

## Chaper 6. System of Particles and Rotational Motion

1. A rope is wound around a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N?  
 (a)  $0.25 \text{ rad s}^{-2}$  (b)  $25 \text{ rad s}^{-2}$   
 (c)  $5 \text{ m s}^{-2}$  (d)  $25 \text{ m s}^{-2}$   
 (NEET 2017)
2. Two discs of same moment of inertia rotating about their regular axis passing through centre and perpendicular to the plane of disc with angular velocities  $\omega_1$  and  $\omega_2$ . They are brought into contact face to face coinciding the axis of rotation. The expression for loss of energy during this process is  
 (a)  $\frac{1}{4} I (\omega_1 - \omega_2)^2$  (b)  $I (\omega_1 - \omega_2)^2$   
 (c)  $\frac{1}{8} I (\omega_1 - \omega_2)^2$  (d)  $\frac{1}{2} I (\omega_1 + \omega_2)^2$   
 (NEET 2017)
3. Which of the following statements are correct?  
 (1) Centre of mass of a body always coincides with the centre of gravity of the body.  
 (2) Centre of mass of a body is the point at which the total gravitational torque on the body is zero.  
 (3) A couple on a body produces both translational and rotational motion in a body.  
 (4) Mechanical advantage greater than one means that small effort can be used to lift a large load.  
 (a) (1) and (2) (b) (2) and (3)  
 (c) (3) and (4) (d) (2) and (4)  
 (NEET 2017)
4. Two rotating bodies A and B of masses  $m$  and  $2m$  with moments of inertia  $I_A$  and  $I_B$  ( $I_B > I_A$ ) have equal kinetic energy of rotation. If  $L_A$  and  $L_B$  be their angular momenta respectively, then  
 (a)  $L_A = \frac{L_B}{2}$  (b)  $L_A = 2L_B$   
 (c)  $L_B > L_A$  (d)  $L_A > L_B$   
 (NEET-II 2016)
5. A solid sphere of mass  $m$  and radius  $R$  is rotating about its diameter. A solid cylinder of the same mass and same radius is also rotating about its geometrical axis with an angular speed twice that of the sphere. The ratio of their kinetic energies of rotation ( $E_{\text{sphere}}/E_{\text{cylinder}}$ ) will be  
 (a) 2 : 3 (b) 1 : 5  
 (c) 1 : 4 (d) 3 : 1  
 (NEET-II 2016)
6. A light rod of length  $l$  has two masses  $m_1$  and  $m_2$  attached to its two ends. The moment of inertia of the system about an axis perpendicular to the rod and passing through the centre of mass is  
 (a)  $\frac{m_1 m_2}{m_1 + m_2} l^2$  (b)  $\frac{m_1 + m_2}{m_1 m_2} l^2$   
 (c)  $(m_1 + m_2) l^2$  (d)  $\sqrt{m_1 m_2} l^2$   
 (NEET-II 2016)
7. A disc and a sphere of same radius but different masses roll off on two inclined planes of the same altitude and length. Which one of the two objects gets to the bottom of the plane first?  
 (a) Both reach at the same time  
 (b) Depends on their masses  
 (c) Disc  
 (d) Sphere  
 (NEET-I 2016)
8. From a disc of radius  $R$  and mass  $M$ , a circular hole of diameter  $R$ , whose rim passes through the centre is cut. What is the moment of inertia of the remaining part of the disc about a perpendicular axis, passing through the centre?  
 (a)  $11MR^2/32$  (b)  $9MR^2/32$   
 (c)  $15MR^2/32$  (d)  $13MR^2/32$   
 (NEET-I 2016)
9. A uniform circular disc of radius 50 cm at rest is free to turn about an axis which is perpendicular to its plane and passes through its centre. It is subjected to a torque which produces a constant angular acceleration of  $2.0 \text{ rad s}^{-2}$ . Its net acceleration in  $\text{m s}^{-2}$  at the end of 2.0 s is approximately

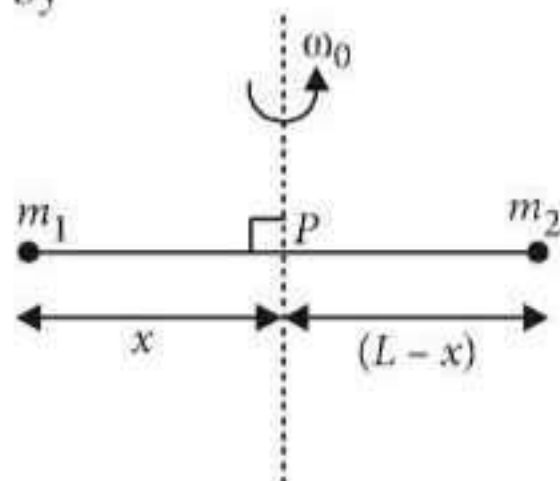


- (a) 6.0 (b) 3.0 (c) 8.0 (d) 7.0

(NEET-I 2016)

10. Point masses  $m_1$  and  $m_2$  are placed at the opposite ends of a rigid rod of length  $L$ , and negligible mass. The rod is to be set rotating about an axis perpendicular to it. The position of point  $P$  on this rod through which the axis should pass so that the work required to set the rod rotating with angular velocity  $\omega_0$  is minimum, is given by

- (a)  $x = \frac{m_2}{m_1} L$   
 (b)  $x = \frac{m_2 L}{m_1 + m_2}$   
 (c)  $x = \frac{m_1 L}{m_1 + m_2}$   
 (d)  $x = \frac{m_1}{m_2} L$



(2015)

11. An automobile moves on a road with a speed of  $54 \text{ km h}^{-1}$ . The radius of its wheels is  $0.45 \text{ m}$  and the moment of inertia of the wheel about its axis of rotation is  $3 \text{ kg m}^2$ . If the vehicle is brought to rest in  $15 \text{ s}$ , the magnitude of average torque transmitted by its brakes to the wheel is

- (a)  $10.86 \text{ kg m}^2 \text{ s}^{-2}$  (b)  $2.86 \text{ kg m}^2 \text{ s}^{-2}$   
 (c)  $6.66 \text{ kg m}^2 \text{ s}^{-2}$  (d)  $8.58 \text{ kg m}^2 \text{ s}^{-2}$

(2015)

12. A force  $\vec{F} = \alpha \hat{i} + 3\hat{j} + 6\hat{k}$  is acting at a point  $\vec{r} = 2\hat{i} - 6\hat{j} - 12\hat{k}$ . The value of  $\alpha$  for which angular momentum about origin is conserved is

- (a) zero (b) 1  
 (c) -1 (d) 2

(2015)

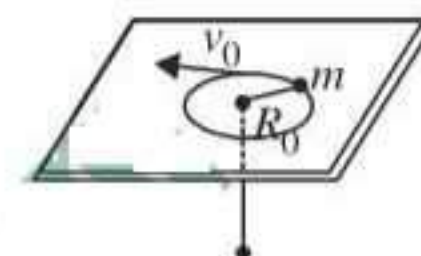
13. A rod of weight  $W$  is supported by two parallel knife edges  $A$  and  $B$  and is in equilibrium in a horizontal position. The knives are at a distance  $d$  from each other. The centre of mass of the rod is at distance  $x$  from  $A$ . The normal reaction on  $A$  is

- (a)  $\frac{W(d-x)}{x}$  (b)  $\frac{W(d-x)}{d}$   
 (c)  $\frac{Wx}{d}$  (d)  $\frac{Wd}{x}$

(2015 Cancelled)

14. A mass  $m$  moves in a circle on a smooth horizontal plane with velocity  $v_0$  at a radius  $R_0$ . The mass is attached to a string which passes through a smooth hole in the plane as shown. The tension in the string is increased gradually and finally  $m$  moves in a circle of radius  $\frac{R_0}{2}$ . The final value of the kinetic energy is

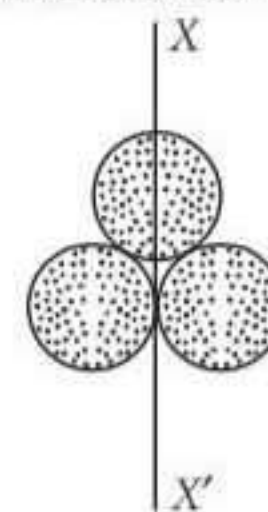
- (a)  $2mv_0^2$   
 (b)  $\frac{1}{2}mv_0^2$   
 (c)  $mv_0^2$   
 (d)  $\frac{1}{4}mv_0^2$



(2015 Cancelled)

15. Three identical spherical shells, each of mass  $m$  and radius  $r$  are placed as shown in figure. Consider an axis  $XX'$  which is touching to two shells and passing through diameter of third shell. Moment of inertia of the system consisting of these three spherical shells about  $XX'$  axis is

- (a)  $\frac{16}{5}mr^2$   
 (b)  $4mr^2$   
 (c)  $\frac{11}{5}mr^2$   
 (d)  $3mr^2$



(2015 Cancelled)

16. A solid cylinder of mass  $50 \text{ kg}$  and radius  $0.5 \text{ m}$  is free to rotate about the horizontal axis. A massless string is wound round the cylinder with one end attached to it and other hanging freely. Tension in the string required to produce an angular acceleration of  $2 \text{ revolutions s}^{-2}$  is

- (a)  $25 \text{ N}$  (b)  $50 \text{ N}$  (c)  $78.5 \text{ N}$  (d)  $157 \text{ N}$

(2014)

17. The ratio of the accelerations for a solid sphere (mass  $m$  and radius  $R$ ) rolling down an incline of angle  $\theta$  without slipping and slipping down the incline without rolling is

- (a)  $5:7$  (b)  $2:3$  (c)  $2:5$  (d)  $7:5$

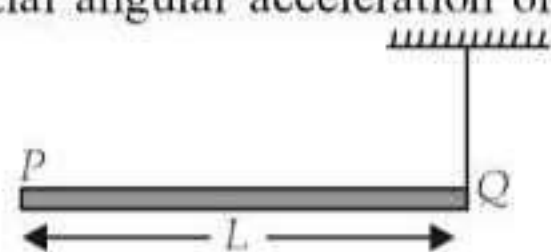
(2014)

18. A rod  $PQ$  of mass  $M$  and length  $L$  is hinged at end  $P$ . The rod is kept horizontal by a massless string tied to point  $Q$  as shown in figure. When



string is cut, the initial angular acceleration of the rod is

- (a)  $\frac{2g}{L}$   
 (b)  $\frac{2g}{2L}$   
 (c)  $\frac{3g}{2L}$   
 (d)  $\frac{g}{L}$  (NEET 2013)



19. A small object of uniform density rolls up a curved surface with an initial velocity 'v'. It reaches up to a maximum height of  $\frac{3v^2}{4g}$  with respect to the initial position. The object is

- (a) hollow sphere (b) disc  
 (c) ring (d) solid sphere (NEET 2013)

20. The ratio of radii of gyration of a circular ring and a circular disc, of the same mass and radius, about an axis passing through their centres and perpendicular to their planes are

- (a)  $1:\sqrt{2}$  (b)  $3:2$  (c)  $2:1$  (d)  $\sqrt{2}:1$

(Karnataka NEET 2013)

21. Two discs are rotating about their axes, normal to the discs and passing through the centres of the discs. Disc  $D_1$  has 2 kg mass and 0.2 m radius and initial angular velocity of  $50 \text{ rad s}^{-1}$ . Disc  $D_2$  has 4 kg mass, 0.1 m radius and initial angular velocity of  $200 \text{ rad s}^{-1}$ . The two discs are brought in contact face to face, with their axes of rotation coincident. The final angular velocity (in  $\text{rad s}^{-1}$ ) of the system is

- (a) 60 (b) 100 (c) 120 (d) 40

(Karnataka NEET 2013)

22. When a mass is rotating in a plane about a fixed point, its angular momentum is directed along

- (a) a line perpendicular to the plane of rotation  
 (b) the line making an angle of  $45^\circ$  to the plane of rotation  
 (c) the radius  
 (d) the tangent to the orbit (2012)

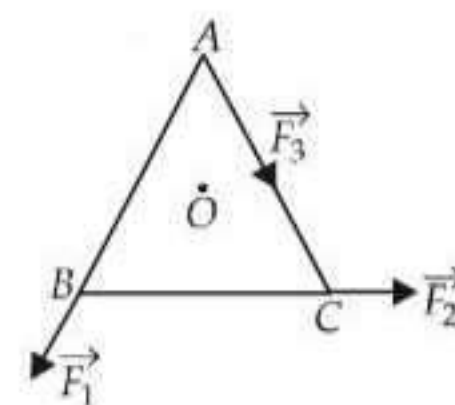
23. Two persons of masses 55 kg and 65 kg respectively, are at the opposite ends of a boat. The length of the boat is 3.0 m and weighs 100 kg. The 55 kg man walks up to the 65 kg man and sits with him. If the boat is in still water the center of mass of the system shifts by

- (a) 3.0 m (b) 2.3 m (c) zero (d) 0.75 m (2012)

24. A car of mass 1000 kg negotiates a banked curve of radius 90 m on a frictionless road. If the banking angle is  $45^\circ$ , the speed of the car is  
 (a)  $20 \text{ m s}^{-1}$  (b)  $30 \text{ m s}^{-1}$   
 (c)  $5 \text{ m s}^{-1}$  (d)  $10 \text{ m s}^{-1}$  (2012)

25. ABC is an equilateral triangle with O as its centre.  $\vec{F}_1$ ,  $\vec{F}_2$  and  $\vec{F}_3$  represent three forces acting along the sides AB, BC and AC respectively. If the total torque about O is zero then the magnitude of  $\vec{F}_3$  is

- (a)  $F_1 + F_2$   
 (b)  $F_1 - F_2$   
 (c)  $\frac{F_1 + F_2}{2}$   
 (d)  $2(F_1 + F_2)$



(2012, 1998)

26. A car of mass  $m$  is moving on a level circular track of radius  $R$ . If  $\mu_s$  represents the static friction between the road and tyres of the car, the maximum speed of the car in circular motion is given by

- (a)  $\sqrt{\mu_s m R g}$  (b)  $\sqrt{\frac{R g}{\mu_s}}$   
 (c)  $\sqrt{\frac{m R g}{\mu_s}}$  (d)  $\sqrt{\mu_s R g}$

(Mains 2012)

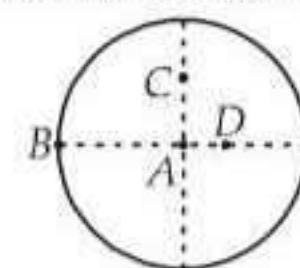
27. A circular platform is mounted on a frictionless vertical axle. Its radius  $R = 2 \text{ m}$  and its moment of inertia about the axle is  $200 \text{ kg m}^2$ . It is initially at rest. A 50 kg man stands on the edge of the platform and begins to walk along the edge at the speed of  $1 \text{ m s}^{-1}$  relative to the ground. Time taken by the man to complete one revolution is

- (a)  $\pi \text{ s}$  (b)  $\frac{3\pi}{2} \text{ s}$  (c)  $2\pi \text{ s}$  (d)  $\frac{\pi}{2} \text{ s}$

(Mains 2012)

28. The moment of inertia of a uniform circular disc is maximum about an axis perpendicular to the disc and passing through

- (a) B  
 (b) C  
 (c) D  
 (d) A



(Mains 2012)



29. Three masses are placed on the x-axis : 300 g at origin, 500 g at  $x = 40$  cm and 400 g at  $x = 70$  cm. The distance of the centre of mass from the origin is

(a) 40 cm (b) 45 cm  
(c) 50 cm (d) 30 cm

(Mains 2012)

30. The instantaneous angular position of a point on a rotating wheel is given by the equation  $\theta(t) = 2t^3 - 6t^2$

The torque on the wheel becomes zero at

(a)  $t = 1$  s (b)  $t = 0.5$  s  
(c)  $t = 0.25$  s (d)  $t = 2$  s (2011)

31. The moment of inertia of a thin uniform rod of mass  $M$  and length  $L$  about an axis passing through its midpoint and perpendicular to its length is  $I_0$ . Its moment of inertia about an axis passing through one of its ends and perpendicular to its length is

(a)  $I_0 + ML^2/2$  (b)  $I_0 + ML^2/4$   
(c)  $I_0 + 2ML^2$  (d)  $I_0 + ML^2$  (2011)

32. A small mass attached to a string rotates on a frictionless table top as shown. If the tension in the string is increased by pulling the string causing the radius of the circular motion to decrease by a factor of 2, the kinetic energy of the mass will

(a) decrease by a factor of 2  
(b) remain constant  
(c) increase by a factor of 2

(d) increase by a factor of 4 (Mains 2011)



33. A circular disk of moment of inertia  $I_t$  is rotating in a horizontal plane, about its symmetry axis, with a constant angular speed  $\omega_i$ . Another disk of moment of inertia  $I_b$  is dropped coaxially onto the rotating disk. Initially the second disk has zero angular speed. Eventually both the disks rotate with a constant angular speed  $\omega_f$ . The energy lost by the initially rotating disc to friction is

(a)  $\frac{1}{2} \frac{I_b^2}{(I_t + I_b)} \omega_i^2$  (b)  $\frac{1}{2} \frac{I_t^2}{(I_t + I_b)} \omega_i^2$   
(c)  $\frac{I_b - I_t}{(I_t + I_b)} \omega_i^2$  (d)  $\frac{1}{2} \frac{I_b I_t}{(I_t + I_b)} \omega_i^2$

(2010)

34. Two particles which are initially at rest, move

towards each other under the action of their internal attraction. If their speeds are  $v$  and  $2v$  at any instant, then the speed of centre of mass of the system will be

(a)  $2v$  (b) zero (c)  $1.5v$  (d)  $v$

(2010)

35. A gramophone record is revolving with an angular velocity  $\omega$ . A coin is placed at a distance  $r$  from the centre of the record. The static coefficient of friction is  $\mu$ . The coin will revolve with the record if

(a)  $r = \mu g \omega^2$  (b)  $r \leq \frac{\mu^2}{\mu g}$

(c)  $r \leq \frac{\mu g}{\omega^2}$  (d)  $r \geq \frac{\mu g}{\omega^2}$  (2010)

36. From a circular disc of radius  $R$  and mass  $9M$ , a small disc of mass  $M$  and radius  $\frac{R}{3}$  is removed

concentrically. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through its centre is

(a)  $\frac{40}{9} MR^2$  (b)  $MR^2$   
(c)  $4MR^2$  (d)  $\frac{4}{9} MR^2$

(Mains 2010)

37. A solid cylinder and a hollow cylinder, both of the same mass and same external diameter are released from the same height at the same time on an inclined plane. Both roll down without slipping. Which one will reach the bottom first?

(a) Both together only when angle of inclination of plane is  $45^\circ$

(b) Both together

(c) Hollow cylinder

(d) Solid cylinder (Mains 2010)

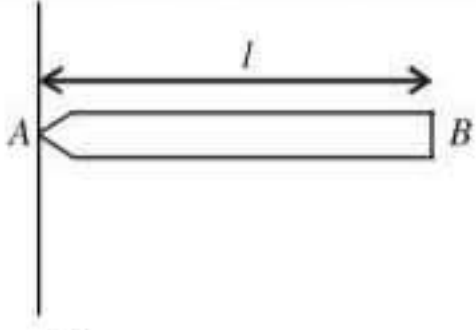
38. A thin circular ring of mass  $M$  and radius  $r$  is rotating about its axis with constant angular velocity  $\omega$ . Two objects each of mass  $m$  are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with angular velocity given by

(a)  $\frac{(M + 2m)\omega}{2m}$  (b)  $\frac{2M\omega}{M + 2m}$

(c)  $\frac{(M + 2m)\omega}{M}$  (d)  $\frac{M\omega}{M + 2m}$

(Mains 2010, 1998)



39. A thin circular ring of mass  $M$  and radius  $R$  is rotating in a horizontal plane about an axis vertical to its plane with a constant angular velocity  $\omega$ . If two objects each of mass  $m$  be attached gently to the opposite ends of a diameter of the ring, the ring will then rotate with an angular velocity
- (a)  $\frac{\omega M}{M+2m}$  (b)  $\frac{\omega(M+2m)}{M}$   
 (c)  $\frac{\omega M}{M+m}$  (d)  $\frac{\omega(M-2m)}{M+2m}$  (2009)
40. If  $\vec{F}$  is the force acting on a particle having position vector  $\vec{r}$  and  $\vec{\tau}$  be the torque of this force about the origin, then
- (a)  $\vec{r} \cdot \vec{\tau} > 0$  and  $\vec{F} \cdot \vec{\tau} < 0$   
 (b)  $\vec{r} \cdot \vec{\tau} = 0$  and  $\vec{F} \cdot \vec{\tau} = 0$   
 (c)  $\vec{r} \cdot \vec{\tau} = 0$  and  $\vec{F} \cdot \vec{\tau} \neq 0$   
 (d)  $\vec{r} \cdot \vec{\tau} \neq 0$  and  $\vec{F} \cdot \vec{\tau} = 0$  (2009)
41. Four identical thin rods each of mass  $M$  and length  $l$ , form a square frame. Moment of inertia of this frame about an axis through the centre of the square and perpendicular to its plane is
- (a)  $\frac{2}{3} Ml^2$  (b)  $\frac{13}{3} Ml^2$   
 (c)  $\frac{1}{3} Ml^2$  (d)  $\frac{4}{3} Ml^2$  (2009)
42. Two bodies of mass 1 kg and 3 kg have position vectors  $\hat{i} + 2\hat{j} + \hat{k}$  and  $-3\hat{i} - 2\hat{j} + \hat{k}$ , respectively. The centre of mass of this system has a position vector
- (a)  $-2\hat{i} - \hat{j} + \hat{k}$  (b)  $2\hat{i} - \hat{j} - 2\hat{k}$   
 (c)  $-\hat{i} + \hat{j} + \hat{k}$  (d)  $-2\hat{i} + 2\hat{k}$  (2009)
43. A thin rod of length  $L$  and mass  $M$  is bent at its midpoint into two halves so that the angle between them is  $90^\circ$ . The moment of inertia of the bent rod about an axis passing through the bending point and perpendicular to the plane defined by the two halves of the rod is
- (a)  $\frac{ML^2}{6}$  (b)  $\frac{\sqrt{2}ML^2}{24}$   
 (c)  $\frac{ML^2}{24}$  (d)  $\frac{ML^2}{12}$  (2008)
44. The ratio of the radii of gyration of a circular disc to that of a circular ring, each of same mass and radius, around their respective axes is
- (a)  $\sqrt{2}:1$  (b)  $\sqrt{2}:\sqrt{3}$   
 (c)  $\sqrt{3}:\sqrt{2}$  (d)  $1:\sqrt{2}$  (2008)
45. A particle of mass  $m$  moves in the  $XY$  plane with a velocity  $v$  along the straight line  $AB$ . If the angular momentum of the particle with respect to origin  $O$  is  $L_A$  when it is at  $A$  and  $L_B$  when it is at  $B$ , then
- (a)  $L_A = L_B$   
 (b) the relationship between  $L_A$  and  $L_B$  depends upon the slope of the line  $AB$   
 (c)  $L_A < L_B$   
 (d)  $L_A > L_B$  (2007)
46. A uniform rod  $AB$  of length  $l$  and mass  $m$  is free to rotate about point  $A$ . The rod is released from rest in the horizontal position. Given that the moment of inertia of the rod about  $A$  is  $\frac{ml^2}{3}$ , the initial angular acceleration of the rod will be
- (a)  $\frac{mgl}{2}$  (b)  $\frac{3}{2} gl$   
 (c)  $\frac{3g}{2l}$  (d)  $\frac{2g}{3l}$  (2007, 2006)
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47. A wheel has angular acceleration of  $3.0 \text{ rad/sec}^2$  and an initial angular speed of  $2.00 \text{ rad/sec}$ . In a time of 2 sec it has rotated through an angle (in radian) of
- (a) 10 (b) 12  
 (c) 4 (d) 6 (2007)
48. The moment of inertia of a uniform circular disc of radius  $R$  and mass  $M$  about an axis touching the disc at its diameter and normal to the disc
- (a)  $\frac{1}{2} MR^2$  (b)  $MR^2$   
 (c)  $\frac{2}{5} MR^2$  (d)  $\frac{3}{2} MR^2$  (2006)
49. A tube of length  $L$  is filled completely with an incompressible liquid of mass  $M$  and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity  $\omega$ . The force exerted by the liquid at the other end is



- (a)  $\frac{ML^2\omega^2}{2}$  (b)  $\frac{ML\omega^2}{2}$   
 (c)  $\frac{ML^2\omega}{2}$  (d)  $ML\omega^2$  (2006)

50. The moment of inertia of a uniform circular disc of radius  $R$  and mass  $M$  about an axis passing from the edge of the disc and normal to the disc is

- (a)  $MR^2$  (b)  $\frac{1}{2}MR^2$   
 (c)  $\frac{3}{2}MR^2$  (d)  $\frac{7}{2}MR^2$  (2005)

51. A drum of radius  $R$  and mass  $M$ , rolls down without slipping along an inclined plane of angle  $\theta$ . The frictional force

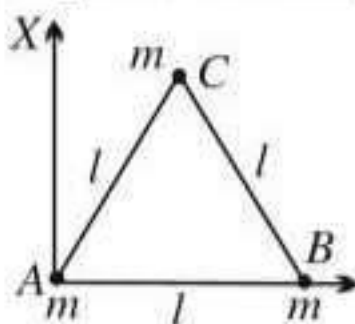
- (a) dissipates energy as heat  
 (b) decreases the rotational motion  
 (c) decreases the rotational and translational motion  
 (d) converts translational energy to rotational energy. (2005)

52. Two bodies have their moments of inertia  $I$  and  $2I$  respectively about their axis of rotation. If their kinetic energies of rotation are equal, their angular velocity will be in the ratio

- (a) 2 : 1 (b) 1 : 2  
 (c)  $\sqrt{2} : 1$  (d)  $1 : \sqrt{2}$  (2005)

53. Three particles, each of mass  $m$  gram, are situated at the vertices of an equilateral triangle  $ABC$  of side  $l$  cm (as shown in the figure). The moment of inertia of the system about a line  $AX$  perpendicular to  $AB$  and in the plane of  $ABC$ , in  $\text{gram-cm}^2$  units will be

- (a)  $\frac{3}{4}ml^2$  (b)  $2ml^2$   
 (c)  $\frac{5}{4}ml^2$  (d)  $\frac{3}{2}ml^2$  (2004)



54. Consider a system of two particles having masses  $m_1$  and  $m_2$ . If the particle of mass  $m_1$  is pushed towards the mass centre of particles through a distance  $d$ , by what distance would be particle of mass  $m_2$  move so as to keep the mass centre of particles at the original position?

- (a)  $\frac{m_1}{m_1 + m_2}d$  (b)  $\frac{m_1}{m_2}d$   
 (c)  $d$  (d)  $\frac{m_2}{m_1}d$  (2004)

55. A wheel having moment of inertia  $2 \text{ kg m}^2$  about its vertical axis, rotates at the rate of 60 rpm about this axis. The torque which can stop the wheel's rotation in one minute would be

- (a)  $\frac{2\pi}{15} \text{ N m}$  (b)  $\frac{\pi}{12} \text{ N m}$   
 (c)  $\frac{\pi}{15} \text{ N m}$  (d)  $\frac{\pi}{18} \text{ N m}$  (2004)

56. A round disc of moment of inertia  $I_2$  about its axis perpendicular to its plane and passing through its centre is placed over another disc of moment of inertia  $I_1$  rotating with an angular velocity  $\omega$  about the same axis. The final angular velocity of the combination of discs is

- (a)  $\frac{I_2\omega}{I_1 + I_2}$  (b)  $\omega$   
 (c)  $\frac{I_1\omega}{I_1 + I_2}$  (d)  $\frac{(I_1 + I_2)\omega}{I_1}$  (2004)

57. The ratio of the radii of gyration of a circular disc about a tangential axis in the plane of the disc and of a circular ring of the same radius about a tangential axis in the plane of the ring is

- (a) 2 : 3 (b) 2 : 1  
 (c)  $\sqrt{5} : \sqrt{6}$  (d)  $1 : \sqrt{2}$  (2004)

58. A stone is tied to a string of length  $l$  and is whirled in a vertical circle with the other end of the string as the centre. At a certain instant of time, the stone is at its lowest position and has a speed  $u$ . The magnitude of the change in velocity as it reaches a position where the string is horizontal ( $g$  being acceleration due to gravity) is

- (a)  $\sqrt{2(u^2 - gl)}$  (b)  $\sqrt{u^2 - gl}$   
 (c)  $u - \sqrt{u^2 - 2gl}$  (d)  $\sqrt{2gl}$  (2003)

59. A ball rolls without slipping. The radius of gyration of the ball about an axis passing through its centre of mass is  $K$ . If radius of the ball be  $R$ , then the fraction of total energy associated with its rotational energy will be

- (a)  $\frac{K^2 + R^2}{R^2}$  (b)  $\frac{K^2}{R^2}$   
 (c)  $\frac{K^2}{K^2 + R^2}$  (d)  $\frac{R^2}{K^2 + R^2}$  (2003)



60. A solid cylinder of mass  $M$  and radius  $R$  rolls without slipping down an inclined plane of length  $L$  and height  $h$ . What is the speed of its centre of mass when the cylinder reaches its bottom?

(a)  $\sqrt{2gh}$  (b)  $\sqrt{\frac{3}{4}gh}$   
(c)  $\sqrt{\frac{4}{3}gh}$  (d)  $\sqrt{4gh}$

(2003, 1989)

61. A thin circular ring of mass  $M$  and radius  $r$  is rotating about its axis with a constant angular velocity  $\omega$ . Four objects each of mass  $m$ , are kept gently to the opposite ends of two perpendicular diameters of the ring. The angular velocity of the ring will be

(a)  $\frac{M\omega}{4m}$  (b)  $\frac{M\omega}{M+4m}$   
(c)  $\frac{(M+4m)\omega}{M}$  (d)  $\frac{(M-4m)\omega}{M+4m}$  (2003)

62. A rod of length is 3 m and its mass acting per unit length is directly proportional to distance  $x$  from one of its end then its centre of gravity from that end will be at

(a) 1.5 m (b) 2 m  
(c) 2.5 m (d) 3.0 m. (2002)

63. A point  $P$  consider at contact point of a wheel on ground which rolls on ground without slipping then value of displacement of point  $P$  when wheel completes half of rotation (If radius of wheel is 1 m)

(a) 2 m (b)  $\sqrt{\pi^2 + 4}$  m  
(c)  $\pi$  m (d)  $\sqrt{\pi^2 + 2}$  m. (2002)

64. A solid sphere of radius  $R$  is placed on smooth horizontal surface. A horizontal force  $F$  is applied at height  $h$  from the lowest point. For the maximum acceleration of centre of mass, which is correct?

(a)  $h = R$   
(b)  $h = 2R$   
(c)  $h = 0$   
(d) no relation between  $h$  and  $R$ . (2002)

65. A disc is rotating with angular speed  $\omega$ . If a child sits on it, what is conserved

(a) linear momentum  
(b) angular momentum  
(c) kinetic energy  
(d) potential energy. (2002)

66. A circular disc is to be made by using iron and aluminium so that it acquired maximum moment of inertia about geometrical axis. It is possible with

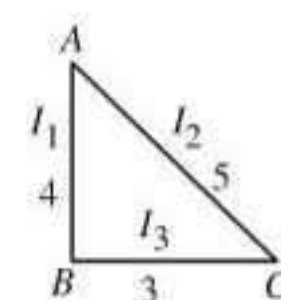
- (a) aluminium at interior and iron surround to it  
(b) iron at interior and aluminium surround to it  
(c) using iron and aluminium layers in alternate order  
(d) sheet of iron is used at both external surface and aluminium sheet as internal layers. (2002)

67. A disc is rolling, the velocity of its centre of mass is  $v_{cm}$ . Which one will be correct?

- (a) the velocity of highest point is  $2v_{cm}$  and point of contact is zero  
(b) the velocity of highest point is  $v_{cm}$  and point of contact is  $v_{cm}$   
(c) the velocity of highest point is  $2v_{cm}$  and point of contact is  $v_{cm}$   
(d) the velocity of highest point is  $2v_{cm}$  and point of contact is  $2v_{cm}$ . (2001)

68. For the adjoining diagram, the correct relation between  $I_1$ ,  $I_2$ , and  $I_3$  is, ( $I$  - moment of inertia)

- (a)  $I_1 > I_2$   
(b)  $I_2 > I_1$   
(c)  $I_3 > I_1$   
(d)  $I_3 > I_2$ .



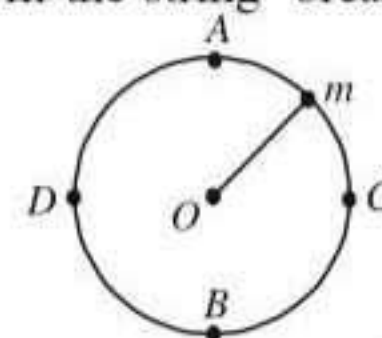
(2000)

69. For a hollow cylinder and a solid cylinder rolling without slipping on an inclined plane, then which of these reaches earlier

- (a) solid cylinder  
(b) hollow cylinder  
(c) both simultaneously  
(d) can't say anything. (2000)

70. As shown in the figure at point  $O$  a mass is performing vertical circular motion. The average velocity of the particle is increased, then at which point will the string break

- (a)  $A$   
(b)  $B$   
(c)  $C$   
(d)  $D$ .



(2000)



71. Three identical metal balls, each of the radius  $r$  are placed touching each other on a horizontal surface such that an equilateral triangle is formed when centres of three balls are joined. The centre of the mass of the system is located at
- line joining centres of any two balls
  - centre of one of the balls
  - horizontal surface
  - point of intersection of the medians

(1999)

72. The moment of inertia of a disc of mass  $M$  and radius  $R$  about an axis, which is tangential to the circumference of the disc and parallel to its diameter is

- $\frac{5}{4}MR^2$
- $\frac{2}{3}MR^2$
- $\frac{3}{2}MR^2$
- $\frac{4}{5}MR^2$

(1999)

73. Find the torque of a force  $\vec{F} = -3\hat{i} + \hat{j} + 5\hat{k}$  acting at the point  $\vec{r} = 7\hat{i} + 3\hat{j} + \hat{k}$

- $-21\hat{i} + 4\hat{j} + 4\hat{k}$
- $-14\hat{i} + 34\hat{j} - 16\hat{k}$
- $14\hat{i} - 38\hat{j} + 16\hat{k}$
- $4\hat{i} + 4\hat{j} + 6\hat{k}$

(1997)

74. The centre of mass of system of particles does not depend on

- position of the particles
- relative distances between the particles
- masses of the particles
- forces acting on the particle

(1997)

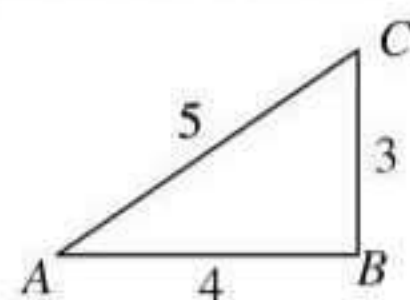
75. A couple produces

- linear and rotational motion
- no motion
- purely linear motion
- purely rotational motion

(1997)

76. The  $ABC$  is a triangular plate of uniform thickness. The sides are in the ratio shown in the figure.  $I_{AB}$ ,  $I_{BC}$  and  $I_{CA}$  are the moments of inertia of the plate about  $AB$ ,  $BC$  and  $CA$  respectively. Which one of the following relations is correct?

- $I_{AB} + I_{BC} = I_{CA}$
- $I_{CA}$  is maximum
- $I_{AB} > I_{BC}$
- $I_{BC} > I_{AB}$



(1995)

77. What is the torque of the force  $\vec{F} = 2\hat{i} - 3\hat{j} + 4\hat{k}$  N acting at the point  $\vec{r} = 3\hat{i} + 2\hat{j} + 3\hat{k}$  m about origin?

- $-6\hat{i} + 6\hat{j} - 12\hat{k}$
- $-17\hat{i} + 6\hat{j} + 13\hat{k}$
- $6\hat{i} - 6\hat{j} + 12\hat{k}$
- $17\hat{i} - 6\hat{j} - 13\hat{k}$

(1995)

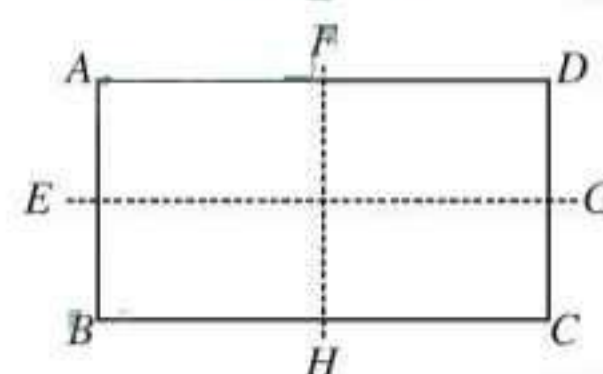
78. A solid spherical ball rolls on a table. Ratio of its rotational kinetic energy to total kinetic energy is

- $\frac{1}{2}$
- $\frac{1}{6}$
- $\frac{7}{10}$
- $\frac{2}{7}$

(1994)

79. In a rectangle  $ABCD$  ( $BC = 2AB$ ). The moment of inertia is minimum along axis through

- $BC$
- $BD$
- $HF$
- $EG$



(1993)

80. A solid sphere, disc and solid cylinder all of the same mass and made of the same material are allowed to roll down (from rest) on the inclined plane, then

- solid sphere reaches the bottom first
- solid sphere reaches the bottom last
- disc will reach the bottom first
- all reach the bottom at the same time

(1993)

81. The speed of a homogenous solid sphere after rolling down an inclined plane of vertical height  $h$  from rest without sliding is

- $\sqrt{\frac{10}{7}gh}$
- $\sqrt{gh}$
- $\sqrt{\frac{6}{5}gh}$
- $\sqrt{\frac{4}{3}gh}$

(1992)

82. If a sphere is rolling, the ratio of the translational energy to total kinetic energy is given by

- 7 : 10
- 2 : 5
- 10 : 7
- 5 : 7

(1991)

83. A particle of mass  $m = 5$  is moving with a uniform speed  $v = 3\sqrt{2}$  in the  $XOY$  plane along the line  $Y = X + 4$ . The magnitude of the angular momentum of the particle about the origin is

- 60 units
- $40\sqrt{2}$  units
- zero
- 7.5 units

(1991)



- 84.** A fly wheel rotating about fixed axis has a kinetic energy of 360 joule when its angular speed is 30 radian/sec. The moment of inertia of the wheel about the axis of rotation is  
 (a)  $0.6 \text{ kgm}^2$  (b)  $0.15 \text{ kgm}^2$   
 (c)  $0.8 \text{ kgm}^2$  (d)  $0.75 \text{ kgm}^2$  (1990)
- 85.** The moment of inertia of a body about a given axis is  $1.2 \text{ kgm}^2$ . Initially, the body is at rest. In order to produce a rotational kinetic energy of 1500 joule, an angular acceleration of  $25 \text{ radian/sec}^2$  must be applied about that axis for a duration of  
 (a) 4 s (b) 2 s  
 (c) 8 s (d) 10 s (1990)
- 86.** Moment of inertia of a uniform circular disc about a diameter is  $I$ . Its moment of inertia about an axis perpendicular to its plane and passing through a point on its rim will be  
 (a)  $5I$  (b)  $3I$   
 (c)  $6I$  (d)  $4I$  (1990)
- 87.** A solid homogenous sphere of mass  $M$  and radius is moving on a rough horizontal surface, partly rolling and partly sliding. During this kind of motion of this sphere  
 (a) total kinetic energy is conserved  
 (b) the angular momentum of the sphere about the point of contact with the plane is conserved  
 (c) only the rotational kinetic energy about the centre of mass is conserved  
 (d) angular momentum about the centre of mass of conserved (1988)
- 88.** A ring of mass  $m$  and radius  $r$  rotates about an axis passing through its centre and perpendicular to its plane with angular velocity  $\omega$ . Its kinetic energy is  
 (a)  $\frac{1}{2}mr^2\omega^2$  (b)  $mr\omega^2$   
 (c)  $mr^2\omega^2$  (d)  $\frac{1}{2}mr\omega^2$  (1988)

## Answer Key

- |         |         |         |         |         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (b)  | 2. (a)  | 3. (*)  | 4. (c)  | 5. (b)  | 6. (a)  | 7. (d)  | 8. (d)  | 9. (c)  | 10. (b) |
| 11. (c) | 12. (c) | 13. (b) | 14. (a) | 15. (b) | 16. (d) | 17. (a) | 18. (c) | 19. (b) | 20. (d) |
| 21. (b) | 22. (a) | 23. (c) | 24. (b) | 25. (a) | 26. (d) | 27. (c) | 28. (a) | 29. (a) | 30. (a) |
| 31. (b) | 32. (d) | 33. (d) | 34. (b) | 35. (c) | 36. (a) | 37. (d) | 38. (d) | 39. (a) | 40. (b) |
| 41. (d) | 42. (a) | 43. (d) | 44. (d) | 45. (a) | 46. (c) | 47. (a) | 48. (d) | 49. (b) | 50. (c) |
| 51. (d) | 52. (c) | 53. (c) | 54. (b) | 55. (c) | 56. (c) | 57. (c) | 58. (a) | 59. (c) | 60. (c) |
| 61. (b) | 62. (b) | 63. (b) | 64. (d) | 65. (b) | 66. (a) | 67. (a) | 68. (b) | 69. (a) | 70. (b) |
| 71. (d) | 72. (a) | 73. (c) | 74. (d) | 75. (d) | 76. (d) | 77. (d) | 78. (d) | 79. (d) | 80. (a) |
| 81. (a) | 82. (d) | 83. (a) | 84. (c) | 85. (b) | 86. (c) | 87. (b) | 88. (a) |         |         |



# EXPLANATIONS

1. (b) :  $m = 3 \text{ kg}$ ,  $r = 40 \text{ cm} = 40 \times 10^{-2} \text{ m}$ ,  $F = 30 \text{ N}$   
 Moment of inertia of hollow cylinder about its axis  
 $= mr^2 = 3 \text{ kg} \times (0.4)^2 \text{ m}^2 = 0.48 \text{ kg m}^2$

The torque is given by,

$$\tau = I\alpha$$

where  $I$  = moment of inertia,

$\alpha$  = angular acceleration

In the given case,  $\tau = rF$ , as the force is acting perpendicularly to the radial vector.

$$\therefore \alpha = \frac{\tau}{I} = \frac{Fr}{mr^2} = \frac{F}{mr} = \frac{30}{3 \times 40 \times 10^{-2}} = \frac{30 \times 100}{3 \times 40}$$

$$\alpha = 25 \text{ rad s}^{-2}$$

2. (a) : Initial angular momentum =  $I\omega_1 + I\omega_2$

Let  $\omega$  be angular speed of the combined system.

Final angular momentum =  $2I\omega$

$\therefore$  According to conservation of angular momentum

$$I\omega_1 + I\omega_2 = 2I\omega \text{ or } \omega = \frac{\omega_1 + \omega_2}{2}$$

Initial rotational kinetic energy

$$E = \frac{1}{2}I(\omega_1^2 + \omega_2^2)$$

Final rotational kinetic energy

$$E_f = \frac{1}{2}(2I)\omega^2 = \frac{1}{2}(2I)\left(\frac{\omega_1 + \omega_2}{2}\right)^2 = \frac{1}{4}I(\omega_1 + \omega_2)^2$$

$\therefore$  Loss of energy  $\Delta E = E_i - E_f$

$$= \frac{1}{2}I(\omega_1^2 + \omega_2^2) - \frac{1}{4}I(\omega_1^2 + \omega_2^2 + 2\omega_1\omega_2)$$

$$= \frac{1}{4}I[\omega_1^2 + \omega_2^2 - 2\omega_1\omega_2] = \frac{1}{4}I(\omega_1 - \omega_2)^2$$

3. (\*) : Centre of gravity of a body is the point at which the total gravitational torque on body is zero. Centre of mass and centre of gravity coincides only for symmetrical bodies.

Hence statements (1) and (2) are incorrect.

A couple of a body produces rotational motion only.

Hence statement (3) is incorrect.

Mechanical advantage greater than one means that the system will require a force that is less than the load in order to move it.

Hence statement (4) is correct.

\*None of the given options is correct.

4. (c) : Here,  $m_A = m$ ,  $m_B = 2m$

Both bodies A and B have equal kinetic energy of rotation

$$k_A = k_B \Rightarrow \frac{1}{2}I_A\omega_A^2 = \frac{1}{2}I_B\omega_B^2$$

$$\Rightarrow \frac{\omega_A^2}{\omega_B^2} = \frac{I_B}{I_A} \quad \dots(i)$$

Ratio of angular momenta,

$$\frac{L_A}{L_B} = \frac{I_A\omega_A}{I_B\omega_B} = \frac{I_A}{I_B} \times \sqrt{\frac{I_B}{I_A}} \quad [\text{Using eqn. (i)}]$$

$$= \sqrt{\frac{I_A}{I_B}} < 1 \quad (\because I_B > I_A)$$

$$\therefore L_B > L_A$$

$$5. (b) : \frac{E_{\text{Sphere}}}{E_{\text{Cylinder}}} = \frac{\frac{1}{2}I_s\omega_s^2}{\frac{1}{2}I_c\omega_c^2} = \frac{I_s\omega_s^2}{I_c\omega_c^2}$$

$$\text{Here, } I_s = \frac{2}{5}mR^2, I_c = \frac{1}{2}mR^2$$

$$\omega_c = 2\omega_s$$

$$\frac{E_{\text{Sphere}}}{E_{\text{Cylinder}}} = \frac{\frac{2}{5}mR^2 \times \omega_s^2}{\frac{1}{2}mR^2 \times (2\omega_s)^2} = \frac{4}{5} \times \frac{1}{4} = \frac{1}{5}$$

6. (a) : Here,  $l_1 + l_2 = l$

Centre of mass of the system,

$$l_1 = \frac{m_1 \times 0 + m_2 \times l}{m_1 + m_2} = \frac{m_2 l}{m_1 + m_2}$$

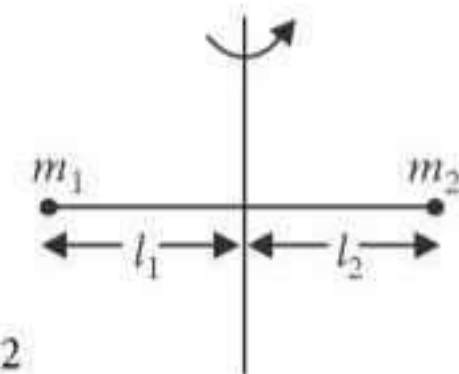
$$l_2 = l - l_1 = \frac{m_1 l}{m_1 + m_2}$$

Required moment of inertia of the system,

$$I = m_1 l_1^2 + m_2 l_2^2$$

$$= (m_1 m_2^2 + m_2 m_1^2) \frac{l^2}{(m_1 + m_2)^2}$$

$$= \frac{m_1 m_2 (m_1 + m_2) l^2}{(m_1 + m_2)^2} = \frac{m_1 m_2}{m_1 + m_2} l^2$$



7. (d) : Time taken by the body to reach the bottom when it rolls down on an inclined plane without slipping is given by

$$t = \sqrt{\frac{2l \left( 1 + \frac{k^2}{R^2} \right)}{g \sin \theta}}$$



Since  $g$  is constant and  $l$ ,  $R$  and  $\sin \theta$  are same for both

$$\therefore \frac{t_d}{t_s} = \frac{\sqrt{1 + \frac{k_d^2}{R^2}}}{\sqrt{1 + \frac{k_s^2}{R^2}}} = \frac{\sqrt{1 + \frac{R^2}{2R^2}}}{\sqrt{1 + \frac{2R^2}{5R^2}}} \quad \left( \because k_d = \frac{R}{\sqrt{2}}, k_s = \sqrt{\frac{2}{5}}R \right)$$

$$= \sqrt{\frac{3}{2} \times \frac{5}{7}} = \sqrt{\frac{15}{14}} \Rightarrow t_d > t_s$$

Hence, the sphere gets to the bottom first.

8. (d) : Mass per unit area of disc =  $\frac{M}{\pi R^2}$

Mass of removed portion of disc,

$$M' = \frac{M}{\pi R^2} \times \pi \left( \frac{R}{2} \right)^2 = \frac{M}{4}$$

Moment of inertia of removed portion about an axis passing through centre of disc  $O$  and perpendicular to the plane of disc,

$$I'_O = I_{O'} + M'd^2$$

$$= \frac{1}{2} \times \frac{M}{4} \times \left( \frac{R}{2} \right)^2 + \frac{M}{4} \times \left( \frac{R}{2} \right)^2$$

$$= \frac{MR^2}{32} + \frac{MR^2}{16} = \frac{3MR^2}{32}$$

When portion of disc would not have been removed, the moment of inertia of complete disc about centre  $O$  is

$$I_O = \frac{1}{2}MR^2$$

So, moment of inertia of the disc with removed portion is

$$I = I_O - I'_O = \frac{1}{2}MR^2 - \frac{3MR^2}{32} = \frac{13MR^2}{32}$$

9. (c) : Given,  $r = 50 \text{ cm} = 0.5 \text{ m}$ ,  $\alpha = 2.0 \text{ rad s}^{-2}$ ,  $\omega_0 = 0$   
At the end of 2 s,

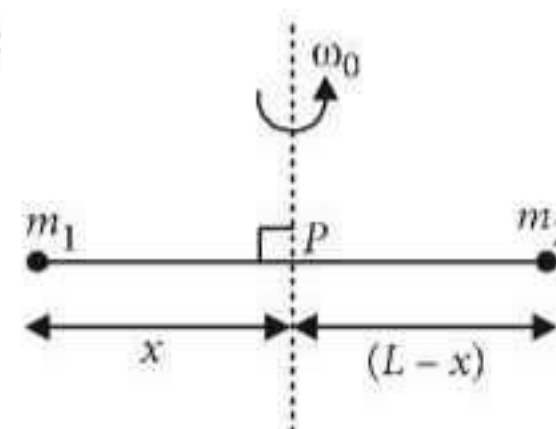
Tangential acceleration,  $a_t = r\alpha = 0.5 \times 2 = 1 \text{ m s}^{-2}$

Radial acceleration,  $a_r = \omega^2 r = (\omega_0 + \alpha t)^2 r$   
 $= (0 + 2 \times 2)^2 \times 0.5 = 8 \text{ m s}^{-2}$

$\therefore$  Net acceleration,

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{1^2 + 8^2} = \sqrt{65} \approx 8 \text{ m s}^{-2}$$

10. (b) :



Moment of inertia of the system about the axis of rotation (through point  $P$ ) is

$$I = m_1 x^2 + m_2 (L-x)^2$$

By work energy theorem,

Work done to set the rod rotating with angular velocity  $\omega_0$  = Increase in rotational kinetic energy

$$W = \frac{1}{2} I \omega_0^2 = \frac{1}{2} [m_1 x^2 + m_2 (L-x)^2] \omega_0^2$$

For  $W$  to be minimum,  $\frac{dW}{dx} = 0$

$$\text{i.e. } \frac{1}{2} [2m_1 x + 2m_2 (L-x)(-1)] \omega_0^2 = 0$$

$$\text{or } m_1 x - m_2 (L-x) = 0 \quad (\because \omega_0 \neq 0)$$

$$\text{or } (m_1 + m_2)x = m_2 L \text{ or } x = \frac{m_2 L}{m_1 + m_2}$$

11. (c) : Here,

Speed of the automobile,

$$v = 54 \text{ km h}^{-1} = 54 \times \frac{5}{18} \text{ m s}^{-1} = 15 \text{ m s}^{-1}$$

Radius of the wheel of the automobile,  $R = 0.45 \text{ m}$

Moment of inertia of the wheel about its axis of rotation,  $I = 3 \text{ kg m}^2$

Time in which the vehicle brought to rest,  $t = 15 \text{ s}$

The initial angular speed of the wheel is

$$\omega_i = \frac{v}{R} = \frac{15 \text{ m s}^{-1}}{0.45 \text{ m}} = \frac{1500}{45} \text{ rad s}^{-1} = \frac{100}{3} \text{ rad s}^{-1}$$

and its final angular speed is

$$\omega_f = 0 \quad (\text{as the vehicle comes to rest})$$

$\therefore$  The angular retardation of the wheel is

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{0 - \frac{100}{3}}{15 \text{ s}} = -\frac{100}{45} \text{ rad s}^{-2}$$

The magnitude of required torque is

$$\tau = I |\alpha| = (3 \text{ kg m}^2) \left( \frac{100}{45} \text{ rad s}^{-2} \right)$$

$$= \frac{20}{3} \text{ kg m}^2 \text{ s}^{-2} = 6.66 \text{ kg m}^2 \text{ s}^{-2}$$

12. (c) : For the conservation of angular momentum about origin, the torque  $\vec{\tau}$  acting on the particle will be zero.



By definition,  $\vec{\tau} = \vec{r} \times \vec{F}$

Here,  $\vec{r} = 2\hat{i} - 6\hat{j} - 12\hat{k}$  and  $\vec{F} = \alpha\hat{i} + 3\hat{j} + 6\hat{k}$

$$\begin{aligned}\therefore \vec{\tau} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -6 & -12 \\ \alpha & 3 & 6 \end{vmatrix} \\ &= \hat{i}(-36 + 36) - \hat{j}(12 + 12\alpha) + \hat{k}(6 + 6\alpha) \\ &= -\hat{j}(12 + 12\alpha) + \hat{k}(6 + 6\alpha)\end{aligned}$$

But  $\vec{\tau} = 0$

$\therefore 12 + 12\alpha = 0$  or  $\alpha = -1$

and  $6 + 6\alpha = 0$  or  $\alpha = -1$

**13. (b) :** Given situation is shown in figure.

$N_1$  = Normal reaction on A

$N_2$  = Normal reaction on B

$W$  = Weight of the rod

In vertical equilibrium,

$$N_1 + N_2 = W \quad \dots(i)$$

Torque balance about centre of mass of the rod,

$$N_1 x = N_2 (d - x)$$

Putting value of  $N_2$  from equation (i)

$$N_1 x = (W - N_1)(d - x)$$

$$\Rightarrow N_1 x = Wd - Wx - N_1 d + N_1 x$$

$$\Rightarrow N_1 d = W(d - x)$$

$$\therefore N_1 = \frac{W(d - x)}{d}$$

**14. (a) :** According to law of conservation of angular momentum

$$mvr = mv'r'$$

$$v_0 R_0 = v \left( \frac{R_0}{2} \right); v = 2v_0 \quad \dots(i)$$

$$\therefore \frac{K_0}{K} = \frac{\frac{1}{2}mv_0^2}{\frac{1}{2}mv^2} = \left( \frac{v_0}{v} \right)^2$$

$$\text{or } \frac{K}{K_0} = \left( \frac{v}{v_0} \right)^2 = (2)^2 \quad (\text{Using (i)})$$

$$K = 4K_0 = 2mv_0^2$$

**15. (b) :** Net moment of inertia of the system,

$$I = I_1 + I_2 + I_3$$

The moment of inertia of a shell about its diameter,

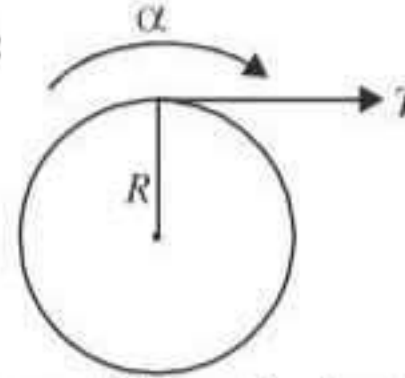
$$I_1 = \frac{2}{3}mr^2$$

The moment of inertia of a shell about its tangent is given by

$$I_2 = I_3 = I_1 + mr^2 = \frac{2}{3}mr^2 + mr^2 = \frac{5}{3}mr^2$$

$$\therefore I = 2 \times \frac{5}{3}mr^2 + \frac{2}{3}mr^2 = \frac{12mr^2}{3} = 4mr^2$$

**16. (d) :**



Here, mass of the cylinder,  $M = 50 \text{ kg}$

Radius of the cylinder,  $R = 0.5 \text{ m}$

Angular acceleration,  $\alpha = 2 \text{ rev s}^{-2}$

$$= 2 \times 2\pi \text{ rad s}^{-2} = 4\pi \text{ rad s}^{-2}$$

Torque,  $\tau = TR$

Moment of inertia of the solid cylinder about its

$$\text{axis, } I = \frac{1}{2}MR^2$$

$\therefore$  Angular acceleration of the cylinder

$$\alpha = \frac{\tau}{I} = \frac{TR}{\frac{1}{2}MR^2}$$

$$T = \frac{MR\alpha}{2} = \frac{50 \times 0.5 \times 4\pi}{2} = 157 \text{ N}$$

**17. (a) :** Acceleration of the solid sphere slipping down the incline without rolling is

$$a_{\text{slipping}} = g \sin \theta \quad \dots(i)$$

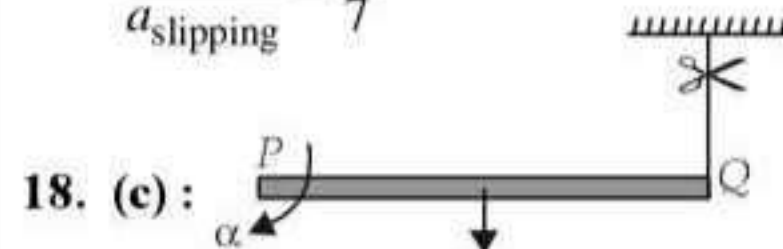
Acceleration of the solid sphere rolling down the incline without slipping is

$$a_{\text{rolling}} = \frac{g \sin \theta}{1 + \frac{k^2}{R^2}} = \frac{g \sin \theta}{1 + \frac{2}{5}}$$

$$= \frac{5}{7} g \sin \theta \quad \left( \because \text{For solid sphere, } \frac{k^2}{R^2} = \frac{2}{5} \right) \quad \dots(ii)$$

Divide eqn. (ii) by eqn. (i), we get

$$\frac{a_{\text{rolling}}}{a_{\text{slipping}}} = \frac{5}{7}$$



**18. (c) :**

When the string is cut, the rod will rotate about P. Let  $\alpha$  be initial angular acceleration of the rod. Then

$$\text{Torque, } \tau = I\alpha = \frac{ML^2}{3}\alpha \quad \dots(i)$$

$$(\text{Moment of inertia of the rod about one end} = \frac{ML^2}{3})$$



Also,  $\tau = Mg \frac{L}{2}$  ... (ii)

Equating (i) and (ii), we get

$$Mg \frac{L}{2} = \frac{ML^2}{3} \alpha \text{ or } \alpha = \frac{3g}{2L}$$

**19. (b) :** The kinetic energy of the rolling object is converted into potential energy at height

$$h \left( = \frac{3v^2}{4g} \right)$$

So by the law of conservation of mechanical energy, we have

$$\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = Mgh \quad \left( \because \omega = \frac{v}{R} \right)$$

$$\frac{1}{2}Mv^2 + \frac{1}{2}I \left( \frac{v}{R} \right)^2 = Mg \left( \frac{3v^2}{4g} \right)$$

$$\frac{1}{2}I \frac{v^2}{R^2} = \frac{3}{4}Mv^2 - \frac{1}{2}Mv^2$$

$$\frac{1}{2}I \frac{v^2}{R^2} = \frac{1}{4}Mv^2 \quad \text{or} \quad I = \frac{1}{2}MR^2$$

Hence, the object is disc.

**20. (d) :** Let  $M$  and  $R$  be mass and radius of the ring and the disc respectively. Then, Moment of inertia of ring about an axis passing through its centre and perpendicular to its plane is

$$I_{\text{ring}} = MR^2$$

Moment of inertia of disc about the same axis is

$$I_{\text{disc}} = \frac{MR^2}{2}$$

As  $I = Mk^2$  where  $k$  is the radius of gyration

$$\therefore I_{\text{ring}} = Mk_{\text{ring}}^2 = MR^2 \text{ or } k_{\text{ring}} = R$$

$$\text{and } I_{\text{disc}} = Mk_{\text{disc}}^2 = \frac{MR^2}{2} \text{ or } k_{\text{disc}} = \frac{R}{\sqrt{2}}$$

$$\therefore \frac{k_{\text{ring}}}{k_{\text{disc}}} = \frac{R}{R/\sqrt{2}} = \sqrt{2}$$

$$k_{\text{ring}} : k_{\text{disc}} = \sqrt{2} : 1$$

**21. (b) :** Moment of inertia of disc  $D_1$  about an axis passing through its centre and normal to its plane is

$$I_1 = \frac{MR^2}{2} = \frac{(2 \text{ kg})(0.2 \text{ m})^2}{2} = 0.04 \text{ kg m}^2$$

Initial angular velocity of disc  $D_1$ ,  $\omega_1 = 50 \text{ rad s}^{-1}$

Moment of inertia of disc  $D_2$  about an axis passing through its centre and normal to its plane is

$$I_2 = \frac{(4 \text{ kg})(0.1 \text{ m})^2}{2} = 0.02 \text{ kg m}^2$$

Initial angular velocity of disc  $D_2$ ,  $\omega_2 = 200 \text{ rad s}^{-1}$

Total initial angular momentum of the two discs is

$$L_i = I_1\omega_1 + I_2\omega_2$$

When two discs are brought in contact face to face (one on the top of the other) and their axes of rotation coincident, the moment of inertia  $I$  of the system is equal to the sum of their individual moment of inertia.

$$I = I_1 + I_2$$

Let  $\omega$  be the final angular speed of the system. The final angular momentum of the system is

$$L_f = I\omega = (I_1 + I_2)\omega$$

According to law of conservation of angular momentum, we get

$$L_i = L_f$$

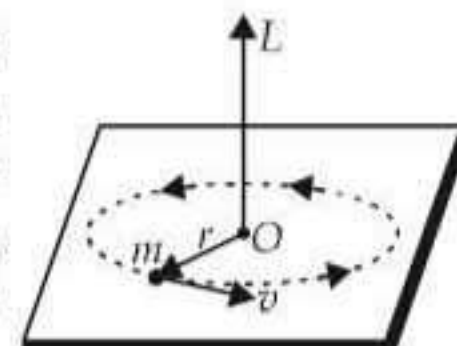
$$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$$

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

$$= \frac{(0.04 \text{ kg m}^2)(50 \text{ rad s}^{-1}) + (0.02 \text{ kg m}^2)(200 \text{ rad s}^{-1})}{(0.04 + 0.02) \text{ kg m}^2}$$

$$= \frac{(2 + 4)}{0.06} \text{ rad s}^{-1} = 100 \text{ rad s}^{-1}$$

**22. (a) :** When a mass is rotating in a plane about a fixed point its angular momentum is directed along a line perpendicular to the plane of rotation.



**23. (c) :** As no external force acts on the system, therefore centre of mass will not shift.

**24. (b) :** Here,  $m = 1000 \text{ kg}$ ,  $R = 90 \text{ m}$ ,  $\theta = 45^\circ$

$$\text{For banking, } \tan \theta = \frac{v^2}{Rg}$$

$$\text{or } v = \sqrt{Rg \tan \theta} = \sqrt{90 \times 10 \times \tan 45^\circ} = 30 \text{ m s}^{-1}$$

**25. (a) :** Let  $x$  be the distance of centre  $O$  of equilateral triangle from each side.

Total torque about  $O = 0$

$$\Rightarrow F_1x + F_2x - F_3x = 0 \text{ or } F_3 = F_1 + F_2$$

**26. (d) :** Force of friction provides the necessary centripetal force.

$$f \leq \mu_s N = \frac{mv^2}{R}$$

$$v^2 \leq \frac{\mu_s RN}{m}$$



$$v^2 \leq \mu_s Rg \quad [\because N = mg]$$

$$\text{or } v \leq \sqrt{\mu_s Rg}$$

$\therefore$  The maximum speed of the car in circular motion is

$$v_{\max} = \sqrt{\mu_s Rg}$$

**27. (c) :** As the system is initially at rest, therefore, initial angular momentum  $L_i = 0$ .

According to the principle of conservation of angular momentum, final angular momentum,  $L_f = 0$ .

$\therefore$  Angular momentum = Angular momentum of man is in opposite direction of platform.

$$\text{i.e., } mvR = I\omega$$

$$\text{or } \omega = \frac{mvR}{I} = \frac{50 \times 1 \times 2}{200} = \frac{1}{2} \text{ rad s}^{-1}$$

Angular velocity of man relative to platform is

$$\omega_r = \omega + \frac{v}{R} = \frac{1}{2} + \frac{1}{2} = 1 \text{ rad s}^{-1}$$

Time taken by the man to complete one revolution is

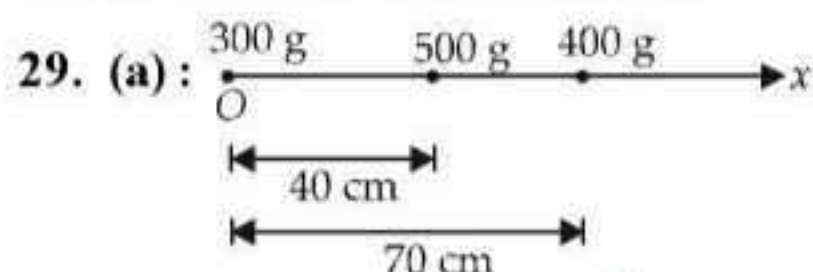
$$T = \frac{2\pi}{\omega_r} = \frac{2\pi}{1} = 2\pi \text{ s}$$

**28. (a) :** According to the theorem of parallel axes,

$$I = I_{\text{CM}} + Ma^2$$

As  $a$  is maximum for point  $B$ ,

Therefore  $I$  is maximum about  $B$ .



The distance of the centre of mass of the system of three masses from the origin  $O$  is

$$\begin{aligned} X_{\text{CM}} &= \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} \\ &= \frac{300 \times 0 + 500 \times 40 + 400 \times 70}{300 + 500 + 400} \\ &= \frac{500 \times 40 + 400 \times 70}{1200} = \frac{400 [50 + 70]}{1200} \\ &= \frac{50 + 70}{3} = \frac{120}{3} = 40 \text{ cm} \end{aligned}$$

**30. (a) :** Given :  $\theta(t) = 2t^3 - 6t^2$

$$\therefore \frac{d\theta}{dt} = 6t^2 - 12t \Rightarrow \frac{d^2\theta}{dt^2} = 12t - 12$$

$$\text{Angular acceleration, } \alpha = \frac{d^2\theta}{dt^2} = 12t - 12$$

When angular acceleration ( $\alpha$ ) is zero, then the torque on the wheel becomes zero ( $\because \tau = I\alpha$ )

$$\Rightarrow 12t - 12 = 0 \text{ or } t = 1 \text{ s}$$

**31. (b) :** According to the theorem of parallel axes, the moment of inertia of the thin rod of mass  $M$  and length  $L$  about an axis passing through one of the ends is

$$I = I_{\text{CM}} + Md^2$$

where  $I_{\text{CM}}$  is the moment of inertia of the given rod about an axis passing through its centre of mass and perpendicular to its length and  $d$  is the distance between two parallel axes.

$$\text{Here, } I_{\text{CM}} = I_0, \quad d = \frac{L}{2}$$

$$\therefore I = I_0 + M\left(\frac{L}{2}\right)^2 = I_0 + \frac{ML^2}{4}$$

**32. (d) :** According to law of conservation of angular momentum

$$mvr = mv'r'$$

$$vr = v'\left(\frac{r}{2}\right)$$

$$v' = 2v \quad \dots(i)$$

$$\therefore \frac{K}{K'} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mv'^2} = \left(\frac{v}{v'}\right)^2$$

$$\text{or } \frac{K'}{K} = \left(\frac{v'}{v}\right)^2 = (2)^2 \quad (\text{Using (i)})$$

$$K' = 4K$$

**33. (d) :** As no external torque is applied to the system, the angular momentum of the system remains conserved.

$$\therefore L_i = L_f$$

According to given problem,

$$I_i \omega_i = (I_i + I_b) \omega_f$$

$$\text{or } \omega_f = \frac{I_i \omega_i}{(I_i + I_b)} \quad \dots(ii)$$

$$\text{Initial energy, } E_i = \frac{1}{2} I_i \omega_i^2 \quad \dots(iii)$$

$$\text{Final energy, } E_f = \frac{1}{2} (I_i + I_b) \omega_f^2 \quad \dots(iv)$$

Substituting the value of  $\omega_f$  from equation (i) in equation (iii), we get

$$\text{Final energy, } E_f = \frac{1}{2} (I_i + I_b) \left( \frac{I_i \omega_i}{I_i + I_b} \right)^2 = \frac{1}{2} \frac{I_i^2 \omega_i^2}{(I_i + I_b)} \quad \dots(iv)$$

$$\text{Loss of energy, } \Delta E = E_i - E_f$$

$$= \frac{1}{2} I_i \omega_i^2 - \frac{1}{2} \frac{I_i^2 \omega_i^2}{(I_i + I_b)} \quad (\text{Using (ii) and (iv)})$$



$$= \frac{\omega_i^2}{2} \left( I_t - \frac{I_t^2}{(I_t + I_b)} \right) = \frac{\omega_i^2}{2} \left( \frac{I_t^2 + I_b I_t - I_t^2}{(I_t + I_b)} \right)$$

$$= \frac{1}{2} \frac{I_b I_t}{(I_t + I_b)} \omega_i^2$$

**34. (b) :** As no external force is acting on the system, the centre of mass must be at rest i.e.  $v_{CM} = 0$ .

**35. (c) :** The coin will revolve with the record, if Force of friction  $\geq$  Centrifugal force

$$\mu mg \geq mr\omega^2$$

$$\text{or } r \leq \frac{\mu g}{\omega^2}$$

**36. (a) :** Mass of the disc =  $9M$

Mass of removed portion of disc =  $M$

The moment of inertia of the complete disc about an axis passing through its centre  $O$  and perpendicular to its plane

$$\text{is } I_1 = \frac{9}{2} MR^2$$

Now, the moment of inertia of the disc with removed portion

$$I_2 = \frac{1}{2} M \left( \frac{R}{3} \right)^2 = \frac{1}{18} MR^2$$

Therefore, moment of inertia of the remaining portion of disc about  $O$  is

$$I = I_1 - I_2 = 9 \frac{MR^2}{2} - \frac{MR^2}{18} = \frac{40MR^2}{9}$$

**37. (d) :** Time taken to reach the bottom of inclined plane.

$$t = \sqrt{\frac{2l \left( 1 + \frac{K^2}{R^2} \right)}{g \sin \theta}}$$

Here,  $l$  is length of incline plane

$$\text{For solid cylinder } K^2 = \frac{R^2}{2}$$

$$\text{For hollow cylinder } K^2 = R^2$$

Hence, solid cylinder will reach the bottom first.

**38. (d) :** As no external torque is acting about the axis, angular momentum of system remains conserved.

$$I_1 \omega_1 = I_2 \omega_2$$

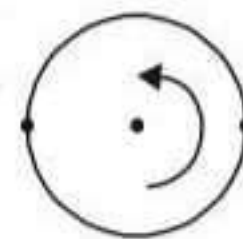
$$\Rightarrow \omega_2 = \frac{I_1 \omega_1}{I_2} = \frac{Mr^2 \omega}{(M+2m)r^2} = \frac{M\omega}{(M+2m)}$$

**39. (a) :** As the masses are added to the ring gently, there is no torque and angular momentum is conserved.

$$I\omega = I'\omega'$$

$$\Rightarrow MR^2\omega = (MR^2 + 2mR^2)\omega'$$

$$\Rightarrow \omega' = \frac{MR^2\omega}{(M+2m)R^2} \Rightarrow \omega' = \frac{M\omega}{M+2m}$$

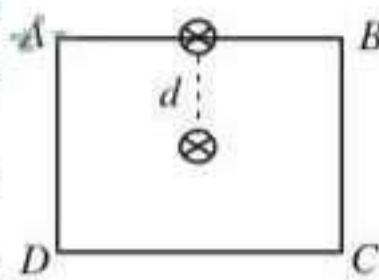


**40. (b) :** Torque is always perpendicular to

$\vec{F}$  as well as  $\vec{r}$ .

$$\therefore \vec{r} \cdot \vec{\tau} = 0 \text{ as well as } \vec{F} \cdot \vec{\tau} = 0$$

**41. (d) :** Moment of inertia for the rod  $AB$  rotating about an axis through the mid-point of  $AB$  perpendicular to the plane of the paper is  $\frac{Ml^2}{12}$ .



$\therefore$  Moment of inertia about the axis through the centre of the square and parallel to this axis,

$$I = I_0 + Md^2 = M \left( \frac{l^2}{12} + \frac{l^2}{4} \right) = \frac{Ml^2}{3}$$

For all the four rods,  $I = \frac{4}{3} Ml^2$ .

**42. (a) :**  $\vec{r}_1 = \hat{i} + 2\hat{j} + \hat{k}$  for  $M_1 = 1$  kg

$$\vec{r}_2 = -3\hat{i} - 2\hat{j} + \hat{k} \text{ for } M_2 = 3 \text{ kg}$$

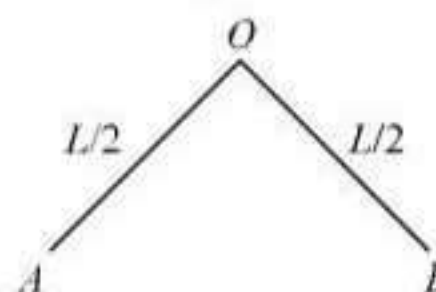
$$r_{C.M.} = \frac{\sum m_i r_i}{\sum m_i}$$

$$\Rightarrow r_{C.M.} = \frac{(1\hat{i} + 2\hat{j} + 1\hat{k}) \times 1 + (-3\hat{i} - 2\hat{j} + \hat{k}) \times 3}{4}$$

$$\Rightarrow r_{C.M.} = \frac{(1\hat{i} + 2\hat{j} + 1\hat{k}) \times 1 + (-9\hat{i} - 6\hat{j} + 3\hat{k})}{4}$$

$$\Rightarrow r_{C.M.} = \frac{-8\hat{i} - 4\hat{j} + 4\hat{k}}{4} = -2\hat{i} - \hat{j} + \hat{k}$$

**43. (d) :**



Total mass =  $M$ , total length =  $L$

Moment of inertia of  $OA$  about  $O$  = Moment of inertia of  $OB$  about  $O$ .

$$\Rightarrow \text{M.I. total} = 2 \times \left( \frac{M}{2} \right) \left( \frac{L}{2} \right)^2 \cdot \frac{1}{3} = \frac{ML^2}{12}$$

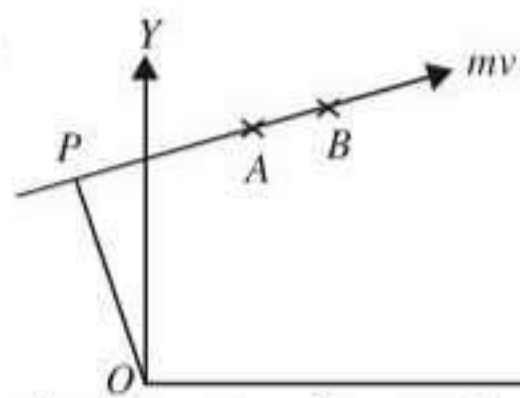
**44. (d) :** M.I. of a circular disc,  $Mk^2 = \frac{M \cdot R^2}{2}$

M.I. of a circular ring =  $MR^2$ .



$\therefore$  Ratio of their radius of gyration =  $\frac{1}{\sqrt{2}}:1$   
or  $1:\sqrt{2}$

45. (a) :



Moment of momentum is angular momentum.  $OP$  is the same whether the mass is at  $A$  or  $B$ .

$\therefore L_A = L_B$ .

46. (c) : Torque about  $A$ ,

$$\tau = mg \times \frac{l}{2} = \frac{mgl}{2}$$

Also  $\tau = I\alpha$

$\therefore$  Angular acceleration,

$$\alpha = \frac{\tau}{I} = \frac{mgl/2}{ml^2/3} = \frac{3g}{2l}$$

47. (a) : Given: Angular acceleration,  $\alpha = 3 \text{ rad/sec}^2$   
Initial angular velocity  $\omega_i = 2 \text{ rad/sec}$   
Time  $t = 2 \text{ sec}$

Using,  $\theta = \omega_i t + \frac{1}{2} \alpha t^2$

$$\therefore \theta = 2 \times 2 + \frac{1}{2} \times 3 \times 4 = 4 + 6 = 10 \text{ radian.}$$

48. (d) : Moment of inertia of a uniform circular disc about an axis through its centre and perpendicular to its plane is  $I_C = \frac{1}{2} MR^2$ .

By the theorem of parallel axes,

$\therefore$  Moment of inertia of a uniform circular disc about an axis touching the disc at its diameter and normal to the disc is  $I$ .

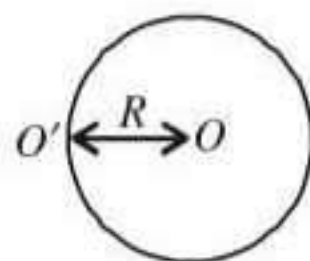
$$I = I_C + Mr^2 = \frac{1}{2} MR^2 + MR^2 = \frac{3}{2} MR^2.$$

49. (b) : The centre of the tube will be at length  $L/2$ . So radius  $r = L/2$ .

The force exerted by the liquid at the other end = centrifugal force

$$\text{Centrifugal force} = Mr\omega^2 = M \left( \frac{L}{2} \right) \omega^2 = \frac{ML\omega^2}{2}$$

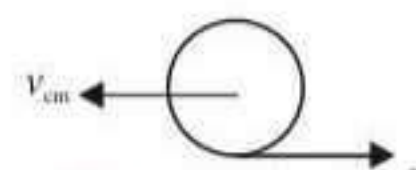
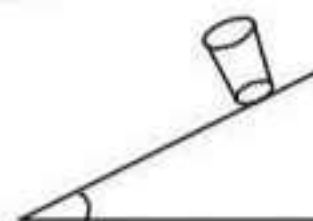
50. (c) : M.I. of disc about its normal =  $\frac{1}{2} MR^2$



M.I. about its one edge =  $MR^2 + \frac{MR^2}{2}$   
(Perpendicular to the plane)

$$\text{Moment of inertia} = \frac{3}{2} MR^2.$$

51. (d) :



Required frictional force converts translational energy into rotational energy.

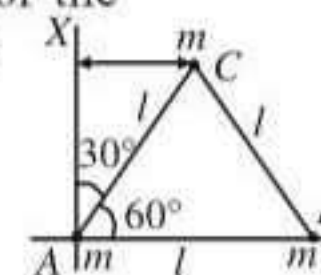
$$52. (c) : \text{K.E.} = \frac{1}{2} I \omega^2$$

$$\therefore \frac{1}{2} I_1 \omega_1^2 = \frac{1}{2} \cdot 2 I_1 \omega_2^2$$

$$\frac{\omega_1^2}{\omega_2^2} = \frac{2}{1} \Rightarrow \frac{\omega_1}{\omega_2} = \frac{\sqrt{2}}{1}$$

53. (c) : The moment of inertia of the

$$\begin{aligned} \text{system} &= m_A r_A^2 + m_B r_B^2 + m_C r_C^2 \\ &= m_A (0)^2 + m(l)^2 + m(l \sin 30^\circ)^2 \\ &= ml^2 + ml^2 \times (1/4) = (5/4) ml^2 \end{aligned}$$



$$54. (b) : \text{C.M.} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \quad \dots (1)$$

After changing position of  $m_1$  and to keep the position of C.M. same

$$\text{C.M.} = \frac{m_1(x_1 - d) + m_2(x_2 + d_2)}{m_1 + m_2}$$

$$0 = \frac{m_1 d + m_2 d_2}{m_1 + m_2}$$

[Substituting value of C.M. from (i)]

$$\Rightarrow d_2 = \frac{m_1}{m_2} d$$

$$55. (c) : \omega_f = \omega_i - \alpha t \Rightarrow 0 = \omega_i - \alpha t$$

$\therefore \alpha = \omega_i/t$ , where  $\alpha$  is retardation.

The torque on the wheel is given by

$$\tau = I\alpha = \frac{I\omega}{t} = \frac{I \cdot 2\pi v}{t} = \frac{2 \times 2 \times \pi \times 60}{60 \times 60}$$

$$\tau = \frac{\pi}{15} \text{ N m}$$

This is the torque required to stop the wheel in 1 min. (or 60 sec.).

56. (c) : Applying conservation of angular momentum.

$$I_1 \omega = (I_1 + I_2) \omega_1$$

$$\omega_1 = \frac{I_1}{(I_1 + I_2)} \omega$$



57. (c) : Radius of gyration of disc about a tangential axis in the plane of disc is  $\frac{\sqrt{5}}{2}R = K_1$ , radius of gyration of circular ring of same radius about a tangential axis in the plane of circular ring is

$$K_2 = \sqrt{\frac{3}{2}}R \quad \therefore \frac{K_1}{K_2} = \frac{\sqrt{5}}{\sqrt{6}}$$

58. (a) : The total energy at A = the total energy at B

$$\Rightarrow \frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgl$$

$$\Rightarrow v = \sqrt{u^2 - 2gl}$$

The change in magnitude of velocity =  $\sqrt{u^2 + v^2}$   
 $= \sqrt{2(u^2 - gl)}$

59. (c) : Total energy

$$= \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = \frac{1}{2}mv^2(1 + K^2/R^2)$$

$$\text{Required fraction} = \frac{K^2/R^2}{1 + K^2/R^2} = \frac{K^2}{R^2 + K^2}$$

60. (c) : Potential energy of the solid cylinder at height  $h = Mgh$

K.E. of centre of mass when reached at bottom

$$= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2 + \frac{1}{2}Mk^2 \frac{v^2}{R^2}$$

$$= \frac{1}{2}Mv^2 \left(1 + \frac{k^2}{R^2}\right)$$

$$\text{For a solid cylinder } \frac{k^2}{R^2} = \frac{1}{2} \quad \therefore \text{K.E.} = \frac{3}{4}Mv^2$$

$$\therefore Mgh = \frac{3}{4}Mv^2 \quad v = \sqrt{\frac{4}{3}gh}$$

61. (b) : According to conservation of angular momentum,  $L = I\omega = \text{constant}$ .

Therefore,  $I_1\omega_1 = I_2\omega_2$

$$\text{or } \omega_2 = \frac{I_1\omega_1}{I_2} = \frac{Mk^2\omega}{(M + 4m)k^2} = \frac{M\omega}{M + 4m}$$

62. (b) : Let us consider an elementary length  $dx$  at a distance  $x$  from one end.

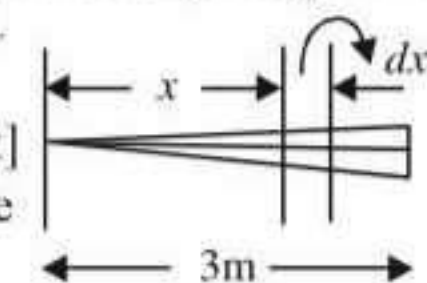
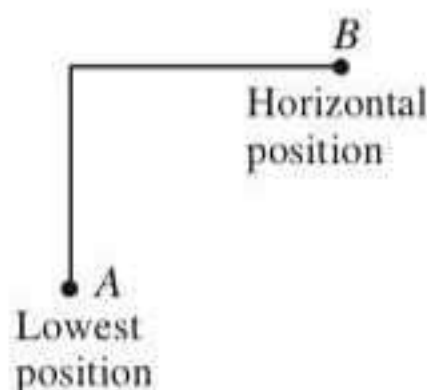
It's mass =  $k \cdot x \cdot dx$

[ $k$  = proportionality constant]

Then centre of gravity of the

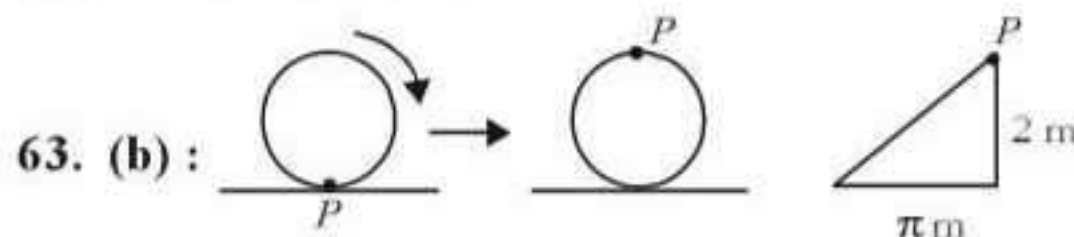
rod  $x_c$  is given by

$$x_c = \frac{\int_0^3 kx dx \cdot x}{\int_0^3 kx dx} = \frac{\int_0^3 x^2 dx}{\int_0^3 x dx} = \frac{\frac{x^3}{3} \Big|_0^3}{\frac{x^2}{2} \Big|_0^3}$$



$$\text{or, } x_c = \frac{27/3}{9/2} = 2.$$

$\therefore$  Centre of gravity of the rod will be at distance of 2 m from one end.



In half rotation point  $P$  has moved horizontally.

$$\frac{\pi d}{2} = \pi r = \pi \times 1 \text{ m} = \pi \text{ m} \quad [\because \text{radius} = 1 \text{ m}]$$

In the same time, it has moved vertically a distance which is equal to its diameter = 2 m.

$$\therefore \text{Displacement of } P = \sqrt{\pi^2 + 2^2} = \sqrt{\pi^2 + 4} \text{ m}$$

64. (d) : Since there is no friction at the contact surface (smooth horizontal surface) there will be no rolling. Hence, the acceleration of the centre of mass of the sphere will be independent of the position of the applied force  $F$ . Therefore, there is no relation between  $h$  and  $R$ .

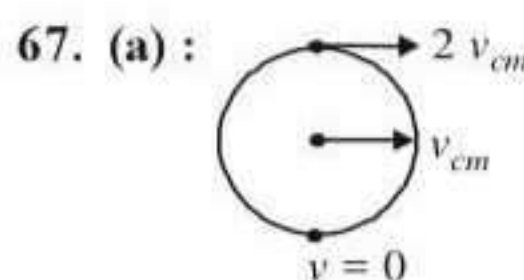
65. (b) : When a child sits on a rotating disc, no external torque is introduced. Hence the angular momentum of the system is conserved. But the moment of inertia of the system will increase and as a result, the angular speed of the disc will decrease to maintain constant angular momentum.

$$[\because \text{angular momentum} = \text{moment of inertia} \times \text{angular velocity}]$$

66. (a) : A circular disc may be divided into a large number of circular rings. Moment of inertia of the disc will be the summation of the moments of inertia of these rings about the geometrical axis.

Now, moment of inertia of a circular ring about its geometrical axis is  $MR^2$ , where  $M$  is the mass and  $R$  is the radius of the ring.

Since the density (mass per unit volume) for iron is more than that of aluminium, the proposed rings made of iron should be placed at a higher radius to get more value of  $MR^2$ . Hence to get maximum moment of inertia for the circular disc, aluminium should be placed at interior and iron at the outside.



68. (b) : As effective distance of mass from  $BC$  is greater than the effective distance of mass from  $AB$ , therefore  $I_2 > I_1$ .



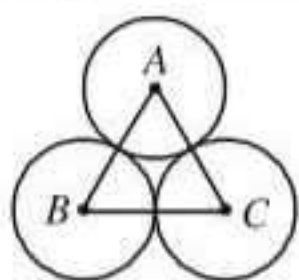
69. (a) : Solid sphere reaches the bottom first because for solid cylinder  $\frac{K^2}{R^2} = \frac{1}{2}$ ,  
and for hollow cylinder  $\frac{K^2}{R^2} = 1$ .

Acceleration down the inclined plane  $\propto \frac{1}{K^2/R^2}$ .  
Solid cylinder has greater acceleration, so it reaches the bottom first.

70. (b) : When a sphere is rotating in a vertical circle, it exerts the maximum outward pull when it is at the lowest point B.

Therefore, tension at B is maximum = Weight +  $\frac{mv^2}{R}$ .  
So, the string breaks at point B.

71. (d) :



Centre of mass of each ball lies on the centre.  
 $\Rightarrow$  Centre of mass of combined body will be at the centroid of equilateral triangle.

72. (a) : Moment of inertia of a disc about its diameter

$$= \frac{1}{4}MR^2$$

Using theorem of parallel axes,

$$I = \frac{1}{4}MR^2 + MR^2 = \frac{5}{4}MR^2$$

73. (c) : Force  $(\vec{F}) = -3\hat{i} + \hat{j} + 5\hat{k}$  and distance of the point  $(\vec{r}) = 7\hat{i} + 3\hat{j} + \hat{k}$ .

$$\text{Torque } \vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 3 & 1 \\ -3 & 1 & 5 \end{vmatrix} = 14\hat{i} - 38\hat{j} + 16\hat{k}$$

74. (d) : The resultant of all forces, on any system of particles, is zero. Therefore their centre of mass does not depend upon the forces acting on the particles.

75. (d)

76. (d) : The intersection of medians is the centre of mass of the triangle. Since the distances of centre of mass from the sides is related as  $x_{BC} < x_{AB} < x_{AC}$ .

Therefore  $I_{BC} > I_{AB} > I_{AC}$  or  $I_{BC} > I_{AB}$ .

77. (d) : Force  $(\vec{F}) = 2\hat{i} - 3\hat{j} + 4\hat{k}$  N and distance of the point from origin  $(r) = 3\hat{i} + 2\hat{j} + 3\hat{k}$  m.

Torque  $\vec{\tau} = \vec{r} \times \vec{F} =$

$$(3\hat{i} + 2\hat{j} + 3\hat{k}) \times (2\hat{i} - 3\hat{j} + 4\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 3 \\ 2 & -3 & 4 \end{vmatrix}$$

$$= 17\hat{i} - 6\hat{j} - 13\hat{k}$$

78. (d) : Linear K.E. of ball =  $\frac{1}{2}mv^2$  and rotational

$$\text{K.E. of ball} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2 = \frac{1}{5}mv^2$$

$$\text{Therefore total K.E.} = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$$

And ratio of rotational K.E. and total K.E.

$$= \frac{(1/5)mv^2}{(7/10)mv^2} = \frac{2}{7}$$

79. (d) : The moment of inertia is minimum about EG because mass distribution is at minimum distance from EG.

80. (a) : For solid sphere,  $\frac{K^2}{R^2} = \frac{2}{5}$

For disc and solid cylinder,  $\frac{K^2}{R^2} = \frac{1}{2}$

As  $\frac{K^2}{R^2}$  for solid sphere is smallest, it takes minimum time to reach the bottom of the incline, disc and cylinder reach together later.

81. (a) : P.E. = total K.E

$$mgh = \frac{7}{10}mv^2, v = \sqrt{\frac{10gh}{7}}$$

82. (d) : Total kinetic energy =

$$E_{\text{trans}} + E_{\text{rot}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2} \times \left(\frac{2}{5}mr^2\right)\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$$

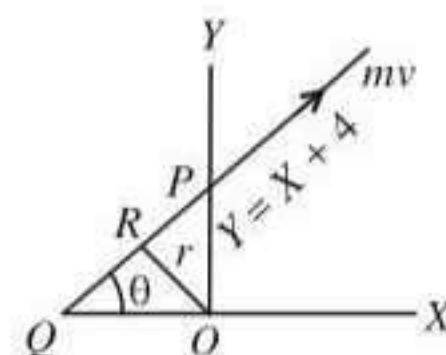
$$\therefore \frac{E_{\text{trans}}}{E_{\text{total}}} = \frac{\frac{1}{2}mv^2}{\frac{7}{10}mv^2} = \frac{5}{7}$$

83. (a) :  $\vec{L} = \vec{r} \times \vec{p}$

$Y = X + 4$  line has been shown in the figure.

When  $X = 0$ ,

$Y = 4$ , So  $OP = 4$ .





The slope of the line can be obtained by comparing with the equation of line

$$y = mx + c$$

$$m = \tan \theta = 1 \Rightarrow \theta = 45^\circ$$

$$\angle OQP = \angle OPQ = 45^\circ$$

If we draw a line perpendicular to this line.

Length of the perpendicular =  $OR$

$$\Rightarrow OR = OP \sin 45^\circ$$

$$= 4 \frac{1}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

Angular momentum of particle going along this line

$$= r \times mv = 2\sqrt{2} \times 5 \times 3\sqrt{2} = 60 \text{ units}$$

$$84. (c) : \text{K.E.} = \frac{1}{2} I \omega^2$$

$$I = \frac{2 \text{K.E.}}{\omega^2} = \frac{2 \times 360}{30 \times 30} = 0.8 \text{ kgm}^2$$

$$85. (b) : I = 1.2 \text{ kgm}^2, E_r = 1500 \text{ J,}$$

$$\alpha = 25 \text{ rad/s}^2, \omega_1 = 0, t = ?$$

$$\text{As } E_r = \frac{1}{2} I \omega^2, \omega = \sqrt{\frac{2E_r}{I}}$$

$$\omega = \sqrt{\frac{2 \times 1500}{1.2}} = 50 \text{ rad/sec}$$

$$\text{From } \omega_2 = \omega_1 + \alpha t.$$

$$50 = 0 + 25t, \quad \text{or } t = 2 \text{ s.}$$

86. (c) : Moment of inertia of uniform circular disc about diameter =  $I$

According to theorem of perpendicular axes.

$$\text{Moment of inertia of disc about axis} = 2I = \frac{1}{2} mr^2$$

Applying theorem of parallel axes

Moment of inertia of disc about the given axis

$$= 2I + mr^2 = 2I + 4I = 6I$$

87. (b) : Angular momentum about the point of contact with the surface includes the angular momentum about the centre. Because of friction, linear momentum will not be conserved.

$$88. (a) : \text{Kinetic energy} = \frac{1}{2} I \omega^2, \text{ and for ring } I = mr^2$$

$$\text{Hence KE} = \frac{1}{2} mr^2 \omega^2$$

