MATHEMATICS

(Maximum Marks: 80)

(Time allowed: Three hours)

(*Candidates are allowed additional 15 minutes for* ***only*** *reading the paper***.**

They must NOT start writing during this time.)

The Question Paper consists of three sections A, B and C.

Candidates are required to attempt all questions from ***Section A*** *and all questions*
EITHER *from* ***Section B OR Section C***

Section A: *Internal choice has been provided in* ***two questions of two marks each, two questions four marks each and two questions of six marks each.***

Section B: *Internal choice has been provided in* ***one question of two marks and one question of four marks.***

Section C: *Internal choice has been provided in* ***one question of two marks and one question of four marks.***

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].
Mathematical tables and graph papers are provided.

SECTION A (65 Marks)

Question 1 **[15×1]**

In sub-parts (i) to (x) choose the correct options and in sub-parts (xi) to (xv), answer the questions as instructed.

(i) Let R be the relation in the set N, given by $R = \{(x, y): x = y + 3 \ \wedge y > 5\}$. Choose the correct answer from the following:

(a) $(7, 4) \in R$.

(b) $(9, 6) \in R$.

(c) $(4, 1) \in R$.

(d) $(8, 5) \in R$.

(ii) If $A = \{1, 2, 3\}, B = \{x, y\}$, then the number of functions from A to B is:

(a) 3

(b) 6

(c) 8

(d) 12

(iii) Simpified value of $\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$ is :

(a) $\frac{1}{2}$

(b) $\frac{1}{\sqrt{2}}$

(c) $\frac{\sqrt{3}}{2}$

(d) $-\frac{\sqrt{3}}{2}$

(iv) Which of the following statements is correct:

(a) Determinanant is number associated to a matrix.

(b) Determinant is a square matrix.

(c) Determinant is a number associated to a square matrix.

(d) None of the above.

(v) If $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, then A^2 is equal to:

(a) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$

(vi) For what value of k inverse does not exist for the matrix $\begin{bmatrix} 1 & 2 \\ k & 6 \end{bmatrix}$:

(a) 0

(b) 3

(c) 6

(d) 2

(vii) Identify from the given options the slope of the normal to the curve $x^2 + 3y + y^2 = 5$ at (1,1):

(a) $-\frac{2}{5}$

(b) $\frac{5}{2}$

(c) $\frac{2}{5}$

(d) $-\frac{5}{2}$

(viii) The function $f(x) = x^3 - 3x$ is strcictly decreasing on:

(a) $-1 < x < 1$

(b) $-1 \leq x \leq 1$

(c) $1 \leq x < \infty$

(d) $-\infty < x < -1$

(ix) The function $f(x) = \sin x + \cos x$, $x \in \left(0, {}^{\pi}\!/_{2}\right)$; Maximum value of the $f(x)$ is:

(a) 1

(b) 2

(c) ${}^{1}\!/_{\sqrt{2}}$

(d) $\sqrt{2}$

(x) If $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then the value of $P\left({}^{B}\!/_{A}\right)$ is :

(a) $\frac{1}{8}$

(b) $\frac{4}{7}$

(c) $\frac{5}{7}$

(d) $\frac{7}{8}$

(xi) Write total number of functions from *set A* to *set B*, where $A = \{1, 2, 3\}$, $B = \{p, q, r, s\}$.

(xii) If A is square matrix of order 3 , with |A| = 4, then write the value of $|-2A|$.

(xiii) Find the sum of order and degree of the differential equation:

$$\left(\frac{dy}{dx}\right)^5 + 3xy\left(\frac{d^3y}{dx^3}\right)^2 + y^2\left(\frac{d^2y}{dx^2}\right)^3 = 0$$

(xiv) A and B appeared for an interview for two vacancies. The probability of A's selection is $\frac{4}{5}$ and B's selection is $\frac{1}{3}$. Find the probability that none of them will be selected.

(xv) If A and B are independent events such that $P(A) = {}^{1}\!/_{4}$ and $P(B) = {}^{2}\!/_{3}$. Find $P(A \cap B)$.

Question 2 **[2]**

(a) Differentiate the function with reference to x:

$$f(x) = \cos^{-1}\left(\sqrt{\frac{1-\cos x}{2}}\right)$$

OR

(b) Find $\frac{dy}{dx}$, if $x = at^2$ and $y = 2at$.

Question 3 **[2]**

Show that the function $f: N \to N$, defined by $f(x) = 5x - 3, \forall x \in N,$ is one-one function but not onto function.

Question 4 **[2]**

Using L'Hospital's Rule, evaluate: $\lim_{x \to 0} \frac{8^x - 4^x}{4x}$

Question 5 **[2]**

(a) Evaluate: $\int \frac{\sec^2 x}{\text{cosec}^2 x} dx$

OR

(b) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$

Question 6 **[2]**

Solve $\frac{dy}{dx} = 1 - xy + y - x$

Question 7 **[4]**

Prove that $tan^{-1}\frac{1}{2} = \frac{\pi}{4} - \frac{1}{2}cos^{-1}\left(\frac{4}{5}\right)$.

Question 8 **[4]**

If $x = tan\left(\frac{1}{a}log\, y\right), prove\ that\ (1 + x^2)\frac{d^2y}{dx^2} + (2x - a)\frac{dy}{dx} = 0$

Question 9 **[4]**

(a) Evaluate: $\int_{-6}^{3} |x+3|\,dx$

OR

(b) Evaluate: $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$

Question 10 **[4]**

(a) Bag A contains 4 white balls and 3 black balls, while Bag B contains 3 white balls and 5 black balls. Two balls are drawn from Bag A and placed in Bag B. Then, what is the probability of drawing a white ball from Bag B?

OR

(b) A word consists of 10 different alphabets, in which there are 4 consonants and 6 vowels. Four alphabets are chosen at random. What is the probability that more than one vowel is selected?

Question 11 **[6]**

(a) Using matrices, solve the following system of equations:
$2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$ and $x + y - 2z = -3$

OR

(b) Solve the matrix equation: $A\begin{pmatrix} 3 & 4 \\ -1 & 2 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 15 \\ 1 & -2 \end{pmatrix}$.

Question 12 **[6]**

(a) Evaluate: $\int tan^{-1}\sqrt{x}\, dx$

OR

(b) Evaluate**:** $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1+e^x} dx$

Question 13 **[6]**

A cone is inscribed in a sphere of radius 12 cm. If the volume of the cone is maximum, find its height.

Question 14 **[6]**

Given three identical Boxes A, B and C, Box A contains 2 gold and 1 silver coin, Box B contains 1 gold and 2 silver coins and Box C contains 3 silver coins. A person chooses a Box at random and takes out a coin. If the coin drawn is of silver, find the probability that it has been drawn from the Box which has the remaining two coins also of silver.

SECTION B (15 Marks)

Question 15 **[5×1]**

In sub-parts (i) and (ii) choose the correct options and in sub-parts (iii) to (v), answer the questions as instructed.

(i) If the vectors $a\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$ and $3\hat{\imath} - 4\hat{\jmath} + b\hat{k}$ are collinear, then $(a, b) =$

(a) $\left(\frac{9}{4}, \frac{8}{3}\right)$

(b) $\left(-\frac{9}{4}, \frac{8}{3}\right)$

(c) $\left(\frac{9}{4}, -\frac{8}{3}\right)$

(d) $\left(-\frac{9}{4}, -\frac{8}{3}\right)$

(ii) The intercepts made by the plane $3x - 2y + 4z = 12$ on the coordinate axes are:

(a) $6, -4,\ 3$

(b) $4, -6,\ 3$

(c) $2, -3,\ 4$

(d) $\frac{1}{4}, -\frac{1}{6}, \frac{1}{3}$

(iii) Find the angle between the vectors $\vec{a} = 6\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = 2\hat{\imath} - 9\hat{\jmath} + 6\hat{k}$.

(iv) Find the unit vector parallel to the vector: $3\hat{\imath} + 6\hat{\jmath} - 2\hat{k}$

(v) Find the equation of the plane passing through $(-2, 1, 3)$ and perpendicular to the line having direction ratios $< 3, 1, 5 >$.

Question 16 **[2]**

(a) If $\vec{a}$, and $\vec{b}$ are perpendicular vectors, $|\vec{a} + \vec{b}| = 13$ and $\vec{a} = 5$, find the value of $|\vec{b}|$.

OR

(b) Find λ if the scalar projection of $\vec{a} = \lambda\hat{\imath} + \hat{\jmath} + 4\hat{k}$ on $\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ is 4 units.

Question 17 **[4]**

(a) Find the distance of the point $A(3,4,4)$ from the point, where the line joining points $P(3,-4,-5)$ and $Q(2,-3,1)$ intersects the plane $2x+y+z=7$.

OR

(b) Find the equation of the lines passing through the point (2, 1, 3) and perpendicular to the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x}{-3}=\frac{y}{2}=\frac{z}{5}$

Question 18 **[4]**

Sketch the graph of $y = |x+4|$. Using integration, find the area of the region bounded by the curve $y = |x+4|$ and $x=-6 \; and \; x=0$.

SECTION C (15 Marks)

Question 19 **[5×1]**

In sub-parts (i) and (ii) choose the correct options and in sub-parts (iii) to (v), answer the questions as instructed.

(i) The demand function of a monopolist is given by $x = 100 - 4p$. The quantity at which MR = 0 will be:

(a) 25

(b) 10

(c) 50

(d) 30

(ii) If the lines of regression are parallel to coordinate axes, then the coefficient of correlation is:

(a) 1

(b) 0

(c) -1

(d) ½

(iii) Find the marginal cost function (MC), if the cost function is :

$C(x)=\frac{x^3}{3}+5x^2-16x+2$.

(iv) If revenue function $R(x)=3x^3+8x-2$, find the average revenue function.

(v) If $\sigma_x=3, \sigma_y=4$ and $b_{xy}=\frac{1}{3}$, then find the value of correlation coefficient (r).

Question 20 **[2]**

(a) A company produces a commodity with ₹ 24,000 as fixed cost. The variable cost estimated to be 25% of the total revenue received on selling the product, is at the rate of ₹ 8 per unit. Find the break-even point.

OR

(b) The total cost function for a production is given by $C(x) = \frac{3}{4}x^2 - 7x + 27$.

Find the number of units produced for which M.C. = A.C.
(M.C.= Marginal Cost and A.C. = Average Cost.)

Question 21 **[4]**

Find the equation of the regression line of y on x, if the observations (x, y) are as follows:

(1, 4), (2, 8), (3, 2), (4, 12), (5, 10), (6, 14), (7, 16), (8, 6), (9, 18)

Also, find the estimated value of y when $x = 14$.

Question 22 **[4]**

(a) A carpenter has 90, 80 and 50 running feet respectively of teak wood, plywood and rosewood which is used to produce product A and product B. Each unit of product A requires 2, 1 and 1 running feet and each unit of product B requires 1, 2 and 1 running feet of teak wood, plywood and rosewood respectively. If product A is sold for ₹ 48 per unit and product B is sold for ₹ 40 per unit, how many units of product A and product B should be produced and sold by the carpenter, in order to obtain the maximum gross income? Formulate the above as a Linear Programming Problem and solve it, indicating clearly the feasible region in the graph.

OR

(b) A farm is engaged in breeding hens. The hens are fed products A and B grown in the farm which contains nutrients P, Q and R. One kilogram of product A contains 36 units, 3 units and 20 units of nutrients P, Q and R respectively, whereas one kilogram of product B contains 6 units, 12 units and 10 units of nutrients P, Q and R respectively. The minimum requirement of nutrients P, Q and R for a hen is 108, 36 and 100 units respectively. Product A costs ₹ 20 per kilogram and product B costs ₹ 40 per kilogram. Using linear programming, find the number of kilograms of products A and B to be produced to minimize the total cost.Identify the feasible region from the rough sketch.

(NOTE: The total weightage of each Unit covered in the question paper shall be as specified in the syllabus, covering all the chapters of each Unit. The weightage of each question in the Question Paper shall be as indicated in this Specimen Paper. However, the number of MCQs given in Question Nos. 1, 15 and 19 may vary from year to year.)
