RELATIONS AND FUNCTIONS

BASIC CONCEPTS

1. **Relation:** If A and B are two non-empty sets, then any subset R of $A \times B$ is called relation from set A to set B.

   i.e., $R : A \to B \Leftrightarrow R \subseteq A \times B$

   If $(x, y) \in R$, then we write $x\,R\,y$ (read as x is R related to y) and if $(x, y) \notin R$, then we write $x \not R\, y$ (read as x is not R related to y).

2. **Domain and Range of a Relation:** If R is any relation from set A to set B then,

   (*a*) **Domain of** R **is** the set of all first coordinates of elements of R and it is denoted by Dom (R).

   (*b*) **Range of** R **is** the set of all second coordinates of R and it is denoted by Range (R).

   A relation R on set A means, the relation from A to A *i.e.*, $R \subseteq A \times A$.

3. **Some Standard Types of Relations:**

   Let A be a non-empty set. Then, a relation R on set A is said to be

   (*a*) **Reflexive:** If $(x, x) \in R$ for each element $x \in A$, *i.e.*, if xRx for each element $x \in A$.

   (*b*) **Symmetric:** If $(x, y) \in R \Rightarrow (y, x) \in R$ for all $x, y \in A$, *i.e.*, if $xRy \Rightarrow yRx$ for all $x, y \in A$.

   (*c*) **Transitive:** If $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$ for all $x, y, z \in A$, *i.e.*, if xRy and $yRz \Rightarrow xRz$.

4. **Equivalence Relation:** Any relation R on a set A is said to be an equivalence relation if R is reflexive, symmetric and transitive.

5. **Antisymmetric Relation:** A relation R in a set A is antisymmetric

   if $(a, b) \in R, (b, a) \in R \Rightarrow a = b\ \forall\ a, b \in R$, or aRb and $bRa \Rightarrow a = b, \forall\ a, b \in R$.

   For example, the relation "greater than or equal to, "≥" is antisymmetric relation as

   $$a \geq b, b \geq a \Rightarrow a = b\ \forall\ a, b$$

   [**Note:** "Antisymmetric" is completely different from not symmetric.]

6. **Equivalence Class:** Let R be an equivalence relation on a non-empty set A. For all $a \in A$, the equivalence class of 'a' is defined as the set of all such elements of A which are related to 'a' under R. It is denoted by $[a]$.

   i.e., $[a]$ = equivalence class of 'a' = $\{x \in A : (x, a) \in R\}$

7. **Function:** Let X and Y be two non-empty sets. Then, a rule f which associates to each element $x \in X$, a unique element, denoted by $f(x)$ of Y, is called a function from X to Y and written as $f : X \to Y$ where, $f(x)$ is called image of x and x is called the **pre-image** of $f(x)$ and the set Y is called the **co-domain** of f and $f(X) = \{f(x): x \in X\}$ is called the range of f.

8. **Types of Function:**

   (*i*) **One-one function (injective function):** A function $f : X \to Y$ is defined to be one-one if the image of distinct element of X under rule f are distinct, *i.e.*, for every $x_1, x_2 \in X$, $f(x_1) = f(x_2)$ implies that $x_1 = x_2$.

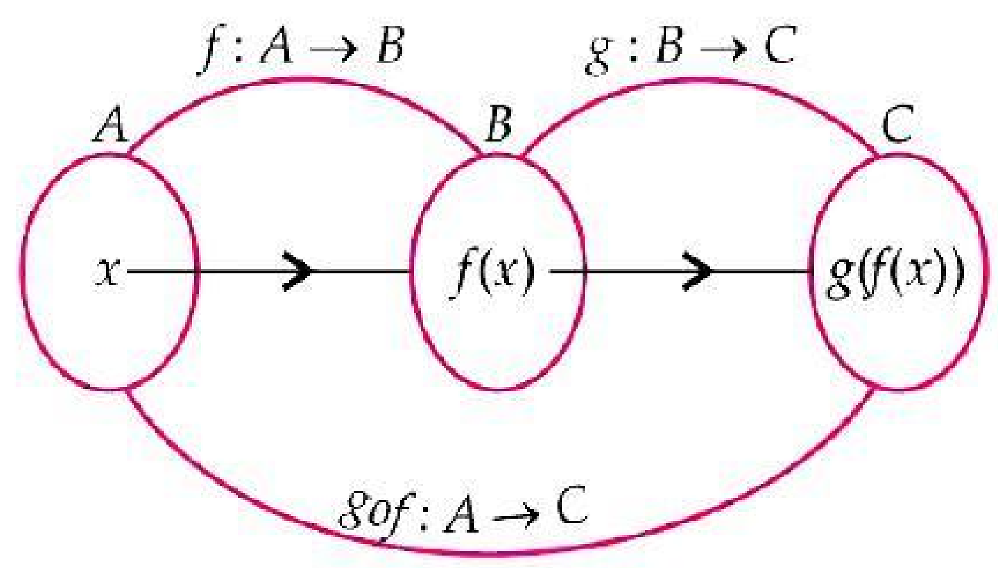
   (*ii*) **Onto function (Surjective function):** A function $f : X \to Y$ is said to be onto function if each element of Y is the image of some element of x *i.e.*, for every $y \in Y$, there exists some $x \in X$, such that $y = f(x)$. Thus f is onto if range of f = co-domain of f.

   (*iii*) **One-one onto function (Bijective function):** A function $f : X \to Y$ is said to be one-one onto, if f is both one-one and onto.

   (*iv*) **Many-one function:** A function $f : X \to Y$ is said to be a many-one function if two or more elements of set X have the same image in Y. *i.e.*,

   $f : X \to Y$ is a many-one function if there exist $a, b \in X$ such that $a \neq b$ but $f(a) = f(b)$.

9. **Composition of Functions:** Let $f : A \to B$ and $g : B \to C$ be two functions. Then, the composition of f and g, denoted by gof, is defined as the function.



$gof : A \to C$ given by

$gof(x) = g(f(x)), \forall\, x \in A$

Clearly, dom(gof) = dom(f)

Also, gof is defined only when range(f) $\subseteq$ dom(g)

10. **Identity Function:** Let R be the set of real numbers. A function $I : R \to R$ such that

$$I(x) = x \ \forall\, x \in R \text{ is called identity function.}$$

Obviously, identity function associates each real number to itself.

11. **Invertible Function:** For $f : A \to B$, if there exists a function $g : B \to A$ such that $gof = I_A$ and $fog = I_B$, where I_A and I_B are identity functions, then f is called an invertible function, and g is called the inverse of f and it is written as $f^{-1} = g$.

12. **Number of Functions:** If X and Y are two finite sets having m and n elements respectively then the number of functions from X to Y is n^m.

13. **Vertical Line Test:** It is used to check whether a relation is a function or not. Under this test, graph of given relation is drawn assuming elements of domain along x-axis. If a vertical line drawn anywhere in the graph, intersects the graph at only one point then the relation is a function, otherwise it is not a function.

14. **Horizontal Line Test:** It is used to check whether a function is one-one or not. Under this test graph of given function is drawn assuming elements of domain along x-axis. If a horizontal line (parallel to x-axis) drawn anywhere in graph, intersects the graph at only one point then the function is one-one, otherwise it is many-one.

MULTIPLE CHOICE QUESTIONS

Choose and write the correct option in the following questions.

1. **The relation R in the set $A = \{1, 2, 3, 4\}$ given by $R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$ is**
   (*a*) reflexive and symmetric but not transitive
   (*b*) reflexive and transitive but not symmetric
   (*c*) symmetric and transitive but not reflexive
   (*d*) an equivalence relation
2. **If $A = \{a, b, c, d\}$, then a relation $R = \{(a, b), (b, a), (a, a)\}$ on A is**
   (*a*) symmetric only (*b*) transitive only
   (*c*) reflexive and transitive (*d*) symmetric and transitive only
3. **For real numbers x and y, define xRy if and only if $x - y + \sqrt{2}$ is an irrational number. Then the relation R is** [*NCERT Exemplar*]
   (*a*) reflexive (*b*) symmetric (*c*) transitive (*d*) none of these
4. **Consider the non-empty set consisting of children in a family and a relation R defined as aRb if a is brother of b. Then R is** [*NCERT Exemplar*]
   (*a*) symmetric but not transitive (*b*) transitive but not symmetric
   (*c*) neither symmetric nor transitive (*d*) both symmetric and transitive
5. **The maximum number of equivalence relation on the set $A = \{1, 2, 3\}$ are** [*NCERT Exemplar*]
   (*a*) 1 (*b*) 2 (*c*) 3 (*d*) 5
6. **Let L denotes the set of all straight lines in a plane. Let a relation R be defined by lRm if and only if l is perpendicular to m $\forall$ $l, m \in L$. Then R is** [*NCERT Exemplar*]
   (*a*) reflexive (*b*) symmetric (*c*) transitive (*d*) none of these
7. **Let $A = \{1, 2, 3\}$. Then number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is**
   (*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4
8. **Let $A = \{1, 2, 3\}$. Then number of equivalence relations containing (1, 2) is/are**
   (*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4
9. **Let A and B be finite sets containing m and n elements respectively. The number of relations that can be defined from A to B is**
   (*a*) 2^{mn} (*b*) 2^{m+n} (*c*) mn (*d*) 0
10. **Set A has 3 elements and the set B has 4 elements. Then the number of injective mapping that can be defined from A to B is** [*NCERT Exemplar*]
    (*a*) 144 (*b*) 12 (*c*) 24 (*d*) 64
11. **The function $f : R \to R$ defined by $f(x) = 2^x + 2^{|x|}$ is**
    (*a*) One-one and onto (*b*) Many-one and onto
    (*c*) One-one and into (*d*) Many-one and into
12. **If the set A contains 5 elements and the set B contains 6 elements, then the number of one-one and onto mapping from A to B is**
    (*a*) 720 (*b*) 120 (*c*) 0 (*d*) none of these
13. **Which of the following functions from Z into Z is bijection?** [*NCERT Exemplar*]
    (*a*) $f(x) = x^3$ (*b*) $f(x) = x + 2$ (*c*) $f(x) = 2x + 1$ (*d*) $f(x) = x^2 + 1$
14. **Let $f : [2, \infty) \to R$ be the function defined by $f(x) = x^2 - 4x + 5$, then the range of f is** [*NCERT Exemplar*]
    (*a*) R (*b*) $[1, \infty)$ (*c*) $[4, \infty)$ (*d*) $[5, \infty)$

15. Let $f : R \to R$ be defined by $f(x) = x^2 + 1$. Then, pre-images of 17 and –3, respectively, are [*NCERT Exemplar*]

(*a*) $\phi, \{4, -4\}$ (*b*) $\{3, -3\}, \phi$ (*c*) $\{4, -4\}, \phi$ (*d*) $\{4, -4\}, \{2, -2\}$

16. Let the function $f : R \to R$ be defined by $f(x) = 2x + \sin x$ for $x \in R$. Then f is

(*a*) one-one but not onto (*b*) onto but not one-one

(*c*) neither one-one nor onto (*d*) one-one and onto

17. Let R be the relation in the set N given by $R = \{(a, b) : a = b - 2, b > 6\}$ choose the correct answer.

(*a*) $(2, 4) \in R$ (*b*) $(3, 8) \in R$ (*c*) $(6, 8) \in R$ (*d*) $(8, 7) \in R$

18. Let $f : R \to R$ be defined as $f(x) = x^4$. Choose the correct answer

(*a*) f is one-one onto (*b*) f is many one onto

(*c*) f is one-one but not onto (*d*) f is neither one-one nor onto.

19. Let $f : R \to R$ be defined as $f(x) = 3x$. Choose the correct answer.

(*a*) f is one-one onto. (*b*) f is many one onto.

(*c*) f is one-one but not onto (*d*) f is neither one-one nor onto.

20. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ defined by
$f(x) = 2x^3 + 2x^2 + 300x + 5 \sin x$ then f is

(*a*) one-one onto (*b*) one-one into (*c*) many one onto (*d*) many one into

21. Let $f : R \to R$ be defined by $f(x) = x^2 + 1$. Then, pre-image of 5 and – 5, respectively are

(*a*) $\phi, \{-2\}$ (*b*) $\{(3, -3\}, \phi$ (*c*) $\{-2, 2\}, \phi$ (*d*) $\{1, -1\}, \{2, -2\}$

22. The domain of the function $f : R \to R$ defined by $f(x) = \sqrt{x^2 - 4}$ is

(*a*) $[-2, 2]$ (*b*) $(-2, 2)$ (*c*) $(-\infty, -2] \cup [2, \infty)$ (*d*) $(-\infty, \infty)$

23. Let $f : R \to R$ be defined by $f(x) = \begin{cases} 3x, & \text{if } x > 3 \\ x^2, & \text{if } 1 < x \le 3 \\ x, & \text{if } x \le 1 \end{cases}$

Then $f(-2) + f(0) + f(2) + f(5)$ is equal to

(*a*) 0 (*b*) 17 (*c*) – 4 (*d*) none of these

24. Let R is reflexive relation on a finite set A having n element, and let there be m ordered pairs in R. Then

(*a*) $m \ge n$ (*b*) $m \le n$ (*c*) $m = n$ (*d*) none of these

25. The domain of the function $f(x) = \log_{3+x}(x^2 - 1)$ is

(*a*) $(-3, -1) \cup (1, \infty)$ (*b*) $[-3, -1) \cup [1, \infty)$

(*c*) $(-3, -2) \cup (-2, -1) \cup (1, \infty)$ (*d*) $[-3, -2) \cup (-2, -1) \cup [1, \infty)$

26. Let $f : R \to [0, \frac{\pi}{2})$ defined by $f(x) = \tan^{-1}(x^2 + x + a)$, then the set of values of a for which f is onto is

(*a*) $[0, \infty)$ (*b*) $[\frac{1}{4}, \infty)$ (*c*) $[2, 1]$ (*d*) none of these

27. If the function $f : R \to R$ and $g : R \to R$ are defined as

$$f(x) = \begin{cases} 0, & x \in \text{rational} \\ x, & x \in \text{irrational} \end{cases}$$

and $$g(x) = \begin{cases} 0, & x \in \text{irrational} \\ x, & x \in \text{rational} \end{cases}$$

then $(f - g)$ is

(*a*) one-one onto. (*b*) many-one onto

(*c*) one-one but not onto (*d*) neither one-one nor onto.

28. If a relation R on the set {1, 2, 3, 4} is defined by $R = \{(1, 2), (3, 4)\}$. Then R is

(*a*) reflexive (*b*) transitive (*c*) symmetric (*d*) none of these

29. If the set A contains 4 elements and the set B contains 5 elements, then the number of one-one and onto mappings from A to B is

(*a*) 0 (*b*) 4^5 (*c*) 5^4 (*d*) none of these

30. Let $A = \{x, y, z)$ and $B = \{a, b\}$ then the number of onto function from A to B is

(*a*) 0 (*b*) 3 (*c*) 6 (*d*) 8

31. If A and B have 4 and 6 elements respectively then the number of one-one function from A to B is

(*a*) 4^6 (*b*) 6^4 (*c*) 360 (*d*) 240

32. If A and B have 4 elements each then the number of one-one onto (bijective) function from A to B is

(*a*) 0 (*b*) 24 (*c*) 4^2 (*d*) None of these

33. If R is an equivalence relation on A, then R^{-1} on A is

(*a*) Transitive only (*b*) Symmetric only (*c*) Reflexive only (*d*) Equivalence relation

34. The relation "greater than" denoted by > in the set of integers is

(*a*) Symmetric (*b*) Reflexive (*c*) Transitive (*d*) None of these

35. If R_1 and R_2 are symmetric relations in a set A, then $R_1 \cup R_2$ is

(*a*) Reflexive (*b*) Symmetric (*c*) Transitive (*d*) None of these

36. The function $f : R \to R$ defined by $f(x) = 4^x + 4^{|x|}$ is

(*a*) one-one and into (*b*) one-one and onto

(*c*) many one and into (*d*) many one and onto

37. Identity relation R on a set A is

(*a*) Reflexive only (*b*) Symmetric only (*c*) Transitive only (*d*) Equivalence

38. The relation "congruence modulo m" on the set $\mathbb{Z}$ of all integers is a relation of type

(*a*) Reflexive only (*b*) Symmetric only (*c*) Transitive only (*d*) Equivalence

39. Let $f : \mathbb{R} \longrightarrow \left[0, \frac{\pi}{2}\right)$ defined by $f(x) = \tan^{-1}(x^2 + x + 2a)$ then the set of values of 'a' for which f is onto, is

(*a*) $\left(-\frac{1}{4}, \infty\right)$ (*b*) $[-1, \infty)$ (*c*) $\left[-\frac{1}{8}, \infty\right)$ (*d*) $\left[\frac{1}{8}, \infty\right)$

40. If the function $f(x)$ satisfying $(f(x))^2 - 4f(x)f'(x) + (f'(x))^2 = 0$ then $f(x)$ equals

(*a*) $\lambda e^{(2+\sqrt{5})x}$ (*b*) $\lambda e^{(2-\sqrt{5})x}$ (*c*) $\lambda e^{(2\pm\sqrt{3})x}$ (*d*) $\lambda e^{(3-\sqrt{3})x}$

41. Let $f : (-1, 1) \longrightarrow B$ where $f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ is one-one and onto, then B equals

(*a*) $\left[0, \frac{\pi}{2}\right]$ (*b*) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (*c*) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (*d*) $\left(0, \frac{\pi}{2}\right)$

42. The function $y = \frac{x}{1+|x|}, x \in \mathbb{R}, y \in \mathbb{R}$ is

(*a*) One-one onto (*b*) Onto but not one-one

(*c*) One-one but not onto (*d*) None of these

43. A relation R in the set of non-zero complexs number is defined by $z_1 R z_2 \Leftrightarrow \frac{z_1 - z_2}{z_1 + z_2}$ is real, then R is

(*a*) Reflexive (*b*) Symmetric (*c*) Transitive (*d*) Equivalence

44. Number of onto (subjective) functions from A to B if $n(A) = 6$ and $n(B) = 3$ are

(*a*) $2^6 - 2$ (*b*) $3^6 - 3$ (*c*) 340 (*d*) None of these

45. Let $A = \{7, 8, 9, 10\}$ and R $\{(8, 8), (9, 9), (10, 10), (7, 8)\}$ be a relation on A, then R is

(*a*) Transitive (*b*) Reflexive (*c*) Symmetric (*d*) None of these

46. Let f, g be a function from the set $\{1, 2, ..., 12\}$ to the set $\{1, 2, 3, ..., 11\}$ then which of the following is correct?

(*a*) Number of onto functions from A to $B = \dfrac{\lfloor 12 \times 11}{2}$

(*b*) Total number of functions from A to $B = 11^{12}$

(*c*) The functions which are not onto $= 11^{12} - \dfrac{\lfloor 12 \times 11}{2}$

(*d*) All of these

47. Let p and q are positive integers, f is a function defined for positive numbers and attains only positive values such that $f[x\ f(y)] = x^a y^b$ then

(*a*) $b^2 = a$ (*b*) $a = b$ (*c*) $a^2 = b$ (*d*) None of these

48. Let $f : (-1, 1) \to B$, be a function defined by $f(x) = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$ then f is both one-one and onto when B is the interval

(*a*) $\left[0, \dfrac{\pi}{2}\right)$ (*b*) $\left(0, \dfrac{\pi}{2}\right)$ (*c*) $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$ (*d*) $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

49. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ defined by $f(x) = \dfrac{e^{x^2} - e^{-x^2}}{e^{x^2} + e^{-x^2}}$ then

(*a*) $f(x)$ is one-one but not onto (*b*) $f(x)$ is neither one-one nor onto

(*c*) $f(x)$ is many one but onto (*d*) $f(x)$ is one-one and onto

50. If $A = \{7, 8, 9\}$, then the relation $R = \{(8, 9)\}$ in A is

(*a*) Symmetric only (*b*) Non-symmetric (*c*) Reflexive only (*d*) Equivalence

51. Let A be the finite set containing n distinct elements . The number of relations that can be defined on A is

(*a*) 2^n (*b*) n^2 (*c*) 2^{n^2} (*d*) 2^{n-1}

52. Let R_1 and R_2 be equivalence relations on a set A, then $R_1 \cup R_2$ may or may not be

(*a*) Reflexive (*b*) Symmetric (*c*) Transitive (*d*) None of these

53. Let R be the relation defined on the set N of natural numbers by the rule $x\ R\ y$ iff $x + 2y = 8$, then domain of R is

(*a*) $\{2,4,8\}$ (*b*) $\{2,4,6\}$ (*c*) $\{2,4,6,8\}$ (*d*) $\{1,2,3,4\}$

54. Let $A = \{a,b,c\}$ and $R = \{(a,a), (b,b), (c,c), (b,c), (a,b)\}$ be a relation on A, then R is

(*a*) Symmetric (*b*) Transitive (*c*) Reflexive (*d*) Equivalence

55. "Every relation is a function and every function is a relation" then which is correct for given statement ?

(*a*) True (*b*) False (*c*) Can't say anything (*d*) None of these

56. If a relation R on the set $\{1, 2, 3\}$ be defined by $R = \{(1, 2)\}$, then R is

(*a*) Reflexive (*b*) Transitive (*c*) Symmetric (*d*) None of these

57. Let us define a relation R in R as $a\ R\ b$ if $a \geq b$. Then, R is

$R = \{(a, b) : a \geq b\}$

(*a*) An equivalence relation

(*b*) Reflexive, transitive but not symmetric

(*c*) Symmetric, transitive but not reflexive

(*d*) Neither transitive nor reflexive but symmetric

58. **If $A = \{1, 2, 3\}$ and consider the relation $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$. Then, R is**

(*a*) Reflexive but not symmetric
(*b*) Reflexive but not transitive
(*c*) Symmetric and transitive
(*d*) Neither Symmetric nor transitive

59. **The relation R defined on the set $A = \{1, 2, 3, 4, 5\}$ by $R = \{(a, b) : |a^2 - b^2| < 7\}$ is given by**

(*a*) $\{(1, 1), (2, 1), (3, 1), (4, 1), (2, 3)\}$
(*b*) $\{(2, 2), (3, 2), (4, 2), (2, 4)\}$
(*c*) $\{(3, 3), (4, 3), (5, 4), (3, 4)\}$
(*d*) $\{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (1, 2), (2, 3)\}$

Answers

1. (*b*)	**2.** (*d*)	**3.** (*a*)	**4.** (*b*)	**5.** (*d*)	**6.** (*b*)
7. (*a*)	**8.** (*b*)	**9.** (*a*)	**10.** (*c*)	**11.** (*c*)	**12.** (*c*)
13. (*b*)	**14.** (*b*)	**15.** (*c*)	**16.** (*d*)	**17.** (*c*)	**18.** (*d*)
19. (*a*)	**20.** (*a*)	**21.** (*c*)	**22.** (*c*)	**23.** (*b*)	**24.** (*a*)
25. (*c*)	**26.** (*b*)	**27.** (*a*)	**28.** (*b*)	**29.** (*a*)	**30.** (*c*)
31. (*c*)	**32.** (*b*)	**33.** (*d*)	**34.** (*c*)	**35.** (*b*)	**36.** (*a*)
37. (*d*)	**38.** (*d*)	**39.** (*d*)	**40.** (*c*)	**41.** (*c*)	**42.** (*c*)
43. (*d*)	**44.** (*d*)	**45.** (*a*)	**46.** (*d*)	**47.** (*c*)	**48.** (*c*)
49. (*b*)	**50.** (*b*)	**51.** (*c*)	**52.** (*c*)	**53.** (*b*)	**54.** (*c*)
55. (*b*)	**56.** (*b*)	**57.** (*b*)	**58.** (*a*)	**59.** (*d*)	

CASE-BASED QUESTIONS

Choose and write the correct option in the following questions.

1. **Read the following and answer any four questions from (*i*) to (*v*).**

A general election of Lok Sabha is a gigantic exercise. About 911 million people were eligible to vote and voter turnout was about 67%, the highest ever

ONE – NATION
ONE – ELECTION
FESTIVAL OF
DEMOCRACY
GENERAL ELECTION– 2019

Let I be the set of all citizens of India who were eligible to exercise their voting right in general election held in 2019. A relation 'R' is defined on I as follows:

$R = \{(V_1, V_2) : V_1, V_2 \in I$ and both use their voting right in general election – 2019$\}$

[*CBSE Question Bank*]

Answer the questions given below.

(*i*) Two neighbours X and $Y \in I$. X exercised his voting right while Y did not cast her vote in general election – 2019. Which of the following is true?

(*a*) $(X, Y) \in R$ (*b*) $(Y, X) \in R$

(*c*) $(X, X) \notin R$ (*d*) $(X, Y) \notin R$

(*ii*) Mr.'X' and his wife 'W' both exercised their voting right in general election -2019, Which of the following is true?

(*a*) both (X,W) and $(W,X) \in R$ (*b*) $(X,W) \in R$ but $(W,X) \notin R$

(*c*) both (X,W) and $(W,X) \notin R$ (*d*) $(W,X) \in R$ but $(X,W) \notin R$

(*iii*) Three friends F_1, F_2 and F_3 exercised their voting right in general election- 2019, then which of the following is true?

(*a*) $(F_1, F_2) \in R$, $(F_2, F_3) \in R$ and $(F_1, F_3) \in R$

(*b*) $(F_1, F_2) \in R$, $(F_2, F_3) \in R$ and $(F_1, F_3) \notin R$

(*c*) $(F_1, F_2) \in R$, $(F_2, F_2) \in R$ but $(F_3, F_3) \notin R$

(*d*) $(F_1, F_2) \notin R$, $(F_2, F_3) \notin R$ and $(F_1, F_3) \notin R$

(*iv*) The above defined relation R is

(*a*) Symmetric and transitive but not reflexive

(*b*) Universal relation

(*c*) Equivalence relation

(*d*) Reflexive but not symmetric and transitive

(*v*) Mr. Shyam exercised his voting right in General Election – 2019, then Mr. Shyam is related to which of the following?

(*a*) All those eligible voters who cast their votes

(*b*) Family members of Mr.Shyam

(*c*) All citizens of India

(*d*) Eligible voters of India

Sol. We have a relation 'R' is defined on I as follows:

$R = \{V_1, V_2\} : V_1, V_2 \in I$ and both use their voting right in general election – 2019}

(*i*) Two neighbors X and $Y \in I$. Since X exercised his voting right while Y did not cast her vote in general election – 2019

Therefore, $(X, Y) \notin R$

$\therefore$ Option (*d*) is correct.

(*ii*) Since Mr. 'X' and his wife 'W' both exercised their voting right in general election – 2019.

$\therefore$ Both (X, W) and $(W, X) \in R$.

$\therefore$ Option (*a*) is correct.

(*iii*) Since three friends F_1, F_2 and F_3 exercised their voting right in general election – 2019, therefore

$(F_1, F_2) \in R$, $(F_2, F_3) \in R$ and $(F_1, F_3) \in R$

$\therefore$ Option (*a*) is correct.

(*iv*) This relation is an equivalence relation

$\therefore$ Option (*c*) is correct.

(*v*) Mr. Shyam exercised his voting right in General election – 2019, then Mr. Shyam is related to all those eligible votes who cast their votes.

$\therefore$ Option (*a*) is correct.

2. **Read the following and answer any four questions from (*i*) to (*v*).**

Sherlin and Danju are playing Ludo at home during Covid-19. While rolling the dice, Sherlin's sister Raji observed and noted the possible outcomes of the throw every time belongs to set {1,2,3,4,5,6}. Let A be the set of players while B be the set of all possible outcomes.

$A = \{S, D\}$, $B = \{1,2,3,4,5,6\}$ [*CBSE Question Bank*]

Answer the questions given below.

(*i*) Let $R : B \to B$ be defined by $R = \{(x, y): y \text{ is divisible by } x\}$ is

(*a*) Reflexive and transitive but not symmetric

(*b*) Reflexive and symmetric and not transitive

(*c*) Not reflexive but symmetric and transitive

(*d*) Equivalence

(*ii*) Raji wants to know the number of functions from *A* to *B*. How many number of functions are possible?

(*a*) 6^2 (*b*) 2^6 (*c*) $6!$ (*d*) 2^{12}

(*iii*) Let *R* be a relation on *B* defined by $R = \{(1,2), (2,2), (1,3), (3,4), (3,1), (4,3), (5,5)\}$. Then *R* is

(*a*) Symmetric (*b*) Reflexive (*c*) Transitive (*d*) None of these three

(*iv*) Raji wants to know the number of relations possible from *A* to *B*. How many numbers of relations are possible?

(*a*) 6^2 (*b*) 2^6 (*c*) $6!$ (*d*) 2^{12}

(*v*) Let $R : B \to B$ be defined by $R=\{(1,1),(1,2), (2,2), (3,3), (4,4), (5,5),(6,6)\}$, then R is

(*a*) Symmetric (*b*) Reflexive and Transitive

(*c*) Transitive and symmetric (*d*) Equivalence

Sol. (*i*) Given $R : B \to B$ be defined by

$R = \{(x, y) : y \text{ is divisible by } x\}$

Reflexive : Let $x \in B$, since x always divide x itself.

$\therefore (x, x) \in R$

It is reflexive.

Symmetric : Let $x, y \in B$ and let $(x, y) \in R$

$\Rightarrow$ y is divisible by x

$\Rightarrow \dfrac{y}{x} = k_1$, where k_1 is an integer.

$\Rightarrow \dfrac{x}{y} = \dfrac{1}{k_1} \neq \text{integer.}$

$\therefore (y, x) \notin R$

It is not symmetric.

Transitive : Let $x, y, z \in B$ and

Let $(x, y) \in R \Rightarrow \frac{y}{x} = k_1$, where k_1 is an integer.

and, $(y, z) \in R \Rightarrow \frac{z}{y} = k_2$, where k_2 is an integer.

$\therefore \quad \frac{y}{x} \times \frac{z}{y} = k_1 . k_2 = k$ (integer)

$\Rightarrow \frac{z}{x} = k \quad \Rightarrow (x, z) \in R$

It is transitive.

Hence, relation is reflexive and transitive but not symmetric.

$\therefore$ Option (*a*) is correct.

(*ii*) We have,

$A = \{S, D\} \Rightarrow n(A) = 2$

and, $B = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(B) = 6$

$\therefore$ Number of functions from A to B is 6^2.

$\therefore$ Option (*a*) is correct.

(*iii*) Given,

R be a relation on B defined by

$R = \{(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)\}$

R is not reflexive since $(1, 1), (3, 3), (4, 4) \notin R$

R is not symmetric as $(1, 2) \in R$ but $(2, 1) \notin R$

and, R is not transitive as $(1, 3) \in R$ and $(3, 1) \in R$ but $(1, 1) \notin R$

$\therefore$ R is neither reflexive nor symmetric nor transitive.

$\therefore$ Option (*d*) is correct.

(*iv*) Total number of possible relations from A to $B = 2^{12}$

$\therefore$ Option (*d*) is correct.

(*v*) Given $R : B \to B$ be defined by $R = \{(1, 1), (1, 2), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

$\therefore$ R is reflexive as each elements of B is related to itself and R is also transitive as $(1, 2) \in R$ and $(2, 2) \in R$

$\Rightarrow (1, 2) \in R$

$\therefore$ R is reflexive and transitive.

$\therefore$ Option (*b*) is correct.

3. Read the following and answer any four questions from (*i*) to (*v*).

An organization conducted bike race under 2 different categories-boys and girls. In all, there were 250 participants. Among all of them finally three from Category 1 and two from Category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}$ $G=\{g_1, g_2\}$ where B represents the set of boys selected and G the set of girls who were selected for the final race. [*CBSE Question Bank*]

Ravi decides to explore these sets for various types of relations and functions

Answer the questions given below.

(*i*) Ravi wishes to form all the relations possible from *B* to *G*. How many such relations are possible?

(*a*) 2^6 (*b*) 2^5 (*c*) 0 (*d*) 2^3

(*ii*) Let $R : B \rightarrow B$ be defined by $R = \{(x, y) : x$ and y are students of same sex\}, Then this relation *R* is

(*a*) Equivalence

(*b*) Reflexive only

(*c*) Reflexive and symmetric but not transitive

(*d*) Reflexive and transitive but not symmetric

(*iii*) Ravi wants to know among those relations, how many functions can be formed from *B* to *G*?

(*a*) 2^2 (*b*) 2^{12} (*c*) 3^2 (*d*) 2^3

(*iv*) Let $R : B \rightarrow G$ be defined by $R = \{ (b_1, g_1), (b_2, g_2), (b_3, g_1)\}$, then *R* is

(*a*) Injective (*b*) Surjective

(*c*) Neither Surjective nor Injective (*d*) Surjective and Injective

(*v*) Ravi wants to find the number of injective functions from B to G. How many numbers of injective functions are possible?

(*a*) 0 (*b*) 2! (*c*) 3! (*d*) 0!

Sol. We have sets

$$B = \{b_1, b_2, b_3\}, G = \{g_1, g_2\}$$

$\Rightarrow n(B) = 3$ and $n(G) = 2$

(*i*) Number of all possible relations from *B* to $G = 2^{3 \times 2} = 2^6$

$\therefore$ Option (*a*) is correct.

(*ii*) Given relation R = \{(*x*, *y*) : *x* and *y* are student of same sex\}

On the set *B*.

Since the set is $B = \{b_1, b_2, b_3\}$ = all boys

$\therefore$ It is ans equivalence relation.

$\therefore$ Option (*a*) is correct.

(*iii*) We have,

$$B = \{b_1, b_2, b_3\} \Rightarrow n(B) = 3$$

$$G = \{g_1, g_2\} \Rightarrow n(G) = 2$$

$\therefore$ Total no. of possible functions from B to $G = 2^3$

$\therefore$ Option (*d*) is correct.

(*iv*) We have,

$R : B \to G$ be defined by

$R = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$

It is not injective because $(b_1, g_1) \in R$ and $(b_3, g_1) \in R$

So $b_1 \neq b_3 \Rightarrow$ same image g_1.

It is surjective because its Co-domain = Range.

$\therefore$ R is Surjective.

$\therefore$ Option (*b*) is correct.

(*v*) Since R is not injective therefore number of injective functions = 0

$\therefore$ Option (*a*) is correct.

ASSERTION-REASON QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(*a*) *Both A and R are true and R is the correct explanation of A.*

(*b*) *Both A and R are true but R is not the correct explanation of A.*

(*c*) *A is true but R is false.*

(*d*) *A is false and R is also false.*

1. Assertion (A) : Let L be the collection of all lines in a plane and R_1 be the relation on L as $R_1 = \{(L_1, L_2) : L_1 \perp L_2\}$ is a symmetric relation.

Reason (R) : A relation R is said to be symmetric if $(a, b) \in R \Rightarrow (b, a) \in R$.

2. Assertion (A) : Let R be the relation on the set of integers Z given by $R = \{(a, b) : 2 \text{ divides } (a - b)\}$ is an equivalence relation.

Reason (R) : A relation R in a set A is said to be an equivalence relation if R is reflexive, symmetric and transitive.

3. Assertion (A) : Let $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x$, then f is a one-one function.

Reason (R) : A function $g : A \to B$ is said to be onto function if for each $b \in B, \exists\, a \in A$ such that $g(a) = b$.

4. Assertion (A) : Let function $f : \{1, 2, 3\} \to \{1, 2, 3\}$ be an onto function. Then it must be one-one function.

Reason (R) : A one-one function $g : A \to B$, where A and B are finite set and having same number of elements, then it must be onto and vice-versa.

Answers

1. (*a*) **2.** (*a*) **3.** (*b*) **4.** (*a*)

HINTS/SOLUTIONS OF SELECTED MCQS

1. Since every element of A is related to itself in the given relation R, therefore R is reflexive and as $(1, 2) \in R$ and $(2, 2) \in R \Rightarrow (1, 2) \in R$ also $(1, 3) \in R$ and $(3, 2) \in R \Rightarrow (1, 2) \in R$. Again $(1, 3) \in R$ and $(3, 3) \in R \Rightarrow (1, 3) \in R$. Thus R is also transitive. Hence relation R is reflexive and transitive but not symmetric because, $(1, 2) \in R$ but $(2, 1) \notin R$, also $(1, 3) \in R$ but $(3, 1) \notin R$ and $(3, 2) \in R$ but $(2, 3) \notin R$.

   Option (b) is correct.

2. On the set $A = \{a, b, c, d\}$ given relation $R = \{(a, b), (b, a), (a, a)\}$ is symmetric and transitive only.

   Since, $(a, b) \in R \Rightarrow (b, a) \in R$, therefore it is symmetric

   Also, $(a, b) \in R$ and $(b, a) \in R \Rightarrow (a, a) \in R$, so it is also transitive. As (b, b), (c, c) and (d, d) does not belong to R hence R is not reflexive.

   Hence relation R is symmetric and transitive only.

   Option (d) is correct.

3. For any $x \in \mathbb{R}$

   $$x - x + \sqrt{2} = \sqrt{2} \text{ is an irrational number} \Rightarrow (x, x) \in R \; \forall \; x \in \mathbb{R}$$

   $\therefore$ R is reflexive

   For $2, \sqrt{2} \in \mathbb{R}$

   $\sqrt{2} - 2 + \sqrt{2} = 2\sqrt{2} - 2$ is an irrational number.

   $\Rightarrow (\sqrt{2}, 2) \in \mathbb{R}$

   But $2 - \sqrt{2} + \sqrt{2} = 2$ which is a rational number

   $\Rightarrow (2, \sqrt{2}) \notin R$

   $\Rightarrow R$ is not reflexive

   R is not transitive

   For $2, \sqrt{3}, \sqrt{2} \in \mathbb{R}$

   $\because$ $2 - \sqrt{3} + \sqrt{2} = 2 - (\sqrt{3} - \sqrt{2})$ is an irrational number

   $\Rightarrow (2, \sqrt{3}) \in R$

   Also $\sqrt{3} - \sqrt{2} + \sqrt{2} = \sqrt{3}$ which is an irrational number

   $\Rightarrow (\sqrt{3}, \sqrt{2}) \in R$

   But $2 - \sqrt{2} + \sqrt{2} = 2$ which is a rational number.

   $\Rightarrow (2, \sqrt{2}) \notin R$

   $\Rightarrow R$ is not transitive

   Option (a) is correct.

4. Given, $aRb \Rightarrow a$ is brother of b

   This does not mean that b is also a brother of a because b can be a sister of a.

   Hence, R is not symmetric.

   Again, $aRb \Rightarrow a$ is brother of b and $bRc \Rightarrow b$ is brother of c.

   So, a is brother of c.

   Hence, R is transitive.

   Option (b) is correct.

5. We are given set A = {1, 2, 3}

Number of equivalance relation on A = number of possible portion of {1, 2, 3}

i.e., 3 = 1 + 1 + 1 Only one combination

3 = 1 + 2 3 Possible combination

3 = 3 1 possible combination

i.e., (*i*) {{1}, {2}, {3}} *i.e.*, {(1, 1), (2, 2), (3, 3)}

(*ii*) {{1, 2}, {3} *i.e.*, {(1, 2), (2, 1), (3, 3)}

(*iii*) {{1, 3}, {2}} *i.e.*, {(1, 3), (3, 1), (2, 2)}

(*iv*) {{2, 3}, {1}} *i.e.*, {(2, 3), (3, 2), (1, 1)}

(*v*) {{1, 2, 3}} *i.e.*, {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}

i.e., Total number of equivalence relation = 5

Option (*d*) is correct.

6. For $l, m \in L$

if $(l, m) \in R \Rightarrow l \perp m \Rightarrow m \perp l \Rightarrow (m, l) \in R$

$\therefore$ R is symmetric.

Option (*b*) is correct.

7. Required relation is reflexive and symmetric but not transitive is given by

$R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (1, 3) (3, 1)\}$

which is reflexive as $(a, a) \in R \ \forall \ a \in A$

which is symmetric as $(a, b) \in R \Rightarrow (b, a) \in R$ for $a, b \in A$

But $(2, 1), (1, 3) \in R \not\Rightarrow (2, 3) \in R$

Hence R is not transitive.

There is only one such relation.

Option (*a*) is correct.

9. We have $n(A) = m$, $n(B) = n$.

$\therefore$ Number of relations defined from A to B

= number of possible subsets of $A \times B = 2^{n(A \times B)} = 2^{mn}$

Option (*a*) is correct.

10. The total number of injective mappings from the set containing n elements into the set containing m elements is ${}^mP_n$. So here it is ${}^4P_3 = 4! = 24$.

Option (*c*) is correct.

11. We have $f : \mathbb{R} \to \mathbb{R}$: such that

$$f(x) = 2^x + 2^{|x|} = \begin{cases} 2^x + 2^x & \text{if } x \geq 0 \\ 2^x + 2^{-x} & \text{if } x < 0 \end{cases} = \begin{cases} 2^{x+1} & \text{if } x \geq 0 \\ 2^x + 2^{-x} & \text{if } x < 0 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} 2^{x+1} \log 2 & \text{if } x \geq 0 \\ 2^x \log 2 + (-2^{-x} \log 2) & \text{if } x < 0 \end{cases} = \begin{cases} 2^{x+1} \log 2 & \text{if } x \geq 0 \\ \log 2\,(2^x - 2^{-x}) & \text{if } x < 0 \end{cases}$$

$\because f'(x) > 0 \ \forall \ x \geq 0$ and $f'(x) < 0 \ \forall \ x < 0$

$\Rightarrow f(x)$ is strictly increasing in $\mathbb{R} \Rightarrow f(x)$ is one-one.

Also, $f(x) \to \infty$ if $x \to \pm\infty$ and $f(x) > 0 \ \forall \ x \in \mathbb{R}$

$\Rightarrow f$ is an into function.

$\therefore$ $f(x)$ is one-one and into function.

Option (*c*) is correct.

12. We have $n(A) = 5$ and $n(B) = 6$

$\therefore$ Number of one-one mopping from A to $B = 6!$

As $n(A) < n(B)$

$\Rightarrow$ There is no onto function from A to B. *i.e.*, number of onto function = 0.

$\therefore$ Number of one-one and onto functions from A to $B = 0$

Option (*c*) is correct.

13. $f : \mathbb{Z} \to \mathbb{Z}$ given by $f(x) = x + 2$

One-one For $x_1, x_2 \in \mathbb{Z}$ such that $x_1 \neq x_2 \quad \Rightarrow \quad x_1 + 2 \neq x_2 + 2$

$\Rightarrow f(x_1) \neq f(x_2)$

$\Rightarrow f$ is one-one.

Onto Let $y \in \mathbb{Z}$ (co-domain) such that

$$f(x) = y \quad \Rightarrow \quad x + 2 = y \quad \Rightarrow \quad x = y - 2$$

For $y \in \mathbb{Z}$ (co-domain), $\exists\ x = y - 2 \in \mathbb{Z}$ (domain) such that

$$f(x) = f(y - 2) = y - 2 + 2 = y$$

$\Rightarrow f$ is onto

As f is one-one and onto.

$\Rightarrow f$ is a bijective function.

Option (*b*) is correct.

14. Given that, $\quad f(x) = x^2 - 4x + 5$

Let $\quad y = x^2 - 4x + 5$

$$y = x^2 - 4x + 4 + 1 = (x - 2)^2 + 1$$

$$(x - 2)^2 = y - 1 \quad \Rightarrow \quad x - 2 = \sqrt{y - 1}$$

$$\Rightarrow \quad x = 2 + \sqrt{y - 1}$$

$\therefore \quad y - 1 \geq 0, y \geq 1$

Range = $[1, \infty)$

Option (*b*) is correct.

15. Let $f(x) = 17 \Rightarrow x^2 + 1 = 17$

$\Rightarrow \quad x = \pm 4 \Rightarrow$ Pre image of 17 are $\{4, -4\}$

and let $f(x) = -3 \Rightarrow x^2 + 1 = -3 \Rightarrow x^2 = -4$ which is not true and hence -3 has no pre image

17. $\because b > 6$ and $a = b - 2$

$\Rightarrow (6, 8) \in R$ as $8 > 6$ and $6 = 8 - 2$

Option (*c*) is correct.

18. f is not one-one because

$$f(-2) = (-2)^4 = 16$$

$$f(2) = (2)^4 = 16$$

i.e., -2 and $2 \in R$ (Domain) have same f-image in R (co-domain)

$\Rightarrow f$ is not one-one.

Also $f(x) = x^4$ never achieve negative value.

$\Rightarrow$ All negative real number of co-domain R have no pre-image in Domain R.

$\Rightarrow f$ is not onto.

Hence, f is neither one-one nor onto.

Option (d) is correct.

19. f is one-one because

$$f(x_1) = f(x_2) \Rightarrow 3x_1 = 3x_2$$

$\Rightarrow \quad x_1 = x_2 \quad \forall\, x_1, x_2 \in R \quad$ (Domain)

Also f is onto as

Let $f(x) = y \quad \Rightarrow \quad 3x = y \quad \Rightarrow \quad x = \dfrac{y}{3}$

$\forall\, y \in R$ (codomain) $\exists x = \dfrac{y}{3} \in R$ (domain)

such that $f(x) = f\left(\dfrac{y}{3}\right) = 3 \times \dfrac{y}{3} = y$

$\Rightarrow \quad f(x)$ is onto

Therefore, f is one-one onto.

Option (a) is correct.

21. Let x be the pre image of 5

$\Rightarrow f(x) = 5$

$\Rightarrow x^2 + 1 = 5$

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

i.e., pre-image of 5 is –2, + 2.

Similarly if x be pre-image of – 5

$\Rightarrow f(x) = -5$

$\Rightarrow x^2 + 1 = -5$

$\Rightarrow x^2 = -6$

$x = \pm\sqrt{-6} \notin R$

i.e., No real number is pre-image of – 5. Hence ϕ is the primage of –5.

Option (c) is correct.

22. To find out the domain of f, we have to find out that value of x for which $f(x)$ is real.

$\Rightarrow \quad x^2 - 4 \geq 0$

$\Rightarrow \quad (x+2)(x-2) \geq 0$

$\Rightarrow \quad (x+2) \geq 0, (x-2) \geq 0$ or $(x+2) \leq 0, (x-2) \leq 0$

$\Rightarrow \quad x \geq -2, x \geq 2$ or $x \leq -2, x \leq 2$

$\Rightarrow \quad x \geq 2$ or $x \leq -2$

Domain of f is $(-\infty, -2] \cup [2, \infty)$

Option (c) is correct.

23. $f(-2) + f(0) + f(2) + f(5) = -2 + 0 + 4 + 15 = 17$

Option (b) is correct.

24. As R is reflexive relation on A, and for being reflexive $(a, a) \in R, \forall\, a \in A$

Therefore, the minimum number of ordered pair in R is n.

$\Rightarrow m \geq n$.

Option (a) is correct.

25. Given function is $f(x) = \log_{3+x}(x^2 - 1)$

It is obvious that $f(x)$ is defined when $x^2 - 1 > 0$, $3 + x > 0$ and $3 + x \neq 1$.

Now, $x^2 - 1 > 0 \Rightarrow x^2 > 1$

$\Rightarrow x < -1$ or $x > 1$

$3 + x > 0 \Rightarrow x > -3$

$3 + x \neq 1 \Rightarrow x \neq -2$

Therefore, domain of the function $f(x) = (-3, -2) \cup (-2, -1) \cup (1, \infty)$

Option (c) is correct.

26. From definition of onto function,

Range of function = Codomain of function = $[0, \frac{\pi}{2})$

$\Rightarrow \quad 0 \leq \tan^{-1}(x^2 + x + a) < \frac{\pi}{2}$

$\rightarrow \quad 0 \leq (x^2 + x + a) < \infty$

$\Rightarrow \quad x^2 + x + a > 0 \ \forall x \in R$

Hence $D \leq 0$

$\Rightarrow \quad 1^2 - 4a \leq 0$

$\Rightarrow \quad 4a \geq 1$

$\Rightarrow \quad a \geq \frac{1}{4}$

$\Rightarrow \quad a \in [\frac{1}{4}, \infty)$

Option (b) is correct.

27. We have,

$f: R \rightarrow R$ and $g: R \rightarrow R$ are such that

$$f(x) = \begin{cases} 0, & x \in \text{rational} \\ x, & x \in \text{irrational} \end{cases}$$

and $$g(x) = \begin{cases} 0, & x \in \text{irrational} \\ x, & x \in \text{rational} \end{cases}$$

$\therefore (f - g): R \rightarrow R$ such that,

$$(f - g)(x) = \begin{cases} -x, & x \in \text{rational} \\ x, & x \in \text{irrational} \end{cases}$$

From definition of $(f - g): R \rightarrow R$, it is obvious that, each rational number of domain of $(f - g)(x)$, associate to its negative rational number in codomain/range and each irrational number of domain of $(f - g)(x)$, associate to same irrational number in codomain/range.

$\Rightarrow$ For each $x \in$ Domain of $(f - g)(x)$, there is only one value in codomain/range of $(f - g)(x)$.

Hence, $(f - g)(x)$ is one-one onto.

Option (a) is correct.

28. We have set $A = \{1, 2, 3, 4\}$ & relation

$R = \{(1, 2), (3, 4)\}$ on A

As for $(a, b) \in R, \nexists (b, c) \in R$ such that

$(a, c) \notin R$.

Hence R is transitive.

So (*b*) is correct option.

Option (*b*) is correct.

29. $\because |A| = 4, |B| = 5$ so there does not exist.

one-one and onto $\because |B| > |A|$ so it is not onto.

So (*a*) is correct option.

Option (*a*) is correct.

30. Number of onto functions are given by

$2^3 - {}^2C_1 (2-1)^3 + {}^2C_2 (2-2)^3$

$= 8 - 2 \times 1 + 0 = 8 - 2 = 6$

Option (*c*) is correct.

31. Number of one-one function $= {}^6P_4$

$= \frac{\underline{|6}}{\underline{|2}} = \frac{720}{2} = 360$

Option (*c*) is correct.

32. Number of one-one and onto function from A to B where $|A| = m$ is $\underline{|m}$.

$\therefore$ Number of one-one onto function $= \underline{|4} = 24$

Option (*b*) is correct.

[**Note:** One-one onto function (bijective) from A to B is possible if A and B have same number of elements.]

34. Let R be a relation on the set of all intergers $\mathbb{Z}$, defined by

$aRb \Leftrightarrow a > b \ \forall \ a, b \in \mathbb{Z}$

(*i*) **Reflexive:** For $1 \in \mathbb{Z}$

$1 \not R 1$ as $1 \not> 1$ so $(1, 1) \notin R \Rightarrow R$ is not reflexive on $\mathbb{Z}$

(*ii*) **Symmetric:** $(3, 2) \subset R$ as $3 > 2$

But $(2, 3) \notin R$ as $2 \not> 3$

Hence R is not symmetric on $\mathbb{Z}$

(*iii*) **Transitive:** Let $(a, b) \in R$ and $(b, c) \in R$ $a > b$ and $b > c$

Now $a > b > c \Rightarrow a > c \Rightarrow (a, c) \in R$

Hence R is a transitive relation on $\mathbb{Z}$

Option (*c*) is correct.

36. $f(x) = 4^x + 4^{|x|}$

One-one

Let $x_1, x_2 \in R$ (domain) such that

$x_1 \neq x_2$

$\Rightarrow 4^{x_1} + 4^{|x_1|} \neq 4^{x_2} + 4^{|x_2|}$

$\Rightarrow f(x_1) \neq f(x_2)$

f is one-one

Onto

For $0 \in R$ (Co-domain) there is no $x \in R$ (domain) such that $f(x) = 0$.

$\therefore$ f is not onto

Range of $f = R - \{0\} \subseteq R$

Hence f is one-one into function.

Option (a) is correct.

39. Here co-domain $= \left[0, \frac{\pi}{2}\right)$

For onto function, we have

Co-domain = Range $= 0 \leq x < \frac{\pi}{2}$

This is valid if $x^2 + x + 2a \geq 0$

[$\because f(x) \geq 0$ *i.e.* $Ax^2 + Bx + C \geq 0$ then $D \leq 0$ if $A > 0$.]

i.e., $x^2 + x + 2a \geq 0 \Rightarrow 1^2 - 4 \times 1 \times 2a \leq 0$

$\rightarrow 1 - 8a \leq 0 \rightarrow 1 \leq 8a \rightarrow 8a \geq 1 \rightarrow a \geq \frac{1}{8}$

$\therefore\ a \in \left[\frac{1}{8}, \infty\right)$

Option (d) is correct.

40. We are given that

$$(f(x))^2 - 4f(x)f'(x) + (f'(x))^2 = 0$$

$$\Rightarrow f'(x) = \frac{4f(x) \pm \sqrt{16(f(x))^2 - 4(f(x))^2}}{2}$$

$$\therefore f'(x) = \frac{4f(x) \pm 2f(x)\sqrt{4-1}}{2}$$

$$= \frac{4f(x) \pm 2\sqrt{3}f(x)}{2}$$

$$= f(x)(2 \pm \sqrt{3})$$

$$\Rightarrow \frac{f'(x)}{f(x)} = (2 \pm \sqrt{3})$$

Integrating, we get

$\Rightarrow \log f(x) = (2 \pm \sqrt{3})x + C$

$\Rightarrow f(x) = e^{(2 \pm \sqrt{3})x + C} = e^C e^{(2 \pm \sqrt{3})x} = \lambda e^{(2 \pm \sqrt{3})x}$

where $\lambda = e^C$

$\Rightarrow f(x) = \lambda e^{(2 \pm \sqrt{3})x} = \lambda e^{(2+\sqrt{3})x}, \lambda e^{(2-\sqrt{3})x}$

Option (c) is correct.

41. $f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1} x$

$f(x)$ is one-one and onto

i.e., $f'(x) > 0$ or $f'(x) < 0$ and co-domain = range of $f(x)$

$B = f(-1, 1) = (2\tan^{-1}(-1), 2\tan^{-1}(1))$

$$= \left(2 \times \left(-\frac{\pi}{4}\right), 2 \times \frac{\pi}{4}\right) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Option (*c*) is correct.

42. Let $f(x) = y = \dfrac{x}{1+|x|}\ \forall x \in \mathbb{R}, y \in \mathbb{R}$

$\therefore f(x) = \dfrac{x}{1+x}$ or $\dfrac{x}{1-x}$ is one-one

Here range of $f(x)$ is $R - \{-1, 1\}$

But y can not have any of the values −1, 1 for some x.

$\therefore f(x)$ is not an onto function.

Option (*c*) is correct.

44. Number of onto function

$$= 3^6 - {}^3C_1 (3-1)^6 + {}^3C_2 (3-2)^6 - {}^3C_3 (3-3)^6$$

$$= 3^6 - 3 \times 2^6 + 3 \times 1 = 3^6 - 3 \times 2^6 + 3$$

$$= 3 \times (3^5 - 2^6 + 1) = 3(243 - 64 + 1)$$

$$= 3 \times (244 - 64) = 3 \times 180 = 540$$

Option (*d*) is correct.

45. As $(7, 7) \notin R$, so R can not be reflexive

Again $(7, 8) \in R$ but $(8, 7) \notin R$, so R is not symmetric.

As $(7, 8), (8, 8) \in R \Rightarrow (7, 8) \in R \Rightarrow R$ is transitive.

Option (*a*) is correct.

46. Let $A = \{1, 2, 3, ..., 12\}$, $n(A) = 12$ (say m)

$B = \{1, 2, 3, ..., 11\}$, $n(B) = 11$ (say n)

$\therefore$ Total number of function from A to $B = 11^{12}$

$\therefore$ Number of onto functions from A to $B = \sum_{r=1}^{n} (-1)^{n-r}\ {}^nC_r r^m$

= coefficient of x^m in $m!\,(e^x - 1)^n$ ...(*i*)

Putting $m = 12$, $n = 11$ and $r = 1, 2, 3, ..., 11$. The number of onto functions is given by

$$= (-1)^{11-1}\,{}^{11}C_1 1^{12} + (-1)^{11-2}\,{}^{11}C_2 2^{12} + (-1)^{11-3}\,{}^{11}C_3 3^{12}$$

$$+ ... + (-1)^1\,{}^{11}C_{10} 10^{12} + (-1)^0\,{}^{11}C_{11} 11^{12}$$

$$= {}^{11}C_{11} 11^{12} - {}^{11}C_{10} 10^{12} + {}^{11}C_9 9^{12} + ... + {}^{11}C_3 3^{12} - {}^{11}C_2 2^{12} + {}^{11}C_1 1^{12}$$

$$= {}^{11}C_0 11^{12} - {}^{11}C_1 10^{12} + {}^{11}C_2 9^{12} + ... + {}^{11}C_8 3^{12} - {}^{11}C_9 2^{12} + {}^{11}C_{10} 1^{12}$$

Also, coefficient x^{12} in $12!\,(e^x - 1)^{11}$

= coefficient of x^{12} **in** $12!\left(1 + \dfrac{x}{1!} + \dfrac{x^2}{2!} + ... + \infty - 1\right)^{11}$

= coefficient of x^{12} **in** $12!\left(\dfrac{x}{1!} + \dfrac{x^2}{2!} + ... + \infty\right)^{11}$

= coefficient of x^{12} **in** $12!\,x^{11}\left(\dfrac{1}{1!} + \dfrac{x}{2!} + \dfrac{x^2}{3!} + ... + \infty\right)^{11}$

= coefficient of x **in** $12! \times \left[1 + \left(\dfrac{x}{2!} + \dfrac{x^2}{3!} + ... + \infty\right)\right]^{11}$

$$= \textbf{coefficient of } x \textbf{ in } 12!\left[{}^{11}C_0\,1 + {}^{11}C_1\left(\frac{x}{2!}+\frac{x^2}{3!}+ .... + \infty\right) + {}^{11}C_2\left(\frac{x}{2!}+\frac{x^2}{3!}+ .... + \infty\right) + ...\right]$$

$$= \textbf{coefficient of } x \textbf{ in } 12!\left[{}^{11}C_1\left(\frac{1}{2!}\right)\right] = \frac{12! \times 11}{2!}$$

Total number of functions which are not onto $= 11^{12} - \dfrac{12! \times 11}{2}$

Option (*d*) is correct.

47. $\because f(x\ f(y)) = x^a y^b \qquad ...(i)$

Replacing x by $\dfrac{1}{f(y)}$, we have from (*i*)

$$f(x \times f(y)) = f\left(x \times \frac{1}{x}\right) = \left(\frac{1}{f(y)}\right)^a y^b$$

$$f(1) = \frac{y^b}{(f(y))^a} \Rightarrow (f(y))^a = \frac{y^b}{f(1)}$$

$$\therefore \text{ Put } y = 1, (f(1))^a = \frac{1^b}{f(1)} = \frac{1}{f(1)}$$

$$\Rightarrow (f(1))^{a+1} - 1$$

$$\Rightarrow f(1) = 1^{\left(\frac{1}{a+1}\right)} = 1$$

$$\Rightarrow f(1) = \frac{y^b}{(f(y))^a} = 1 \Rightarrow (f(y))^a = y^b$$

$$\Rightarrow f(y) = y^{b/a}$$

Replacing y as x, we have

$$f(x) = x^{b/a} \qquad ...(ii)$$

$$\therefore f(x\,.\,y^{b/a}) = x^a y^b$$

Let $y^{b/a} = t \Rightarrow y = t^{a/b}$

$$f(x\,.\,t) = x^a t^a \Rightarrow f(x) = x^a \qquad ...(iii)$$

Now from (*ii*) and (*iii*), we get

$$x^{b/a} = x^a \Rightarrow \frac{a}{b} = \frac{1}{a} \Rightarrow b = a^2$$

Option (*c*) is correct.

51. Number of relations that can be defined on $A = 2^{n^2}$

Option (*c*) is correct.

54. We have $R = \{(a, a), (b, b), (c, c), (b, c), (a, b)\}$

For $(b, c) \in R$, but $(c, b) \notin R$.

Hence R is not symmetric.

Also for $(a, b), (b, c) \in R$ but $(a, c) \notin R$.

$\Rightarrow$ R is not transitive.

As $(a, a) \in R\ \forall\ a \in A$

Hence R is reflexive.

Option (*c*) is correct.

55. Let $A = \{1, 2\}, B = \{a, b\}$

Let $R = \{(1, a), (1, b), (2, a), (2, b)\}$

Clearly R is a relation from A to B

But R is not a function.

As $(1, a), (1, b) \in R$ and $(2, a), (2, b) \in R$

Option (b) is correct.

56. $R = \{(1, 2)\}, A = \{1, 2, 3\}$

Clearly R is neither reflexive nor symmetric.

As $(1, 2) \in R$ but $\nexists\ (2, b) \in R$ for $b \in A$ such that $(1, b) \notin R$.

Hence R is a transitive relation on A.

Option (b) is correct.

57. $R = \{(a, b) : a \geq b\}$

Reflexive

Clearly $(a, a) \in R\ \forall a \in R$.

Hence R is reflexive.

Symmetric

$\because (2, 1) \in R$ but $(1, 2) \notin R$

Hence R is not symmetric.

Transitive

Let (a, b) and $(b, c) \in R$

$\Rightarrow \quad a \geq b$ and $b \geq c$

$\Rightarrow \quad a \geq c$

Hence (a, b) and $(b, c) \in R \Rightarrow (a, c) \in R$

$\Rightarrow \quad R$ is a transitive relation on R.

Option (b) is correct.

59. $R = \{(x, y) : |x^2 - y^2| < 7\}$

$R = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (1, 2), (2,3)\}$

Option (d) is correct.