UNIT 3

LAWS OF MOTION

"In the beginning there was a mechanics" – Von Laue

Learning Objectives

In this unit, the student is exposed to

- Newton's laws
- logical connection between laws of Newton
- free body diagram and related problems
- law of conservation of momentum
- role of frictional forces
- centripetal and centrifugal forces
- origin of centrifugal force

3.1 INTRODUCTION

Each and every object in the universe interacts with every other object. The cool breeze interacts with the tree. The tree interacts with the Earth. In fact, all species interact with nature. But, what is the difference between a human's interaction with nature and that of an animal's. Human's interaction has one extra quality. We not only interact with nature but also try to understand and explain natural phenomena scientifically.

In the history of mankind, the most curiosity driven scientific question asked was about motion of objects–'How things move?' and 'Why things move?' Surprisingly, these simple questions have paved the way for development from early civilization to the modern technological era of the 21st century.

Objects move because something pushes or pulls them. For example, if a book is at rest, it will not move unless a force is applied on it. In other words, to move an object a force must be applied on it. About 2500 years ago, the famous philosopher, Aristotle, said that '*Force causes motion*'. This statement is based on common sense. But any scientific answer cannot be based on common sense. It must be endorsed with quantitative experimental proof.

In the 15th century, Galileo challenged Aristotle's idea by doing a series of experiments. He said force is not required to maintain motion.

Galileo demonstrated his own idea using the following simple experiment. When a ball rolls from the top of an inclined plane to its bottom, after reaching the ground it moves some distance and continues

UNIT-3(XI-Physics_Vol-1).indd 105 05-01-2022 22:17:21

to move on to another inclined plane of same angle of inclination as shown in the Figure 3.1(a). By increasing the smoothness of both the inclined planes, the ball reach almost the same height(h) from where it was released (L1) in the second plane (L2) (Figure 3.1(b)). The motion of the ball is then observed by varying the angle of inclination of the second plane keeping the same smoothness. If the angle of inclination is reduced, the ball travels longer distance in the second plane to reach the same height (Figure 3.1 (c)). When the angle of inclination is made zero, the ball moves forever in the horizontal direction (Figure 3.1(d)). If the Aristotelian idea were true, the ball would not have moved in the second plane even if its smoothness is made maximum since no

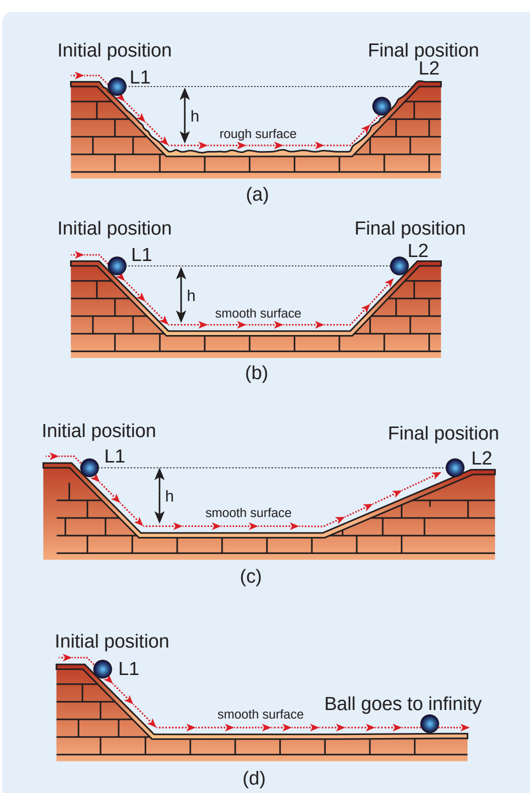

Figure 3.1 Galileo's experiment with the second plane (a) at same inclination angle as the first (b) with increased smoothness (c) with reduced angle of inclination (d) with zero angle of inclination

force acted on it in the horizontal direction. From this simple experiment, Galileo proved that force is not required to maintain motion. An object can be in motion even without a force acting on it.

In essence, Aristotle coupled the motion with force while Galileo decoupled the motion and force.

3.2 NEWTON'S LAWS

Newton analysed the views of Galileo, and other scientist like Kepler and Copernicus on motion and provided much deeper insights in the form of three laws.

3.2.1 Newton's First Law

Every object continues to be in the state of rest or of uniform motion (constant velocity) unless there is external force acting on it.

This inability of objects to move on its own or change its state of motion is called inertia. Inertia means resistance to change its state. Depending on the circumstances, there can be three types of inertia.

1. **Inertia of rest:** When a stationary bus starts to move, the passengers experience a sudden backward push. Due to inertia, the body (of a passenger) will try to

Figure 3.2 Passengers experience a backward push due to inertia of rest

continue in the state of rest, while the bus moves forward. This appears as a backward push as shown in Figure 3.2. *The inability of an object to change its state of rest is called inertia of rest.*

2. **Inertia of motion:** When the bus is in motion, and if the brake is applied suddenly, passengers move forward and hit against the front seat. In this case, the bus comes to a stop, while the body (of a passenger) continues to move forward due to the property of inertia as shown in Figure 3.3. *The inability of an object to change its state of uniform speed (constant speed) on its own is called inertia of motion.*
3. **Inertia of direction:** When a stone attached to a string is in whirling motion, and if the string is cut suddenly, the stone will not continue to move in circular motion but moves tangential to the circle as illustrated in Figure 3.4. This is because the body cannot change its direction of motion without any force acting on it. *The inability of an object to change its direction of motion on its own is called inertia of direction.*

Figure 3.3 Passengers experience a forward push due to inertia of motion

When we say that an object is at rest or in motion with constant velocity, it has a meaning only if it is specified with respect to some reference frames. In physics, any motion has to be stated with respect to a reference frame. It is to be noted that Newton's first law is valid only in certain special reference frames called inertial frames. In fact, Newton's first law defines an inertial frame.

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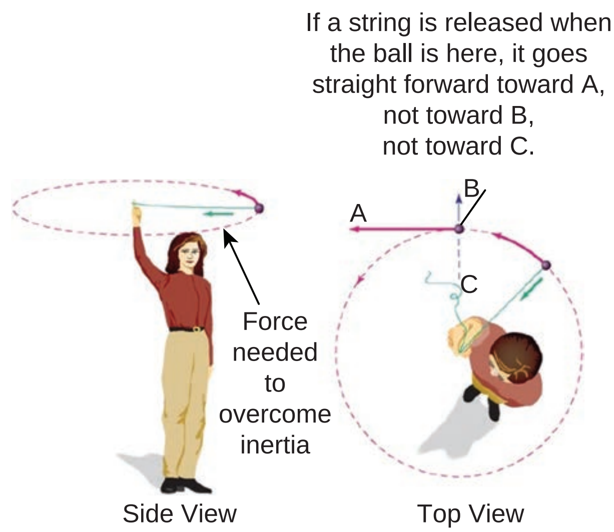


Figure 3.4 A stone moves tangential to circle due to inertia of direction

Inertial Frames

If an object is free from all forces, then it moves with constant velocity or remains at rest when seen from inertial frames. Thus, there exists some special set of frames in which, if an object experiences no force, it moves with constant velocity or remains at rest. But how do we know whether an object is experiencing a force or not? All the objects in the Earth experience Earth's gravitational force. In the ideal case, if an object is in deep space (very far away from any other object), then Newton's first law will be certainly valid. Such deep space can be treated as an inertial frame. But practically it is not possible to reach such deep space and verify Newton's first law.

For all practical purposes, we can treat Earth as an inertial frame because an object on the table in the laboratory appears to be at rest always. This object never picks up acceleration in the horizontal direction since no force acts on it in the horizontal direction. So the laboratory can be taken as an inertial frame for all physics experiments and calculations. For making these conclusions, we analyse only the horizontal motion of the object as there is no horizontal force that acts on it. We should not analyse the motion in vertical direction as the two forces (gravitational force in the downward direction and normal force in upward direction) that act on it makes the net force is zero in vertical direction. Newton's first law deals with the motion of objects in the absence of any force and not the motion under zero net force. Suppose a train is moving with constant velocity with respect to an inertial frame, then an object at rest in the inertial frame (outside the train) appears to move with constant velocity with respect to the train (viewed from within the train). So the train can be treated as an inertial frame. All inertial frames are moving with constant velocity relative to each other. If an object appears to be at rest in one inertial frame, it may appear to move with constant velocity with respect to another inertial frame. For example, in Figure 3.5, the car is moving with uniform velocity v with respect to a person standing (at rest) on the ground. As the car is moving with constant velocity with respect to the person at rest on the ground, both frames (with respect to the car and to the ground) are inertial frames.

Figure 3.5 The person and vehicle are inertial frames

Suppose an object remains at rest on a smooth table kept inside the train, and if the train suddenly accelerates (which we may not sense), the object appears to accelerate backwards even without any force acting on it. It is a clear violation of Newton's first law as the object gets accelerated without being acted upon by a force. It implies that the train is not an inertial frame when it is accelerated. For example, Figure 3.6 shows that car 2 is a non-inertial frame since it moves with acceleration $\vec{a}$ with respect to the ground.

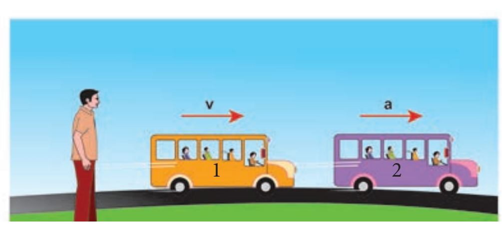


Figure 3.6 Car 2 is a non-inertial frame

These kinds of accelerated frames are called non-inertial frames. A rotating frame is also a non inertial frame since rotation requires acceleration. In this sense, Earth is not really an inertial frame since it has self-rotation and orbital motion. But these rotational effects of Earth can be ignored for the motion involved in our day-to-day life. For example, when an object is thrown, or the time period of a simple pendulum is measured in the physics laboratory, the Earth's self-rotation has very negligible effect on it. In this sense, Earth can be treated as an inertial frame. But at the same time, to analyse the motion of satellites and wind patterns around the Earth, we cannot treat Earth as an inertial frame since its self-rotation has a strong influence on wind patterns and satellite motion.

3.2.2 Newton's Second Law

This law states that

The force acting on an object is equal to the rate of change of its momentum

$$\vec{F} = \frac{d\vec{p}}{dt} \qquad (3.1)$$

In simple words, whenever the momentum of the body changes, there must be a force acting on it. The momentum of the object is defined as $\vec{p} = m\vec{v}$. In most cases, the mass of the object remains constant during the motion. In such cases, the above equation gets modified into a simpler form

$$\vec{F} = \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}.$$

$$\vec{F} = m\vec{a}. \qquad (3.2)$$

The above equation conveys the fact that if there is an acceleration $\vec{a}$ on the body, then there must be a force acting on it. This implies that if there is a change in velocity, then there must be a force acting on the body. The force and acceleration are always in the same direction. Newton's second law was a paradigm shift from Aristotle's idea of motion. According to Newton, the force need not cause the motion but only a change in motion. It is to be noted that *Newton's second law is valid only in inertial frames*. In non-inertial frames Newton's second law cannot be used in this form. It requires some modification.

In the SI system of units, the unit of force is measured in newtons and it is denoted by symbol 'N'.

One Newton is defined as the force which acts on 1 kg of mass to give an acceleration 1 m s^{-2} in the direction of the force.

Aristotle vs. Newton's approach on sliding object

Newton's second law gives the correct explanation for the experiment on the inclined plane that was discussed in section 3.1. In normal cases, where friction is not negligible, once the object reaches the bottom of the inclined plane (Figure 3.1), it travels some distance and stops. Note that it stops because there is a frictional force acting in the direction opposite to its velocity. It is this frictional force that reduces the velocity of the object to zero and brings it to rest. As per Aristotle's idea, as soon as the body reaches the bottom of the plane, it can travel only a small distance and stops because there is no force acting on the object. Essentially, he did not consider the frictional force acting on the object.

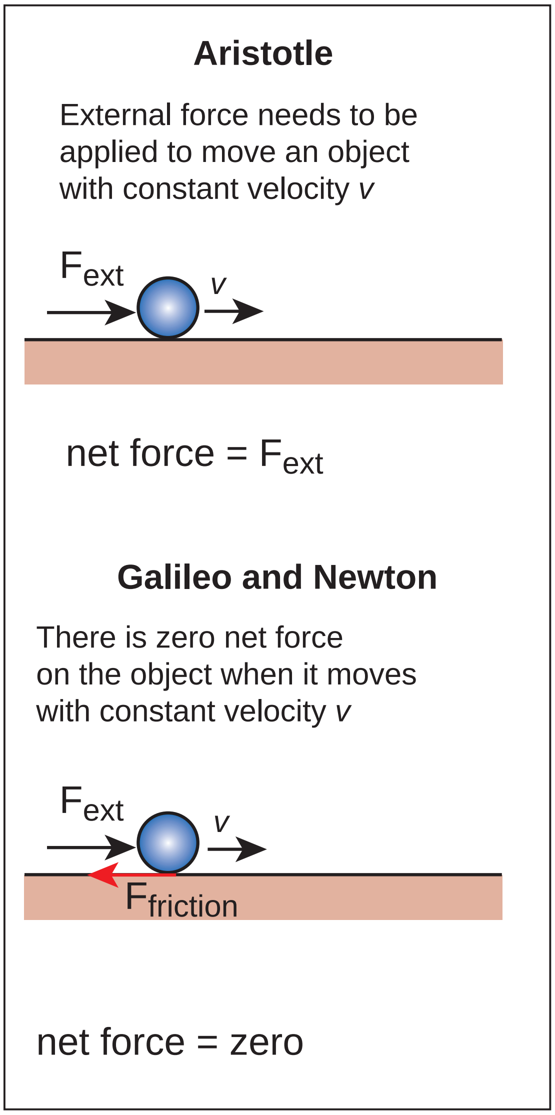


Figure 3.7 Aristotle, Galileo and Newton's approach

3.2.3 Newton's Third Law

Consider Figure 3.8(a) whenever an object 1 exerts a force on the object 2 ($\vec{F}_{21}$), then object 2 must also exert equal and opposite force on the object 1 ($\vec{F}_{12}$). These forces must lie along the line joining the two objects.

$$\vec{F}_{12} = -\vec{F}_{21}$$

Newton's third law assures that the forces occur as equal and opposite pairs. An isolated force or a single force cannot exist in nature. *Newton's third law states that for every action there is an equal and opposite reaction*. Here, action and reaction pair of forces do not act on the same body but on two different bodies. Any one of the forces can be called as an action force and the other the reaction force. Newton's third law is valid in both inertial and non-inertial frames.

These action-reaction forces are not cause and effect forces. It means that when the object 1 exerts force on the object 2, the object 2 exerts equal and opposite force on the body 1 at the same instant.

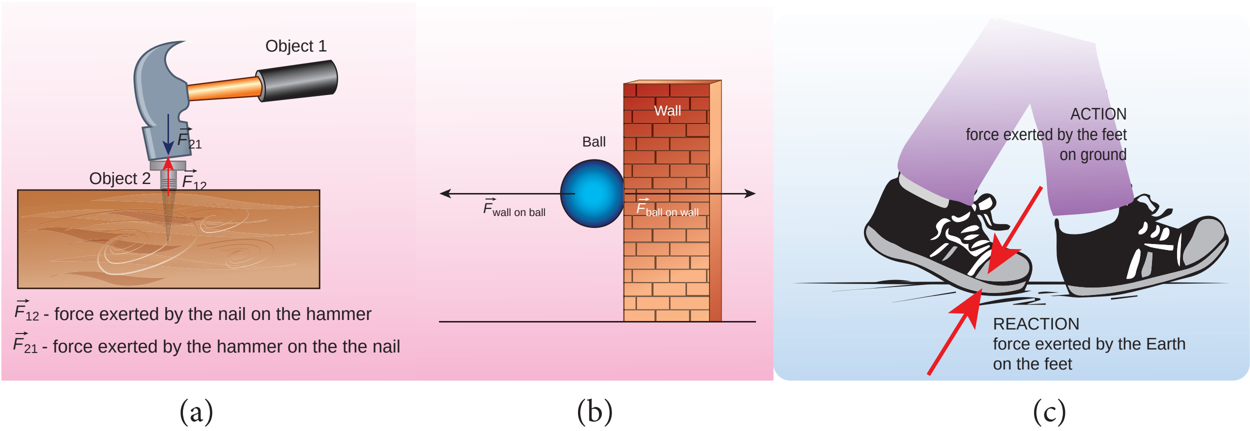


Figure 3.8 Demonstration of Newton's third law (a) Hammer and the nail (b) Ball bouncing off the wall (c) Walking on the floor with friction

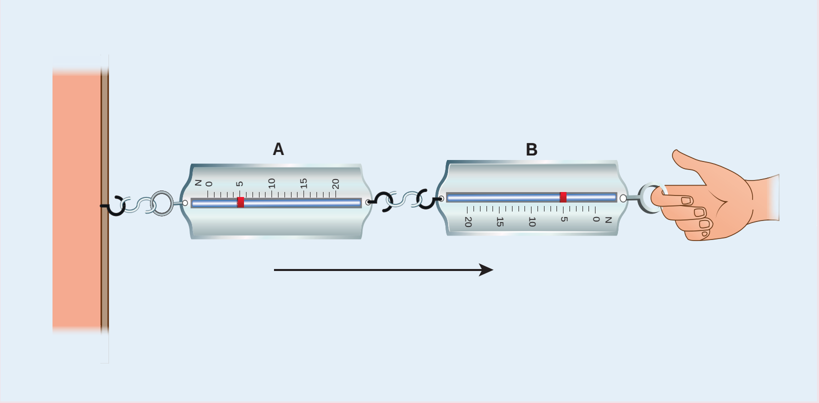
ACTIVITY

Verification of Newton's third law

Attach two spring balances as shown in the figure. Fix one end with rigid support and leave the other end free, which can be pulled with the hand.

Pull one end with some force and note the reading on both the balances.

Repeat the exercise a number of times.



Note

The reading in the spring balance A is due to the force given by spring balance B. The reading in the spring balance B is due to the reaction force given by spring balance A. Note that according to Newton's third law, both readings (force) are equal.

3.2.4 Discussion on Newton's Laws

1. Newton's laws are vector laws. The equation $\vec{F} = m\vec{a}$ is a vector equation and essentially it is equivalent to three scalar equations. In Cartesian coordinates, this equation can be written as $F_x\hat{i} + F_y\hat{j} + F_z\hat{k} = ma_x\hat{i} + ma_y\hat{j} + ma_z\hat{k}$.

   By comparing both sides, the three scalar equations are

   $F_x = ma_x$ The acceleration along the x direction depends only on the component of force acting along the x-direction.

   $F_y = ma_y$ The acceleration along the y direction depends only on the component of force acting along the y-direction.

   $F_z = ma_z$ The acceleration along the z direction depends only on the component of force acting along the z-direction.

   From the above equations, we can infer that the force acting along y direction cannot alter the acceleration along x direction. In the same way, F_z cannot affect a_y and a_x. This understanding is essential for solving problems.

2. The acceleration experienced by the body at time t depends on the force which acts on the body at that instant of time. It does not depend on the force which acted on the body before the time t. This can be expressed as

$$\vec{F}(t) = m\vec{a}(t)$$

Acceleration of the object does not depend on the previous history of the force. For example, when a spin bowler or a fast bowler throws the ball to the batsman, once the ball leaves the hand of the bowler, it experiences only gravitational force and air frictional force. The acceleration of the ball is independent of how the ball was bowled (with a lower or a higher speed).

3. In general, the direction of a force may be different from the direction of motion. Though in some cases, the object may move in the same direction as the direction of the force, it is not always true. A few examples are given below.

Case 1: Force and motion in the same direction

When an apple falls towards the Earth, the direction of motion (direction of velocity) of the apple and that of force are in the same downward direction as shown in the Figure 3.9 (a).

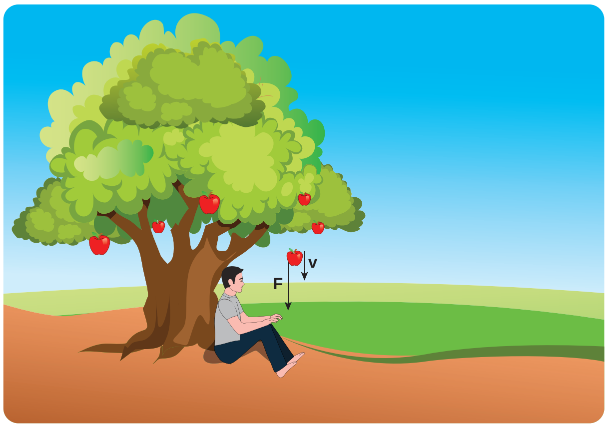


Figure 3.9 (a) Force and motion in the same direction

Case 2: Force and motion not in the same direction

The Moon experiences a force towards the Earth. But it actually moves in elliptical orbit. In this case, the direction of the force is different from the direction of motion as shown in Figure 3.9 (b).

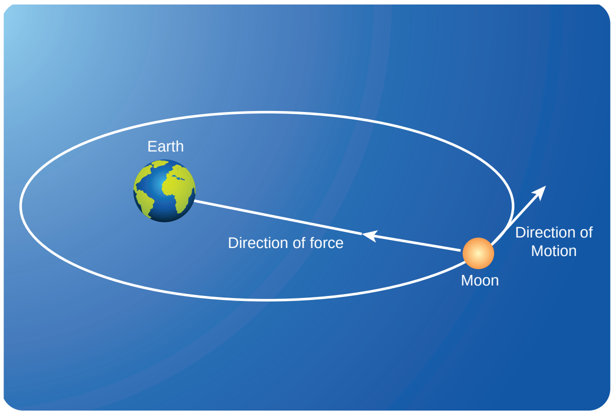


Figure 3.9 (b) Moon orbiting in elliptical orbit around the Earth

Case 3: Force and motion in opposite direction

If an object is thrown vertically upward, the direction of motion is upward, but gravitational force is downward as shown in the Figure 3.9 (c).

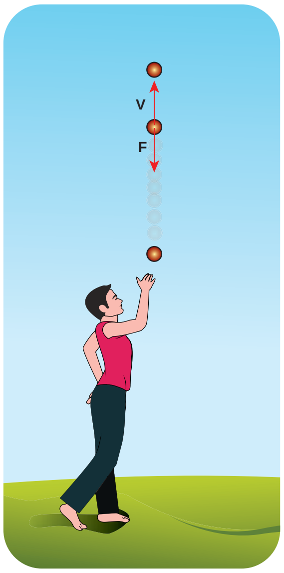


Figure 3.9 (c) Force and direction of motion are in opposite directions

Case 4: Zero net force, but there is motion

When a raindrop gets detached from the cloud it experiences both downward gravitational force and upward air drag force. As it descends towards the Earth, the upward air drag force increases and after a certain time, the upward air drag force cancels the downward gravity. From then on the raindrop moves at constant velocity till it touches the surface of the Earth. Hence the raindrop comes with zero net force, therefore with zero acceleration but with non-zero terminal velocity. It is shown in the Figure 3.9 (d).

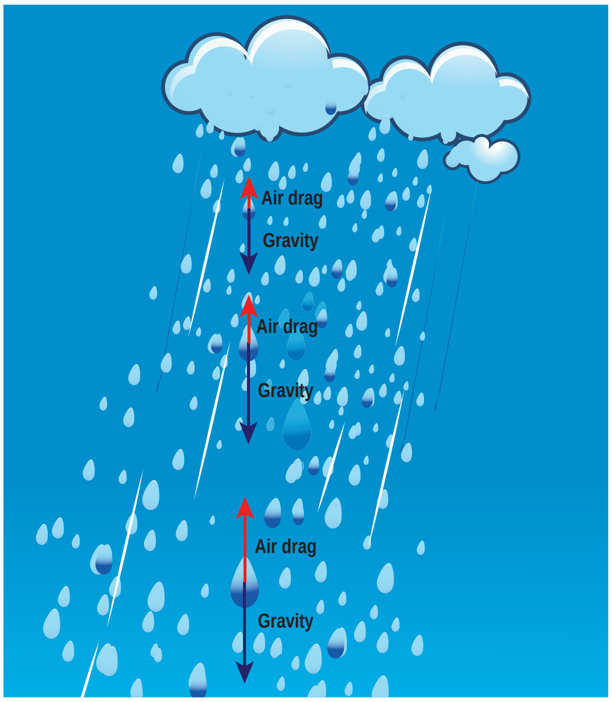


Figure 3.9 (d) Zero net force and non zero terminal velocity

4. If multiple forces $\vec{F}_1, \vec{F}_2, \vec{F}_3 \ldots \vec{F}_n$ act on the same body, then the total force $(\vec{F}_{net})$ is equivalent to the vectorial sum of the individual forces. Their net force provides the acceleration.

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots + \vec{F}_n$$

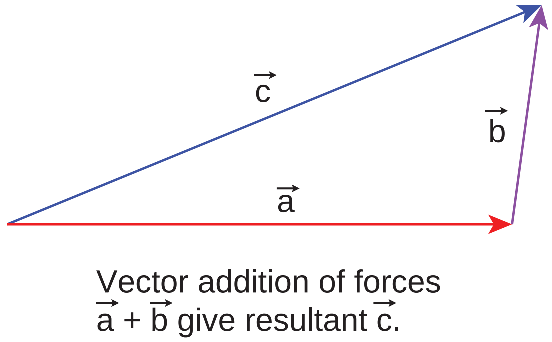


Figure 3.10 Vector addition of forces

Newton's second law for this case is

$$\vec{F}_{net} = m\vec{a}$$

In this case the direction of acceleration is in the direction of net force.

Example

Bow and arrow

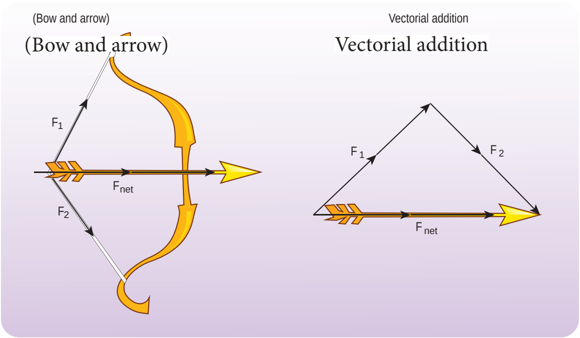


Figure 3.11 Bow and arrow – Net force is on the arrow

5. Newton's second law can also be written in the following form.

Since the acceleration is the second derivative of position vector of the body $\left(\vec{a} = \frac{d^2\vec{r}}{dt^2}\right)$, the force on the body is

$$\vec{F} = m\frac{d^2\vec{r}}{dt^2}.$$

From this expression, we can infer that Newton's second law is basically a second order ordinary differential equation and whenever the second derivative of position vector is not zero, there must be a force acting on the body.

6. If no force acts on the body then Newton's second law, $m\frac{d\vec{v}}{dt} = 0$.

   It implies that $\vec{v} = constant$. It is essentially Newton's first law. It implies that the second law is consistent with the first law. However, it should not be thought of as the reduction of second law to the first when no force acts on the object. Newton's first and second laws are independent laws. They can internally be consistent with each other but cannot be derived from each other.

7. Newton's second law is cause and effect relation. Force is the cause and acceleration is the effect. Conventionally, the effect should be written on the left and cause on the right hand side of the equation. So the correct way of writing Newton's second law is $m\vec{a} = \vec{F}$ or $\frac{d\vec{p}}{dt} = \vec{F}$

3.3 APPLICATION OF NEWTON'S LAWS

3.3.1 Free Body Diagram

Free body diagram is a simple tool to analyse the motion of the object using Newton's laws.

The following systematic steps are followed for developing the free body diagram:

1. Identify the forces acting on the object.
2. Represent the object as a point.
3. Draw the vectors representing the forces acting on the object.

When we draw the free body diagram for an object or a system, the forces exerted by the object should not be included in the free body diagram.

EXAMPLE 3.1

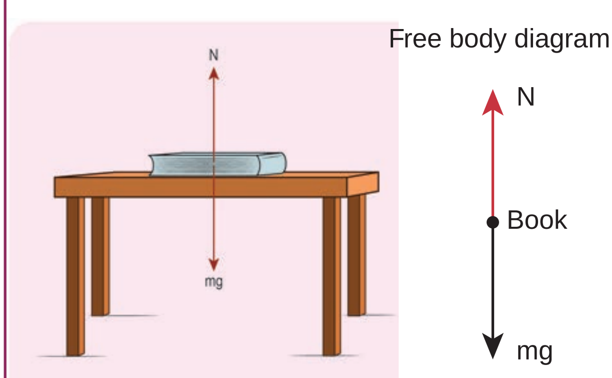
A book of mass m is at rest on the table. (1) What are the forces acting on the book? (2) What are the forces exerted by the book? (3) Draw the free body diagram for the book.

Solution

(1) There are two forces acting on the book.

   (i) Gravitational force (mg) acting downwards on the book

   (ii) Normal contact force (N) exerted by the surface of the table on the book. It acts upwards as shown in the figure.



> **Note** In the free body diagram, as the magnitudes of the normal force and the gravitational force are same, the lengths of both these vectors are also same.

(2) According to Newton's third law, there are two reaction forces exerted by the book.

(i) The book exerts an equal and opposite force (mg) on the Earth which acts upwards.

(ii) The book exerts a force which is equal and opposite to normal force on the surface of the table (N) acting downwards.

Note

It is to be emphasized that while applying Newton's third law it is wrong to conclude that the book on the table is at rest due to the downward gravitational force exerted by the Earth and the equal and opposite reacting normal force exerted by the table on the book. Action and reaction forces never act on the same body.

(3) The free body diagram of the book is shown in the figure.

EXAMPLE 3.2

If two objects of masses 2.5 kg and 100 kg experience the same force 5 N, what is the acceleration experienced by each of them?

Solution

From Newton's second law (in magnitude form), F = ma

For the object of mass 2.5 kg, the acceleration is $a = \frac{F}{m} = \frac{5}{2.5} = 2\ m\ s^{-2}$

For the object of mass 100 kg, the acceleration is $a = \frac{F}{m} = \frac{5}{100} = 0.05\ m\ s^{-2}$

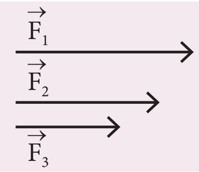
Note

Even though the force applied on both the objects is the same, acceleration experienced by each object differs. The acceleration is inversely proportional to mass. For the same force, the heavier mass experiences lesser acceleration and the lighter mass experiences greater acceleration.

When an apple falls, it experiences Earth's gravitational force. According to Newton's third law, the apple exerts equal and opposite force on the Earth. Even though both the apple and Earth experience the same force, their acceleration is different. The mass of Earth is enormous compared to that of an apple. So an apple experiences larger acceleration and the Earth experiences almost negligible acceleration. Due to the negligible acceleration, Earth appears to be stationary when an apple falls.

EXAMPLE 3.3

Which is the greatest force among the three force $\vec{F}_1, \vec{F}_2, \vec{F}_3$ shown below



Solution

Force is a vector and magnitude of the vector is represented by the length of the vector. Here $\vec{F}_1$ has greater length compared to other two. So $\vec{F}_1$ is largest of the three.

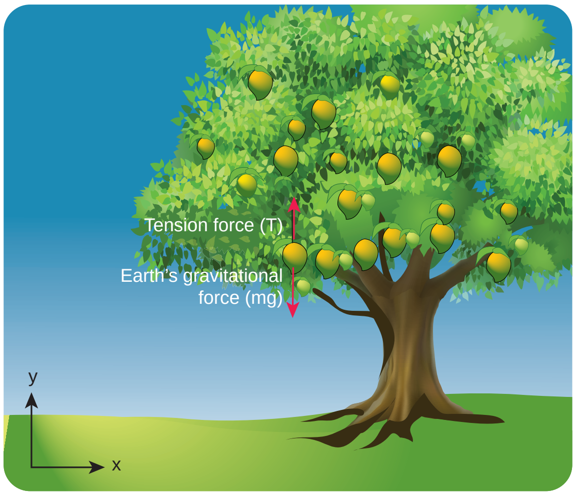
EXAMPLE 3.4

Apply Newton's second law to a mango hanging from a tree. (Mass of the mango is 400 gm)

Solution

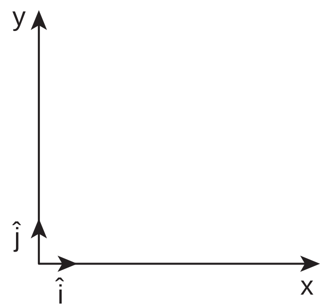
Note: Before applying Newton's laws, the following steps have to be followed:

1) Choose a suitable inertial coordinate system to analyse the problem. For most of the cases we can take Earth as an inertial coordinate system.
2) Identify the system to which Newton's laws need to be applied. The system can be a single object or more than one object.
3) Draw the free body diagram.
4) Once the forces acting on the system are identified, and the free body diagram is drawn, apply Newton's second law. In the left hand side of the equation, write the forces acting on the system in vector notation and equate it to the right hand side of equation which is the product of mass and acceleration. Here, acceleration should also be in vector notation.
5) If acceleration is given, the force can be calculated. If the force is given, acceleration can be calculated.



By following the above steps:

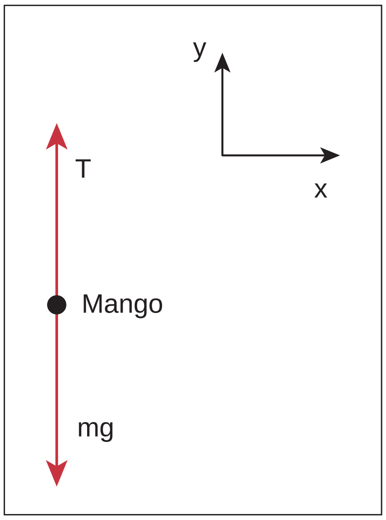
We fix the inertial coordinate system on the ground as shown in the figure.



The forces acting on the mango are

i) Gravitational force exerted by the Earth on the mango acting downward along negative y axis
ii) Tension (in the cord attached to the mango) acts upward along positive y axis.

The free body diagram for the mango is shown in the figure



$$\vec{F}_g = mg\left(-\hat{j}\right) = -mg\hat{j}$$

Here, mg is the magnitude of the gravitational force and $\left(-\hat{j}\right)$ represents the unit vector in negative y direction

$$\vec{T} = T\hat{j}$$

Here T is the magnitude of the tension force and $(\hat{j})$ represents the unit vector in positive y direction

$$\vec{F}_{net} = \vec{F}_g + \vec{T} = -mg\hat{j} + T\hat{j} = (T - mg)\hat{j}$$

From Newton's second law $\vec{F}_{net} = m\vec{a}$

Since the mango is at rest with respect to us (inertial coordinate system) the acceleration is zero ($\vec{a} = 0$).

So $\vec{F}_{net} = m\vec{a} = 0$

$$(T - mg)\hat{j} = 0$$

By comparing the components on both sides of the above equation, we get $T - mg = 0$

So the tension force acting on the mango is given by $T = mg$

Mass of the mango m = $400g$ and $g = 9.8\ m\ s^{-2}$

Tension acting on the mango is $T = 0.4 \times 9.8 = 3.92$ N

EXAMPLE 3.5

A person rides a bike with a constant velocity $\vec{v}$ with respect to ground and another biker accelerates with acceleration $\vec{a}$ with respect to ground. Who can apply Newton's second law with respect to a stationary observer on the ground?

Solution

Second biker cannot apply Newton's second law, because he is moving with acceleration $\vec{a}$ with respect to Earth (he is not in inertial frame). But the first biker can apply Newton's second law because he is moving at constant velocity with respect to Earth (he is in inertial frame).

EXAMPLE 3.6

The position vector of a particle is given by $\vec{r} = 3t\hat{i} + 5t^2\hat{j} + 7\hat{k}$. Find the direction in which the particle experiences net force?

Solution

Velocity of the particle,

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3t)\hat{i} + \frac{d}{dt}(5t^2)\hat{j} + \frac{d}{dt}(7)\hat{k}$$

$$\frac{d\vec{r}}{dt} = 3\hat{i} + 10t\hat{j}$$

Acceleration of the particle

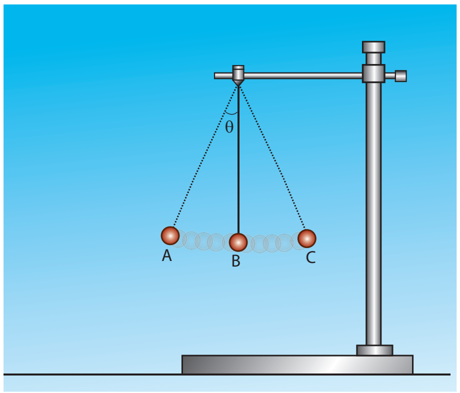
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = 10\hat{j}$$

Here, the particle has acceleration only along positive y direction. According to Newton's second law, net force must also act along positive y direction. In addition, the particle has constant velocity in positive x direction and no velocity in z direction. Hence, there are no net force along x or z direction.

EXAMPLE 3.7

Consider a bob attached to a string, hanging from a stand. It oscillates as shown in the figure.

a) Identify the forces that act on the bob?
b) What is the acceleration experienced by the bob?

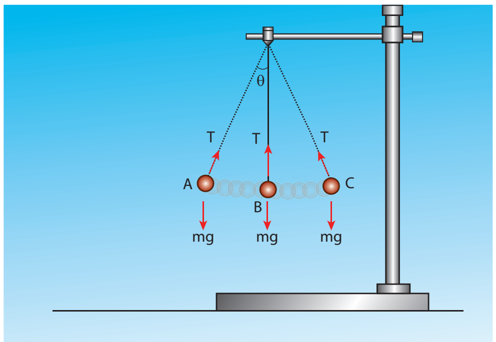


Solution

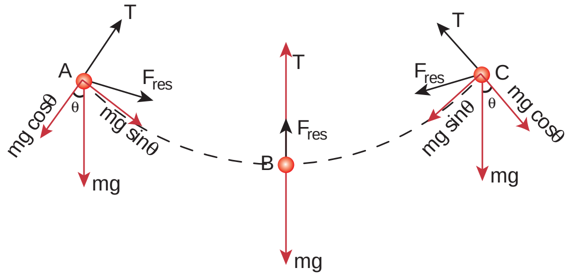
Two forces act on the bob.

(i) Gravitational force (mg) acting downwards

(ii) Tension (T) exerted by the string on the bob, whose position determines the direction of T as shown in figure.



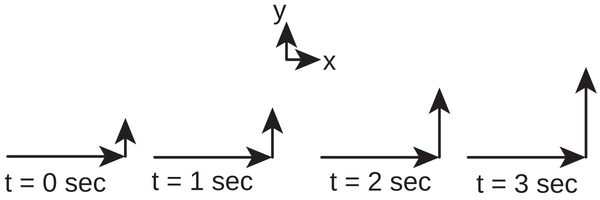
The bob is moving in a circular arc as shown in the above figure. Hence it has centripetal acceleration. At a point A and C, the bob comes to rest momentarily and then its velocity increases when it moves towards point B. Hence, there is a tangential acceleration along the arc. The gravitational force can be resolved into two components (mg cosθ, mg sinθ) as shown below



Note Note that the bob does not move in the direction of the resultant force. At the points A and C, tension T = mg cosθ. At all other points, tension T is greater than mg cosθ, since it has non zero centripetal acceleration. At point B, the resultant force acts upward along the string. It is an example of a non uniform circular motion because the bob has both the centripetal and tangential accelerations.

EXAMPLE 3.8

The velocity of a particle moving in a plane is given by the following diagram. Find out the direction of force acting on the particle?



Solution

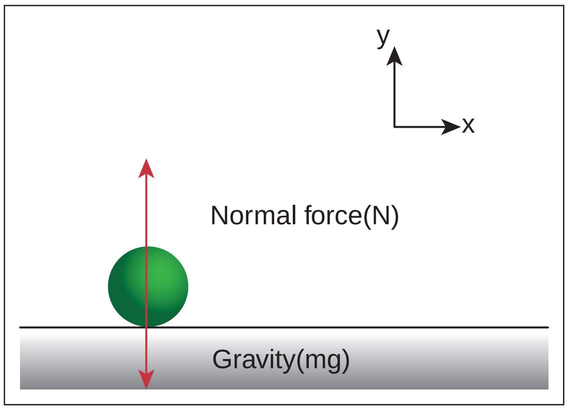
The velocity of the particle is $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$. As shown in the figure, the particle is moving in the xy plane, there is no motion in the z direction. So velocity in the z direction is zero ($v_z = 0$). The velocity of the particle has x component (v_x) and y component (v_y). From figure, as time increases from t = 0 sec to t = 3 sec, the length of the vector in y direction is changing (increasing). It means y component of velocity $\left(v_y\right)$ is increasing with respect to time. According to Newton's second law, if velocity changes with respect to time then there must be acceleration. In this case, the particle has acceleration in the y direction since the y component of velocity changes. So the particle experiences force in the y direction. The length of the vector in x direction does not change. It means that the particle has constant velocity in the x direction. So no force or zero net force acts in the x direction.

EXAMPLE 3.9

Apply Newton's second law for an object at rest on Earth and analyse the result.

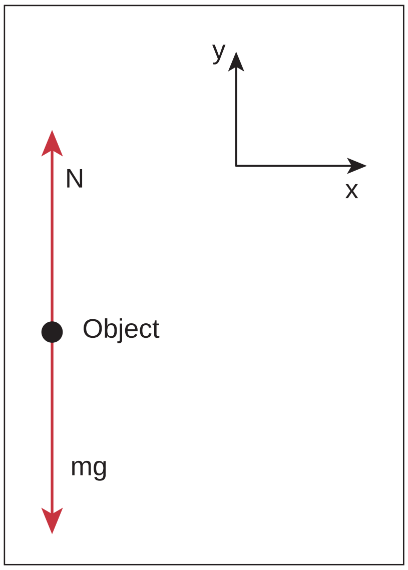
Solution

The object is at rest with respect to Earth (inertial coordinate system). There are two forces that act on the object.



i) Gravity acting downward (negative y-direction)
ii) Normal force by the surface of the Earth acting upward (positive y-direction)

The free body diagram for this object is



$$\vec{F}_g = -mg\hat{j}$$
$$\vec{N} = N\hat{j}$$

Net force $\vec{F}_{net} = -mg\hat{j} + N\hat{j}$

But there is no acceleration on the ball. So $\vec{a} = 0$. By applying Newton's second law $(\vec{F}_{net} = m\vec{a})$

Since $\vec{a} = 0, \vec{F}_{net} = -mg\hat{j} + N\hat{j}$

$$(-mg + N)\hat{j} = 0$$

By comparing the components on both sides of the equation, we get

$$-mg + N = 0$$
$$N = mg$$

We can conclude that if the object is at rest, the magnitude of normal force is exactly equal to the magnitude of gravity.

EXAMPLE 3.10

A particle of mass 2 kg experiences two forces, $\vec{F}_1 = 5\hat{i} + 8\hat{j} + 7\hat{k}$ and $\vec{F}_2 = 3\hat{i} - 4\hat{j} + 3\hat{k}$. What is the acceleration of the particle?

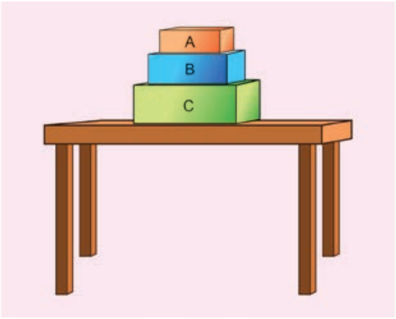
Solution

We use Newton's second law, $\vec{F}_{net} = m\vec{a}$ where $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$. From the above equations the acceleration is $\vec{a} = \frac{\vec{F}_{net}}{m}$, where

$$\vec{F}_{net} = (5+3)\hat{i} + (8-4)\hat{j} + (7+3)\hat{k}$$
$$\vec{F}_{net} = 8\hat{i} + 4\hat{j} + 10\hat{k}$$
$$\vec{a} = \left(\frac{8}{2}\right)\hat{i} + \left(\frac{4}{2}\right)\hat{j} + \left(\frac{10}{2}\right)\hat{k}$$
$$\vec{a} = 4\hat{i} + 2\hat{j} + 5\hat{k}$$

EXAMPLE 3.11

Identify the forces acting on blocks A, B and C shown in the figure.



Solution

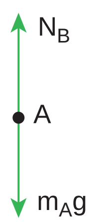
Forces on block A:

(i) Downward gravitational force exerted by the Earth ($m_A g$)

(ii) Upward normal force exerted by block B (N_B)

The free body diagram for block A is as shown in the following picture.

Force on block A



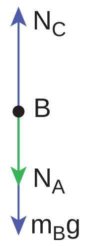
Forces on block B :

(i) Downward gravitational force exerted by Earth ($m_B g$)

(ii) Downward force exerted by block A (N_A)

(iii) Upward normal force exerted by block C (N_C)

Force on block B



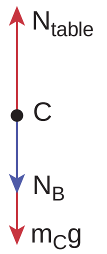
Forces onblock C:

(i) Downward gravitational force exerted by Earth ($m_C g$)

(ii) Downward force exerted by block B (N_B)

(iii) Upward force exerted by the table (N_{table})

Force on block C



EXAMPLE 3.12

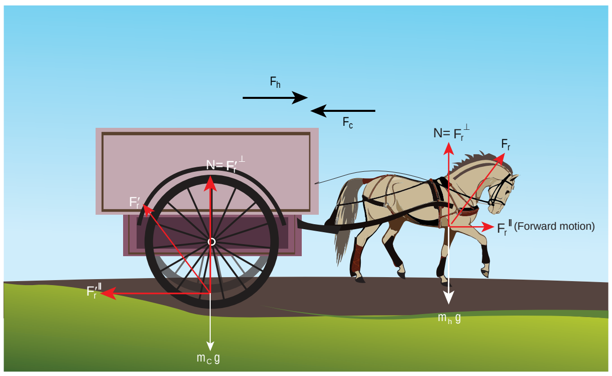
Consider a horse attached to the cart which is initially at rest. If the horse starts walking forward, the cart also accelerates in the forward direction. If the horse pulls the cart with force F_h in forward direction, then according to Newton's third law, the cart also pulls the horse by equivalent opposite force $F_c = F_h$ in backward direction. Then total force on 'cart+horse' is zero. Why is it then the 'cart+horse' accelerates and moves forward?

Solution

This paradox arises due to wrong application of Newton's second and third laws. Before applying Newton's laws, we should decide 'what is the system?'. Once we identify the 'system', then it is possible to identify all the forces acting on the system. We should not consider the force exerted by the system. If there is an unbalanced force acting on the system, then it should have acceleration in the direction of the resultant force. By following these steps we will analyse the horse and cart motion.

If we decide on the cart+horse as a 'system', then we should not consider the force exerted by the horse on the cart or the force exerted by cart on the horse. Both are internal forces acting on each other. According to Newton's third law, total internal force acting on the system is zero and it cannot accelerate the system. The acceleration of the system is caused by some external force. In this case, the force exerted by the road on the system is the external force acting on the system. It is wrong to conclude that the total force acting on the system (cart+horse) is zero without including all the forces acting on the system. The road is pushing the horse and cart forward with acceleration. As there is an external force acting on the system, Newton's second law has to be applied and not Newton's third law.

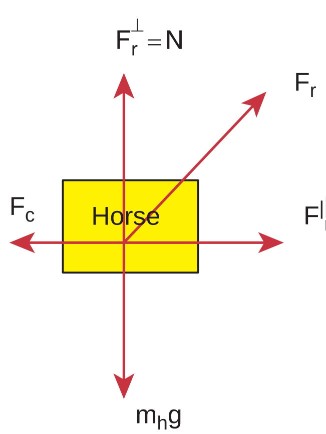
The following figures illustrates this.



If we consider the horse as the 'system', then there are three forces acting on the horse.

(i) Downward gravitational force $(m_h g)$
(ii) Force exerted by the road (F_r)
(iii) Backward force exerted by the cart (F_c)

It is shown in the following figure.



F_r – Force exerted by the road on the horse

F_c – Force exerted by the cart on the horse

$F_r^{\perp}$ – Perpendicular component of $F_r = N$

$F_r^{\parallel}$ – Parallel component of F_r which is reason for forward movement

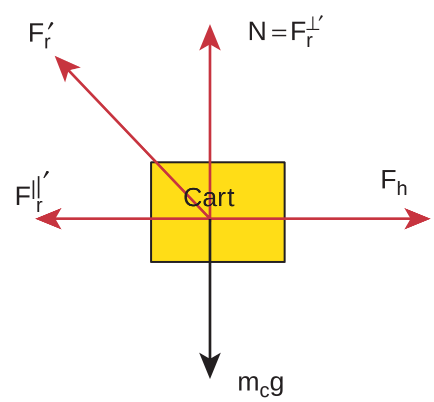
The force exerted by the road can be resolved into parallel and perpendicular components. The perpendicular component balances the downward gravitational force. There is parallel component along the forward direction. It is greater than the backward force (F_c). So there is net force along the forward direction which causes the forward movement of the horse.

If we take the cart as the system, then there are three forces acting on the cart.

(i) Downward gravitational force $(m_c g)$
(ii) Force exerted by the road $(F_r^{'})$
(iii) Force exerted by the horse (F_h)

It is shown in the figure

Force on the cart



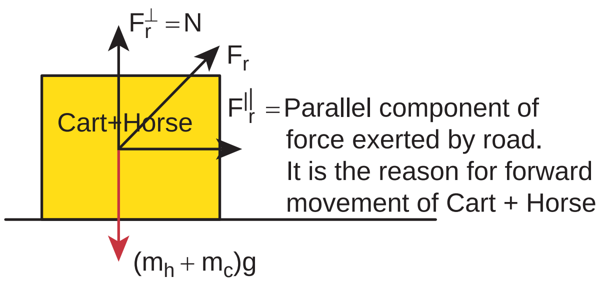
The force exerted by the road $(\vec{F_r^{'}})$ can be resolved into parallel and perpendicular components. The perpendicular component cancels the downward gravity $(m_c g)$. Parallel component acts backwards and the force exerted by the horse $(\vec{F_h})$ acts forward. Force $(\vec{F_h})$ is greater than the parallel component acting in the opposite direction. So there is an overall unbalanced force in the forward direction which causes the cart to accelerate forward.

If we take the cart+horse as a system, then there are two forces acting on the system.

(i) Downward gravitational force $(m_h + m_c)g$

(ii) The force exerted by the road (F_r) on the system.

It is shown in the following figure.



(iii) In this case the force exerted by the road (F_r) on the system (cart+horse) is resolved in to parallel and perpendicular components. The perpendicular component is the normal force which cancels the downward gravitational force $(m_h + m_c)$g. The parallel component of the force is not balanced, hence the system (cart+horse) accelerates and moves forward due to this force.

EXAMPLE 3.13

The position of the particle is represented by $y = ut - \frac{1}{2}gt^2$.

a) What is the force acting on the particle?
b) What is the momentum of the particle?

Solution

To find the force, we need to find the acceleration experienced by the particle.

The acceleration is given by $a = \frac{d^2y}{dt^2}$

(or) $a = \frac{dv}{dt}$

Here

v =velocity of the particle in y direction

$$v = \frac{dy}{dt} = u - gt$$

The momentum of the particle = mv = m (u-gt).

$$a = \frac{dv}{dt} = -g$$

The force acting on the object is given by $F = ma = -mg$

The negative sign implies that the force is acting on the negative y direction. This is exactly the force that acts on the object in projectile motion.

3.3.2 Particle Moving in an Inclined Plane

When an object of mass m slides on a frictionless surface inclined at an angle θ as shown in the Figure 3.12, the forces acting on it decides the

a) acceleration of the object
b) speed of the object when it reaches the bottom

The force acting on the object is

(i) Downward gravitational force (mg)
(ii) Normal force perpendicular to inclined surface (N)

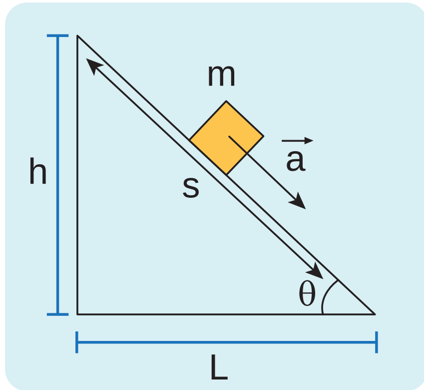


Figure 3.12 Object moving in an inclined plane

To draw the free body diagram, the block is assumed to be a point mass (Figure 3.13 (a)). Since the motion is on the inclined surface, we have to choose the coordinate system parallel to the inclined surface as shown in Figure 3.13 (b).

The gravitational force mg is resolved in to parallel component $mg\sin\theta$ along the inclined plane and perpendicular component $mg\cos\theta$ perpendicular to the inclined surface (Figure 3.13 (b)).

Note that the angle made by the gravitational force (mg) with the perpendicular to the surface is equal to the angle of inclination θ as shown in Figure 3.13 (c).

There is no motion(acceleration) along the y axis. Applying Newton's second law in the y direction

$$-mg\cos\theta\hat{j} + N\hat{j} = 0(No\ acceleration)$$

By comparing the components on both sides, $N - mg\cos\theta = 0$

$$N = mg\cos\theta$$

The magnitude of normal force (N) exerted by the surface is equivalent to $mg\,cos\theta$.

The object slides (with an acceleration) along the x direction. Applying Newton's second law in the x direction

$$mg\sin\theta\hat{i} = ma\hat{i}$$

By comparing the components on both sides, we can equate

$$mg\sin\theta = ma$$

The acceleration of the sliding object is

$$a = g\ sin\theta$$

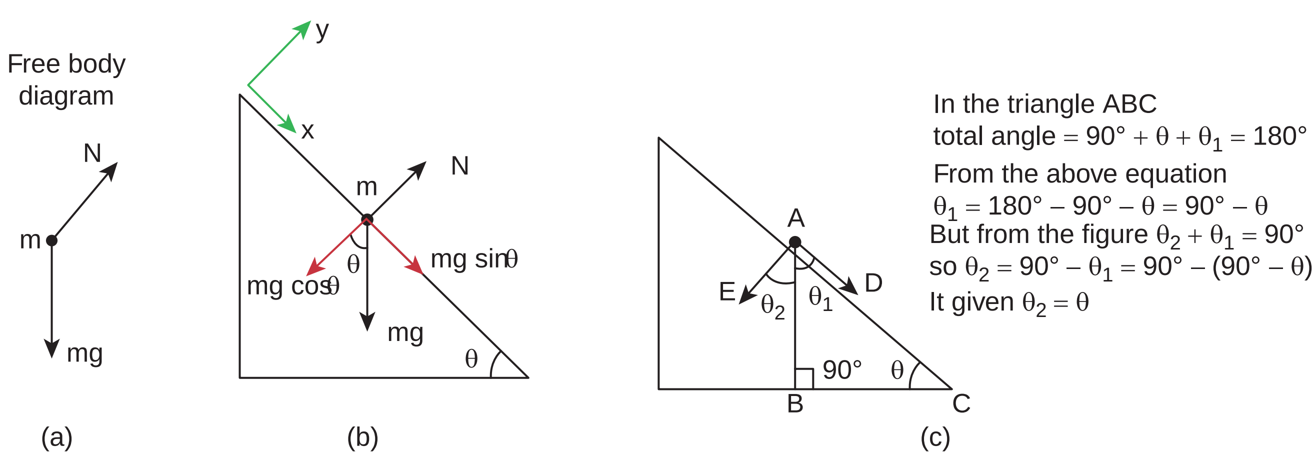


Figure 3.13 (a) Free body diagram, (b) mg resolved into parallel and perpendicular components (c) The angle θ_2 is equal to θ

Note that the acceleration depends on the angle of inclination θ. If the angle θ is 90 degree, the block will move vertically with acceleration $a = g$.

Newton's kinematic equation is used to find the speed of the object when it reaches the bottom. The acceleration is constant throughout the motion.

$$v^2 = u^2 + 2as \text{ along the x direction} \quad (3.3)$$

The acceleration a is equal to g sinθ. The initial speed (u) is equal to zero as it starts from rest. Here s is the length of the inclined surface.

The speed (v) when it reaches the bottom is (using equation (3.3))

$$v = \sqrt{2sg \sin\theta} \quad (3.4)$$

Note Here we choose the coordinate system along the inclined plane. Even if we choose the coordinate system parallel to the horizontal surface, we will get the same result. But the mathematics will be quite complicated. Choosing a suitable inertial coordinate system for the given problem is very important.

3.3.3 Two Bodies in Contact on a Horizontal Surface

Consider two blocks of masses m_1 and m_2 ($m_1 > m_2$) kept in contact with each other on a smooth, horizontal frictionless surface as shown in Figure 3.14.

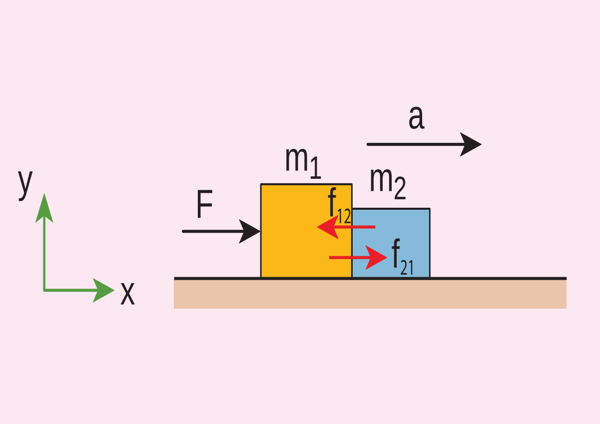


Figure 3.14 (a) Two blocks of masses m_1 and m_2 ($m_1 > m_2$) kept in contact with each other on a smooth, horizontal frictionless surface

By the application of a horizontal force F, both the blocks are set into motion with acceleration 'a' simultaneously in the direction of the force F.

To find the acceleration $\vec{a}$, Newton's second law has to be applied to the system (combined mass $m = m_1 + m_2$)

$$\vec{F} = m\vec{a}$$

If we choose the motion of the two masses along the positive x direction,

$$F\hat{i} = ma\hat{i}$$

By comparing components on both sides of the above equation

$$F = ma \quad \text{where } m = m_1 + m_2$$

The acceleration of the system is given by

$$\therefore a = \frac{F}{m_1 + m_2} \quad (3.5)$$

The force exerted by the block m_1 on m_2 due to its motion is called force of contact ($\vec{f}_{21}$). According to Newton's third law, the block m_2 will exert an equivalent opposite reaction force ($\vec{f}_{12}$) on block m_1.

Figure 3.14 (b) shows the free body diagram of block m_1.

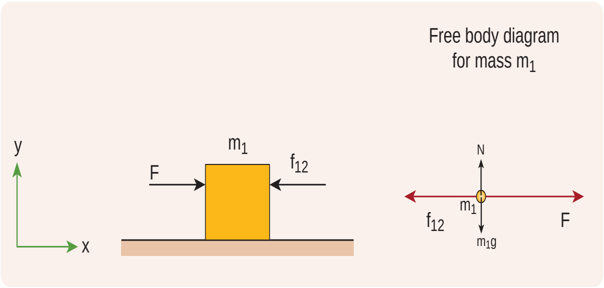


Figure 3.14 (b) Free body diagram of block of mass m_1

$$\therefore F\hat{i} - f_{12}\hat{i} = m_1 a\hat{i}$$

By comparing the components on both sides of the above equation, we get

$$F - f_{12} = m_1 a$$

$$f_{12} = F - m_1 a \qquad (3.6)$$

Substituting the value of acceleration from equation (3.5) in (3.6) we get

$$f_{12} = F - m_1\left(\frac{F}{m_1 + m_2}\right)$$

$$f_{12} = F\left[1 - \frac{m_1}{m_1 + m_2}\right]$$

$$f_{12} = \frac{Fm_2}{m_1 + m_2} \qquad (3.7)$$

Equation (3.7) shows that the magnitude of contact force depends on mass m_2 which provides the reaction force. Note that this force is acting along the negative x direction.

In vector notation, the reaction force on mass m_1 is given by $\vec{f}_{12} = -\frac{Fm_2}{m_1 + m_2}\hat{i}$

For mass m_2 there is only one force acting on it in the x direction and it is denoted by $\vec{f}_{21}$. This force is exerted by mass m_1. The free body diagram for mass m_2 is shown in Figure 3.14 (c).

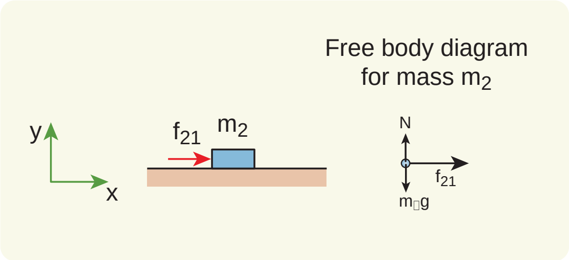


Figure 3.14 (c) Free body diagram of block of mass m_2

Applying Newton's second law for mass m_2

$$f_{21}\hat{i} = m_2 a\hat{i}$$

By comparing the components on both sides of the above equation

$$f_{21} = m_2 a \qquad (3.8)$$

Substituting for acceleration from equation (3.5) in equation (3.8), we get $f_{21} = \frac{Fm_2}{m_1 + m_2}$

In this case the magnitude of the contact force is

$f_{21} = \frac{Fm_2}{m_1 + m_2}$ The direction of this force is along the positive x direction.

In vector notation, the force acting on mass m_2 exerted by mass m_1 is $\vec{f}_{21} = \frac{Fm_2}{m_1+m_2}\hat{i}$

Note $\vec{f}_{12} = -\vec{f}_{21}$ which confirms Newton's third law.

3.3.4 Motion of Connected Bodies

When objects are connected by strings and a force F is applied either vertically or horizontally or along an inclined plane, it produces a tension T in the string, which affects the acceleration to an extent. Let us discuss various cases for the same.

Case 1: Vertical motion

Consider two blocks of masses m_1 and m_2 ($m_1 > m_2$) connected by a light and inextensible string that passes over a pulley as shown in Figure 3.15.

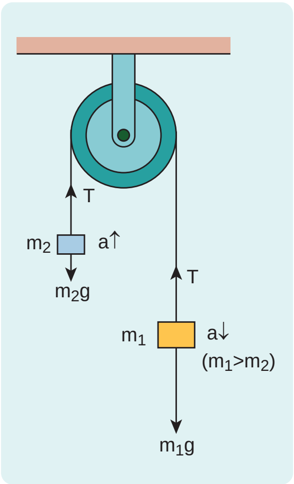


Figure 3.15 Two blocks connected by a string over a pulley

Let the tension in the string be T and acceleration a. When the system is released, both the blocks start moving, m_2 vertically upward and m_1 downward with same acceleration a. The gravitational force m_1g on mass m_1 is used in lifting the mass m_2.

The upward direction is chosen as y direction. The free body diagrams of both masses are shown in Figure 3.16.

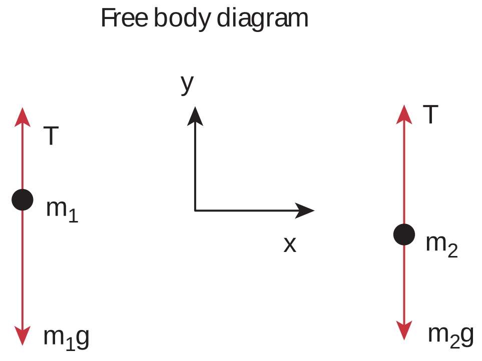


Figure 3.16 Free body diagrams of masses m_1 and m_2

Applying Newton's second law for mass m_2

$$T\hat{j} - m_2g\hat{j} = m_2a\hat{j}$$

The left hand side of the above equation is the total force that acts on m_2 and the right hand side is the product of mass and acceleration of m_2 in y direction.

By comparing the components on both sides, we get

$$T - m_2g = m_2a \qquad (3.9)$$

Similarly, applying Newton's second law for mass m_1

$$T\hat{j} - m_1g\hat{j} = -m_1a\hat{j}$$

As mass m_1 moves downward ($-\hat{j}$), its acceleration is along ($-\hat{j}$)

By comparing the components on both sides, we get

$$T - m_1 g = -m_1 a$$

$$m_1 g - T = m_1 a \qquad (3.10)$$

Adding equations (3.9) and (3.10), we get

$$m_1 g - m_2 g = m_1 a + m_2 a$$

$$(m_1 - m_2) g = (m_1 + m_2) a \qquad (3.11)$$

From equation (3.11), the acceleration of both the masses is

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g \qquad (3.12)$$

If both the masses are equal ($m_1 = m_2$), from equation (3.12)

$$a = 0$$

This shows that if the masses are equal, there is no acceleration and the system as a whole will be at rest.

To find the tension acting on the string, substitute the acceleration from the equation (3.12) into the equation (3.9).

$$T - m_2 g = m_2 \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$$

$$T = m_2 g + m_2 \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g \qquad (3.13)$$

By taking $m_2 g$ common in the RHS of equation (3.13)

$$T = m_2 g \left(1 + \frac{m_1 - m_2}{m_1 + m_2}\right)$$

$$T = m_2 g \left(\frac{m_1 + m_2 + m_1 - m_2}{m_1 + m_2}\right)$$

$$T = \left(\frac{2 m_1 m_2}{m_1 + m_2}\right) g$$

Equation (3.12) gives only magnitude of acceleration.

For mass m_1, the acceleration vector is given by $\vec{a} = -\left(\frac{m_1 - m_2}{m_1 + m_2}\right) g \hat{j}$

For mass m_2, the acceleration vector is given by $\vec{a} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g \hat{j}$

Case 2: Horizontal motion

In this case, mass m_2 is kept on a horizontal table and mass m_1 is hanging through a small pulley as shown in Figure 3.17. Assume that there is no friction on the surface.

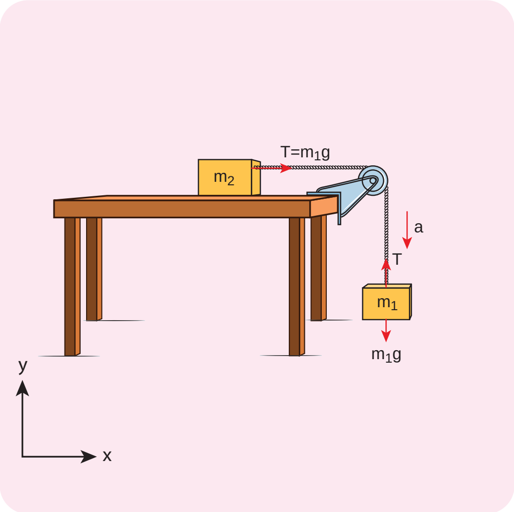


Figure 3.17 Blocks in horizontal motion

As both the blocks are connected to the unstretchable string, if m_1 moves with an acceleration a downward then m_2 also moves with the same acceleration a horizontally.

The forces acting on mass m_2 are

(i) Downward gravitational force (m_2g)
(ii) Upward normal force (N) exerted by the surface
(iii) Horizontal tension (T) exerted by the string

The forces acting on mass m_1 are

(i) Downward gravitational force (m_1g)
(ii) Tension (T) acting upwards

The free body diagrams for both the masses is shown in Figure 3.18.

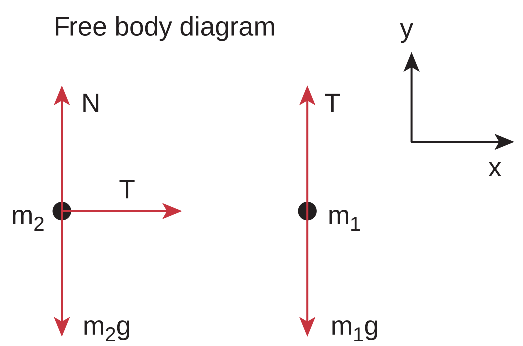


Figure 3.18 Free body diagrams of masses m_1 and m_2

Applying Newton's second law for m_1

$$T\hat{j} - m_1 g\hat{j} = -m_1 a\hat{j} \text{ (along y direction)}$$

By comparing the components on both sides of the above equation,

$$T - m_1 g = -m_1 a \qquad (3.14)$$

Applying Newton's second law for m_2

$$T\hat{i} = m_2 a\hat{i} \text{ (along x direction)}$$

By comparing the components on both sides of above equation,

$$T = m_2 a \qquad (3.15)$$

There is no acceleration along y direction for m_2.

$$N\hat{j} - m_2 g\hat{j} = 0$$

By comparing the components on both sides of the above equation

$$N - m_2 g = 0$$
$$N = m_2 g \qquad (3.16)$$

By substituting equation (3.15) in equation (3.14), we can find the tension T

$$m_2 a - m_1 g = -m_1 a$$
$$m_2 a + m_1 a = m_1 g$$
$$a = \frac{m_1}{m_1 + m_2} g \qquad (3.17)$$

Tension in the string can be obtained by substituting equation (3.17) in equation (3.15)

$$T = \frac{m_1 m_2}{m_1 + m_2} g \qquad (3.18)$$

Comparing motion in both cases, it is clear that the tension in the string for horizontal motion is half of the tension for vertical motion for same set of masses and strings.

This result has an important application in industries. The ropes used in conveyor belts (horizontal motion) work for longer duration than those of cranes and lifts (vertical motion).

3.3.5 Concurrent Forces and Lami's Theorem

A collection of forces is said to be concurrent, if the lines of forces act at a common point. Figure 3.19 illustrates concurrent forces.

Concurrent forces need not be in the same plane. If they are in the same plane, they are concurrent as well as coplanar forces.

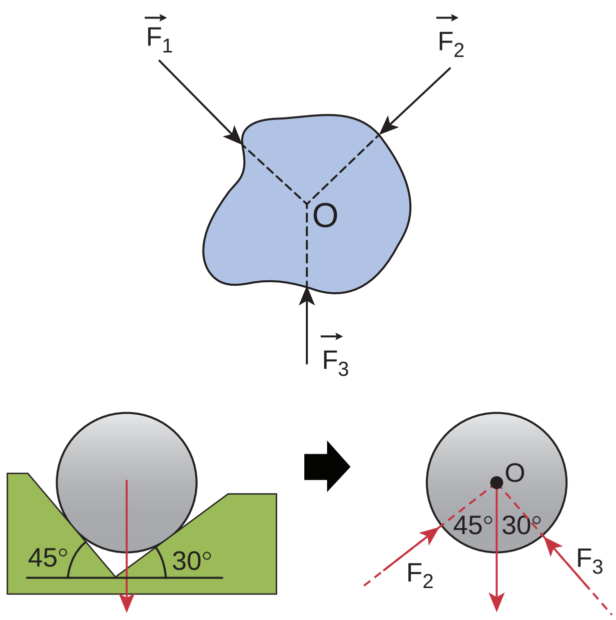


Figure 3.19 Concurrent forces

3.4 LAMI'S THEOREM

If a system of three concurrent and coplanar forces is in equilibrium, then Lami's theorem states that the magnitude of each force of the system is proportional to sine of the angle between the other two forces. The constant of proportionality is same for all three forces.

Let us consider three coplanar and concurrent forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ which act at a common point O as shown in Figure 3.20. If the point is at equilibrium, then according to Lami's theorem

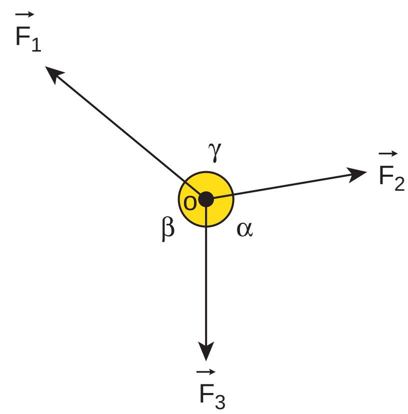


Figure 3.20 Three coplanar and concurrent forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ acting at O

$$|\vec{F}_1| \propto \sin\alpha$$

$$|\vec{F}_2| \propto \sin\beta$$

$$|\vec{F}_3| \propto \sin\gamma$$

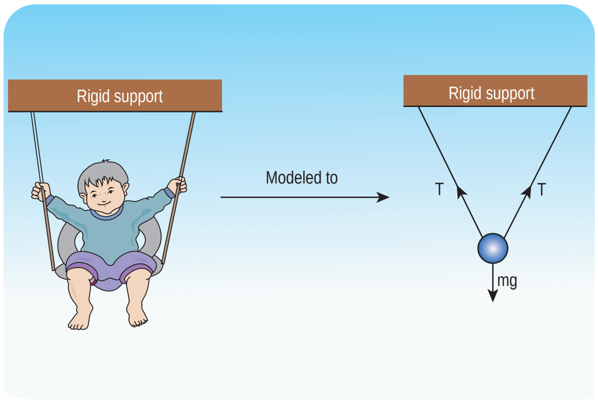
Therefore, $$\frac{|\vec{F}_1|}{\sin\alpha} = \frac{|\vec{F}_2|}{\sin\beta} = \frac{|\vec{F}_3|}{\sin\gamma} \quad (3.19)$$

Lami's theorem is useful to analyse the forces acting on objects which are in static equilibrium.

Application of Lami's Theorem

EXAMPLE 3.14

A baby is playing in a swing which is hanging with the help of two identical chains is at rest. Identify the forces acting on the baby. Apply Lami's theorem and find out the tension acting on the chain.



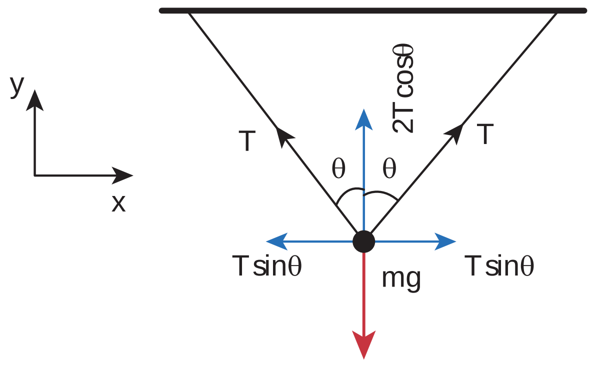
Solution

The baby and the chains are modeled as a particle hung by two strings as shown in the figure. There are three forces acting on the baby.

i) Downward gravitational force along negative y direction (mg)

ii) Tension (T) along the two strings

These three forces are coplanar as well as concurrent as shown in the following figure.



By using Lami's theorem

$$\frac{T}{\sin(180-\theta)} = \frac{T}{\sin(180-\theta)} = \frac{mg}{\sin(2\theta)}$$

Since $\sin(180-\theta) = \sin\theta$ and $\sin(2\theta) = 2\sin\theta\cos\theta$

We get $$\frac{T}{\sin\theta} = \frac{mg}{2\sin\theta\cos\theta}$$

From this, the tension on each string is $T = \dfrac{mg}{2\cos\theta}$.

Note When $\theta = 0°$, the strings are vertical and the tension on each string is $T = \dfrac{mg}{2}$

3.5 LAW OF CONSERVATION OF TOTAL LINEAR MOMENTUM

In nature, conservation laws play a very important role. The dynamics of motion of bodies can be analysed very effectively using conservation laws. There are three conservation laws in mechanics. Conservation of total energy, conservation of total linear momentum, and conservation of angular momentum. By combining Newton's second and third laws, we can derive the law of conservation of total linear momentum.

When two particles interact with each other, they exert equal and opposite forces on each other. The particle 1 exerts force $\vec{F}_{21}$ on particle 2 and particle 2 exerts an exactly equal and opposite force $\vec{F}_{12}$ on particle 1, according to Newton's third law.

$$\vec{F}_{21} = -\vec{F}_{12} \quad (3.20)$$

In terms of momentum of particles, the force on each particle (Newton's second law) can be written as

$$\vec{F}_{12} = \frac{d\vec{p}_1}{dt} \text{ and } \vec{F}_{21} = \frac{d\vec{p}_2}{dt}. \quad (3.21)$$

Here $\vec{p}_1$ is the momentum of particle 1 which changes due to the force $\vec{F}_{12}$ exerted by particle 2. Further $\vec{p}_2$ is the momentum of particle 2. This changes due to $\vec{F}_{21}$ exerted by particle 1.

Substitute equation (3.21) in equation (3.20)

$$\frac{d\vec{p}_1}{dt} = -\frac{d\vec{p}_2}{dt} \quad (3.22)$$

$$\frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = 0 \quad (3.23)$$

$$\frac{d}{dt}\left(\vec{p}_1 + \vec{p}_2\right) = 0$$

It implies that $\vec{p}_1 + \vec{p}_2 =$ constant vector (always).

$\vec{p}_1 + \vec{p}_2$ is the total linear momentum of the two particles ($\vec{p}_{tot} = \vec{p}_1 + \vec{p}_2$). It is also called as total linear momentum of the system. Here, the two particles constitute the system. From this result, the law of conservation of linear momentum can be stated as follows.

If there are no external forces acting on the system, then the total linear momentum of the system ($\vec{p}_{tot}$) is always a constant vector. In otherwords, the total linear momentum of the system is conserved in time. Here the word 'conserve' means that $\vec{p}_1$ and $\vec{p}_2$ can vary,in such a way that $\vec{p}_1 + \vec{p}_2$ is a constant vector.

The forces $\vec{F}_{12}$ and $\vec{F}_{21}$ are called the internal forces of the system, because they act only between the two particles. There is no external force acting on the two particles from outside. In such a case the total linear momentum of the system is a constant vector or is conserved.

EXAMPLE 3.15

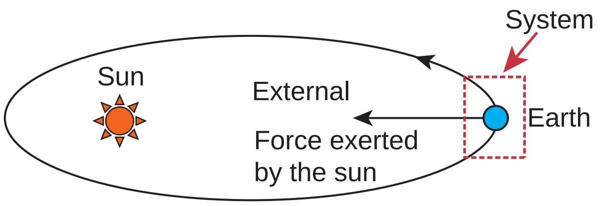
Identify the internal and external forces acting on the following systems.

a) Earth alone as a system
b) Earth and Sun as a system
c) Our body as a system while walking
d) Our body + Earth as a system

Solution

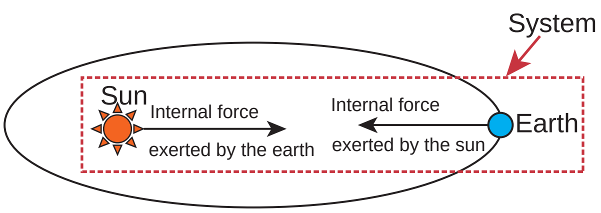
a) Earth alone as a system

Earth orbits the Sun due to gravitational attraction of the Sun. If we consider Earth as a system, then Sun's gravitational force is an external force. If we take the Moon into account, it also exerts an external force on Earth.



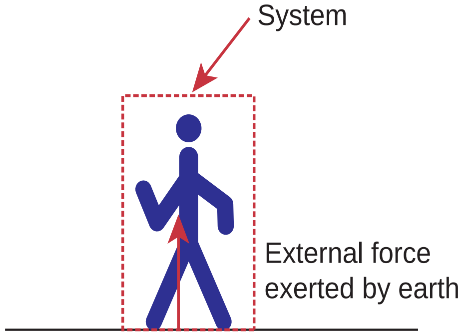
b) (Earth + Sun) as a system

In this case, there are two internal forces which form an action and reaction pair- the gravitational force exerted by the Sun on Earth and gravitational force exerted by the Earth on the Sun.



c) Our body as a system

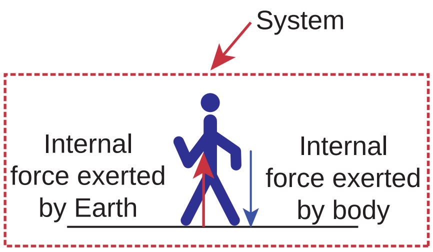
While walking, we exert a force on the Earth and Earth exerts an equal and opposite force on our body. If our body alone is considered as a system, then the force exerted by the Earth on our body is external.



Our body as a system

d) (Our body + Earth) as a system

In this case, there are two internal forces present in the system. One is the force exerted by our body on the Earth and the other is the equal and opposite force exerted by the Earth on our body.

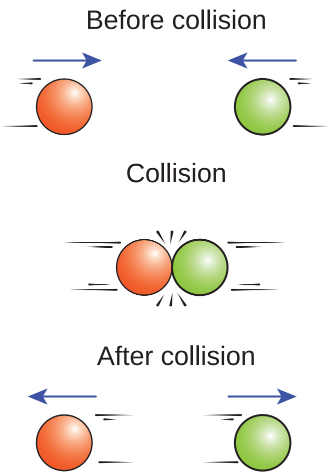


Our body + Earth as a system

Meaning of law of conservation of momentum

1) The Law of conservation of linear momentum is a vector law. It implies that both the magnitude and direction of total linear momentum are constant. In some cases, this total momentum can also be zero.
2) To analyse the motion of a particle, we can either use Newton's second law or the law of conservation of linear momentum. Newton's second law requires us to specify the forces involved in the process. This is difficult to specify in real situations. But conservation of linear momentum does not require any force involved in the process. It is covenient and hence important.

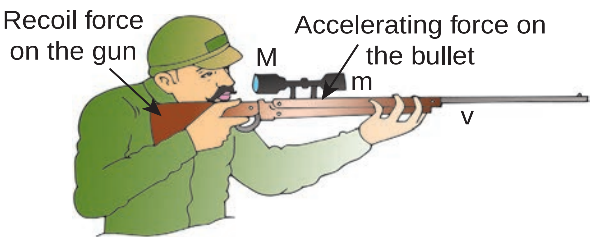
For example, when two particles collide, the forces exerted by these two particles on each other is difficult to specify. But it is easier to apply conservation of linear momentum during the collision process.



Collision of two particles

Examples

- Consider the firing of a gun. Here the system is Gun+bullet. Initially the gun and bullet are at rest, hence the total linear momentum of the system is zero. Let $\vec{p}_1$ be the momentum of the bullet and $\vec{p}_2$ the momentum of the gun before firing. Since initially both are at rest,



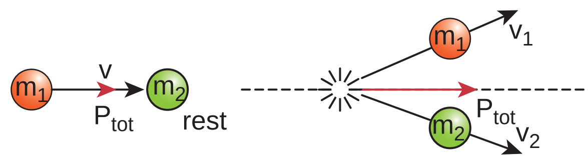
$$\vec{p}_1 = 0, \vec{p}_2 = 0.$$

Total momentum before firing the gun is zero, $\vec{p}_1 + \vec{p}_2 = 0$.

According to the law of conservation of linear momentum, total linear momemtum has to be zero after the firing also.

When the gun is fired, a force is exerted by the gun on the bullet in forward direction. Now the momentum of the bullet changes from $\vec{p}_1$ to $\vec{p}_1'$. To conserve the total linear momentum of the system, the momentum of the gun must also change from $\vec{p}_2$ to $\vec{p}_2'$. Due to the conservation of linear momentum, $\vec{p}_1' + \vec{p}_2' = 0$. It implies that $\vec{p}_1' = -\vec{p}_2'$, the momentum of the gun is exactly equal, but in the opposite direction to the momentum of the bullet. This is the reason after firing, the gun suddenly moves backward with the momentum $\left(-\vec{p}_2'\right)$. It is called 'recoil momemtum'. This is an example of conservation of total linear momentum.

• Consider two particles. One is at rest and the other moves towards the first particle (which is at rest). They collide and after collison move in some arbitrary directions. In this case, before collision, the total linear momentum of the system is equal to the initial linear momentum of the moving particle. According to conservation of momentum, the total linear momentum after collision also has to be in the forward direction. The following figure explains this.



Before collision   After collision

A more accurate calculation is covered in section 4.4. It is to be noted that the total momentum vector before and after collison points in the same direction. This simply means that the total linear momentum is constant before and after the collision. At the time of collision, each particle exerts a force on the other. As the two particles are considered as a system, these forces are only internal, and the total linear momentum cannot be altered by internal forces.

3.5.1 Impulse

If a very large force acts on an object for a very short duration, then the force is called impulsive force or impulse.

If a force (F) acts on the object in a very short interval of time (Δt), from Newton's second law in magnitude form

$$Fdt = dp$$

Integrating over time from an initial time t_i to a final time t_f, we get

$$\int_i^f dp = \int_{t_i}^{t_f} F\,dt$$

$$p_f - p_i = \int_{t_i}^{t_f} F\,dt$$

p_i = initial momentum of the object at time t_i

p_f = final momentum of the object at time t_f

$p_f - p_i = \Delta p =$ change in momentum of the object during the time interval $t_f - t_i = \Delta t$.

The integral $\int_{t_i}^{t_f} F\,dt = J$ is called the impulse and it is equal to change in momentum of the object.

If the force is constant over the time interval, then

$$\int_{t_i}^{t_f} F\,dt = F\int_{t_i}^{t_f} dt = F\left(t_f - t_i\right) = F\Delta t$$

$$F\,\Delta t = \Delta p \qquad (3.24)$$

Equation (3.24) is called the 'impulse-momentum equation'.

For a constant force, the impulse is denoted as $J = F\Delta t$ and it is also equal to change in momentum (Δp) of the object over the time interval Δt.

Impulse is a vector quantity and its unit is Ns.

The average force acted on the object over the short interval of time is defined by

$$F_{avg} = \frac{\Delta p}{\Delta t} \qquad (3.25)$$

From equation (3.25), the average force that act on the object is greater if Δt is smaller. Whenever the momentum of the body changes very quickly, the average force becomes larger.

The impulse can also be written in terms of the average force. Since Δp is change in momentum of the object and is equal to impulse (J), we have

$$J = F_{avg}\Delta t \qquad (3.26)$$

The graphical representation of constant force impulse and variable force impulse is given in Figure 3.21.

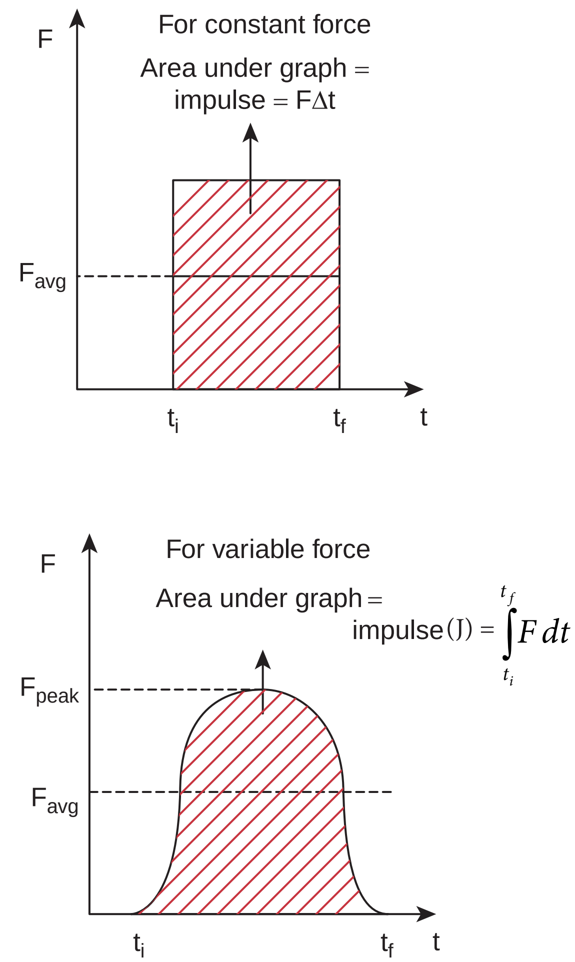


Figure 3.21 Constant force impulse and variable force impulse

Illustration

1. When a cricket player catches the ball, he pulls his hands gradually in the direction of the ball's motion. Why?

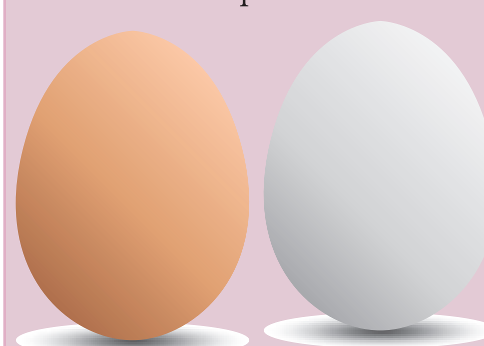
   If he stops his hands soon after catching the ball, the ball comes to rest very quickly. It means that the momentum of the ball is brought to rest very quickly. So the average force acting on the body will be very large. Due to this large average force, the hands will get hurt. To avoid getting hurt, the player brings the ball to rest slowly.

2. When a car meets with an accident, its momentum reduces drastically in a very short time. This is very dangerous for the passengers inside the car since they will experience a large force. To prevent this fatal shock, cars are designed with air bags in such a way that when the car meets with an accident, the momentum of the passengers will reduce slowly so that the average force acting on them will be smaller.

3. The shock absorbers in two wheelers play the same role as airbags in the car. When there is a bump on the road, a sudden force is transferred to the vehicle. The shock absorber prolongs the period of transfer of force on to the body of the rider. Vehicles without shock absorbers will harm the body due to this reason.

4. Jumping on a concrete cemented floor is more dangerous than jumping on the sand. Sand brings the body to rest slowly than the concrete floor, so that the average force experienced by the body will be lesser.

Impulse



If an egg is thrown, can you catch the egg safely without breaking it? How?

EXAMPLE 3.16

An object of mass 10 kg moving with a speed of 15 $m\ s^{-1}$ hits the wall and comes to rest within

a) 0.03 second
b) 10 second

Calculate the impulse and average force acting on the object in both the cases.

Solution

Initial momentum of the object $p_i = 10 \times 15 = 150\ k\ gm\ s^{-1}$

Final momentum of the object $p_f = 0$

$$\Delta p = 150 - 0 = 150\ kg\ m s^{-1}$$

(a) Impulse $J = \Delta p = 150\, N\, s$.

(b) Impulse $J = \Delta p = 150\, N\, s$

(a) Average force $F_{avg} = \frac{\Delta p}{\Delta t} = \frac{150}{0.03} = 5000\, N$

(b) Average force $F_{avg} = \frac{150}{10} = 15\, N$

We see that, impulse is the same in both cases, but the average force is different.

3.6 FRICTION

3.6.1 Introduction

If a very gentle force in the horizontal direction is given to an object at rest on the table, it does not move. It is because of the opposing force exerted by the surface on the object which resists its motion. This force is called the *frictional force which always opposes the relative motion between an object and the surface where it is placed.* If the force applied is increased, the object moves after a certain limit.

Relative motion: when a force parallel to the surface is applied on the object, the force tries to move the object with respect to the surface. This 'relative motion' is opposed by the surface by exerting a frictional force on the object in a direction opposite to applied force. Frictional force always acts on the object parallel to the surface on which the object is placed. There are two kinds of friction namely 1) Static friction and 2) Kinetic friction.

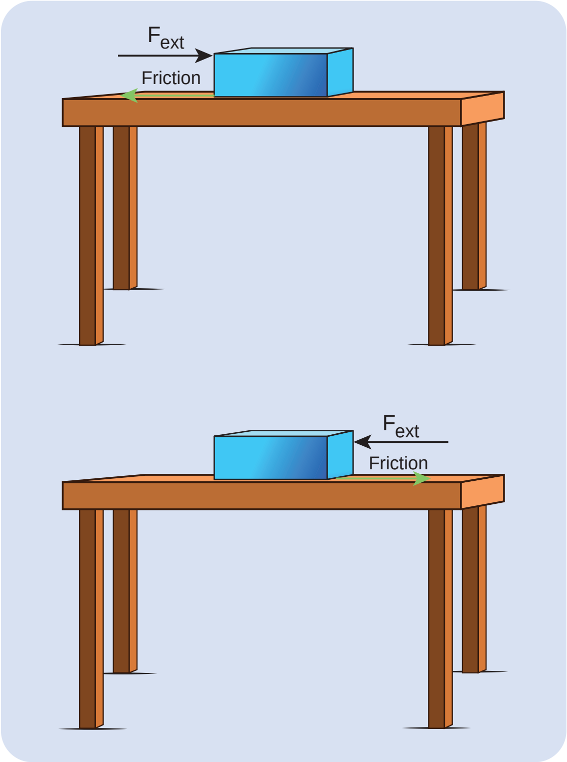


Figure 3.22 Frictional force

3.6.2 Static Friction ($\vec{f}_s$)

Static friction is the force which opposes the initiation of motion of an object on the surface. When the object is at rest on the surface, only two forces act on it. They are the downward gravitational force and upward normal force. The resultant of these two forces on the object is zero. As a result the object is at rest as shown in Figure 3.23.

If some external force F_{ext} is applied on the object parallel to the surface on which the object is at rest, the surface exerts exactly an equal and opposite force on the object to resist its motion and tries to keep the object at rest. It implies that external force and frictional force are exactly equal and opposite. Therefore, no motion parallel to the surface takes place. But if the external force is increased above a particular limit, the surface cannot provide sufficient opposing frictional force to balance the external force on the object. Then the object starts to slide. This is the maximal static friction that can be exerted by the surface. Experimentally, it is found that the magnitude of static frictional force f_s satisfies the following empirical relation.

$$0 \le f_s \le \mu_s N, \qquad (3.27)$$

where μ_s is the coefficient of static friction. It depends on the nature of the surfaces in contact. N is normal force exerted by the surface on the body and sometimes it is equal to mg. But it need not be equal to *mg* always.

Equation (3.27) implies that the force of static friction can take any value from zero to $\mu_s N$.

If the object is at rest and no external force is applied on the object, the static friction acting on the object is zero ($f_s = 0$).

If the object is at rest, and there is an external force applied parallel to the surface, then the force of static friction acting on the object is exactly equal to the external force applied on the object ($f_s = F_{ext}$). But still the static friction f_s is less than $\mu_s N$.

When object begins to slide, the static friction (f_s) acting on the object attains maximum,

The static and kinetic frictions (which we discuss later) depend on the normal force acting on the object. If the object is pressed hard on the surface then the normal force acting on the object will increase. As a consequence it is more difficult to move the object. This is shown in Figure 3.23 (a) and (b). The static friction does not depend upon the area of contact.

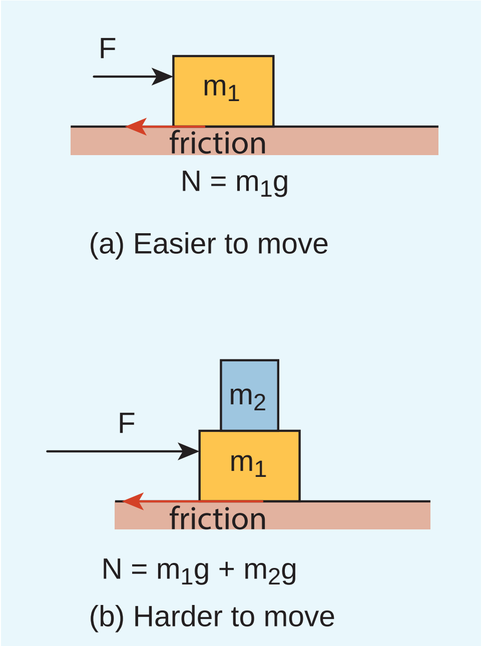


Fig 3.23 Static friction and kinetic friction (a) Easier to move (b) Harder to move

EXAMPLE 3.17

Consider an object of mass 2 kg resting on the floor. The coefficient of static friction between the object and the floor is $\mu_s = 0.8$. What force must be applied on the object to move it?

Solution

Since the object is at rest, the gravitational force experienced by an object is balanced by normal force exerted by floor.

$$N = mg$$

The maximum static frictional force $f_s^{max} = \mu_s N = \mu_s mg$

$$f_s^{max} = 0.8 \times 2 \times 9.8 = 15.68\,N$$

Therefore to move the object the external force should be greater than maximum static friction.

$$F_{ext} > 15.68\,N$$

EXAMPLE 3.18

Consider an object of mass 50 kg at rest on the floor. A Force of 5 N is applied on the object but it does not move. What is the frictional force that acts on the object?

Solution

When the object is at rest, the external force and the static frictional force are equal and opposite.

The magnitudes of these two forces are equal, $f_s = F_{ext}$

Therefore, the static frictional force acting on the object is

$$f_s = 5N.$$

The direction of this frictional force is opposite to the direction of F_{ext}.

EXAMPLE 3.19

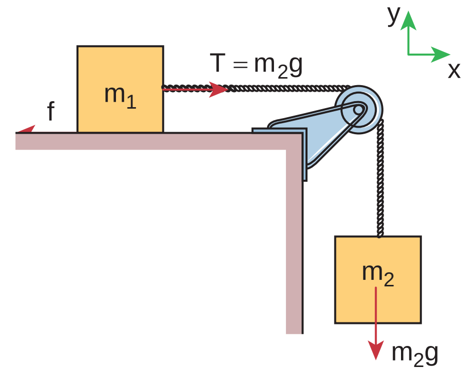
Two bodies of masses 7 kg and 5 kg are connected by a light string passing over a smooth pulley at the edge of the table as shown in the figure. The coefficient of static friction between the surfaces (body and table) is 0.9. Will the mass m_1 = 7 kg on the surface move? If not what value of m_2 should be used so that mass 7 kg begins to slide on the table?

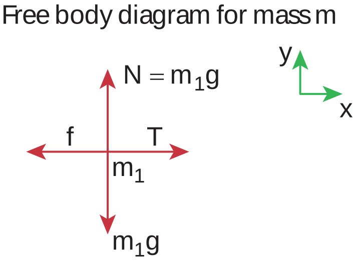
Solution

As shown in the figure, there are four forces acting on the mass m_1

a) Downward gravitational force along the negative y-axis (m_1g)
b) Upward normal force along the positive y axis (N)
c) Tension force due to mass m_2 along the positive x axis
d) Frictional force along the negative x axis

Since the mass m_1 has no vertical motion, $m_1g = N$





Free body diagram for mass m_1

To determine whether the mass m_1 moves on the surface, calculate the maximum static friction exerted by the table on the mass m_1. If the tension on the mass m_1 is equal to or greater than this maximum static friction, the object will move.

$$f_s^{max} = \mu_s N = \mu_s m_1 g$$

$$f_s^{max} = 0.9 \times 7 \times 9.8 = 61.74\ N$$

The tension $T = m_2 g = 5 \times 9.8 = 49\ N$

$$T < f_s^{max}$$

The tension acting on the mass m_1 is less than the maximum static friction. So the mass m_1 will not move.

To move the mass m_1, $T > f_s^{max}$ where $T = m_2 g$

$$m_2 = \frac{\mu_s m_1 g}{g} = \mu_s m_1$$

$$m_2 = 0.9 \times 7 = 6.3\ kg$$

If the mass m_2 is greater than 6.3 kg then the mass m_1 will begin to slide. Note that if there is no friction on the surface, the mass m_1 will move even when m_2 is just 1 kg.

The values of coefficient of static friction for pairs of materials are presented in Table 3.1. Note that the ice and ice pair have very low coefficient of static friction. This means a block of ice can move easily over another block of ice.

Table 3.1 Coefficient of Static Friction for a Pair of Materials

Material	Coefficient of Static Friction
Glass and glass	1.0
Ice and ice	0.10
Steel and steel	0.75
Wood and wood	0.35
Rubber tyre and dry concrete road	1.0
Rubber tyre and wet road	0.7

3.6.3 Kinetic Friction

If the external force acting on the object is greater than maximum static friction, the objects begin to slide. When an object slides, the surface exerts a frictional force called **kinetic friction** $\vec{f}_k$ (also called sliding friction or dynamic friction). To move an object at constant velocity we must apply a force which is equal in magnitude and opposite to the direction of kinetic friction.

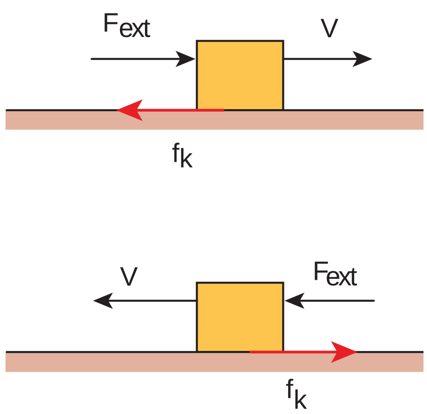


Figure 3.24 Kinetic friction

Experimentally it was found that the magnitude of kinetic friction satisfies the relation

$$f_k = \mu_k N \qquad (3.28)$$

where μ_k is the coefficient of kinetic friction and N the normal force exerted by the surface on the object,

and $$\mu_k < \mu_s$$

This implies that starting of a motion is more difficult than maintaining it. The salient features of static and kinetic friction are given in Table 3.2.

Table 3.2 Salient Features of Static and Kinetic Friction

Static friction	Kinetic friction
It opposes the starting of motion	It opposes the relative motion of the object with respect to the surface
Independent of surface area of contact	Independent of surface area of contact
μ_s depends on the nature of materials in mutual contact	μ_k depends on nature of materials and temperature of the surface
Depends on the magnitude of applied force	Independent of magnitude of applied force
It can take values from zero to $\mu_s N$	It can never be zero and always equals to $\mu_k N$ whatever be the speed (true v <10 ms^{-1})
$f_s^{max} > f_k$	It is less than maximum value of static friction
$\mu_s > \mu_k$	Coefficient of kinetic friction is less than coefficient of static friction

The variation of both static and kinetic frictional forces with external applied force is graphically shown in Figure 3.25.

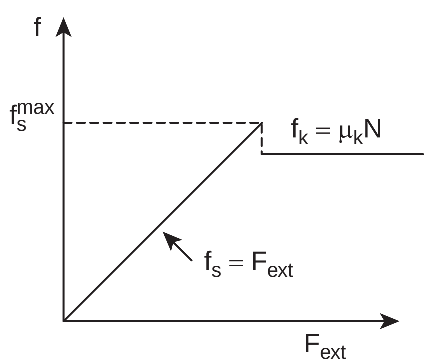


Figure 3.25 Variation of static and kinetic frictional forces with external applied force

The Figure 3.25 shows that static friction increases linearly with external applied force till it reaches the maximum. If the object begins to move then the kinetic friction is slightly lesser than the maximum static friction. Note that the kinetic friction is constant and it is independent of applied force.

Note The relation $f_s = \mu_s N$ is not a vector relation. This is because the normal force N and f_s are not in the same direction even though f_s is equal to μ_s times the normal force. This is also true in the case of kinetic friction.

3.6.4 To Move an Object - Push or pull? Which is easier?

When a body is pushed at an arbitrary angle θ $\left(0 \text{ to } \frac{\pi}{2}\right)$, the applied force F can be resolved into two components as F sinθ parallel to the surface and F cosθ perpendicular to the surface as shown in Figure 3.26. The total downward force acting on the body is mg + Fcosθ. It implies that the normal force acting on the body increases. Since there is no acceleration along the vertical direction the normal force N is equal to

$$N_{push} = mg + F\cos\theta \quad (3.29)$$

As a result the maximal static friction also increases and is equal to

$$f_s^{max} = \mu_s N_{push} = \mu_s \left(mg + F\cos\theta\right) \quad (3.30)$$

Equation (3.30) shows that a greater force needs to be applied to push the object into motion.

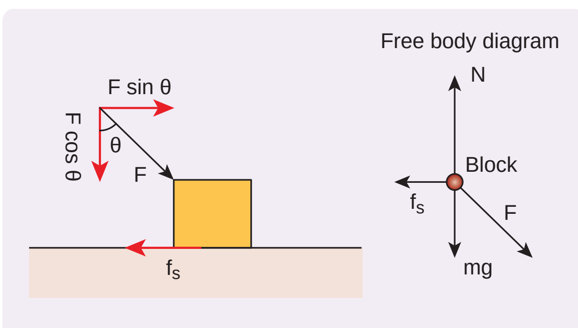


Figure 3.26 An object is pushed at an angle θ

When an object is pulled at an angle θ, the applied force is resolved into two components as shown in Figure 3.27. The total downward force acting on the object is

$$N_{pull} = mg - F\cos\theta \quad (3.31)$$

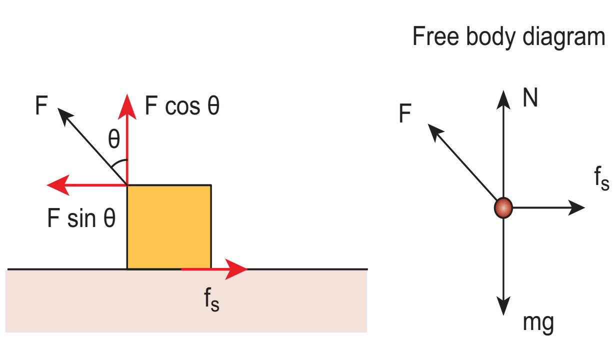


Figure 3.27 An object is pulled at an angle θ

Equation (3.31) shows that the normal force is less than N_{push}. From equations (3.29) and (3.31), it is easier to pull an object than to push to make it move.

3.6.5 Angle of Friction

The angle of friction is defined as the angle between the normal force (N) and the resultant force (R) of normal force and maximum friction force (f_s^{max})

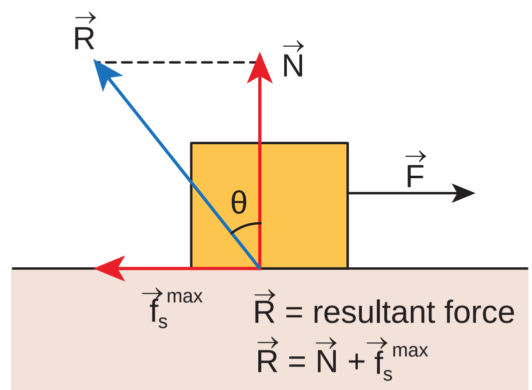


Figure 3.28 Angle of Friction

In Figure 3.28 the resultant force is $R = \sqrt{\left(f_s^{max}\right)^2 + N^2}$

$$\tan\theta = \frac{f_s^{max}}{N} \quad (3.32)$$

But from the frictional relation, the object begins to slide when $f_s^{max} = \mu_s N$

$$\text{or when } \frac{f_s^{max}}{N} = \mu_s \quad (3.33)$$

From equations (3.32) and (3.33) the coefficient of static friction is

$$\mu_s = \tan\theta \quad (3.34)$$

UNIT-3(XI-Physics_Vol-1).indd 141 05-01-2022 22:18:19

The coefficient of static friction is equal to tangent of the angle of friction

3.6.6 Angle of Repose

Consider an inclined plane on which an object is placed, as shown in Figure 3.29. Let the angle which this plane makes with the horizontal be θ. For small angles of θ, the object may not slide down. As θ is increased, for a particular value of θ, the object begins to slide down. This value is called angle of repose. Hence, the angle of repose is the angle of inclined plane with the horizontal such that an object placed on it begins to slide.

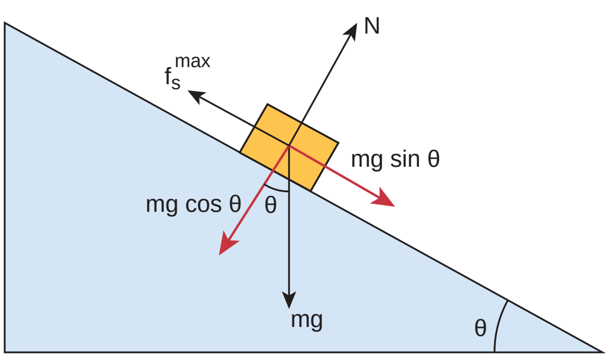


Figure 3.29 Angle of repose

Let us consider the various forces in action here. The gravitational force mg is resolved into components parallel $(mg\sin\theta)$ and perpendicular $(mg\cos\theta)$ to the inclined plane.

The component of force parallel to the inclined plane (mg sinθ) tries to move the object down.

The component of force perpendicular to the inclined plane (mg cosθ) is balanced by the Normal force (N).

$$N = mg\cos\theta$$

When the object just begins to move, the static friction attains its maximum value

$$f_s = f_s^{max} = \mu_s N = \mu_s mg\cos\theta \qquad (3.35)$$

This friction also satisfies the relation

$$f_s^{max} = mg\sin\theta \qquad (3.36)$$

Dividing equations (3.35) and (3.36), we get

$$\mu_s = \sin\theta / \cos\theta$$

From the definition of angle of friction, we also know that

$$\tan\theta = \mu_s, \qquad (3.37)$$

in which θ is the angle of friction.

Thus the angle of repose is the same as angle of friction. But the difference is that the angle of repose refers to inclined surfaces and the angle of friction is applicable to any type of surface.

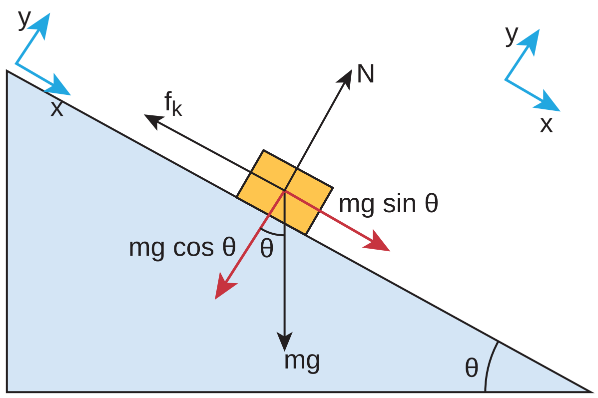
EXAMPLE 3.20

A block of mass m slides down the plane inclined at an angle 60° with an acceleration $\frac{g}{2}$. Find the coefficient of kinetic friction?

Solution

Kinetic friction comes to play as the block is moving on the surface.

The forces acting on the mass are the normal force perpendicular to surface, downward gravitational force and kinetic friction f_k along the surface.



Along the x-direction

$$mg\sin\theta - f_k = \text{ma}$$

But a = g/2

$$mg\sin 60° - f_k = \text{mg/2}$$

$$\frac{\sqrt{3}}{2}\text{mg} - f_k = \text{mg/2}$$

$$f_k = mg\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)$$

$$f_K = \left(\frac{\sqrt{3}-1}{2}\right)\text{mg}$$

There is no motion along the y-direction as normal force is exactly balanced by the mg $\cos\theta$.

$$\text{mg}\cos\theta = \text{N} = \text{mg/2}$$

$$f_K = \mu_K \text{N} = \mu_K \text{mg/2}$$

$$\mu_K = \frac{\left(\frac{\sqrt{3}-1}{2}\right)mg}{\frac{mg}{2}}$$

$$\mu_K = \sqrt{3} - 1$$

3.6.7 Application of Angle of Repose

1. Antlions make sand traps in such a way that when an insect enters the edge of the trap, it starts to slide towards the bottom where the antilon hide itself. The angle of inclination of sand trap is made to be equal to angle of repose. It is shown in the Figure 3.30.

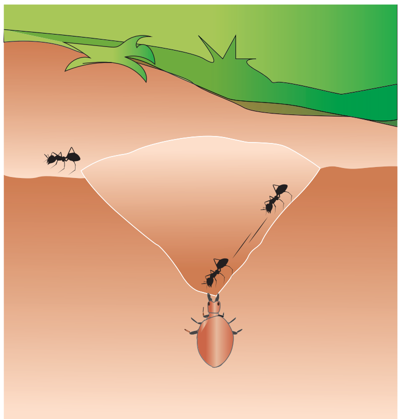


Figure 3.30 Sand trap of antlions

2. Children are fond of playing on sliding board (Figure 3.31). Sliding will be easier

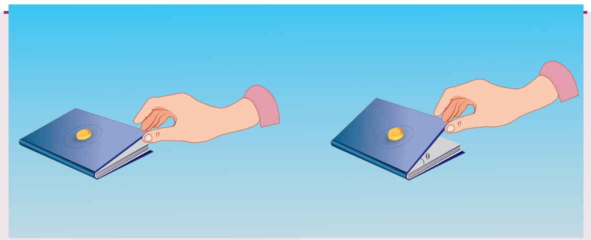
when the angle of inclination of the board is greater than the angle of repose. At the same time if inclination angle is much larger than the angle of repose, the slider will reach the bottom at greater speed and get hurt.

Figure 3.31 Sliding board

ACTIVITY

Measuring the coefficients of friction

Take a hard bound note book and a coin. Keep the coin on the note book. The note book cover has to be in an inclined position as shown in the figure. Slowly increase the angle of inclination of the cover with respect to rest of the pages. When the angle of inclination reaches the angle of repose, the parallel component of gravitational force (mg sinθ) to book surface becomes equal to the frictional force and the coin begins to slide down. Measure the angle of inclination and take the tangent of this angle. It gives the coefficient of static friction between the surface of the cover and coin. The same can be repeated with other objects such as an eraser in



order to observe that the coefficient of static friction differs from case to case.

Note At the point of sliding $\tan\theta_s = \mu_s$

To measure the coefficient of kinetic friction, reduce the inclination of the book after it starts sliding, such that the coin/eraser moves with uniform velocity. Now measure the angle from which coefficient of kinetic friction can be calculated as

$$\mu_K = \tan\theta_k$$

Observe that $\theta_K < \theta_S$

3.6.8 Rolling Friction

The invention of the wheel plays a crucial role in human civilization. One of the important applications is suitcases with rolling on coasters. Rolling wheels makes it easier than carrying luggage. When an object moves on a surface, essentially it is sliding on it. But wheels move on the surface through rolling motion. In rolling motion when a wheel moves on a surface, the point of contact with surface is always at rest. Since the point of contact is at rest, there is no relative motion between the wheel and surface. Hence the frictional force is very less. At the same time if an object moves without a wheel, there is a relative motion between the object and the surface. As a result frictional force is larger. This makes it difficult to move the object. The Figure 3.32 shows the difference between rolling and kinetic friction.

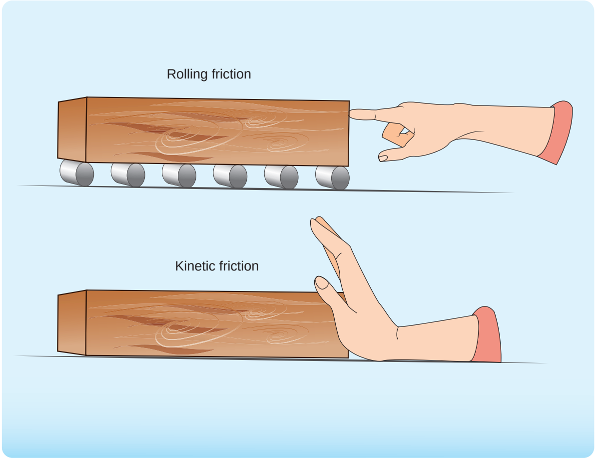


Figure 3.32 Rolling and kinetic friction

Ideally in pure rolling, motion of the point of contact with the surface should be at rest, but in practice it is not so. Due to the elastic nature of the surface at the point of contact there will be some deformation on the object at this point on the wheel or surface as shown in Figure 3.33. Due to this deformation, there will be minimal friction between wheel and surface. It is called 'rolling friction'. In fact, 'rolling friction' is much smaller than kinetic friction.

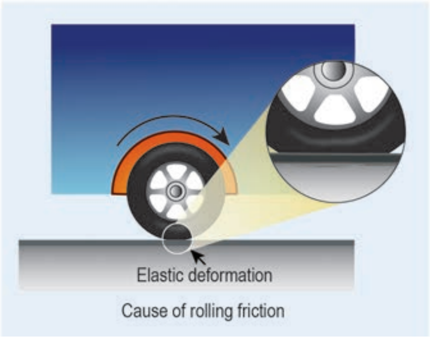


Figure 3.33 Rolling friction

3.6.9 Methods to Reduce Friction

Frictional force has both positive and negative effects. In some cases it is absolutely necessary. Walking is possible because of frictional force. Vehicles (bicycle, car) can move because of the frictional force between the tyre and the road. In the braking system, kinetic friction plays a major role. As we have already seen, the frictional force comes into effect whenever there is relative motion between two surfaces. In big machines used in industries, relative motion between different parts of the machine produce unwanted heat which reduces its efficiency. To reduce this kinetic friction lubricants are used as shown in Figure 3.34.

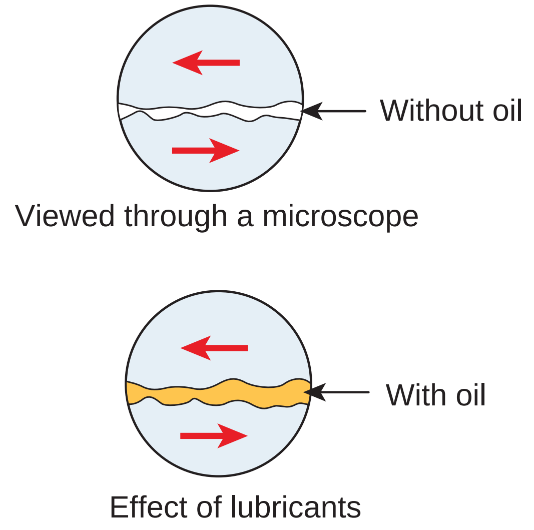


Figure 3.34 Reducing kinetic friction using lubricant

Ball bearings provides another effective way to reduce the kinetic friction (Figure 3.35) in machines. If ball bearings are fixed between two surfaces, during the relative motion only the rolling friction comes to effect and not kinetic friction. As we have seen earlier, the rolling friction is much smaller than kinetic friction; hence the machines are protected from wear and tear over the years.

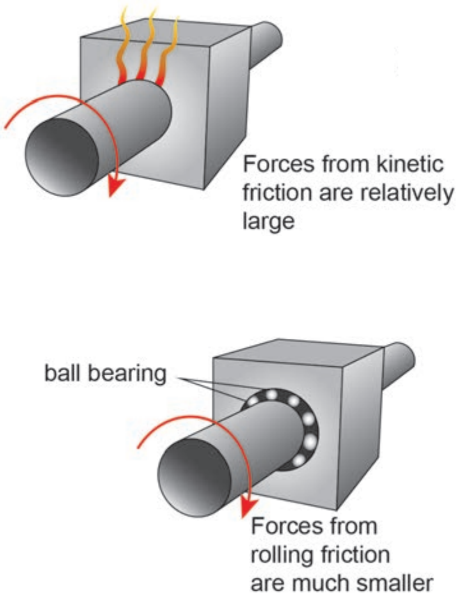


Figure 3.35 Reducing kinetic friction using ball bearing

During the time of Newton and Galileo, frictional force was considered as one of the natural forces like gravitational force. But in the twentieth century, the understanding on atoms, electron and protons has changed the perspective. The frictional force is actually the electromagnetic force between the atoms on the two surfaces. Even well polished surfaces have irregularities on the surface at the microscopic level as seen in the Figure 3.36.

POINTS TO PONDER

When you walk on the tiled floor where water is spilled, you are likely to slip. Why?

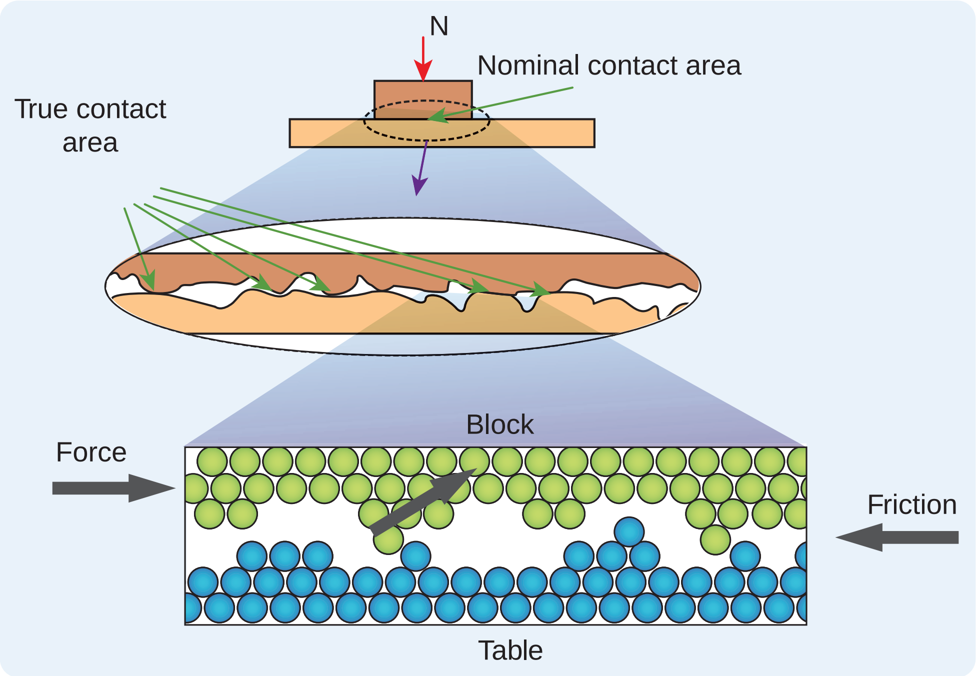
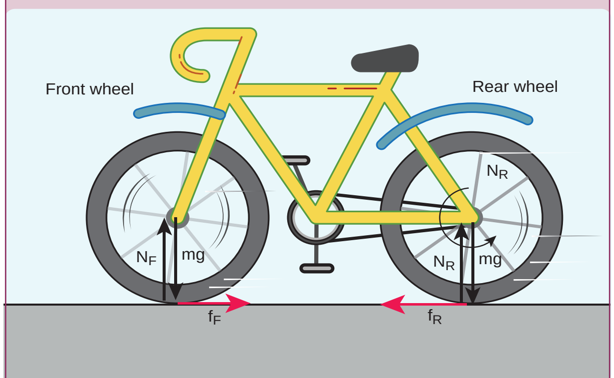


Figure 3.36 Irregularities on the surface at the microscopic level

Frictional force in the motion of a bicycle

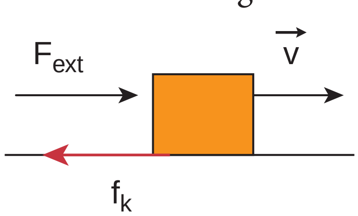
When a bicycle moves in the forward direction, what is the direction of frictional force in the rear and front wheels?



When we pedal a bicycle, we try to push the surface backward and the velocity of point of contact in the rear wheel is backwards. So, the frictional force pushes the rear wheel to move forward. But as the front wheel is connected with a rigid support to the back wheel, the forward motion of back wheel pushes the front wheel in the forward direction. So, the frictional forces act backward. Remember both frictional forces correspond to only static friction and not kinetic friction. If the wheel slips then kinetic friction comes into effect. In addition to static friction, the rolling friction also acts on both wheels in the backward direction.

EXAMPLE 3.21

Consider an object moving on a horizontal surface with a constant velocity. Some external force is applied on the object to keep the object moving with a constant velocity. What is the net force acting on the object?



Solution

If an object moves with constant velocity, then it has no acceleration. According to Newton's second law there is no net force acting on the object. The external force is balanced by the kinetic friction.

Note It is not that 'no force acts on the object'. In fact there are two forces acting on the object. Only the net force acting on the object is zero.

3.7 DYNAMICS OF CIRCULAR MOTION

In the previous sections we have studied how to analyse linear motion using Newton's laws. It is also important to know how to apply Newton's laws to circular motion, since circular motion is one of the very common types of motion that we come across in our daily life. A particle can be in linear motion with or without any external force. But when circular motion occurs there must necessarily be some force acting on the object. There is no Newton's first law for circular motion. In other words without a force, circular motion cannot occur in nature. A force can change the velocity of a particle in three different ways.

1. The magnitude of the velocity can be changed without changing the direction of the velocity. In this case the particle will move in the same direction but with acceleration.

Examples

Particle falling down vertically, bike moving in a straight road with acceleration.

2. The direction of motion alone can be changed without changing the magnitude (speed). If this happens continuously then we call it 'uniform circular motion'.
3. Both the direction and magnitude (speed) of velocity can be changed. If this happens non circular motion occurs. For example oscillation of a swing or simple pendulum, elliptical motion of planets around the Sun.

In this section we will deal with uniform circular motion and non-uniform circular motion.

3.7.1 Centripetal force

If a particle is in uniform circular motion, there must be centripetal acceleration towards the centre of the circle. If there is acceleration then there must be some force acting on it with respect to an inertial frame. This force is called centripetal force.

As we have seen in chapter 2, the centripetal acceleration of a particle in the circular motion is given by $a = \frac{v^2}{r}$ and it acts towards centre of the circle. According to Newton's second law, the centripetal force is given by

$$F_{cp} = ma_{cp} = \frac{mv^2}{r}$$

The word Centripetal force means centre seeking force.

In vector notation $\vec{F}_{cp} = -\frac{mv^2}{r}\hat{r}$

For uniform circular motion $\vec{F}_{cp} = -m\omega^2 r\,\hat{r}$

The direction $-\hat{r}$ points towards the centre of the circle which is the direction of centripetal force as shown in Figure 3.38.

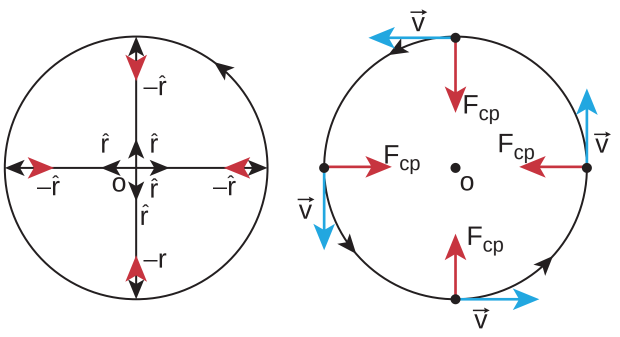


Figure 3.38 Centripetal force

It should be noted that 'centripetal force' is not other forces like gravitational force or spring force. It can be said as 'force towards centre'. The origin of the centripetal force can be gravitational force, tension in the string, frictional force, Coulomb force etc. Any of these forces can act as a centripetal force.

1. In the case of whirling motion of a stone tied to a string, the centripetal force on the particle is provided by the tensional force on the string. In circular motion in an amusement park, the centripetal force is provided by the tension in the iron ropes.
2. In motion of satellites around the Earth, the centripetal force is given by Earth's gravitational force on the satellites. Newton's second law for satellite motion is

$$F = \text{earth's gravitational force} = \frac{mv^2}{r}$$

Where r- distance of the planet from the centre of the Earth.

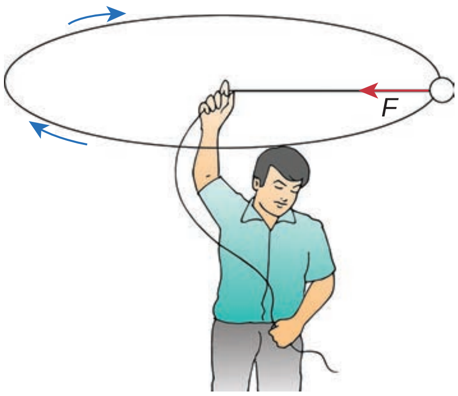


Figure 3.39 Whirling motion of objects

m-mass of the satellite

v-speed of the satellite

3. When a car is moving on a circular track the centripetal force is given by the frictional force between the road and the tyres.

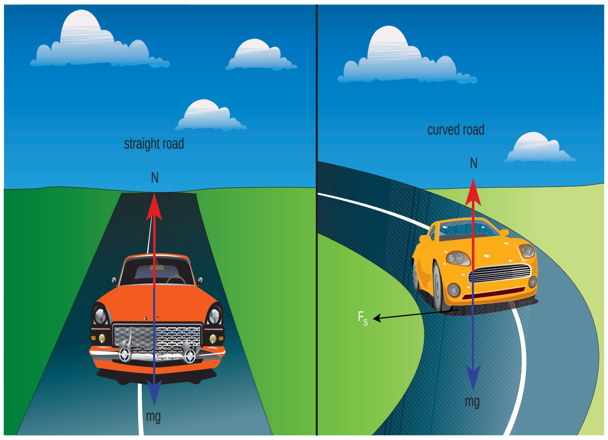


Figure 3.40 Car in the circular track

Newton's second law for this case is

$$\text{Frictional force} = \frac{mv^2}{r}$$

m-mass of the car

v-speed of the car

r-radius of curvature of track

Even when the car moves on a curved track, the car experiences the centripetal force which is provided by frictional force between the surface and the tyre of the car. This is shown in the Figure 3.41.

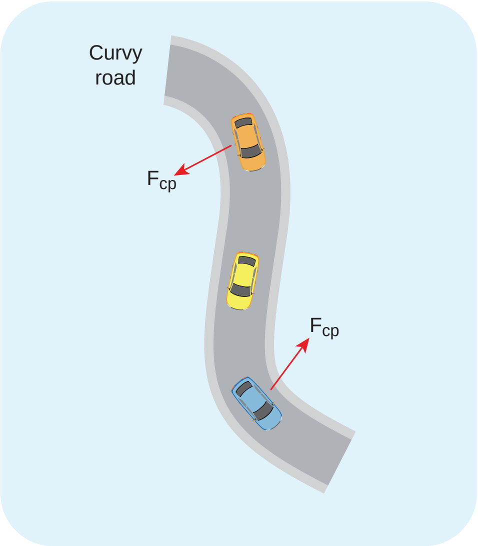


Figure 3.41 Centripetal force due to frictional force between the road and tyre

4. When the planets orbit around the Sun, they experience centripetal force towards the centre of the Sun. Here gravitational force of the Sun acts as centripetal force on the planets as shown in Figure 3.42

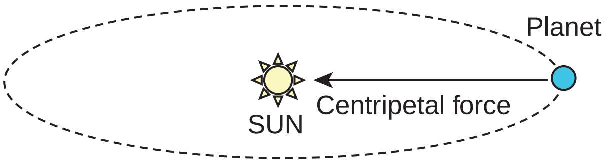


Figure 3.42 Centripetal force on the orbiting planet due Sun's gravity

Newton's second law for this motion

Gravitational force of Sun on the planet $= \frac{mv^2}{r}$

EXAMPLE 3.22

If a stone of mass 0.25 kg tied to a string executes uniform circular motion with a speed of 2 m s^{-1} of radius 3 m, what is the magnitude of tensional force acting on the stone?

Solution: $F_{cp} = \frac{mv^2}{r}$

$$F_{cp} = \frac{\frac{1}{4}\times(2)^2}{3} = 0.333 \text{ N.}$$

EXAMPLE 3.23

The Moon orbits the Earth once in 27.3 days in an almost circular orbit. Calculate the centripetal acceleration experienced by the Moon? (Radius of the Earth is 6.4 × 10^6 m)

Solution

The centripetal acceleration is given by $a = \frac{v^2}{r}$. This expression explicitly depends on Moon's speed which is non trivial. We can work with the formula

$$\omega^2 R_m = a_m$$

a_m is centripetal acceleration of the Moon due to Earth's gravity.

ω is angular velocity.

R_m is the distance between Earth and the Moon, which is 60 times the radius of the Earth.

$$R_m = 60R = 60\times6.4\times10^6 = 384\times10^6\ m$$

As we know the angular velocity $\omega = \frac{2\pi}{T}$ and T = 27.3 days = 27.3 × 24 × 60 × 60 second = 2.358 × 10^6 sec

By substituting these values in the formula for acceleration

$$a_m = \frac{(4\pi^2)(384\times10^6)}{(2.358\times10^6)^2} = 0.00272 \text{ m s}^{-2}$$

The centripetal acceleration of Moon towards the Earth is 0.00272 m s^{-2}

Note: This result was calculated by Newton himself. In unit 6 we will use this result.

3.7.2 Vehicle on a leveled circular road

When a vehicle travels in a curved path, there must be a centripetal force acting on it. This centripetal force is provided by the frictional force between tyre and surface of the road. Consider a vehicle of mass 'm' moving at a speed 'v' in the circular track of radius 'r'. There are three forces acting on the vehicle when it moves as shown in the Figure 3.43

1. Gravitational force (mg) acting downwards
2. Normal force (N) acting upwards
3. Frictional force (F_s) acting horizontally inwards along the road

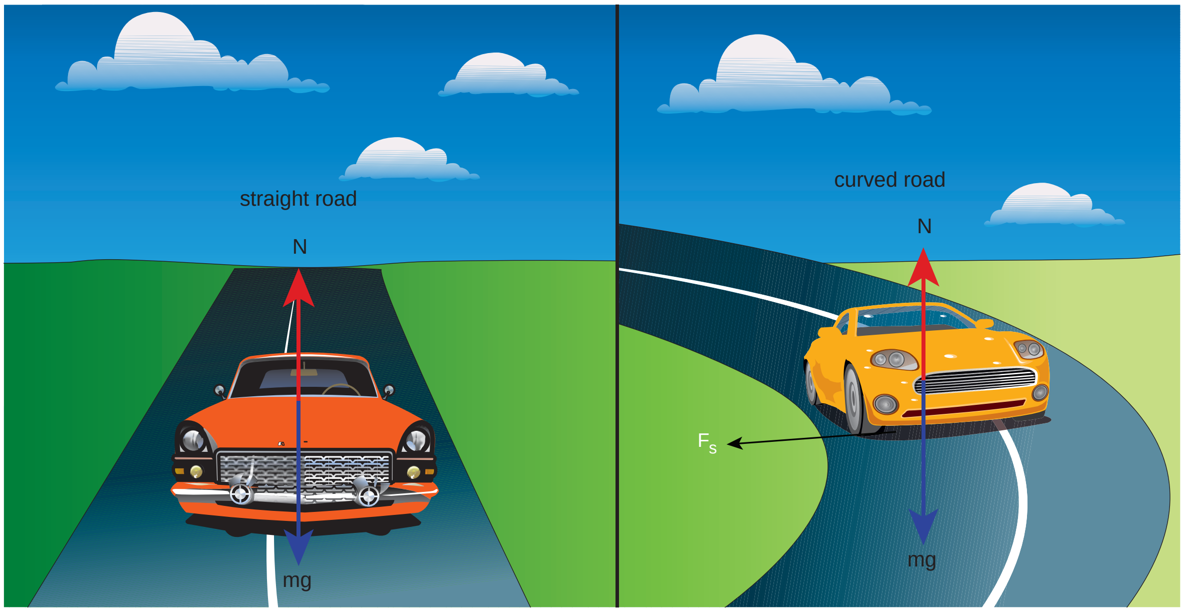


Figure 3.43 Forces acting on the vehicle on a leveled circular road

Suppose the road is horizontal then the normal force and gravitational force are exactly equal and opposite. The centripetal force is provided by the force of static friction F_s between the tyre and surface of the road which acts towards the centre of the circular track,

$$\frac{mv^2}{r} = F_s$$

As we have already seen in the previous section, the static friction can increase from zero to a maximum value

$$F_s \leq \mu_s mg.$$

There are two conditions possible:

a) If $\frac{mv^2}{r} \leq \mu_s mg$, or $\mu_s \geq \frac{v^2}{rg}$ or $\sqrt{\mu_s rg} \geq v$
(Safe turn)

The static friction would be able to provide necessary centripetal force to bend the car on the road. So the coefficient of static friction between the tyre and the surface of the road determines what maximum speed the car can have for safe turn.

b) If $\frac{mv^2}{r} > \mu_s mg$, or $\mu_s < \frac{v^2}{rg}$ (skid)

If the static friction is not able to provide enough centripetal force to turn, the vehicle will start to skid.

EXAMPLE 3.24

Consider a circular leveled road of radius 10 m having coefficient of static friction 0.81. Three cars (A, B and C) are travelling with speed 7 m s^{-1}, 8 m s^{-1} and 10 ms^{-1} respectively. Which car will skid when it moves in the circular level road? (g =10 m s^{-2})

Solution

From the safe turn condition the speed of the vehicle (v) must be less than or equal to $\sqrt{\mu_s rg}$

$$v \le \sqrt{\mu_s rg}$$

$$\sqrt{\mu_s rg} = \sqrt{0.81 \times 10 \times 10} = 9\, m\, s^{-1}$$

For Car C, $\sqrt{\mu_s rg}$ is less than v

The speed of car A, B and C are 7 m s^{-1}, 8 m s^{-1} and 10 m s^{-1} respectively. The cars A and B will have safe turns. But the car C has speed 10 m s^{-1} while it turns which exceeds the safe turning speed. Hence, the car C will skid.

3.7.3 Banking of Tracks

In a leveled circular road, skidding mainly depends on the coefficient of static friction μ_s The coefficient of static friction depends on the nature of the surface which has a maximum limiting value. To avoid this problem, usually the outer edge of the road is slightly raised compared to inner edge as shown in the Figure 3.44. This is called banking of roads or tracks. This introduces an inclination, and the angle is called banking angle.

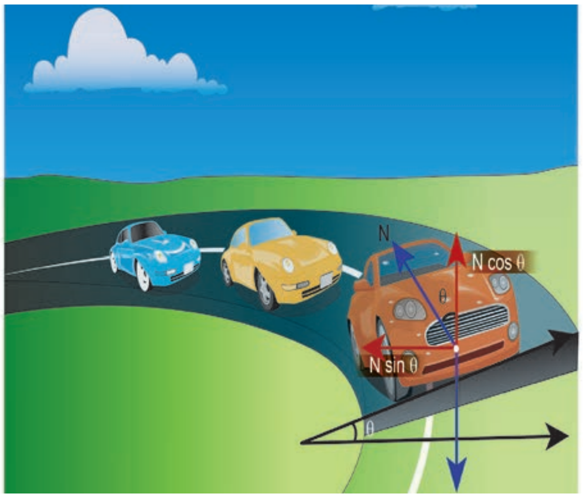


Figure 3.44 Outer edge of the road is slightly raised to avoid skidding

Let the surface of the road make angle θ with horizontal surface. Then the normal force makes the same angle θ with the vertical. When the car takes a turn, there are two forces acting on the car:

a) Gravitational force mg (downwards)

b) Normal force N (perpendicular to surface)

We can resolve the normal force into two components. $N\cos\theta$ and $N\sin\theta$ as shown in Figure 3.46. The component $N\cos\theta$ balances the downward gravitational force 'mg' and component $N\sin\theta$ will provide the necessary centripetal acceleration. By using Newton second law

$$N\cos\theta = mg$$

$$N\sin\theta = \frac{mv^2}{r}$$

By dividing the equations we get $\tan\theta = \frac{v^2}{rg}$

$$v = \sqrt{rg\tan\theta}$$

The banking angle θ and radius of curvature of the road or track determines the safe speed of the car at the turning. If the speed of car exceeds this safe speed, then it starts to skid outward but frictional force comes into effect and provides an additional centripetal force to prevent the outward skidding. At the same time, if the speed of the car is little lesser than safe speed, it starts to skid inward and frictional force comes into effect, which reduces centripetal force to prevent inward skidding. However if the speed of the vehicle is sufficiently greater than the correct speed, then frictional force cannot stop the car from skidding.

EXAMPLE 3.25

Consider a circular road of radius 20 meter banked at an angle of 15 degree. With what speed a car has to move on the turn so that it will have safe turn?

Solution

$$v = \sqrt{(rg\tan\theta)} = \sqrt{20 \times 9.8 \times \tan 15°}$$

$$= \sqrt{20 \times 9.8 \times 0.26} = 7.1 \text{ m s}^{-1}$$

The safe speed for the car on this road is 7.1 m s^{-1}

3.7.4 Centrifugal Force

Circular motion can be analysed from two different frames of reference. One is the inertial frame (which is either at rest or in uniform motion) where Newton's laws are obeyed. The other is the rotating frame of reference which is a non-inertial frame of reference as it is accelerating. When we examine the circular motion from these frames of reference the situations are entirely different. To use Newton's first and second laws in the rotational frame of reference, we need to include a pseudo force called 'centrifugal force'. This 'centrifugal force' appears to act on the object with respect to rotating frames. To understand the concept of centrifugal force, we can take a specific case and discuss as done below.

Consider the case of a whirling motion of a stone tied to a string. Assume that the stone has angular velocity ω in the inertial frame (at rest). If the motion of the stone is observed from a frame which is also rotating along with the stone with same angular velocity ω then, the stone appears to be at rest. This implies that in addition to the inward centripetal force $-m\omega^2 r$ there must be an equal and opposite force that acts on the stone outward with value $+m\omega^2 r$. So the total force acting on the stone in a rotating frame is equal to zero $(-m\omega^2 r + m\omega^2 r = 0)$. This outward force $+m\omega^2 r$ is called the centrifugal force. The word 'centrifugal' means 'flee from centre'. Note that the 'centrifugal force' appears to act on the particle, only when we analyse the motion from a rotating frame. With respect to an inertial frame there is only centripetal force which is given by the tension in the string. For this reason centrifugal force is called as a 'pseudo force'. A pseudo force has no origin. It arises due to the non inertial nature of the frame considered. When circular motion problems are solved from a rotating frame of reference, while drawing free body diagram of a particle, the centrifugal force should necessarily be included as shown in the Figure 3.45.

3.7.5 Effects of Centrifugal Force

Although centrifugal force is a pseudo force, its effects are real. When a car takes a turn in a curved road, person inside the car feels an outward force which pushes the person away. This outward force is also called centrifugal force. If there is sufficient friction between the person and the seat, it will prevent the person from moving outwards. When a car moving in a straight line suddenly takes a turn, the objects not fixed to the car try to continue in linear motion due to their inertia of direction. While observing this motion from an inertial frame, it appears as a straight line as shown in Figure 3.46. But, when it is observed from the rotating frame it appears to move outwards.

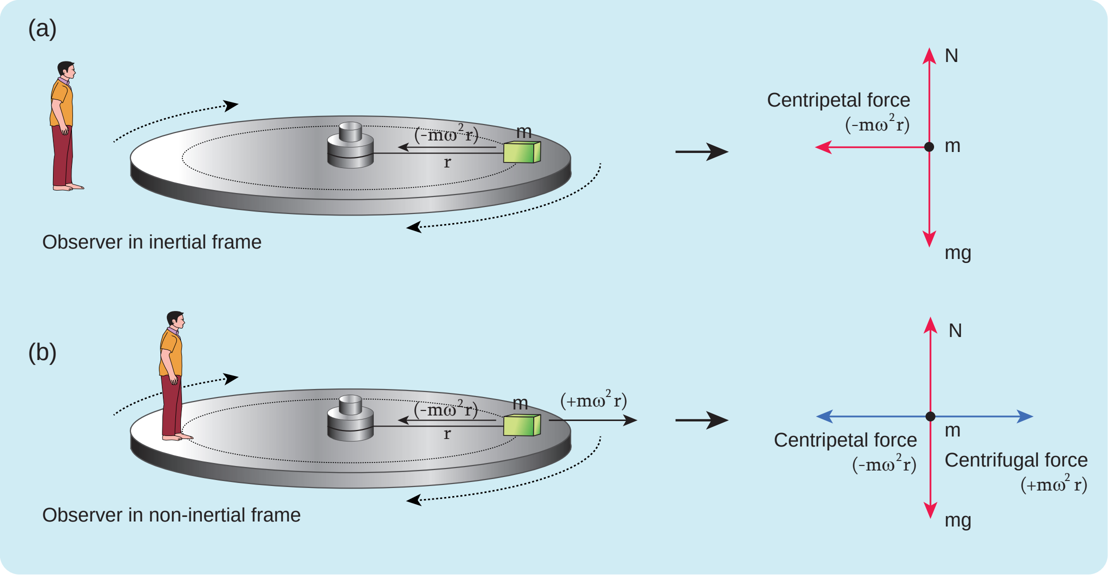


Figure 3.45 Free body diagram of a particle including the centrifugal force

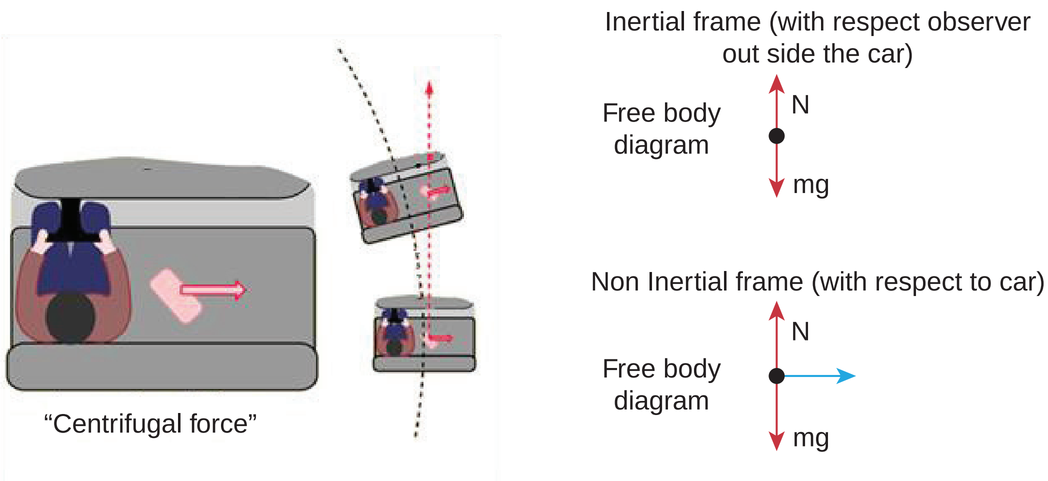


Figure 3.46 Effects of centrifugal force

A person standing on a rotating platform feels an outward centrifugal force and is likely to be pushed away from the platform. Many a time the frictional force between the platform and the person is not sufficient to overcome outward push. To avoid this, usually the outer edge of the platform is little inclined upwards which exerts a normal force on the person which prevents the person from falling as illustrated in Figures 3.47.

Caution!

It is dangerous to stand near the open door (or) steps while travelling in the bus. When the bus takes a sudden turn in a curved road, due to centrifugal force the person is pushed away from the bus. Even though centrifugal force is a pseudo force, its effects are real.

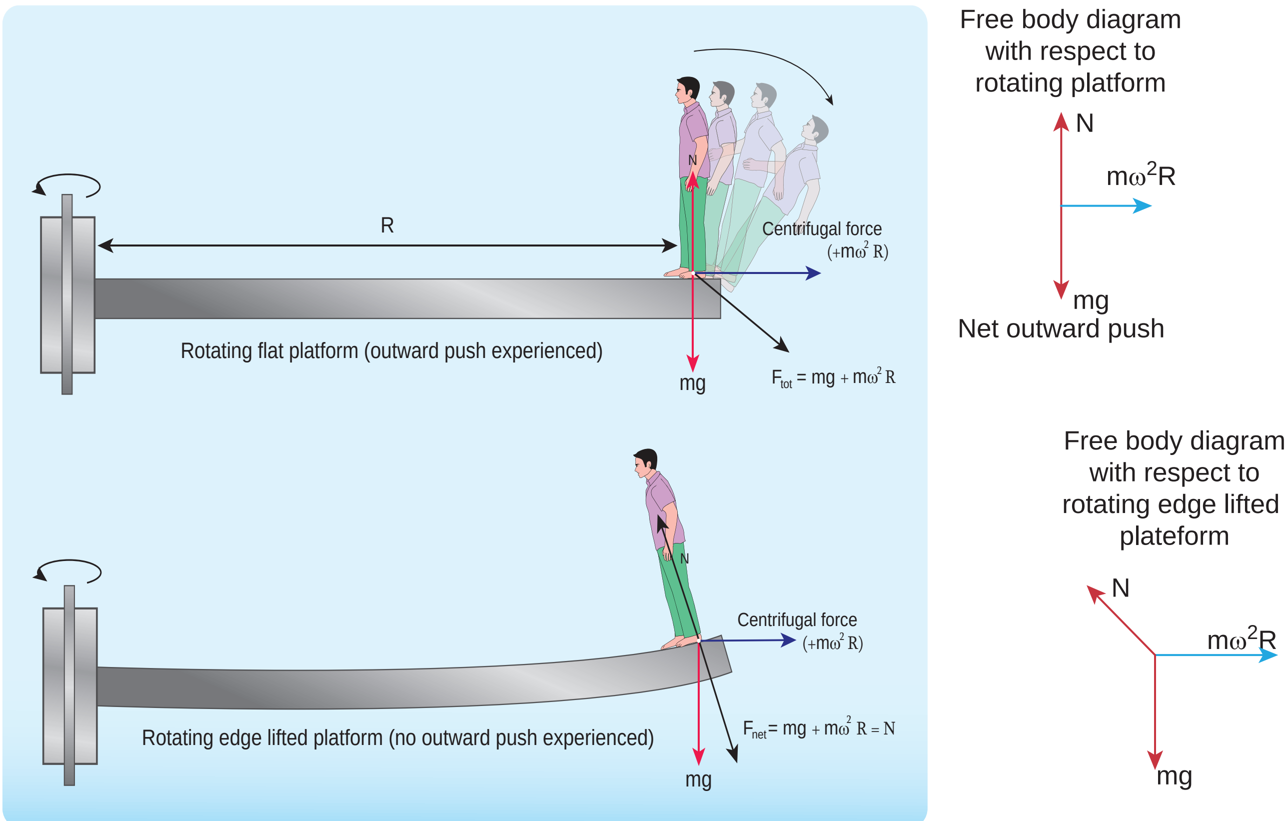


Figure 3.47 Outward centrifugal force in rotating platform

3.7.6 Centrifugal Force due to Rotation of the Earth

Even though Earth is treated as an inertial frame, it is actually not so. Earth spins about its own axis with an angular velocity ω. Any object on the surface of Earth (rotational frame) experiences a centrifugal force. The centrifugal force appears to act exactly in opposite direction from the axis of rotation. It is shown in the Figure 3.48.

The centrifugal force on a man standing on the surface of the Earth is $F_{cf} = m\omega^2 r$

where r is perpendicular distance of the man from the axis of rotation. By using right angle triangle as shown in the Figure 3.48, the distance $r = R\cos\theta$

Here R = radius of the Earth

and θ = latitude of the Earth where the man is standing.

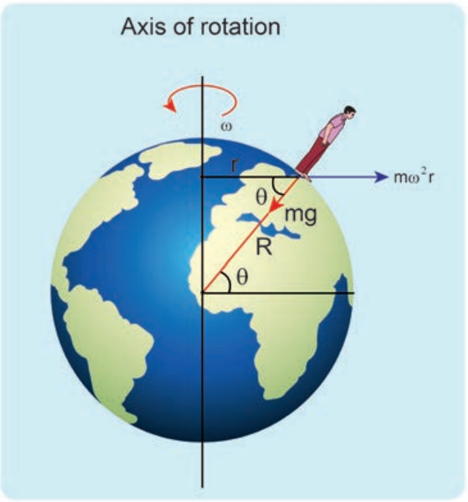


Figure 3. 48 Centrifugal force acting on a man on the surface of Earth

EXAMPLE 3.26

Calculate the centrifugal force experienced by a man of 60 kg standing at Chennai? (Given: Latitude of Chennai is $13°$

Solution

The centrifugal force is given by $F_c = m\omega^2 R\cos\theta$

The angular velocity (ω) of Earth $= \frac{2\pi}{T}$, where T is time period of the Earth (24 hours)

$$\omega = \frac{2\pi}{24\times60\times60} = \frac{2\pi}{86400}$$
$$= 7.268\times10^{-5}\, rad sec^{-1}$$

The radius of the Earth R = 6400 Km = 6400×10^3 m

Latitude of Chennai $= 13°$

$$F_{cf} = 60\times\left(7.268\times10^{-5}\right)^2 \times 6400\times10^3$$
$$\times\cos\left(13°\right) = 1.9678\,N$$

A 60 kg man experiences centrifugal force of approximately 2 Newton. But due to Earth's gravity a man of 60 kg experiences a force = mg = 60 × 9.8 = 588N. This force is very much larger than the centrifugal force.

3.7.7 Centripetal Force Versus Centrifugal Force

Salient features of centripetal and centrifugal forces are compared in Table 3.4.

Table 3.4 Salient Features of Centripetal and Centrifugal Forces

Centripetal force	Centrifugal force
It is a real force which is exerted on the body by the external agencies like gravitational force, tension in the string, normal force etc.	It is a pseudo force or fictitious force which cannot arise from gravitational force, tension force, normal force etc.
Acts in both inertial and non-inertial frames	Acts only in rotating frames (non-inertial frame)
It acts towards the axis of rotation or centre of the circle in circular motion	It acts outwards from the axis of rotation or radially outwards from the centre of the circular motion
$\left\|F_{cp}\right\| = m\omega^2 r = \frac{mv^2}{r}$	$\left\|F_{cf}\right\| = m\omega^2 r = \frac{mv^2}{r}$
Real force and has real effects	Pseudo force but has real effects
Origin of centripetal force is interaction between two objects.	Origin of centrifugal force is inertia. It does not arise from interaction. In an inertial frame the object's inertial motion appears as centrifugal force in the rotating frame.
In inertial frames centripetal force has to be included when free body diagrams are drawn.	In inertial frames there is no centrifugal force. In rotating frames, both centripetal and centrifugal force have to be included when free body diagrams are drawn.

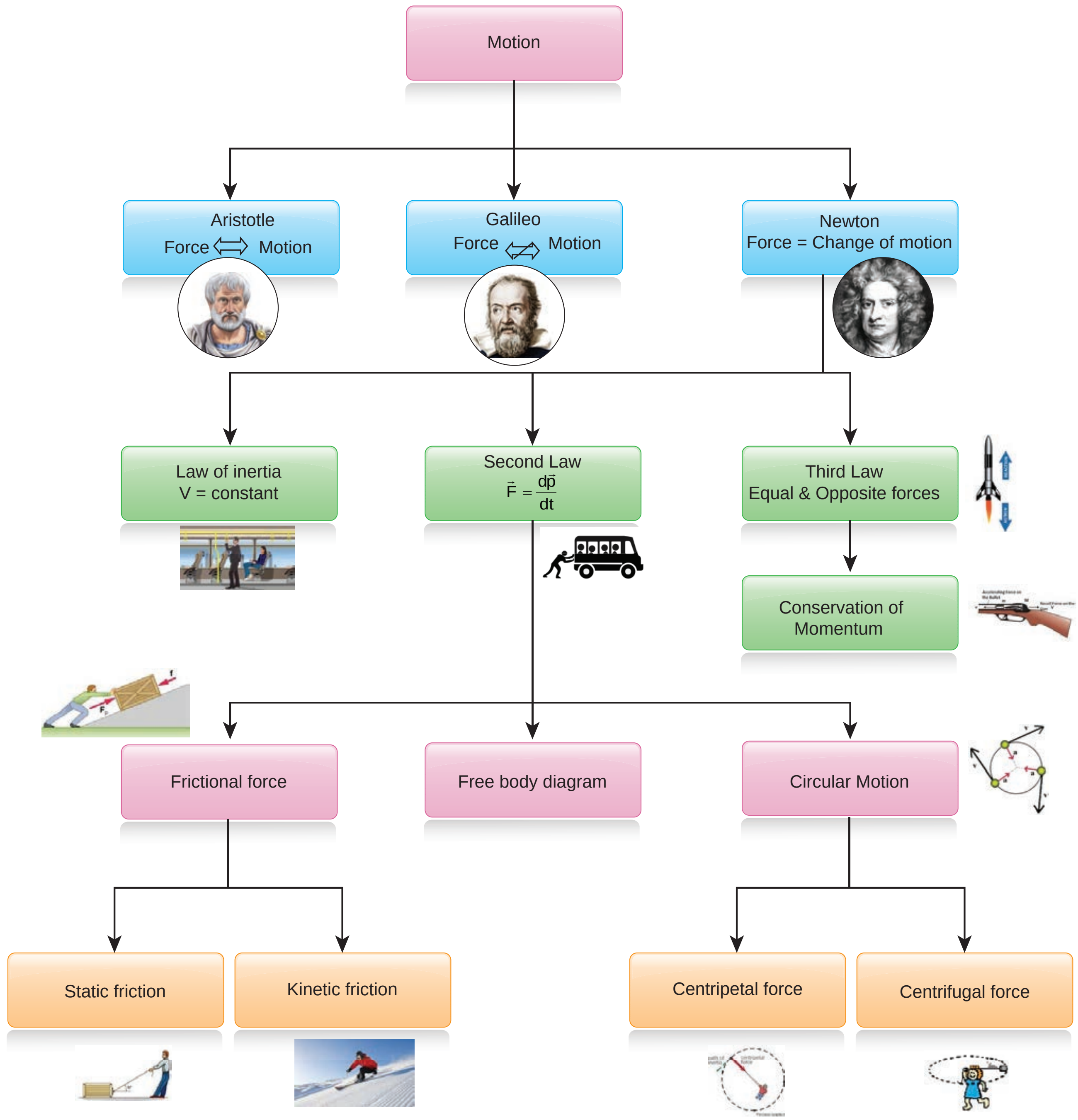
SUMMARY

- Aristotle's idea of motion: To maintain motion, a force is required
- Galileo's idea of motion: To maintain motion, a force is not required
- Mass is a measure of inertia of the body
- Newton's first law states that under no external force, the object continues its state of motion or state of rest.
- Newton's second law states that to change the momentum of the body, external force is required
  Mathematically it is defined as $\vec{F} = \frac{d\vec{p}}{dt}$
- Both Newton's first and second laws are valid only in inertial frames
- Inertial frame is the one in which if there is no force on the object, the object moves at constant velocity.
- Newton's third law states that for every force there is an equivalent and opposite force and such a pair of forces is called action and reaction pair.
- To draw a free body diagram for an object,
  - Isolate the object from other objects and identify the forces acting on it
  - The force exerted by that object should not be taken into account
  - Draw the direction of each force with relative magnitude
  - Apply Newton's second law in each direction
- If no net external force acts on a collection of particles (system), then the total momentum of the collection of particles (system) is a constant vector.
- Internal forces acting in the system cannot change the total momentum of the system.
- Lami's theorem states that if an object is in equilibrium under the concurrent forces, then the ratio of each force with the sine of corresponding opposite angle is same.
- An impulse acting on a body is equal to the change in momentum of the body. Whenever a force acts on the object for a very short time, it is difficult to calculate the force. But impulse can be calculated.
- Static friction is the force which always opposes the movement of the object from rest. It can take values from zero to $\mu_s N$. If an external force is greater than $\mu_s N$ then object begins to move.
- If the object begins to move, kinetic friction comes into effect. To move an object with constant velocity, the external force must be applied to overcome the kinetic friction. The kinetic friction is $\mu_k N$.
- Rolling friction is much smaller than static and kinetic friction. This is the reason that to move an object roller coaster is fixed in the bottom of the object. Example: Rolling suitcase

SUMMARY (*cont*)

- The origin of friction is electromagnetic interaction between the atoms of two surfaces which are touching each other.
- Whenever there is a motion along a curve, there must be a centripetal force that acts towards the centre of the curve. In uniform circular motion the centripetal force acts at the centre of the circle.
- The centripetal force is not a separate natural force. Any natural force can behave as centripetal force. In planetary motion, Sun's gravitational force acts as centripetal force. In the whirling motion of a stone attached to a string, the centripetal force is given by the string. When Moon orbits the Earth, it experiences Earth's gravitational force as centripetal force.
- Centrifugal force arises whenever the motion is analysed from rotating frame. It is a pseudo force. The inertial motion of the object appears as centrifugal force in the rotating frame.
- The magnitude of centrifugal and centripetal force is $m\omega^2 r$. But centripetal force acts towards centre of the circular motion and centrifugal force appears to acts in the opposite direction to centripetal force.

CONCEPT MAP


Motion
Aristotle
Force ⟺ Motion
Galileo
Force ⇎ Motion
Newton
Force = Change of motion
Law of inertia
V = constant
Second Law
$\vec{F} = \frac{d\vec{p}}{dt}$
Third Law
Equal & Opposite forces
Conservation of Momentum
Frictional force
Free body diagram
Circular Motion
Static friction
Kinetic friction
Centripetal force
Centrifugal force

EVALUATION

I. Multiple Choice Questions

1. When a car takes a sudden left turn in the curved road, passengers are pushed towards the right due to

(a) inertia of direction

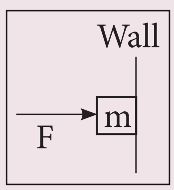
(b) inertia of motion

(c) inertia of rest

(d) absence of inertia

2. An object of mass m held against a vertical wall by applying horizontal force F as shown in the figure. The minimum value of the force F is

(IIT JEE 1994)



(a) Less than mg

(b) Equal to mg

(c) Greater than mg

(d) Cannot determine

3. A vehicle is moving along the positive x direction, if sudden brake is applied, then

(a) frictional force acting on the vehicle is along negative x direction

(b) frictional force acting on the vehicle is along positive x direction

(c) no frictional force acts on the vehicle

(d) frictional force acts in downward direction

4. A book is at rest on the table which exerts a normal force on the book. If this force is considered as reaction force, what is the action force according to Newton's third law?

(a) Gravitational force exerted by Earth on the book

(b) Gravitational force exerted by the book on Earth

(c) Normal force exerted by the book on the table

(d) None of the above

5. Two masses m_1 and m_2 are experiencing the same force where $m_1 < m_2$. The ratio of their acceleration $\frac{a_1}{a_2}$ is

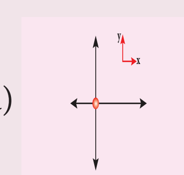
(a) 1

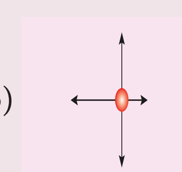
(b) less than 1

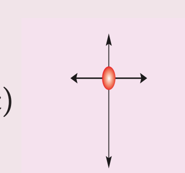
(c) greater than 1

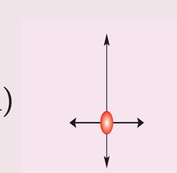
(d) all the three cases

6. Choose appropriate free body diagram for the particle experiencing net acceleration along negative y direction. (Each arrow mark represents the force acting on the system).

a) 

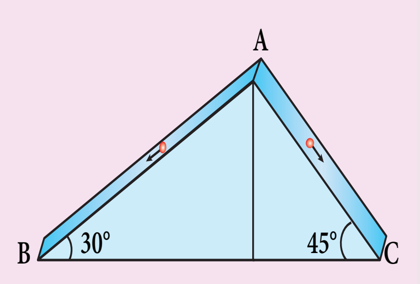
b) 

c) 

d) 

7. A particle of mass m sliding on the smooth double inclined plane (shown in figure) will experience

UNIT-3(XI-Physics_Vol-1).indd 160 05-01-2022 22:18:46



(a) greater acceleration along the path AB

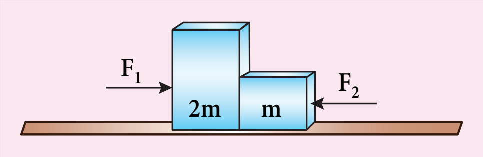
(b) greater acceleration along the path AC

(c) same acceleration in both the paths

(d) no acceleration in both the paths.

8. Two blocks of masses m and 2m are placed on a smooth horizontal surface as shown. In the first case only a force F_1 is applied from the left. Later only a force F_2 is applied from the right. If the force acting at the interface of the two blocks in the two cases is same, then $F_1 : F_2$ is

(Physics Olympiad 2016)



(a) 1:1 (b) 1:2

(c) 2:1 (d) 1:3

9. Force acting on the particle moving with constant speed is

(a) always zero

(b) need not be zero

(c) always non zero

(d) cannot be concluded

10. An object of mass m begins to move on the plane inclined at an angle θ. The coefficient of static friction of inclined surface is μ_s. The maximum static friction experienced by the mass is

(a) mg

(b) $\mu_s$mg

(c) $\mu_s$mg sinθ

(d) μ_s mg cos θ

11. When the object is moving at constant velocity on the rough surface,

(a) net force on the object is zero

(b) no force acts on the object

(c) only external force acts on the object

(d) only kinetic friction acts on the object

12. When an object is at rest on the inclined rough surface,

(a) static and kinetic frictions acting on the object is zero

(b) static friction is zero but kinetic friction is not zero

(c) static friction is not zero and kinetic friction is zero

(d) static and kinetic frictions are not zero

13. The centrifugal force appears to exist

(a) only in inertial frames

(b) only in rotating frames

(c) in any accelerated frame

(d) both in inertial and non-inertial frames

14. Choose the correct statement from the following
   (a) Centrifugal and centripetal forces are action reaction pairs
   (b) Centripetal forces is a natural force
   (c) Centrifugal force arises from gravitational force
   (d) Centripetal force acts towards the centre and centrifugal force appears to act away from the centre in a circular motion

15. If a person moving from pole to equator, the centrifugal force acting on him
   (a) increases
   (b) decreases
   (c) remains the same
   (d) increases and then decreases

Answers

1) a	**2)** c	**3)** a	**4)** c	**5)** c
6) c	**7)** b	**8)** c	**9)** b	**10)** d
11) a	**12)** c	**13)** b	**14)** d	**15)** a

II. Short Answer Questions

1. Explain the concept of inertia. Write two examples each for inertia of motion, inertia of rest and inertia of direction.
2. State Newton's second law.
3. Define one newton.
4. Show that impulse is the change of momentum.
5. Using free body diagram, show that it is easy to pull an object than to push it.
6. Explain various types of friction. Suggest a few methods to reduce friction.
7. What is the meaning by 'pseudo force'?
8. State the empirical laws of static and kinetic friction.
9. State Newton's third law.
10. What are inertial frames?
11. Under what condition will a car skid on a leveled circular road?

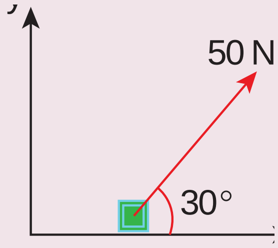
III. Long Answer Questions

1. Prove the law of conservation of linear momentum. Use it to find the recoil velocity of a gun when a bullet is fired from it.
2. What are concurrent forces? State Lami's theorem.
3. Explain the motion of blocks connected by a string in i) Vertical motion ii) Horizontal motion.
4. Briefly explain the origin of friction. Show that in an inclined plane, angle of friction is equal to angle of repose.
5. State Newton's three laws and discuss their significance.
6. Explain the similarities and differences of centripetal and centrifugal forces.
7. Briefly explain 'centrifugal force' with suitable examples.
8. Briefly explain 'rolling friction'.
9. Describe the method of measuring angle of repose.
10. Explain the need for banking of tracks.
11. Calculate the centripetal acceleration of Moon towards the Earth.

UNIT-3(XI-Physics_Vol-1).indd 162 05-01-2022 22:18:47

IV. Numerical Problems

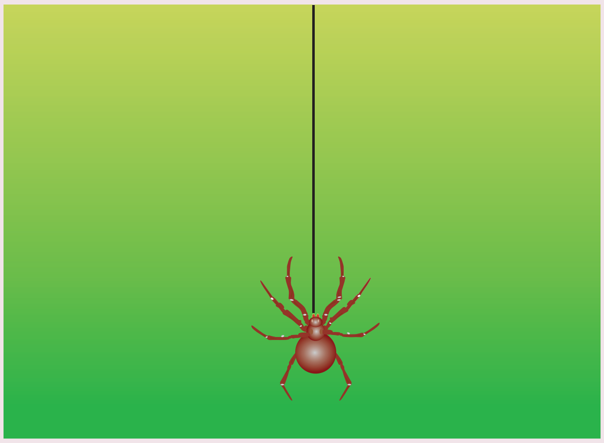
1. A force of 50N act on the object of mass 20 kg. shown in the figure. Calculate the acceleration of the object in x and y directions.



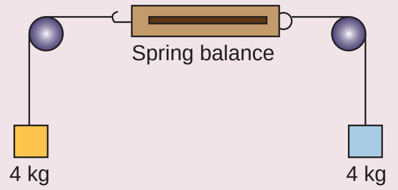
Ans: a_x=2.165 ms^{-2}; a_y=1.25 ms^{-2}

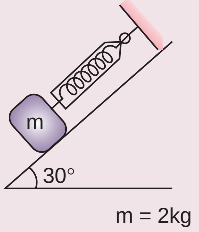
2. A spider of mass 50 g is hanging on a string of a cob web as shown in the figure. What is the tension in the string?

Ans: T=0.49N



3. What is the reading shown in spring balance?





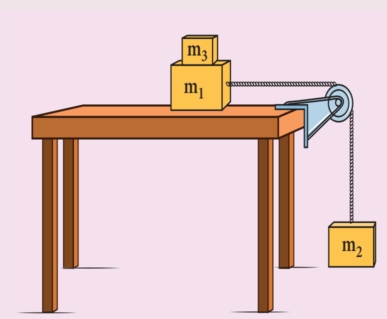
Ans: Zero, 9.8 N

4. The physics books are stacked on each other in the sequence: +1 volumes 1 and 2; +2 volumes 1 and 2 on a table.
   a) Identify the forces acting on each book and draw the free body diagram.
   b) Identify the forces exerted by each book on the other.

5. A bob attached to the string oscillates back and forth. Resolve the forces acting on the bob in to components. What is the acceleration experience by the bob at an angle θ.

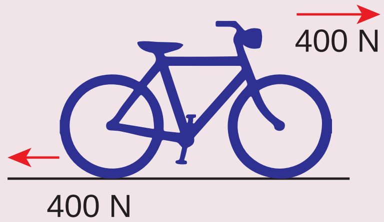
   Ans: Tangential acceleration = g sinθ ;
   centripetal acceleration = $\frac{(T - mg\cos\theta)}{m}$.

6. Two masses m_1 and m_2 are connected with a string passing over a frictionless pulley fixed at the corner of the table as shown in the figure. The coefficient of static friction of mass m_1 with the table is μ_s. Calculate the minimum mass m_3 that may be placed on m1to prevent it from sliding. Check if m_1=15 kg, m_2=10 kg, m_3=25 kg and μ_s =0.2



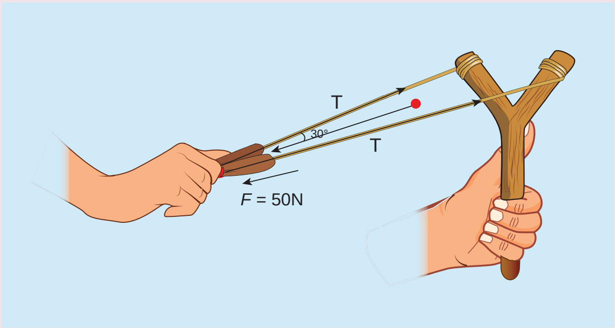
Ans: $m_3 = \frac{m_2}{\mu_s} - m_1$, the combined masses $m_1 + m_3$ will slide.

7. Calculate the acceleration of the bicycle of mass 25 kg as shown in Figures 1 and 2.



Ans: a=4 ms^{-2}, zero

8. Apply Lami's theorem on sling shot and calculate the tension in each string ?



Ans: $T = 28.868N$.

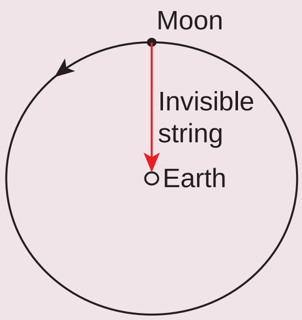
9. A football player kicks a 0.8 kg ball and imparts it a velocity 12 ms^{-1}. The contact between the foot and ball is only for one- sixtieth of a second. Find the average kicking force.

Ans: $576N$.

10. A stone of mass 2 kg is attached to a string of length 1 meter. The string can withstand maximum tension 200 N. What is the maximum speed that stone can have during the whirling motion?

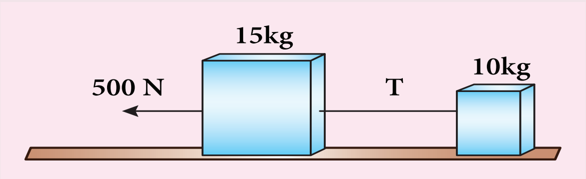
Ans: $v_{max} = 10ms^{-1}$

11. Imagine that the gravitational force between Earth and Moon is provided by an invisible string that exists between the Moon and Earth. What is the tension that exists in this invisible string due to Earth's centripetal force? (Mass of the Moon = 7.34×10^{22} kg, Distance between Moon and Earth = 3.84×10^{8} m)



Ans: $T \approx 2 \times 10^{20} N$.

12. Two bodies of masses 15 kg and 10 kg are connected with light string kept on a smooth surface. A horizontal force F=500 N is applied to a 15 kg as shown in the figure. Calculate the tension acting in the string



Ans: $T = 200N$.

13. People often say "For every action there is an equivalent opposite reaction". Here they meant 'action of a human'. Is it correct to apply Newton's third law to human actions? What is mean by 'action' in Newton third law? Give your arguments based on Newton's laws.

Ans: Newton's third law is applicable to only human's actions which involves physical force. Third law is not applicable to human's psychological actions or thoughts

14. A car takes a turn with velocity 50 ms^{-1} on the circular road of radius of curvature 10 m. calculate the centrifugal force experienced by a person of mass 60kg inside the car?

Ans: 15,000 N

15. A long stick rests on the surface. A person standing 10 m away from the stick. With what minimum speed an object of mass 0.5 kg should he thrown so that it hits the stick. (Assume the coefficient of kinetic friction is 0.7).

Ans: 11.71 ms^{-1}

BOOKS FOR REFERENCE

1. Charles Kittel, Walter Knight, Malvin Ruderman, Carl Helmholtz and Moyer, *Mechanics,* 2nd edition, Mc Graw Hill Pvt Ltd,

2. A.P.French, *Newtonian Mechanics,* Viva-Norton Student edition

3. SomnathDatta, *Mechanics,* Pearson Publication

4. H.C.Verma, *Concepts of physics* volume 1 and Volume 2, Bharati Bhawan Publishers

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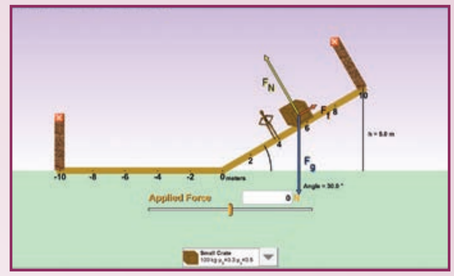
6. Halliday, Resnick & Walker, *Fundamentals of Physics*, Wiley Publishers, 10th edition

ICT CORNER

Force and motion

Through this activity you will understand the Force and motion

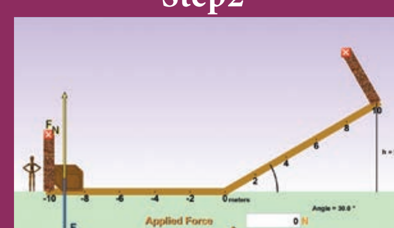


STEPS:

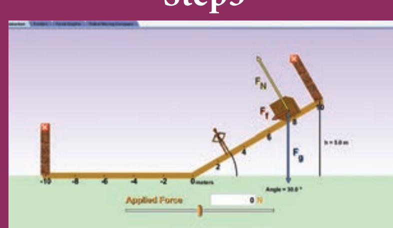
- Open the browser and type the given URL to open the PhET simulation on force and motion. Click OK to open the java applet.
- Select the values of the applied force to observe the change.
- Observe the change of the ramp angle by changing the position of the object.
- You can also observe the variations in force and ramp angle by changing the weights.

Step1

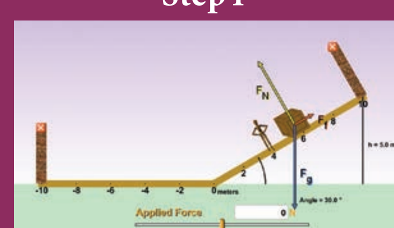
Step2



Step3



Step4



PhET simulation's URL:

https://phet.colorado.edu/en/simulation/ramp-forces-and-motion

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.