Sample Question Paper - 23
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours ***Maximum Marks : 40***

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *All questions are compulsory.*
3. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
4. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
5. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

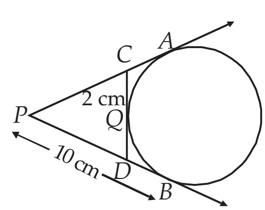
1. Find the roots of the quadratic equation : $x^2 - 3\sqrt{5}x + 10 = 0$

2. The mean of the given distribution is 18. Find the frequency *f* of the class 19-21.

Class	Frequency
11 - 13	3
13 - 15	6
15 - 17	9
17 - 19	13
19 - 21	f
21 - 23	5
23 - 25	4

3. The 17^{th} term of an A.P. is 5 more than twice its 8^{th} term. If the 11^{th} term of the A.P. is 43, then find its n^{th} term.

4. In the given fig., *PA* and *PB* are tangents to be drawn from an external point *P*. *CD* is a third tangent touching the circle at *Q*. If *PB* = 10 cm, and *CQ* = 2 cm, what is the length of *PC* ?



OR

What is the distance between two parallel tangents to a circle of the radius 4 cm ?

5. The total surface area of a cube is $32\frac{2}{3}\text{m}^2$. Find the volume of cube.

OR

The side of solid metallic cube is 50 cm. The cube is melted and recast into 8000 equal solid cubical dice. Determine the side of the dice.

6. In a village, number of members in 50 families are given in the following frequency distribution :

Number of members	1-3	3-5	5-7	7-9	9-11	11-13	13-15	15-17	17-19
Number of families	2	8	6	10	5	5	7	4	3

Find the mean of the above data.

SECTION - B

7. In which of the following situations, do the lists of numbers involved form an A.P.? Give reasons for your answers.

(i) The fee charged every month by a school from Classes I to XII, when the monthly fee for Class I is ₹ 250, and it increases by ₹ 50 for the next higher class.

(ii) The number of bacteria in a certain food item after each second, when they double in every second.

OR

If the m^{th} term of an A.P. is $1/n$ and n^{th} term is $1/m$, then show that its $(mn)^{th}$ term is 1.

8. A man in a boat rowing away from a light house 100 m high takes 2 minutes to change the angle of elevation of the top of the light house from 60° to 30°. Find the speed of the boat in metres per minute. [Use $\sqrt{3} = 1.732$]

9. Draw a circle of radius 4 cm. Draw two tangents to the circle inclined at an angle of 60° to each other.

10. If $x = -2$ is a root of the equation $3x^2 + 7x + p = 0$, find the values of k so that the roots of the equation $x^2 + k(4x + k - 1) + p = 0$ are equal.

SECTION - C

11. A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled into it. The diameter of the pencil is 7 mm, the diameter of the graphite is 1 mm and the length of the pencil is 10 cm. Calculate the weight of the whole pencil, if the specific density of the wood is 0.7 gm/cm^3 and that of graphite is 2.1 gm/cm^3.

OR

Water flows at the rate of 10 metre per minute from a cylindrical pipe 5 mm in diameter. How long will it take to fill up a conical vessel whose diameter at the base is 40 cm and depth 24 cm?

12. A boy standing on a horizontal plane finds a bird flying at a distance of 100 m from him at an elevation of 30°. A girl standing on the roof of a 20 m high building, finds the elevation of the same bird to be 45°. The boy and the girl are on the opposite sides of the bird. Find the distance of the bird from the girl. (Given $\sqrt{2} = 1.414$)

Case Study - 1

13. An agency has decided to install customised playground equipments at various colony parks. For that they decided to study the age-group of children playing in a park of the particular colony. The classification of children according to their ages, playing in a park is shown in the following table.

Age group of children (in years)	6-8	8-10	10-12	12-14	14-16
Number of children	43	58	70	42	27

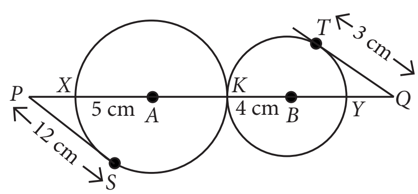
Based on the above information, answer the following questions.

(i) Find the frequency of the class succeeding the modal class.

(ii) Find the mode of the ages of children playing in the park?

Case Study - 2

14. In a maths class, the teacher draws two circles that touch each other externally at point *K* with centres *A* and *B* and radii 5 cm and 4 cm respectively as shown in the figure.



Based on the above information, answer the following questions.

(i) Find the value of *PK*.

(ii) Find the value of *QY*.

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

1. Given, $x^2 - 3\sqrt{5}x + 10 = 0$

Using quadratic formula,

$$x = \frac{3\sqrt{5} \pm \sqrt{(-3\sqrt{5})^2 - 4(1)(10)}}{2(1)} = \frac{3\sqrt{5} \pm \sqrt{5}}{2}$$

$\Rightarrow\ x = \frac{4\sqrt{5}}{2}$ or $x = \frac{2\sqrt{5}}{2}$

$\Rightarrow\ x = 2\sqrt{5}$ or $x = \sqrt{5}$

2. We have the following table :

Class interval	Class mark (x_i)	Frequency (f_i)	$f_i x_i$
11-13	12	3	36
13-15	14	6	84
15-17	16	9	144
17-19	18	13	234
19-21	20	f	$20 \times f$
21-23	22	5	110
23-25	24	4	96
		$N = \Sigma f_i = 40 + f$	$\Sigma f_i x_i = 704 + 20f$

Mean, $\bar{x} = \frac{\Sigma f_i x_i}{N}\ \Rightarrow\ 18 = \frac{704 + 20f}{40 + f}$

$\Rightarrow\ 18(40 + f) = 704 + 20f$

$\Rightarrow\ 720 + 18f = 704 + 20f$

$\Rightarrow\ 20f - 18f = 720 - 704\ \Rightarrow\ 2f = 16\ \Rightarrow\ f = 8.$

$\therefore$ The frequency of class 19-21 is 8.

3. Let a be the first term and d be the common difference of the given A.P.

According to the question, $a_{17} = 2a_8 + 5$

$\Rightarrow\ a + 16d = 2[a + 7d] + 5$

$\Rightarrow\ a + 16d = 2a + 14d + 5\ \Rightarrow\ a = 2d - 5$ ...(i)

Also, $a_{11} = 43 \Rightarrow a + 10d = 43$

$\Rightarrow\ (2d - 5) + 10d = 43$ [Using (i)]

$\Rightarrow\ 12d = 48\ \Rightarrow\ d = 4$

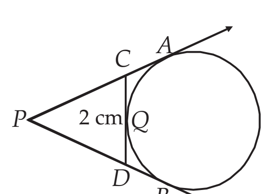
$\therefore\ a = 2d - 5 = 2(4) - 5 = 8 - 5 = 3$

Thus, n^{th} term of the A.P., $a_n = a + (n - 1)d$

$= 3 + (n - 1)\,4 = 3 + 4n - 4 = 4n - 1$

4. Given : $PB = 10$ cm,

$CQ = 2$ cm



Since PA and PB are tangents to be drawn from an external point P.

$\therefore\ PA = PB$

Similarly, $CA = CQ$ and $DB = DQ$

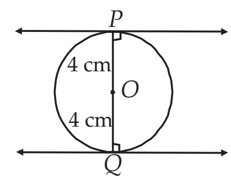
$\therefore\ PA = PB = 10$ cm

$PA = PC + CA$

$\Rightarrow\ PA = PC + CQ$

$\Rightarrow\ PC = PA - CQ = 10 - 2 = 8$ cm

OR



We know that two tangents are parallel if and only if tangents are drawn at the end point of diameter.

So PQ is a diameter of circle.

$PQ = 2$(radius of circle)

$= 2 \times 4 = 8$ cm

Hence distance between two parallel tangents is 8 cm.

5. Let the side of the cube be x m.

$\therefore$ Total surface area $= 6x^2$ m^2

$\Rightarrow\ 6x^2 = 32\frac{2}{3} \Rightarrow 6x^2 = \frac{98}{3}$

$\Rightarrow\ x^2 = \frac{98}{3} \times \frac{1}{6} = \frac{49}{9} \Rightarrow x = \frac{7}{3}$ m.

$\therefore$ Volume of the cube $= x^3$m^3

$= \left(\frac{7}{3}\right)^3$ m^3 $= \frac{343}{27}$m^3 $= 12\frac{19}{27}$m^3.

OR

The side of the given metallic cube = 50 cm

Then the volume of the melted metal $= (50)^3$ cm^3

$= 125000$ cm^3

Let x cm be the side of a cubical dice.

Then volume of 8000 dice $= 8000 \times (x^3)$ cm^3

We have, the volume of 8000 cubical dice = The volume of the given solid metallic cube

$\Rightarrow\ 8000\,(x)^3 = 125000$

$\Rightarrow\ x^3 = \frac{125}{8} = \left(\frac{5}{2}\right)^3\ \Rightarrow\ x = \frac{5}{2} = 2.5$

Hence, side of the dice = 2.5 cm

6. The frequency distribution table from the given data can be drawn as :

Number of members	(x_i)	(f_i)	$f_i x_i$
1-3	2	2	4
3-5	4	8	32
5-7	6	6	36
7-9	8	10	80

9-11	10	5	50
11-13	12	5	60
13-15	14	7	98
15-17	16	4	64
17-19	18	3	54
		$\Sigma f_i = 50$	$\Sigma f_i x_i = 478$

$\therefore$ Mean $= \dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{478}{50} = 9.56$

7. (i) The fee charged (in ₹) every month by a school from classes I to XII is 250, (250 + 50), (250 + 2 × 50), (250 + 3 × 50),... *i.e.*, 250, 300, 350, 400, ...
which forms an A.P., with common difference, $d = 50$
(ii) Let the number of bacteria in a certain food = x
Since, they double in every second
$\therefore$ $x, 2x, 2(2x), 2[2(2x)], \ldots$
i.e., $x, 2x, 4x, 8x, \ldots$
Here, $a_1 = x$, $a_2 = 2x$, $a_3 = 4x$ and $a_4 = 8x$
Now, $a_2 - a_1 = 2x - x = x$
$a_3 - a_2 = 4x - 2x = 2x$, $a_4 - a_3 = 8x - 4x = 4x$
Since, the difference between two successive terms is not same. So, the list of numbers does not form an A.P.

OR

Let a be the first term and d be the common difference of the given A.P.
r^{th} term of A.P., $a_r = a + (r - 1)d$
According to question,

$$a_m = a + (m-1)d = \frac{1}{n} \qquad \ldots(i)$$

and $$a_n = a + (n-1)d = \frac{1}{m} \qquad \ldots(ii)$$

Subtracting (ii) from (i), we get

$$(m-n)d = \frac{m-n}{mn} \Rightarrow d = \frac{1}{mn}$$

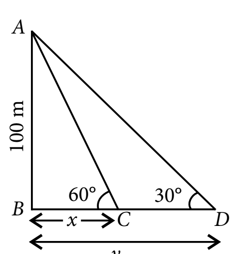
Putting $d = \dfrac{1}{mn}$ in (i), we get

$$a + (m-1)\frac{1}{mn} = \frac{1}{n} \Rightarrow a + \frac{1}{n} - \frac{1}{mn} = \frac{1}{n} \Rightarrow a = \frac{1}{mn}$$

$$\therefore\ a_{mn} = a + (mn-1)d = \frac{1}{mn} + (mn-1)\frac{1}{mn}$$

$$= \frac{1 + mn - 1}{mn} = 1$$

8. Let $AB = 100$ m be the height of the light house.
Let the initial distance be x m and angle is 60°.
In ΔABC,



$$\tan 60° = \frac{AB}{BC} = \frac{100}{x}$$

$$\Rightarrow \sqrt{3} = \frac{100}{x} \Rightarrow x = \frac{100}{\sqrt{3}}$$

Now, after two minutes, new distance be y m and angle is 30°.
In ΔABD,

$$\tan 30° = \frac{AB}{BD} = \frac{100}{y}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{y} \Rightarrow y = 100\sqrt{3}$$

$\therefore$ Distance travelled in 2 minutes = $y - x$

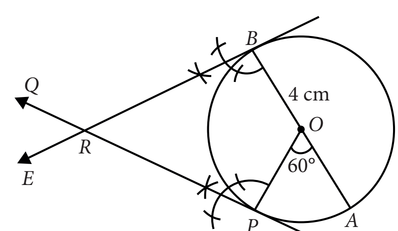
$$= 100\sqrt{3} - \frac{100}{\sqrt{3}} = \frac{300 - 100}{\sqrt{3}}$$

$$= \frac{200}{\sqrt{3}} = \frac{200}{1.732} = 115.47 \text{ m}$$

Speed of boat $= \dfrac{\text{Distance}}{\text{Time}}$

$$= \frac{115.47}{2} = 57.74 \text{ metres/minute}$$

9. Steps of construction :
Step-I : Draw a circle with centre O and radius 4 cm.
Step-II : Draw any diameter AOB.
Step-III : Take a point P on the circle such that $\angle AOP = 60°$.
Step-IV : Draw $PQ \perp OP$ and $BE \perp OB$. Let PQ and BE intersect at R.
Hence, RB and RP are the required tangents.



10. $\because x = -2$ is a root of the equation $3x^2 + 7x + p = 0$
$\therefore$ $3(-2)^2 + 7(-2) + p = 0$
$\Rightarrow$ $12 - 14 + p = 0 \Rightarrow p = 2$
We have, $x^2 + k(4x + k - 1) + p = 0$
$\Rightarrow$ $x^2 + 4kx + k^2 - k + 2 = 0$ $\quad (\because p = 2)$
$\because$ Roots are equal. $\therefore D = 0$
$\Rightarrow$ $(4k)^2 - 4(1)(k^2 - k + 2) = 0$
$\Rightarrow$ $16k^2 - 4k^2 + 4k - 8 = 0 \Rightarrow 12k^2 + 4k - 8 = 0$
$\Rightarrow$ $3k^2 + k - 2 = 0 \Rightarrow (k + 1)(3k - 2) = 0$
$\Rightarrow$ $k + 1 = 0$ or $3k - 2 = 0 \Rightarrow k = -1$ or $k = \dfrac{2}{3}$

11. We have,
Diameter of the graphite cylinder = 1 mm $= \dfrac{1}{10}$ cm

$\therefore$ Radius $= \frac{1}{20}$ cm

Length of the graphite cylinder = 10 cm

Volume of the graphite cylinder

$$= \left(\frac{22}{7} \times \frac{1}{20} \times \frac{1}{20} \times 10\right) \text{cm}^3$$

Weight of graphite = Volume × Specific density

$$= \left(\frac{22}{7} \times \frac{1}{20} \times \frac{1}{20} \times 10 \times 2.1\right) \text{gm}$$

$$= \left(\frac{22}{7} \times \frac{1}{20} \times \frac{1}{20} \times 10 \times \frac{21}{10}\right) \text{gm} = 0.165 \text{ gm}$$

Diameter of pencil $= 7\text{mm} = \frac{7}{10}$ cm

$\therefore$ Radius of pencil $= \frac{7}{20}$ cm and

length of pencil = 10 cm

$\therefore$ Volume of pencil $= \frac{22}{7} \times \frac{7}{20} \times \frac{7}{20} \times 10 \text{ cm}^3$

Volume of wood

$$= \left(\frac{22}{7} \times \frac{7}{20} \times \frac{7}{20} \times 10 - \frac{22}{7} \times \frac{1}{20} \times \frac{1}{20} \times 10\right) \text{cm}^3$$

$$= \frac{22}{7} \times \frac{1}{20} \times \frac{1}{20} \times 10\ (7 \times 7 - 1) \text{cm}^3 = \frac{11}{7} \times \frac{1}{20} \times 48 \text{ cm}^3$$

$\therefore$ Weight of wood $= \left(\frac{11}{7} \times \frac{1}{20} \times 48 \times 0.7\right) \text{gm}$

$$= \left(\frac{11}{7} \times \frac{1}{20} \times 48 \times \frac{7}{10}\right) \text{gm} = 2.64 \text{ gm}$$

Total weight = (2.64 + 0.165)gm = 2.805 gm.

OR

We have,

r = Radius of the base of the conical vessel = 20 cm

h = Height of the conical vessel = 24 cm

$\therefore$ Volume of the conical vessel

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 20 \times 20 \times 24 \text{ cm}^3 \quad \text{...(i)}$$

Suppose the conical vessel is filled in x minutes.

Then, length of the water column

$= (10 \times x)$ m $= 1000x$ cm

Clearly, water column forms a cylinder of length $1000x$ cm, and radius $= \frac{5}{2}\text{mm} = \frac{5}{20}\text{cm} = \frac{1}{4}\text{cm}$

$\therefore$ Volume of the water that flows in x minutes

$$= \left\{\frac{22}{7} \times \left(\frac{1}{4}\right)^2 \times 1000x\right\} \text{cm}^3 \quad \text{...(ii)}$$

From (i) and (ii), we have

$$\frac{22}{7} \times \left(\frac{1}{4}\right)^2 \times 1000x = \frac{1}{3} \times \frac{22}{7} \times 20 \times 20 \times 24$$

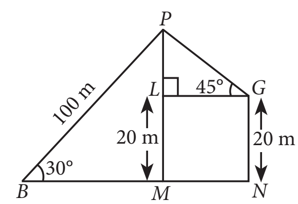
$$\Rightarrow x = \frac{20 \times 20 \times 24 \times 16}{3 \times 1000} \Rightarrow x = \frac{256}{5} = 51\frac{1}{5} \text{ minutes}$$

= 51 minutes 12 secs

Hence, the conical vessel is filled in 51 minutes 12 secs.

12. Let P be the position of bird, B and G be the position of the boy and the Girl respectively.

GN be the building at which the girl is standing.



In ΔPMB,

$$\frac{PM}{BP} = \sin 30^\circ \Rightarrow PM = 100 \times \frac{1}{2} = 50 \text{ m}$$

Now, $PL = PM - LM = 50 - 20 = 30$ m

In ΔPLG, $\frac{PL}{PG} = \sin 45^\circ$

$$\Rightarrow \frac{30}{PG} = \frac{1}{\sqrt{2}} \Rightarrow PG = 30\sqrt{2} = 30 \times 1.414 = 42.42 \text{ m}$$

Hence, the bird is flying at a distance of 42.42 m from the girl.

13. (i) Since, the highest frequency is 70, therefore the maximum number of children are of the age-group 10-12.

Since, the modal class is 10-12.

$\therefore$ Lower limit of modal class = 10

Here, $f_0 = 58$, $f_1 = 70$ and $f_2 = 42$

Thus, the frequency of the class succeeding the modal class is 42.

(ii) Mode $= l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

$$= 10 + \left[\frac{70 - 58}{140 - 58 - 42}\right] \times 2$$

$$= 10 + \frac{12}{40} \times 2 = 10 + \frac{24}{40} = 10.6 \text{ years}$$

14. Here, $AS = 5$ cm, $BT = 4$ cm [$\because$ Radii of circles]

(i) Since, radius at point of contact is perpendicular to tangent.

$\therefore$ By Pythagoras theorem, we have

$$PA = \sqrt{PS^2 + AS^2} = \sqrt{12^2 + 5^2} = \sqrt{169} = 13 \text{ cm}$$

(ii) Again by Pythagoras theorem, we have

$$BQ = \sqrt{TQ^2 + BT^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5 \text{ cm}$$

and $QY = BQ - BY = 5 - 4 = 1$ cm