Chapter

6

Work, Energy, Power and Collision

Introduction

The terms 'work', 'energy' and 'power' are frequently used in everyday language. A farmer clearing weeds in his field is said to be working hard. A woman carrying water from a well to her house is said to be working. In a drought affected region she may be required to carry it over large distances. If she can do so, she is said to have a large stamina or energy. Energy is thus the capacity to do work. The term power is usually associated with speed. In karate, a powerful punch is one delivered at great speed. In physics we shall define these terms very precisely. We shall find that there is a loose correlation between the physical definitions and the physiological pictures these terms generate in our minds.

Work is said to be done when a force applied on the body displaces the body through a certain distance in the direction of force.

Work Done by a Constant Force

Let a constant force $\vec{F}$ be applied on the body such that it makes an angle θ with the horizontal and body is displaced through a distance s

By resolving force $\vec{F}$ into two components :

(i) $F\cos\theta$ in the direction of displacement of the body.

(ii) $F\sin\theta$ in the perpendicular direction of displacement of the body.

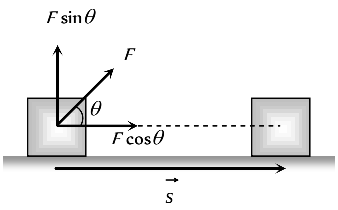


Fig. 6.1

Since body is being displaced in the direction of $F\cos\theta$, therefore work done by the force in displacing the body through a distance s is given by

$$W = (F\cos\theta)s = Fs\cos\theta$$

or $W = \vec{F}.\vec{s}$

Thus work done by a force is equal to the scalar (or dot product) of the force and the displacement of the body.

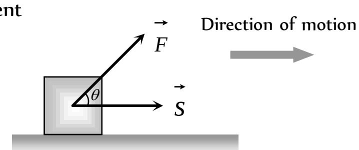
If a number of forces $\vec{F}_1, \vec{F}_2, \vec{F}_3 ...... \vec{F}_n$ are acting on a body and it shifts from position vector $\vec{r}_1$ to position vector $\vec{r}_2$ then

$$W = (\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ... \vec{F}_n).(\vec{r}_2 - \vec{r}_1)$$

Nature of Work Done

Positive work

Positive work means that force (or its component) is parallel to displacement



$0^o \le \theta < 90^o$

Fig. 6.2

The positive work signifies that the external force favours the motion of the body.

Example: (i) When a person lifts a body from the ground, the work done by the (upward) lifting force is positive

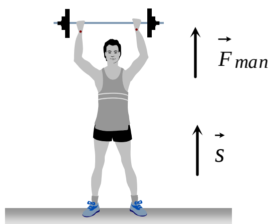


Fig. 6.3

(ii) When a lawn roller is pulled by applying a force along the handle at an acute angle, work done by the applied force is positive.

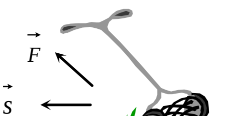


Fig. 6.4

(iii) When a spring is stretched, work done by the external (stretching) force is positive.

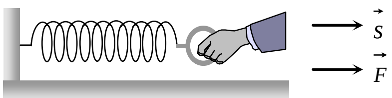
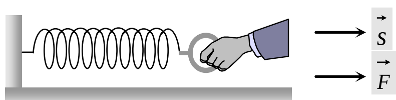


Fig. 6.5



Maximum work : $W_{\max} = F\,s$

When $\cos\theta = \text{maximum} = 1$ *i.e.* $\theta = 0^o$

It means force does maximum work when angle between force and displacement is zero.

Negative work

Negative work means that force (or its component) is opposite to displacement *i.e.*

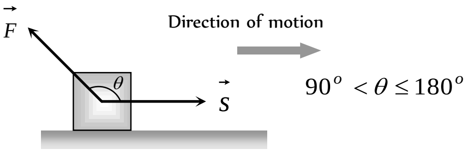


Fig. 6.6

The negative work signifies that the external force opposes the motion of the body.

Example: (i) When a person lifts a body from the ground, the work done by the (downward) force of gravity is negative.

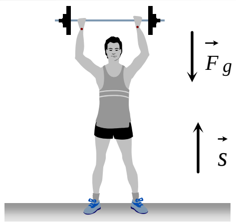


Fig. 6.7

(ii) When a body is m[illegible]ver a rough surface, the work done by the frictional force is negative.

Minimum work : $W_{\min} = -F\,s$

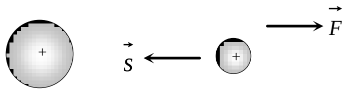


Fig. 6.8

When $\cos\theta = \text{minimum} = -1$ *i.e* $\theta = 180^o$

It means force does minimum [maximum negative] work when angle between force and displacement is 180°.

(iii) When a positive charge is moved towards another positive charge. The work done by electrostatic force between them is negative.

Zero work

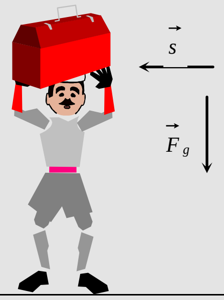
Under three condition, work done becomes zero $W = Fs\cos\theta = 0$

(1) **If the force is perpendicular to the displacement** $[\vec{F} \perp \vec{s}\,]$

Example: (i) When a coolie travels on a horizontal platform with a load on his head, work done against gravity by the coolie is zero.

(ii) When a body moves in a circle the work done by the centripetal force is always zero.

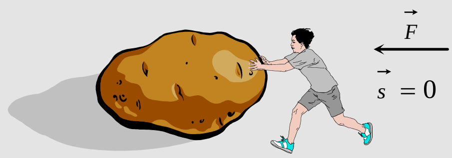
(iii) In case of motion of a charged particle in a magnetic field as force $[\vec{F} = q(\vec{v} \times \vec{B})]$ is always perpendicular to motion, work done by this force is always zero.



(2) **If there is no displacement** [$\boldsymbol{s}$ = **0**]

Example: (i) When a person tries to displace a wall or heavy stone by applying a force and it does not move, then work done is zero.

(ii) A weight lifter does work in lifting the weight off the ground but does not work in holding it up.



(3) **If there is no force acting on the body** [$\boldsymbol{F}$ = **0**]

Example: Motion of an isolated body in free space.

Work Done by a Variable Force

When the magnitude and direction of a force varies with position, the work done by such a force for an infinitesimal displacement is given by $dW = \vec{F}.\vec{ds}$

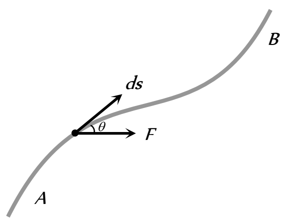


Fig. 6.9

The total work done in going from A to B as shown in the figure is

$W = \int_A^B \vec{F}.\vec{ds} = \int_A^B (F\cos\theta)ds$

In terms of rectangular component $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$\vec{ds} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$\therefore\ W = \int_A^B (F_x\hat{i} + F_y\hat{j} + F_z\hat{k}).(dx\hat{i} + dy\hat{j} + dz\hat{k})$

or $W = \int_{x_A}^{x_B} F_x\,dx + \int_{y_A}^{y_B} F_y\,dy + \int_{z_A}^{z_B} F_z\,dz$

Dimension and Units of Work

Dimension : As work = Force × displacement

$$[W] = [MLT^{-2}] \times [L] = [ML^2T^{-2}]$$

Units : The units of work are of two types

Absolute units	Gravitational units
Joule [S.I.]: Work done is said to be one *Joule*, when 1 *Newton* force displaces the body through 1 *metre* in its own direction. From, $W = F.s$ 1 *Joule* = 1 *Newton* ×1 *m*	*kg-m* [S.I.]: 1 *kg-m* of work is done when a force of 1*kg-wt.* displaces the body through 1*m* in its own direction. From $W = F\,s$ 1 *kg-m* = 1 *kg-wt* × 1 *m* = 9.81 *N* × 1 *metre* = 9.81 *Joule*
erg [C.G.S.] : Work done is said to be one *erg* when 1 *dyne* force displaces the body through 1 *cm* in its own direction. From $W = F\,s$ $1\,erg = 1 dyne \times 1 cm$ **Relation between Joule and erg** 1 *Joule* = 1 *N* × 1 *m* = 10^5 *dyne* × 10^2 *cm* = 10^7 *dyne* × *cm* = 10^7 *erg*	*gm-cm* [C.G.S.] : 1 *gm-cm* of work is done when a force of 1*gm-wt* displaces the body through 1*cm* in its own direction. From $W = F\,s$ 1 *gm-cm* = 1*gm-wt* × 1*cm.* = 981 *dyne* × 1*cm* = 981 *erg*

Work Done Calculation by Force Displacement Graph

Let a body, whose initial position is x_i, is acted upon by a variable force (whose magnitude is changing continuously) and consequently the body acquires its final position x_f.

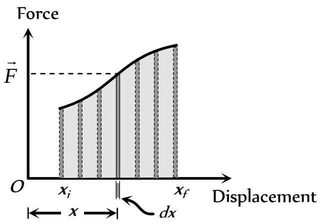


Fig. 6.10

Let F be the average value of variable force within the interval dx from position x to $(x + dx)$ *i.e.* for small displacement dx. The work done will be the area of the shaded strip of width dx. The work done on the body in displacing it from position x_i to x_f will be equal to the sum of areas of all the such strips

$$dW = \vec{F}\,dx$$

$$\therefore W = \int_{x_i}^{x_f} dW = \int_{x_i}^{x_f} F\,dx$$

$$\therefore W = \int_{x_i}^{x_f} (\text{Area of strip of width } dx)$$

$$\therefore W = \text{Area under curve between } x_i \text{ and } x_f$$

i.e. Area under force-displacement curve with proper algebraic sign represents work done by the force.

Work Done in Conservative and Non-conservative Field

(1) In conservative field, work done by the force (line integral of the force *i.e.* $\int \vec{F}.\vec{dl}$) is independent of the path followed between any two points.

$$\underset{\text{Path I}}{W_{A\to B}} = \underset{\text{Path II}}{W_{A\to B}} = \underset{\text{Path III}}{W_{A\to B}}$$

or $$\underset{\text{Path I}}{\int \vec{F}.\vec{dl}} = \underset{\text{Path II}}{\int \vec{F}.\vec{dl}} = \underset{\text{Path III}}{\int \vec{F}.\vec{dl}}$$

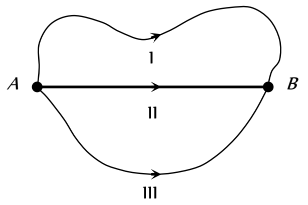


Fig. 6.11

(2) In conservative field work done by the force (line integral of the force *i.e.* $\int \vec{F}.\vec{dl}$) over a closed path/loop is zero.

$$W_{A\to B} + W_{B\to A} = 0$$

or $$\oint \vec{F}.\vec{dl} = 0$$

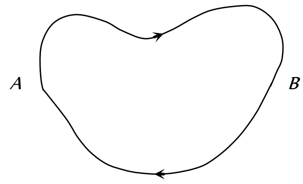


Fig. 6.12

Conservative force : The forces of these type of fields are known as conservative forces.

Example : Electrostatic forces, gravitational forces, elastic forces, magnetic forces *etc* and all the central forces are conservative in nature.

If a body of mass m lifted to height h from the ground level by different path as shown in the figure

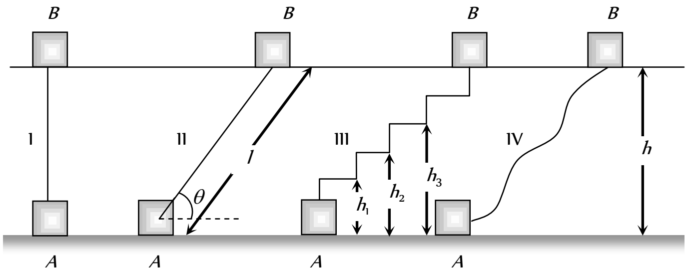


Fig. 6.13

Work done through different paths

$$W_I = F.s = mg \times h = mgh$$

$$W_{II} = F.s = mg\sin\theta \times l = mg\sin\theta \times \frac{h}{\sin\theta} = mgh$$

$$W_{III} = mgh_1 + 0 + mgh_2 + 0 + mgh_3 + 0 + mgh_4$$

$$= mg(h_1 + h_2 + h_3 + h_4) = mgh$$

$$W_{IV} = \int \vec{F}.\vec{ds} = mgh$$

It is clear that $W_I = W_{II} = W_{III} = W_{IV} = mgh$.

Further if the body is brought back to its initial position *A*, similar amount of work (energy) is released from the system, it means $W_{AB} = mgh$ and $W_{BA} = -mgh$.

Hence the net work done against gravity over a round trip is zero.

$$W_{Net} = W_{AB} + W_{BA} = mgh + (-mgh) = 0$$

i.e. the gravitational force is conservative in nature.

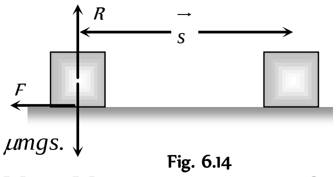
Non-conservative forces : A force is said to be non-conservative if work done by or against the force in moving a body from one position to another, depends on the path followed between these two positions and for complete cycle this work done can never be zero.

Example: Frictional force, Viscous force, Airdrag *etc.*

If a body is moved from position *A* to another position *B* on a rough table, work done against frictional force shall depend on the length of the path between *A* and *B* and not only on the position *A* and *B*.

$$W_{AB} = \mu mgs$$

Further if the body is brought back to its initial position *A*, work has to be done against the frictional force, which opposes the motion. Hence the net work done against the friction over a round trip is not zero.



$W_{BA} = \mu mgs.$

Fig. 6.14

$\therefore W_{Net} = W_{AB} + W_{BA} = \mu mgs + \mu mgs = 2\mu mgs \neq 0.$

i.e. the friction is a non-conservative force.

Work Depends on Frame of Reference

With change of frame of reference (inertial), force does not change while displacement may change. So the work done by a force will be different in different frames.

Examples : (1) If a porter with a suitcase on his head moves up a staircase, work done by the upward lifting force relative to him will be zero (as displacement relative to him is zero) while relative to a person on the ground will be *mgh*.

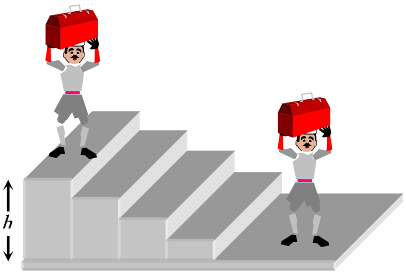


Fig. 6.15

(2) If a person is pushing a box inside a moving train, the work done in the frame of train will $\vec{F}.\vec{s}$ while in the frame of earth will be $\vec{F}.(\vec{s} + \vec{s}_0)$ where $\vec{s}_0$ is the displacement of the train relative to the ground.

Energy

The energy of a body is defined as its capacity for doing work.

(1) Since energy of a body is the total quantity of work done, therefore it is a scalar quantity.

(2) Dimension: $[ML^2T^{-2}]$ it is same as that of work or torque.

(3) Units : *Joule* [S.I.], *erg* [C.G.S.]

Practical units : *electron volt* (*eV*), Kilowatt hour (*KWh*), Calories (*cal*)

Relation between different units:

1 *Joule* = 10^7 *erg*

1 *eV* = 1.6×10^{-19} *Joule*

1 *kWh* = 3.6×10^6 *Joule*

1 *calorie* = 4.18 *Joule*

(4) Mass energy equivalence : Einstein's special theory of relativity shows that material particle itself is a form of energy.

The relation between the mass of a particle *m* and its equivalent energy is given as

$E = mc^2$ where *c* = velocity of light in vacuum.

If $m = 1\,amu = 1.67 \times 10^{-27}\ kg$

then $E = 931\,MeV = 1.5 \times 10^{-10}\ Joule$.

If $m = 1kg$ then $E = 9 \times 10^{16}\ Joule$

Examples : (i) Annihilation of matter when an electron (e^-) and a positron (e^+) combine with each other, they annihilate or destroy each other. The masses of electron and positron are converted into energy. This energy is released in the form of γ -rays.

$$e^- + e^+ \rightarrow \gamma + \gamma$$

Each γ photon has energy = 0.51 *MeV*.

Here two γ photons are emitted instead of one γ photon to conserve the linear momentum.

(ii) Pair production : This process is the reverse of annihilation of matter. In this case, a photon (γ) having energy equal to 1.02 *MeV* interacts with a nucleus and give rise to electron (e^-) and positron (e^+). Thus energy is converted into matter.

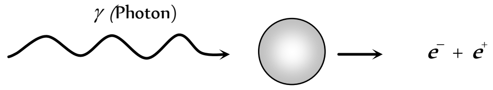


Fig. 6.16

(iii) Nuclear bomb : When the nucleus is split up due to mass defect (The difference in the mass of nucleons and the nucleus), energy is released in the form of γ -radiations and heat.

(5) Various forms of energy

(i) Mechanical energy (Kinetic and Potential)

(ii) Chemical energy

(iii) Electrical energy

(iv) Magnetic energy

(v) Nuclear energy

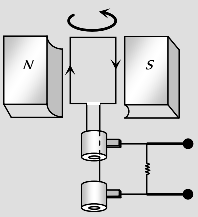
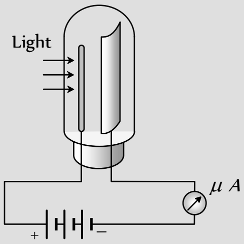
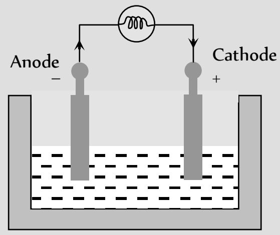
(vi) Sound energy

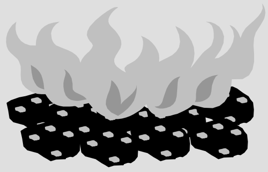
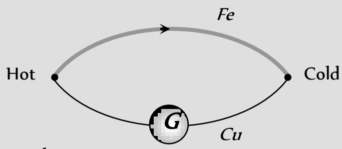
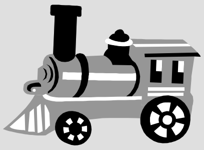
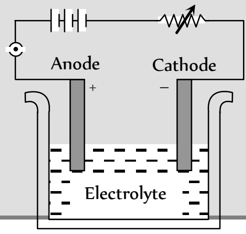
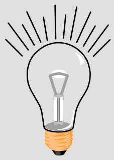
(vii) Light energy

(viii) Heat energy

(6) Transformation of energy : Conversion of energy from one form to another is possible through various devices and processes.

Table : 6.1 Various devices for energy conversion from one form to another

Mechanical → electrical	Light → Electrical	Chemical → electrical
Dynamo 	Photoelectric cell 	Primary cell 

Chemical → heat	Sound → Electrical	Heat → electrical
Coal Burning 	Microphone 	Thermo-couple 
Heat → Mechanical	**Electrical → Mechanical**	**Electrical → Heat**
Engine 	Motor 	Heater 
Electrical → Sound	**Electrical → Chemical**	**Electrical → Light**
Speaker 	Voltameter 	Bulb 

Kinetic Energy

The energy possessed by a body by virtue of its motion, is called kinetic energy.

Examples : (i) Flowing water possesses kinetic energy which is used to run the water mills.

(ii) Moving vehicle possesses kinetic energy.

(iii) Moving air (*i.e.* wind) possesses kinetic energy which is used to run wind mills.

(iv) The hammer possesses kinetic energy which is used to drive the nails in wood.

(v) A bullet fired from the gun has kinetic energy and due to this energy the bullet penetrates into a target.

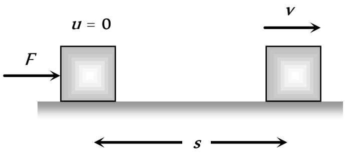


Fig. 6.17

(1) **Expression for kinetic energy :**

Let m = mass of the body,

u = Initial velocity of the body (= 0)

F = Force acting on the body,

a = Acceleration of the body,

s = Distance travelled by the body,

v = Final velocity of the body

From $v^2 = u^2 + 2as$

$$\Rightarrow v^2 = 0 + 2as \quad \therefore s = \frac{v^2}{2a}$$

Since the displacement of the body is in the direction of the applied force, then work done by the force is

$$W = F \times s = ma \times \frac{v^2}{2a}$$

$$\Rightarrow W = \frac{1}{2}mv^2$$

This work done appears as the kinetic energy of the body

$$KE = W = \frac{1}{2}mv^2$$

(2) **Calculus method :** Let a body is initially at rest and force $\vec{F}$ is applied on the body to displace it through small displacement $d\vec{s}$ along its own direction then small work done

$$dW = \vec{F}.d\vec{s} = F\,ds$$

$$\Rightarrow \quad dW = m\,a\,ds \qquad [\text{As } F = ma]$$

$$\Rightarrow \quad dW = m\frac{dv}{dt}ds \qquad \left[\text{As } a = \frac{dv}{dt}\right]$$

$$\Rightarrow dW = mdv.\frac{ds}{dt}$$

$$\Rightarrow dW = m\,v\,dv \qquad \text{...(i)}$$

$$\left[As\ \frac{ds}{dt}=v\right]$$

Therefore work done on the body in order to increase its velocity from zero to v is given by

$$W=\int_0^v mv\,dv=m\int_0^v v\,dv=m\left[\frac{v^2}{2}\right]_0^v=\frac{1}{2}mv^2$$

This work done appears as the kinetic energy of the body $KE=\frac{1}{2}mv^2$.

In vector form $KE=\frac{1}{2}m(\vec{v}.\vec{v})$

As m and $\vec{v}.\vec{v}$ are always positive, kinetic energy is always positive scalar *i.e.* kinetic energy can never be negative.

(3) **Kinetic energy depends on frame of reference :** The kinetic energy of a person of mass m, sitting in a train moving with speed v, is zero in the frame of train but $\frac{1}{2}mv^2$ in the frame of the earth.

(4) **Kinetic energy according to relativity :** As we know $E=\frac{1}{2}mv^2$.

But this formula is valid only for ($v \ll c$) If v is comparable to c (speed of light in free space = $3\times10^8\ m/s$) then according to Einstein theory of relativity

$$E=\frac{mc^2}{\sqrt{1-(v^2/c^2)}}-mc^2$$

(5) **Work-energy theorem:** From equation (i) $dW=mv\,dv$.

Work done on the body in order to increase its velocity from u to v is given by

$$W=\int_u^v mv\,dv=m\int_u^v v\,dv=m\left[\frac{v^2}{2}\right]_u^v$$

$$\Rightarrow W=\frac{1}{2}m[v^2-u^2]$$

Work done = change in kinetic energy

$W=\Delta E$

This is work energy theorem, it states that work done by a force acting on a body is equal to the change in the kinetic energy of the body.

This theorem is valid for a system in presence of all types of forces (external or internal, conservative or non-conservative).

If kinetic energy of the body increases, work is positive *i.e.* body moves in the direction of the force (or field) and if kinetic energy decreases, work will be negative and object will move opposite to the force (or field).

Examples : (i) In case of vertical motion of body under gravity when the body is projected up, force of gravity is opposite to motion and so kinetic energy of the body decreases and when it falls down, force of gravity is in the direction of motion so kinetic energy increases.

(ii) When a body moves on a rough horizontal surface, as force of friction acts opposite to motion, kinetic energy will decrease and the decrease in kinetic energy is equal to the work done against friction.

(6) **Relation of kinetic energy with linear momentum:** As we know

$$E=\frac{1}{2}mv^2=\frac{1}{2}\left[\frac{P}{v}\right]v^2 \qquad [\text{As } P=mv]$$

$$\therefore\ E=\frac{1}{2}Pv$$

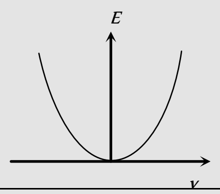
or $$E=\frac{P^2}{2m} \qquad \left[\text{As } v=\frac{P}{m}\right]$$

So we can say that kinetic energy $E=\frac{1}{2}mv^2=\frac{1}{2}Pv=\frac{p^2}{2m}$

and Momentum $P=\frac{2E}{v}=\sqrt{2mE}$

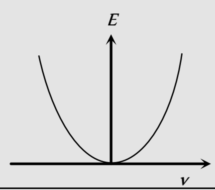
From above relation it is clear that a body can not have kinetic energy without having momentum and vice-versa.

(7) **Various graphs of kinetic energy**



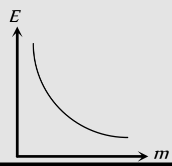
$E\propto v$

m = constant



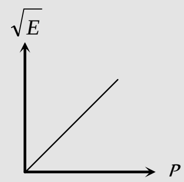
$E\propto P^2$

m = constant



$E\propto\frac{1}{m}$

P = constant



$P\propto\sqrt{E}$

m = constant

Stopping of Vehicle by Retarding Force

If a vehicle moves with some initial velocity and due to some retarding force it stops after covering some distance after some time.

(1) **Stopping distance :** Let m = Mass of vehicle,

v = Velocity, P = Momentum, E = Kinetic energy

F = Stopping force, x = Stopping distance,

t = Stopping time

Then, in this process stopping force does work on the vehicle and destroy the motion.

By the work- energy theorem

$$W = \Delta K = \frac{1}{2}mv^2$$

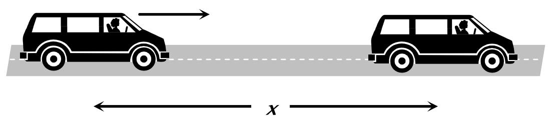


Fig. 6.18

$\Rightarrow$ Stopping force (F) $\times$ Distance (x) = Kinetic energy (E)

$\Rightarrow$ Stopping distance (x) $= \dfrac{\text{Kinetic energy } (E)}{\text{Stopping force } (F)}$

$$\Rightarrow \; x = \frac{mv^2}{2F} \qquad \text{...(i)}$$

(2) **Stopping time :** By the impulse-momentum theorem

$$F \times \Delta t = \Delta P \Rightarrow F \times t = P$$

$$\therefore \; t = \frac{P}{F}$$

$$\text{or } \; t = \frac{mv}{F} \qquad \text{...(ii)}$$

(3) **Comparison of stopping distance and time for two vehicles :** Two vehicles of masses m_1 and m_2 are moving with velocities v_1 and v_2 respectively. When they are stopped by the same retarding force (F).

The ratio of their stopping distances $\dfrac{x_1}{x_2} = \dfrac{E_1}{E_2} = \dfrac{m_1 v_1^2}{m_2 v_2^2}$

and the ratio of their stopping time $\dfrac{t_1}{t_2} = \dfrac{P_1}{P_2} = \dfrac{m_1 v_1}{m_2 v_2}$

(i) If vehicles possess same velocities

$v_1 = v_2$

$$\frac{x_1}{x_2} = \frac{m_1}{m_2} \; ; \; \frac{t_1}{t_2} = \frac{m_1}{m_2}$$

(ii) If vehicle possess same kinetic momentum

$P_1 = P_2$

$$\frac{x_1}{x_2} = \frac{E_1}{E_2} = \left(\frac{P_1^2}{2m_1}\right)\left(\frac{2m_2}{P_2^2}\right) = \frac{m_2}{m_1}$$

$$\frac{t_1}{t_2} = \frac{P_1}{P_2} = 1$$

(iii) If vehicle possess same kinetic energy

$$\frac{x_1}{x_2} = \frac{E_1}{E_2} = 1$$

$$\frac{t_1}{t_2} = \frac{P_1}{P_2} = \frac{\sqrt{2m_1E_1}}{\sqrt{2m_2E_2}} = \sqrt{\frac{m_1}{m_2}}$$

Note : ❑ If vehicle is stopped by friction then

Stopping distance $x = \dfrac{\frac{1}{2}mv^2}{F} = \dfrac{\frac{1}{2}mv^2}{ma} = \dfrac{v^2}{2\mu g}$

[As $a = \mu g$]

Stopping time $t = \dfrac{mv}{F} = \dfrac{mv}{m\mu g} = \dfrac{v}{\mu g}$

Potential Energy

Potential energy is defined only for conservative forces. In the space occupied by conservative forces every point is associated with certain energy which is called the energy of position or potential energy. Potential energy generally are of three types : Elastic potential energy, Electric potential energy and Gravitational potential energy.

(1) **Change in potential energy :** Change in potential energy between any two points is defined in the terms of the work done by the associated conservative force in displacing the particle between these two points without any change in kinetic energy.

$$U_2 - U_1 = -\int_{r_1}^{r_2} \vec{F}.d\vec{r} = -W \qquad \text{...(i)}$$

We can define a unique value of potential energy only by assigning some arbitrary value to a fixed point called the reference point. Whenever and wherever possible, we take the reference point at infinity and assume potential energy to be zero there, *i.e.* if we take $r_1 = \infty$ and $r_2 = r$ then from equation (i)

$$U = -\int_{\infty}^{r} \vec{F}.d\vec{r} = -W$$

In case of conservative force (field) potential energy is equal to negative of work done by conservative force in shifting the body from reference position to given position.

This is why, in shifting a particle in a conservative field (say gravitational or electric), if the particle moves opposite to the field, work done by the field will be negative and so change in potential energy will be positive *i.e.* potential energy will increase. When the particle moves in the direction of field, work will be positive and change in potential energy will be negative *i.e.* potential energy will decrease.

(2) **Three dimensional formula for potential energy:** For only conservative fields $\vec{F}$ equals the negative gradient $(-\vec{\nabla})$ of the potential energy.

So $\vec{F} = -\vec{\nabla}U$ ($\vec{\nabla}$ read as Del operator or Nabla operator and $\vec{\nabla} = \dfrac{\partial}{\partial x}\hat{i} + \dfrac{\partial}{\partial y}\hat{j} + \dfrac{\partial}{\partial z}\hat{k}$)

$$\Rightarrow \vec{F} = -\left[\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right]$$

where,

$\dfrac{\partial U}{\partial x}$ = Partial derivative of U *w.r.t.* x (keeping y and z constant)

$\dfrac{\partial U}{\partial y}$ = Partial derivative of U *w.r.t.* y (keeping x and z constant)

$\dfrac{\partial U}{\partial z}$ = Partial derivative of U *w.r.t.* z (keeping x and y constant)

(3) **Potential energy curve :** A graph plotted between the potential energy of a particle and its displacement from the centre of force is called potential energy curve.

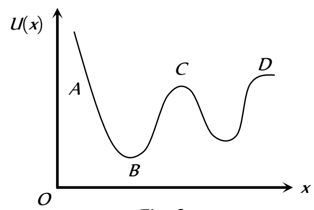


Fig. 6.19

Figure shows a graph of potential energy function $U(x)$ for one dimensional motion.

As we know that negative gradient of the potential energy gives force.

$$\therefore \; -\frac{dU}{dx} = F$$

(4) **Nature of force**

(i) Attractive force :

On increasing *x*, if *U* increases,

$\frac{dU}{dx} = \text{positive}$, then *F* is in negative direction

i.e. force is attractive in nature.

In graph this is represented in region *BC*.

(ii) Repulsive force :

On increasing *x*, if *U* decreases,

$\frac{dU}{dx} = \text{negative}$, then *F* is in positive direction

i.e. force is repulsive in nature.

In graph this is represented in region *AB*.

(iii) Zero force :

On increasing *x*, if *U* does not change,

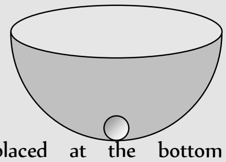
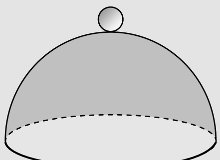
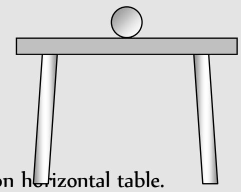
$\frac{dU}{dx} = 0$ then *F* is zero

i.e. no force works on the particle.

Point *B*, *C* and *D* represents the point of zero force or these points can be termed as position of equilibrium.

(5) **Types of equilibrium :** If net force acting on a particle is zero, it is said to be in equilibrium.

For equilibrium $\frac{dU}{dx} = 0$, but the equilibrium of particle can be of three types :

Stable	Unstable	Neutral
When a particle is displaced slightly from its present position, then a force acting on it brings it back to the initial position, it is said to be in stable equilibrium position.	When a particle is displaced slightly from its present position, then a force acting on it tries to displace the particle further away from the equilibrium position, it is said to be in unstable equilibrium.	When a particle is slightly displaced from its position then it does not experience any force acting on it and continues to be in equilibrium in the displaced position, it is said to be in neutral equilibrium.
Potential energy is minimum.	Potential energy is maximum.	Potential energy is constant.
$F = -\frac{dU}{dx} = 0$	$F = -\frac{dU}{dx} = 0$	$F = -\frac{dU}{dx} = 0$
$\frac{d^2U}{dx^2} = \text{positive}$ *i.e.* rate of change of $\frac{dU}{dx}$ is positive.	$\frac{d^2U}{dx^2} = \text{negative}$ *i.e.* rate of change of $\frac{dU}{dx}$ is negative.	$\frac{d^2U}{dx^2} = 0$ *i.e.* rate of change of $\frac{dU}{dx}$ is zero.
Example :  A marble placed at the bottom of a hemispherical bowl.	*Example* :  A marble balanced on top of a hemispherical bowl.	*Example* :  A marble placed on horizontal table.

Elastic Potential Energy

(1) **Restoring force and spring constant :** When a spring is stretched or compressed from its normal position ($x = 0$) by a small distance *x*, then a restoring force is produced in the spring to bring it to the normal position.

According to Hooke's law this restoring force is proportional to the displacement *x* and its direction is always opposite to the displacement.

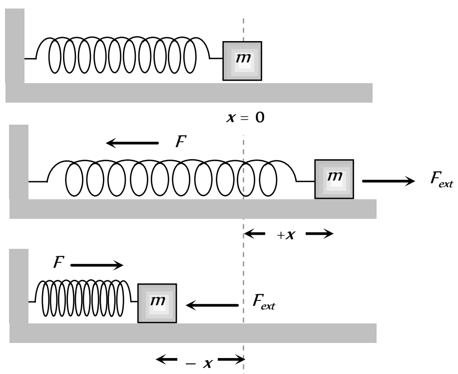


Fig. 6.20

i.e. $\vec{F} \propto -\vec{x}$

or $\vec{F} = -k\,\vec{x}$ ...(i)

where *k* is called spring constant.

If $x = 1$, $F = k$ (Numerically)

or $k = F$

Hence spring constant is numerically equal to force required to produce unit displacement (compression or extension) in the spring. If required force is more, then spring is said to be more stiff and vice-versa.

Actually *k* is a measure of the stiffness/softness of the spring.

Dimension : As $k = \frac{F}{x}$

$\therefore\ [k] = \frac{[F]}{[x]} = \frac{[MLT^{-2}]}{L} = [MT^{-2}]$

Units : S.I. unit *Newton/metre*, C.G.S unit *Dyne/cm*.

Note : ❑ Dimension of force constant is similar to surface tension.

(2) **Expression for elastic potential energy :** When a spring is stretched or compressed from its normal position (x = 0), work has to be done by external force against restoring force. $\vec{F}_{ext} = -\vec{F}_{restoring} = k\vec{x}$

Let the spring is further stretched through the distance *dx*, then work done

$$dW = \vec{F}_{ext} . d\vec{x} = F_{ext} . dx \cos 0^o = kx\ dx \quad [\text{As } \cos 0^\circ = 1]$$

Therefore total work done to stretch the spring through a distance x from its mean position is given by

$$W = \int_0^x dW = \int_0^x kx\ dx = k\left[\frac{x^2}{2}\right]_0^x = \frac{1}{2}kx^2$$

This work done is stored as the potential energy in the stretched spring.

$\therefore$ Elastic potential energy $U = \frac{1}{2}kx^2$

$$U = \frac{1}{2}Fx \qquad \left[\text{As } k = \frac{F}{x}\right]$$

$$U = \frac{F^2}{2k} \qquad \left[\text{As } x = \frac{F}{k}\right]$$

$\therefore$ Elastic potential energy $U = \frac{1}{2}kx^2 = \frac{1}{2}Fx = \frac{F^2}{2k}$

Note : ❑ If spring is stretched from initial position x_1 to final position x_2 then work done

= Increment in elastic potential energy

$= \frac{1}{2}k(x_2^2 - x_1^2)$

❑ Work done by the spring-force on the block in various situation are shown in the following table

Table : 6.2 Work done for spring

Initial state of the spring	Final state of the spring	Initial position (x_1)	Final position (x_2)	Work done (W)
Natural	Compressed	0	$-x$	$-1/2\ kx^2$
Natural	Elongated	0	x	$-1/2\ kx^2$
Elongated	Natural	x	0	$1/2\ kx^2$
Compressed	Natural	$-x$	0	$1/2\ kx^2$
Elongated	Compressed	x	$-x$	0
Compressed	Elongated	$-x$	x	0

(3) **Energy graph for a spring :** If the mass attached with spring performs simple harmonic motion about its mean position then its potential energy at any position (x) can be given by

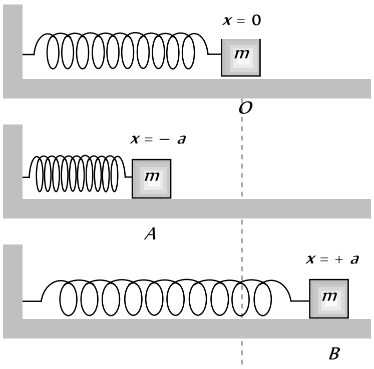
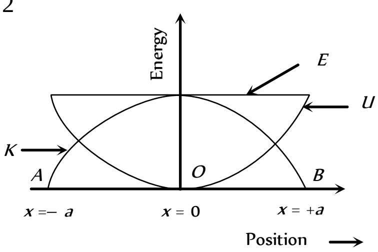


Fig. 6.21

$$U = \frac{1}{2}kx^2 \qquad ...(i)$$

So for the extreme position

$$U = \frac{1}{2}ka^2 \qquad [\text{As } x = \pm a \text{ for extreme}]$$



This is maximum potential energy or the total energy of mass.

$$\therefore \text{ Total energy } E = \frac{1}{2}ka^2 \qquad ...(ii)$$

[Because velocity of mass is zero at extreme position]

$\therefore\ K = \frac{1}{2}mv^2 = 0$]

Now kinetic energy at any position

$$K = E - U = \frac{1}{2}ka^2 - \frac{1}{2}kx^2$$

$$K = \frac{1}{2}k(a^2 - x^2) \qquad ...(iii)$$

From the above formula we can check that

$$U_{max} = \frac{1}{2}ka^2 \qquad [\text{At extreme } x = \pm a]$$

and $$U_{min} = 0 \qquad [\text{At mean } x = 0]$$

$$K_{max} = \frac{1}{2}ka^2 \qquad [\text{At mean } x = 0]$$

and $$K_{min} = 0 \qquad [\text{At extreme } x = \pm a]$$

$$E = \frac{1}{2}ka^2 = \text{constant (at all positions)}$$

It means kinetic energy and potential energy changes parabolically *w.r.t.* position but total energy remain always constant irrespective to position of the mass

Electrical Potential Energy

It is the energy associated with state of separation between charged particles that interact via electric force. For two point charge q_1 and q_2, separated by distance *r*.

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r}$$

While for a point charge *q* at a point in an electric field where the potential is *V*

$U = qV$

As charge can be positive or negative, electric potential energy can be positive or negative.

Gravitational Potential Energy

It is the usual form of potential energy and this is the energy associated with the state of separation between two bodies that interact via gravitational force.

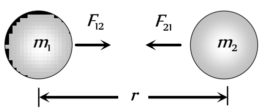


Fig. 6.23

For two particles of masses m_1 and m_2 separated by a distance *r*

Gravitational potential energy $U = -\dfrac{G m_1 m_2}{r}$

(1) If a body of mass *m* at height *h* relative to surface of earth then

Gravitational potential energy $U = \dfrac{mgh}{1 + \dfrac{h}{R}}$

Where *R* = radius of earth, *g* = acceleration due to gravity at the surface of the earth.

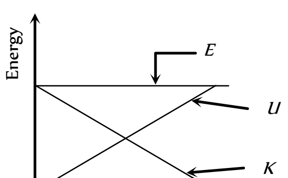
(2) If $h << R$ then above formula reduces to $U = mgh$.

(3) If *V* is the gravitational potential at a point, the potential energy of a particle of mass *m* at that point will be

$U = mV$

(4) Energy height graph : When a body projected vertically upward from the ground level with some initial velocity then it possess kinetic energy but its initial potential energy is zero.

As the body moves upward its potential energy increases due to increase in height but kinetic energy decreases (due to decrease in velocity). At maximum height its kinetic energy becomes zero and potential energy maximum but through out the complete motion, total energy remains constant as shown in the figure.



Work Done in Pulling the Chain Against Gravity

A chain of length *L* and mass *M* is held on a frictionless table with $(1/n)^{th}$ of its length hanging over the edge.

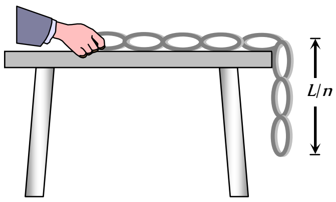


Fig. 6.25

Let $m = \dfrac{M}{L}$ = mass per unit length of the chain and *y* is the length of the chain hanging over the edge. So the mass of the chain of length *y* will be *ym* and the force acting on it due to gravity will be *mgy*.

The work done in pulling the *dy* length of the chain on the table.

$dW = F(-dy)$ [As *y* is decreasing]

i.e. $dW = mgy\,(-dy)$

So the work done in pulling the hanging portion on the table.

$$W = -\int_{L/n}^{0} mgy\,dy = -mg\left[\frac{y^2}{2}\right]_{L/n}^{0} = \frac{mg\,L^2}{2n^2}$$

$$\therefore\ W = \frac{MgL}{2n^2} \qquad [\text{As } m = M/L]$$

Alternative method :

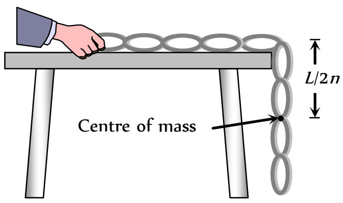


Fig. 6.26

If point mass *m* is pulled through a height *h* then work done $W = mgh$

Similarly for a chain we can consider its centre of mass at the middle point of the hanging part *i.e.* at a height of $L/(2n)$ from the lower end and mass of the hanging part of chain $= \dfrac{M}{n}$

So work done to raise the centre of mass of the chain on the table is given by

$$W = \frac{M}{n} \times g \times \frac{L}{2n} \qquad [\text{As } W = mgh]$$

or $$W = \frac{MgL}{2n^2}$$

Velocity of Chain While Leaving the Table

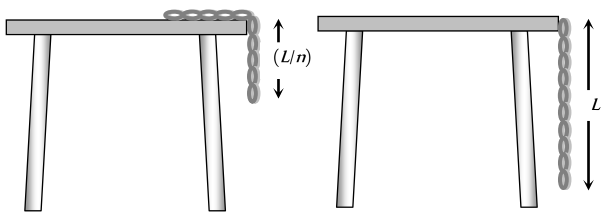


Fig. 6.27

Taking surface of table as a reference level (zero potential energy)

Potential energy of chain when $1/n$ length hanging from the edge $= \frac{-MgL}{2n^2}$

Potential energy of chain when it leaves the table $= -\frac{MgL}{2}$

Kinetic energy of chain = loss in potential energy

$$\Rightarrow \frac{1}{2}Mv^2 = \frac{MgL}{2} - \frac{MgL}{2n^2}$$

$$\Rightarrow \frac{1}{2}Mv^2 = \frac{MgL}{2}\left[1 - \frac{1}{n^2}\right]$$

$\therefore$ Velocity of chain $v = \sqrt{gL\left(1 - \frac{1}{n^2}\right)}$

Law of Conservation of Energy

(1) **Law of conservation of energy**

For a body or an isolated system by work-energy theorem we have

$$K_2 - K_1 = \int \vec{F}.d\vec{r} \quad ...(i)$$

But according to definition of potential energy in a conservative field

$$U_2 - U_1 = -\int \vec{F}.d\vec{r} \quad ...(ii)$$

So from equation (i) and (ii) we have

$$K_2 - K_1 = -(U_2 - U_1)$$

or $K_2 + U_2 = K_1 + U_1$

i.e. $K + U$ = constant.

For an isolated system or body in presence of conservative forces, the sum of kinetic and potential energies at any point remains constant throughout the motion. It does not depend upon time. This is known as the law of conservation of mechanical energy.

$$\Delta(K + U) = \Delta E = 0$$

[As E is constant in a conservative field]

$$\therefore \Delta K + \Delta U = 0$$

i.e. if the kinetic energy of the body increases its potential energy will decrease by an equal amount and vice-versa.

(2) **Law of conservation of total energy :** If some non-conservative force like friction is also acting on the particle, the mechanical energy is no more constant. It changes by the amount equal to work done by the frictional force.

$$\Delta(K + U) = \Delta E = W_f$$

[where W_f is the work done against friction]

The lost energy is transformed into heat and the heat energy developed is exactly equal to loss in mechanical energy.

We can, therefore, write $\Delta E + Q = 0$

[where Q is the heat produced]

This shows that if the forces are conservative and non-conservative both, it is not the mechanical energy which is conserved, but it is the total energy, may be heat, light, sound or mechanical *etc.*, which is conserved.

In other words : "*Energy may be transformed from one kind to another but it cannot be created or destroyed. The total energy in an isolated system remain constant*". This is the law of conservation of energy.

Power

Power of a body is defined as the rate at which the body can do the work.

Average power $(P_{av.}) = \frac{\Delta W}{\Delta t} = \frac{W}{t}$

Instantaneous power $(P_{inst.}) = \frac{dW}{dt} = \frac{\vec{F}.d\vec{s}}{dt}$ [As $dW = \vec{F}.d\vec{s}$]

$P_{inst} = \vec{F}.\vec{v}$ [As $\vec{v} = \frac{d\vec{s}}{dt}$]

i.e. power is equal to the scalar product of force with velocity.

Important Points

(1) Dimension : $[P] = [F][v] = [MLT^{-2}][LT^{-1}]$

$\therefore [P] = [ML^2T^{-3}]$

(2) Units : *Watt* or *Joule/sec* [S.I.]

Erg/sec [C.G.S.]

Practical units : *Kilowatt* (*KW*), *Mega watt* (*MW*) and Horse power (*hp*)

Relations between different units :

$1\,Watt = 1\,Joule/\sec = 10^7\,erg/\sec$

$1\,hp = 746\,Watt$

$1\,MW = 10^6\,Watt$

$1\,KW = 10^3\,Watt$

(3) If work done by the two bodies is same then power $\propto \frac{1}{\text{time}}$

i.e. the body which perform the given work in lesser time possess more power and vice-versa.

(4) As power = work/time, any unit of power multiplied by a unit of time gives unit of work (or energy) and not power, *i.e.* Kilowatt-hour or watt-day are units of work or energy.

$$1\,KWh = 10^3\,\frac{J}{sec} \times (60 \times 60 sec) = 3.6 \times 10^6\,Joule$$

(5) The slope of work time curve gives the instantaneous power. As $P = dW/dt = \tan\theta$

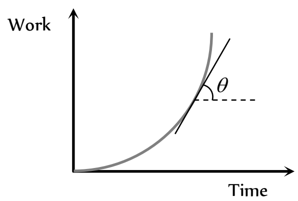


Fig. 6.28

(6) Area under power-time curve gives the work done as $P = \frac{dW}{dt}$

$\therefore W = \int P\,dt$

$\therefore$ W = Area under P-t curve

Position and Velocity of an Automobile w.r.t Time

An automobile of mass m accelerates, starting from rest, while the engine supplies constant power P, its position and velocity changes *w.r.t* time.

(1) **Velocity :** As $Fv = P$ = constant

i.e. $m\frac{dv}{dt}v = P$ $\left[\text{As}\,F = \frac{mdv}{dt}\right]$

or $\int v\,dv = \int \frac{P}{m} dt$

By integrating both sides we get $\frac{v^2}{2} = \frac{P}{m} t + C_1$

As initially the body is at rest *i.e.* $v = 0$ at $t = 0$, so $C_1 = 0$

$\therefore\ v = \left(\frac{2Pt}{m}\right)^{1/2}$

(2) **Position :** From the above expression $v = \left(\frac{2Pt}{m}\right)^{1/2}$

or $\frac{ds}{dt} = \left(\frac{2Pt}{m}\right)^{1/2}$ $\qquad \left[\text{As} v = \frac{ds}{dt}\right]$

i.e. $\int ds = \int \left(\frac{2Pt}{m}\right)^{1/2} dt$

By integrating both sides we get

$s = \left(\frac{2P}{m}\right)^{1/2} . \frac{2}{3} t^{3/2} + C_2$

Now as at $t = 0$, $s = 0$, so $C_2 = 0$

$s = \left(\frac{8P}{9m}\right)^{1/2} t^{3/2}$

Collision

Collision is an isolated event in which a strong force acts between two or more bodies for a short time as a result of which the energy and momentum of the interacting particle change.

In collision particles may or may not come in real touch e.g. in collision between two billiard balls or a ball and bat, there is physical contact while in collision of alpha particle by a nucleus (*i.e.* Rutherford scattering experiment) there is no physical contact.

(1) **Stages of collision :** There are three distinct identifiable stages in collision, namely, before, during and after. In the before and after stage the interaction forces are zero. Between these two stages, the interaction forces are very large and often the dominating forces governing the motion of bodies. The magnitude of the interacting force is often unknown, therefore, Newton's second law cannot be used, the law of conservation of momentum is useful in relating the initial and final velocities.

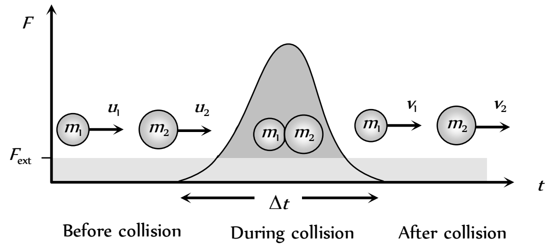


Fig. 6.29

(2) **Momentum and ener... n in collision**

(i) Momentum conservation : In a collision, the effect of external forces such as gravity or friction are not taken into account as due to small duration of collision (Δt) average impulsive force responsible for collision is much larger than external force acting on the system and since this impulsive force is 'Internal' therefore the total momentum of system always remains conserved.

(ii) Energy conservation : In a collision 'total energy' is also always conserved. Here total energy includes all forms of energy such as mechanical energy, internal energy, excitation energy, radiant energy or even mass energy.

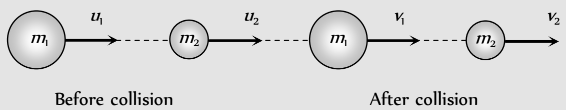
These laws are the fundamental laws of physics and applicable for any type of collision but this is not true for conservation of kinetic energy.

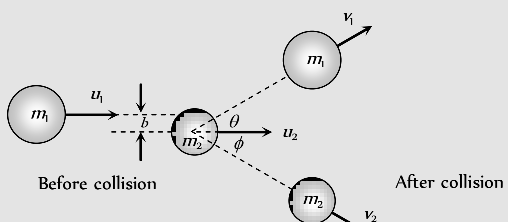
(3) **Types of collision :** (i) On the basis of conservation of kinetic energy.

Perfectly elastic collision	Inelastic collision	Perfectly inelastic collision
If in a collision, kinetic energy after collision is equal to kinetic energy before collision, the collision is said to be perfectly elastic.	If in a collision kinetic energy after collision is not equal to kinetic energy before collision, the collision is said to inelastic.	If in a collision two bodies stick together or move with same velocity after the collision, the collision is said to be perfectly inelastic.
Coefficient of restitution $e = 1$	Coefficient of restitution $0 < e < 1$	Coefficient of restitution $e = 0$
$(KE)_{final} = (KE)_{initial}$	Here kinetic energy appears in other forms. In some cases $(KE)_{final} < (KE)_{initial}$ such as when initial KE is converted into internal energy of the product (as heat, elastic or excitation) while in other cases $(KE)_{final} > (KE)_{initial}$ such as when internal energy stored in the colliding particles is released	The term 'perfectly inelastic' does not necessarily mean that all the initial kinetic energy is lost, it implies that the loss in kinetic energy is as large as it can be. (Consistent with momentum conservation).
Examples : (1) Collision between atomic particles (2) Bouncing of ball with same velocity after the collision with earth.	*Examples* : (1) Collision between two billiard balls. (2) Collision between two automobile on a road. In fact all majority of collision belong to this category.	*Example* : Collision between a bullet and a block of wood into which it is fired. When the bullet remains embedded in the block.

(ii) On the basis of the direction of colliding bodies

Head on or one dimensional collision	Oblique collision
In a collision if the motion of colliding particles before and after the collision is along the same line, the collision is said to be head on or one dimensional.	If two particle collision is 'glancing' *i.e.* such that their directions of motion after collision are not along the initial line of motion, the collision is called oblique. If in oblique collision the particles before and after collision are in same plane, the collision is called 2-dimensional otherwise 3-dimensional.
Impact parameter b is zero for this type of collision.	Impact parameter b lies between 0 and $(r_1 + r_2)$ *i.e.* $0 < b < (r_1 + r_2)$ where r_1 and r_2 are radii of colliding bodies.





Example : collision of two gliders on an air track.	*Example* : Collision of billiard balls.

Perfectly elastic head on collision

Let two bodies of masses m_1 and m_2 moving with initial velocities u_1 and u_2 in the same direction and they collide such that after collision their final velocities are v_1 and v_2 respectively.

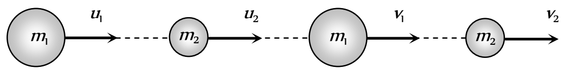


Fig. 6.30

According to law of conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \text{... (i)}$$

$$\Rightarrow \ m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad \text{...(ii)}$$

According to law of conservation of kinetic energy

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad \text{...(iii)}$$

$$\Rightarrow \ m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \quad \text{...(iv)}$$

Dividing equation (iv) by equation (ii)

$$v_1 + u_1 = v_2 + u_2 \quad \text{...(v)}$$

$$\Rightarrow \ u_1 - u_2 = v_2 - v_1 \quad \text{...(vi)}$$

Relative velocity of separation is equal to relative velocity of approach.

Note : ❑ The ratio of relative velocity of separation and relative velocity of approach is defined as coefficient of restitution.

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

or $\ v_2 - v_1 = e(u_1 - u_2)$

❑ For perfectly elastic collision, $e = 1$

$\therefore \ v_2 - v_1 = u_1 - u_2$ [As shown in eq. (vi)]

❑ For perfectly inelastic collision, $e = 0$

$\therefore \ v_2 - v_1 = 0$ or $v_2 = v_1$

It means that two body stick together and move with same velocity.

❑ For inelastic collision, $0 < e < 1$

$\therefore \ v_2 - v_1 = e(u_1 - u_2)$

In short we can say that e is the degree of elasticity of collision and it is dimensionless quantity.

Further from equation (v) we get

$$v_2 = v_1 + u_1 - u_2$$

Substituting this value of v_2 in equation (i) and rearranging

we get, $$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2} \quad \text{...(vii)}$$

Similarly we get,

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1u_1}{m_1 + m_2} \quad \text{...(viii)}$$

(1) **Special cases of head on elastic collision**

(i) **If projectile and target are of same mass *i.e.* $m_1 = m_2$**

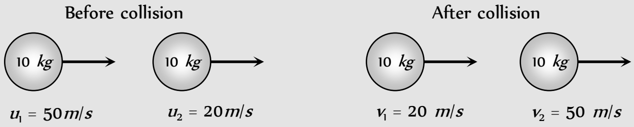
Since $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2}{m_1 + m_2}u_2$ and $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1u_1}{m_1 + m_2}$

Substituting $m_1 = m_2$ we get

$v_1 = u_2$ and $v_2 = u_1$

It means when two bodies of equal masses undergo head on elastic collision, their velocities get interchanged.

Example : Collision of two billiard balls



Sub case : $u_2 = 0$ *i.e.* target is at rest

$v_1 = 0$ and $v_2 = u_1$

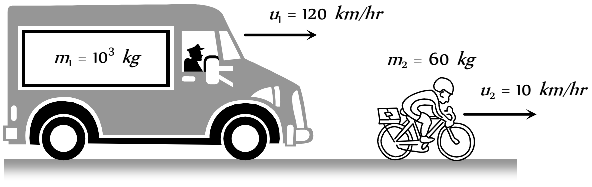
(ii) **If massive projectile collides with a light target *i.e.* $m_1 >> m_2$**

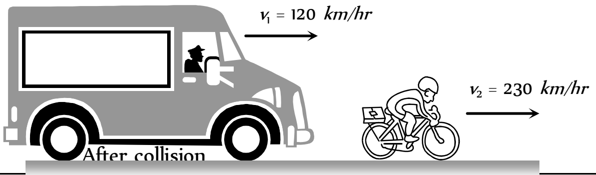
Since $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2}$ and $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1u_1}{m_1 + m_2}$

Substituting $m_2 = 0$, we get

$v_1 = u_1$ and $v_2 = 2u_1 - u_2$

Example : Collision of a truck with a cyclist





Sub case : $u_2 = 0$ i.e. target is at rest

$v_1 = u_1$ and $v_2 = 2u_1$

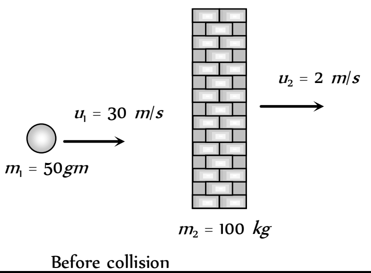
(iii) **If light projectile collides with a very heavy target *i.e.* $m_1 << m_2$**

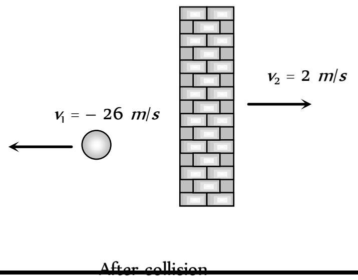
Since $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2}$ and $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1u_1}{m_1 + m_2}$

Substituting $m_1 = 0$, we get

$v_1 = -u_1 + 2u_2$ and $v_2 = u_2$

Example : Collision of a ball with a massive wall.





Sub case : $u_2 = 0$ i.e. target is at rest

$v_1 = -u_1$ and $v_2 = 0$

i.e. the ball rebounds with same speed in opposite direction when it collide with stationary and very massive wall.

(2) **Kinetic energy transfer during head on elastic collision**

Kinetic energy of projectile before collision $K_i = \frac{1}{2}m_1u_1^2$

Kinetic energy of projectile after collision $K_f = \frac{1}{2}m_1v_1^2$

Kinetic energy transferred from projectile to target ΔK = decrease in kinetic energy in projectile

$$\Delta K = \frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1(u_1^2 - v_1^2)$$

Fractional decrease in kinetic energy

$$\frac{\Delta K}{K} = \frac{\frac{1}{2}m_1(u_1^2 - v_1^2)}{\frac{1}{2}m_1u_1^2} = 1 - \left(\frac{v_1}{u_1}\right)^2 \quad \text{...(i)}$$

We can substitute the value of v_1 from the equation

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2}$$

If the target is at rest *i.e.* $u_2 = 0$ then $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$

From equation (i) $\frac{\Delta K}{K} = 1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$ ...(ii)

or $\frac{\Delta K}{K} = \frac{4m_1m_2}{(m_1 + m_2)^2}$ ...(iii)

or $\frac{\Delta K}{K} = \frac{4m_1m_2}{(m_1 - m_2)^2 + 4m_1m_2}$ ...(iv)

Note : ❑ Greater the difference in masses, lesser will be transfer of kinetic energy and vice versa

❑ Transfer of kinetic energy will be maximum when the difference in masses is minimum

i.e. $m_1 - m_2 = 0$ or $m_1 = m_2$ then

$$\frac{\Delta K}{K} = 1 = 100\%$$

So the transfer of kinetic energy in head on elastic collision (when target is at rest) is maximum when the masses of particles are equal i.e. mass ratio is 1 and the transfer of kinetic energy is 100%.

❑ If $m_2 = n m_1$ then from equation (iii) we get

$$\frac{\Delta K}{K} = \frac{4n}{(1+n)^2}$$

❑ Kinetic energy retained by the projectile

$$\left(\frac{\Delta K}{K}\right)_{\text{Retained}} = 1 - \text{ kinetic energy transferred by projectile}$$

$$\Rightarrow \left(\frac{\Delta K}{K}\right)_{\text{Retained}} = 1 - \left[1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2\right] = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$$

(3) **Velocity, momentum and kinetic energy of stationary target after head on elastic collision**

(i) Velocity of target : We know

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1u_1}{m_1 + m_2}$$

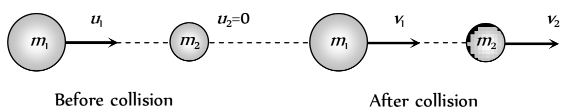


Fig. 6.31

$$\Rightarrow v_2 = \frac{2m_1u_1}{m_1 + m_2}$$

$$= \frac{2u_1}{1 + m_2/m_1} \quad \text{As } u_2 = 0 \text{ and}$$

Assuming $\frac{m_2}{m_1} = n$

$$\therefore v_2 = \frac{2u_1}{1+n}$$

(ii) Momentum of target : $P_2 = m_2v_2 = \dfrac{2nm_1u_1}{1+n}$

$$\left[\text{As } m_2 = m_1 n \text{ and } v_2 = \frac{2u_1}{1+n}\right]$$

$$\therefore P_2 = \frac{2m_1u_1}{1 + (1/n)}$$

(iii) Kinetic energy of target :

$$K_2 = \frac{1}{2}m_2v_2^2 = \frac{1}{2}n m_1\left(\frac{2u_1}{1+n}\right)^2 = \frac{2m_1u_1^2 n}{(1+n)^2}$$

$$= \frac{4(K_1)n}{(1-n)^2 + 4n} \qquad \left[\text{As } K_1 = \frac{1}{2}m_1u_1^2\right]$$

(iv) Relation between masses for maximum velocity, momentum and kinetic energy

Velocity	$v_2 = \dfrac{2u_1}{1+n}$	For v_2 to be maximum n must be minimum *i.e.* $n = \dfrac{m_2}{m_1} \to 0 \therefore m_2 << m_1$	Target should be very light.
Momentum	$P_2 = \dfrac{2m_1u_1}{(1+1/n)}$	For P_2 to be maximum, $(1/n)$ must be minimum or n must be maximum. *i.e.* $n = \dfrac{m_2}{m_1} \to \infty \therefore m_2 >> m_1$	Target should be massive.
Kinetic energy	$K_2 = \dfrac{4K_1 n}{(1-n)^2 + 4n}$	For K_2 to be maximum $(1-n)^2$ must be minimum. *i.e.* $1 - n = 0 \Rightarrow n = 1 = \dfrac{m_2}{m_1} \therefore m_2 = m_1$	Target and projectile should be of equal mass.

Perfectly Elastic Oblique Collision

Let two bodies moving as shown in figure.

By law of conservation of momentum

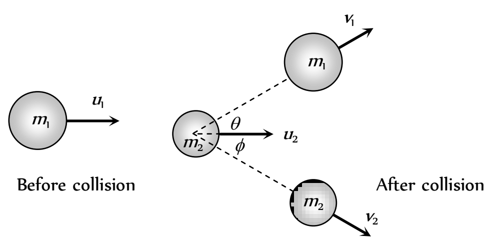


Fig. 6.32

Along *x*-axis, $m_1u_1 + m_2u_2 = m_1v_1\cos\theta + m_2v_2\cos\phi$ ...(i)

Along *y*-axis, $0 = m_1v_1\sin\theta - m_2v_2\sin\phi$ ...(ii)

By law of conservation of kinetic energy

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad \text{...(iii)}$$

In case of oblique collision it becomes difficult to solve problem unless some experimental data is provided, as in these situations more unknown variables are involved than equations formed.

Special condition : If $m_1 = m_2$ and $u_2 = 0$ substituting these values in equation (i), (ii) and (iii) we get

$$u_1 = v_1 \cos\theta + v_2 \cos\phi \quad \text{...(iv)}$$

$$0 = v_1 \sin\theta - v_2 \sin\phi \quad \text{...(v)}$$

and $u_1^2 = v_1^2 + v_2^2$ ...(vi)

Squaring (iv) and (v) and adding we get

$$u_1^2 = v_1^2 + v_2^2 + 2v_1v_2\cos(\theta + \phi) \quad \text{...(vii)}$$

Using (vi) and (vii) we get $\cos(\theta + \phi) = 0$

$\therefore\ \theta + \phi = \pi/2$

i.e. after perfectly elastic oblique collision of two bodies of equal masses (if the second body is at rest), the scattering angle $\theta + \phi$ would be 90^o.

Head on Inelastic Collision

(1) **Velocity after collision :** Let two bodies *A* and *B* collide inelastically and coefficient of restitution is *e*.

Where

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$$

$\Rightarrow v_2 - v_1 = e(u_1 - u_2)$

$$\therefore\ v_2 - v_1 = e(u_1 - u_2) \quad \text{...(i)}$$

From the law of conservation of linear momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \text{...(ii)}$$

By solving (i) and (ii) we get

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \left(\frac{(1+e)m_2}{m_1 + m_2}\right)u_2$$

Similarly $v_2 = \left[\frac{(1+e)m_1}{m_1 + m_2}\right]u_1 + \left(\frac{m_2 - e\,m_1}{m_1 + m_2}\right)u_2$

By substituting $e = 1$, we get the value of v_1 and v_2 for perfectly elastic head on collision.

(2) **Ratio of velocities after inelastic collision :** A sphere of mass *m* moving with velocity *u* hits inelastically with another stationary sphere of same mass.

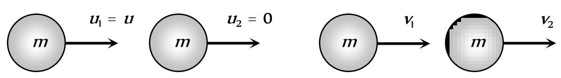


Fig. 6.33

$$\therefore\ e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{v_2 - v_1}{u - 0}$$

$$\Rightarrow\ v_2 - v_1 = eu \quad \text{...(i)}$$

By conservation of momentum :

Momentum before collision = Momentum after collision

$mu = mv_1 + mv_2$

$$\Rightarrow v_1 + v_2 = u \quad \text{...(ii)}$$

Solving equation (i) and (ii) we get $v_1 = \frac{u}{2}(1 - e)$

and $v_2 = \frac{u}{2}(1 + e)$

$\therefore \frac{v_1}{v_2} = \frac{1 - e}{1 + e}$

(3) **Loss in kinetic energy**

Loss in K.E. (ΔK) = Total initial kinetic energy – Total final kinetic energy

$$= \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right)$$

Substituting the value of v_1 and v_2 from the above expressions

$$\text{Loss } (\Delta K) = \frac{1}{2}\left(\frac{m_1m_2}{m_1 + m_2}\right)(1 - e^2)(u_1 - u_2)^2$$

By substituting $e = 1$ we get $\Delta K = 0$ *i.e.* for perfectly elastic collision, loss of kinetic energy will be zero or kinetic energy remains same before and after the collision.

Rebounding of Ball After Collision With Ground

If a ball is dropped from a height *h* on a horizontal floor, then it strikes with the floor with a speed.

$v_0 = \sqrt{2gh_0}$ [From $v^2 = u^2 + 2gh$]

and it rebounds from the floor with a speed

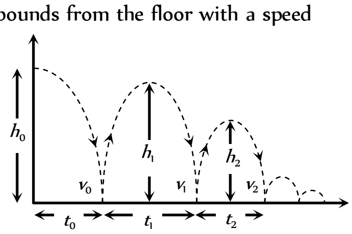


Fig. 6.34

$v_1 = e\,v_0 = e\sqrt{2gh_0}$

$$\left[\text{As } e = \frac{\text{velocity after collision}}{\text{velocity before collision}}\right]$$

(1) **First height of rebound :** $h_1 = \frac{v_1^2}{2g} = e^2h_0$

$\therefore\ h_1 = e^2h_0$

(2) **Height of the ball after n^{th} rebound :** Obviously, the velocity of ball after n^{th} rebound will be

$$v_n = e^n v_0$$

Therefore the height after n rebound will be

$$h_n = \frac{v_n^2}{2g} = e^{2n} h_0$$

$$\therefore\ h_n = e^{2n} h_0$$

(3) **Total distance travelled by the ball before it stops bouncing**

$$H = h_0 + 2h_1 + 2h_2 + 2h_3 + \dots = h_0 + 2e^2 h_0 + 2e^4 h_0 + 2e^6 h_0 + \dots$$

$$H = h_0[1 + 2e^2(1 + e^2 + e^4 + e^6 \dots)]$$

$$= h_0\left[1 + 2e^2\left(\frac{1}{1-e^2}\right)\right]$$

$$\left[\text{As } 1 + e^2 + e^4 + \dots = \frac{1}{1-e^2}\right]$$

$$\therefore\ H = h_0\left[\frac{1+e^2}{1-e^2}\right]$$

(4) **Total time taken by the ball to stop bouncing**

$$T = t_0 + 2t_1 + 2t_2 + 2t_3 + .. = \sqrt{\frac{2h_0}{g}} + 2\sqrt{\frac{2h_1}{g}} + 2\sqrt{\frac{2h_2}{g}} + ..$$

$$= \sqrt{\frac{2h_0}{g}}\ [1 + 2e + 2e^2 + \dots\dots] \quad [\text{As } h_1 = e^2 h_0\ ;\ h_2 = e^4 h_0\ ]$$

$$= \sqrt{\frac{2h_0}{g}}\ [1 + 2e(1 + e + e^2 + e^3 + \dots\dots)]$$

$$= \sqrt{\frac{2h_0}{g}}\left[1 + 2e\left(\frac{1}{1-e}\right)\right] = \sqrt{\frac{2h_0}{g}}\left(\frac{1+e}{1-e}\right)$$

$$\therefore\ T = \left(\frac{1+e}{1-e}\right)\sqrt{\frac{2h_0}{g}}$$

Perfectly Inelastic Collision

In such types of collisions, the bodies move independently before collision but after collision as a one single body.

(1) **When the colliding bodies are moving in the same direction**

By the law of conservation of momentum

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v_{\text{comb}}$$

$$\Rightarrow\ v_{\text{comb}} = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$$

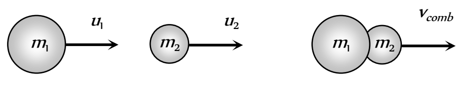


Fig. 6.35

Loss in kinetic energy

$$\Delta K = \left(\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2\right) - \frac{1}{2}(m_1 + m_2) v_{comb}^2$$

$$\Delta K = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(u_1 - u_2)^2$$

[By substituting the value of v_{comb}]

(2) **When the colliding bodies are moving in the opposite direction**

By the law of conservation of momentum

$$m_1 u_1 + m_2(-u_2) = (m_1 + m_2) v_{\text{comb}}$$

(Taking left to right as positive)

$$\therefore\ v_{\text{comb}} = \frac{m_1 u_1 - m_2 u_2}{m_1 + m_2}$$

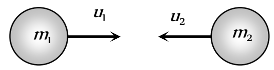


Fig. 3.36

when $m_1 u_1 > m_2 u_2$ then $v_{\text{comb}} > 0$ (positive)

i.e. the combined body will move along the direction of motion of mass m_1.

when $m_1 u_1 < m_2 u_2$ then $v_{\text{comb}} < 0$ (negative)

i.e. the combined body will move in a direction opposite to the motion of mass m_1.

(3) **Loss in kinetic energy**

ΔK = Initial kinetic energy – Final kinetic energy

$$= \left(\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2\right) - \left(\frac{1}{2}(m_1 + m_2) v_{\text{comb}}^2\right)$$

$$= \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(u_1 - u_2)^2$$

Collision Between Bullet and Vertically Suspended Block

A bullet of mass m is fired horizontally with velocity u in block of mass M suspended by vertical thread.

After the collision bullet gets embedded in block. Let the combined system raised upto height h and the string makes an angle θ with the vertical.

(1) **Velocity of system**

Let v be the velocity of the system (block + bullet) just after the collision.

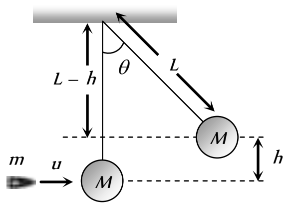


Fig. 3.37

$\text{Momentum}_{\text{bullet}} + \text{Momentum}_{\text{block}} = \text{Momentum}_{\text{bullet and block system}}$

$$mu + 0 = (m + M)v$$

$$\therefore\ v = \frac{mu}{(m + M)} \qquad \dots\text{(i)}$$

(2) **Velocity of bullet :** Due to energy which remains in the bullet-block system, just after the collision, the system (bullet + block) rises upto height h.

By the conservation of mechanical energy

$$\frac{1}{2}(m + M)v^2 = (m + M)gh \Rightarrow v = \sqrt{2gh}$$

Now substituting this value in the equation (i) we get $\sqrt{2gh} = \frac{mu}{m+M}$

$$\therefore \quad u = \left[\frac{(m+M)\sqrt{2gh}}{m}\right]$$

(3) **Loss in kinetic energy :** We know that the formula for loss of kinetic energy in perfectly inelastic collision

$$\Delta K = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(u_1 - u_2)^2$$ (When the bodies are moving in same direction.)

$$\therefore \quad \Delta K = \frac{1}{2}\frac{mM}{m+M}u^2$$

[As $u_1 = u$, $u_2 = 0$, $m_1 = m$ and $m_2 = M$]

(4) **Angle of string from the vertical**

From the expression of velocity of bullet $u = \left[\frac{(m+M)\sqrt{2gh}}{m}\right]$ we can get $h = \frac{u^2}{2g}\left(\frac{m}{m+M}\right)^2$

From the figure $\cos\theta = \frac{L-h}{L} = 1 - \frac{h}{L} = 1 - \frac{u^2}{2gL}\left(\frac{m}{m+M}\right)^2$

or $\theta = \cos^{-1}\left[1 - \frac{1}{2gL}\left(\frac{mu}{m+M}\right)^2\right]$

Tips & Tricks

- The area under the force-displacement graph is equal to the work done.
- Work done by gravitation or electric force does not depend on the path followed. It depends on the initial and final positions of the body. Such forces are called conservative. When a body returns to the starting point under the action of conservative force, the net work done is zero that is $\oint dW = 0$.
- Work done against friction depends on the path followed. Viscosity and friction are not conservative forces. For non conservative forces, the work done on a closed path is not zero. That is $\oint dW \neq 0$.
- Work done is path independent only for a conservative field.
- Work done depends on the frame of reference.
- Work done by a centripetal force is always zero.
- Energy is a promise of work to be done in future. It is the stored ability to do work.
- Energy of a body is equal to the work done by the body and it has nothing to do with the time taken to perform the work. On the other hand, the power of the body depends on the time in which the work is done.
- When work is done on a body, its kinetic or potential energy increases.
- When the work is done by the body, its potential or kinetic energy decreases.
- According to the work energy theorem, the work done is equal to the change in energy. That is $W = \Delta E$.
- Work energy theorem is particularly useful in calculation of minimum stopping force or minimum stopping distance. If a body is brought to a halt, the work done to do so is equal to the kinetic energy lost.
- Potential energy of a system increases when a conservative force does work on it.
- The kinetic energy of a body is always positive.
- When the momentum of a body increases by a factor *n*, then its kinetic energy is increased by factor *n*.
- If the speed of a vehicle is made *n* times, then its stopping distance becomes *n* times.
- The total energy (including mass energy) of the universe remains constant.
- One form of energy can be changed into other form according to the law of conservation of energy. That is amount of energy lost of one form should be equal to energy or energies produced of other forms.
- Kinetic energy can change into potential energy and vice versa. When a body falls, potential energy is converted into kinetic energy.
- Pendulum oscillates due to conversion of kinetic energy into potential energy and vice versa. Same is true for the oscillations of mass attached to the spring.
- Conservation laws can be used to describe the behaviour of a mechanical system even when the exact nature of the forces involved is not known.
- Although the exact nature of the nuclear forces is not known, yet we can solve problems regarding the nuclear forces with the help of the conservation laws.
- Violation of the laws of conservation indicates that the event cannot take place.
- The gravitational potential energy of a mass *m* at a height *h* above the surface of the earth (radius *R*) is given by $U = \frac{mgh}{1 + h/R}$. When $h \ll R$, we find $U = mgh$.
- Electrostatic energy in capacitor - $U = \frac{1}{2}CV^2$, where C is capacitance, V = potential difference between the plates.
- Electric potential energy of a test charge q_0 at a place where electric potential is V, is given by : $U = q_0 V$.
- Electric potential energy between two charges (q_1 and q_2) separated by a distance r is given by $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$. Here ε_0 is permittivity of vacuum and $1/4\pi\varepsilon_0 = 9 \times 10^9\ Nm^2C^{-2}$.
- Magnetic energy stored in an inductor – $U = \frac{1}{2}LI^2$, where L = inductance, I = current.

✍ Energy gained by a body of mass *m*, specific heat *C*, when its temperature changes by $\Delta\theta$ is given by : $Q = mC\Delta\theta$.

✍ The Potential energy associated with a spring of constant *k* when extended or compressed by distance *x* is given by $U = \frac{1}{2}kx^2$.

✍ Kinetic energy of a particle executing *SHM* is given by : $K = \frac{1}{2}m\omega^2(a^2 - y^2)$ where *m* = mass, ω = angular frequency, *a* = amplitude, *y* = displacement.

✍ Potential energy of a particle executing SHM is given by : $U = \frac{1}{2}m\omega^2 y^2$.

✍ Total energy of a particle executing SHM is given by : $E = K + U = \frac{1}{2}m\omega^2 a^2$.

✍ Energy density associated with a wave $= \frac{1}{2}\rho\omega^2 a^2$ where ρ =density of medium, ω = angular frequency, *a* = amplitude of the of the wave.

✍ Energy associated with a photon : $E = h\nu = hc/\lambda$, where *h* = planck's constant, ν = frequency of the light wave, *c* = velocity of light, λ = wave length.

✍ Mass and energy are interconvertible. That is mass can be converted into energy and energy can be converted into mass.

✍ A mass *m* (in *kg*) is equivalent to energy (in *J*) which is equal to *mc* where *c* = speed of light.

✍ A stout spring has a large value of force constant, while for a delicate spring, the value of spring constant is low.

✍ The term energy is different from power. Whereas energy refers to the capacity to perform the work, power determines the rate of performing the work. Thus, in determining power, time taken to perform the work is significant but it is of no importance for measuring energy of a body.

✍ Collision is the phenomenon in which two bodies exert mutual force on each other.

✍ The collision generally occurs for very small interval of time.

✍ Physical contact between the colliding bodies is not essential for the collision.

✍ The mutual forces between the colliding bodies are action and reaction pair. In accordance with the Newton's third law of motion, they are equal and opposite to each other.

✍ The collision is said to be elastic when the kinetic energy is conserved.

✍ In the elastic collisions the forces involved are conservative.

✍ In the elastic collisions, the kinetic or mechanical energy is not converted into any other form of energy.

✍ Elastic collisions produce no sound or heat.

✍ There is no difference between the elastic and perfectly elastic collisions.

✍ In the elastic collisions, the relative velocity before collision is equal to the relative velocity after the collision. That is $\vec{u}_1 - \vec{u}_2 = \vec{v}_2 - \vec{v}_1$ where $\vec{u}_1$ and $\vec{u}_2$ are initial velocities and $\vec{v}_1$ and $\vec{v}_2$ are the velocities of the colliding bodies after the collision. This is called Newton's law of impact.

✍ The collision is said to be inelastic when the kinetic energy is not conserved.

✍ In the perfectly inelastic collision, the colliding bodies stick together. That is the relative velocity of the bodies after the collision is zero.

✍ In an elastic collision of two equal masses, their kinetic energies are exchanged.

✍ If a body of mass *m* moving with velocity *v*, collides elastically with a rigid wall, then the change in the momentum of the body is 2*mv*.

✍ $e = \frac{\vec{v}_2 - \vec{v}_1}{\vec{u}_1 - \vec{u}_2}$ is called coefficient of restitution. Its value is 1 for elastic collisions. It is less than 1 for inelastic collisions and zero for perfectly inelastic collision.

✍ During collision, velocity of the colliding bodies changes.

✍ Linear momentum is conserved in all types of collisions.

✍ Perfectly elastic collision is a rare physical phenomenon.

✍ Collisions between two ivory or steel or glass balls are nearly elastic.

✍ The force of interaction in an inelastic collision is non-conservative in nature.

✍ In inelastic collision, the kinetic energy is converted into heat energy, sound energy, light energy etc.

✍ In head on collisions, the colliding bodies move along the same straight line before and after collision.

✍ Head on collisions are also called one dimensional collisions.

✍ In the oblique collisions the colliding bodies move at certain angles before and/or after the collisions.

✍ The oblique collisions are two dimensional collisions.

✍ When a heavy body collides head-on elastically with a lighter body, then the lighter body begins to move with a velocity nearly double the velocity of the heavier body.

✍ When a light body collides with a heavy body, the lighter body returns almost with the same speed.

✍ If a light and a heavy body have equal momenta, then lighter body has greater kinetic energy.

✍ Suppose, a body is dropped form a height h_0 and it strikes the ground with velocity v_0. After the (inelastic) collision let it rise to a height h_1. If v_1 be the velocity with which the body rebounds, then

$$e = \frac{v_1}{v_0} = \left[\frac{2gh_1}{2gh_0}\right]^{1/2} = \left[\frac{h_1}{h_0}\right]^{1/2}$$

✍ If after *n* collisions with the ground, the velocity is v_n and the height to which it rises be h_n, then

$$e^n = \frac{v_n}{v_0} = \left[\frac{h_n}{h_0}\right]^{1/2}$$

✍ $P = \vec{F}.\vec{v} = Fv\cos\theta$ where $\vec{v}$ is the velocity of the body and θ is the angle between $\vec{F}$ and $\vec{v}$.

✍ Area under the $F - v$ graph is equal to the power dissipated.

✍ Power dissipated by a conservative force (gravitation, electric force etc.) does not depend on the path followed. It depends on the initial and final positions of the body. That is $\oint dP = 0$.

✍ Power dissipated against friction depends on the path followed. That is $\oint dP \neq 0$.

✍ Power is also measured in horse power (*hp*). It is the *fps* unit of power. 1 *hp* = 746 *W*.

✍ An engine pulls a train of mass *m* with constant velocity. If the rails are on a plane surface and there is no friction, the power dissipated by the engine is zero.

✍ In the above case if the coefficient of friction for the rail is μ, the power of the engine is $P = \mu mgv$.

✍ In the above case if the engine pulls on a smooth track on an inclined plane (inclination θ), then its power $P = (mg\sin\theta)v$.

✍ In the above case if the engine pulls upwards on a rough inclined plane having coefficient of friction μ, then power of the engine is

$P = (\mu\cos\theta + \sin\theta)mg\,v$.

✍ If the engine pulls down on the inclined plane then power of the engine is

$P = (\mu\cos\theta - \sin\theta)mg\,v$.