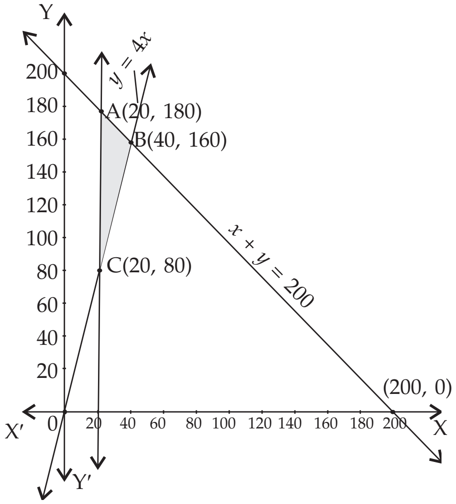
x	200	0	40
y	0	200	160

Let $x = 20,$

Let $y = 4x$

x	20	40	0
y	80	160	0

Corner Points	*Z = 1000x + 600y*	
A(20, 180)	Z = 1000(20) + 600(180) = 20000 + 108000 = 128000	
B(40, 160)	Z = 1000(40) + 600(160) = 40000 + 96000 = **136000**	← **Maximum**
C(20, 80)	Z = 1000(20) + 600(80) = 20000 + 48000 = 68000	



∴ The vertices of feasible region are A(20, 180), B(40, 160) and C(20, 80).

Maximum Profit = ₹ 1,36,000

Or

Given. $2x + 3y \le 120$

and $\frac{x}{50} + \frac{y}{80} \le 1 \quad \Rightarrow \frac{8x + 5y}{400} \le 1 \quad \Rightarrow 8x + 5y \le 400, x \ge 0, \ y \ge 0$

$2x + 3y \le 120$

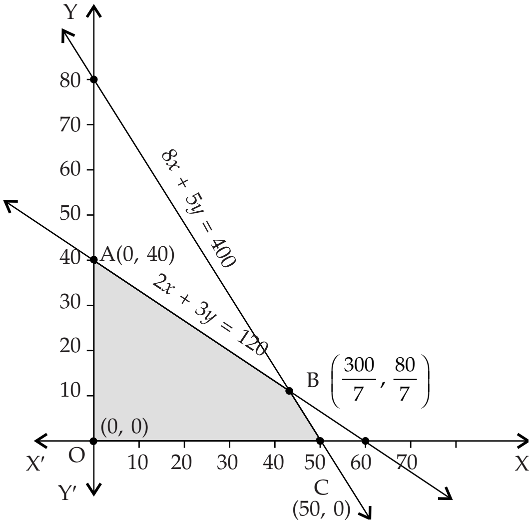
Let $2x + 3y = 120$ ...(*i*)

x	0	60
y	40	0

$8x + 5y \le 400$

Let $8x + 5y = 400$ ...(*ii*)

x	0	50
y	80	0



Multiplying (*i*) by 4 and (*ii*) by 1, we get

$$\begin{array}{r} 8x + 12y = 480 \\ 8x + 5y = 400 \\ - \quad - \quad - \\ \hline 7y = 80 \end{array} \quad \Rightarrow y = \frac{80}{7}$$

Putting the value of $y = \frac{80}{7}$ in (*ii*), we have $8x + 5\left(\frac{80}{7}\right) = 400$

$$8x = 400 - \frac{400}{7} \quad \Rightarrow 8x = \frac{2400}{7} \quad \Rightarrow x = \frac{2400}{8 \times 7} = \frac{300}{7}$$

∴ Point of intersection $B\left(\frac{300}{7}, \frac{80}{7}\right)$

Corner Points	*Z = x + y*	
A(0, 40)	0 + 40 = 40	
$B\left(\frac{300}{7}, \frac{80}{7}\right)$	$\frac{300}{7} + \frac{80}{7} = \frac{380}{7} = \mathbf{54\frac{2}{7}}$	← **Maximum**
C(50, 0)	50 + 0 = 50	
O(0, 0)	0 + 0 = 0	

∴ **Maximum Profit =** $\mathbf{54\frac{2}{7}}$

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