

# 266

# I

Total No. of Questions—24

Total No. of Printed Pages—4

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Part III

MATHEMATICS

Paper II(A)

(English Version)

Time : 3 Hours

Max. Marks : 75

Note :—This question paper consists of THREE sections A, B and C.

SECTION A

10×2=20

I. Very Short Answer Type Questions

(i) Answer ALL questions.

(ii) Each question carries TWO marks.

1. Find the complex conjugate of  $(2 + 5i)(-4 + 6i)$ .
2. If  $x + iy = \text{cis } \alpha \cdot \text{cis } \beta$ , then find the value of  $x^2 + y^2$ .
3. If ABC are angles of a triangle such that  $x = \text{cis } A$ ,  $y = \text{cis } B$ ,  $z = \text{cis } C$ , then find the value of  $xyz$ .
4. For what values of  $x$  the expression  $x^2 - 5x - 14$  is positive ?
5. If 1, 1,  $\alpha$  are the roots of  $x^3 - 6x^2 + 9x - 4 = 0$ , then find  $\alpha$ .
6. Find the number of ways of arranging the letter of the word "MATHEMATICS".
7. Find the value of  ${}^{10}C_5 + {}^{2 \cdot 10}C_4 + {}^{10}C_3$ .
8. Find the 8th term of  $\left(1 - \frac{5x}{2}\right)^{-3/5}$ .
9. Find the mean deviation about the mean for the following data :  
3, 6, 10, 4, 9, 10.

10. If the mean and variance of a binomial variable  $X$  are 2.4 and 1.44 respectively, find the parameters of the distribution  $X$ . (Binomial).

**SECTION B**

5×4=20

**II. Short Answer Type Questions :**

- (i) Answer ANY FIVE questions.  
(ii) Each question carries FOUR marks.

11. If  $(x - iy)^{1/3} = a - ib$ , then show that :

$$\frac{x}{a} + \frac{y}{b} = 4(a^2 - b^2).$$

12. Find the maximum value of the function  $\frac{x^2 + 14x + 9}{x^2 + 2x + 3}$  over  $\mathbf{R}$ .
13. Find the sum of all 4-digit numbers that can be formed using the digits 1, 3, 5, 7, 9.
14. Prove that :

$${}^{25}C_4 + \sum_{r=0}^4 ({}^{29-r}C_3) = {}^{30}C_4.$$

15. Resolve  $\frac{x^2 + 5x + 7}{(x - 3)^3}$  into partial fractions.
16. A and B are events with  $P(A) = 0.5$ ,  $P(B) = 0.4$  and  $P(A \cap B) = 0.3$ . Find the probability that :
- (i) A does not occur  
(ii) Neither A nor B occurs.
17. A problem in calculus is given to two students A and B whose chances of solving it are  $\frac{1}{3}$  and  $\frac{1}{4}$  respectively. Find the probability of the problem being solved if both of them try independently.

**SECTION C**

5×7=35

**III. Long Answer Type Questions :**

- (i) Answer ANY FIVE questions.  
 (ii) Each question carries SEVEN marks.

18. If  $n$  is a positive integer, show that :

$$(P + iQ)^{\frac{1}{n}} + (P - iQ)^{\frac{1}{n}} = 2(P^2 + Q^2)^{\frac{1}{2n}} \cdot \cos \left[ \frac{1}{n} \tan^{-1} \frac{Q}{P} \right].$$

19. Solve the equation :

$$6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0.$$

20. For  $r = 0, 1, 2, \dots, n$ , prove that :

$$C_0 C_r + C_1 C_{r+1} + C_2 C_{r+2} + \dots + C_{n-r} \cdot C_n = {}^{2n}C_{(n+r)} \text{ and hence deduce that :}$$

- (i)  $C_0^2 + C_1^2 + C_2^2 + \dots + C_n^2 = {}^{2n}C_n$   
 (ii)  $C_0 C_1 + C_1 C_2 + C_2 C_3 + \dots + C_{n-1} \cdot C_n = {}^{2n}C_{n+1}$ .

21. Find the sum to infinite terms of the series :

$$\frac{7}{5} \left( 1 + \frac{1}{10^2} + \frac{1.3}{1.2} \cdot \frac{1}{10^4} + \frac{1.3.5}{1.2.3} \cdot \frac{1}{10^6} + \dots \infty \right)$$

22. Find the mean deviation from the mean of the following data, using the step deviation method :

Marks	No. of Students
0—10	6
10—20	5
20—30	8
30—40	15
40—50	7
50—60	6
60—70	3

23. (a) State and prove Addition theorem on probability.
- (b) Find the probability of drawing an ace or a spade from a well shuffled pack of 52.
24. If  $X$  is a random variable with probability distribution  $P(X = k) = \frac{(k + 1)C}{2^k}$ ,  $k = 0, 1, 2, 3, \dots$ , then find  $C$ .