MIND MAP : LEARNING MADE SIMPLE CHAPTER - 7

Permutations & Combinations

Combinations under Certain Conditions

- The no. of arrangements of n distinct objects taken r at a time, so that k particular objects are
  (i) Always included: ${}^{n-k}C_{r-k}.r!$
  (ii) Never included: ${}^{n-k}C_{r}.r!$
- The no. of selections from 'n' different objects, taking at least one $= {}^{n}C_1 + {}^{n}C_2 + {}^{n}C_3 + \ldots\ldots\ldots\ldots {}^{n}C_n = 2^n - 1$

Combinations

Each of different selections made by taking source or all of a number of distinct objects or item, irrespective of their arrangements or order in which they are placed, is called a combination.

The no. of combinations of n different things taken r at a time, denoted by ${}^{n}C_r$ is given by

$${}^{n}C_r = \frac{n!}{r!(n-r)!};\ 0 \le r \le n$$

Fundamental Principle of Counting (F.P.C.)

F.P.C. of Multiplication: If an event can occur in m different ways, following which another event in n different ways, then the total number of occurance of the events in the given order is $m \times n$

F.P.C. of Addition: If there are two events such that they can performed independently in m and n ways respectively, then either of the two events can be performed in $(m+n)$ ways.

Permutation

Each of the different arrangements which can be made by taking some or all of a number of distinct objects is called a **permutation**.

The no. of permutations of n different things taken r at a time, where repetition is not allowed, is denoted by ${}^{n}P_r$ and is given by

$${}^{n}P_r = \frac{n!}{(n-r)!}$$

Permutations under given Conditions

The no. of all permutations of 'n' different objects taken r at a time:

(i) When a particular object is to be always included in each arrangement is ${}^{n-1}P_r$

(ii) When a particular object is never taken in each arrangement in ${}^{n-1}P_r$

(iii) The no. of permutation of n different things taken r at a time, where repetition is allowed is $(n)^r$

Permutations of alike Objects

The no. of permutations of n objects taken all at a time, where p_1 objects are of first kind, p_2 objects are of the second kind, ...p_k objects are of the kth kind and rest, if any one are all different is $\dfrac{n!}{p_1!\,p_2!\,p_3!\ldots p_k!}$

Circular Permutations

1. The no. of circular permutations of 'n' distinct objects is $(n-1)!$
2. If anti-clockwise and clockwise order of arrangements are not distinct then the no. of circular permutations of n distinct items is $\frac{1}{2}(n-1)!$

e.g.: arrangements of beads in a necklace, arrangements of flower in a garland etc.

Some Special Results of ${}^{n}C_r$

Since,

$${}^{n}C_r = \frac{n!}{r!(n-r)!}$$

- In particular, $r = n$
- ${}^{n}C_r = \dfrac{n!}{n!0!} = 1$
- ${}^{n}C_{n-r} = {}^{n}C_r$
- ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

Relation between ${}^{n}P_r$ and ${}^{n}C_r$

${}^{n}P_r = {}^{n}C_r \times r!$ $\quad\because 0 < r \le n$

i.e., Corresponding to each combination of ${}^{n}C_r$, we have r! permutations, because r objects in every combination can be rearranged in r! ways