CHAPTER 5

Squares and Square Roots

Learning Objectives

In this chapter you will learn:

- *To find Square of a number.*
- *About perfect square number and different properties of squared numbers.*
- *To find square root of a number using different methods.*
- *To use concept of square and square roots in solving practical life problems.*

5.1 Introduction:-

In Geometry, you have learnt about square. You also know that area of a square = side × side (where side means, the length of a side)

Now, see the following numbers : 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, ... etc.

What is special in these numbers,

Since 1 = 1 × 1 ; 4 = 2 × 2 ; 9 = 3 × 3 ; 16 = 4 × 4 ; 25 = 5 × 5 ; 36 = 6 × 6 and so on, so all such numbers can be expressed as the product of some number with itself.

Such numbers like 1, 4, 9, 16, 25, 36, ... are known as **Perfect squares** or **Square numbers.**

In general, if a natural number m can be expressed as n^2, where n is also a natural number, then, that number m is a perfect square or a square number.

Example Is 49 a perfect square?

Sol. First we shall express 49 as a product of two natural numbers.

i.e. 49 = 7× 7

Here we see 49 can be expressed as the product of number 7 with itself.

So, 49 is a perfect square.

Example Is 27 a perfect square number?

Sol. First we shall express 27 as a product of two natural numbers.

i.e. 27 = 3 × 9

= 1× 27

Here we see that 27 cannot be expressed as the product of some number with itself.

Hence 27 is not a perfect square number, or we can say that 27 is not a square of any natural number.

Similarly 8, 10, 13, 58, 176 etc are not perfect square numbers.

So we can say that all the natural numbers are not Perfect Squares.

5.2 Properties of Square Numbers:

Study the following table, which shows the squares of first 30 natural numbers.

Natural Number	Square of the Number	Natural Numbers	Square of the Number	Natural Numbers	Square of the Number
1	1	11	121	21	441
2	4	12	144	22	484
3	9	13	169	23	529
4	16	14	196	24	576
5	25	15	225	25	625
6	36	16	256	26	676
7	49	17	289	27	729
8	64	18	324	28	784
9	81	19	361	29	841
10	100	20	400	30	900

Property 1 : Square numbers (perfect square numbers) always end with the digits 0, 1, 4, 5, 6 or 9.

From above table, we observe that digits at ones place of square number which are Red in colour, are 0, 1, 4, 5, 6 and 9. None of these have 2, 3, 7 or 8 at ones place.

Can we say that if a number ends with 0, 1, 4, 5, 6 and 9 then it must be a square number?

No, it is not always true.

For example, numbers 10 and 20 end with 0 but they are not perfect squares.

Similarly 11, 21, 31 etc end with 1 but they are also not perfect squares.

Number 14, 15, 24, 26, 29 etc. are some more examples whose ones digits are 4, 5, 6 and 9, but they are not perfect squares.

So, we can say that if a number ends with 0, 1, 4, 5, 6 or 9 then it may or may not be a perfect square.

A number ending in 2, 3, 7 or 8 can never be a perfect square.

For example 22, 33, 237, 2378, 3542, 15437 etc. are not perfect squares.

Example 5.1 Write five numbers on which you can decide by looking at their ones digit that they are not square numbers. (perfect square)

Sol. We know the numbers ending with 2, 3, 7 or 8, can never be perfect square.

So five numbers are 62, 93, 147, 228, 222 etc.

Example 5.2 Write five numbers on which you cannot decide just by looking at their ones digit that whether they are square numbers or not.

Sol. We know the numbers ending with 0, 1, 4, 5, 6 or 9 may or may not be a square number. So, we cannot decide just by looking at the numbers ending with these digits. Some examples are 120, 221, 534, 565, 216, 219 etc.

Property 2: The number of zeroes at the end of a square number is always even.

Observe the last row of the given table, which is in Green Colour.

We see that, all the perfect squares have even number of zeroes at the end.

For Example:	10^2	=	10×10	=	100
	20^2	=	20×20	=	400
	30^2	=	30×30	=	900
Some other example	60^2	=	60×60	=	3600
	100^2	=	100×100	=	10000

(Therefore, the number of zeroes at the end of the square of a number is twice the number of zeroes at the end of the number.)

Note: It is not always true that a number ending with even number of zeroes is always a perfect square. It may or may not be a perfect square.

For example, 400 is a perfect square but 300, 500 and 700 are not.

A number ending with odd number of zeroes is never a perfect square.

For example 10, 110, 1000, 5000, ends with odd number of zeroes. Hence none of these numbers is a perfect square.

Example 5.3. What will be the number of zeroes in the square of the following numbers.

(i) 60 (ii) 200 (iii) 8000

Sol. (i) As 60 has one zero at end, so its square will have two zeroes at end.

(ii) As 200 has two zeroes at end, so its square will have four zeroes at end.

(iii) As 8000 has 3 zeroes at end, so its squares will have 6 zeroes at end.

Property 3: Square of an even number is always even.

See the table, numbers marked with magenta colour.

4^2	=	4×4	=	16
8^2	=	8×8	=	64
12^2	=	12×12	=	144
24^2	=	24×24	=	576

Property 4: Square of an odd number is always odd.

See the table, numbers marked with blue colour.

1^2	=	1×1	=	1
5^2	=	5×5	=	25
11^2	=	11×11	=	121
19^2	=	19×19	=	361

Property 5: Ones digit of square of a number :

Observe the ones digit of the number and ones digit of square of that number. We have the following result :

Ones digit of Number	Ones digit of squared number (Square ends)
1 or 9	1
2 or 8	4
3 or 7	9
4 or 6	6
5	5

We can observe that from the ones digit of a number, we can find the ones digit of its square number.

Also from the ones digit of the square number, we can find the ones digit of the number.

Example 5.4 : What will be the ones digit in the square of following numbers?

(i) 211 (ii) 299 (iii) 1018 (iv) 1687 (v) 4204

Sol. (i) As ones digit of 211 is 1, so ones digit of its square will be 1.

(ii) As ones digit of 299 is 9, so ones digit of its square will be 1.

(iii) As ones digit of 1018 is 8, so ones digit of its square will be 4.

(iv) As ones digit of 1687 is 7, so ones digit of its square will be 9.

(v) As ones digit of 4204 is 4, so ones digit of its square will be 6.

Property 6: There are 2n natural numbers between the squares of two consecutive numbers i.e. n and n + 1.

For example, take $n = 1$ and $n + 1=2$. Now between $(1)^2$ and $(2)^2$ (between 1 and 4) there are $2 \times n = 2\times1=2$ natural numbers (i.e. 2 and 3)

Consider $n = 4$ and $n + 1 = 5$. Now between $(4)^2 = 16$ and $(5)^2=25$ there are $2 \times n = 2 \times 4 = 8$ natural numbers (i.e.17, 18, 19, 20, 21, 22, 23, 24)

Example 5.5 : How many natural numbers lie between

(i) 8^2 and 9^2 (ii) 11^2 and 12^2

Sol. As we know that there are 2n numbers between n^2 and $(n+1)^2$, So

(i) $8^2 = 64$ and $9^2 = 81$,

there are $2 \times 8 = 16$ natural numbers between them

[as $n = 8$, $n + 1 = 8 + 1 = 9$]

(ii) $11^2 = 121$ and $12^2 = 144$

there are $2 \times 11 = 22$ natural numbers between them

[as $n = 11$, $n+1 = 12$]

Property 7: Square of a number n can be expressed as the sum of the first n odd natural numbers.

i.e. sum of first n odd natural numbers = n^2

e.g. $2^2 = 4 = 1 + 3$ (Here 1 and 3 are the first two odd natural number)

$3^2 = 9 = 1 + 3 + 5$ (Here 1, 3 and 5 are the first 3 odd natural numbers)

$4^2 = 16 = 1 + 3 + 5 + 7$ (Here 1, 3, 5 and 7 are the first 4 odd natural numbers)

$5^2 = 25 = 1 + 3 + 5 + 7 + 9$ (Here 1, 3, 5, 7 and 9 are the first 5 odd natural numbers)

So we can say

$1 + 3 + 5 + \ldots\ldots\ldots\ldots + n = n^2$

i.e. sum of first n odd natural numbers = n^2

Example 5.6 : Find the sum of 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 without actually adding them.

Sol. As we know that a sum of first *n* odd natural numbers can be expressed as a square of number n.

∴ Sum of first 8 odd natural numbers can be expressed as square of 8.

$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = 8^2 = 64$ (sum of first 8 odd natural numbers)

Example 5.7 : Write 100 as a sum of odd numbers.

Sol. We know that $100 = 10^2$

∴ 100 can be expressed as a sum of first 10 odd natural numbers.

∴ $100 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$

Property 8: The square of an odd number (except 1) can be expressed as the sum of two consecutive natural numbers.

For example $3^2 = 9 = 4 + 5$

$5^2 = 25 = 12 + 13$

$7^2 = 49 = 24 + 25$

In general, for any odd number n,

$$n^2 = \left(\frac{n^2 - 1}{2}\right) + \left(\frac{n^2 + 1}{2}\right)$$

Example 5.8: Express the following as a sum of two consecutive numbers

(a) 19^2 (b) 23^2

Sol. (a) We know that $19^2 = 361$ [As $n^2 = \left(\frac{n^2 - 1}{2}\right) + \left(\frac{n^2 + 1}{2}\right)$

where n = 19 (odd natural number)

$$19^2 = \left(\frac{19^2 - 1}{2}\right) + \left(\frac{19^2 + 1}{2}\right)$$

$$= \left(\frac{361 - 1}{2}\right) + \left(\frac{361 + 1}{2}\right)$$

$$= \frac{360}{2} + \frac{362}{2}$$

$$= 180 + 181$$

(b) We know that $(23)^2 = 529$ [As $n^2 = \left(\frac{n^2 - 1}{2}\right) + \left(\frac{n^2 + 1}{2}\right)$

where n = 23 (odd natural number)

$$= \left(\frac{23^2 - 1}{2}\right) + \left(\frac{23^2 + 1}{2}\right)$$

$$= \left(\frac{529 - 1}{2}\right) + \left(\frac{529 + 1}{2}\right)$$

$$= \frac{528}{2} + \frac{530}{2}$$

$$= 264 + 265$$

Property 9: Product of two consecutive even or odd natural numbers.

If $n-1$ and $n+1$ are two consecutive even or odd natural numbers then

$(n-1) \times (n+1) = n^2-1$,

For example

$5 \times 7 = 35 = (6 - 1) \times (6 + 1)$ and $5 \times 7 = 35 = 6^2 - 1$

Here n is 6,

$10 \times 12 = 120 = (11 - 1) \times (11 + 1)$ and $10 \times 12 = 120 = (11^2-1)$

Here n is 11

Property 10: The difference of the squares of two consecutive natural numbers is equal to the sum of both the numbers.

For example $5^2 - 4^2 = 25 - 16 = 9 = 5 + 4$

$10^2 - 9^2 = 100 - 81 = 19 = 10 + 9$

Or we can say

$(n + 1)^2 - (n)^2 = (n + 1) + n$ or $2n + 1$

Example 5.9 : Find the difference between $21^2 - 20^2$ without calculations.

Sol. We know that the difference of the squares of two consecutive natural numbers is equal to the sum of the two given numbers.

$\therefore \quad 21^2 - 20^2 = 21 + 20 = 41$

Property 11: The square of a negative integer is always positive.

For example $(-3)^2 = -3 \times -3 = 9$

$(-5)^2 = -5 \times -5 = 25$

$(-10)^2 = -10 \times -10 = 100$

Property 12: The square of a proper fraction is always less than the given fraction.

For example $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$

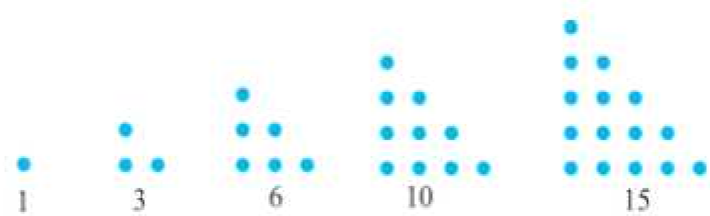
And $\frac{4}{9} < \frac{2}{3}$

$\Rightarrow 0.444... < 0.666.....$

Some Interesting Patterns

1. Adding Triangular Numbers

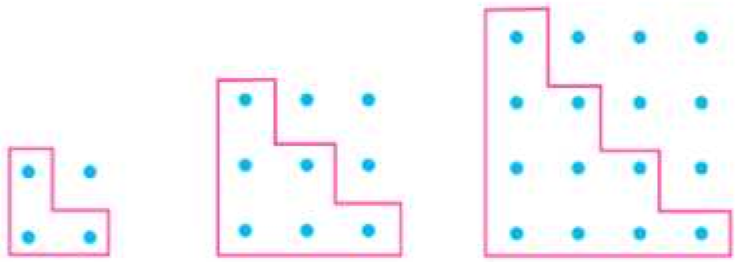
Triangular numbers are those numbers whose dot patterns can be arranged as triangles.



These numbers are 1, 3, 6, 10, 15, 21, 28................

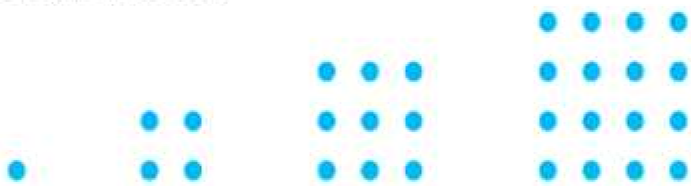
n^{th} triangular number is given by $\frac{n(n+1)}{2}$

Note: Here we observe that, the sum of any two consecutive triangular numbers is a perfect square.



$3+1=4=2^2$ $6+3=9=3^2$ $10+6=16=4^2$

2. Square numbers of the form n^2 can be represented geometrically in the form of square with n rows and n columns of dots.



1^2 $2^2 = 4$ $3^2 = 9$ $4^2 = 16$

3. Let us observe the pattern made by square of numbers such as 1^2, 11^2, 111^2 and so on

$1^2 = 1$

$11^2 = 1\ 2\ 1$

$111^2 = 1\ 2\ 3\ 2\ 1$

$1111^2 = 1\ 2\ 3\ 4\ 3\ 2\ 1$

$11111^2 = 1\ 2\ 3\ 4\ 5\ 4\ 3\ 2\ 1$

On observing the above pattern, the square of any number of the type 1111111...... can be easily calculated.

Here we observe that

(i) The digits in the squares is a combination of ascending and descending order of digits.

(ii) There are odd numbers of digits in the squares.

(iii) The central digit in the square number is equal to the number of 1s in the number.

(iv) The first and last digits are always 1.

4. Observe the squares of 7, 67, 667....... . Here also observe a very intersting pattern.

$7^2 = 49$

$67^2 = 4\ 4\ 8\ 9$

$667^2 = 4\ 4\ 4\ 8\ 8\ 9$

$6667^2 = 4\ 4\ 4\ 4\ 8\ 8\ 8\ 9$

Here we observe that

(i) All the squares end with digit 9.

(ii) The numbes of 4s in the square of number is one more than the number of 6s in number.

(iii) The number of 8s is the square of number is same as the number of 6s in the number.

5. Observe the squares of 5, 15, 25, 35, 45............ i.e. the number having 5 as ones digit, it also follow a very interesting pattern.

$5^2 = 25 = 0 \times 1(\text{hundred}) + 5^2$

$15^2 = 225 = 1 \times 2\ (\text{hundred}) + 5^2$

$25^2 = 625 = 2 \times 3\ (\text{hundred}) + 5^2$

$35^2 = 1225 = 3 \times 4\ (\text{hundred}) + 5^2$

$105^2 = 11025 = 10 \times 11\ (\text{hundred}) + 5^2$

Or

$(a5)^2 = a \times (a+1)$ hundred $+ 5^2$

6. Observe the following interesting patterns :

$1^2 + 2^2 + 2^2 = 3^2$

$2^2 + 3^2 + 6^2 = 7^2$

$3^2 + 4^2 + 12^2 = 13^2$

$4^2 + 5^2 + 20^2 = 21^2$ etc.

Here we observe, ignoring the squares, the first two numbers on the LHS are consecutive numbers and third number is the product of first two numbers. The number on the RHS is the successor of third number of LHS.

i.e. $a^2 + (a + 1)^2 + (a\ (a + 1))^2 = [a\ (a + 1) + 1]^2$

Example 5.10 : Express 21^2 as a sum of squares of 3 numbers.

Sol. Here we have to express 21^2 as a sum of squares of 3 numbers.

$\therefore \quad a^2 + b^2 + c^2 = 21^2$

If we ignore the square, c will be the predecessor of 21 i.e. 20.

Now 20 is equal to product of two consecutives numbers i.e 4 and 5.

So $\quad 21^2 = (4)^2 + (5)^2 + (20)^2$

Pythagorean Triplets

We know that, in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides, which is the Pythagoras Theorem.

So Pythagorean triplet refers to a group of 3 numbers which follows the Pythagoras Theorem.

3 numbers, named a, b and c are known as Pythagorean triplet if $a^2 + b^2 = c^2$

Consider three numbers 3, 4 and 5

Now $(3)^2 = 9$, $(4)^2 = 16$ and $(5)^2 = 25$

Here $\quad 3^2 + 4^2 = 5^2$

i.e. $\quad 9 + 16 = 25$

$\quad 25 = 25$

Hence 3, 4 and 5 forms a Pythagorean triplet.

There are uncountable number of Pythagorean triplets.

Some of them are (6, 8, 10), (5, 12, 13) etc.

For any natural number n>1, we have

$(2n)^2 + (n^2-1)^2 = (n^2+1)^2$

So $(2n, n^2-1, n^2+1)$ forms a Pythagorean triplet.

By this we can easily found a Pythagorean triplet.

Note: All Pythagorean triplets may not be obtained using this method.

Example 5.11: Write a Pythagorean triplet with 6 as one of the numbers of the triplet.

Sol. We can get Pythagorean triplet by using the general form $2n, n^2-1, n^2+1$

Let us take $2n = 6 \Rightarrow n = 3$

So $n^2 - 1 = 3^2 - 1 = 8$ and $n^2 + 1 = 3^2 + 1 = 10$

So Pythagorean triplet is 6, 8 and 10.

Example 5.12: Find a Pythagorean triplet whose one of the number is 15.

Sol. We can get Pythagorean triplet by using the general form $2n, n^2-1, n^2+1$

Let us take $2n = 15 \Rightarrow n = \frac{15}{2}$, which is not a natural number

So we cannot takes $2n = 15$

So, let us take $n^2 - 1 = 15$

$\Rightarrow n^2 = 16$

$\Rightarrow n = 4$

$\therefore \quad 2n = 2 \times 4 = 8$

also $n^2 - 1 = (4)^2 - 1 = 16 - 1 = 15$

also $n^2 + 1 = (4)^2 + 1 = 16 + 1 = 17$

So required triplet is (8, 15, 17)

Exercise 5.1

1. Find the square of the following numbers :

(i) 19 (ii) 41 (iii) –11 (iv) $\frac{3}{7}$ (v) $1\frac{2}{3}$

(vi) 1.7 (vii) 0.02 (viii) 0.014

2. The following numbers are not perfect squares, give reasons.

(i) 177 (ii) 1058 (iii) 7928 (iv) 23453 (v) 42222

(vi) 64000 (vii) 222000 (viii) 42977 (ix) 5000 (x) 100000

3. What will be the number of zeroes in the square of following numbers?

(i) 90 (ii) 120 (iii) 400 (iv) 6000 (v) 80000

(vi) 1600

4. The square of which of the following would be an odd number or an even number?

(i) 431 (ii) 2826 (iii) 7779 (iv) 82004 (v) 473

(vi) 4096 (vii) 9267 (viii) 27916

5. What will be the ones digit in the squares of following numbers?

(i) 41 (ii) 321 (iii) 89 (iv) 439 (v) 62

(vi) 4012 (vii) 88 (viii) 348 (ix) 93 (x) 703

(xi) 57 (xii) 1327 (xiii) 44 (xiv) 1024 (xv) 26

(xvi) 2226 (xvii) 55 (xviii) 125

6. How many natural numbers lie between squares of the following numbers?

(i) 14 and 15 (ii) 21 and 22 (iii) 30 and 31 (iv) 10 and 11

7. Express the following as indicated

(i) 81 as the sum of first 9 odd numbers.

(ii) 144 as the sum of first 12 odd numbers.

(iii) 256 as the sum of first 16 odd numbers.

8. Without actually adding, find the sum of:

(i) $1 + 3 + 5 + 7 + 9$

(ii) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25$

9. Express the following as the sum of two consecutive numbers:

(i) $(15)^2$ (ii) $(21)^2$ (iii) $(33)^2$ (iv) $(37)^2$

10. Solve following without actually calculating:

(i) $8^2 - 7^2$ (ii) $13^2 - 12^2$ (iii) $25^2 - 24^2$ (iv) $80^2 - 79^2$

(v) $110^2 - 109^2$

11. Observe the following patterns and find the missing terms:

(i)

$1^2 + 2^2 + 2^2 = 3^2$

$2^2 + 3^2 + 6^2 = 7^2$

$3^2 + 4^2 + \ldots = 13^2$

$\ldots + 5^2 + \ldots = 21^2$

$5^2 + \ldots + 30^2 = \ldots$

$6^2 + 7^2 + \ldots = \ldots$

(ii)

$1^2 = 1 = 1$

$2^2 = 4 = 1 + 2 + 1$

$3^2 = 9 = 1 + 2 + 3 + 2 + 1$

$\ldots = \ldots = 1 + 2 + 3 + 4 + 3 + 2 + 1$

$\ldots = \ldots = 1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1$

(iii)

$21^2 = 441$

$201^2 = 40401$

$2001^2 = 4004001$

$\boxed{}^2 = 400040001$

$(2000001)^2 = 4000004000001$

$(20000001)^2 = \boxed{}$

12. Using the pattern $1^2 = 1$, $11^2 = 121$, $111^2 = 12321$, $1111^2 = 1234321$ find $(1111111)^2$.

13. Find the squares of the following numbers having 5 at ones digit place?

(i) 45 (ii) 75 (iii) 95 (iv) 125 (v) 205

14. Which of the following are Pythagorean triplets?

(i) 3, 4, 5 (ii) 6, 8, 10 (iii) 8, 15, 17 (iv) 13, 17, 19

15. Write a Pythagorean triplet whose one of the number is:

(i) 8 (ii) 12 (iii) 16 (iv) 18 (v) 20

16. **Multiple Choice Questions :**

(i) Square of an odd number is always ....

(a) Even (b) Odd (c) Even or Odd (d) None of these

(ii) The number of zeros in the square of 600 will be

(a) 1 (b) 2 (c) 3 (d) 4

(iii) The ones digit in the square of 52698 is

(a) 1 (b) 4 (c) 6 (d) 9

(iv) How many natural numbers lie between 6^2 and 7^2 ?

(a) 6 (b) 8 (c) 10 (d) 12

(v) 1 + 3 + 5 + 7 + ... upto n terms is equal to

(a) $n^2 - 1$ (b) $n^2 + 1$ (c) $(n + 1)^2$ (d) n^2

(vi) The square of a proper fraction is :

(a) Less than the fraction (b) Greater than the fraction

(c) Equal to the fraction (d) None of these

(vii) The value of $(111111)^2$ is

(a) 1234564321 (b) 1234455321

(c) 12345654321 (d) 1234554321

(viii) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = ...

(a) $(6)^2$ (b) $(5)^2$ (c) $(7)^2$ (d) $(8)^2$

(ix) Which of the following is a perfect square ?

(a) 4000 (b) 40000 (c) 40 (d) 400000

(x) The nth triangular number is given by

(a) $\frac{n(n+1)}{2}$ (b) $\frac{n(n-1)}{2}$ (c) $\frac{n-1}{2}$ (d) $\frac{n}{2}$

5.3 Square Root

We have already learnt how to find square of a number. Now suppose we have a squared number (perfect square). Now how can we find the original number whose square we have ? For this purpose we have concept of square root.

Let we are given a number K, we need to find a number which when squared gives K. In other words we are simply going to reverse the process.

$\therefore$ Square root can be defined as inverse of doing square:

Symbol used for square root is $\sqrt{\ }$. Now observe the table and apply above definition.

$2^2 = 4$	Square Root of 4 = 2	i.e. $\sqrt{4} = 2$
$6^2 = 36$	Square Root of 36 = 6	i.e. $\sqrt{36} = 6$
$13^2 = 169$	Square Root of 169 = 13	i.e. $\sqrt{169} = 13$
$21^2 = 441$	Square Root of 441 = 21	i.e. $\sqrt{441} = 21$

As $4^2 = 16$

And $(-4)^2 = 16$

$\therefore$ $\sqrt{16}$ can have both 4 and –4 as its answers but here, in this chapter we will only discuss to positive square root of a number.

5.3.1 Finding Square Roots

Before we learn how to find square root of a number we must know why we need to find square root of a number? Consider the following situations:

1. We know that area of square = $(\text{Side})^2$

   If you are given area of square as 3125cm^2 how will you find its side?
2. Suppose you are given with sides of a rectangle, how will you find the diagonal?
3. Suppose you have a right triangle whose adjacent sides are given, how will you calculate its hypotenuse?

In all above situations and many other similar situations we need to calculate square root at some stage.

Before we learn to calculate square root of a number. Let's connect square and square root of a number through properties already learnt.

1. Square root of even number is even and that of odd number is odd.

   i.e. $\sqrt{196} = 14$ as $14^2 = 196$

   and $\sqrt{225} = 15$ as $15^2 = 225$
2. Ones place digit of square root of any perfect square number ending with 1 is either 1 or 9.

   i.e. $\sqrt{121} = 11$ as $11^2 = 121$

   and $\sqrt{361} = 19$ as $19^2 = 361$
3. Ones place digit of square root of any perfect square number ending with 4 is either 2 or 8.

   i.e. $\sqrt{144} = 12$ as $12^2 = 144$

   and $\sqrt{324} = 18$ as $18^2 = 324$

4. Ones place digit of square root of a perfect square ending with 9 is either 3 or 7.

i.e. $\sqrt{169} = 13$ as $13^2 = 169$

and $\sqrt{729} = 27$ as $27^2 = 729$

5. Ones place digit of square root of any perfect square ending with 5 is 5.

i.e. $\sqrt{225} = 15$ as $15^2 = 225$

and $\sqrt{625} = 25$ as $25^2 = 625$

6. Any number which ends with 2, 3, 7, 8 or have odd number of zeros at its end cannot be a perfect square. Square root of these type numbers will not be a natural number.

e.g. 232, 407, 1603, 1008 and 1690 can never be a perfect square numbers.

Example 5.13 Fill in the blanks

(i) $11^2 = 121$ So $\sqrt{121}$ = ... (ii) $14^2 = 196$ So $\sqrt{......}$ = 14

Sol. (i) As $11^2 = 121$ So $\sqrt{121} = 11$

(ii) As $14^2 = 196$ So $\sqrt{196} = 14$

5.3.2 Finding square root by repeated subtraction

As we have learnt earlier that the sum of the first n odd natural numbers is n^2. That is, every square number can be expressed as sum of consecutive odd natural number starting from 1. consider $\sqrt{49}$

Step (i) $49 - 1 = 48$ Step (ii) $48 - 3 = 45$ Step (iii) $45 - 5 = 40$

Step (iv) $40 - 7 = 33$ Step (v) $33 - 9 = 24$ Step (vi) $24 - 11 = 13$

Step (vii) $13 - 13 = 0$

We can write $49 = 1 + 3 + 5 + 7 + 5 + 9 + 11 + 13$

Se here we have started subtracting successive odd natural numbers starting from 1 and obtained 0 at 7th step. So $\sqrt{49} = 7$

Example 5.14 By repeated subtraction of odd numbers starting from 1, find whether the following numbers are perfect square or not? if number is perfect square, find its square root.

(a) 36 (b) 55 (c) 121

Sol. (a) The given number is 36, Start subtracting the odd numbers starting from 1

(i) $36 - 1 = 35$ (ii) $35 - 3 = 32$ (iii) $32 - 5 = 27$

(iv) $27 - 7 = 20$ (v) $20 - 9 = 11$ (vi) $11 - 11 = 0$

As we have obtained 0, so 36 is a perfect square and we obtained 0 at 6th step

so $\sqrt{36} = 6$

(b) (i) 55 – 1 = 54 (ii) 54 – 3 = 51 (iii) 51 – 5 = 46

(iv) 46 – 7 = 39 (v) 39 – 9 = 30 (vi) 30 – 11 = 19

(vii) 19 – 13 = 06 (viii) 6 – 15 = –9

As we did not get zero, hence 55 is not a perfect square number.

(c) (i) 121 – 1 = 120 (ii) 120 – 3 = 117 (iii) 117 – 5 = 112

(iv) 112 – 7 = 105 (v) 105 – 9 = 96 (vi) 96 – 11 = 85

(vii) 85 – 13 = 72 (viii) 72 – 15 = 57 (ix) 57 – 17 = 40

(x) 40 – 19 = 21 (xi) 21 – 21 = 0

As we got zero, so 121 is a perfect square number.

and we obtained 0 at 11th step so $\sqrt{121} - 11$

Now can you find $\sqrt{625}$ using this method? Yes, but it will be time consuming. We have another methods for finding the square root. We will discuss in next section.

5.3.3 Finding square root through prime factorisation :

Consider the prime factorisation of some numbers and their squares, as shown in table 6.7

Number	Prime factorisation of number	Square of number	Prime factorization of square of number
4	2 × 2	16	2 × 2 × 2 × 2
6	2 × 3	36	2 × 2 × 3 × 3
8	2 × 2 × 2	64	2 × 2 × 2 × 2 × 2 × 2
9	3 × 3	81	3 × 3 × 3 × 3
10	2 × 5	100	2 × 2 × 5 × 5
12	2 × 2 × 3	144	2 × 2 × 2 × 2 × 3 × 3
15	3 × 5	225	3 × 3 × 5 × 5

On observing given table, you will see that 2 occurs twice in prime factorisation of 4 and 2 occurs 4 times in the prime factorisation of 16. Similarly, How many times 2 occur in prime factorisation of 6? It is one time. Now how many times does 2 occur in the prime factorisation of 36? It is two times. Similarly observe the occurrence of 3 in 6 and 36.

You will find that each prime factor in the prime factorisation of the square of a number, occurs twice the number of times it occurs in the prime factorisation of the number itself. Let us use this to find the square root of a given number say 100.

The prime factorisation of 100 is as $100 = 2 \times 2 \times 5 \times 5$

2	100
2	50
5	25
5	5
	1

By pairing the prime factors,we get

$100 = 2 \times 2 \times 5 \times 5 = 2^2 \times 5^2 = (2 \times 5)^2$

So $\sqrt{100} = 2 \times 5 = 10$

Similarly, we can find the square root of 144.

2	144
2	72
2	36
2	18
3	9
3	3
	1

The Prime Factorisation of 144 is

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$= 2^2 \times 2^2 \times 3^2 = (2 \times 2 \times 3)^2$

So $\sqrt{144} = 2 \times 2 \times 3 = 12$

Is 48 a perfect square?

We know $48 = 2 \times 2 \times 2 \times 2 \times 3$

Since one factor (3) of 48 is not in pair, so 48 is not a perfect square.

Suppose we want to find the smallest number which will make 48 a perfect square, how should we proceed? In the prime factorisation of 48 we see that 3 is the only factor that does not have a pair. So we need to multiply 48 by 3 to complete the pair.

Hence $48 \times 3 = 144$ is a perfect square.

Now can you tell by which smallest number should we divide 48 to get a perfect square?

As we have seen the factor 3 is not in pair, so if we divide 48 by 3,

we get $48 \div 3 = 16$ which is a perfect square

Example 5.15 Find the square root of the following numbers by method of Prime factorisation (i) 729 (ii) 9604

Sol. Let us find the prime factoisation of the given numbers is :

(i)

3	729
3	243
3	81
3	27
3	9
3	3
	1

So $729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$

i.e. $729 = 3^2 \times 3^2 \times 3^2$

$729 = (3 \times 3 \times 3)^2$

Hence $\sqrt{729} = 3 \times 3 \times 3 = 27$

(ii)

2	9604
2	4802
7	2401
7	343
7	49
7	7
	1

So $9604 = 2 \times 2 \times 7 \times 7 \times 7 \times 7$

i.e. $9604 = 2^2 \times 7^2 \times 7^2$

$9604 = (2 \times 7 \times 7)^2$

Hence $\sqrt{9604} = 2 \times 7 \times 7 = 98$

Example 5.16 For each of the following numbers, find the smallest number by which it should be multiplied to get a perfect square number. Also, find the square root of the perfect square number so obtained. (i) 180 (ii) 768

Sol. (i) Let us find the prime factorisation of 180.

2	180
2	90
3	45
3	15
5	5
	1

Now 180 $= 2 \times 2 \times 3 \times 3 \times 5$

$= 2^2 \times 3^2 \times 5^1$

The prime factor 5, does not occur in pair. So we need to multiply 180 by 5 to get a perfect square number.

Now $180 \times 5 = 900 = 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 2^2 \times 3^2 \times 5^2$

i.e. $900 = (2 \times 3 \times 5)^2 = (30)^2$

so $\sqrt{900} = 30$

(ii) Let us find the Prime factorisation of 768.

2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

$768 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$

Here 3 is the only factor that does not occur in pair

So we need to multiply 768 by 3 to complete the pair. So new number is

$768 \times 3 = 2304$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$

i.e. $2304 = 2^2 \times 2^2 \times 2^2 \times 2^2 \times 3^2$

$2304 = (2 \times 2 \times 2 \times 2 \times 3)^2 = (48)^2$

Hence $\sqrt{2304} = 48$

Example 5.17 Find the smallest number by which 9408 must be divided so that the quotient is a perfect square. Also find the square root of the quotient.

Sol. Let us find the Prime factorisation of 9408

2	9408
2	4704
2	2352
2	1176
2	588
2	294
3	147
7	49
7	7
	1

$9408 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 7 \times 7$

Here except 3, all factors are in pairs.

So if we divide 9408 by 3, then

$9408 \div 3 = 3136$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7 = 2^2 \times 2^2 \times 2^2 \times 7^2$

$= (2 \times 2 \times 2 \times 7)^2$

which is a perfect square and

$\sqrt{3136} = 2 \times 2 \times 2 \times 7 = 56$

Example 5.18 2025 Plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows and number of plants in each row.

Sol. As we have to plant as many plants in each row as the number of rows. So number of plants will be a squared number.

Let number of plants in each row = number of rows = x

As per question $x \times x = 2025$

i.e. $x^2 = 2025$

To find x, we have to find a number whose square is 2025.

i.e. x is square root of 2025

Now $2025 = 3\times3\times3\times3\times5\times5 = 3^2 \times 3^2 \times 5^2$

i.e. $2025 = (3 \times 3 \times 5)^2$

So $\sqrt{2025} = 3 \times 3 \times 5 = 45$

3	2025
3	675
3	225
3	75
5	25
5	5
	1

Hence number of rows and number of plants in each row are 45.

Example 5.19: Find the smallest square number which is divisible by each of numbers 8, 12, 50.

Sol. As required number is divisible by each of 8, 12 and 50

∴ We have to find L.C.M. of 8, 12 and 50

L.C.M (8, 12, 50) $= 2 \times 2 \times 2 \times 3 \times 5 \times 5$

$= 600$

2	8 - 12 - 50
2	4 - 6 - 25
2	2 - 3 - 25
3	1 - 3 - 25
5	1 - 1 - 25
5	1 - 1 - 5
	1 - 1 - 1

But 600 is not a perfect square.

So we have to make perfect square:

We have $600 = 2\times2\times2\times3\times5\times5$

Since, to make 600 a perfect square, we have to multiply it with 2 and 3.

i.e. $600 \times 2 \times 3 = 3600$

∴ 3600 is a perfect square which is divisible by 8, 12 and 50.

Exercise 5.2

1. Tell the ones place digit of square root of following numbers:

(i) 121 (ii) 729 (iii) 676 (iv) 1936

(v) 484 (vi) 2401 (vii) 1600 (viii) 3025

2. From the following numbers find the number which cannot be a perfect square number.

100, 512, 1728, 529, 1024, 441, 1320, 3617

3. Find the square root of following numbers by method of repeated subtraction.

(i) 64 (ii) 49 (iii) 121 (iv) 100

4. Find square root of following numbers using method of prime factorisation:

(i) 3600 (ii) 676 (iii) 9216 (iv) 2916

(v) 6400 (vi) 1764 (vii) 12100 (viii) 1024

5. Find the smallest number by which following number must be multiplied so that the resultant is a perfect square. Also find square root of that number:

(i) 243 (ii) 240 (iii) 2662 (iv) 972

(v) 3087 (vi) 5000

6. Find the smallest number by which following numbers must be divided so that quotient is a perfect square. Also find square root of the number obtained:

(i) 108 (ii) 3125 (iii) 2400 (iv) 5103

(v) 2205 (vi) 12168

7. Find the smallest perfect square which is divisible by:

(i) 8, 12, 28 (ii) 27, 24, 15

8. During a plantation drive 256 plants were planted in a such a way that each row contains as many plants as number of rows. Find number of rows.

9. Area of a square is equal to the area of rectangle whose sides are 27cm and 12cm. Find the side of square.

10. Some students raised funds to help an orphange. If total funds collected was 3136 and each student contributed amount equal to number of students. Find amount contributed by each student.

11. **Multiple Choice Questions :**

(i) What will be ones place digit of $\sqrt{961}$?

(a) 1 or 7 (b) 1 or 9 (c) 3 or 6 (d) 7 or 8

(ii) Guess ones place digit of square root of 625.

(a) 1 (b) 4 (c) 9 (d) 5

(iii) Which of the following cannot be perfect square ?

(a) 625 (b) 728 (c) 729 (d) 144

(iv) What will be square root of 144?

(a) 10 (b) 12 (c) 18 (d) 22

(v) By what number 32 should be multiplied to make it a perfect square?

(a) 2 (b) 3 (c) 4 (d) 5

(vi) By what number 288 should be divided to make it a perfect square?

(a) 5 (b) 4 (c) 3 (d) 2

5.3.4 Finding Square Root by Long Division Method

Consider a number 16777216. If we find its square root by prime factorisation it involve division by 2, 24 times which makes it quite lengthy that is why we use long division method for such problems. Before solving such question let us learn to find number of digits in square root of a number. Observe the following table.

Number	No. of digits	Square root of number	No. of digits in square root
64	2	8	1
144	3	12	2
961	3	31	2
1024	4	32	2
262144	6	512	3
16777216	8	4096	4

If you observe carefully we can draw following inference from above table.

(a) If number of digits in squared number are even, then number of digits in square root of number = $\frac{n}{2}$

(b) If number of digits in number are odd, then number of digits in its square root of number = $\frac{n+1}{2}$

Estimating the number :

We know, in the square root of a perfect square having n digits, the number of digits are $\frac{n}{2}$ (if n is even) and $\frac{n+1}{2}$ (if n is odd). As we use bar from unit digit of the number by taking two digits at a time to find the number of digits in the square root of a perfect square number

$$\sqrt{\overline{6}\,\overline{25}} = 25 \;;\; \sqrt{\overline{12}\,\overline{96}} = 36$$

Both the numbers $\overline{6}\,\overline{25}$ and $\overline{12}\,\overline{96}$ have two bars and the number of digits in their square root are 2. Can you tell the number of digits in the square root of 14400? By Placing bars we get $\overline{1}\,\overline{44}\,\overline{00}$. Since there are 3 bars, the square root will be a 3 digit number.

Example 5.20: Find number of digits in square roots of following

(i) 7744 (ii) 15625 (iii) 25600

Sol. (i) Number of digits in 7744 = 4 (Even)

Number of digits in its square root = $\frac{4}{2} = 2$

(ii) Number of digits in 15625 = 5 (odd)

Number of digits in its square root = $\frac{5+1}{2} = \frac{6}{2} = 3$

(iii) Number of digits in 25600 = 5 (odd)

Number of digits in square root of 25600 = $\frac{5+1}{2} = \frac{6}{2} = 3$

Now we can move further to find square root of a number by method of long division.

It is very useful method to find the square root of a given square number. Consider the following steps to find the square root of 625.

Step 1 Place a bar over every Pair of digits starting from the digit at ones place. If the number of digits are odd, then the left most single digit too will have a bar. So we have $\overline{6}\ \overline{25}$

Step 2 Find the largest number whose square is less than or equal to the number under the extreme left bar ($2^2 < 6 < 3^2$). Take this number as divisor and the quotient with the number under extreme left bar (here 6). Divide and get the remainder (2 in this case)

$$\begin{array}{r|l} & 2 \\ \hline 2 & \overline{6}\ \overline{25} \\ & -4 \\ \hline & 2 \end{array}$$

Step 3 Bring down the number under the next bar (25 in this case) to the right of remainder

$$\begin{array}{r|l} & 2 \\ \hline 2 & \overline{6}\ \overline{25} \\ & -4 \\ \hline & 2\ 25 \end{array}$$

Setp 4 Double the quotient and enter it with a blank on its right

$$\begin{array}{r|l} & 2 \\ \hline 2 & \overline{6}\ \overline{25} \\ & -4 \\ \hline 4\_ & 2\ 25 \end{array}$$

Step 5 Guess a largest possible digit to fill in the blank which will also become the new digit in the quotient, such that when the new divisor is multiplied to the new digit in the quotient, the product is less than or equal to the dividend. In this case $45 \times 5 = 225$, so we choose the new digit as 5, to get the remainder.

$$\begin{array}{r|l} & 2\ 5 \\ \hline 2 & \overline{6}\ \overline{25} \\ & -4 \\ \hline 45 & 225 \\ & -225 \\ \hline & 0 \end{array}$$

Step 6 Since the remainder is 0 and no digit left in the given number so $\sqrt{625} = 25$

Example 5.21 Find the square root of 1296 using long division method

Sol. Following the above steps

$$\begin{array}{r|l} & 3\ 6 \\ \hline 3 & \overline{12}\ \overline{96} \\ & -9 \\ \hline 66 & 396 \\ & -396 \\ \hline & 0 \end{array}$$

Therefore, $\sqrt{1296} = 36$

Example 5.22 Find the least number that must be subtracted from 1308 so as to get a perfect square. Also find the square root of the perfect square.

Sol. Let us try to find the $\sqrt{1308}$ by long division method

we get the remainder 12.

It shows that $(36)^2$ is less than 1308 by 12.

This means if we subtract 12 from the number, we get a perfect square. so the required perfect square number is

```
       3 6
   3 | 13 08
     | - 9
  66 |   408
     | - 396
     |    12
```

$1308 - 12 = 1296$ and $\sqrt{1296} = 36$

Example 5.23 Find the greatest four digits number which is a perfect square.

Sol. Greatest 4 digit number is 9999.

Now we will check whether this number is a perfect square. Otherwise we have to find the nearest number less than 9999, which is a perfect square. Here 9999 is not a perfect square. As as we get remainder 198. So If we subtract 198 from 9999, the new number will be a perfect square. So largest four digit perfect square number = 9999 – 198 = 9801 and

```
        9 9
    9 | 99 99
      | -81
  189 |  1899
      | -1701
      |   198
```

$\sqrt{9801} = 99$

Example 5.24 Find the least number that must be added to 5615 so as the get a perfect square. Also, find the square root of the perfect square.

Sol. We find $\sqrt{5615}$ by long division method,

This shows $(74)^2 < 5615$

next perfect square is $(75)^2 = 5625$

Hence the number to be added is = $(75)^2 - 5615$

$= 5625 - 5615 = 10$

```
        7 4
    7 | 56 15
      | -49
  144 |   715
      |   576
      |   139
```

So number is 5625 and its square root is 75

Example 5.25 Find the smallest four digit number which is a perfect square?

Sol. The smallest four digit number is 1000. We will check whether this number is a perfect square. Otherwise we have to find its nearest four digit number which is a perfect square. By long division method, we get remainder 39. This show that 1000 is not a perfect square also.

```
       3 1
   3 | 10 00
     | - 9
  61 |   100
     |  - 61
     |    39
```

$(31)^2 < 1000$

Next perfect square is $(32)^2 = 1024$

Hence the number to be added is $= 1024 - 1000 = 24$

and square root of 1024 = $\sqrt{1024} = 32$

Example 5.26 **Area of a square Plot is 3136m². Find the side of square**

Sol. We know that area of a square = $(\text{side})^2$

Let side of Plot is = x m

So $x^2 = 3136$ i.e $x = \sqrt{3136}$

So, side of square = 56m

```
         56
     ---------
   5 | 31 36
     |-25
 ---------
 106 |  636
     | -636
     ---------
     |    0
```

Example 5.27 **There are 505 students in a school. For a P.T. drill, they have to stand in such a manner that the number of rows is equal to number of columns. How many children will be left out in the arrangement.**

Sol. As number of rows and columns are equal, so it is a square arrangement. we have to find nearest number (smaller) to 505, which is a perfect square

By using long division method, we get the remainder 21.

It shows that $(22)^2$ is less than 505 by 21.

This means if we subtract 21 from 505, we get a perfect square.

Hence 505 – 21 = 484 students can be arranged in equal number of rows and columns. 21 students will be left out in the arrangement.

```
        22
     --------
   2 | 5 05
     |-4
 --------
  42 | 105
     | -84
     --------
     |  21
```

Exercise 5.3

1. Find number of digits in square root of following numbers.

   (i) 12996 (ii) 6084 (iii) 698896 (iv) 72900 (v) 1806336

2. Using Long division method, find the square root of following :

   (i) 9216 (ii) 8100 (iii) 50176 (iv) 4761

   (v) 421201 (vi) 16900 (vii) 5184 (viii) 86436

   (ix) 16777216 (x) 46656

3. Find the least number that must be added to the following numbers to get a perfect square. Also; find the square root of new number.

   (i) 540 (ii) 1765 (iii) 3260 (iv) 4000 (v) 5200 (vi) 790

4. Find the least number that must be subtracted from the following numbers so as to get a perfect square number. Also find the square root of the perfect square number.

   (i) 696 (ii) 1140 (iii) 6021 (iv) 10204 (v) 126441 (vi) 788501

5. Find the greatest five digits number which is a perfect square. Also find the square root.

6. Find the smallest four digits number which is a perfect square. Also find the square root.

7. Find the length of side of the following square field whose area is

   (i) 3136m² (ii) 7225m² (iii) 12100m² (iv) 18225m²

8. Find the length of hypotenuse of a right angle triangle whose other two sides are 6cm and 8cm.

9. A gardener has 1100 Plants. He wants to plant these in such a way that the number of rows and the number of columns remain same. Find the minimum number of plants he needs more for this.

10. Choose the correct answer:

(i) What will be number of digits in square root of 676.

(a) 1 (b) 2 (c) 3 (d) 4

(ii) What will be number of digits in square root of 186624.

(a) 1 (b) 3 (c) 2 (d) 4

(iii) What smallest number must be added to 140 to make it a perfect square?

(a) 4 (b) 8 (c) 12 (d) 16

(iv) What smallest number must be subtracted from 750 to make it a perfect square?

(a) 11 (b) 21 (c) 31 (d) 41

(v) If area of square is $384cm^2$. Find its side.

(a) 12cm (b) 14cm (c) 16cm (d) 18cm

(vi) If 404 children are arranged in rows and columns such that number of rows and number of columns are equal. Find how many children are left out.

(a) 10 (b) 4 (c) 8 (d) 6

5.4 Square Root of Decimal Numbers :

To find the square root of a decimal number, we will follow the steps given below. Consider the number is 51.84

Step 1 We put bars on the integral part (here 51) of the number in the usual manner and place bars on the decimal part (here 84) on every pair of digits beginning with the first decimal place. proceed as usual, we get $\overline{51}.\overline{84}$

Step 2 Now proceed in similar manner. The left most bar is 51 and $7^2 < 51 < 8^2$, take 7 as divisor and the number under the left most bar as the divided i.e. 51.

```
      7
  7 | 51.84
    | -49
    |   2
```

Divide and get the remainder

Step 3 The remainder is 2. Write the number under the next bar (i.e. 84) to the right of remainder, we get 284

```
       7
   7 | 51.84
     | -49
 14_ |  284
```

Step 4 Double the quotient (i.e. 7) and enter a blank on its right. Since 84 is the decimal part so put a decimal point in the quotient i.e (after 7)

```
          7.
     7 | 51.84
       |-49
   14_ |  284
```

```
          7.2
     7 | 51.84
       |-49
   142 |  284
       | -284
       |    0
```

Step 5 We know 142 × 2 = 284, So the new digit is 2, Divide and get the remainder

Step 6 **Since the remainder is zero and no bar left, therefore $\sqrt{51.84} = 7.2$**

Example 6.28: Find the square root of 31.36

Sol.

```
          5.6
     5 | 31.36
       |-25
   106 |  636
       | -636
       |    0
```

$\sqrt{31.36} = 5.6$

> **Note :** Let us learn how to put bars in a decimal numbers. Consider a number say 325.732, it has two parts : integral and decimal parts. For integral part 325, we start from the unit's place (here 5) and move towards left. The first bar is over 25 and the second bar is over 3. For decimal part 732, we start from decimal and move towards right. First bar is over 73 and for second bar we put 0 after 2 and make $.\overline{73}\,\overline{20}$

Exercise 5.4

1. Find the square root of the following decimal numbers

   (i) 9.61 (ii) 11.56 (iii) 466.56 (iv) 1.4641 (v) 1354.24

   (vi) 1.218816

2. Find the square root of the followings:

   (i) $\frac{64}{169}$ (ii) $\frac{144}{441}$ (iii) $\frac{81}{784}$ (iv) $\frac{196}{625}$

3. Find the square root of 2, 3 and 5 upto three digits of decimals.

4. Multiple choice questions :

(i) Choose the correct way of placing bars from following:

(a) $\sqrt{15625}$ (b) $\sqrt{15625}$ (c) $\sqrt{15625}$ (d) $\sqrt{15625}$

(ii) After how many places decimal will appear in square root of 24.01.

(a) 1 (b) 2 (c) 3 (d) 4

(iii) Find square root of 39.0625

(a) 6.25 (b) 62.5 (c) 0.625 (d) 6.6251

(iv) Find the length of hypotenuse of a right triangle having other two sides as 6cm and 8cm.

(a) 6cm (b) 8cm (c) 10cm (d) $10cm^2$

Activity

AIM :- To observe some given number pattern and write their next three steps/rows.

Objectives : To understand number pattern and to generalise them

Previous knowledge :- Knowledge of number patterns.

Material Required : Some patterns involving numbers.

Procedure :

1. Observe the following number pattern:

(a)
$1^2 = 1$
$(11)^2 = 121$
$(111)^2 = 12321$

(b)
$1^2 = 1$
$2^2 = 1 + 3$
$3^2 = 1 + 3 + 5$

2. Identify the rule involved in each pattern.
3. Complete the next three rows of each pattern on the basis of rule described in step 2.

Observations:

(i) 4th row in pattern (a) is $(1111)^2$ = 1234321

5th row in pattern (a) is $(11111)^2$ = ...

6th row in pattern (a) is .... = 123456....................

(ii) 4th row in pattern (b) is 4^2 = 1+3+5+7

5th row in pattern (b) is .... = 1+3+5+7+9

6th row in pattern (b) is .... = 1+3+5+7+.............

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Q1. What is the value of $(1111111)^2$?

Ans: 1234567654321

Q2. Express 12^2 as a sum of first 12 odd natural numbers.

Ans: $12^2 = 1+3+5+7+9+11+13+15+17+19+21+23$

Q3. What is value of 1+3+5+7+9+11+13+15 ?

Ans: $(8)^2 = 64$

Learning Outcomes

After completion of the chapter, students are now able to:

- *Find square of a number.*
- *Understand different properties of square numbers.*
- *Find square root of a number using different methods.*
- *Understand perfect square numbers.*
- *Use concept of square and square root in solving practical life problems.*

Answers

Exercise 5.1

1. (i) 361 (ii) 1681 (iii) 121 (iv) $\frac{9}{49}$

(v) $\frac{25}{9}$ (vi) 2.89 (vii) 0.004 (viii) 0.000196

2. (i) Ones digit is 7 (ii) Ones digit in 8 (iii) Ones digit is 8

(iv) Ones digit is 3 (v) Ones digit is 2 (vi) No of zeros are odd

(vii) No of zeros are odd (viii) Ones digit is 7 (ix) No. of zeros are odd

(x) No. of zeros are odd

3. (i) Two zeros (ii) Two zeros (iii) Four zeros

(iv) Six zeros (v) Eight zeros (vi) Four zeros

4. (i), (iii), (v), (vii) would be odd numbers

(ii), (iv), (vi), (viii) would be even numbers.

5. (i) 1 (ii) 1 (iii) 1 (iv) 1 (v) 4 (vi) 4 (vii) 4
(viii)4 (ix) 9 (x) 9 (xi) 9 (xii) 9 (xiii) 6 (xiv) 6
(xv) 6 (xvi) 6 (xvii) 5 (xviii) 5

6. (i) 28 (ii) 42 (iii) 60 (iv) 20

7. (i) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17
(ii) 1 + 3 + 5 + 7 + 9 + 11+13+15+17+19+21+23
(iii) 1+3+5+7+9+11+13+15+17+19+21+23+25+27+29+31

8. (i) $(5)^2$ (ii) $(13)^2$

9. (i) (112, 113) (ii) (220, 221) (iii) (544, 545)
(iv) (684, 685)

10. (i) 15 (ii) 25 (iii) 49 (iv) 159 (v) 219

11. (i)
$1^2 + 2^2 + 2^2 = 3^2$
$2^2 + 3^2 + 6^2 = 7^2$
$3^2 + 4^2 + \underline{12}^2 = 13^2$
$\underline{4}^2 + 5^2 + \underline{20}^2 = 21^2$
$5^2 + \underline{6}^2 + 30^2 = \underline{31}^2$
$6^2 + 7^2 + \underline{42}^2 = \underline{43}^2$

(ii)
$1^2 = 1 = 1$
$2^2 = 4 = 1 + 2 + 1$
$3^2 = 9 = 1 + 2 + 3 + 2 + 1$
$\underline{4}^2 = \underline{16} = 1 + 2 + 3 + 4 + 3 + 2 + 1$
$\underline{5}^2 = \underline{25} = 1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1$

(iii)
$21^2 = 441$
$201^2 = 40401$
$2001^2 = 4004001$
$\underline{(20001)^2} = 400040001$
$(2000001)^2 = 4000004000001$
$(20000001)^2 = \underline{400000040000001}$

12. 1234567654321'

13. (i) 2025 (ii) 5625 (iii) 9025 (iv) 15625 (v) 42025

14. a, b, c

15. (i) 8, 15, 17 (ii) 12, 35, 37 (iii) 16, 63, 65 (iv) 18, 80, 82
(v) 20, 99, 101

16. (i) b (ii) d (iii) b (iv) d (v) d
(vi) a (vii) c (viii) d (ix) b (x) a

Exercise 5.2

1. (i) 1 or 9 (ii) 3 or 7 (iii) 4 or 6 (iv) 4 or 6
(v) 2 or 8 (vi) 1 or 9 (vii) 0 (viii) 5

2. 512, 1320, 3617

3. (i) 8 (ii) 7 (iii) 11 (iv) 10

4. (i) 60 (ii) 26 (iii) 96 (iv) 54
(v) 80 (vi) 42 (vii) 110 (viii) 32

5. (i) 3, 27 (ii) 15, 60 (iii) 22, 242 (iv) 3, 54
(v) 7, 147 (vi) 2, 100

6. (i) 3, 6 (ii) 5, 25 (iii) 6, 20 (iv) 7, 27
(v) 5, 21 (vi) 2, 78

7. (i) 7056 (ii) 32400

8. 16 rows 9. 18cm 10. 56

11. (i) b (ii) d (iii) b (iv) b (v) a (vi) d

Exercise 5.3

1. (i) 3 (ii) 2 (iii) 3 (iv) 3 (v) 4

2. (i) 96 (ii) 90 (iii) 224 (iv) 69 (v) 649
(vi) 130 (vii) 72 (viii) 294 (ix) 4096 (x) 216

3. (i) 36, 24 (ii) 84, 43 (iii) 104, 58 (iv) 96, 64 (v) 129, 73
(vi) 51, 29

4. (i) 20, 26 (ii) 51, 33 (iii) 92, 77 (iv) 3, 101 (v) 416, 355
(vi) 1732, 887

5. 99856, 316 6. 1024, 32

7. (i) 56cm (ii) 85m (iii) 110m (iv) 135m

8. 10cm 9. 56 plants

10. (i) b (ii) b (iii) a (iv) b (v) d (vi) b

Exercise 5.4

1. (i) 3.1 (ii) 3.4 (iii) 21.6 (iv) 1.21
(v) 36.8 (vi) 1.104

2. (i) $\frac{8}{13}$ (ii) $\frac{12}{21}$ (iii) $\frac{9}{28}$ (iv) $\frac{14}{25}$

3. (i) 1.414, 1.732, 2.236

4. (i) a (ii) a (iii) a (iv) c