# CUET (UG)

## Mathematics Sample Paper - 04

## Solved

**Time Allowed: 50 minutes** **Maximum Marks: 200**

**General Instructions:**

1. There are 50 questions in this paper.
2. Section A has 15 questions. Attempt all of them.
3. Attempt any 25 questions out of 35 from section B.
4. Marking Scheme of the test:

a. Correct answer or the most appropriate answer: Five marks (+5).
b. Any incorrectly marked option will be given minus one mark (-1).
c. Unanswered/Marked for Review will be given zero mark (0).

### Section A

1. If A = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then $A^2$ is equal to **[5]**

a) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

2. If a diagonal matrix is commutative with every matrix of the same order then it is necessarily **[5]**

a) A diagonal matrix with atleast two different diagonal elements

b) A unit matrix

c) A diagonal matrix with exactly two different diagonal elements

d) A scalar matrix

3. Total number of possible matrices of order $3 \times 3$ with each entry 2 or 0 is **[5]**

a) 27

b) 81

c) 9

d) 512

4. The equation of tangent at those points where the curve $y = x^2 - 3x + 2$ meets x-axis are **[5]**

a) $x + y + 2 = 0 = x - y - 1$

b) $x - y = 0 = 0\ x + y$

c) $x - y - 1 = 0 = x - y$

d) $x + y - 1 = 0 = x - y - 2$

5. $f(x) = \operatorname{cosec} x$ in $(-\pi, 0)$ has a maxima at **[5]**

a) $x = \frac{-\pi}{2}$

b) $x = 0$

c) $x = \frac{-\pi}{3}$

d) $x = \frac{-\pi}{4}$

6. The tangent to the curve given by $x = e^t \times \cos t$, $y = e^t \times \sin t$ at $t = \frac{\pi}{4}$ makes with x-axis an angle: **[5]**

a) 0

b) $\frac{\pi}{4}$

c) $\frac{\pi}{3}$

d) $\frac{\pi}{2}$

7. The primitive of the function $f(x) = \left(1 - \frac{1}{x^2}\right)a^{x + \frac{1}{x}}, a > 0$ is **[5]**

a) $\frac{a^{x+\frac{1}{x}}}{x}\log_e a$

b) $\log_e a \cdot a^{x+\frac{1}{x}}$

c) $\frac{a^{x+\frac{1}{x}}}{\log_e a}$

d) $\frac{a^{\frac{1}{x}}}{x \log_e a}$

8. $\int \sec^2(7 - 4x)\, dx = ?$ **[5]**

a) $-4 \tan (7 - 4x) + C$

b) $\frac{-1}{4}\tan(7 - 4x) + C$

c) $4 \tan (7 - 4x) + C$

d) $\frac{1}{4}\tan(7 - 4x) + C$

9. $\int x \sin^3 x^2 \cos x^2\, dx = ?$ **[5]**

a) $\frac{1}{8} \sin^4 x^2 + C$

b) $\frac{1}{4} \sin^4 x^2 + C$

c) None of these

d) $\frac{1}{2} \sin^4 x^2 + C$

10. The area bounded by $y = 2 - x^2$ and $x + y = 0$ is **[5]**

a) $\frac{9}{2}$ sq. units

b) $\frac{7}{2}$ sq. units

c) None of these

d) 9 sq. units

11. Which of the following is the general solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$? **[5]**

a) y = A cos x + B sin x

b) $y = (Ax + B)e^{-x}$

c) $y = Ae^{x} + Be^{-x}$

d) $y = (Ax + B)e^{x}$

12. The solution of the DE xdy + ydx = 0 is **[5]**

a) log (x + y) = C

b) x + y = C

c) xy = C

d) none of these

13. The corner points of the feasible region for a Linear Programming problem are P(0, 5), Q(1, 5), R(4, 2)
and S(12, 0). The minimum value of the objective function Z = 2x + 5y is at the point. **[5]**

a) Q

b) S

c) R

d) P

14. The probability distribution of a discrete random variable X is given below: **[5]**

| x | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| P(x) | $\frac{5}{k}$ | $\frac{7}{k}$ | $\frac{9}{k}$ | $\frac{11}{k}$ |

a) 32

b) 16

c) 48

d) 8

15. Find the mean number of heads in three tosses of a fair coin. **[5]**

a) 1.2

b) 1.4

c) 1.5

d) 1.0

## Section B

## Attempt any 25 questions

16. Identity relation R on a set A is **[5]**

a) Reflexive only

b) Transitive only

c) Equivalence

d) Symmetric only

17. If $\cot^{-1}\left(\frac{-1}{5}\right) = x$ then sin x = ? **[5]**

a) $\frac{7}{\sqrt{26}}$

b) None of these

c) $\frac{1}{\sqrt{26}}$

d) $\frac{5}{\sqrt{26}}$

18. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = (A^2 + B^2)$ then **[5]**

a) None of these

b) a = -2, b = 3

c) a = 1, b = 4

d) a = 2, b = -3

19. Let $f(x) = \begin{vmatrix} x & -4 & 5 \\ 1 & 1 & -2 \\ 2 & x & 1 \end{vmatrix}$, then f'(5) is equal to **[5]**

a) 40

b) 1

c) 26

d) 24

20. If A is an invertible matrix of order 2, then det $(A^{-1})$ is equal to **[5]**

a) 0

b) $\frac{1}{\det(A)}$

c) det (A)

d) 1

21. If A and B are invertible matrices of order 3, then det(adj A) = **[5]**

a) $|A|^2$

b) None of these

c) $(\det A)^2$

d) 1

22. What is the rate of change of $\sqrt{x^2+16}$ w.r.t. $x^2$ at x = 3? **[5]**

a) $\frac{1}{5}$

b) $\frac{1}{15}$

c) $\frac{1}{10}$

d) $\frac{1}{20}$

23. The function f(x) $= \begin{cases} \frac{\sin x}{x} + \cos x, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ is continuous at x = 0, then the value of k is **[5]**

a) 1

b) 3

c) 1.5

d) 2

24. Derivative of $\sin^3 x$ w.r.t $\cos^3 x$ is **[5]**

a) cot x

b) – tan x

c) $\tan^3 x$ d) tan x

25. If $y = x^{x^{x+\infty}}$ then $\frac{dy}{dx} = ?$ **[5]**

a) None of these

b) $\frac{y}{x(1-\log x)}$

c) $\frac{y^2}{x(1-y\log x)}$

d) $\frac{y^2}{x(1-\log x)}$

26. If a function f is derivable at x = a, then $\underset{h\to 0}{Lt} \frac{f(a-h)-f(a)}{h}$ is equal to **[5]**

a) f ‘(a)

b) None of these

c) does not exist

d) – f ‘(a)

27. f(x) = sin x - kx is decreasing for all x ∈ R, when **[5]**

a) $k \geq 1$

b) k < 1

c) k > 1

d) $k \leq 1$

28. The curve y = $a\,x^3 + bx^2 + c\,x$ is inclined at 45° to the X – axis at (0, 0) but it touches X – axis at (1, 0) , then the values of a, b, c, are given by **[5]**

a) a = 1, b = – 2, c = 1

b) a = 1, b = 1, c = – 2

c) a = – 2, b = 1, c = 1

d) a = – 1, b = 2, c = 1.

29. Tangents to the curve y = $x^3$ at the points (1, 1) and ( – 1, – 1) are **[5]**

a) perpendicular

b) parallel

c) none of these

d) intersecting but not at right angles

30. Let f(x) = x|x|, then f (x) has **[5]**

a) point of inflexion at x = 0

b) none of these

c) local minima at x = 0

d) local maxima at x = 0

31. $\int \frac{(1+\sin x)}{(1+\cos x)} dx = ?$ **[5]**

a) $\tan\frac{x}{2} - 2\log\left|\cos\frac{x}{2}\right| + C$

b) none of these

c) $-\tan\frac{x}{2} + 2\log\left|\cos\frac{x}{2}\right| + C$

d) $\tan\frac{x}{2} + 2\log\left|\cos\frac{x}{2}\right| + C$

32. If $\int \frac{3e^{x}-5e^{-x}}{4e^{x}+5e^{-x}} dx = ax + b \cdot log|4e^{x} + 5e^{-x}| + C$, then **[5]**

a) $a = \frac{-1}{8}, b = \frac{7}{8}$

b) $a = \frac{-1}{8}, b = \frac{-7}{8}$

c) $a = \frac{1}{8}, b = \frac{7}{8}$

d) $a = \frac{1}{8}, b = \frac{-7}{8}$

33. $\int \sin^3 x \cos^3 x dx = ?$ **[5]**

a) $\frac{(\sin x)^4}{4} - \frac{(\sin x)^6}{6} + c$

b) $\frac{1}{4}\cos^4 x + \frac{1}{6}\cos^6 x + C$

c) $-\frac{1}{4}\cos^4 x + \frac{1}{6}\cos^6 x + C$

d) None of these

34. $\int\{(2x+1)\sqrt{x^2+x+1}\}dx = ?$ **[5]**

a) None of these

b) $\frac{2}{3}\left(x^2+x+1\right)^{3/2} + C$

c) $\frac{3}{2}\left(x^2+x+1\right)^{3/2} + C$

d) $\frac{3}{2}(2x+1)^{3/2} + C$

35. The area of the region bounded by the curve y = x + 1 and the lines x = 2 and x = 3 is **[5]**

a) $\frac{11}{2}$ sq units

b) $\frac{13}{2}$ sq units

c) $\frac{9}{2}$ sq units

d) $\frac{7}{2}$ sq units

36. General solution of $\frac{dy}{dx} + (\sec x)y = \tan x \left(0 \leqslant x < \frac{\pi}{2}\right)$ is **[5]**

a) $y(\sec x + \tan x) = \sec x + \tan x - x + C$

b) $y(\sec x - \tan x) = \sec x + \tan x - x + C$

c) $y(\sec x + \tan x) = \sec x - \tan x - x + C$

d) $y(\sec x - \tan x) = \sec x - \tan x - x + C$

37. For the differential equation $xy\frac{dy}{dx} = (x + 2)(y + 2)$ find the solution curve passing through the point (1, –1). **[5]**

a) $y + x + 2 = log\left(x^2(y + 2)^2\right)$

b) $y - x - 2 = log\left(x^2(y - 2)^2\right)$

c) $y - x + 2 = log\left(x^2(y + 2)^2\right)$

d) $y - x - 2 = log\left(x^2(y + 2)^2\right)$

38. What is the general solution of the differential equation $e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0$? **[5]**

a) $\cos y = C(1 - e^x)$

b) $\sin y = C(1 - e^x)$

c) $\cot y = C(1 - e^x)$

d) None of these

39. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that $\vec{C}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to **[5]**

a) $\frac{1}{4}|\vec{a}|^2|\vec{b}|^2$

b) $\frac{3}{4}|\vec{a}|^2|\vec{b}|^2$

c) 0

d) 1

40. Which one of the following vectors is normal to the vector $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$? **[5]**

a) None of these

b) $\hat{\mathrm{i}} - \hat{\mathrm{j}} - \hat{\mathrm{k}}$

c) $\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$

d) $\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$

41. If $\vec{a}, \vec{b}$ represent the diagonals of a rhombus, then **[5]**

a) $\vec{a} \times \vec{b} = \vec{0}$

b) $\vec{a} + \vec{b} = 1$

c) $\vec{a} \times \vec{b} = \vec{a}$

d) $\vec{a} \cdot \vec{b} = 0$

42. If $|\vec{a}| = 3$ and $-1 \leq k \leq 2$, then $|k\vec{a}|$ lies in the interval. **[5]**

a) [- 3, 6]

b) [3, 6]

c) [0, 6]

d) [1, 2]

43. If $\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors then $(3\vec{a} + 2\vec{b}) \cdot (5\vec{a} - 6\vec{b}) = ?$ **[5]**

a) 6

b) 12

c) 5

d) 3

44. The distance d from a point $P(x_1, y_1, z_1)$ to the plane $Ax + By + Cz + D = 0$ is **[5]**

a) $d = \left| \frac{Ax_1 + By_1 + Cz_1 + D}{\sqrt{A^2 + B^2 + C^2}} \right|$

b) $d = \left| \frac{Ax_1 + By_1 + 2Cz_1 + D}{\sqrt{A^2 + B^2 + C^2}} \right|$

c) $d = \left| \frac{Ax_1 + 2By_1 + Cz_1 + D}{\sqrt{A^2 + B^2 + C^2}} \right|$

d) $d = \left| \frac{Ax_1 + By_1 + Cz_1 + 2D}{\sqrt{A^2 + B^2 + C^2}} \right|$

45. Find the distance of the point (0, 0, 0) from the plane 3x – 4y + 12 z = 3 **[5]**

a) $\frac{3}{13}$

b) $\frac{7}{13}$

c) $\frac{9}{13}$

d) $\frac{5}{13}$

46. Find the equation of the plane with intercept 3 on the y – axis and parallel to ZOX plane. **[5]**

a) y = 3

b) y = 5

c) y = 4

d) y = 2

47. A bag contains 3 white, 4 black and 2 red balls. If 2 balls are drawn at random (without replacement), then the probability that both the balls are white is: **[5]**

a) $\frac{1}{18}$

b) $\frac{1}{12}$

c) $\frac{1}{24}$

d) $\frac{1}{36}$

48. A machine operates only when all of its three components function. The probabilities of the failures of the first, second and third component are 0.2, 0.3 and 0.5 respectively. What is the probability that the machine will fail? **[5]**

a) None of these

b) 0.07

c) 0.72

d) 0.70

49. Five cards are drawn successively with replacement from a well – shuffled deck of 52 cards. What is the probability that only 3 cards are spades? **[5]**

a) $\frac{77}{512}$

b) $\frac{45}{512}$

c) $\frac{57}{512}$

d) $\frac{41}{512}$

50. A problem in Statistics is given to three students A, B and C whose chances of solving it independently are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, respectively. The probability that the problem will be solved, is **[5]**

a) $\frac{3}{4}$

b) $\frac{1}{2}$

c) $\frac{11}{12}$

d) $\frac{1}{12}$

# Solutions

## Section A

1.

**(d)** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

**Explanation:** $A^2 = A . A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

2.

**(d)** A scalar matrix

**Explanation:** By definition of scalar matrix, it should be a scalar matrix.

3.

**(d)** 512

**Explanation:** Since each element $a_{ij}$ can be filled in two ways (with either '2' or "0'), total number of possible matrices is 8x8x8 = 512

4.

**(d)** x + y - 1 = 0 = x - y - 2

**Explanation:** $y = x^2 - 3x + 2$

Slope of tangent

$\frac{dy}{dx} = 2x - 3$

Tangent meets at x-axis hence y = 0

$x^2 - 3x + 2 = 0$

(x - 2)(x - 1) = 0

x = 2 or x = 1

For x = 2 $\Rightarrow m = \left(\frac{dy}{dx}\right)_{12,0} = 1$

Equation of tangent m = -1, point (1, 0)

y - 0 = x - 2

$\Rightarrow$ x - y - 2 = 0

For x = 1 $\Rightarrow m = \left(\frac{dy}{dx}\right)_{(1,0)} = -1$

Equation of tangent m = -1, point (1, 0)

y - 0 = -1(x - 1)

$\Rightarrow$ x + y - 1 = 0

5. **(a)** $x = \frac{-\pi}{2}$

**Explanation:** We can go through options for this question

Option a is wrong because 0 is not included in $(-\pi, 0)$

At $x = \frac{-\pi}{4}$ value of f(x) is $-\sqrt{2}$ = -1.41

At $x = \frac{-\pi}{3}$ value of f(x) is -2.

At $x = \frac{-\pi}{2}$ value of f(x) = -1.

$\therefore$ f(x) has max value at $x = \frac{-\pi}{2}$

Which is -1.This is the required solution.

6.

**(d)** $\frac{\pi}{2}$

**Explanation:** $\frac{dx}{dt} = -e^t \cdot \sin t + e^t \cos t,$

$\frac{dy}{dt} = e^t \cos t + e^t \sin t$

Therefore, $\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}} = \frac{\cos t + \sin t}{\cos t - \sin t} = \frac{\sqrt{2}}{0}$

and hence this option is correct

7.

**(c)** $\frac{a^{x+\frac{1}{x}}}{\log_e a}$

**Explanation:** $f(x) = \left(1 - \frac{1}{x^2}\right)a^{x+\frac{1}{x}}$

$\Rightarrow \int f(x)dx = \int \left(1 - \frac{1}{x^2}\right)a^{x+\frac{1}{x}}dx$

Put $x + \frac{1}{x} = t$

$\Rightarrow \left(1 - \frac{1}{x^2}\right)$dx = dt

$I = \int a^t dt$

$I = \frac{a^t}{\log_e a} + c$

$I = \frac{a^{x+\frac{1}{x}}}{\log_e a} + c$

8.

**(b)** $\frac{-1}{4}\tan(7-4x) + C$

**Explanation:** Given integral is ∫ $\sec^2$(7-4 x) dx=?

Let, 7 – 4x = z

⇒ -4dx = dz

So,

∫ $\sec^2$(7-4 x) dx=?

$= \int \sec^2 z \frac{dz}{-4}$

$= -\frac{1}{4}\int \sec^2 z dz$

$\int \sec^2(7-4x)dx$ where c is the integrating constant.

$= \int \sec^2 z \frac{dz}{-4}$

$= -\frac{1}{4}\int \sec^2 z dz$

$= -\frac{1}{4}\tan z + c$

$= -\frac{1}{4}\tan(7-4\text{x}) + \text{c}$

9. **(a)** $\frac{1}{8}$ sin $^4$ $x^2$+C

**Explanation: Formula :-** $\int x^n dx = \frac{x^{n+1}}{n+1} + c; \int e^x dx = e^x + c$

Therefore ,

Put sin $x^2$ = t

⇒ 2 x cos $x^2$ dx = dt

$= \frac{1}{2}\int t^3 dt$

$= \frac{1}{2}\frac{t^4}{4} + c \Rightarrow \frac{t^4}{8} + c$

$= \frac{\left(\sin x^2\right)^4}{8} + c$

10. **(a)** $\frac{9}{2}$ sq. units

**Explanation:** The area bounded by $y = 2 - x^2$ and $x + y = 0 \Rightarrow y = -x$

$2 - x^2 = -x$

$x^2 - x - 2 = 0$

$\Rightarrow (x - 2)(x + 1) = 0$

$\Rightarrow x = 2$ or $x = -1$

$\int_{-1}^{2}\left(2 - x^2 - x\right)dx$

$\left[2x - \frac{x^3}{3} + \frac{x^2}{2}\right]_1^2$

$2(2 + 1) - \left(\frac{8}{3} + \frac{1}{3}\right) + \left(2 - \frac{1}{2}\right)$

$6 - 3 + \frac{3}{2}$

$\frac{9}{2}$ sq. units

11.

**(d)** $y = (Ax + B)e^x$

**Explanation:** For $y = (Ax + B)e^x$

$\frac{dy}{dx} = (Ax + B)e^x + Ae^x = (Ax + A + B)e^x$

$\Rightarrow \frac{d^2y}{dx^2} = (Ax + A + B)e^x + Ae^x = (Ax + 2A + B)e^x$

$\therefore \frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx}\right) + y$

$= (Ax + 2A + B)e^x - 2(Ax + A + B)e^x + (Ax + B)e^x$

$= 0$

12.

**(c)** $xy = C$

**Explanation:** Given, $xdy + ydx = 0$

$xdy = -ydx$

$-\frac{dy}{y} = \frac{dx}{x}$

On integrating on both sides ,we obtain

-log y = log x +log c

log x + log y =log c

log xy =log c

xy = C

13.

**(c)** R

**Explanation:**

| Corner points | Value of Z = 2x + 5y |
|---|---|
| P(0, 5) | Z = 2(0) + 5(5) = 25 |
| Q(1, 5) | Z = 2(1) + 5(5) = 27 |
| R(4, 2) | Z = 2(4) + 5(2) = 18 $\rightarrow$ Minimum |
| S(12, 0) | Z = 2(12) + 5(0) = 24 |

Thus, minimum value of Z occurs ar R(4, 2)

14. **(a)** 32

**Explanation:** We know that, $\sum P(X) = 1$

$\Rightarrow \frac{5}{k} + \frac{7}{k} + \frac{9}{k} + \frac{11}{k} = 1$

$\Rightarrow \frac{32}{k} = 1$

$\therefore k = 32$

15.

**(c)** 1.5

**Explanation:** Let X is the random variable of "number of heads" X = 0, 1, 2, 3.

P(X = 0) = P($\bar{H}\bar{H}\bar{H}$)= $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

P(X = 1) = P( $H\bar{H}\bar{H}$ or $\bar{H}H\bar{H}$ or $\bar{H}\bar{H}H$) = $3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$

P(X = 2) = P( $HH\bar{H}$ or $H\bar{H}H$ or $\bar{H}HH$) = $3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$

P(X = 3) = P(HHH) = $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

Therefore, the probability distribution is:

| **X** | **0** | **1** | **2** | **3** |
|---|---|---|---|---|
| **P(X)** | $\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{3}{8}$ | $\frac{1}{8}$ |

Therefore, Mean of Heads is :

E(X) = $\sum_{i=1}^{n} X_i P(X_i) = 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8} = \frac{12}{8} = 1.5$

## Section B

16.

**(c)** Equivalence

**Explanation:** Identity relation R in a set A is defined as

R = {(a, a) : a $\in$ A}

**Important Note:** A Identity relation is **Reflexive**, **Symmetric** and **Transitive**.

Thus Identity relation is always an **Equivalence Relation**.

17.

**(d)** $\frac{5}{\sqrt{26}}$

**Explanation:** Given: $\cot^{-1}(\frac{-1}{5}) = x$

$$\Rightarrow \cot x = \frac{-1}{5} = \frac{\text{adjacent side}}{\text{opposite side}}$$

By pythagorus theroem,

$(\text{Hypotenuse})^2 = (\text{opposite side})^2 + (\text{adjacent side})^2$

Therefore, Hypotenuse = $\sqrt{26}$

$$\Rightarrow \sin x = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{5}{\sqrt{26}}$$

18.

**(c)** a = 1, b = 4

**Explanation:** $A = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix} B = \begin{pmatrix} a & 1 \\ b & -1 \end{pmatrix}$

$$A + B = \begin{pmatrix} 1+a & 0 \\ 2+b & -2 \end{pmatrix}$$

$$(A+B)^2 = \begin{pmatrix} 1+a & 0 \\ 2+b & -2 \end{pmatrix}\begin{pmatrix} 1+a & 0 \\ 2+b & -2 \end{pmatrix}$$

$$= \begin{pmatrix} (1+a)^2 & 0 \\ (2+b)(1+a) - 4 - 2b & -4 \end{pmatrix}$$

$$= \begin{pmatrix} (1+a)^2 & 0 \\ 2 + 2a + b + ab - 4 - 2b & 4 \end{pmatrix}$$

$$= \begin{pmatrix} (1+a)^2 & 0 \\ 2a + ab - b - 2 & 4 \end{pmatrix}$$

$A^2 = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}$

$= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$B^2 = \begin{pmatrix} \mathbf{a} & \mathbf{1} \\ \mathbf{b} & \mathbf{-1} \end{pmatrix}\begin{pmatrix} \mathbf{a} & \mathbf{1} \\ \mathbf{b} & \mathbf{-1} \end{pmatrix}$

$= \begin{pmatrix} a^2 + b & a - 1 \\ ab - b & b + 1 \end{pmatrix}$

Given that; $(A + B)^2 = (A^2 + B^2)$

$\Rightarrow \begin{pmatrix} (1 + a)^2 & 0 \\ 2a + ab - b - 2 & 4 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} a^2 + b & a - 1 \\ ab - b & b + 1 \end{pmatrix}$

$= \begin{pmatrix} -1 + a^2 + b & a - 1 \\ ab - b & b \end{pmatrix}$

By comparison

a - 1 = 0

a = 1

b = 4

19.

**(c)** 26

**Explanation:** $f(x) = \begin{vmatrix} x & -4 & 5 \\ 1 & 1 & -2 \\ 2 & x & 1 \end{vmatrix}$ = x (1 + 2x) - 1 (- 4 - 5x) + 2 (8 - 5)

$= x + 2x^2 + 4 + 5x + 6 = 2x^2 + 6x + 10$

f'(x) = 4x + 6

f'(5) = 20 + 6 = 26

20.

**(b)** $\frac{1}{\det(A)}$

**Explanation:** We know that, $A^{-1} = \frac{1}{|A|}$ Adj (A)

So, $\left|A^{-1}\right| = \left|\frac{1}{|A|}\text{Adj}(A)\right|$

$= \frac{1}{|A|^n}|\text{Adj}(A)|$

$= \frac{1}{|A|^n} |A|^{n-1} = \frac{1}{|A|^1}$

$= \frac{1}{|A|^1}$

{since adj(A) is of order n and $|Adj(A)| = |A|^{n-1}$}

21. **(a)** $|A|^2$

**Explanation:** Let A be a non singular square matrix of order n then, $\det(adjA) = |A|^{n-1}$

Here order is 3 so $\det(adj\ A) = |A|^{3-1} = |A|^2$

22.

**(c)** $\frac{1}{10}$

**Explanation:** Let $u = \sqrt{x^2 + 16}$ and $v = x^2$

Now, $\frac{du}{dx} = \frac{1}{2\sqrt{x^2+16}} \times 2x = \frac{x}{\sqrt{x^2+16}}$ and $\frac{dv}{dx} = 2x$

Now, rate of change of u w.r.t. v is

$$\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{x}{\sqrt{x^2+16}} \times \frac{1}{2x}$$

$$\frac{du}{dv} = \frac{1}{2\sqrt{x^2+16}}$$

$$\Rightarrow \frac{du}{dv} \text{ at}(x = 3) = \frac{1}{2\sqrt{9+16}}$$

$$= \frac{1}{2\sqrt{25}}$$

$$= \frac{1}{2 \times 5} = \frac{1}{10}$$

$$\therefore \frac{d\sqrt{\left(x^2+16\right)}}{d\left(x^2\right)} = \frac{1}{10} \text{ at } x = 3$$

23.

**(d)** 2

**Explanation:** Since the given function is continuous,

$$\therefore k = \lim_{x \to 0} \frac{Sinx}{x} + Cosx$$

$$\Rightarrow k = 1 + 1 = 2$$

24.

**(b)** – tan x

**Explanation:** Let y = $\sin^3$x and z = $\cos^3$x, then , $\frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{3\sin^2 x\cos x}{3\cos^2 x(-\sin x)} = -\tan x.$

Which is the required solution.

25.

**(c)** $\frac{y^2}{x(1-y\log x)}$

**Explanation:** Given:

$y = x^{x^{x+\infty}}$

We can write it as

$\Rightarrow$ y = $x^y$

Taking log of both sides we obtain

log y = y log x

Differentiating with respect to x,we get

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}\log x + y \cdot \frac{1}{x}$

$\Rightarrow \left(\frac{1}{y} - \log x\right)\frac{dy}{dx} = \frac{y}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left(\frac{y}{1-y\log x}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{y^2}{x(1-y\log x)}.$

Which is the required solution.

26.

**(d)** – f ‘(a)

**Explanation:** $\lim_{h\to 0}\frac{f(a-h)-f(a)}{h} = \lim_{t\to 0}\frac{f(a+t)-f(a)}{-t}(put..h = -t, as..h \to 0, t \to 0)$

$\Rightarrow -\lim_{t\to 0}\frac{f(a+t)-f(a)}{t} = -f'(a)$

27. **(a)** $k \geq 1$

**Explanation:** Given,f(x) = sin x - kx

f'(x) = cos x - k

$\therefore$ f decreases, if f'$(x) \leq 0$

$\Rightarrow \cos x - k \leq 0$

$\Rightarrow \cos x \leq k$

Therefore, for decreasing $k \geq 1$

28. **(a)** a = 1, b = – 2, c = 1

**Explanation:** $y = ax^3 + bx^2 + cx$

$\Rightarrow \frac{dy}{dx} = 3ax^2 + 2bx + c.$

At ( 0 , 0) , slope of tangent =tan45 ° = 1. ⇒ c = 1. At ( 1 ,0 ) , slope of tangent = 0. ⇒ 3a+2b+c=0.
From this,we get ,3a+2b=-1........(1)
Also, when x = 1 , y = 0 , therefore , a + b + c = 0 . From this,we get, a+b=-1.......(2)
From(1) and (2),we get,
a=1 ,b= -2 and c =1

29.

**(b)** parallel

**Explanation:** y = $x^3$

$\frac{dy}{dx} = \frac{d}{dx}(x^3) = 3x^2$

So, $\frac{dy}{dx}$ $at$ $(1, 1) = 3$ and $\frac{dy}{dx}$ $at\ (-1, -1) = 3$

Since the slopes are equal, the tangents are parallel.

30. **(a)** point of inflexion at x = 0

**Explanation:** Given, f(x) = x|x| $= \begin{cases} -x^2, & x < 0 \\ x^2, & x > 0 \end{cases}$

$\Rightarrow f'(x) = -2x$ when x < 0 and 2x when x > 0 $f'(x) = 0 \Rightarrow x = 0$

Hence f(x) has a point of inflexion at x = 0.
But, x = 0 is not a local extreme as we cannot find an interval I around x = 0 such that
$f(0) \geq f(x)\ or\ f(0) \leq f(x) \quad \forall x \in I$

31. **(a)** $\tan\frac{x}{2} - 2\log\left|\cos\frac{x}{2}\right| + C$

**Explanation: Formula :-** $\int x^n dx = \frac{x^{n+1}}{n+1} + c; \int \sec^2 x dx = \tan x$

Therefore ,

$\Rightarrow \int \frac{1+\sin x}{2\cos^2\frac{x}{2}} dx$

$= \int \frac{1}{2\cos^2\frac{x}{2}} + \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}} dx = \frac{1}{2}\int \sec^2\frac{x}{2} dx + \int \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} dx$

$= \frac{1}{2}\tan\frac{x}{2} \times 2 + \int \tan\frac{x}{2} dx$

$= \tan\frac{x}{2} + 2\left(-\log\cos\frac{x}{2}\right) + c$

$= \tan\frac{x}{2} - 2\log\left|\cos\frac{x}{2}\right| + c$

32. **(a)** a = $\frac{-1}{8}$, b = $\frac{7}{8}$

**Explanation:** On differentiating both sides, we have

$$\frac{3e^{x}-5e^{-x}}{4e^{x}+5e^{-x}} = a + b\frac{\left(4e^{x}-5e^{-x}\right)}{4e^{x}+5e^{-x}}$$

giving $3e^{x} - 5e^{-x} = a\,(4e^{x} + 5e^{-x}) + b\,(4e^{x} - 5e^{-x})$

$=(4a + 4b)e^{x} + (5a - 5b)e^{-x}$

Comparing coefficients on both sides, we obtain

3 = 4a + 4b

and -5 = 5a - 5b

This verifies a = $\frac{-1}{8}$, b = $\frac{7}{8}$.

Which is the required solution.

33. **(a)** $\frac{(\sin x)^{4}}{4} - \frac{(\sin x)^{6}}{6} + c$

**Explanation: Formula :-** $\int x^{n}dx = \frac{x^{n+1}}{n+1} + c; \int \frac{1}{1+x^{2}}dx = \tan^{-1}x + c$

Therefore ,

$\Rightarrow \int \cos x(\cos^{2} x \sin^{3} x)dx$

$= \int \cos x((1-\sin^{2} x) \sin^{3} x)\,dx$

$= \int\cos x(\sin^{3} x-\sin^{5} x)dx$

$= \int \sin^{3} x \cos xdx$

$=\int \sin^{5} x \cos xdx$

Put sin x = t

$\Rightarrow \cos xdx = dt$

$= \int t^{3}\,dt-\int t^{5}\,dt$

$=\frac{t^{4}}{4} - \frac{t^{6}}{6} + c$

$= \frac{(\sin x)^{4}}{4} - \frac{(\sin x)^{6}}{6} + c$

34.

**(b)** $\frac{2}{3}\left(x^{2} + x + 1\right)^{3/2} + C$

**Explanation: Formula :-** $\int x^n dx = \frac{x^{n+1}}{n+1} + c; \int \frac{1}{x\sqrt{x^2-1}}dx = \sec^{-1}x + c$

Therefore ,

Put $x^2 + x + 1 = t$, $(2x + 1)\,dx = dt$

$\Rightarrow \int \sqrt{t}dt = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$

$\Rightarrow \frac{2}{3}t^{\frac{3}{2}} + c$

$\Rightarrow \frac{2}{3}\left(x^2 + x + 1\right)^{\frac{3}{2}} + c$

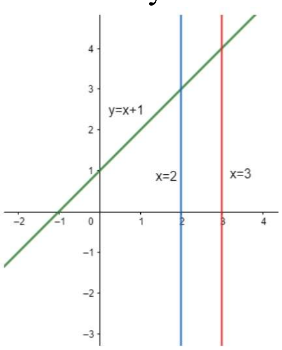
35.

**(d)** $\frac{7}{2}$ sq units

**Explanation:**

Given;

The curve y = x + 1 and the lines x = 2 and x = 3




Required area = $\int_2^3 (x + 1)dx$

$= \left[\frac{x^2}{2} + x\right]_2^3$

$= \left(\frac{9}{2} + 3 - 2 - 2\right)$

$= \frac{7}{2}$ sq. units

36. **(a)** $y(\sec x + \tan x) = \sec x + \tan x - x + C$

**Explanation:** $\frac{dy}{dx} + \sec x . y = \tan x \Rightarrow P = \sec x, Q = \tan x$

$\Rightarrow I.F = e^{\int \sec x dx} = e^{\log|\sec x + \tan x|} = \log|\sec x + \tan x|$

$\Rightarrow y.\log|\sec x + \tan x| = \int \tan x . \log|\sec x + \tan x|dx$

$\Rightarrow y.\log|\sec x+\tan x| = \sec x+\tan x - x + C$

37.

**(c)** $y - x + 2 = log\left(x^2(y+2)^2\right)$

**Explanation:** $\frac{ydy}{y+2} = \frac{(x+2)dx}{x}$

$\int\frac{ydy}{y+2} = \int\frac{(x+2)dx}{x}$

$\int\frac{y+2-2dy}{y+2} = \int\frac{(x+2)dx}{x}$

$\int dy - \int\frac{2}{y+2} = \int dx + \int\frac{2}{x}$

$y - 2log|y+2| = x + 2log|x| + c$

Here x=1 and y=-1 implies

$-1 - 2log|-1+2| = 1 + 2log|1| + c \Rightarrow -1 - 2log|1| = 1 + c \because log|1| = 0 \Rightarrow \therefore c = -2$

Hence,

$y - 2log|y+2| = x + 2log|x| - 2$

$y - x + 2 = 2log|x| + 2log|y+2|$

$y - x + 2 = 2log|x(y+2)|$

$y - x + 2 = log|x^2(y+2)^2|$

38.

**(d)** None of these

**Explanation:** Given,

$e^x \tan y\, dx + (1 - e^x)\sec^2 y\, dy = 0$

$\Rightarrow \frac{e^x}{1-e^x}\cdot dx + \frac{\sec^2 y}{\tan y}\cdot dy = 0$

On integrating, we get

$\int\frac{e^x dx}{1-e^x} + \int\frac{\sec^2 y}{\tan y}dy = 0$

$\Rightarrow -\log(1 - e^x) + \log\tan y = \log C$

$\Rightarrow \log\tan y = \log C + \log(1 - e^x)$

$= \log C(1 - e^x)$

$\therefore \tan y = C(1 - e^x)$

39. **(a)** $\frac{1}{4}|\vec{a}|^2|\vec{b}|^2$

**Explanation:** $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$

$= ((\vec{a} \times \vec{b}) \cdot \vec{c})^2$ ( $\vec{c}$ is perpendicular to both $\vec{a}$ and $\vec{b}$ i.e $\vec{c}$ is parallel to $\vec{a} \times \vec{b}$ )

$= ((\vec{a} \times \vec{b})]^2$ ( $\because$ All are unit vectors and cos 0 = 1)

$= \left( |\vec{a}| \vec{b} | \sin\frac{\pi}{6} \right)^2$

$= \frac{|\vec{a}|^2 |\vec{b}|^2}{4}$

40. **(a)** None of these

**Explanation:** Let $\mathbf{\vec{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

Let any vector normal to $\vec{a}$, then the dot product of both vectors should be zero.

i. $(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) = 1 + 1 - 1 = 1 \neq 0$

ii. $(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 1 - 1 + 1 = 1 \neq 0$

iii. $(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}) = 1 - 1 - 1 = -1 \neq 0$

41.

**(d)** $\vec{a} \cdot \vec{b} = 0$

**Explanation:** Diagonals of a rhombus are perpendicular to each other

$\Rightarrow \vec{a} \cdot \vec{b} = 0$

42.

**(c)** [0, 6]

**Explanation:** [0, 6] is the correct answer. The smallest value of $|k\vec{a}|$ will exist at numerically smallest value of k, i.e., at k = 0, which gives $|k\vec{a}| = |k||\vec{a}| = 0 \times 3 = 0$ The numerically greatest value of k is 2 at which $|k\vec{a}| = 6$.

43.

**(d)** 3

**Explanation:** 3

Given $|\vec{a}|^2 = |\vec{b}|^2 = 1$ and $\vec{a} \cdot \vec{b} = 0$

$(3\vec{a} + 2\vec{b}) \cdot (5\vec{a} - 6\vec{b}) = 15|\vec{a}|^2 - 12|\vec{b}|^2 - 8\vec{a}.\vec{b}$

$= (15 \times 1) - (12 \times 1) - (8 \times 0)$

$= (15 - 12 - 0) = 3$

44. **(a)** $d = \left| \frac{Ax_1 + By_1 + Cz_1 + D}{\sqrt{A^2 + B^2 + C^2}} \right|$

**Explanation:** The distance d from a point $P(x_1, y_1, z_1)$ to the plane Ax + By + Cz + D = 0 is given by : $d = \left| \frac{Ax_1 + By_1 + Cz_1 + D}{\sqrt{A^2 + B^2 + C^2}} \right|$.

45. **(a)** $\frac{3}{13}$

**Explanation:** As we know that the length of the perpendicular from point $P(x_1, y_1, z_1)$ from the plane $a_1 x + b_1 y + c_1 z + d_1 = 0$ is given by:

$$\frac{\left|a_1x+b_1y+c_1z+d_1\right|}{\sqrt{a^2{}_1+b^2{}_1+c^2{}_1}}.$$

Here, $P(0, 0, 0)$ is the point and equation of plane is 3x – 4y + 12 z – 3.

Therefore, the perpendicular distance is: $\frac{|0-0+0-3|}{\sqrt{9+16+144}} = \frac{|-3|}{\sqrt{169}} = \frac{3}{13}$ units.

46. **(a)** y = 3

**Explanation:** The required equation of plane is y = 3.

47.

**(b)** $\frac{1}{12}$

**Explanation:** Total balls = 3 + 4 + 2 = 9

n(S) = 9 $C_2$

n(E) = 3$C_2$

$$P(E) = \frac{n(E)}{n(S)} = \frac{3C_2}{9C_2} = \frac{\frac{3\times 2}{2\times 1}}{\frac{9\times 8}{2\times 1}}$$

$$= \frac{3\times 2}{9\times 8} = \frac{1}{3\times 4}$$

$$= \frac{1}{12}$$

48.

**(c)** 0.72

**Explanation:** The probability of failure of the first component = 0.2 = P(A)

The probability of failure of second component = 0.3 = P(B)

The probability of failure of third component = 0.5 = P(C)

As the events are independent,

The machine will operate only when all the components work, i.e.,

(1 - 0.2)(1 - 0.3)(1 - 0.5) = P(A’)P(B’)P(C’)

In rest of the cases, it won’t work,

So P(A ∪ B ∪ C) = 1 - P(A’ ∩ B’ ∩ C’) = 1 - (0.8).(0.7).(0.5)

⇒ 1 - 0.28 = 0.72

49.

**(b)** $\frac{45}{512}$

**Explanation:** Here, probability of getting a spade from a deck of 52 cards = $\frac{13}{52} = \frac{1}{4}$. p = $\frac{1}{4}$, q = $\frac{3}{4}$.

let, x is the number of spades, then x has the binomial distribution with n = 5, $p = \frac{1}{4}$, $q = \frac{3}{4}$ .

P(only 3 cards are spades) = P(x = 3) = $^5C_3\left(\frac{3}{4}\right)^{5-3}\left(\frac{1}{4}\right)^3 = \frac{45}{512}$

50. **(a)** $\frac{3}{4}$

**Explanation:** P (problem will be solved)

= 1 - P (problem will not solved by A, B and C)

$= 1 - \left\{\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\right\}$

$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = 1 - \frac{1}{4} = \frac{3}{4}$