JEE MAIN 2025

Sample Paper - 3

Time Allowed: 3 hours **Maximum Marks: 300**

General Instructions:

1. There are three subjects in the question paper consisting of Physics (Q. no. 1 to 25), Chemistry (Q, no. 26 to 50), and Mathematics (Q. no. 51 to 75).

2. Each subject is divided into two sections. Section A consists of 20 multiple-choice questions & Section B consists of 5 numerical value-type questions.

3. There will be only one correct choice in the given four choices in Section A. For each question for Section A, 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice questions and zero marks will be awarded for not attempted questions.

4. For Section B questions, 4 marks will be awarded for correct answers and zero for unattempted and incorrect answers.

5. Any textual, printed, or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.

6. All calculations/written work should be done in the rough sheet is provided with the Question Paper.

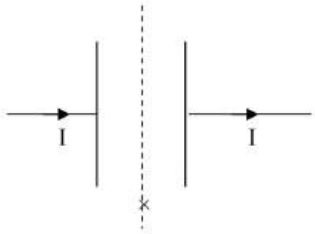
PHYSICS **MAX.MARKS: 100**

SECTION – I
(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

1. A circular parallel plate capacitor of radius R and distance d between the plate is given. A capacitor is being charged with a current I flowing through the wire. Neglect fringing effect.



What is the rate of change of electric flux through plane in middle of capacitor with respect to time (i.e. $\frac{d\phi}{dt}$) –

A) $\frac{2I}{\varepsilon_0}$ B) $\frac{I}{\varepsilon_0}$ C) $\frac{4I}{\varepsilon_0}$ D) $\frac{6I}{\varepsilon_0}$

2. A particle is moving in a circular path. The acceleration and momentum of the particle at a certain moment are $\vec{a} = (4\hat{i} + 3\hat{j})\ m/s^2$ and $\vec{p} = (8\hat{i} - 6\hat{j})$ kg-m/s. The motion of the particle at that instant is

A) Uniform circular motion

B) accelerated circular motion

C) decelerated circular motion

D) We cannot say anything with $\vec{a}$ and $\vec{p}$ only

3. A block starts moving up a fixed inclined plane of inclination 60° with a velocity of 20 m/s and stops after 2 sec. The approximate value of coefficient of friction is ($g = 10\ m/s^2$)

A) 3 B) 3.3 C) 0.27 D) 0.33

4. A ball of mass m approaches a wall of mass M (>> m) with speed 4 m/s along the normal to the wall. The speed of wall is 1m/s towards the ball. The speed of the ball after an elastic collision with the wall is -

A) 5 m/s away from the wall B) 9 m/s away from the wall

C) 3 m/s away from the wall D) 6 m/s away from the wall

5. The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth, is

A) 2R B) $\frac{R}{\sqrt{2}}$ C) R / 2 D) $\sqrt{2}R$

6. An ideal gas is expanded so that amount of heat given is equal to the decrease in internal energy. The gas undergoes the process $TV^{1/5}$ = constant. The adiabatic compressibility of gas when pressure is P, is –

A) $\frac{7}{5P}$ B) $\frac{5}{7P}$ C) $\frac{2}{5P}$ D) $\frac{7}{3P}$

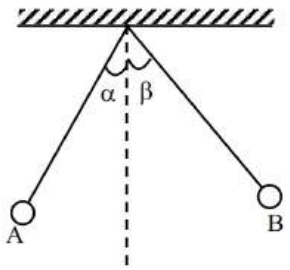
7. An ideal gas is held in a container of volume V at pressure P. The average speed of a gas molecule under these conditions is v. If now the volume and pressure are changed to 2V and 2P, the average speed of a molecule will be

A) 1/2 v B) v C) 2v D) 4v

8. A wire is 4 m long and has a mass 0.2 kg. The wire is kept horizontally. A transverse pulse is generated by plucking one end of the taut (tight) wire. The pulse makes four trips back and forth along the cord in 0.8 sec. The tension in the cord will be -

A) 80 N B) 160 N C) 240 N D) 320 N

9. Two identical simple pendulums A and B are fixed at same point. They are displaced by very small angles α and $\beta(\beta > \alpha)$ and released from rest. Find the time after which B reaches its initial position for the first time. Collisions are elastic and length of strings is λ.



A) $\pi\sqrt{\frac{\ell}{g}}$ B) $2\pi\sqrt{\frac{\ell}{g}}$ C) $\frac{\pi\beta}{\alpha}\sqrt{\frac{\ell}{g}}$ D) $\frac{2\pi\beta}{\alpha}\sqrt{\frac{\ell}{g}}$

10. The figure shows a charge q placed inside a cavity C in an uncharged conductor. Now if an external electric field is switched on then :

A) only induced charge on outer surface will redistribute.

B) only induced charge on inner surface will redistribute

C) Both induced charge on outer and inner surface will redistribute.

D) force on charge q placed inside the cavity will change

11. STATEMENT – 1

If potential difference between two points is zero and the resistance between the same two points is also zero, then current may flow between those two points

STATEMENT – 2

Kirchhoff's 1^{st} law is conservation of charge.

A) Statement – 1 is True. Statement – 2 is True; Statement – 2 is a correct explanation for Statement – 1.

B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is not a correct explanation for Statement – 1.

C) Statement – 1 is True, Statement is False.

D) Statement – 1 is False, Statement is True.

12. An α particle is moving along a circle of radius R with a constant angular velocity ω. Point A lies in the same plane at a distance 2R from the centre. Point A records magnetic field produced by α particle. If the minimum time interval between two successive times at which A records zero magnetic field is 't', then the angular speed ω is :

A) $\frac{2\pi}{t}$ B) $\frac{2\pi}{3t}$ C) $\frac{\pi}{3t}$ D) $\frac{\pi}{t}$

13. Curie temperature is the temperature above which -

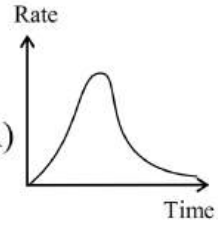
A) a ferromagnetic material behaves like paramagnetic

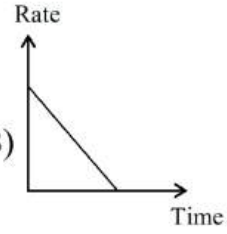
B) a paramagnetic material behaves like diamagnetic

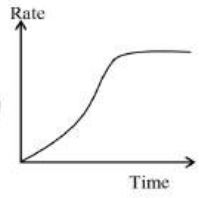
C) a ferromagnetic material behaves like diamagnetic

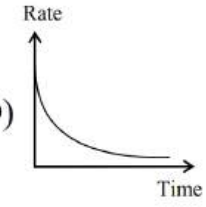
D) a paramagnetic material behaves like ferromagnetic

14. In an LR circuit connected to a battery, the time rate of change of energy stored in the inductor is plotted against time during the growth of current in the circuit. Which of the following best represents the resulting curve?

A) 

B) 

C) 

D) 

15. When an object is at distance x and y from a lens, a real image and a virtual image is formed respectively having same magnification. The focal length of the lens is –

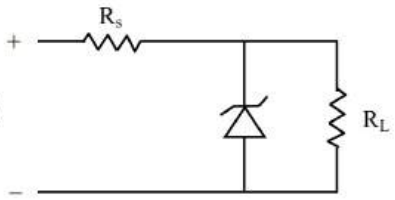
A) $\frac{x+y}{2}$ B) $x - y$ C) $\sqrt{xy}$ D) $x + y$

16. If the zero of the Vernier lies on the right hand side of zero of the main scale and fourth division coincide with the main scale division when the jaws are in contact, the zero correction will be __ (assume standard Vernier Calipers)

A) + 0.04 cm B) + 0.06 cm C) –0.04 cm D) –0.06 cm

17. A zener diode is to be used as a voltage regulator. Identify the correct set up –

A)



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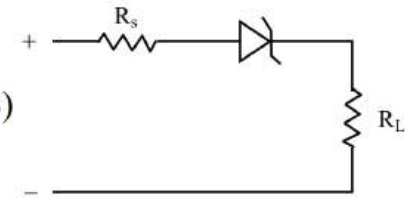
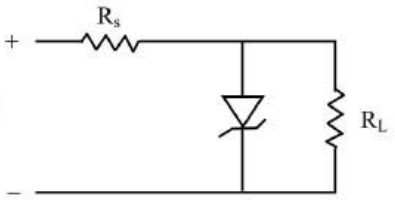
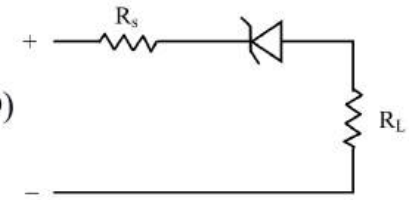


C)



D)



18. The graph of $\log\left(\frac{R}{R_0}\right)$ versus log A (R = radius of a nucleus and A = mass number) is -

A) a circle B) an ellipse C) a parabola D) a straight line

19. Match the entries of Column I with those in Column II

Column I		**Column II**	
(A)	Mass of products less than mass of reactants	(P)	Most stable
(B)	Iron nucleus	(Q)	Total internal reflection take place
(C)	Angle of contact is obtuse	(R)	Nuclear fission
(D)	Incidence angle more than critical angle	(S)	Stronger cohesive forces

A) A – R; B – P; C – Q ; D – S B) A – R; B – P; C – S ; D – Q

C) A – Q; B – S; C – R ; D – P D) A – Q; B – S; C – P ; D – R

20. In a YDSE experiment, I_0 is given to be the intensity of the central bright fringe & β is the fringe width. Then, at a distance y from central bright fringe, the intensity will be -

(y is very small)

A) $I_0 \cos\left(\frac{\pi y}{\beta}\right)$ B) $I_0 \cos^2\left(\frac{\pi y}{\beta}\right)$ C) $I_0 \cos\left(\frac{2\pi y}{\beta}\right)$ D) $I_0 \cos^2\left(\frac{\pi y}{2\beta}\right)$

SECTION-II
(NUMERICAL VALUE ANSWER TYPE)

This section contains 5 questions. The answer to each question is a Numerical value. If the Answer in the decimals , **Mark nearest Integer only.**

Marking scheme: +4 for correct answer, -1 in all other cases.

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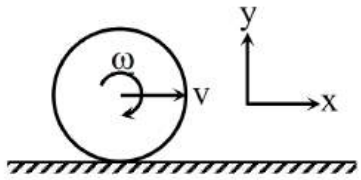
21. A particle of mass 10^{-2} kg is moving along the positive x-axis under the influence of a force $F(x) = -K/(2x^2)$ where $K = 10^{-2}$ Nm^2. At time $t = 0$ it is at $x = 1.0$ m and its velocity is $v = 0$. Find its velocity when it reaches $x = 0.50$ m. (in m/s)

22. A uniform ball of radius R = 10 cm rolls without slipping between two rails such that the horizontal distance is d = 16 cm between two contact points of the rail to the ball. If the angular velocity is

5 rad/s, then find the velocity of centre of mass of the ball in cm/s.

23. A disc of radius '5cm' rolls on a horizontal surface with linear velocity $v = 1\hat{i}$ m/s and angular velocity 50 rad/sec. Height of particle from ground on rim of disc which has velocity in vertical direction is (in cm) -



24. A liquid flows out drop by drop from a vessel through a vertical tube with an internal diameter of 2 mm, then the total number of drops that flows out during 10 grams of the liquid flow out: [Assume that the diameter of the neck of a drop at the moment it breaks away is equal to the internal diameter of tube and surface tension is 0.02 N/m, $g = 9.8$ m/s^2]

25. A tuning fork is in unison with a sonometer wire vibrating in its fourth overtone. Mass hanged with wire is 9 kg. When additional mass is hanged wire vibrates in unison with tuning fork in its 3rd harmonic. Additional mass hanged in kg is.

CHEMISTRY **MAX.MARKS: 100**

SECTION – I
(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

26. The element with $[Xe]4f^{14}5d^{1}6s^{2}$ belongs to

A) 7^{th} period and Group 4 of the modern periodic table

B) 6^{th} period and Group 3 of the modern periodic table

C) 5^{th} period and Group 3 of the modern periodic table

D) 6^{th} period and Group 5 of the modern periodic table

27. The relative stability of +1 oxidation state of group 13 elements follows the order:

A) $A\ell < Ga < T\ell < In$ B) $Ga < A\ell < In < T\ell$

C) $T\ell < In < Ga < A\ell$ D) $A\ell < Ga < In < T\ell$

28. Assertion (A) : SO_2 turns acidified $K_2Cr_2O_7$ green.

Reason (R) : $K_2Cr_2O_7$ is reduced to Cr^{3+} (green) salt by SO_2.

A) Both A and R are true and R is the correct explanation of A

B) Both A and R are true but R is not the correct explanation of A

C) A is true but R is false

D) A is false but R is true

29. Product (X and Y) of the following reactions (1 and 2) are

1) $2NaOH + C\ell_2 \xrightarrow[\text{(Cold and dilute)}]{} NaC\ell + X + H_2O$

2) $6NaOH + 3C\ell_2 \xrightarrow[\text{(Hot and conc.)}]{} NaC\ell + Y + H_2O$

A) $X = NaC\ell O_3$ and $Y = NaOC\ell$ B) $X = NaOC\ell$ and $Y = NaC\ell O_3$

C) $X = NaHC\ell O_3$ and $Y = NaOC\ell$ D) $X = NaC\ell O_3$ and $Y = NaHC\ell O_3$

30. Arrange $Ce^{+3}, La^{+3}, Pm^{+3}, Yb^{+3}$ in increasing order of their ionic radii

A) $Yb^{+3} < Pm^{+3} < Ce^{+3} < La^{+3}$ B) $Ce^{+3} < Yb^{+3} < Pm^{+3} < La^{+3}$

C) $Yb^{+3} < Pm^{+3} < La^{+3} < Ce^{+3}$ D) $Pm^{+3} < La^{+3} < Ce^{+3} < Yb^{+3}$

31. Which one of the following does not contain electrons in 5f subshell?

A) $Lr(Z=103)$ B) $U(Z=92)$ C) $Cm(Z=96)$ D) $Th(Z=90)$

32. Consider octahedral complex of Ca^{2+} with $EDTA^{-4}$.

x = co-ordination number of complex

y = Total number of O – donors. Find (x+y)?

A) 13 B) 14 C) 10 D) 11

33. In which of the following species maximum atoms can lie in same plane?

A) $PC\ell_5$ B) AsH_4^+ C) XeF_4 D) XeO_2F_2

34. Which of following involves maximum C – O bond length:

A) $Ni(CO)_4$ B) $[Fe(CO)_5]$ C) $[Mn(CO)_6]^+$ D) $[V(CO)_6]^-$

35. Assertion (A) : Coloured cations can be identified by borax – bead test

Reason (R) : Transparent bead $(NaBO_2 + B_2O_3)$ forms coloured bead with coloured cation.

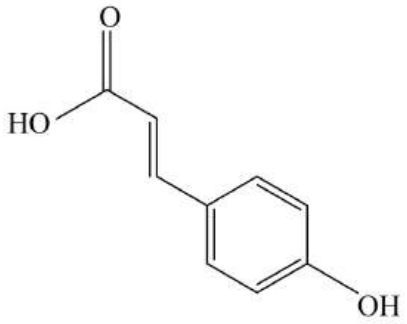
A) Both A and R are true and R is the correct explanation of A

B) Both A and R are true but R is not the correct explanation of A

C) A is true but R is false

D) A is false but R is true

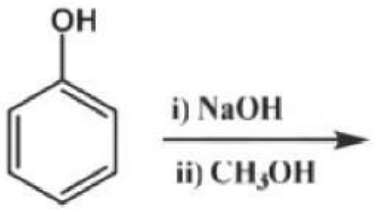
36. Para-coumaric acid, an antioxidant in coffee, is treated with aqueous solution of bromine. What is the maximum number of bromine atoms that can be incorporated into a molecule of para-coumaric acid?

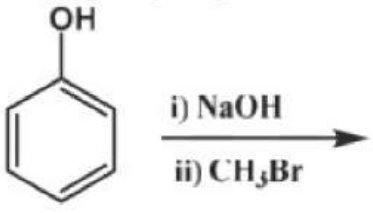


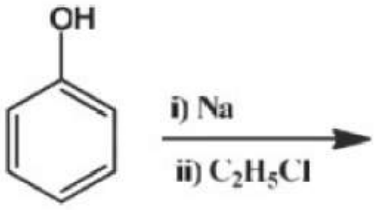
para – coumaric acid

A) 1 B) 2 C) 3 D) 4

37. Which of the following reaction produces anisole as a major product?

A) 

B) 

C) 

D) All

38. The oxidation of toluene with hot $KMnO_4$ gives

A) Benzoic acid

B) Benzaldehyde

C) Benzene

D) Benzyl alcohol

39. Primary, secondary and tertiary amines can be separated by

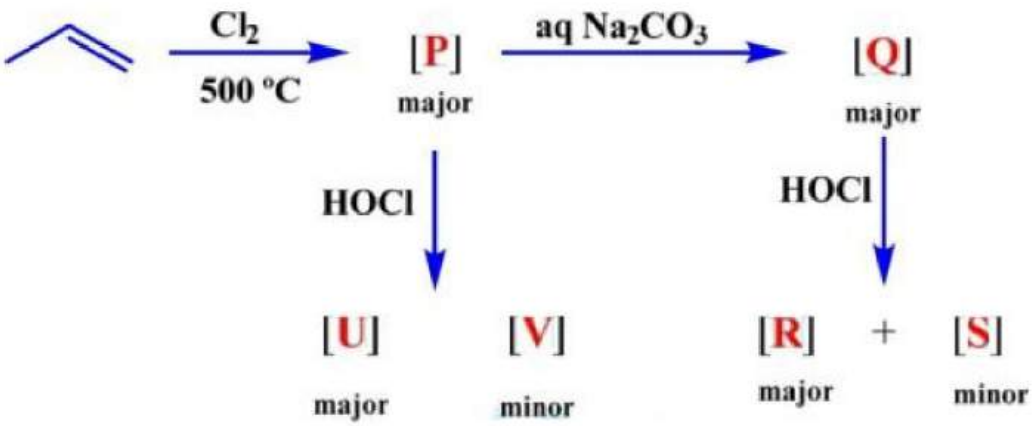
A) Hinsberg's test

B) HNO_2 test

C) Carbylamine test

D) Lucas reagent

40. Identify the **CORRECT** option(s) among the following ?



A) [R] & [S] on treatment with excess of dil NaOH gives same compound

B) Position of double bond in [P] & [Q] in same as per IUPAC

C) [U] & [V] on treatment with excess of dilute NaOH gives different product as [R] & [S] on same treatment

D) [U] & [R] are constitutional isomers

41. A $10\,cm^3$ sample of an unknown gaseous hydrocarbon was mixed with $70\,cm^3$ of oxygen and the mixture was set on fire by means of an electric spark. When the reaction was over and water vapours were liquefied, the final volume of gases decreased to $65\,cm^3$. This mixture then reacted with a potassium hydroxide solution and the volume of gases decreased to $45\,cm^3$. Find the molar mass of hydrocarbon in gm/mol, if volumes of gases were measured at standard temperature and pressure (STP) conditions ?

A) 26 B) 36 C) 16 D) 46

42. Radial wave function for 2s orbital of hydrogen – like atoms is given as

$$\psi = 2^{-3/2}\left[\frac{z}{a_0}\right]^{\frac{3}{2}}\left[2-\frac{zr}{a_0}\right]e^{\frac{zr}{2a_0}}$$

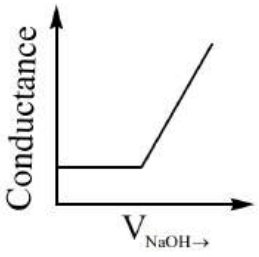
The radial probability $\left(4\pi r^2\psi^2\right)$ for an electron in He^+ at $r = a_0$ is

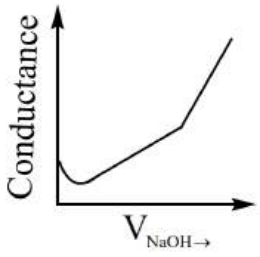
A) $\left[\frac{1}{a_0}\right]^{\frac{3}{2}}$ B) $2^{-3/2}\left[\frac{1}{a_0}\right]^{\frac{3}{2}}$ C) Zero D) $\left[\frac{1}{a_0}\right]^3$

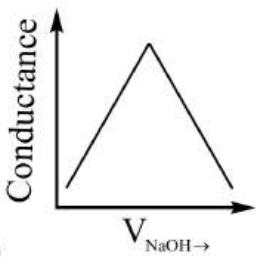
43. Benzene and toluene are nearly ideal solutions. If at a temperature $P^0_{Benzene} = 120\,mm\,Hg$ and $P^0_{Toluene} = 50\,mm\,Hg$. Then what will be the mole fraction of Toluene in vapour phase of this solution if mole fraction of Benzene in liquid phase of solution is 0.6.

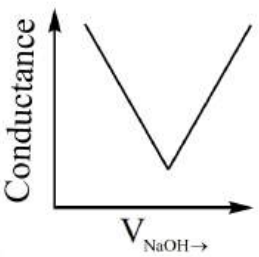
A) 0.70 B) 0.6 C) 0.4 D) 0.22

44. Choose the correct representation of conductometric titration of benzoic acid vs sodium hydroxide.

A)


B)


C)


D)


45. Consider the reaction $2H_2(g)+2NO(g)\rightarrow N_2(g)+2H_2O(g)$. The rate law for this reaction is: rate $=k[H_2][NO]^2$ under what conditions could these steps represents the mechanism?

Step: 1 $2NO \rightleftharpoons N_2O_2$

Step: 2 $N_2O_2 + H_2 \rightarrow N_2O + H_2O$

Step: 3 $N_2O + H_2 \rightarrow N_2 + H_2O$

A) These steps cannot be the mechanism under any circumstances

B) These steps cannot be the mechanism if Step 1 is sloe step

C) These steps cannot be the mechanism if Step 2 is sloe step

D) These steps cannot be the mechanism if Step 3 is sloe step

SECTION-II
(NUMERICAL VALUE ANSWER TYPE)

This section contains 5 questions. The answer to each question is a Numerical value. If the Answer in the decimals , **Mark nearest Integer only.**
Marking scheme: +4 for correct answer, -1 in all other cases.

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46. How many of these elements have lower electron affinity than fluorine ?

Cl, S, O, N, P, Br, I, C

47.

Complex	Spin-only magnetic moment
$\left[Fe(CO)_4(C_2O_4)\right]^+$	a
$[FeCl_4]^-$	b
$\left[Fe(CN)_6\right]^{3-}$	c
$\left[Cu(NH_3)_4\right]^{2+}$	d

Then (a+b+c+d) = _____

(Given : $\sqrt{3}=1.7;\sqrt{2}=1.4;\sqrt{5}=2.2;\sqrt{15}=3.8,\sqrt{35}=5.9;\sqrt{6}=2.4$)

48. The number of correct statements is /are :

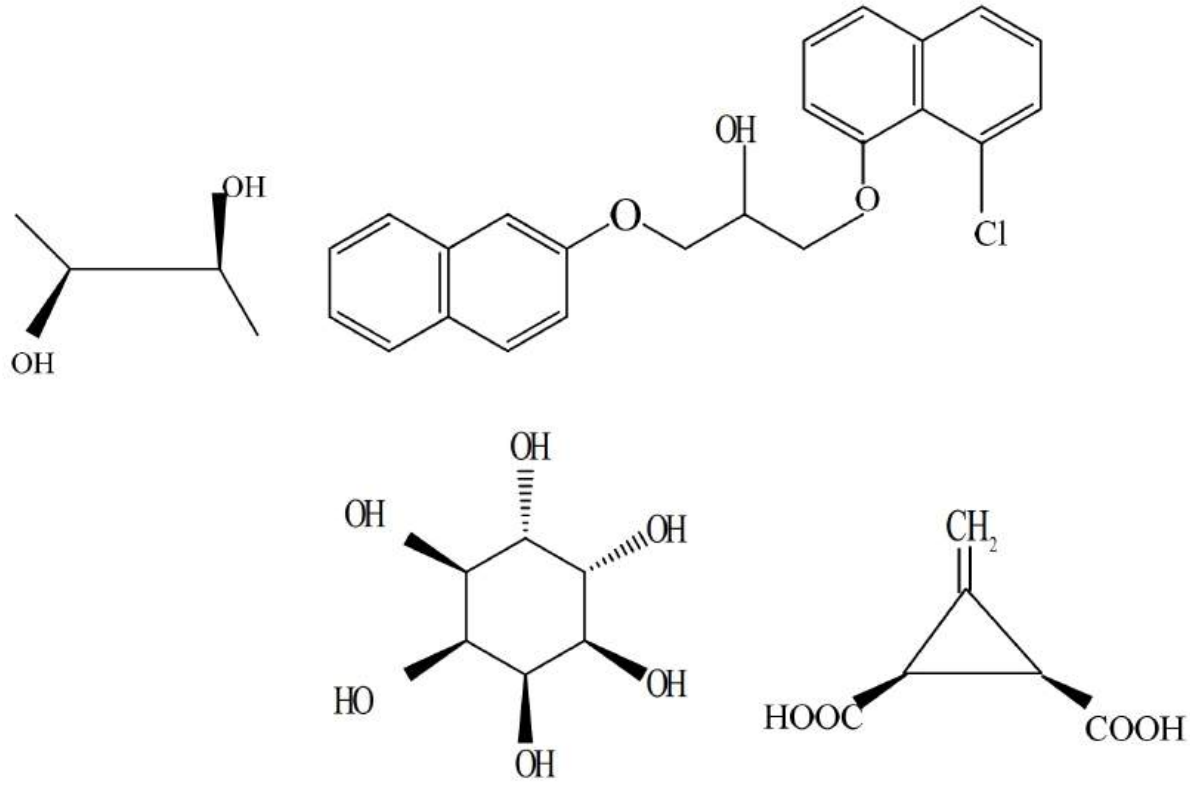
(i) In manganate and permanganate ions. The π - bonding takes place by overlap of p-orbitals of oxygen and d-orbitals of manganese

(ii) Manganate ion is green in colour and permanganate ion in purple in colour

(iii) Manganate and permanganate ions are paramagnetic

(iv) Manganate and permanganate ions are tetrahedral

49. The total number of chiral compound/s from the following is_____



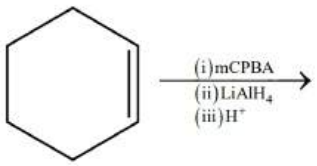
50. How many of the following reactions will give an alcohol as the major product ?

(i) $CH_3CH_2C\ell \xrightarrow[aq]{NaOH}$

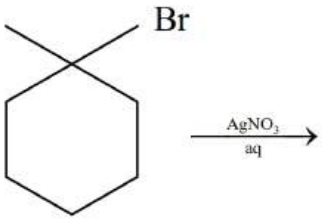
(ii) $CH_3CH = CH_2 \xrightarrow{aq\,Na_2CO_3}$

(iii) $CH_3CH_2 - C\ell \xrightarrow{CH_3OH}$

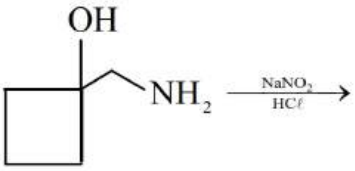
(iv)


(i)mCPBA
(ii)LiAlH4
(iii)H+

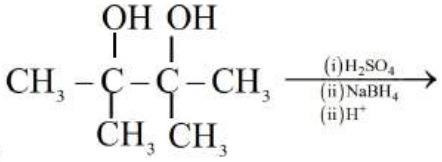
(v)


Br
AgNO3
aq

(vi)


OH
NH2
NaNO2
HCℓ

(vii)


OH OH
CH3 – C – C – CH3
CH3 CH3
(i)H2SO4
(ii)NaBH4
(ii)H+

MATHEMATICS **MAX.MARKS: 100**

SECTION – I
(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

51. Let z_1, z_2, z_3 be three distinct complex numbers lying in a circle with a centre at the origin such that $z_1+z_2\, z_3, z_2+z_3 z_1$ and $z_3+z_1\, z_2$ are real numbers, then $z_1\, z_2 z_3$ equals to

A) -1 B) 0 C) 1 D) None of these

52. The number of irrational solutions of the equation $\sqrt{x^2+\sqrt{x^2+11}}+\sqrt{x^2-\sqrt{x^2+11}}=4$ is

A) 0 B) 2 C) 4 D) 11

53. If $H_1, H_2, ...H_n$ are n harmonic means between a and $b, (b \neq a)$, then the value $\frac{H_1+a}{H_1-a}+\frac{H_n+b}{H_n-b}$ is equal to

A) $n+1$ B) $n-1$ C) $2n$ D) $2n+3$

54. Let $H_n=1+\frac{1}{2}+\cdots+\frac{1}{n}$ then the sum to n terms of the series $\frac{1^2}{1^3}+\frac{1^2+2^2}{1^3+2^3}+\frac{1^2+2^2+3^2}{1^3+2^3+3^3}+....$ is

A) $\frac{4}{3}H_n-1$ B) $\frac{4}{3}H_n+\frac{1}{n}$ C) $\frac{4}{3}H_n$ D) $\frac{4}{3}H_n-\frac{2}{3}\frac{n}{n+1}$

55. If $a=\log_{12}18, b=\log_{24}54$ then the value of $ab+5(a-b)$ is

A) 0 B) 4 C) 1 D) 6

56. The number of ordered pairs $(m,n), m,n \in \{1,2....100\}$ such that 7^m+7^n is divided by 5 is

A) 1250 B) 2000 C) 2500 D) 5000

57. The number of irrational terms in the expansion of $\left(5^{1/6}+2^{1/8}\right)^{100}$ is

A) 96 B) 97 C) 98 D) 99

58. If A, B and C are three sets such that $A\cap B=A\cap C$ and $A\cup B=A\cup C$, then

A) $A=C$ B) $B=C$ C) $A\cap B=\phi$ D) A=B

59. Two $n \times n$ square matrices A and B are said to be similar if there exists a non-singular matrix Psuch that $P^{-1}AP = B$. If A and B are similar and B and C are similar, then

A) AB and BC are similar

B) A and C are similar

C) $A+C$ and B are similar

D) None of these

60. A letter is taken at random from the letters of the word 'STATICSTICS' and another letter is taken at random from the letters of the word 'ASSISTANT'. The probability that they are the same letter is

A) $\frac{1}{45}$ B) $\frac{13}{90}$ C) $\frac{19}{90}$ D) $\frac{3}{16}$

61. $\sin 47^0 + \sin 61^0 - \sin 11^0 - \sin 25^0$ is equal to

A) $\sin 36^0$ B) $\cos 36^0$ C) $\sin 7^0$ D) $\cos 7^0$

62. $A_1, A_2, A_3, \ldots A_n$ are n points in a plane whose coordinates are $(x_1, y_1), (x_2, y_2), (x_3, y_3) \ldots, (x_n, y_n)$ respectively. A_1A_2 is bisected at the point G_1, G_1A_3 is divided in the ratio 1:2 at G_2, G_2A_4 is divided in the ratio 1:3 at G_3, G_3A_5 is divided in the ratio 1:4 at G_4, and so on until all n points are exhausted. The coordinates of the final point G so obtained are

A) $\left(n(x_1 + x_2 + \ldots + x_n), n(y_1 + y_2 + \ldots . y_n)\right)$

B) $\left(\frac{x_1 + x_2 + \ldots + x_n}{n}, \frac{y_1 + y_2 + \ldots . + y_n}{n}\right)$

C) $\left(\frac{x_1 + 2x_2 + 3x_3 \ldots + nx_n}{n}, \frac{y_1 + 2y_2 + 3y_3 \ldots . + ny_n}{n}\right)$

D) None of these

63. Let PQ and RS be tangents at the extremities of a diameter PR of a circle of radius r. Such that $PS$and RQ intersect at a point X on the circumference of the circle, then diameter of the circle equals to

A) $\sqrt{PQ.RS}$ B) $\frac{PQ + RS}{2}$ C) $\frac{2PQ \cdot RS}{PQ + RS}$ D) $\sqrt{\frac{PQ^2 + RS^2}{2}}$

64. The locus of the vertices of the family of parabolas $y=\frac{a^3x^2}{3}+\frac{a^2x}{2}-2a$ is

A) $xy=\frac{105}{64}$ B) $xy=\frac{3}{4}$ C) $xy=\frac{35}{16}$ D) $xy=\frac{64}{105}$

65. A triangle ABC is placed so that the mid-point of its sides are on the x, y and z axes respectively. Lengths of the intercepts made by the plane containing the triangle on these axes are respectively α,β,γ, then the coordinates of the centroid of the triangle ABC are

A) $(-\alpha/3,\beta/3,\gamma/3)$ B) $(\alpha/3,-\beta/3,\gamma/3)$

C) $(\alpha/3,\beta/3,-\gamma/3)$ D) $(\alpha/3,\beta/3,\gamma/3)$

66. Let $f:\mathrm{R}\to\mathrm{R}$ be a function defined by $f(x)=\frac{e^{|x|}-e^{-x}}{e^{x}+e^{-x}}$. Then

A) f is both one-one and onto B) f is one-one but not onto

C) f is onto but not one-one D) f is neither one-one nor onto

67. The value of $f(0)$ so that the function $f(x)=\frac{\cos(\sin x)-\cos x}{x^4}$ is continuous at each point in its domain, is equal to

A) 2 B) 1/6 C) 2/3 D) -1/3

68. If $f(x)=(1+x)^n$ $(n\in\mathrm{N},\ x\in\mathrm{R})$, then the value of $f(0)+f'(0)+\frac{f''(0)}{2!}+\frac{f'''(0)}{2!}+\cdots+\frac{f^n(0)}{n!}$ is (where $f^n(0)$ represents n^{th} derivates of f at 0)

A) n B) 2^n C) 2^{n-1} D) None of these

69. $\int\frac{dx}{x^{1/2}+x^{1/3}}=$

A) $2\sqrt{x}-3x^{1/3}+6x^{1/6}-\log\left(1+x^{1/6}\right)+C$ B) $3x^{1/3}-6x^{1/6}-\log\left(1+x^{1/6}\right)+C$

C) $\sqrt{x}+3x^{1/3}-6x^{1/6}+\log\left(1+x^{1/6}\right)+C$ D) $3x^{1/3}-\sqrt{x}+6x^{1/6}-\log\left(1+x^{1/6}\right)+C$

70. The solution of the differential equation $y'-Ay=-By^2(A,B>0)$ is (C is arbitrary constant)

A) $y=\left((B/A)e^{Ax}+C\right)^{-1}$ B) $y=(B/A)+Ce^{-Ax}$

C) $y=e^{-Ax}\left((B/A)e^{Ax}+C\right)^{-1}$ D) $y=e^{Ax}\left((B/A)e^{Ax}+C\right)^{-1}$

SECTION-II
(NUMERICAL VALUE ANSWER TYPE)

This section contains 5 questions. The answer to each question is a Numerical value. If the Answer in the decimals , **Mark nearest Integer only.**

Marking scheme: +4 for correct answer, -1 in all other cases.

71. If $\tan^{-1}\left(x+\frac{2}{x}\right)-\tan^{-1}\left(x-\frac{2}{x}\right)=\tan^{-1}\frac{4}{x}$ then the value of $250x^4+320x^2+137$ is equal to

72. If O is the origin and the coordinates of A and B are (51, 65) and (75, 81) respectively. Then $OA\times OB\cos\angle AOB$ is equal to

73. Let $0\le\beta_r\le 1\,(r=1,2,\cdots)$ and $\sum_{r=1}^{k}\cos\beta_r=\frac{k\pi}{2}$ for any $k\ge 1$ and $A=\sum_{r=1}^{k}(\beta_r)^r$, then

$$\lim_{x\to A} 498.\frac{\left(1+x^2\right)^{1/3}-\left(1-2x\right)^{1/4}}{x+x^2}$$

74. A positive integer $n\le 5$ such that $\int_0^1 e^{2x-1}(x-1)^n\,dx=\frac{7}{4}e^{-1}-\frac{1}{16}\left(e^2-1\right)$ is

75. Let a,b,c are given vectors, λ,μ,v are scalars If $d=\lambda(a\times b)+\mu(b\times c)+v(c\times a),[abc]=1/8$ and $d\cdot(a+b+c)=8$ then $\lambda+\mu+v$ is equal to

KEY SHEET

PHYSICS

1	**B**	2	**B**	3	**C**	4	**D**	5	**A**
6	**B**	7	**C**	8	**A**	9	**B**	10	**A**
11	**B**	12	**B**	13	**A**	14	**A**	15	**A**
16	**C**	17	**A**	18	**D**	19	**B**	20	**B**
21	**1**	22	**30**	23	**3**	24	**780**	25	**16**

CHEMISTRY

26	**B**	27	**D**	28	**A**	29	**B**	30	**A**
31	**D**	32	**C**	33	**C**	34	**D**	35	**A**
36	**D**	37	**B**	38	**A**	39	**A**	40	**A**
41	**A**	42	**C**	43	**D**	44	**B**	45	**C**
46	**7**	47	**11**	48	**3**	49	**2**	50	**4**

MATHEMATICS

51	**C**	52	**C**	53	**C**	54	**D**	55	**C**
56	**C**	57	**B**	58	**B**	59	**B**	60	**C**
61	**D**	62	**B**	63	**A**	64	**A**	65	**D**
66	**D**	67	**B**	68	**B**	69	**A**	70	**D**
71	**1777**	72	**9090**	73	**249**	74	**3**	75	**64**

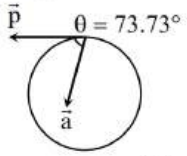
SOLUTIONS
PHYSICS

1. $\phi = \dfrac{Q}{A\varepsilon_0} \times A$

$\phi = \dfrac{Q}{\varepsilon_0}$

$\dfrac{d\phi}{dt} = \dfrac{1}{\varepsilon_0} \times \dfrac{dQ}{dt} = \dfrac{I}{\varepsilon_0}$

2. Angle between $\vec{a}$ and $\vec{p}$ is :



$\theta = \cos^{-1} \dfrac{\vec{a}.\vec{p}}{|\vec{a}||\vec{p}|}$

$= \cos^{-1} \left\{ \dfrac{32-18}{\sqrt{(16+9)}\sqrt{(64+36)}} \right\}$

$= \cos^{-1} \left(\dfrac{14}{50} \right)$

$\theta = 73.73°$

Since $0° < 90°$, the motion is an acceleration one.

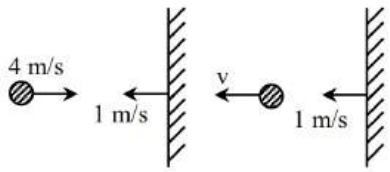
3. Retardation = $g(\sin\theta + \mu\cos\theta) = 5(\sqrt{3}+\mu)$

Now $v = u - at$

$\therefore \quad a = \dfrac{u}{t}$ as $v = 0$

$\therefore \quad 5(\sqrt{3}+\mu) = 10 \Rightarrow \mu = 0.27$

4.



Before collision After collision

Let v be the velocity of ball after collision, collision is elastic

$\therefore$ e =1

or

relative velocity of separation = relative velocity of approach

$\therefore$ v – 1 = 4 + 1

or v = 6 m/s (away from the wall)

5. $\because \quad g = \dfrac{GH}{(R+h)^2}$

$\therefore \quad \dfrac{GM}{9R^2} = \dfrac{GM}{(R+h)^2}$

$\Rightarrow \quad 3R = R + h$

$\Rightarrow \quad h = 2R$

So option (1) is correct.

6. $Dq = -Du$

$$C = -C_V = \frac{-R}{\gamma - 1} = \frac{+R}{\gamma - 1} + \frac{P}{n}\frac{dV}{dT}$$

$$\boxed{-\frac{P}{n}\frac{dV}{dT} = \frac{2R}{\gamma - 1}}$$

$T^5V = \text{const.}$

$$V = \frac{\text{const.}}{T^5}$$

$$\frac{dV}{dT} = -5\frac{\text{const}}{T^6}$$

$PV = Nrt$

$P/n = RT/V$

$$+\frac{RT}{\text{const.}}T^5 \times \left(-5\frac{\text{const}}{T^6}\right) = \frac{2R}{\gamma - 1}$$

$$\frac{5}{2} = \frac{1}{\gamma - 1} \Rightarrow \gamma - 1 = 2/5$$

$\gamma = 7/5$

adiabatic compressibility

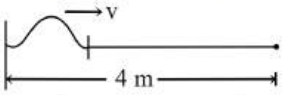
$$\beta = \frac{1}{\gamma P} = \frac{5}{7P}$$

7. $$PV = \frac{1}{3}m_o N v_{rms}^2$$

$$(2P)(2V) = \frac{1}{3}m_o N v'^2_{rms}$$

$$\varpi'_{rms} = 2v_{rms} = 2v$$

8. 4 trips means 32 m



$$t = \frac{d}{v} \Rightarrow v = \frac{d}{t} = \frac{32}{0.8} = 40 \text{ m/s}$$

$$v = \sqrt{\frac{T}{\mu}}$$

$$\Rightarrow T = \mu v^2$$

$$T = \frac{0.2}{4} \times (40)^2 = \frac{2 \times 16 \times 10}{4}$$

$T = 80$ N

9. Time period of both A and B $T = 2\pi\sqrt{\frac{\ell}{g}}$

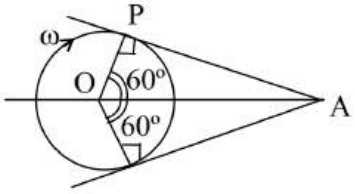
After first collision, B acquires amplitude of A and after second collision it acquires its own amplitude in this process time taken is

$$= \frac{T}{4} + \frac{T}{4} + \frac{T}{4} + \frac{T}{4} = T = 2\pi\sqrt{\frac{\ell}{g}}$$

10. The distribution of charge on the outer surface, depends only on the charges outside, and it distributes itself such that the net electric field inside the outer surface due to the charge on outer surface and all the outer charges is zero. Similarly the distribution of charge on the inner surface, depends only on the charges inside the inner surface, and it distributes itself such that the net, electric field outside the inner surface due to the charge on inner surface and all the inner charges is zero.
Also the force on charge inside the cavity is due to the charge on the inner surface. Hence answer is option (A).

11. Conceptual

12. Point A shall record zero magnetic field (due to α-particle) is at position P and Q as shown in figure. The time taken by α-particle to go from P to Q is –



$t = \frac{1}{3}\frac{2\pi}{\omega}$ or $\omega = \frac{2\pi}{3t}$

13. Curie temperature is temperature above which Ferromagnetic materials obey Curie's law.

14. $U = \frac{1}{2}LI^2$

$\text{Rate} = \frac{dU}{dt} = LI\left(\frac{dI}{dt}\right)$

At $t = 0$, $I = 0$

$\therefore$ Rate $= 0$

At $t = \infty$, $I = I_0$ but $\frac{dI}{dt} = 0$, therefore, rate $= 0$

15. The given lens is a convex lens. Let the magnification be m, then for real image

$\frac{1}{mx} + \frac{1}{x} = \frac{1}{f}$ ... (i)

and for virtual image $\frac{1}{-my} + \frac{1}{y} = \frac{1}{f}$ ... (ii)

From Eq. (i) and Eq. (ii), we get

$f = \frac{x+y}{2}$

16. $e = 4 \times LC = 4 \times 0.01$ cm $= 0.04$ cm

$c = -0.04$ cm

17. Zener diode is in parallel to load resistance and is connected in reverse bias.

18. $R = R_0 A^{1/3} \Rightarrow \frac{R}{R_0} = A^{1/3}$

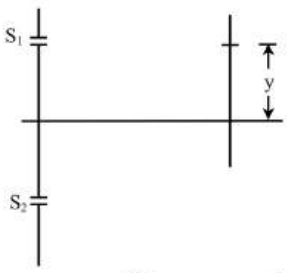
$\log\left(\frac{R}{R_0}\right) = \frac{1}{3}\log A$

or $y = \frac{1}{3}x$

a straight line passing through origin with slope 1/3.

19. Conceptual

20.



$\Delta x = \frac{y}{\beta} \Rightarrow \Delta\phi = \frac{2\pi}{\lambda} \times \Delta x = \frac{2\pi y}{\beta}$

$I_{net} = I + I + 2I\cos\Delta\phi$

$= 2I\left(1+\cos\left(\frac{2\pi y}{\beta}\right)\right) = 4I\cos^2\left(\frac{\pi y}{\beta}\right)$

$= I_0\cos^2\left(\frac{\pi y}{\beta}\right)$ [□ $I_0 = 4I$]

21. $F = \frac{-k}{2x^2} \Rightarrow F = -\frac{10^{-2}}{2x^2}$

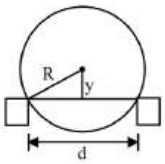
$a = \frac{F}{m} = \frac{10^{-2}}{(10^{-2})2x^2} \Rightarrow a = \frac{-1}{2x^2} = v\frac{dv}{dx}$

$-\int_1^{0.5}\frac{dx}{2x^2} = \int_0^v vdv \Rightarrow -\frac{1}{2}\int_1^{0.5}x^{-2}dx = \frac{v^2}{2}$

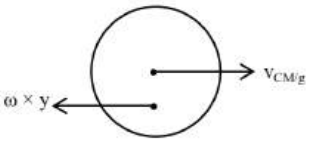
$\Rightarrow \frac{-1}{2}\left[\frac{-1}{x}\right]_1^{0.5} = \frac{v^2}{2} \Rightarrow \frac{1}{2}\left[\frac{1}{0.5}-1\right] = \frac{v^2}{2}$

$\Rightarrow \quad v = 1$ m/s

22.



$y^2 = R^2 - \left(\frac{d}{2}\right)^2$



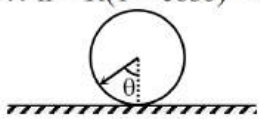
$v_{CM/g} = \omega \times y$

$= 30$ cm/s

23. $v = R\omega\cos\theta$

$\Rightarrow \cos\theta = \frac{v}{R\omega} = \frac{2}{5}$ cm

$\therefore h = R(1-\cos\theta) = 3$ cm.



24. If m is mass of single drop then as it drops

$mg = 2\pi Rt$

If number of drops in M = 10 grams is N then,

$N = \frac{M}{m} = \frac{Mg}{mg} = \frac{Mg}{2\pi rT} \approx 779.86 \approx 780$

25. For sonometer wire

$\nu_n = \frac{n\sqrt{\frac{F}{\mu}}}{2\ell}$

$\Rightarrow n\sqrt{F}$ = constant

[$\therefore \nu, \mu, \lambda$ are constant for two cases of comparison]

$\Rightarrow F_2 = \frac{n_1^2}{n_2^2} \cdot F_1$

$\Rightarrow m_2 = 25$ kg

$\therefore$ Additional mass = 16 kg

CHEMISTRY

26. $[Xe]4f^{14}5d^{1}6s^{2}$ ---------- Lu

27. INERT PAIR effect NCERT (Physical properties of group 13 elements)

28. $K_2Cr_2O_7 + SO_2 + H_2SO_4 \longrightarrow K_2SO_4 + Cr_2(SO_4)_3 + H_2O$

29. $X: NaC\ell O$

$Y: NaC\ell O_3$.

30. By using Lanthanoid contraction concept we get correct answer.

31. $Th(Z=90): {}_{86}[Rn]6d^2 7s^2$

32. $y = 4, x = 6$

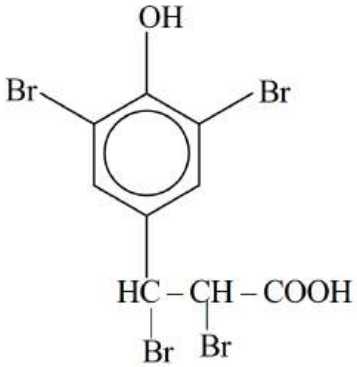
33. $PC\ell_5 - sp^3(TBP)$; $AsH_4^+ - sp^3(Tetrahedral)$; $XeF_4 - sp^3d^2$ -Square planar;

$XeO_2F_2 - sp^3d(Seesaw)$

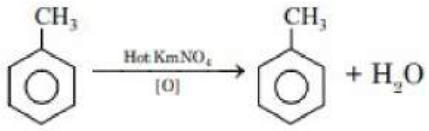
34. synergic bond concept in complex compounds

35. Conceptual

36. Product is



38.



Benzoic acid

39. Hinsberg's test is used for separation of $1^\circ, 2^\circ$ and 3° Amines.

41. $2 + n + 3(-2) = 0 \quad n = -4$

$$C_xH_y(g) + \left(x + \frac{y}{4}\right)O_2(g) \longrightarrow xCO_2(g) + \frac{y}{2}H_2O(\ell)$$

10 70 - -

- $70 - 10\left(x + \frac{y}{4}\right)$ $10x$ $-70 - 10x - \frac{5}{2}y + 10x = 65$

$5y = 5 \times 2 \quad y = 210 \quad x = 20 \quad x = 2$

$C_xH_y = C_2H_2$ Molar mass $= 26\,g/mol$

42. At $r = a_0 \quad \psi = 0 \Rightarrow 4\pi r^2 \psi^2 = 0$

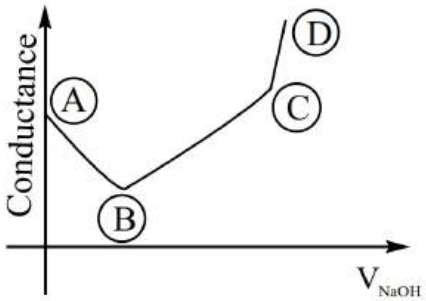
43. In liquid phase

$P_T = P_B^0 . \chi_B + P_T^0 . \chi_T = 120 \times 0.6 + 50 \times 0.4$

$= 72 + 20 = 92$ mmHg

In vapour phase $\chi_T = \frac{P_{Toluene}}{P_{Total}}\frac{20}{92} = 0.22$

44. $\underset{(WA)}{C_3H_5COOH} + \underset{(SB)}{NaOH} \longrightarrow \underset{(Salt)}{C_6H_5COONa} + H_2O$



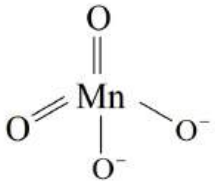
45.

46. NCERT data

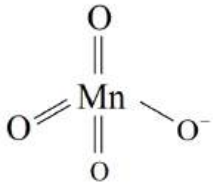
47. $a = \sqrt{3}; b = \sqrt{35}; c = \sqrt{3}; d = \sqrt{3}$

48. Option i) Manganate $\Rightarrow MnO_4^{2-}$

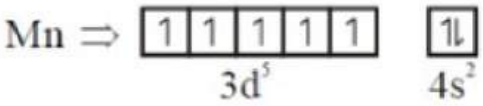
Permanganate $\Rightarrow MnO_4^-$



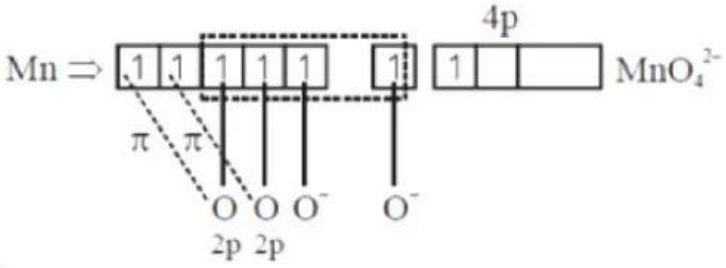
hybridisation of Mn $\Rightarrow d^3s$



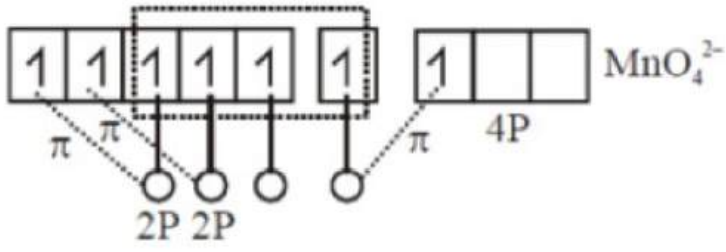
hybridisation of Mn $\Rightarrow d^3s$



After excitation



$2 \times 2p_\pi - 3d_{\pi\sigma}$



$2 \times 2p_\pi - 3d_\pi$

$1 \times 2p_\pi - 4p_\pi$

(ii) $MnO_4^{2-} \Rightarrow$ green

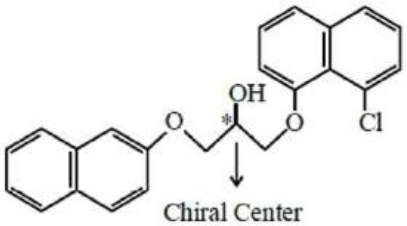
$MnO_4^- \Rightarrow$ purple / violet

(iii) Manganate contains I unpaired electron hence it is paramagnetic Where as permanganetic contains no unpaired electrons hence it is diamagnetic.

(iv) Both have d^3s hybridization hence both have tetrahedral geometry.

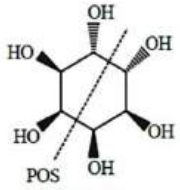
49.

No POS, COS
(Chiral)

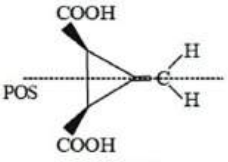


Chiral Center

No POS, COS
(Chiral)



(Achiral)



(Achiral)

50.

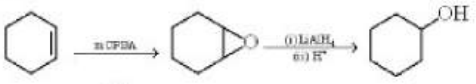
(i) $CH_3CH_2Cl \xrightarrow[aq]{NaOH}$

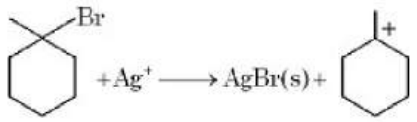
(ii) $CH_3CH = CH_2 \xrightarrow[\text{(Basic medium)}]{aq\ Na_2CO_3}$ No reaction

(iii)

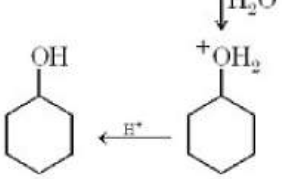
$CH_3O^- + CH_3CH_2 - Cl \rightarrow CH_3OCH_2CH_3 + Cl^-$

(iv)

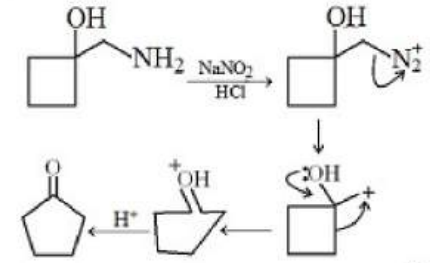




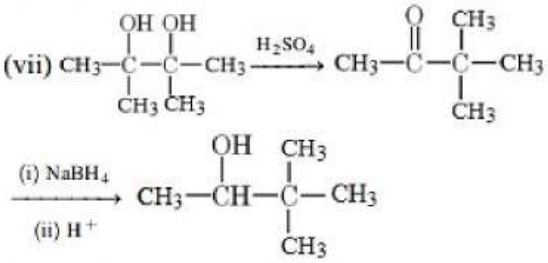
(v)



(vi)



(vii)



(Pinacol Pinacolone rearrangement)

MATHS

51.

Solution As z_1, z_2, z_3 lie on a circle with centre at the origin,

$$|z_1| = |z_2| = |z_3| = r \text{(say)}.$$

As $z_1 + z_2 z_3 \in \mathbf{R}$, $z_1 + z_2 z_3 = \bar{z}_1 + \bar{z}_2\,\bar{z}_3$

$$= \frac{r^2}{z_1} + \frac{r^4}{z_2 z_3}$$

$$= \frac{r^2(z_2 z_3 + r^2 z_1)}{z_1 z_2 z_3}$$

$$\Rightarrow \quad \therefore \quad \frac{r^2}{z_1 z_2 z_3} = \frac{z_1 + z_2 z_3}{z_2 z_3 + r^2 z_1} \qquad (1)$$

Similarly, $$\frac{r^2}{z_1 z_2 z_3} = \frac{z_2 + z_3 z_1}{z_3 z_1 + r^2 z_2} \qquad (2)$$

and $$\frac{r^2}{z_1 z_2 z_3} = \frac{z_3 + z_1 z_2}{z_1 z_2 + r^2 z_3} \qquad (3)$$

From (1), (2) and (3), we get

$$\frac{r^2}{z_1 z_2 z_3} = \frac{z_1 + z_2 z_3}{z_2 z_3 + r^2 z_1} = \frac{z_2 + z_3 z_1}{z_3 z_1 + r^2 z_2} = \frac{z_3 + z_1 z_2}{z_1 z_2 + r^2 z_3}$$

$$= \frac{z_1 + z_2 z_3 - (z_2 + z_3 z_1)}{z_2 z_3 + r^2 z_1 - (z_3 z_1 + r^2 z_2)}$$

$$= \frac{(z_1 - z_2)(z_3 - 1)}{(z_1 - z_2)(z_3 - r^2)} = \frac{z_3 - 1}{z_3 - r^2}$$

$$\therefore \quad \frac{r^2}{z_1 z_2 z_3} = \frac{z_3 - 1}{z_3 - r^2} = \frac{z_2 - 1}{z_2 - r^2} = \frac{z_1 - 1}{z_1 - r^2}$$

$$= \frac{(z_2 - 1) - (z_1 - 1)}{(z_2 - r^2) - (z_1 - r^2)}$$

$$= \frac{z_2 - z_1}{z_2 - z_1} = 1$$

$$\Rightarrow \quad z_3 - 1 = z_3 - r^2 \Rightarrow r^2 = 1.$$

Hence, $z_1 z_2 z_3 = 1$

52.

Solution Put $\sqrt{x^2 + 11} = t$ so that (1) can be written as

$$\sqrt{t^2 + t - 11} + \sqrt{t^2 - t - 11} = 4 \qquad (2)$$

We also have the identity

$$(t^2 + t - 11) - (t^2 - t - 11) = 2t \qquad (3)$$

Dividing (2) by (3) we get

$$\sqrt{t^2 + t - 11} - \sqrt{t^2 - t - 11} = \frac{t}{2} \qquad (4)$$

Adding (2) and (4), we get

$$2\left(\sqrt{t^2 + t - 11}\right) = 4 + t/2$$

$$\Rightarrow \quad t^2 + t - 11 = 4 + t + t^2/16$$

$$\Rightarrow \quad t = 4 \Rightarrow x = \pm\sqrt{5}.$$

53.

Solution As $a, H_1, H_2, \ldots, H_n, b$ are in H.P., $\frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2} \cdots \frac{1}{H_n}, \frac{1}{b}$ are in A.P.

Let d be the common difference of this A.P., then

$$\frac{1}{b} = \frac{1}{a} + (n + 1)d$$

and $$\frac{1}{H_n} - \frac{1}{H_1} = (n - 1)d$$

Now, $$\frac{H_1 + a}{H_1 - a} = \frac{1/a + 1/H_1}{1/a - 1/H_1} = \frac{1/a + 1/H_1}{-d}$$

and $$\frac{H_n + b}{H_n - b} = \frac{1/b + 1/H_1}{1/b - 1/H_n} = \frac{1/b + 1/H_n}{d}$$

$$\therefore \quad \frac{H_1 + a}{H_1 - a} + \frac{H_n + b}{H_n - b} = \frac{1}{d}\left[\frac{1}{b} - \frac{1}{a} + \frac{1}{H_n} - \frac{1}{H_1}\right]$$

$$= \frac{1}{d}[(n + 1)d + (n - 1)d] = 2n$$

54.

Solution We have

$$t_n = \frac{1^2+2^2+3^2+\cdots+n^2}{1^3+2^3+3^3+\cdots+n^3} = \frac{\frac{1}{6}n(n+1)(2n+1)}{\frac{1}{4}n^2(n+1)^2}$$

$$= \frac{2(2n+1)}{3n(n+1)} = \frac{2}{3}\left[\frac{1}{n}+\frac{1}{n+1}\right]$$

$$\Rightarrow \sum_{k=1}^{n} t_k = \frac{2}{3}\left\{\left(1+\frac{1}{2}\right)+\left(\frac{1}{2}+\frac{1}{3}\right)+\cdots+\left(\frac{1}{n}+\frac{1}{n+1}\right)\right\}$$

$$= \frac{4}{3}H_n - \frac{2}{3}\frac{n}{n+1}.$$

55.

$$a = \log_{12} 18 = \frac{\log_2 18}{\log_2 12} = \frac{1+2\log_2 3}{2+\log_2 3} \text{ and}$$

$$b = \log_{24} 54 = \frac{\log_2 54}{\log_2 24} = \frac{1+3\log_2 3}{3+\log_2 3}.$$

Putting $x = \log_2 3$, we have

$$ab + 5(a-b) = \frac{1+2x}{2+x}\cdot\frac{1+3x}{3+x} + 5\left(\frac{1+2x}{2+x} - \frac{1+3x}{3+x}\right)$$

$$= \frac{6x^2+5x+1+5(-x^2+1)}{(x+2)(x+3)} = \frac{x^2+5x+6}{(x+2)(x+3)} = 1$$

56.

Solution Note that 7^r $(r \in N)$ ends in 7, 9, 3 or 1 (corresponding to $r = 1, 2, 3$ and 4 respectively.)

Thus, $7^m + 7^n$ cannot end in 5 for any values of m, $n \in \mathbf{N}$. In other words, for $7^m + 7^n$ to be divisible by 5, it should end in 0.

For $7^m + 7^n$ to end in 0, the forms of m and n should be as follows:

	m	n
1	$4r$	$4s+2$
2	$4r+1$	$4s+3$
3	$4r+2$	$4s$
4	$4r+3$	$4s+1$

Thus, for a given value of m there are just 25 values of n for which $7^m + 7^n$ ends in 0. [For instance, if $m = 4r$, then $n = 2, 6, 10, \ldots, 98$]

$\therefore$ there are $100 \times 25 = 2500$ ordered pairs (m, n) for which $7^m + 7^n$ is divisible by 5.

57.

Solution $(r+1)$th in the expansion of $(5^{1/6} + 2)^{100}$ is

$$t_{r+1} = {}^{100}C_r (5^{1/6})^{100-r} (2^{1/8})^r$$

As 5 and 2 are relatively prime, t_{r+1} will be rational if $\frac{100-r}{6}$ and $\frac{r}{8}$ are both integers. i.e. if $100 - r$ is a multiple of 6 and r is a multiple of 8. As $0 \le r \le 100$, multiples of 8 upto 100 and corresponding value of $100 - r$ are

$$r = 0, 8, 16, 24, \ldots, 88, 96$$

$$100 - r = 100, 92, 84, 76, \ldots, 12, 4$$

Out of values of $100 - r$, multiples of 6 are 84, 60, 36, 12.

$\therefore$ there are just four rational terms.

59. CONCEPTUAL

60.

Solution Letters of the word STATISTICS ar

AIICSSSTTT

Letters of the word ASSISTANT are

AAINSSSTT

Common letters are *A*, *I*, *S* and *T*

Probability of choosing *A* is $\frac{1}{10}\times\frac{2}{9}=\frac{2}{90}$

Probability of choosing *I* is $\frac{2}{10}\times\frac{1}{9}=\frac{2}{90}$

Probability of choosing *S* is $\frac{3}{10}\times\frac{3}{9}=\frac{9}{90}$

Probability of choosing *T* is $\frac{3}{10}\times\frac{2}{9}=\frac{6}{90}$

$\therefore$ Probability of required event

$$=\frac{2}{90}+\frac{2}{90}+\frac{9}{90}+\frac{6}{90}=\frac{19}{90}.$$

61.

Solution The given expression is equal to

$$(\sin 47^\circ + \sin 61^\circ) - (\sin 11^\circ + \sin 25^\circ$$

$$= 2\sin 54^\circ \cos 7^\circ - 2\sin 18^\circ \cos 7^\circ$$

$$= 2\cos 7^\circ(\sin 54^\circ - \sin 18^\circ)$$

$$= 2\cos 7^\circ\left[\frac{\sqrt{5}+1}{4}-\frac{\sqrt{5}-1}{4}\right] = \cos 7^\circ.$$

62.CONCEPTUAL

63.

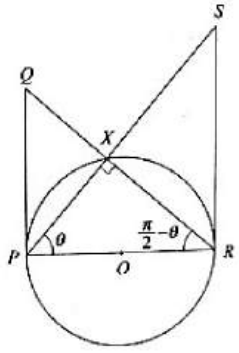
olution From the Fig. 16.20 we have

$$\frac{PQ}{PR} = \tan(\pi/2 - \theta) = \cot\theta$$

nd $$\frac{RS}{PR} = \tan\theta$$

$$\Rightarrow \quad \frac{PQ}{PR}\cdot\frac{RS}{PR} = 1$$

$$\Rightarrow \quad (PR)^2 = PQ \cdot RS$$



64.

Solution Equation of the parabola is

$$\left(y+\frac{35a}{16}\right) = \frac{a^3}{3}\left(x+\frac{3}{4a}\right)^2 \text{ whose vertex is}$$

$$x = -\frac{3}{4a},\ y = -\frac{35a}{16} \Rightarrow xy = \frac{105}{64}.$$

65.

Solution Equation of the plane containing the triang

ABC is $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1$

which meets the axes in $(\alpha, 0, 0)$, $(0, \beta, 0)$ and $(0, 0, \gamma)$.
Let the coordinates of A be (x_1, y_1, z_1)
Since the middle point of AB lies on the z-axis it is $(0, 0, \gamma)$
and thus the coordinates of B are $(-x_1, -y_1, 2\gamma - z_1)$
Similarly the coordinates of C are $(-x_1, 2\beta - y_1, -z_1)$
So that the middle point of $BC = (-x_1, \beta - y_1, \gamma - z_1)$
$= (\alpha, 0, 0)$

$\Rightarrow$ $x_1 = -\alpha,\ y_1 = \beta,\ z_1 = \gamma.$

and thus the coordinates of A are $(-\alpha, \beta, \gamma)$
similarly the coordinates of B are $(\alpha, -\beta, \gamma)$
and those of C are $(\alpha, \beta, -\gamma)$.
Hence the coordinates of the centroid of the triangle ABC are $(\alpha/3, \beta/3, \gamma/3)$.

66.

Solution f is not one-one as $f(0) = 0$ and $f(-1) = 0$
f is also not onto as for $y = 1$ there is no $x \in R$ such tha
$f(x) = 1$. If there is such a $x \in \mathbf{R}$ then $e^{|x|} - e^{-x} = e^x + e^{-x}$
Clearly $x \neq 0$. For $x > 0$, this equation gives $-e^{-x} = e^{-}$
which is not possible. For $x < 0$, the above equation gives
$e^x = -e^{-x}$ which is also not possible.

67.

Solution For f to be continuous, we must have

$$f(0) = \lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4}$$

$$= \lim_{x\to 0} \frac{-2\sin\frac{\sin x + x}{2}\sin\frac{\sin x - x}{2}}{x^4}$$

$$= \frac{1}{2}\lim_{x\to 0} \frac{\sin\left(\frac{\sin x + x}{2}\right)\sin\left(\frac{\sin x - x}{2}\right)}{\left(\frac{\sin x + x}{2}\right)\left(\frac{\sin x - x}{2}\right)} \times \frac{\sin x + x}{x} \times \frac{\sin x - x}{x^3}$$

$$= -\frac{1}{2}\lim_{u\to 0}\frac{\sin u}{u}\lim_{v\to 0}\frac{\sin v}{v}\lim_{x\to 0}\left(\frac{\sin x}{x}+1\right)\times$$

$$\lim\frac{\frac{x^3}{31} - \frac{x^5}{51} + \cdots}{x^3} \left(u = \frac{\sin x + x}{2}, v = \frac{\sin x - x}{2}\right)$$

$$= -\frac{1}{2}\times 1\times 1\times 2\times\frac{1}{31} = \frac{1}{6}.$$

68.

Solution $f(0) = 1, f'(x) = n(1+x)^{n-1}, f''(x) = n(n-1)(1+x)^{n-2}, \cdots, f^n(x) = n(n-1)\cdots 1(1+x)^{n-n}$. So $f'(0) = n$, $f''(0) = n(n-1)$, $\cdots$, $f^n(0) = n!$. Hence the given expression is equal to

$$1 + n + \frac{n(n-1)}{2!} + \cdots + \frac{n!}{n!} = {}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n = 2^n.$$

69.

Solution Write $I = \int \frac{dx}{x^{1/3}\left(1+x^{1/6}\right)}$

Put $x^{1/6} = t$ or $x = t^6$, so that

$$I = 6\int \frac{t^5 dt}{t^2(1+t)} = 6\int \frac{t^3}{t+1} dt$$

$$= 6\int \frac{t^3+1-1}{t+1} dt$$

$$= 6\int \left[\left(t^2-t+1\right) - \frac{1}{t+1}\right] dt$$

$$= 6\left[\frac{t^3}{3} - \frac{t^2}{2} + t - \log|t+1|\right] + C$$

$$= 2\sqrt{x} - 3x^{1/3} + 6x^{1/6} - \log\left(1+x^{1/6}\right) + C$$

70. conceptual

71.

Solution We have

$$\frac{x+\frac{2}{x}-\left(x-\frac{2}{x}\right)}{1+\left(x^2-\frac{4}{x^2}\right)} = \frac{4}{x} \Rightarrow x^2 - \frac{4}{x^2} = 0$$

$$\Rightarrow \quad x^2 = 2$$

72.

Solution $(OA)^2 = (51)^2 + (65)^2$, $(OB)^2 = (75)^2 + (81)^2$

$(AB)^2 = (75-51)^2 + (81-65)^2$

and from triangle OAB

$(AB)^2 = (OA)^2 + (OB)^2 - 2OA \times OB \cos \angle AOB$.

So $2OA \times OB \cos \angle AOB = 2(75 \times 51 + 81 \times 65)$

$\Rightarrow$ $OA \times OB \cos \angle AOB = 3825 + 5265 = 9090.$

73. *Solution* Since maximum value of $\cos^{-1} x$ in $[0, 1] = \pi/2$. $\sum_{r=1}^{k} \cos^{-1}\beta_r = k\pi/2$ is possible if and only if each $\cos^{-1}\beta_r = \pi/2 \Leftrightarrow \beta_r = 0$.

$$A = \sum_{r=1}^{k} (\beta_r)^r = 0$$

Thus $\lim_{x\to 0} \frac{(1+x^2)^{1/3} - (1-2x)^{1/4}}{x+x^2}$

$$= \lim_{x\to 0} \frac{(1+x^2/3+0(x^4)) - (1-x/2+0(x^2))}{x(1+x)}$$

$$= \lim_{x\to 0} \frac{1/2 + x/3 + 0(x^2)}{1+x} = 1/2.$$ Thus the reqd. limit is 249.

74. *Solution* Let $I_n = \int_0^1 e^{2x}(x-1)^n\,dx$. Put $x - 1 = t$, so that $dx = dt$.

$$I_n = \int_{-1}^{0} e^{2t+1}\,t^n\,dt = e\int_{-1}^{0} t^n e^{2t}\,dt$$

$$= e\left(\frac{t^n e^{2t}}{2}\Big|_{-1}^{0} - \frac{n}{2}\int_{-1}^{0} t^{n-1}e^{2t}\,dt\right)$$

$$= e\left(0 - (-1)^n e^{-2} - \frac{n}{2}\int_{-1}^{0} t^{n-1}e^{2t}\,dt\right)$$

$$= (-1)^{n+1}e^{-1} - \frac{ne}{2}\int_{-1}^{0} t^{n-1}e^{2t}\,dt$$

$$= (-1)^{n+1}e^{-1} - \frac{n}{2}I_{n-1}$$

For $n = 1$, $I_1 = e^{-1} - \frac{1}{2}I_0 = e^{-1} - \frac{1}{4}(e^2 - 1)$

For $n = 2$, $I_2 = -e^{-1} - \frac{1}{2}I_1 = \frac{-3}{2}e^{-1} + \frac{1}{8}(e^2 - 1)$

and for $n = 3$, $I_3 = e^{-1} - \frac{1}{2}I_2 = \frac{7}{4}e^{-1} - \frac{1}{16}(e^2 - 1)$

75. *Solution* $\mathbf{d}\cdot\mathbf{c} = \lambda(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c} + \mu(\mathbf{b}\times\mathbf{c})\cdot\mathbf{c} + \nu(\mathbf{c}\times\mathbf{a})\cdot\mathbf{c}$

$= \lambda[\mathbf{a}\ \mathbf{b}\ \mathbf{c}] + 0 + 0 = \lambda\,[\mathbf{a}\ \mathbf{b}\ \mathbf{c}] = \lambda/8$

Hence $\lambda = 8(\mathbf{d}\cdot\mathbf{c})$. Similarly, $\mu = 8\,(\mathbf{d}\cdot\mathbf{a})$ and $\nu = 8(\mathbf{d}\cdot\mathbf{b})$

$\therefore\ \lambda + \mu + \nu = 8\mathbf{d}\cdot\mathbf{c} + 8\mathbf{d}\cdot\mathbf{a} + 8\mathbf{d}\cdot\mathbf{b}$

$= 8\mathbf{d}\cdot(\mathbf{a} + \mathbf{b} + \mathbf{c}) = 64.$