Class- X Session- 2022-23

Subject- Mathematics (Standard)

Sample Question Paper - 32

Time Allowed: 3 Hrs. **Maximum Marks : 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section **A** has 20 MCQs carrying 1 mark each
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. Two chords PQ and RS intersect at T outside the circle. If PQ = 5 cm, QT = 3 cm, TR = 2 cm. length of RS is : **[1]**

a) 8 cm b) 15 cm

c) 12 cm d) 10 cm

2. The length of the median through A of $\triangle$ABC with vertices A(7, -3), B(5, 3) and C(3, -1) is **[1]**

a) 5 units b) 3 units

c) 7 units d) 25 units

3. A die is thrown twice. The probability that 5 will come up at least once is **[1]**

a) $\frac{11}{36}$ b) 0

c) 1 d) $\frac{25}{36}$

4. If A(4, 9), B(2, 3) and C(6, 5) are the vertices of $\triangle ABC$, then the length of median through C is [1]

a) 10 units
b) 5 units
c) $\sqrt{10}$units
d) 25 units

5. The coordinates of a point on x-axis which lies on the perpendicular bisector of the line segment joining the points (7, 6) and (-3, 4) are [1]

a) (3, 0)
b) (0, 2)
c) (0, 3)
d) (2, 0)

6. The area of the triangle formed by y = x, x = 6 and y = 0 is [1]

a) 18 sq. units
b) 72 sq. units
c) 36 sq. units
d) 9 sq. units

7. A card is selected at random from a well shuffled deck of 52 playing cards. The probability of its being a face card is [1]

a) $\frac{3}{26}$
b) $\frac{3}{13}$
c) $\frac{1}{26}$
d) $\frac{4}{13}$

8. Cards marked with numbers 1, 2, 3, ..., 25 are placed in a box and mixed thoroughly and one card is drawn at random from the box. The probability that the number on the card is a multiple of 3 or 5 is [1]

a) $\frac{8}{25}$
b) $\frac{12}{25}$
c) $\frac{4}{25}$
d) $\frac{1}{5}$

9. $9x^2 + 12x + 4 = 0$ have [1]

a) Real and Distinct roots
b) No real roots
c) Distinct roots
d) Real and Equal roots

10. If a marble of radius 2.1 cm is put into a cylindrical cup full of water of radius 5cm and height 6 cm, then how much water flows out of the cylindrical cup? [1]

a) 38.8 cm^3
b) 471.4 cm^3
c) 19.4 cm^3
d) 55.4 cm^3

11. $4x^2 - 20x + 25 = 0$ have [1]

a) Real roots
b) No Real roots

c) Real and Equal roots | d) Real and Distinct roots

12. If HCF of two numbers is 1, the two numbers are called relatively ________ or ________. **[1]**

a) composite, co-prime
b) composite, prime
c) prime, co-prime
d) twin primes, square numbers

13. If $\sin\theta - \cos\theta = 0$ then the value of $\left(\sin^4\theta + \cos^4\theta\right)$ is **[1]**

a) $\frac{1}{2}$
b) 1
c) $\frac{3}{4}$
d) $\frac{1}{4}$

14. From a lighthouse, the angles of depression of two ships on opposite sides of the lighthouse are observed to be 30° and 45°. If the height of the lighthouse is h meters, the distance between the ships is **[1]**

a) $1 + \left(1 + \frac{1}{\sqrt{3}}\right)$ h metres
b) $\sqrt{3}$ h metres
c) ($\sqrt{3}$ + 1) h metres
d) ($\sqrt{3}$ - 1) h metres

15. Two vertices of $\triangle$ABC are A (-1, 4) and B(5, 2) and its centroid is G(0, -3). Then, the coordinates of C are **[1]**

a) (4, 3)
b) (4, 15)
c) (-4, -15)
d) (-15, -4)

16. ________ is neither prime nor composite. **[1]**

a) 4
b) 1
c) 2
d) 3

17. The sum of the digits of a two digit number is 9. If 27 is added to it, the digits of the number get reversed. The number is **[1]**

a) 63
b) 27
c) 36
d) 72

18. The algebraic sum of the deviations of a frequency distribution from its mean is: **[1]**

a) 0
b) a non-zero number
c) always positive
d) always negative

19. **Assertion (A):** If a number x is divided by y(x, y) (both x and y are positive) then remainder will be less than x. **[1]**
Reason (R): Dividend = Divisor × Quotient + Remainder.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** D and E are points on the sides AB and AC respectively of a $\triangle$ ABC such that AB = 10.8 cm, AD = 6.3 cm, AC = 9.6 cm and EC = 4 cm then DE is parallel to BC. **[1]**
Reason (R): If a line is parallel to one side of a triangle then it divides the other two sides in the same ratio.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. **Tree Plantation Drive** **[2]**

A Group Housing Society has 600 members, who have their houses on the campus and decided to hold a Tree Plantation Drive on the occasion of New Year. Each household was given the choice of planting a sapling of its choice. The number of different types of saplings planted was:

i. Neem - 125

ii. Peepal - 165

iii. Creepers - 50

iv. Fruit plants - 150

v. Flowering plants - 110

On the opening ceremony, one of the plants is selected randomly for a prize. After reading the above passage, answer the following questions.
What is the probability that the selected plant is

i. A fruit plant or a flowering plant?

ii. Either a Neem plant or a Peepal plant?

22. Is the pair of linear equations consistent? Justify your answer. **[2]**
$-3x - 4y = 12$, $4y + 3x = 12$

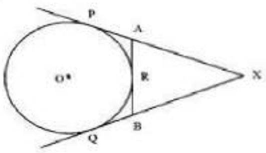
23. If the distance between points (x, 0) and (0,3) is 5, what are the values of x? **[2]**

OR

Find the points of trisection of the line segment joining the points (5, -6) and (-7, 5).

24. Find the zeroes of the polynomial $x^2 + \frac{1}{6}x - 2$ and verify the relation between the coefficients and the zeroes of the polynomial. **[2]**

25. In given Fig. XP and XQ are tangents from X to the circle with centre O. R is a point on the circle. Prove that XA + AR = XB + BR. **[2]**



OR

Two concentric circles are of radii 7 cm and r cm respectively where r > 7. A chord of the larger circle of the length 48 cm, touches the smaller circle. Find the value of r.

Section C

26. Draw the graph of the pair of equations 2x + y = 4 and 2x – y = 4. Write the vertices of the triangle formed by these lines and the y-axis. Also find the area of this triangle. **[3]**

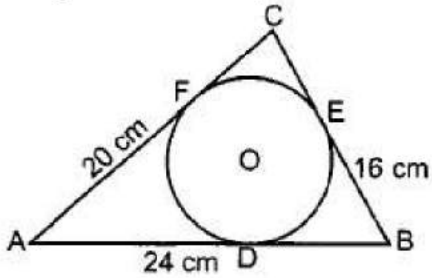
27. The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they all change simultaneously at 8 a.m. then at what time will they again change simultaneously? **[3]**

OR

Find the largest number that will divide 398, 436 and 542 leaving remainders 7,11 and 15 respectively.

28. If, $\cot B = \frac{12}{5}$ prove that $\tan^2 B - \sin^2 B = \sin^4 B \cdot \sec^2 B$ **[3]**

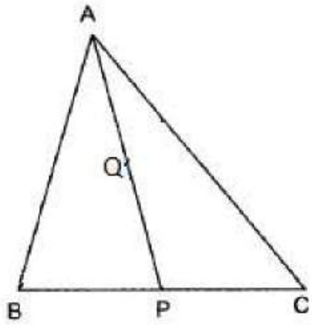
29. A circle is inscribed in a $\triangle$ABC having sides 16 cm, 20 cm and 24 cm as shown in figure. Find AD, BE and CF. **[3]**



OR

If radii of the two circles are equal, prove that AB = CD where AB and CD are common tangents.

30. In Fig. P is the mid-point of BC and Q is the mid-point of AP. If BQ, when produced meets AC at R, prove that RA = $\frac{1}{3}$ CA. **[3]**



31. A person observed the angle of elevation of the top of a tower is 30°. He walked 50 m towards the foot of the tower along level ground and found the angle of elevation of the top of the tower as 60°. Find the height of the tower. **[3]**

Section D

32. In a trapezium ABCD, AB | | DC and DC = 2AB. EF | | AB, where E and F lie on BC and AD respectively such that $\frac{BE}{EC} = \frac{4}{3}$. Diagonal DB intersects EF at G. Prove that, 7EF = 11AB. **[5]**

33. Sum of the areas of two squares is 544 m^2. If the difference of their perimeters is 32 m, find the sides of the two squares. **[5]**

OR

The perimeter of a rectangular field is 82 m and its area is 400 square metre. Find the length and breadth of the rectangle.

34. To find out the concentration of SO_2 in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below: **[5]**

Concentration of SO_2 (in ppm)	**Frequency**
0.00 - 0.04	4
0.04 - 0.08	9
0.08 - 0.12	9
0.12 - 0.16	2
0.16 - 0.20	4
0.20 - 0.24	2

find the mean concentration of SO_2 in the air.

35. Four equal circles are described at the four corners of a square so that each touches two of the others. The shaded area enclosed between the circles is $\frac{24}{7}\text{cm}^2$. Find the radius of each circle. **[5]**

OR

A chord of a circle of radius 10cm subtends a right angle at the center. Find the area of the corresponding: (Use $\pi = 3.14$)

i. minor sector
ii. major sector
iii. minor segment
iv. major segment

Section E

36. **Read the text carefully and answer the questions:** **[4]**

An ice-cream seller used to sell different kinds and different shapes of ice-cream like rectangular shaped with one end hemispherical, cone-shaped and rectangular brick, etc. One day Sheetal and her brother came to his shop. Sheetal purchased an ice-cream which has the following shape: ice-cream cone as the union of a right circular cone and a hemisphere that has the same (circular) base as the cone. The height of the cone is 9 cm and the radius of its base is 2.5 cm. her brother purchased rectangular brick shaped ice cream with length 9 cm, width 4cm and thickness 2 cm.


(i) The volume of the ice-cream without hemispherical end.

(ii) The volume of the ice-cream with a hemispherical end.

(iii) Find the volume her brother ice cream?

OR

Whose quantity of ice cream is more and by how much?

37. **Read the text carefully and answer the questions:** **[4]**

Sehaj Batra gets pocket money from his father every day. Out of pocket money, he saves money for poor people in his locality. On 1st day he saves ₹27.5 On each succeeding day he increases his saving by ₹2.5.

(i) Find the amount saved by Sehaj on 10^{th} day.

(ii) Find the amount saved by Sehaj on 25^{th} day.

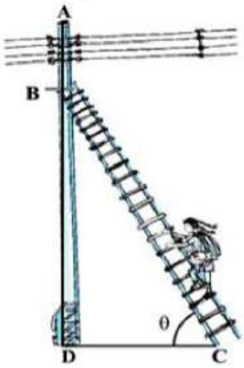
OR

Find in how many days Sehaj saves ₹1400.

(iii) Find the total amount saved by Sehaj in 30 days.

38. **Read the text carefully and answer the questions:** **[4]**

In a village, group of people complained about an electric fault in their area. On their complaint, an electrician reached village to repair an electric fault on a pole of height 10 m. She needs to reach a point 1.5 m below the top of the pole to undertake the repair work (see the adjoining figure). She used ladder, inclined at an angle of θ to the horizontal such that $\cos\theta = \frac{\sqrt{3}}{2}$, to reach the required position.



(i) Find the length BD?

(ii) Find the length of ladder.

OR

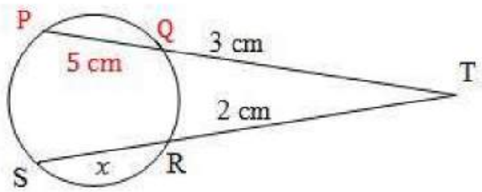
If the height of pole and distance BD is doubled, then what will be the length of the ladder?

(iii) How far from the foot of the pole should she place the foot of the ladder?

SOLUTION

Section A

1. **(d)** 10 cm

Explanation: 

We know that if two chords intersect each other at T outside the circle, then TP $\times$ TQ = TS $\times$ TR Let SR = $x$cm

$\Rightarrow (5 + 3) \times 3 = (x + 2) \times 2$

$\Rightarrow x + 2 = 12$

$\Rightarrow x = 10$ cm x = 10 cm

$\therefore$ SR = 10 cm

2. **(a)** 5 units

Explanation: ABC is a triangle with A(7, - 3), B(5, 3) and C(3, - 1)

Let median on BC bisects BC at D. (AD is given as the median)

$\therefore$ Coordinates of D are $\left(\frac{5+3}{2}, \frac{3-1}{2}\right) = (4, 1)$

$\therefore$ AD = $\sqrt{(4-7)^2 + (1+3)^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25} = 5$ units

3. **(a)** $\frac{11}{36}$

Explanation: Elementary events are

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

∴ Number of Total outcomes = 36

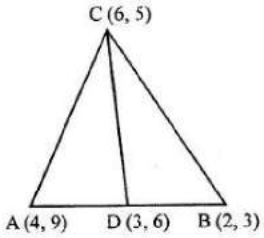
And Number of possible outcomes = 11

∴ Required Probability = $\frac{11}{36}$

4. **(c)** $\sqrt{10}$units

Explanation: A(4, 9), B(2, 3) and C(6, 5) are the vertices of $\triangle ABC$

Let median CD has been drawn C(6, 5)



∴ D is mid point of AB

$$D = \left(\frac{4+2}{2}, \frac{9+3}{2}\right)$$

∴ D(3, 6)

∴ Length of CD

$= \sqrt{(6-3)^2 + (5-6)^2} = \sqrt{3^2 + (-1)^2}$

$= \sqrt{9+1} = \sqrt{10}$ units

5. **(a)** (3, 0)

Explanation: The given point P lies on x-axis

Let the co-ordinates of P be (x, 0)

The point P lies on the perpendicular bisector of the line segment joining the points A(7, 6), B(-3, 4)

$\therefore PA = PB \Rightarrow PA^2 = PB^2$

$\Rightarrow (x-7)^2 + (0-6)^2 = (x+3)^2 + (0-4)^2$

$\Rightarrow x^2 - 14x + 49 + 36 = x^2 + 6x + 9 + 16$

$\Rightarrow$ -14x + 85 = 6x + 25

$\Rightarrow$ 6x + 14x = 85 - 25 $\Rightarrow$ 20x = 60

$x = \frac{60}{20} = 3$

∴ co-ordinates of P will be (3, 0)

6. **(a)** 18 sq. units

Explanation: The triangle formed by the lines y = x, x = 6 and y = 0 is shaded.

The area of the shaded region, i.e., $x = y$

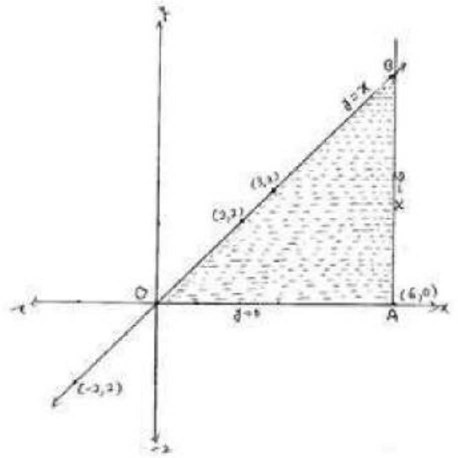
We got a right-angled triangle with base 6 units and height 6 units

Triangle OAB = $\frac{1}{2}$ × OA × AB

Hence the area of triangle = $\frac{1}{2} \times 6 \times 6 = 18$ sq.units

x	2	-2	3

y	2	-2	3



7. **(b)** $\frac{3}{13}$

Explanation: Face Cards are = 4 kings + 4 queens + 4 jacks = 12
Number of possible outcomes = 12
Number of Total outcomes = 52

$\therefore$ Required Probability = $\frac{12}{52} = \frac{3}{13}$

8. **(b)** $\frac{12}{25}$

Explanation: Number of multiples of 3 = 8 (3 6 9 12 15 18 21 24)
Number of multiples of 5 = 5 (5 10 15 20 25)
Number of possible outcomes (multiples of 3 or 5) = 12 (3,5,6,9,10,12,15,18,20,21,24,25)
Number of Total outcomes = 25

$\therefore$ Required Probability = $\frac{12}{25}$

9. **(d)** Real and Equal roots

Explanation: Comparing the given equation to the below equation
$ax^2 + bx + c = 0$
a = 9, b = 12, c = 4
$D = b^2 - 4ac$
$D = 12^2 - 4 \times 9 \times 4$
D = 144 - 144
D = 0
If $b^2 - 4ac = 0$ then equation have equal and real roots.

10. **(a)** 38.8 cm^3

Explanation: We have,
radius of spherical marble = r = 2.1 cm
Now, volume of spherical marble $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{21}{10} \times \frac{21}{10} \times \frac{21}{10} = 38.808 cm^3$
When a marble is dropped into the cylindrical cup full of water, then
volume of water that flows out of the cup = volume of marble = 38.808 cm^3

11. **(c)** Real and Equal roots

Explanation: $D = b^2 - 4ac$

$D = (-20)^2 - 4 \times 4 \times 25$

$D = 400 - 400$

$D = 0$. Hence Real and equal roots.

12. **(c)** prime, co-prime

Explanation: If their greatest common factor is 1, then one of the two numbers must be a prime or co-prime. Their least common multiple must be the product of the two numbers.

13. **(a)** $\frac{1}{2}$

Explanation: It is given that,

$\sin\theta - \cos\theta = 0$

$\Rightarrow \sin\theta = \cos\theta$

$\Rightarrow \frac{\sin\theta}{\cos\theta} = 1$

$\Rightarrow \tan\theta = 1$

$\Rightarrow \tan\theta = \tan 45°$

$\Rightarrow \theta = 45°$

$\therefore \sin^4\theta + \cos^4\theta$

$= \sin^4 45° + \cos^4 45°$

$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4$

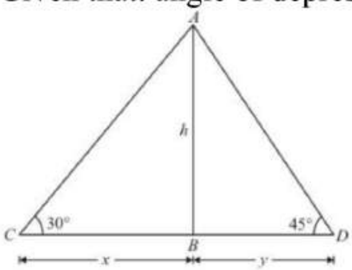
$= \frac{1}{4} + \frac{1}{4}$

$= \frac{1}{2}$

14. **(c)** $(\sqrt{3} + 1)$ h metres

Explanation: Let the height of the light house AB be h meters

Given that: angle of depression of ship are $\angle C = 30^o$ and $\angle D = 45^o$



Distance of the ship C = BC = x and distance of the ship D = BD = y

Here, we have to find distance between the ships.

So we use trigonometric ratios.

In a triangle ABC,

$\Rightarrow \tan C = \frac{AB}{BC}$

$\Rightarrow \tan 30^\circ = \frac{h}{x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$

$\Rightarrow x = \sqrt{3}h$

Again in a triangle ABD,

$\tan D = \frac{AB}{BD}$

$\Rightarrow \tan 45^o = \frac{h}{y}$

$\Rightarrow 1 = \frac{h}{y}$

$\Rightarrow y = h$

Now, distance between the ships = $x + y = \sqrt{3}h + h = (\sqrt{3} + 1)\, h$

15. **(c)** (-4, -15)

Explanation: Let the vertex C be C (x,y). Then

$\frac{-1+5+x}{1} = 0$ and $\frac{4+2+y}{3} = -3 \Rightarrow x + 4 = 0$ and $6 + y = -9$

$\therefore \quad x = -4$ and $y = -15$

so, the coordinates of C are (-4, -15).

16. **(b)** 1

Explanation: 1 is neither prime nor composite.

A prime is a natural number greater than 1 that has no positive divisors other than 1 and itself

e.g. 5 is prime because 1 and 5 are its only positive integers factors but 6 is composite because it has divisors 2 and 3 in addition to 1 and 6.

17. **(c)** 36

Explanation: Let the original No is 10x + y

The sum of the digits of a two digit no. Is 9.

If the digits are reversed,

the new no. Is 27 less than the given no.

x + y = 9

(10x + y) = (10y + x) - 27

9x - 9y = -27

x - y = -3

x = 3

y = 6

Then the number Is 36

18. **(a)** 0

Explanation: The algebraic sum of the deviations of a frequency distribution from its mean is zero.

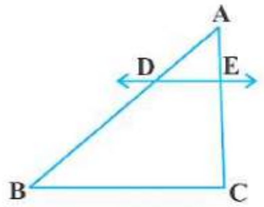
Let $x_1, x_2, x_3, ...x_n$ are observations and $\bar{X}$ is the mean

$\therefore (\bar{x} - x_1) + (\bar{x} - x_2) + (\bar{x} - x_3) + ...(\bar{x} - xn)$

$= n\bar{x} - (x_1 + x_2 + x_3 + ...x_n)$

$= n\bar{x} - n\bar{x} = 0$

19. **(d)** A is false but R is true.
Explanation: Remainder is less than by divisor not by divident.
20. **(b)** Both A and R are true but R is not the correct explanation of A.
Explanation:
We know that if a line is parallel to one side of a triangle then it divides the other two sides in the same ratio. This is the Basic Proportionality theorem.



So, Reason is correct.
DB = 10.8 - 6.3 = 4.5 cm and AE = 9.6 - 4 = 5.6 cm

Now, $\frac{AD}{DB} = \frac{6.3}{4.5} = \frac{63}{45} = \frac{7}{5}$ and $\frac{AE}{EC} = \frac{5.6}{4} = \frac{56}{40} = \frac{7}{5}$

$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$

By Converse of Basic Proportionality theorem, DE||BC
So, Assertion is correct.
But reason (R) is not the correct explanation of assertion (A).

Section B

21. We have, total plant = 125 + 165 + 50 + 150 +110
= 600
i.e. total outcomes = 600
i. Fruit or flowering plant = 150 +110
= 260
i.e. favourable outcomes = 260

Prob. $= \frac{260}{600}$

$= \frac{13}{20}$

ii. Either a Neem or peepal plant = 125 + 165
= 290

Prob. $= \frac{290}{600}$

$= \frac{29}{60}$

22. Conditions for pair of linear equations to be consistent is

$a_1/a_2 \neq b_1/b_2$... [unique solution]

and $a_1/a_2 = b_1/b_2 = c_1/c_2$ ...[coincident or infinitely many solutions]

Comparing the given pair of linear equations

$-3x - 4y - 12 = 0$ and $4y + 3x - 12 = 0$

with standard form we get:

$a_1 = -3, b_1 = -4, c_1 = -12;$

And $a_2 = 3, b_2 = 4, c_2 = -12;$

$a_1/a_2 = -3/3 = -1$

$b_1/b_2 = -4/4 = -1$

$c_1/c_2 = -12/-12 = 1$

Here, $a_1/a_2 = b_1/b_2 \neq c_1/c_2$

Hence, the pair of linear equations has no solution, i.e., inconsistent.

23. Distance between (x, 0) and (0, 3) = 5

$$\Rightarrow \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 5$$

$$\Rightarrow \sqrt{(0 - x)^2 + (3 - 0)^2} = 5$$

$$\Rightarrow \sqrt{x^2 + 9} = 5$$

Squaring,

$x^2 + 9 = 25 \Rightarrow x^2 = 25 - 9 = 16$

$\Rightarrow x^2 - 16 = 0 \Rightarrow (x + 4)(x - 4) = 0$

Either $x + 4 = 0$, then $x = -4$

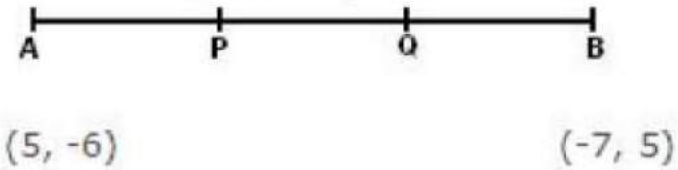
or $x - 4 = 0$, then $x = 4$

Hence $x = 4, -4$

OR

According to the question, A(5, -6) and B(-7, 5).

Let P and Q be the point of trisection of AB i.e. $AP = PQ = QB$



P divides AB internally in the ratio of 1:2, by applying section formula, we get the coordinates of P.

$$= \left(\frac{1(-7)+2(5)}{1+2}\right), \left(\frac{1(5)+2(-6)}{1+2}\right) \therefore P\left(1, \frac{-7}{3}\right)$$

Q also divides AB internally in the ratio of 2:1, by applying section formula, we get the coordinates of Q.

$$= \left(\frac{2(-7)+1(5)}{2+1}\right), \left(\frac{2(5)+1(-6)}{2+1}\right) \therefore Q\left(-3, \frac{4}{3}\right)$$

24. $x^2 + \frac{1}{6}x - 2 = \frac{1}{6}(6x^2 + x - 12) = \frac{1}{6}[6x^2 + 9x - 8x - 12]$

$= \frac{1}{6}[3x(2x + 3) - 4(2x + 3)] = \frac{1}{6}(3x - 4)(2x + 3)$

Hence, $\frac{4}{3}$ and $-\frac{3}{2}$ are the zeroes of the given polynomial.

The given polynomial is $x^2 + \frac{1}{6}x - 2$

The sum of zeroes $= \frac{4}{3} + -\frac{3}{2} = \frac{-1}{6} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$ and

the product of zeroes $= \frac{4}{3} \times \frac{-3}{2} = -2 = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

25. Since the lengths of tangents from an exterior point to a circle are equal.

$\therefore$ XP = XQ ....(i)

AP = AR .... (ii)

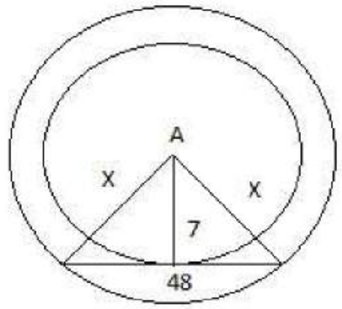
BQ = BR .... (iii)

Now Xp = XQ i.e. XA + AP = XB + BQ

$\Rightarrow$ XA + AR = XB + bR

Hence proved.

OR



Let us take r = x

Now using Pythagoras theorem

$(x)^2 = 24^2 + 7^2$

$(x)^2 = 576 + 49$

$(x)^2 = 625$

Therefore, x = 25 cm.

r = 25 cm.

Section C

26. The given pair of linear equations

2x + y = 4

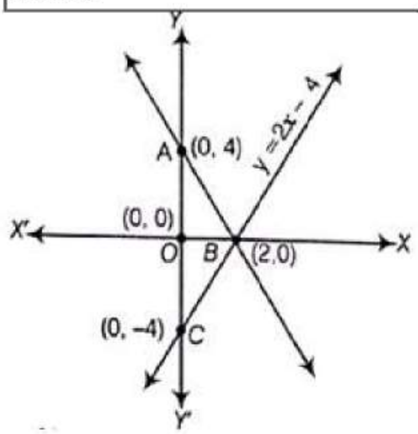
and 2x-y = 4

Table for line 2x + y = 4

x	0	2
y	4	0
Points	A	B

and table for line 2x - y = 4

x	0	2
y	- 4	0
Points	C	B



So the Graphical representation of both lines is as above.
Here, both lines and Y - axis from a $\triangle ABC$.
Hence, the vertices of a $\triangle ABC$ are A (0,4), B(2,0) and C(0,- 4) where A and C are obtained by putting x = 0 in the given equations abd B is obtained by solving them together.
$\therefore$ Required area of $\triangle ABC = 2 \times$ Area of $\triangle AOB$

$$\triangle ABC = 2 \times \left(\frac{1}{2} \times 4 \times 2\right) = 8 \text{ sq. units.}$$

Hence, the required area of the triangle is 8 sq units.

27. We have to find Prime Factors of the following numbers

$48 = 2^4 \times 3$

$72 = 2^3 \times 3^2$

$108 = 2^2 \times 3^3$

so the LCM of 48, 72 and 108is

$LCM = 2^4 \times 3^3$

$LCM = 16 \times 27 = 432$

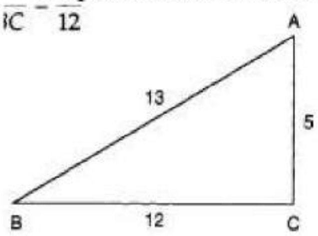
$432 \text{ seconds} = \frac{432}{60} \text{ mins}$

432 seconds = 7.2 mins
So the time it will change together again is
= 8 : 07 : 12 am

OR

The largest positive integer that will divide 398, 436 and 542 leaving reminders 7, 11 and 15 respectively is the HCF of the numbers (398 – 7), (436 – 11) and (542 – 15) i.e. 391, 425 and 527.
HCF of 391,425 and 527:
HCF of 425 and 391:
$425 = 391 \times 1 + 34$
$391 = 34 \times 11 + 17$
$34 = 17 \times 2 + 0$
HCF of 425 and 391 = 17
$527 = 17 \times 31$
Similarly, HCF of 17 and 527 = 17
So, HCF of (391,425 ,527) = 17
$\therefore$ Required number is 17.

28.



We have

$$\cot B = \frac{\text{Base}}{\text{Perpendicular}} = \frac{12}{5}$$

Let us draw a right triangle ABC, in which $\angle C = 90°$ such that
Base = BC = 12 units and, Perpendicular = AC = 5 units.
Applying Pythagoras Theorem in,ΔBCA we get

$AB^2 = BC^2 + AC^2$

$\Rightarrow \quad AB^2 = 12^2 + 5^2 = 169$

$\Rightarrow \quad AB = \sqrt{169} = 13$

$\therefore \quad \sin B = \frac{AC}{AB} = \frac{5}{13}, \tan B = \frac{AC}{BC} = \frac{5}{12}$ and, $\sec B = \frac{AB}{BC} = \frac{13}{12}$

Now, L.H.S = $\tan^2 B - \sin^2 B$

$\Rightarrow \quad \text{L. H. S} = (\tan B)^2 - (\sin B)^2$

$\Rightarrow \quad \text{L. H. S} = \left(\frac{5}{12}\right)^2 - \left(\frac{5}{13}\right)^2 = \frac{25}{144} - \frac{25}{169}$

$\Rightarrow \quad \text{L. H. S} = 25\left(\frac{1}{144} - \frac{1}{169}\right) = 25\left(\frac{169-144}{144 \times 169}\right)$

$\Rightarrow \quad \text{L. H. S} = 25 \times \frac{25}{144 \times 169} = \frac{25 \times 25}{144 \times 169} = \frac{5^2 \times 5^2}{12^2 \times 13^2}$.......(i)

and, R. H. S = $\sin^4 B \cdot \sec^2 B$

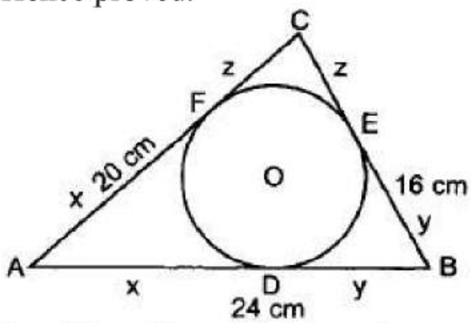
$\Rightarrow \quad \text{R. H. S} = (\sin B)^4 (\sec B)^2 = \left(\frac{5}{13}\right)^4 \times \left(\frac{13}{12}\right)^2 = \frac{5^4 \times 13^2}{13^4 \times 12^2} = \frac{5^4}{13^2 \times 12^2} = \frac{5^2 \times 5^2}{13^2 \times 12^2}$

.....(ii)

From (i) and (ii), we have

$\tan^2 B - \sin^2 B = \sin^4 B \cdot \sec^2 B$

Hence proved.

29. 

Let AD = AF = x [Tangents from external point are equal]

$BD = BE = y$ and $CE = CF = z$

According to the question,

$AB = x + y = 24$ cm ...(i)

$BC = y + z = 16$cm ...(ii)

$AC = x + z = 20$cm ...(iii)

Subtracting (iii) from (i). we get

y - z = 4 ...(iv)

Adding (ii) and (iv), we get

$2y = 20 \Rightarrow y = 10$cm

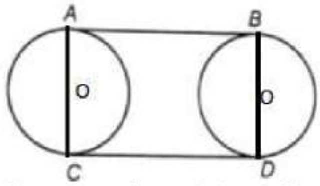
Substituting the value of y in (ii) and (i) we get $z = 6cm; x = 14cm$

$\therefore \; AD = 14cm, BE = 10cm \; and \; CF = 6cm.$

OR

Given: AB and CD are two common tangents to two circles of equal radii.

To prove



Construction: OA, OC, O'B and O'D proof

Now, $\angle OAB = 90^\circ$ and $\angle OCD = 90^\circ$ as $OA \perp AB$ and $OC \perp CD$

A tangent at any point of a circle is perpendicular to radius through the point of contact

Thus, AC is a straight line.

Also, $\angle O'BA = \angle O'DC = 90^\circ$

A tangent at a point on the circle is perpendicular to the radius through point of contact

so ABCD is a quadrilateral with four sides as AB, BC, CD and AD

But as $\angle A = \angle B = \angle C = \angle D = 90^\circ$

so, ABCD is a rectangle.

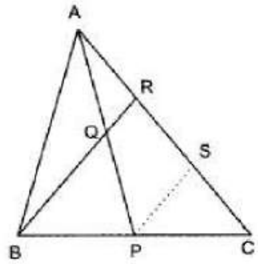
Hence, AB = CD opposite sides of the rectangle are equal.

30. **GIVEN**: A ΔABC in which P is the mid-point of BC, Q is the mid-point of BR and, Q is also the mid-point of AP such that BQ produced meets AC at R.

TO PROVE $RA = \frac{1}{3}CA.$

CONSTRUCTION: Draw $PS \| BR$, meeting AC at S.

PROOF: In ΔBCR, P is the mid-point of BC and $PS \| BR$.



$\therefore$ S is the mid-point of CR.

$\Rightarrow$ CS = SR ...(i)

In ΔAPS, Q is the mid-point of AP and $QR \| PS$.

$\therefore$ R is the mid-point of AS.

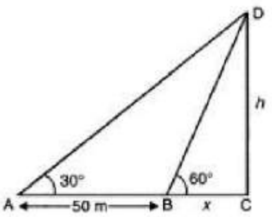
$\Rightarrow$ AR = RS ...(ii)

From (i) and (ii), we get

AR = RS = SC

$\Rightarrow$ AC = AR + RS + SC = 3 AR

$\Rightarrow \quad AR = \frac{1}{3}AC = \frac{1}{3}CA$

31. 

Let height of the tower be DC = h m and BC = x m AC = (50 + x) m

In $\triangle DBC$, $\frac{h}{x} = \tan 60^\circ = \sqrt{3}$

$\Rightarrow \quad h = \sqrt{3}x$ ..(i)

In $\triangle$DAC, $\frac{h}{x+50} = \tan 30^\circ = \frac{1}{\sqrt{3}}$,

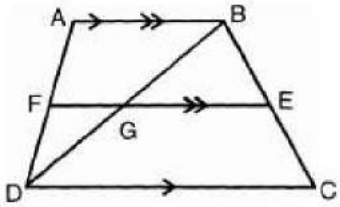
$\Rightarrow \quad \sqrt{3}h = x + 50$ ...(ii)

Substituting the value of h from (i) in (ii), we get
3x = x + 50
or, 3x - x = 50
$\Rightarrow$ 2x = 50
$\Rightarrow$ x = 25 m

$h = 25\sqrt{3} = 25 \times 1 \cdot 732$m

= 43.3 m
Hence, Height of tower = 43.3 m.

Section D

32. 

In a trapezium ABCD, AB|| DC ,. EF || AB and CD=2AB

and also $\frac{BE}{EC} = \frac{4}{3}$ -------------(1)

AB || CD and AB || EF

$\therefore \frac{AF}{FD} = \frac{BE}{EC} = \frac{4}{3}$

In ΔBGE and ΔBDC
$\angle BEG = \angle BCD$ ($\because$ corresponding angles)
$\angle GBE = \angle DBC$ (Common)
$\therefore \Delta BGE \sim \Delta BDC$ [By AA similarity]

$\Rightarrow \frac{EG}{CD} = \frac{BE}{BC}$ ....(2)

Now, from (1) $\frac{BE}{EC} = \frac{4}{3}$

$\Rightarrow \frac{EC}{BE} = \frac{3}{4}$

$\Rightarrow \frac{EC}{BE} + 1 = \frac{3}{4} + 1$

$\Rightarrow \frac{EC+BE}{BE} = \frac{7}{4}$

$\Rightarrow \frac{BC}{BE} = \frac{7}{4}$ or $\frac{BE}{BC} = \frac{4}{7}$

from equation (2), $\frac{EG}{CD} = \frac{4}{7}$

So $EG = \frac{4}{7}CD$ ......(3)

Similarly, $\Delta DGF \sim \Delta DBA$ (by AA similarity)

$\Rightarrow \frac{DF}{DA} = \frac{FG}{AB}$

$\Rightarrow \frac{FG}{AB} = \frac{3}{7}$

$\Rightarrow FG = \frac{3}{7}AB$ ...(4)

$$\left[\begin{array}{l} \because \frac{AF}{AD} = \frac{4}{7} = \frac{BE}{BC} \\ \Rightarrow \frac{EC}{BC} = \frac{3}{7} = \frac{DE}{DA} \end{array} \right]$$

Adding equations (3) and (4),we get,

$EG + FG = \frac{4}{7}CD + \frac{3}{7}AB$

$\Rightarrow EF = \frac{4}{7} \times (2AB) + \frac{3}{7}AB$

$= \frac{8}{7}AB + \frac{3}{7}AB = \frac{11}{7}AB$

$\therefore \quad 7EF = 11AB$

33. Let sides of two squares be a cm and b cm

Sum of areas of squares = $a^2 + b^2$

Sum of Perimater = 4a + 4b

A.T.Q $a^2 + b^2 = 544$

4a - 4b = 32

or, a - b = 8

a = b + 8

$\Rightarrow a^2 + b^2 = 544$

$(b + 8)^2 + b^2 = 544$

$\Rightarrow b^2 + 64 + 16b + b^2 = 544$

$\Rightarrow 2b^2 + 16b + 64 = 544$

$\Rightarrow b^2 + 8b + 32 = 272$

$\Rightarrow b^2 + 8b - 240 = 0$

$\Rightarrow b^2 + 20b - 12b - 240 = 0$

$\Rightarrow b(b + 20) - 12(b + 20) = 0$

b = 12 or b = -20

Sides cant be -ve

b = 12

a = 20

Therefore, sides of two squares are 20 cm and 12 cm respectively

OR

Perimeter = 82 m

$\Rightarrow$ 2 (l +b) = 82 m

or, l + b = 41 m

Area = 400 m^2

$\Rightarrow$ l $\times$ b = 400 m^2

Let length be x m. Then,

breadth = (41- x) m

Now, $x(41 - x) = 400$

$41x - x^2 = 400$

$x^2 - 41x + 400 = 0$

(x - 16)(x - 25) = 0

x = 16 or x = 25

Hence, if length = 16 m, then breadth = 25 m

or, if length = 25 m, then breadth = 16 m

34. We may find class marks for each interval by using the relation

$$x = \frac{\text{upperlimit} + \text{lowerclasslimit}}{2}$$

Class size of this data = 0.04

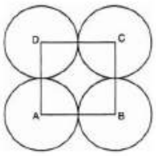
Concentration of SO_2	**Frequency f_i**	**Class interval x_i**	**$d_i = x_i - 0.14$**	**u_i**	**f_iu_i**
0.00 – 0.04	4	0.02	-0.12	-3	-12
0.04 – 0.08	9	0.06	-0.08	-2	-18
0.08 – 0.12	9	0.10	-0.04	-1	-9
0.12 – 0.16	2	0.14	0	0	0
0.16 – 0.20	4	0.18	0.04	1	4
0.20 – 0.24	2	0.22	0.08	2	4
Total	$\sum f_i = 30$				$\sum f_iu_i = -31$

let a = 0.14

$$\text{Mean } \bar{x} = a + \left(\frac{\sum f_iu_i}{\sum f_i}\right) \times h$$

$$= 0.14 + (0.04)\left(\frac{-31}{30}\right)$$

= 0.099 ppm

35. 

Let r cm be the radius of each circle.

Area of square - Area of 4 sectors = $\frac{24}{7}\text{cm}^2$

$$(\text{side})^2 - 4\left[\frac{\theta}{360}\pi r^2\right] = \frac{24}{7}\text{cm}^2$$

or, $(2r)^2 - 4\left(\frac{90^\circ}{360^\circ} \times \pi r^2\right) = \frac{24}{7}$

or, $(2r)^2 - 4\left(\frac{1}{4^\circ} \times \pi r^2\right) = \frac{24}{7}$

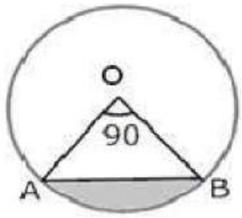
or, $(2r)^2 - \left(\pi r^2\right) = \frac{24}{7}$

or, $4r^2 - \frac{22}{7}r^2 = \frac{24}{7}$

or, $\frac{28r^2 - 22r^2}{7} = \frac{24}{7}$

or, $6r^2 = 24$
or, $r^2 = 4$
or, $r = \pm 2$
or, Radius of each circle is 2 cm (r cannot be negative)

OR



i. Area of minor sector = $\frac{\theta}{360}\pi r^2$

$= \frac{90}{360}(3.14)(10)^2$

$= \frac{1}{4} \times 3.14 \times 100$

$= \frac{314}{4}$

$= 78.50 = 78.5 \text{ cm}^2$

ii. Area of major sector = Area of circle - Area of minor sector

$= \pi(10)^2 - \frac{90}{360}\pi(10)^2 = 3.14\,(100) - \frac{1}{4}(3.14)\,(100)$

$= 314 - 78.50 = 235.5 \text{ cm}^2$

iii. We know that area of minor segment

= Area of minor sector OAB - Area of ΔOAB

$\because$ area of $\triangle OAB = \frac{1}{2}(OA)(OB)\sin\angle AOB$

$= \frac{1}{2}(OA)(OB)\left(\because \angle AOB = 90^\circ\right)$

Area of sector $= \frac{\theta}{360}\pi r^2$

$= \frac{1}{4}(3.14)\,(100) - 50 = 25(3.14) - 50 = 78.50 - 50 = 28.5 \text{ cm}^2$

iv. Area of major segment = Area of the circle - Area of minor segment

$= \pi(10)^2 - 28.5$

$= 100(3.14) - 28.5$

$= 314 - 28.5 = 285.5 \text{ cm}^2$

Section E

36. **Read the text carefully and answer the questions:**

An ice-cream seller used to sell different kinds and different shapes of ice-cream like rectangular shaped with one end hemispherical, cone-shaped and rectangular brick, etc. One day Sheetal and her brother came to his shop. Sheetal purchased an ice-cream which has the following shape: ice-cream cone as the union of a right circular cone and a hemisphere that has the same (circular) base as the cone. The height of the cone is 9 cm and the radius of its base is 2.5 cm. her brother purchased rectangular brick shaped ice cream with length 9 cm, width 4cm and thickness 2 cm.

(i)

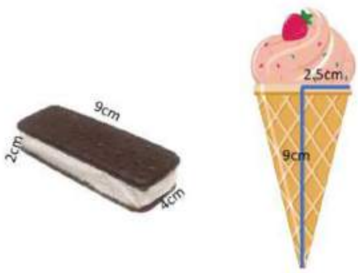
For cone, radius of the base (r) = 2.5cm = $\frac{5}{2}$cm

Height (h) = 9 cm

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9$

$= \frac{825}{14} \text{cm}^3$



For hemisphere,

Radius (r) = 2.5cm = $\frac{5}{2}$cm

$\therefore$ Volume $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{1375}{42} \text{cm}^3$

The volume of the ice-cream without hemispherical end = Volume of the cone

$= \frac{825}{14} \text{cm}^3$

(ii) For cone, radius of the base (r) = 2.5cm = $\frac{5}{2}$cm

Height (h) = 9 cm

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9$

$= \frac{825}{14}\text{cm}^3$

For hemisphere,

Radius (r) = 2.5cm = $\frac{5}{2}$cm

$\therefore$ Volume $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{1375}{42}\text{cm}^3$

Volume of the ice-cream with hemispherical end = Volume of the cone + Volume of the hemisphere

$= \frac{825}{14} + \frac{1375}{42} = \frac{2475 + 1375}{42}$

$= \frac{3850}{42} = \frac{275}{3} = 91\frac{2}{3}\text{cm}^3$

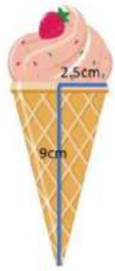
(iii) For cone, Radius of the base (r) = 2.5cm = $\frac{5}{2}$cm

Height (h) = 9 cm

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9$

$= \frac{825}{14}\text{cm}^3$



For hemisphere,

Radius (r) = 2.5cm = $\frac{5}{2}$cm

$\therefore$ Volume = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{1375}{42}\text{cm}^3$

Volume of rectangular brick shaped ice cream = $9 \times 4 \times 2 = 72$ cm^3

OR

For cone, Radius of the base (r)

= 2.5cm = $\frac{5}{2}$cm

Height (h) = 9 cm

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9$

$= \frac{825}{14}\text{cm}^3$

For hemisphere,

Radius (r) = 2.5cm = $\frac{5}{2}$cm

$\therefore$ Volume = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{1375}{42}\text{cm}^3$

Sheetal ice cream quantity is more than her brother

Volume of Sheeta's ice cream - Volume her brother's ice cream

= 91.66 - 72 = 19.66 cm^3

37. **Read the text carefully and answer the questions:**

Sehaj Batra gets pocket money from his father every day. Out of pocket money, he saves money for poor people in his locality. On 1st day he saves ₹27.5 On each succeeding day he increases his saving by ₹2.5.

(i) Money saved on 1st day = ₹27.5

∵ Sehaj increases his saving by a fixed amount of ₹2.5

∴ His saving form an AP with a = 27.5 and d = 2.5

∴ Money saved on 10th day,

$a_{10} = a + 9d = 27.5 + 9(2.5)$

$= 27.5 + 22.5 =$ ₹50

(ii) $a_{25} = a = 24d$

$= 27.5 + 24(2.5)$

$= 27.5 + 60 =$ ₹ 87.5

OR

Let $S_n = 387.5$, a = 27.5 and d = 2.5

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow 387.5 = \frac{n}{2}[2 \times 27.5 + (n-1)2.5]$$

$$\Rightarrow 387.5 = \frac{n}{2}[55 + (n-1) \times 2.5]$$

$\Rightarrow$ $775 = 55n + 2.5n^2 - 2.5n$

$\Rightarrow$ $25n^2 + 525n = 7750 = 0$

$\Rightarrow$ $n^2 + 21n - 310 = 0$

$\Rightarrow$ $(n + 31)(n - 10) = 0$

$\Rightarrow$ n = -31 reject n = 10 accept

So in 10 years Sehaj saves ₹ 387.5.

(iii)Total amount saved by Sehaj in 30 days.

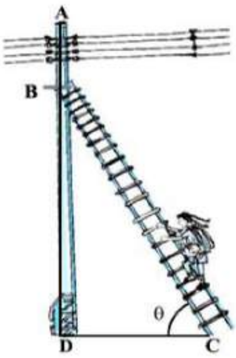
$$= \frac{30}{2}[2 \times 27.5 + (30-1) \times 2.5]$$

$= 15(55 + 29(2.5))$

$=$ ₹1912.5

38. **Read the text carefully and answer the questions:**

In a village, group of people complained about an electric fault in their area. On their complaint, an electrician reached village to repair an electric fault on a pole of height 10 m. She needs to reach a point 1.5 m below the top of the pole to undertake the repair work (see the adjoining figure). She used ladder, inclined at an angle of θ to the horizontal such that $\cos\theta = \frac{\sqrt{3}}{2}$, to reach the required position.



(i) Length BD = AD - AB = 10 - 2.5 = 8.5

(ii) The length of ladder BC

In △BDC

$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = 30^{o}$$

$$\sin 30^{o} = \frac{BD}{BC}$$

$$\Rightarrow \frac{1}{2} = \frac{8.5}{BC}$$

$$\Rightarrow BC = 2 \times 8.5 = 17 \text{ m}$$

OR

If the height of pole and distance BD is doubled, then the length of the ladder is

$$\sin 30^{o} = \frac{BD}{BC}$$

$$\Rightarrow \frac{1}{2} = \frac{17}{BC}$$

$$\Rightarrow BC = 2 \times 17 = 34 \text{ m}$$

(iii)Distance between foot of ladder and foot of wall CD

In △BDC

$$\cos 30^{o} = \frac{CD}{BC}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{CD}{17}$$

$$\Rightarrow CD = 8.5\sqrt{3} \text{ m}$$