Class X Session 2023-24
Subject - Mathematics (Basic)
Sample Question Paper - 3

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

1. If a and b are two positive integers such that the least prime factor of a is 3 and the least prime factor of b is 5. Then, the least prime factor of (a + b) is **[1]**

a) 5 b) 3

c) 2 d) 8

2. $3 + 2\sqrt{5}$ is a/an: **[1]**

a) natural Number b) integer

c) irrational number d) rational number

3. The roots of the equation $ax^2 + bx + c = 0$ will be reciprocal of each other if **[1]**

a) None of these b) a = b

c) b = c d) c = a

4. The value of k for which the system of linear equations x + 2y = 3, 5x + ky + 7 = 0 is inconsistent is: **[1]**

a) $-\frac{14}{3}$ b) 5

c) $\frac{2}{5}$ d) 10

5. If x = 1 is a common root of $ax^2 + ax + 2 = 0$ and $x^2 + x + b = 0$ then, ab **[1]**

a) 2 b) 1

c) 3 d) 4

6. The ordinate of the point on the y-axis, which is equidistant from (-4, 1) and (2, 3) is: **[1]**

a) 1 b) 2

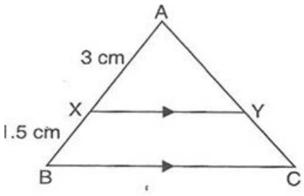
c) -1 d) -2

7. $\triangle$ABC $\sim$ $\triangle$DEF and their perimeters are 32 cm and 24 cm respectively. If AB = 10 cm, then DE equals: **[1]**

a) 8 cm b) 7.5 cm

c) $5\sqrt{3}$ cm d) 15 cm

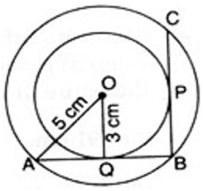
8. In the given figure $XY||BC$. If AX = 3cm, XB = 1.5 cm and BC = 6cm, then XY is equal to **[1]**



a) 6 cm. b) 4.5 cm

c) 3 cm. d) 4 cm.

9. Two concentric circles of radii 3 cm and 5 cm are given. The length of chord BC which touches the inner circle at P is equal to **[1]**



a) 4 cm. b) 8 cm

c) 6 cm. d) 10 cm.

10. If in a $\Delta ABC, \angle C = 90^\circ$ and $\angle B = 45^\circ$, then state which of the following is true? **[1]**

a) Perpendicular = Hypotenuse b) Base = Hypotenuse

c) Base = Hypotenuse + Perpendicular d) Base = Perpendicular

11. A kite is flying at a height of 30 m from the ground. The length of string from the kite to the ground is 60 m. Assuming that there is no slack in the string, the angle of elevation of the kite at the ground is **[1]**

a) 30° b) 45°

c) 90° d) 60°

12. The value of sin 45° + cos 45° is **[1]**

a) $\sqrt{2}$ b) $\frac{1}{\sqrt{2}}$

c) 1 d) $\frac{1}{\sqrt{3}}$

13. If the area of a sector of a circle is $\frac{5}{18}$ of the area of the circle, then the sector angle is equal to **[1]**

a) 100° b) 120°

c) 90° d) 60°

14. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. The area of the sector formed by the arc is: **[1]**

a) $231\ cm^2$

b) $250\ cm^2$

c) $220\ cm^2$

d) $200\ cm^2$

15. Cards marked with numbers 1, 2, 3, ..., 25 are placed in a box and mixed thoroughly and one card is drawn at random from the box. The probability that the number on the card is a multiple of 3 or 5 is **[1]**

a) $\frac{8}{25}$

b) $\frac{12}{25}$

c) $\frac{4}{25}$

d) $\frac{1}{5}$

16. For a symmetrical frequency distribution, we have **[1]**

a) mean < mode < median

b) mode = $\frac{1}{2}$(mean + median)

c) mean = mode = median

d) mean > mode > median

17. The maximum volume of a cone that can be carved out of a solid hemisphere of radius 'r' is **[1]**

a) πr^3

b) $\frac{2}{3}\pi r^3$

c) $\frac{1}{3}\pi r^3$

d) $\frac{1}{3}\pi r^2 h$

18. If $u_i = \frac{x_i - 25}{10}$, $\Sigma f_i u_i = 20$, $\Sigma f_i = 100$, then $\overline{x}$ = **[1]**

a) 24

b) 25

c) 23

d) 27

19. **Assertion (A):** Distance of point (a, b) from origin is $\sqrt{b^2 - a^2}$ **[1]**

Reason (R): Distance of point (x, y) from origin is $\sqrt{(x-0)^2 + (y-0)^2}$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The HCF of two numbers is 18 and their product is 3072. Then their LCM = 169. **[1]**

Reason (R): If a, b are two positive integers, then HCF × LCM = a × b.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

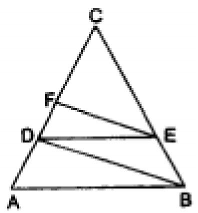
c) A is true but R is false.

d) A is false but R is true.

Section B

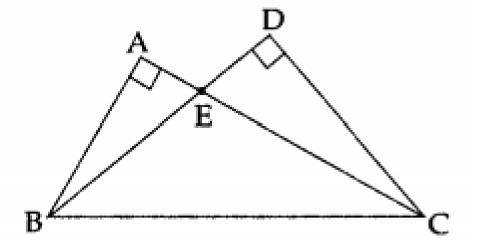
21. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the pair of linear equations intersect at a point, are parallel or coincident: 5x – 4y – 8 = 0; 7x + 6y – 9 = 0. **[2]**

22. In the given figure, $AB \| DE$ and $BD \| EF$ Prove that $\mathrm{DC}^2 = \mathrm{CF} \times \mathrm{AC}$. **[2]**



OR

In the adjoining figure, two triangles ABC and DBC are on the same base BC in which $\angle A = \angle D = 90^\circ$. If CA and BD meet each other at E, show that AE × CE = BE × DE.



23. Prove that the tangents drawn at the ends of a diameter of a circle are parallel. **[2]**

24. If tan A = 1 and sin B = $\frac{1}{\sqrt{2}}$, find the value of cos(A+B) where A and B are both acute angles. **[2]**

25. Find the area of the segment of a circle of radius 14 cm, if the length of the corresponding arc APB is 22 cm. **[2]**

OR

A chord 10 cm long is drawn in a circle whose radius is $5\sqrt{2}$ cm. Find the areas of both the segments. [Take π = 3.14.]

Section C

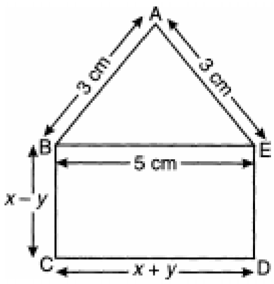
26. Show that $5-\sqrt{3}$ is irrational. **[3]**

27. Find the zeros of f(x) = x^2 - 2x - 8 and verify the relationship between the zeros and its coefficients. **[3]**

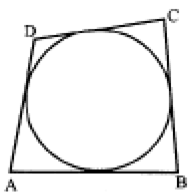
28. The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the number. Find the number. Solve the pair of the linear equation obtained by the elimination method. **[3]**

OR

In the figure below ABCDE is a pentagon with BE || CD and BC || DE. BC is perpendicular to CD. If the perimeter of ABCDE is 21 cm, find the Values of x and y.



29. In the adjoining figure, a quadrilateral ABCD is drawn to circumscribe a circle, Prove that AB + CD = AD + BC **[3]**



30. Prove that $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=\frac{1}{\sec\theta-\tan\theta}$, using identity $sec^2\theta=1+tan^2\theta$. **[3]**

OR

If sin θ+ cos θ = $\sqrt{2}$, then evaluate tan θ + cot θ.

31. What is the probability that a randomly taken leap year has 52 Sundays? **[3]**

Section D

32. The length of the sides forming right angle of a right triangle are 5x cm and (3x -1)cm. If the area of the triangle is 60 cm^2. Find its hypotenuse. **[5]**

OR

A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ₹

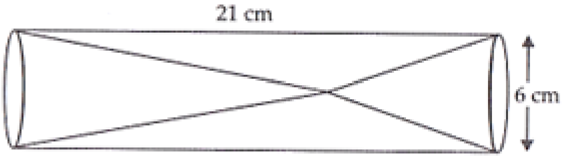
750. We would like to find out the number of toys produced on that day. Represent the situations mathematically (quadratic equation).

33. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$. **[5]**

34. A toy is in the form of a cone mounted on a hemisphere. The diameter of the base of the cone is 7 cm and its height is 15.5 cm. Find the volume of the toy. (Use π = 3.14). **[5]**

OR

Two solid cones A and B placed in a cylindrical tube as shown in the figure. The ratio of their capacities are 2 : 1. Find the heights and capacities of cones. Also, find the volume of the remaining portion of the cylinder.



35. If the median of the distribution given below is 28.5, then find the values of x and y. **[5]**

Class Interval	**frequency**
0-10	5
10-20	x
20-30	20
30-40	15
40-50	y
50-60	5
Total	60

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Suman is celebrating his birthday. He invited his friends. He bought a packet of toffees/candies which contains 360 candies. He arranges the candies such that in the first row there are 3 candies, in second there are 5 candies, in third there are 7 candies and so on.

(i) Find the total number of rows of candies.

(ii) How many candies are placed in last row?

OR

Find the number of candies in 12th row.

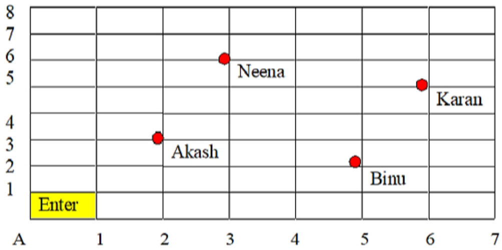
(iii) If Aditya decides to make 15 rows, then how many total candies will be placed by him with the same arrangement?

37. **Read the text carefully and answer the questions:** **[4]**

Karan went to the Lab near to his home for COVID 19 test along with his family members.

The seats in the waiting area were as per the norms of distancing during this pandemic (as shown in the figure).

His family member took their seats surrounded by red circular area.



(i) What is the distance between Neena and Karan?

(ii) What are the coordinates of seat of Akash?

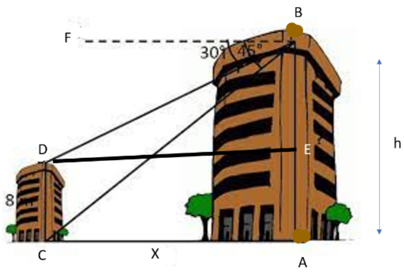
OR

Find distance between Binu and Karan.

(iii) What will be the coordinates of a point exactly between Akash and Binu where a person can be?

38. **Read the text carefully and answer the questions:** **[4]**

Basant and Vinod lives in a housing society in Dwarka, New Delhi. There are two building in their housing society. The first building is 8 meter tall. One day, both of them were just trying to guess the height of the other multi-storeyed building. Vinod said that it might be a 45 degree angle from the bottom of our building to the top of multi-storeyed building so the height of the building and distance from our building to this multi-storeyed building will be same. Then, both of them decided to estimate it using some trigonometric tools. Let's assume that the first angles of depression of the top and bottom of an 8 m tall building from top of a multi-storeyed building are 30^o and 45^o, respectively.



(i) Now help Vinod and Basant to find the height of the multistoried building.

(ii) Also, find the distance between two buildings.

OR

Find the distance between top of multistoried building and bottom of first building.

(iii) Find the distance between top of multistoried building and top of first building.

Solution

Section A

1.

(c) 2

Explanation: Since 5 + 3 = 8, the least prime factor of a + b has to be 2, unless a + b is a prime number greater than 2.

If a + b is a prime number greater than 2, then a + b must be an odd number. So, either a or b must be an even number. If a is even, then the least prime factor of a is 2, which is not 3 or 5. So, neither a nor b can be an even number. Hence, a + b cannot be a prime number greater than 2 if the least prime factor of a is 3 or 5.

2.

(c) irrational number

Explanation: Here, 3 is rational and $2\sqrt{5}$ is irrational.

We know that the sum of a rational and an irrational is an irrational number, therefore, $3+2\sqrt{5}$ is irrational.

3.

(d) c = a

Explanation: Product of roots = $\frac{c}{a}$. Also $\left(\alpha \times \frac{1}{\alpha}\right)=1$.

$\therefore \frac{c}{a}=1 \Rightarrow c=a$.

4.

(d) 10

Explanation: For a system of equations $a_1x + b_1y + c_1 = 0$; $a_2x + b_2y + c_2 = 0$ to have no solution, the condition to be satisfied is

$\frac{a_1}{a_2}=\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{1}{5}=\frac{2}{k} \neq \frac{-3}{7}$

$\therefore$ For k = 10, the given system of equation is inconsistent.

5. **(a)** 2

Explanation: Here, $ax^2 + ax + 2 = 0$….. (1)

$x^2 + x + b = 0$….. (2)

Putting the value of x = 1 in equation (2) we get

$1^2 + 1 + b = 0$

2 + b = 0

b = −2

Now, putting the value of x = 1 in equation (1) we get

a + a + 2 = 0

2 a + 2 = 0

$a=\frac{-2}{2}$

= - 1

Then,

ab = (−1) $\times$ (−2) = 2

6.

(c) -1

Explanation: A(2, 3) and B(-4, 1) are the given points.

Let C(0.y) be the points are y-axis

AC = $\sqrt{(0-2)^2+(y-3)^2}$

$\Rightarrow$ AC = $\sqrt{4+y^2+9-6y}$

$\Rightarrow$ AC = $\sqrt{y^2-6y+13}$

BC = $\sqrt{(0+4)^2+(y-1)^2}$

$\Rightarrow$ BC = $\sqrt{16+y^2+1-2y}$

$\Rightarrow$ BC = $\sqrt{y^2-2y+17}$

Since AC = BC

$AC^2 = BC^2$

$y^2 - 6y + 13 = y^2 - 2y + 17$

$\Rightarrow$ -6y + 2y = 17 - 13

$\Rightarrow$ -4y = 4

$\Rightarrow$ y = -1

Therefore, the point on y-axis is (0, -1) and here ordinate is -1.

7.

(b) 7.5 cm

Explanation: $\because \triangle ABC \sim \triangle DEF$

$\therefore \frac{\text{Perimeter }(\triangle\text{ABC})}{\text{Perimeter }(\triangle\text{DEF})} = \frac{\text{AB}}{\text{DE}}$

$\Rightarrow \frac{32}{24} = \frac{10}{DE}$

$\Rightarrow DE = \frac{10\times24}{32}$ =7.5 cm

8.

(d) 4 cm.

Explanation: Since $XY||BC$, then using Thales theorem,

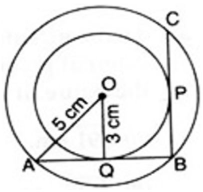
$\Rightarrow \frac{\text{AX}}{\text{AB}} = \frac{\text{XY}}{\text{BC}}$

$\Rightarrow \frac{3}{4.5} = \frac{\text{XY}}{6}$

$\Rightarrow$XY = 4 cm

9.

(b) 8 cm

Explanation: 

Construction: Joined OP.

In right angled triangle AOQ,

AQ = $\sqrt{(5)^2-(3)^2} = \sqrt{25-9} = \sqrt{16}$ = 4 cm

Since perpendicular from centre bisect opposite sides.

$\therefore$AQ = QB = 4 cm

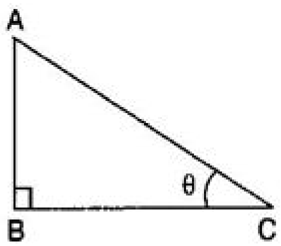
Also QB = PB = 4 cm [Tangents to a circle]

And PB = PC = 4 cm [OP $\perp$ BC]

$\therefore$ BC = PB + PC = 4 + 4 = 8 cm

10.

(d) Base = Perpendicular

Explanation: 

Given: in triangle ABC, $\angle C = 45°$, and $\angle B = 90°$,

Since, $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$

$\Rightarrow \tan 45° = \frac{\text{Perpendicular}}{\text{Base}}$

$\Rightarrow 1 = \frac{\text{Perpendicular}}{\text{Base}}$

$\Rightarrow$ Base = perpendicular

11. **(a)** 30°

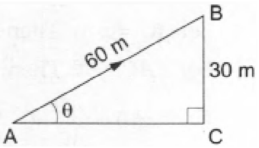
Explanation: Let AB be the tower and B be the kite.

Let AC be the horizontal and let BC $\perp$ AC.

Let $\angle$CAB = θ.

BC = 30 m and AB = 60 m. Then,

$\frac{BC}{AB}$ = sin $\theta \Rightarrow$ sin θ = $\frac{30}{60} = \frac{1}{2} \Rightarrow$ sin θ = sin $30^o \Rightarrow \theta = 30^o$.



12. **(a)** $\sqrt{2}$

Explanation: Given: $\sin 45^\circ + \cos 45^\circ$

= $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

= $\frac{2}{\sqrt{2}} = \sqrt{2}$

13. **(a)** 100°

Explanation: We have given that area of the sector is $\frac{5}{18}$ of the area of the circle.

Therefore, area of the sector $= \frac{5}{18} \times$ area of the circle

$\Rightarrow \frac{\theta}{360} \times \pi r^2 = \frac{5}{18} \times \pi r^2$

Now we will simplify the equation as below,

$\Rightarrow \frac{\theta}{360} = \frac{5}{18}$

$\therefore \theta = \frac{5}{18} \times 360$

$\therefore \theta = 100$

Therefore, sector angle is 100°.

14. **(a)** 231 cm^2

Explanation: The angle subtended by the arc = 60^o

So, area of the sector = $(\frac{60^o}{360^o}) \times \pi r^2$ cm^2

= $(\frac{441}{6}) \times (\frac{22}{7})cm^2$

= 231 cm^2

15.

(b) $\frac{12}{25}$

Explanation: Number of multiples of 3 = 8 (3 6 9 12 15 18 21 24)

Number of multiples of 5 = 5 (5 10 15 20 25)

Number of possible outcomes (multiples of 3 or 5) = 12 (3,5,6,9,10,12,15,18,20,21,24,25)

Number of Total outcomes = 25

$\therefore$ Required Probability = $\frac{12}{25}$

16.

(c) mean = mode = median

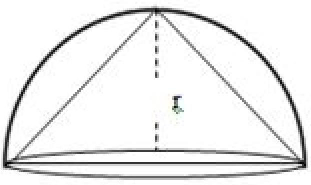
Explanation: For a symmetrical distribution,

we have Mean = mode = median

17.

(c) $\frac{1}{3}\pi r^3$

Explanation:



Volume of cone = $\frac{1}{3}\pi r^2 h$

Here height of the carved out cone = Radius of the hemisphere

$\therefore$ Volume of cone = $\frac{1}{3}\pi r^2 \times r = \frac{1}{3}\pi r^3$

18.

(d) 27

Explanation: Given that, $u_i = \frac{x_i - 25}{10}$, $\Sigma f_i u_i = 20$, $\Sigma f_i = 100$

Here assumed mean = 25 and class interval (h) = 10

$\therefore \bar{x} = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h = 25 + \frac{20}{100} \times 10$

= 25 + 2 = 27

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: It will be $\sqrt{(a-0)^2 + (b-0)^2} = \sqrt{a^2 + b^2}$

20.

(d) A is false but R is true.

Explanation: We know that for any two numbers, Product of the two numbers = HCF $\times$ LCM

HCF $\times$ LCM = 18 $\times$ 169 = 3042 $\neq$ 3072

So, A is false but R is true.

Section B

21. 5 x - 4 y - 8 = 0

7 x + 6 y - 9 = 0

Here, $a_1 = 5$, $b_1 = -4$, $c_1 = 8$

$a_2 = 7$, $b_2 = 6$, $c_2 = 9$

We see that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the lines representing the given pair of linear equations intersect at the point and the equations are consistent having unique solution.

22. In$\triangle ABC$, $AB \| DE$.

$\therefore \quad \frac{CD}{DA} = \frac{CE}{EB}$ ...(i) [by Thales' theorem]

In $\triangle CDB$, $BD \| EF$

$\therefore \quad \frac{CF}{FD} = \frac{CE}{EB}$ ...(ii) [by Thales' theorem]

From (i) and (ii) we get

$\frac{CD}{DA} = \frac{CF}{FD}$

$\Rightarrow \quad \frac{DA}{DC} = \frac{FD}{CF}$ [taking reciprocals]

$\Rightarrow \quad \frac{DA}{DC} + 1 = \frac{FD}{CF} + 1$

$\Rightarrow \frac{DA+DC}{DC} = \frac{FD+CF}{CF}$

$\Rightarrow \quad \frac{AC}{DC} = \frac{DC}{CF}$

$\Rightarrow \quad DC^2 = CF \times AC$

OR

In triangles ABC and DBC

$\angle$A = $\angle$D = 90°

AC and BD intersect each other at E

AE $\times$ EC = ED $\times$BE

In triangles AEB and EDC,

$\angle$AEB = $\angle$DEC ...(Vertically opposite angles)

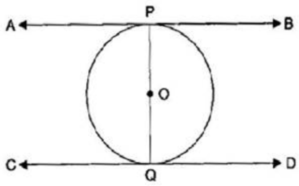
triangle ABE $\sim$ EDC

(EB/EC) = (AE/DE)

EB $\times$ DE = EC $\times$ AE

Hence, AE $\times$ EC = ED $\times$ BE

23.



Given: PQ is a diameter of a circle with centre O.

The lines AB and CD are the tangents at P and Q respectively.

To Prove: AB $\|$ CD

Proof: Since AB is a tangent to the circle at P and OP is the radius through the point of contact.

$\therefore \angle OPA = 90^o$ .... (i)

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\because$ CD is a tangent to the circle at Q and OQ is the radius through the point of contact.

$\therefore \angle OQD = 90^o$ .... (ii)

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

From eq. (i) and (ii), $\angle OPA = \angle OQD$

But these form a pair of equal alternate angles also,

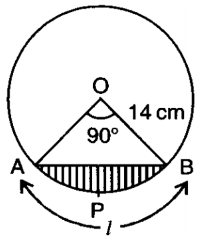
$\therefore AB \parallel CD$

24. Given tan A = 1 and sin B = $\frac{1}{\sqrt{2}}$

$\Rightarrow$ tan A = tan 45^o and sin B = sin 45°

$\Rightarrow$ A = 45° and B = 45^o

Now cos(A+ B) = cos($45^o + 45^o$) = cos 90^o = 0.

25. 

l = APB = 22 cm

$\frac{\theta}{180^\circ} \times \frac{22}{7} \times 14 = 22\text{cm}$

$\Rightarrow \quad \theta = 90^\circ$

Area of the sector = $\frac{lr}{2} = \frac{22\times14}{2}$ = 154 cm^2

Area of triangle AOB= $\frac{1}{2} \times \text{OA} \times \text{OB} = \frac{1}{2} \times 14 \times 14$ = 98 cm^2

Area of the segment = (154 - 98) cm^2 = 56 cm^2

OR

Given Radius = r = $5\sqrt{2}$cm

= OA = OB

Length of chord AB = 10 cm

In $\triangle OAB$, OA = OB = $= 5\sqrt{2}$

AB = 10cm

$OA^2 + OB^2 = \ = (5\sqrt{2})^2 + (5\sqrt{2})^2$

= 50 + 50 = 100 = $(AB)^2$

Pythagoras theorem is satisfied OAB is right triangle

= angle subtended by chord = $\angle AOB = 90^\circ$

Area of segment (minor) = shaded region

= area of sector - area of $\triangle OAB$

$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times OA \times OB$

$= \frac{90}{360} \times \frac{22}{7}(5\sqrt{2})^2 - \frac{1}{2} \times 5\sqrt{2} \times 5\sqrt{2}$

$= \frac{275}{7} - 25 = \frac{100}{7}\text{cm}^2$

Area of major segment = (area of circle) - (area of minor segment)

$= \pi r^2 - \frac{100}{7}$

$= \frac{22}{7} \times (5\sqrt{2})^2 - \frac{100}{7}$

$= \frac{1100}{7} - \frac{100}{7}$

$= \frac{1000}{7}\text{cm}^2$

Section C

26. Let us assume, to the contrary, that $5-\sqrt{3}$ is rational.

That is, we can find coprime numbers a and b (b ≠ 0) such that $5-\sqrt{3}=\frac{a}{b}$

Therefore, $5-\frac{a}{b}=\sqrt{3}$

Rearranging this equation, we get $\sqrt{3}=5-\frac{a}{b}=\frac{5b-a}{b}$

Since a and b are integers, we get $5-\frac{a}{b}$ is rational, and so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational

This contradiction has arisen because of our incorrect assumption that $5-\sqrt{3}$ is rational.

So, we conclude that $5-\sqrt{3}$ is irrational.

27. $f(x) = x^2 - 2x - 8$

$= x^2 - 4x + 2x - 8$

$= x(x - 4) + 2(x - 4)$

$= (x + 2)(x - 4)$

f(x) = 0 if x+2 = 0 or x-4 = 0

x = -2 or 4

So the zeroes of the polynomials are -2 and 4.

For the Polynomial $f(x)=x^2 - 2x - 8$

a=1,b=-2, c=-8

Sum of the zeroes = - 2 + 4 = 2 = $-\frac{b}{a}$

Product of zeros =(-2)(4) = -8 = $\frac{c}{a}$

Hence, the relationship between the zeros and coefficients is verified.

28. Let the unit's digit and the ten's digit in the two-digit number be x and y respectively.

Then the number = 10y + x

Also, the number obtained by reversing the order of the digits = 10x + y

According to the question,

x + y = 9...............(1)

9(10y + x) = 2(10x + y)

⇒ 90y + 9x = 20x + 2y

⇒ 11x - 88y = 0

⇒ x - 8y = 0 ....(2)

Subtracting equation(2) from equation(1), we get

9y = 9

$\Rightarrow \quad y=\frac{9}{9}=1$

Substituting this value of y in equation (1), we get

x + 1 = 9

$\Rightarrow x=9-1=8$

Hence, the required number is 18.

Verification: substituting x = 8 and y = 1,

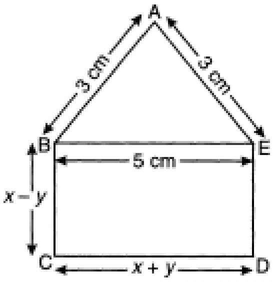
we find that both the equations (1) and (2) are satisfied as shown below:

x + y = 8 + 1 = 9

x - 8y = 8 - 8(1) = 0

Hence, the solution is correct.

OR



Since BC | | DE and BE | | CD with BC ⊥ CD, BCDE is a rectangle.

Since, BE = CD

∴ x + y = 5 ..(i)

Also, DE = BC = x - y

Since, perimeter of ABCDE is 21

∴AB + BC + CD+ DE + EA = 21

⇒3 + x - y + x + y + x - y + 3 = 21

⇒6 + 3x - y = 21

⇒3x - y = 15.....(ii)

Adding eqns. (i) and (ii), we get

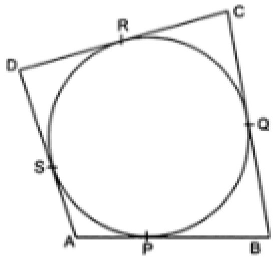
4x = 20

⇒x = 5

On substituting the value of x in (i), we get

y = 0

∴x = 5 and y = 0.

29. 

It is given that ABCD is the quadrilateral circumscribing the circle.

Let the quadrilateral touches the circle at points P, Q, R and S.

As we know that length of tangents drawn from an external point are always equal),

therefore, AP = AS ....(i)

BP = BQ ....(ii)

CR = CQ ....(iii)

DR = DS ....(iv)

Adding (i) + (ii) + (iii) + (iv), we obtain

AP + BP + CR + DR = AS + BQ + CQ + DS

(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

AB + CD = AD + BC

Hence proved

30. We have to prove that, $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=\frac{1}{\sec\theta-\tan\theta}$ using identity $sec^2\theta=1+tan^2\theta$

LHS = $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=\frac{\tan\theta-1+\sec\theta}{\tan\theta+1-\sec\theta}$ [dividing the numerator and denominator by $\cos\theta$.]

$=\frac{(\tan\theta+\sec\theta)-1}{(\tan\theta-\sec\theta)+1}=\frac{\{(\tan\theta+\sec\theta)-1\}(tan\theta-\sec\theta)}{\{(\tan\theta-\sec\theta)+1\}(\tan\theta-\sec\theta)}$ [Multiplying and dividing by $(\tan\theta-\sec\theta)$]

$=\frac{(\tan^2\theta-\sec^2\theta)-(tan\theta-\sec\theta)}{\{(\tan\theta-\sec\theta)+1\}(\tan\theta-\sec\theta)}$ [$\because(a-b)(a+b)=a^2-b^2$]

$=\frac{-1-\tan\theta+\sec\theta}{(\tan\theta-\sec\theta+1)(\tan\theta-\sec\theta)}$[$\because\tan^2\theta-\sec^2\theta=-1$]

$=\frac{-(\tan\theta-\sec\theta+1)}{(\tan\theta-\sec\theta+1)(\tan\theta-\sec\theta)}=\frac{-1}{\tan\theta-\sec\theta}$

$=\frac{1}{\sec\theta-\tan\theta}$ =**RHS**

Hence Proved.

OR

Given that, sin θ+ cos θ =$\sqrt{2}$

On squaring both the sides, we get

$(\sin\theta+\cos\theta)^2=(\sqrt{2})^2$

$\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta=2$ [$\because(a+b)^2=a^2+2ab+b^2$]

⇒ 1 + 2 sin θ cos θ = 2

⇒ 2 sin θ cos θ = 2 - 1 = 1

⇒ $\frac{1}{\sin\theta\cos\theta}=2$ ....(i)

Now, $\tan\theta+\cot\theta=\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$

$\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}=\frac{1}{\cos\theta\sin\theta}$ ....(ii)

From (i) and (ii) we get

$\tan\theta + \cot\theta = 2$

31. No. of days in a leap = 366

(i.e. $\frac{366}{7}$ = 52 weeks + 2 days)

So, there will be 52 weeks and 2 days

So, every leap year has 52 Sundays

Now, the probability depends on the remaining 2 days

The possible pairing of days are:

Sunday-Monday

Monday-Tuesday

Tuesday-Wednesday

Wednesday-Thursday

Thursday-Friday

Friday-Saturday

Saturday-Sunday

There are total of 7 pairs and out of 7 pairs, only 2 pairs have Sunday.

The remaining 5 pairs do not include Sunday.

hence, the probability of not getting Sunday in the last 2 days = $\frac{5}{7}$

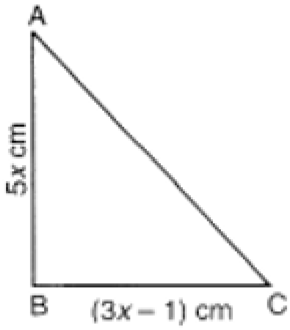
Therefore, the probability of only 52 Sundays in a Leap year = $\frac{5}{7}$

Section D

32. Area of triangle = $\frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 5x \times (3x - 1)$

According to the question,



$15x^2 - 5x = 120$

or, $3x^2 - x - 24 = 0$

or, $3x^2 - 9x + 8x - 24 = 0$

or, 3x (x - 3) + 8 (x - 3) = 0

or, (x - 3) (3x + 8) = 0

$\therefore \quad x = 3, x = -\frac{8}{3}$

Length can't be negative, so x = 3

AB = 5 × 3 = 15 cm, BC = 3x - 1 = 9 - 1

= 8 cm

$AC = \sqrt{15^2 + 8^2}$

$= \sqrt{225 + 64}$

$= \sqrt{289} = 17\text{cm}$

Hence hypotenuse = 17 cm

OR

Let the number of toys produced be *x*.

∴ Cost of production of each toy = Rs (55 − *x*)

It is given that, total production of the toys = Rs 750

∴ x(55 – x) = 750

⇒ $x^2 - 55x + 750 = 0$

Now to factorize this equation we have to find two numbers such that their product is 750 and sum is 55

⇒ $x^2 - 25x - 30x + 750 = 0$

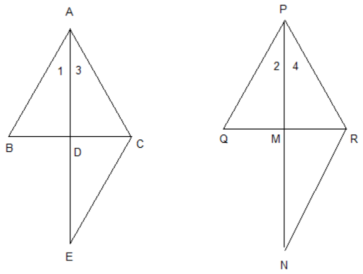
⇒ x(x – 25) – 30(x – 25) = 0

$\Rightarrow (x - 25)(x - 30) = 0$

Either $x - 25 = 0$ or $x - 30 = 0$

i.e., $x = 25$ or $x = 30$

Hence, the number of toys will be either 25 or 30.

33. 

Given : In ΔABC and ΔPQR The AD and PM are their medians,

such that $\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$

To prove : $\Delta ABC \sim \Delta PQR$

Construction : Produce AD to E such that AD = DE and produce PM to N such that PM = MN. Join CE and RN.

Proof : In ΔABD and ΔEDC

$AD = DE$

$\angle ADB = \angle EDC$ (vertically opposite angles)

$BD = DC$(as AD is a median)

$\therefore \quad \Delta ABD \equiv \Delta EDC$ (By SAS congruency)

or, $AB = CE$ (By CPCT)

Similarly, PQ = RN

$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$ (Given)

or, $\frac{CE}{RN} = \frac{2AD}{2PM} = \frac{AC}{PR}$

or $\frac{CE}{RN} = \frac{AE}{PN} = \frac{AC}{PR}$

So $\Delta ACE \sim \Delta PRN$

$\angle 3 = \angle 4$

Similarly $\angle 1 = \angle 2$

$\angle 1 + \angle 3 = \angle 2 + \angle 4$

So $\angle A = \angle P$ and

$\frac{AB}{PQ} = \frac{AC}{PR}$(given)

Hence $\Delta ABC \sim \Delta PQR$

34. According to question it is given that

Diameter of the base of the cone is = 7cm

Therefore radius = $\frac{7}{2}$ = 3.5cm

Total height of the toy = 14.5 cm

Height of the cone = 15.5 - 3.5 = 12 cm

Height of the hemisphere = 3.5 cm

According to question it is also given that

Volume of the toy = Volume of cone + Volume of hemisphere

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^2$

$= \frac{1}{3}\pi r^2 (2r + h)$

$= \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 [2 \times 3.5 + 12]$

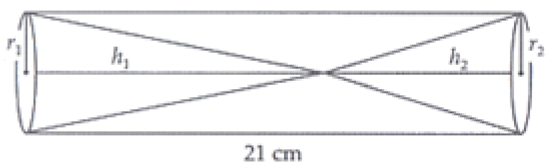
$= \frac{1}{3} \times 22 \times 1.75 \times 19$

= 243.83 cm^3

OR

Let height of the cone 1 be 'h' cm and the height of the cone 2 be (21 cm - h) .

As the ratio of volumes of cone c_1 and c_2 is 2 : 1, their radii are same equal to r = $\frac{6}{2}$cm = 3 cm.



$\therefore \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$

$\Rightarrow \frac{2}{1} = \frac{h}{21cm - h}$

or 42 cm - 2h = h

or, 3h = 42 cm

$\Rightarrow$ h = 42/3

$\Rightarrow$ h = 14 cm

Hence, height of cone 1 = 14 cm and height of cone 2 = 7 cm

Cone I	**Cone II**	**Cylinder**
$r_1 = \frac{6}{3}$ = 3 cm	r_2 = 3 cm	r = 3 cm
h_1 = 14 cm	h_2 = 7 cm	h = 21 cm

Volume of cone 1$= \frac{1}{3}\pi r_1^2 h_1 = \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 14$ = 132 cm^3

Volume of cone 2$= \frac{1}{3}\pi r_2^2 h_2 = \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 7$ = 22 × 3 = 66 cm^3

Volume of remaining portion of tube = Vol. of cylinder – Vol. of cone 1 – Vol. of cone 2

= $\pi r^2 h$ – 132 – 66

= $\frac{22}{7} \times 3 \times 3 \times 21 - 198$

= 22 × 27 – 198 = 594 – 198 = 396 cm^3

Hence, the required volume is 396 cm^3.

35.

Monthly Consumption	**Number of consumers** (f_i)	**Cumulative Frequency**
0-10	5	5
10-20	x	5 + x
20-30	20	25 + x
30-40	15	40 + x
40-50	y	40 + x + y
50-60	5	45 + x + y
Total	$\sum f_i = n = 60$	

Here, $\sum f_i = n = 60$, then $\frac{n}{2} = \frac{60}{2} = 30$, also, median of the distribution is 28.5, which lies in interval 20 – 30.

$\therefore$ Median class = 20 – 30

So, l = 20, n = 60, f = 20, cf = 5 + x and h = 10

$\because 45 + x + y = 60$

$\Rightarrow x + y = 15$ ……..........(i)

Now, Median = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

$\Rightarrow 28.5 = 20 + \left[\frac{30-(5+x)}{20}\right] \times 10$

$\Rightarrow 28.5 = 20 + \frac{30-5-x}{2}$

$\Rightarrow 28.5 = \frac{40+25-x}{2}$

$\Rightarrow 57.0 = 65 - x$

$\Rightarrow x = 65 - 57 = 8$

$\Rightarrow$ x = 8

Putting the value of x in eq. (i), we get,

8 + y = 15

$\Rightarrow$ y = 7

Hence the value of x and y are 8 and 7 respectively.

Section E

36. **Read the text carefully and answer the questions:**

Suman is celebrating his birthday. He invited his friends. He bought a packet of toffees/candies which contains 360 candies. He arranges the candies such that in the first row there are 3 candies, in second there are 5 candies, in third there are 7 candies and so on.

(i) Let there be 'n' number of rows

Given 3, 5, 7... are in AP

First term a = 3 and common difference d = 2

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 360 = \frac{n}{2}[2 \times 3 + (n-1) \times 2]$

$\Rightarrow 360 = n[3 + (n - 1) \times 1]$

$\Rightarrow n^2 + 2n - 360 = 0$

$\Rightarrow (n + 20)(n - 18) = 0$

$\Rightarrow n = -20$ reject

n = 18 accept

(ii) Since there are 18 rows number of candies placed in last row (18^{th} row) is

$a_n = a + (n - 1)d$

$\Rightarrow a_{18} = 3 + (18 - 1)2$

$\Rightarrow a_{18} = 3 + 17 \times 2$

$\Rightarrow a_{18} = 37$

OR

The number of candies in 12th row.

$a_n = a + (n - 1)d$

$\Rightarrow a_{12} = 3 + (12 - 1)2$

$\Rightarrow a_{12} = 3 + 11 \times 2$

$\Rightarrow a_{12} = 25$

(iii) If there are 15 rows with same arrangement

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{15} = \frac{15}{2}[2 \times 3 + (15-1) \times 2]$

$\Rightarrow S_{15} = 15[3 + 14 \times 1]$

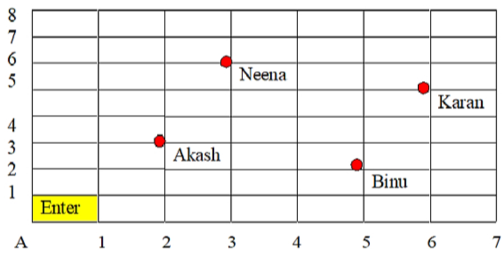
$\Rightarrow S_{15} = 255$

There are 255 candies in 15 rows.

37. **Read the text carefully and answer the questions:**

Karan went to the Lab near to his home for COVID 19 test along with his family members.

The seats in the waiting area were as per the norms of distancing during this pandemic (as shown in the figure). His family member took their seats surrounded by red circular area.



(i) Position of Neena = (3, 6)

Position of Karan = (6, 5)

Distance between Neena and Karan = $\sqrt{(6-3)^2 + (5-6)^2}$

= $\sqrt{9 + (-1)^2}$

= $\sqrt{10}$

(ii) Co-ordinate of seat of Akash = 2, 3

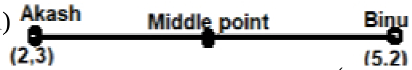
OR

Binu = (5, 5); Karan = (6, 5)

Distance = $\sqrt{(6-5)^2 + (5-2)^2}$

$= \sqrt{1 + 9}$

$= \sqrt{10}$

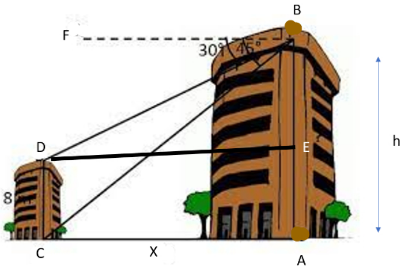
(iii) 

Co-ordinate of middle point = $\left(\frac{2+5}{2}, \frac{3+2}{2}\right)$

= 3.5, 2.5

38. **Read the text carefully and answer the questions:**

Basant and Vinod lives in a housing society in Dwarka, New Delhi. There are two building in their housing society. The first building is 8 meter tall. One day, both of them were just trying to guess the height of the other multi-storeyed building. Vinod said that it might be a 45 degree angle from the bottom of our building to the top of multi-storeyed building so the height of the building and distance from our building to this multi-storeyed building will be same. Then, both of them decided to estimate it using some trigonometric tools. Let's assume that the first angles of depression of the top and bottom of an 8 m tall building from top of a multi-storeyed building are 30^o and 45^o, respectively.



(i) Let h is height of big building, here as per the diagram.

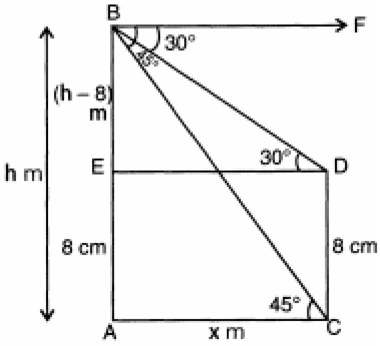
AE = CD = 8 m (Given)

BE = AB - AE = (h - 8) m

Let AC = DE = x

Also, $\angle FBD = \angle BDE = 30^\circ$

$\angle FBC = \angle BCA = 45^\circ$



In $\triangle$ACB, $\angle A = 90^\circ$

$\tan 45^\circ = \frac{AB}{AC}$

$\Rightarrow$ x = h, ...(i)

In $\triangle$BDE, $\angle E = 90^\circ$

$\tan 30^\circ = \frac{BE}{ED}$

$\Rightarrow x = \sqrt{3}(h - 8)$.(ii)

From (i) and (ii), we get

$h = \sqrt{3}h - 8\sqrt{3}$

h($\sqrt{3}$ - 1) = 8$\sqrt{3}$

h = $\frac{8\sqrt{3}}{\sqrt{3}-1} = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

= $\frac{1}{2} \times (24 + 8\sqrt{3}) = \frac{1}{2} \times (24 + 13.84)$ = 18.92 m

(ii) Let h is height of big building, here as per the diagram.

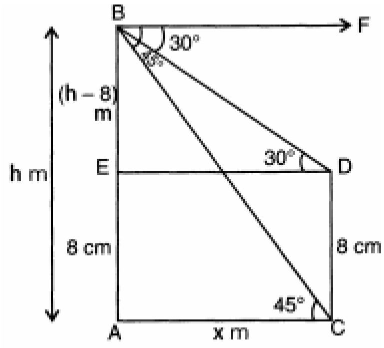
AE = CD = 8 m (Given)

BE = AB - AE = (h - 8) m

Let AC = DE = x

Also, $\angle FBD = \angle BDE = 30^\circ$

$\angle FBC = \angle BCA = 45^\circ$



In $\triangle$ACB, $\angle A = 90^\circ$

$\tan 45^\circ = \frac{AB}{AC}$

$\Rightarrow$ x = h, ...(i)

In $\triangle$BDE, $\angle E = 90^\circ$

$\tan 30^\circ = \frac{BE}{ED}$

$\Rightarrow x = \sqrt{3}(h-8)$.(ii)

From (i) and (ii), we get

$h = \sqrt{3}h - 8\sqrt{3}$

h($\sqrt{3}$ - 1) = 8$\sqrt{3}$

h = $\frac{8\sqrt{3}}{\sqrt{3}-1} = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

= $\frac{1}{2} \times (24 + 8\sqrt{3}) = \frac{1}{2} \times (24 + 13.84)$ = 18.92 m

Hence height of the multistory building is 18.92 m and the distance between two buildings is 18.92 m.

OR

In $\triangle$ABC

sin $45^\circ = \frac{AB}{BC}$

$\Rightarrow BC = \frac{AB}{\sin 45^0}$

$\Rightarrow BC = \frac{18.92}{\frac{1}{\sqrt{2}}}$

$\Rightarrow$ BC = 26.76 m

Hence the distance between top of multistoried building and bottom of first building is 26.76 m.

(iii) In $\triangle$BDE

cos 30^o = $\frac{ED}{BD}$

$\Rightarrow BD = \frac{ED}{\cos 30^0}$

$\Rightarrow BD = \frac{\frac{8\sqrt{3}}{\sqrt{3}-1}}{\frac{\sqrt{3}}{2}} = \frac{16}{\sqrt{3}-1}$

$\Rightarrow BD = 8\left(\sqrt{3}+1\right)$ = 21.86 m

Hence, the distance between top of multistoried building and top of first building is 21.86 m.