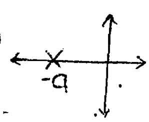
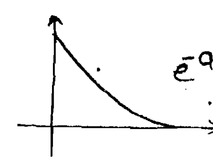
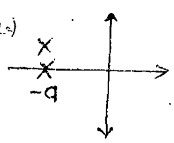
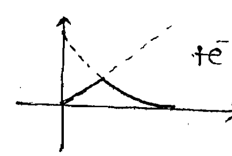
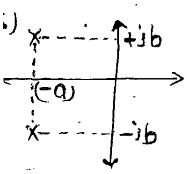
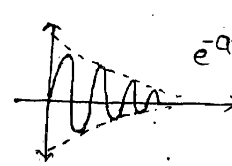
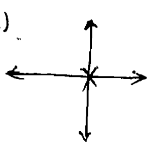
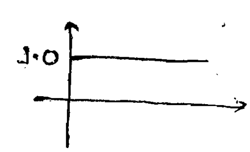
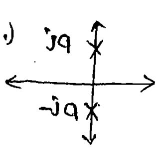
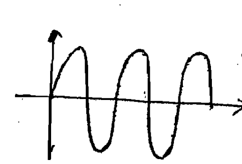
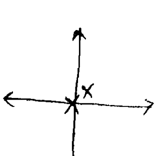
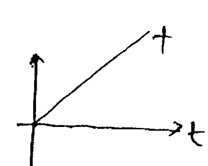
Chapter-03
Stability

* The stability of LTI system may be defined as when the sys. is subjected to bounded i/p the o/p should be bounded.
* BIBO implies the IR of the sys. should tend to zero as time t approaches ∞.
* The stability of a sys. depands on roots of the c/s eqn

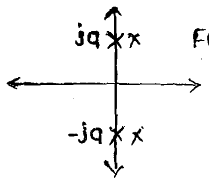
$$1+G(s)H(s)=0$$

i.e. closed loop poles.

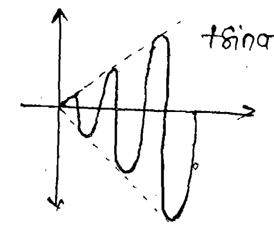
* IR & stability →

Closed loop pole locn		Stability Criteria	Impulse Responce
1.) 	$F(s)=\frac{1}{s+a}$	absolutely stable	 e^{-at}
2.) 	$F(s)=\frac{1}{(s+a)^2}$	absolutety stable	 te^{-at}
) 	$F(s)=\frac{1}{(s+a)^2+b^2}$	absolutely stable	 $e^{-at}\sin bt$
) 	$F(s)=\frac{1}{s}$	marginaly stable/ criticaly stable	
) 	$F(s)=\frac{1}{s^2+a^2}$	marginaly/criticaly	 $\sin at$
) 	$F(s)=\frac{1}{s^2}$	Unstable	

(7.)



$F(S) = \dfrac{1}{(S^2+a^2)^2}$ Unstable



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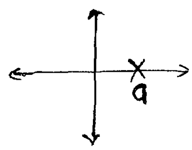
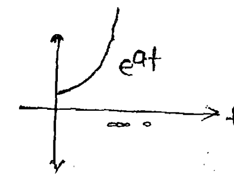
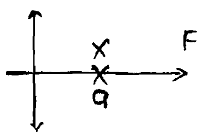


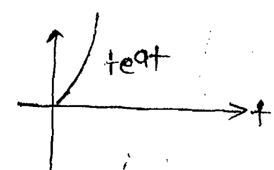
$F(S) = \dfrac{1}{(S-a)}$ Unstabe



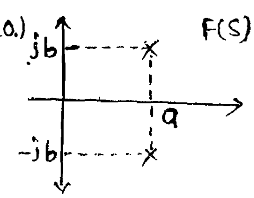
9.)



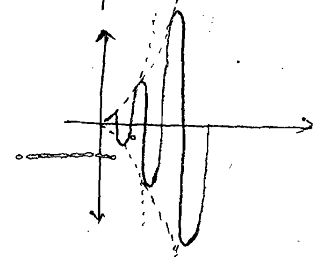
$F(S) = \dfrac{1}{(S-a)^2}$ Unstable



0.)



$F(S) = \dfrac{1}{(S-a)^2+b^2}$ Unstable



* Routh-Hurwitz Criteria →

(2/5) $P(S) = S^4 + 8S^3 + 18S^2 + 16S + 5 = 0$

ROUTH = ARRAY

s^4	1	18	5
s^3	8	16	0
s^2	$b_1 = 16$	$b_2 = 5$	0
s	$c_1 = 13.5$	0	0
s^0	$d_1 = 5$	0	0

$b_1 = \dfrac{8\times18 - 1\times16}{8} = 16$

$b_2 = \dfrac{8\times5 - 1\times0}{8} = 5$

$c_1 = \dfrac{16\times16 - 8\times5}{16} = 13.5$

$d_1 = \dfrac{13.5\times5 - 16\times0}{13.5} = 5$

(9/5) $P(S) = S^5 + S^4 + 2S^3 + 2S^2 + 3S + 15$

s^5	1	2	3
s^4	1	2	15
s^3	$\cancel{0}\ \epsilon$	-12	0
s^2	$\frac{2\epsilon+12}{\epsilon}$	15	0
s^1	$\frac{-24\epsilon-144-15\epsilon^2}{2\epsilon+12}$		
s^0	15		

To check for sign changes

(1.) $\lim_{\epsilon\to 0} \frac{2\epsilon+12}{\epsilon} = \frac{2(0)+12}{0} = +\infty$

$\lim_{\epsilon\to 0} \frac{-24\epsilon-144-15\epsilon^2}{2\epsilon+12} = \frac{-144}{12} = -12$

sign changes: $+\infty \to -12$

$-12 \to +15$

2 poles are in RHS

Difficulty-1 → when the 1st element of any row is 0 while the rest of row has atleast one non-zero term then in such cases substitute $+\epsilon$ (small +ve no) in place of zero & evaluate the rest of RA (Routh array) in terms of ϵ.

Check for sign changes by taking $\lim_{\epsilon\to 0}$ for the 1st column elements to comment on stability.

(4/65) $P(s) = s^6+2s^5+8s^4+12s^3+20s^2+16s+16=0$

s^6	1	8	20	16
s^5	2	12	16	20
s^4	2	12	16	0
s^3	$\cancel{0}$ 8	$\cancel{0}$ 24	0	0
s^2	6	16	0	0
s^1	2.6	0	0	0
s^0	16	0	0	0

(i) Construct an auxiliary eq^n A(s)

$A(s) = 2s^4+12s^2+16$

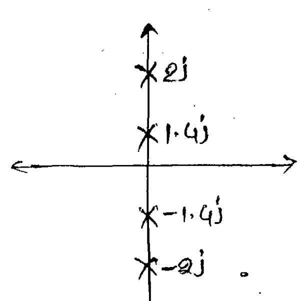
(ii) $\frac{d}{ds}A(s) = \cancel{8s^3+36s^2+16}\ 8s^3+24s$

$\boxed{\text{The roots of } A(s) = \text{poles symmetric about origin}}$

$\frac{-12 \pm \sqrt{144-8\times16}}{4} = -2, -4$

$(s^2+2)(s^2+4)=0$

$s = \pm j1.4, \pm j2$



$\frac{5}{65}$

s^5	2	4	2
s^4	1	2	1
s^3	$\cancel{0}$ 4	$\cancel{0}$ 4	0
s^2	1	1	0
s^1	$\cancel{0}$ 2	0	0
s^0	1	0	0

$A_1(s) = s^4 + 2s^2 + 1$

$\frac{d}{ds} A_1(s) = 4s^3 + 4s$

$A_2(s) = s^2 + 1$

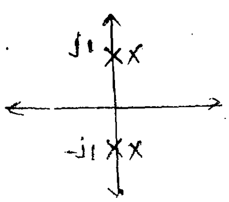
$\frac{d}{ds} A_2(s) = 2s$

The roots of $A_1(s)$

$$\frac{-2 \pm \sqrt{4-4}}{2} = -1, -1$$

$(s^2+1)(s^2+1) = 0$

$s = \pm j, \pm j$

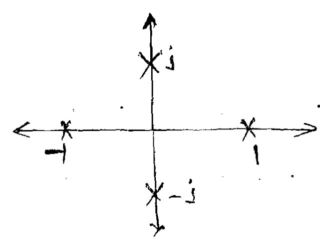


Unstable

Poles symmetric about origin

$(s^2+1)(s^2-1) = 0$

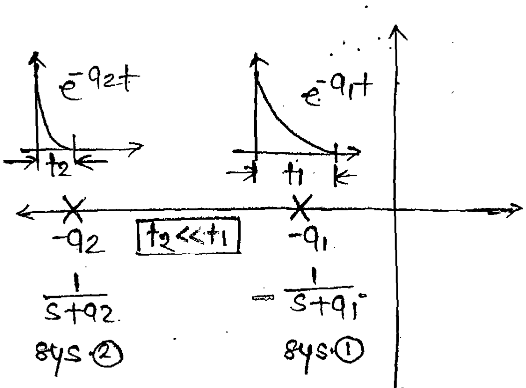
$s = \pm j, \ s = \pm 1$



Difficulty (02) → when one compete row of Routh array is 0, then in such cases construct an AE, A(s) differentiated to get new coffiicient & evaluate rest of the RA.

* Check the roots of AE which are poles symmetric about origin to comment on stability.

Relative stability Analysis →

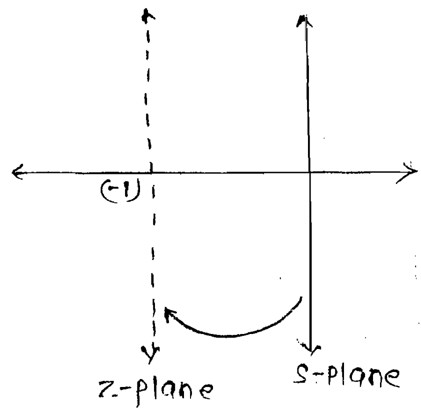


Both sys ① & sys ② are said to be absolutely stable

sys (2) is said to be relatively more stable than sys ① bcoz $t_2 << t_1$

$P(s) = s^3 + 7s^2 + 25s + 39 = 0$

check weather the roots are lying more -vely wrt $= 1$?



$s+1=z$

$s=z-1$

$P(z) = (z-1)^3 + (z-1)^2 7 + 25(z-1) + 39 = 0$

$P(z) = z^3 + 4z^2 + 14z + 20 = 0$

z^3	1	14
z^2	4	20
z	9	0
z^0	20	0

<u>Shortcut</u> → * Put $s=-1$ in given $P(s)$ & if +ve value is coming means all the roots are lying in LHS

If $P(s)=0$; then only 4 values are lying on LHS..

* <u>Conditionaly stable</u> → A sys. is said to be conditionaly stable if its stability depends on one (or) more parameters.

7/56) $P(s) = s^4 + 2s^3 + 3s^2 + 2s + k = 0$

s^4	1	3	k
s^3	2	2	0
s^2	2	k	0
s^1	$\frac{4-2k}{2}$	0	0
s^0	k	0	0

(i) $\frac{4-2k}{2} > 0$; $k < 2$

(ii) $k > 0$

$\boxed{0 < k < 2}$

at $k = k_{mar} = 2$

s' row $= 0$

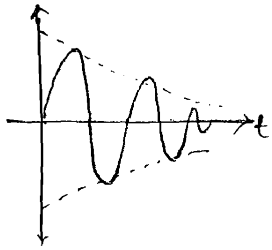
$A(s) = 2s^2 + k = 0$

$2s^2 + 2 = 0$

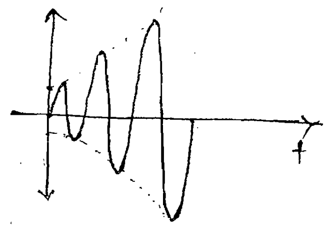
$s = \pm j \rightarrow (j\omega)$

$\boxed{\omega = \omega_{mar} = 1 \ r/s}$

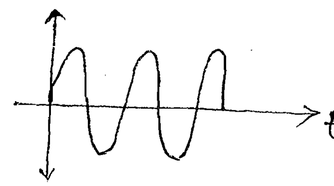
case(1) $\rightarrow$ $s = -\alpha \pm j\omega$



case(2) $\rightarrow$ $s = \alpha \pm j\omega$



case(3) $\rightarrow$ $s = \pm j\omega$



9/66

$$1 + \frac{K(s-2)^2}{(s+2)^2} = 0$$

$$(s+2)^2 + K(s-2)^2 = 0$$

$$s^2(1+K) + s(4-4K) + (4+4K) = 0$$

s^2	$1+K$	$4+4K$
s^1	$4-4K$	0
s^0	$4+4K$	0

(i) $1+K>0$, $K>-1$

(2.) $4-4K>0$, $K<1$

$\boxed{-1<K<1}$ $\boxed{0 \le K < 1}$

1/6

$$1 + \frac{10(K_p s + K_I)}{s(s^2+s+20)} = 0$$

$$s^3 + s^2 + s(20+10K_p) + 10K_I = 0$$

s^3	1	$20+10K_p$
s^2	1	$10K_I$
s	$20+10K_p-10K_I$	0
s^0	$10K_I$	0

(i) $10K_I>0$, $\boxed{K_I>0}$

(ii) $20+10K_p-10K_I>0$

$\boxed{K_p > K_I - \alpha}$

2/66

$$1 + \frac{K(s+2)^2}{s(s^2+1)(s+4)} = 0$$

$$s^4 + 4s^3 + s^2(1+K) + s(4+4K) + 4K = 0$$

s^4	1	$1+K$	$4K$
s^3	4	$(4+4K)$	0
s^2	$\cancel{0}\ \epsilon$	$4K$	0
s^1	$\frac{(4+4K)\epsilon-16K}{\epsilon}$	0	0
s^0	$4K'$	0	0

$\boxed{\text{the sys. is unstable for all } K>0}$

DATE-19/11/14

8/66 $\quad 1+\dfrac{K}{(s^2+2s+2)(s+2)}=0$

Soln →

$$(s^2+2s+2)(s+2)+K=0$$

$$s^3+4s^2+6s+(4+K)=0$$

s^3	1	6	0
s^2	4	$(4+K)$	0
s^1	$\frac{24-(4+K)}{4}$	0	0
s^0	$4+K$	0	0

(i) $\dfrac{24-(4+K)}{4}>0$

$K<20$

(ii) $4+K>0$

$K>-4$

$\boxed{-4<K<20}$

at $K=K_{max}=20$

$$A(s)=4s^2+(4+K)=0$$

$$4s^2+(4+20)=0$$

$$s^2=-6$$

$$s=\pm j\sqrt{6}=j\omega$$

$$\boxed{\omega=\sqrt{6}\ r/s}$$

Shortcut → (3rd order system) (only when all coefficients of eqn are +ve)

$$s^3+\overbrace{4s^2+6s+(4+K)}^{A(s)}=0$$

(i) Product of external coefficients < product of internal = stable

(ii) Product of external coefficients > product of internal = unstabe

(iii) = = marginaly stable

$$4+k=24$$
$$k_{max}=20$$
$$\cancel{4+20=24}$$
$$A(s)=4s^2+(4+k)=0$$
$$4s^2+(4+20)=0$$
$$s^2=-6$$
$$s=\pm j\sqrt{6} \approx j\omega$$
$$\boxed{\omega=\sqrt{6}\ r/s}$$

10/6b

$$1+\frac{K(s+1)}{s^3+as^2+2s+1}=0$$

Sol^n →

$$s^3+as^2+s(2+k)+(k+1)=0$$

$$k+1=a(k+2) \quad \text{(Given)}$$
$$a=\frac{k+1}{k+2}$$
$$A(s)=as^2+(k+1)=0$$
$$s^2=\frac{-(k+1)}{a}$$
$$s^2=\frac{-(\cancel{k+1})\times(k+2)}{(\cancel{k+1})}$$
$$s=\pm j\sqrt{k+2} \approx j\omega$$
$$\omega=\sqrt{k+2}$$
$$2=\sqrt{k+2}$$
$$k=2$$
$$a=\frac{2+1}{2+2}=\frac{3}{4}$$
$$\boxed{k=2,\ a=\frac{3}{4}}$$

conv(3)
66

Soln→ (a.) $1 + \frac{K(s+\alpha)}{s(s+2)(s+4)^2} = 0$

$$s^4 + 10s^3 + 32s^2 + (K+32)s + K\alpha = 0$$

s^4	1	32	$K\alpha$
s^3	10	$32+K$	0
s^2	$\frac{288-K}{10}$	$K\alpha$	0
s^1	A	0	0
s^0	$K\alpha$	0	0

where $A = \dfrac{\frac{(288-K)(K+32)}{10} - 10K\alpha}{\left(\frac{288-K}{10}\right)}$

* (i) $\frac{288-K}{10} > 0$

$\boxed{K < 288}$

* (ii) $\dfrac{\frac{(288-K)(K+32)}{10} - 10K\alpha}{\left(\frac{288-K}{10}\right)} > 0$

$(288-K)(K+32) - 100K\alpha > 0$

$K\alpha < \frac{(288-K)(K+32)}{100}$

$\boxed{0 < K\alpha < \frac{(288-K)(K+32)}{100}}$

(b.) $e_{ss} = \lim_{s \to 0} \dfrac{s \cdot \frac{1}{s^2}}{1 + \frac{K(s+\alpha)}{s(s+2)(s+4)^2}}$

$e_{ss} = \frac{32}{K\alpha}$ let $e_{ss} = 0.16$ (16%)

$0.16 = \frac{32}{200 \times \alpha}$ $\boxed{\because \alpha = 1}$ $\boxed{K = 200}$

$K\alpha = 200 \times 1 = 200$

$200 < \frac{(288-200)(200+32)}{100}$

$\boxed{200 < 204}$

THE ROOT LOCUS Technique

* The Root locus is defined as the Locus of closed loop poles obtained when sys. gain k is varied from 0 to ∞.
* The RL determines relative stability of the sys.

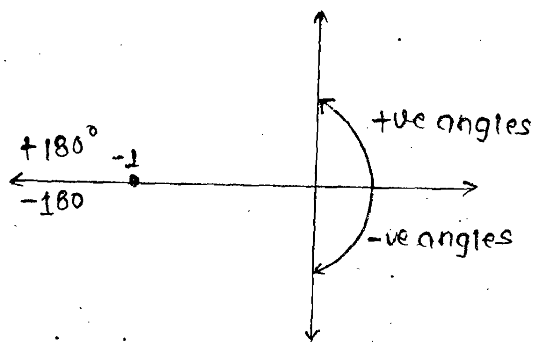
Angle & magnitude condⁿ →

* The angle condⁿ is used for checking weather certain points lie on RL (or) not & also the validity of RL for closed loop poles.

$$1 + G(S)\cdot H(S) = 0$$

$$G(S)\cdot H(S) = -1 + j0$$

$$\underline{\angle G(S)\cdot H(S)} = +180^\circ - \tan^{-1}\left(\frac{0}{1}\right)\left(\frac{Img}{Real}\right) = 180^\circ \text{ or } \pm 180^\circ \text{ or } \pm(2q+1)\,180^\circ$$



* The angle condⁿ may be stated as for a point to lie on RL the angle evaluated at that point must be odd multiple of $\pm 180^\circ$.
- The magnitude condⁿ is used for finding the sys. ~~gain~~ K at any point on RL.

$$1 + G(S)\cdot H(S) = 0$$

$$G(S)\cdot H(S) = -1 + j0$$

$$|G(S)\cdot H(S)| = \sqrt{(-1)^2 + 0^2} = 1$$

$$\boxed{|G(S)\cdot H(S)| = 1}$$

4/68 $S_1 = -3+4j$; $S_2 = -3-2j$

$$G(s).H(s) = \frac{k}{(s+1)^4}$$

$$G(s)\cdot H(s)\Big|_{(s=s_1=-3+4j)} = \frac{k+j0}{(-3+4j+1)^4} = \frac{0^\circ}{(-2+4j)^4} = \frac{0^\circ}{116.5\times 4} = -466^\circ$$

$$G(s).H(s)\Big|_{(s=s_2=-3-2j)} = \frac{k+j0}{(-3-2j+1)^4} = \frac{0^\circ}{-135\times 4} = +540^\circ$$

Because of the odd multiple is present in the s2 then $180\times 3 = 540^\circ$ it is lying on RL.

5/68
$$G(s) = \frac{k}{s(s^2+7s+12)}$$

$s = -1+j1$

$$G(s)\Big|_{s=-1+j1} = \frac{k}{(-1+j)[(-1+j)^2+7(-1+j)+12]} = \frac{k+j0}{(-1+j)(5+5j)}$$

$$\angle G(s)\Big|_{s=-1+j} = \frac{0^\circ}{(135^\circ)(45^\circ)} = -180^\circ \text{ (lying on the R-L)}$$

$$|G(s)|_{s=-1+j} = 1 \Rightarrow \frac{\sqrt{k^2+0^2}}{\sqrt{(-1)^2+(1)^2}\sqrt{5^2+5^2}} \Rightarrow \frac{k}{\sqrt{2}\times\sqrt{50}} = 1 \quad \boxed{\therefore k=10}$$

Que.→ The OLTF of U.FB sys. is $G(s) = \frac{k}{s(s+1)(s+3)}$

A zero is added to the sys. so that the locus passes through $-1+j$.

The locn of zero would be?

(a.) -1.33 (b) -2.33 (c) -1.66 (d) -2.66.

Soln→ Locus passes through $-1+j$ means it is lying on R-L.

$$G(s) = \frac{k}{s(s+1)(s+3)}$$

$$G(s)\Big|_{s=-1+j} = \frac{k(-1+j+z)}{(-1+j)(-1+j+1)(-1+j+3)}$$

$$= \frac{(k+j0)(z-1+j)}{(-1+j)(0+j)(2+j)}$$

$$\angle G(j\omega) = \frac{(0^\circ)\tan^{-1}\left(\frac{1}{z-1}\right)}{(135^\circ)(90^\circ)(26.5^\circ)}$$

$$\tan^{-1}\left(\frac{1}{z-1}\right) - 251.5^\circ = 180^\circ$$

$$\tan^{-1}\left(\frac{1}{z-1}\right) = 180 + 251.5^\circ$$

$$\left(\frac{1}{z-1}\right) = \tan(431.5^\circ)$$

$$\left(\frac{1}{z-1}\right) = 3$$

$$z = 1.33$$

$$(s+z) = 0$$

$$(s+1.33) = 0$$

$$\boxed{s = -1.33}$$

* Construction Rules of R-L →

(1.) The R-L is symmetrical about real axis.

(2.) Let p = no. of open Loop poles ; &
z = no. of open Loop zeros
& $p > z$; then the no. of branches of R-L $= p$

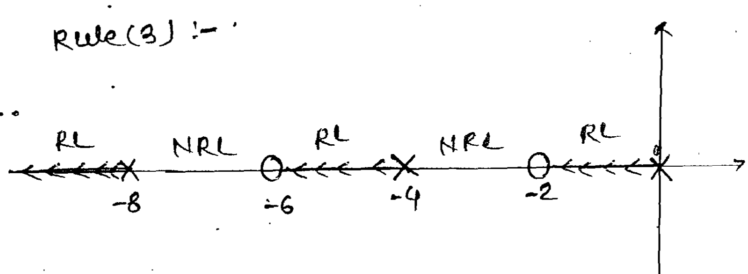
(3.) The no. of branches terminating at zeros $= z$
The no. of branches terminating at $\infty = p - z$

(3.) A point on Real axis is said to be on RL if to the right side of this point the sum of open loop poles & zeros is odd.

Eg:- $$G(s) = \frac{K(s+2)(s+6)}{s(s+4)(s+8)}$$

Rule (2) :- $p = 3, z = 2, \; p - z = 1$

Rule (3) :-



$$\frac{G(s)}{1+G(s)} = \frac{\dfrac{k(s+2)(s+6)}{s(s+4)(s+8)}}{1+\dfrac{k(s+2)(s+6)}{s(s+4)(s+8)}}$$

$$= \frac{k(s+2)(s+6)}{s(s+4)(s+8)+k(s+2)(s+6)}$$

Closed loop poles $= s(s+4)(s+8)+k(s+2)(s+6)$

when $k=0$

closed loop poles $= 0, -4, -8$

(4.) Angle of Asymptodes → The p-z branches terminate at ∞ along certain straight line known as asymptodes of RL.

Therefore no. of asymptodes $= P-Z$.

$$\theta = \frac{(2q+1)\,180^\circ}{(P-Z)} \qquad q = 0,1,2,3\ldots$$

eg:- $P-Z = 2$

$$\theta_1 = \frac{[2(0)+1]\times 180^\circ}{2} = 90^\circ \quad ; \quad \theta_2 = \frac{[2(1)+1]\times 180^\circ}{2} = 270^\circ$$

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$$s(s+4)(s^2+2s+s)+k(s+1)=0$$

$$1+\frac{k(s+1)}{s(s+4)(s^2+3s)} = 0$$

$$1+G(s)\cdot H(s) = 0$$

$$G(s)\cdot H(s) = \frac{k(s+1)}{s(s+4)(s^2+3s)} = \frac{k(s+1)}{s^2(s+4)(s+3)}$$

(2.) $P=4,\ Z=1,\ P-Z=3$

(4.) $$\theta_1 = \frac{[2(0)+1]\times 180^\circ}{3} = 60^\circ \qquad \theta_2 = \frac{[2(2)+1]\times 180^\circ}{3} = 300^\circ$$

$$\theta_2 = \frac{[2(1)+1]\times 180^\circ}{3} = 180^\circ$$

★★★ $$\boxed{\text{Angle b/n asymptodes} = \frac{2\pi}{P-Z}}$$

(5.) **Centoriod →** It is the intersection point of asymptodes on the real axis. It may (or) may not be a part of RL.

$$\text{Centoriod} = \frac{\sum \text{Real part of open Loop poles} - \sum \text{Real part of open Loop zeros}}{P-Z}$$

(7/8)

$$s^3 + 5s^2 + (K+6)s + K = 0$$

$$s^3 + 5s^2 + 6s + Ks + K = 0$$

$$s^3 + 5s^2 + 6s + K(s+1) = 0$$

$$1 + \frac{K(s+1)}{s^3+5s^2+6s} = 0$$

$$1 + G(s)H(s) = 0$$

$$G(s).H(s) = \frac{K(s+1)}{s(s^2+5s+6)} = \frac{K(s+1)}{s(s+3)(s+2)}$$

(2.) $P = 3;\ Z = 1,\quad P-Z = 2$

(5.) Centoriod →

Zero at $s = -1+j0 = -1$

Poles at $s = 0+j0$
$-2+j0$
$-3+j0$

-5

$$\text{Centoriod} = \frac{-5-(-1)}{2} = -2$$

Centoriod $= -2, 0$

K(6.) **Break away points →** They are those points where multiple roots of the c/s eqⁿ occur.

Procedure → (1.) Construct $1+G(s).H(s) = 0$

(2.) write 'k' in terms of "s"

(3.) find $\frac{dk}{ds} = 0$.

(4.) The roots of $\frac{dk}{ds} = 0$ will give break away points.

(5.) To test valid BA points substitute in step (2.)

If k is +ve $\Rightarrow$ valid BA points.

General predictions about BA points →

(1.) The branches of RL either approach (or) ~~live~~ leave the BA points at an angle of $\frac{\pm 180}{n}$, where n = no of branches approaching (or) leaving the BA point.

(2.) The complex congugate path for the branches of RL approaching (or) leaving the BA point is a circle.

(3.) whenever there are 2 adjecently placed poles on the real axis with the section of real axis b/n them as a part of RL then there exist some BA point b/n the adjecently placed poles.

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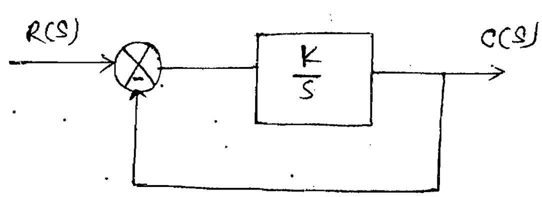
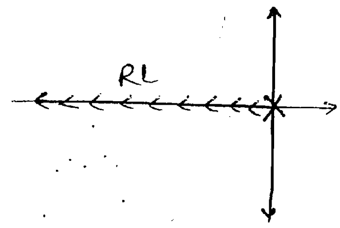
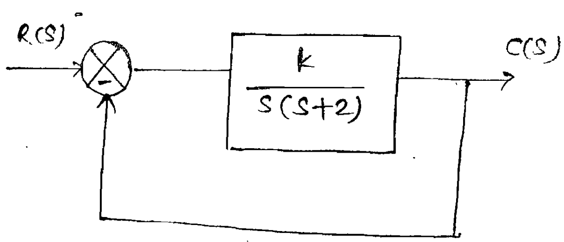
First order system →



$$\frac{C(S)}{R(S)} = \frac{k}{S+k} \qquad G(S) = \frac{k}{S}$$



* 2nd order system →

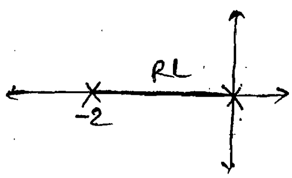


$$\frac{C(s)}{R(s)} = \frac{k}{s^2+2s+k}$$

$$G(s) = \frac{k}{s(s+2)}$$

(2.) $P = 2$; $Z = 0$; $P-Z = 2$

(3.)



(4.) $\theta_1 = 90°$, $\theta_2 = 270°$

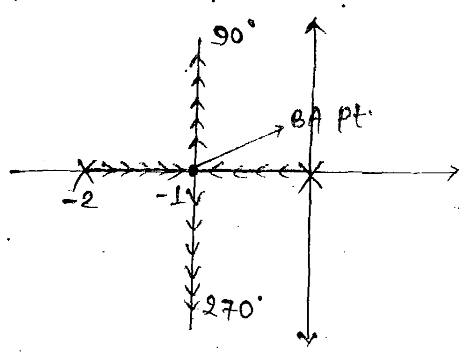
(5.) Centoriod $= \frac{0+(-2)}{2} = -1$

(6) BA point →

$s^2+2s+k = 0$

$k = -s^2-2s$

$\frac{dk}{ds} = 0$, $-2s-2 = 0$; $\boxed{s=-1}$

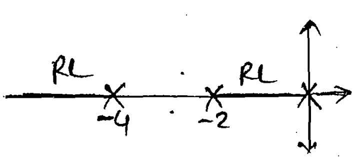


* 3rd order system →

$$G(s) = \frac{k}{s(s+2)(s+4)}$$

Effect of adding poles to a TF :-

(2) $P=3$, $Z=0$, $P-Z=3$ (3)



(4.) $\theta_1 = 60°$, $\theta_2 = 180°$, $\theta_3 = 300°$

(5.) $\frac{0+(-2)+(-4)-0}{3} = -2$

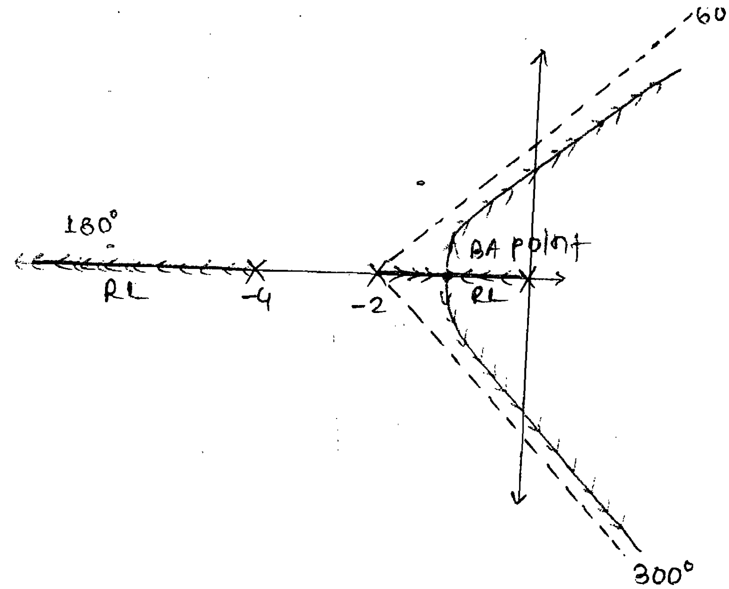
(6.) BA point

$s^3+6s^2+8s+k = 0$

$k = -s^3-6s^2-8s$

$\frac{dk}{ds} = 0$; $3s^2+12s+8 = 0$

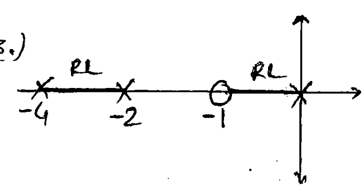
$s = -0.8, -3.15$ ✗



$*$ $G(s) = \dfrac{k(s+1)}{s(s+2)(s+4)}$ <u>Effect of adding zero to a TF.</u>

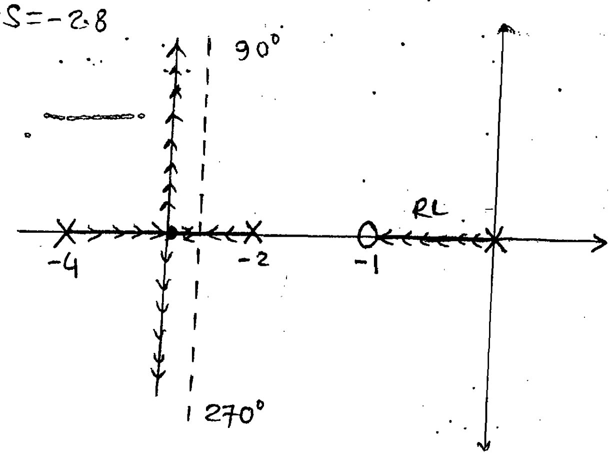
(2.) $P=3,\ Z=1,\ P-Z=2$

(3.)



(4.) $\theta_1 = 90°,\ \theta_2 = 270°$ (5.) $\dfrac{0+(-2)+(-4)-(-1)}{2} = -2.5$

(6.) BA point $s=-2.8$



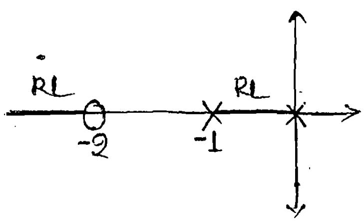
$***$ Stability ↑
RL shifts to LHS

<u>Prediction (4)</u> Whenever there is a zero on real axis & to the left side of that zero there are no poles (or) zeros on the real axis with the entire section of real axis to the left side of zero as a part of RL then there exists a BA points to the left side of that zero.

Eg:- $G(s) = \dfrac{k(s+2)}{s(s+1)}$

(2.) $P=2, \; Z=1, \; P-Z=1$

(3.)



(6.) BA points:-

$$s(s+1)+k(s+2)=0$$

$$k = \frac{-s^2-s}{s+2}$$

$$\frac{dk}{ds} = 0$$

$$\frac{(s+2)(-2s-1)-[(-s^2-s)\cdot 1]}{(s+2)^2} = 0$$

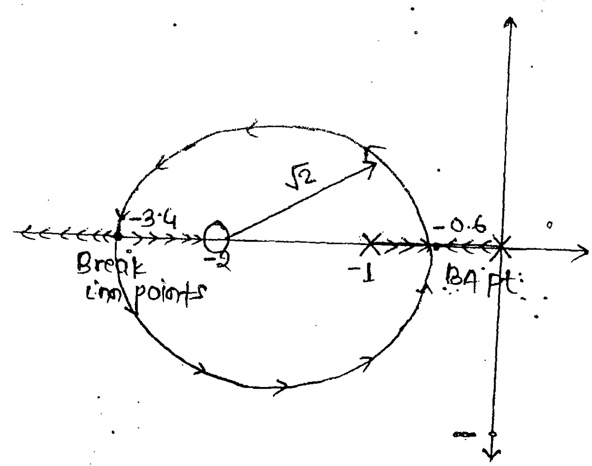
$$-2s^2+s-4s-2+s^2+s=0$$

$$-s^2-4s-2=0$$

$$s^2+4s+2=0$$

$$s = \frac{-4 \pm \sqrt{16-8}}{2}$$

$$s = -2 \pm \sqrt{2} = -0.6, -3.4$$



To evaluate the center & radius →

$$G(s) = \frac{k(s+b)}{s(s+a)}$$

Let $s = x+jy$

$$G(s) = \frac{k[x+jy+b]}{(x+jy)(x+jy+a)}$$

$$G(s) = \frac{k[(x+b)+jy]}{x^2+xjy+jyx-y^2+ax+ajy}$$

$$G(s) = \frac{K[(x+b)+jy]}{x^2+ax-y^2+j(2xy+ay)}$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x+b}\right) - \tan^{-1}\left(\frac{2xy+ay}{x^2+ax-y^2}\right) = 180^\circ$$

$$\Rightarrow \tan^{-1}\left(\frac{A-B}{1+AB}\right) = 180^\circ$$

$$\Rightarrow \frac{A-B}{1+AB} = \tan(180^\circ)$$

$$\Rightarrow \frac{A-B}{1+AB} = 0$$

$$\Rightarrow A-B = 0$$

$$\Rightarrow \left(\frac{\cancel{y}}{x+b}\right) - \left(\frac{2x\cancel{y}+a\cancel{y}}{x^2+ax-y^2}\right) = 0$$

$$\Rightarrow x^2+ax-y^2-(2x^2+2xb+ax+ab) = 0$$

$$\Rightarrow (x+b)^2+y^2 = b(b-a)$$

$$\Rightarrow \text{center} = -b, 0, = -2, 0$$

$$\Rightarrow \text{Radius} = \sqrt{b(b-a)} = \sqrt{2(2-1)} = \sqrt{2}$$

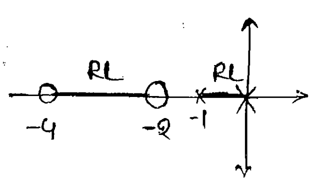
$$\boxed{\text{BA points Shortcuts} = \text{Center} \pm \text{Radius}}$$

Prediction (5) whenever there are 2 adjecently placed zeros on the real axis with the section of real axis b/n them as a part of RL then there exists BA point b/n the adjecently placed zeros.

eg:- $G(s) = \dfrac{K(s+2)(s+4)}{s(s+1)}$

(2.) $P=2$, $Z=2$; $P-Z=0$

(3.)



(6.) BA. points →

$$s(s+1)+K(s^2+6s+8)=0$$

$$K=\frac{-s^2-s}{s^2+6s+8}$$

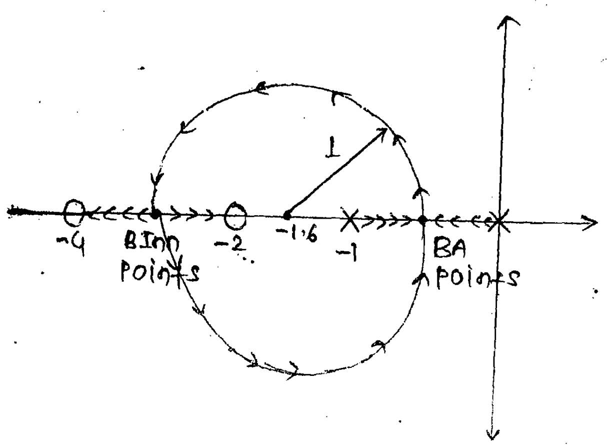
$$\frac{dK}{ds}=0$$

$$\frac{(s^2+6s+8)(-2s-1)-[(-s^2-s)(2s+6)]}{(s^2+6s+8)^2}=0$$

$$\Rightarrow 5s^2+16s+8=0$$

$$s=\frac{-16\pm\sqrt{256-160}}{10}$$

$s=-1.6\pm 1 = -2.6, -0.6$
(Center Radius)



Rule(7) → Intersection of RL with imaginary axis :- The roots of the AE A(s) at K = Kmax from Routh Array gives the intersection of RL with imaginary axis.

eg:-

$$G(s)=\frac{K}{s(s+2)(s+4)}$$

$$s^3+6s^2+8s+K=0$$

s^3	1	8
s^2	6	K
s^1	$\frac{48-K}{6}$	0
s^0	K	0

(i.) $\frac{48-K}{6}>0$ (ii) $K>0$

$K<48$

$\boxed{0<K<48}$

$K=K_{max}=48$

$A(s)=6s^2+K=0$

$6s^2+48=0$

$s=\pm j\sqrt{8}$

$\boxed{s=\pm j2.8}$

Intersection of asymptodes with jω axis

$$\tan\theta = \frac{y}{x}$$

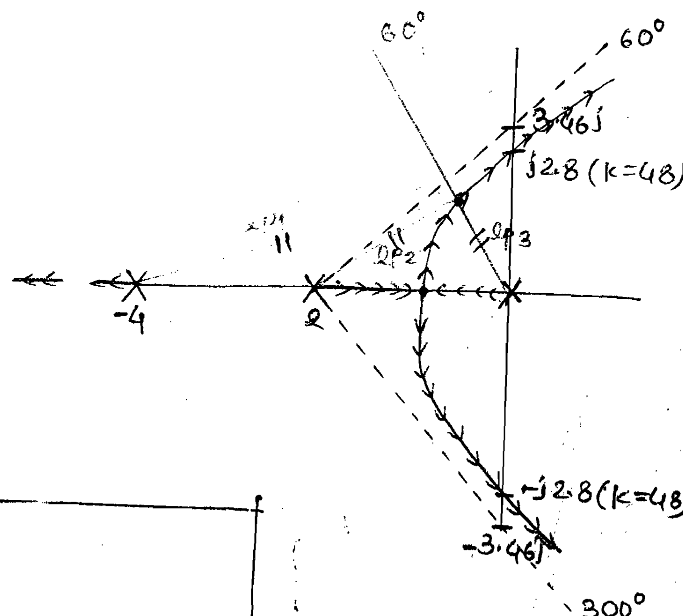
$$\tan 60^\circ = \frac{y}{2}$$

$$y = \tan 60^\circ \times 2$$

$$y = \sqrt{3} \times 2$$

$$= 3.46$$

$$= \pm 3.48j$$



Short cut method →

$$G(s) = \frac{K}{s(s+a)(s+b)}$$

Intersection of RL with jω axis $= \pm j\sqrt{ab}$

Q.→ Find k when $\xi = 0.5$ from RL?

Soln →

$$\theta = \cos^{-1}(0.5)$$

$$\theta = 60^\circ$$

$$k = \frac{\text{Product of vector lengths of poles}}{\text{Product of vector lengths of zeros}}$$

$$= \frac{l_{p1} \times l_{p2} \times l_{p3}}{1}$$

Linearity	$a_1x_1(t)+a_2x_2(t) \rightleftharpoons a_1c_1(n)+a_2c_2(n)$	$a_1x_1(t)+a_2x_2(t) \rightleftharpoons a_1X_1(\omega)+a_2X_2(\omega)$	$a_1f_1(t)+a_2f_2(t) \rightleftharpoons a_1F_1(S)+a_2F_2(S)$	$a_1x_1(n)+a_2x_2(n) \rightleftharpoons a_1X_1(z)+a_2X_2(z)$
Time-Reversal	$x(-t) \rightleftharpoons c_{(-n)}$	$x(-t) \rightleftharpoons X(-\omega)$	$f(-t) \rightleftharpoons F(-s)$	$x(-n) \rightleftharpoons X(z^{-1})$
Congugation	$x^*(t) \rightleftharpoons c^*_{-n}$	$x^*(t) \rightleftharpoons X^*(-\omega)$	$f^*(t) \rightleftharpoons f^*(s^*)$	$x^*(n) \rightleftharpoons X^*(z^*)$
Time-shifting	$x(t-t_0) \rightleftharpoons c_n e^{-jn\omega_0 t_0}$	$x(t-t_0) \rightleftharpoons X(\omega)e^{-j\omega t_0}$	$f(t-t_0) \rightleftharpoons F(S)\cdot e^{-St_0}$	$x(n-n_0) \rightleftharpoons X(z)z^{-n_0}$ $n_0=1 \Rightarrow x(n-1) \rightleftharpoons X(z)z^{-1}$
Freq. shifting	$x(t)e^{jm\omega_0 t} \rightleftharpoons c_{n-m}$ $x(t)e^{-jm\omega_0 t} \rightleftharpoons c_{n+m}$	$x(t)\cdot e^{j\omega_0 t} \rightleftharpoons X(\omega-\omega_0)$	$e^{-at}f(t) \rightleftharpoons F(S+a)$	
Convolution in time	$x_1(t)*x_2(t) \rightleftharpoons T_0(c_{1n}\cdot c_{2n})$ where $T_0=\text{LCM}(T_1,T_2)$	$x_1(t)*x_2(t)=X_1(\omega)\cdot X_2(\omega)$	$F_1(t)*F_2(t) \rightleftharpoons F_1(S)\cdot F_2(S)$	$x_1(n)*x_2(n) \rightleftharpoons X_1(z)X_2(z)$
Multiplication in time	$x_1(t)\cdot x_2(t) \rightleftharpoons (c_{1n}*c_{2n})$	$x_1(t)\cdot x_2(t)=\frac{1}{2\pi}\left[X_1(\omega)\overset{*}{\cdot}X_2(\omega)\right]$	$f_1(t)\cdot f_2(t) \rightleftharpoons \frac{1}{2\pi j}\left[F_1 s * F_2(S)\right]$	$x_1(n)\cdot x_2(n) \rightleftharpoons \frac{1}{2\pi j}\left[X_1(z)*X_2(z)\right]$
Integration in time.	$\int_{-\infty}^{t}x(k)dk=\frac{c_n}{jn\omega_0}$	$\int_{-\infty}^{t}x(k)dk=\frac{X(\omega)}{j\omega}+\pi X(0)\delta(\omega)$ $X(0)=x(\omega)\vert_{\omega=0}$	$\int_{-\infty}^{t}x(t)\cdot dt \rightleftharpoons \begin{cases}\frac{F(S)}{S}; \text{ Bilateral LT.}\\ \frac{F(S)}{S}+\frac{\int_{-\infty}^{0}f(t)dt}{S} \text{ Unilateral LT.}\end{cases}$	$\sum_{k=-\infty}^{n}x(n) \rightleftharpoons \frac{X(z)}{1-z^{-1}}$
Diff in time	$\frac{d^m x(t)}{dt^m}=(jn\omega_0)^m c_n$	$\frac{d^n x(t)}{dt^n}=(j\omega)^n X(\omega)$	$\frac{d^n f(t)}{dt^n} \rightleftharpoons \begin{cases}S^n F(S) \text{ Bilateral LT.}\\ S^nF(S)-S^{n-1}f(0^-)-S^{n-2}f'(0^-)-S^{n-3}f''(0^-) \text{ Uni LT}\end{cases}$	$\frac{dx(n)}{dn}=\frac{x(n)-x(n-1)}{n-(n-1)}$ $x(n)-x(n-1) \rightleftharpoons (1-z^{-1})X(z)$

Passeval's Power theorem	$P=\frac{1}{T_0}\int_{T_0} \lvert x(t)\rvert^2 dt$ $P=\sum_{n=-\infty}^{\infty} \lvert c_n\rvert^2$	$E=\int_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt$ $=\frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2 d\omega.$	Initial value theorem:- $F(0)=\lim_{s\to\infty} s\cdot F(s)$ applicable only causal sys. $f(t)=0, t<0$	Initial value theorem $x(0)=\lim_{z\to\infty} X(z)$ $x(0)=\lim_{s\to\infty} sX(s)$
Time Scaling		$x(at)_{a\neq 0}=\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$	$f(at)_{a\neq 0} \rightleftharpoons \frac{1}{\lvert a\rvert}F\left(\frac{s}{a}\right)$	Condn:- applicable only for causal type of sys. i.e. $x(n)=0, n<0$
modulation		$x(t)\cdot\cos\omega_0 t=\frac{1}{2}[X(\omega+\omega_0)+X(\omega-\omega_0)]$ $x(t)\cdot\sin\omega_0 t=\frac{j}{2}[X(\omega+\omega_0)-X(\omega-\omega_0)]$		Scaling of z:- $a^n x(n) \rightleftharpoons X(a^{-1}z)$
Differentiation in freq.		$t^n x(t)=j^n\frac{d^n X(\omega)}{d\omega^n}$	$t^n f(t) \rightleftharpoons (-1)^n\frac{d^n F(s)}{ds^n}$	$n\cdot x(n) \rightleftharpoons -z\frac{dX(z)}{dz}$
Area under time domain		$X(\omega)=\int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t}dt$ $\downarrow \omega=0$ $X(0)=\int_{-\infty}^{\infty} x(t)dt$ $x(t)=X(\omega)\rvert_{\omega=0}$	Final value theorem→ $F(\infty)=\lim_{s\to 0} s\cdot F(s)$ (i) causal type $f(t)=0, t<0$ (ii) $sF(s)$ should have only LHS poles in s-plane.	Final value theorem→ $x(\infty)=\lim_{z\to 1}(1-z^{-1})X(z)$ $x(\infty)=\lim_{s\to 0} sX(s)$ Condn:- (i) applicable for causal signals i.e. $x(n)=0$, $n<0$
Area under freq. domain		$X(\omega)=2\pi x(t)\rvert_{t=0}$		
Integration in freq.			$\frac{f(t)}{t} \rightleftharpoons \int_s^{\infty} F(s)ds$	(ii) $(1-z^{-1})X(z)$ should have poles inside unit circle in z-plane.

*Rule no.(8.) → Angle of departure & arrival → The angle of departure is obtained when complex poles terminate at ∞

* The angle of arrival is obtained at complex zeros.

$$\phi_D = 180° + \phi \quad ; \quad \phi_A = 180° - \phi$$

where; $\phi = \Sigma\phi_Z - \Sigma\phi_P$

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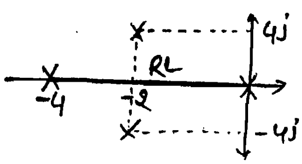
$$G(s).H(s) = \frac{K}{s(s+4)(s^2+4s+20)}$$

(2.) $P=4$; $Z=0$; $P-Z=4$

(3.)



(4.) $\theta_1 = 45°$, $\theta_2 = 135°$, $\theta_3 = 225°$, $\theta_4 = 315°$

(5.) $\frac{0+(-2)+(-2)+(-4)-0}{4} = -2$

(6.) BA points →

(9.) Shortcut methods →

Avg. value of real poles $= \frac{0+(-4)}{2} = -2$

* If the avg. value of real poles = Real part of complex pole
There will be 3 B.A points.

* The avg. value of real poles ≠ Real part of complex poles.
There will be 1 BA. points

(b.) nature of BA points →

Absolute value of avg. value of real poles. $= 2$

$2 \times x = 20$

$\boxed{x = 10}$

$x \geq 5 \Rightarrow$ There will be 1 real & 2 complex BA points

$x < 5 \Rightarrow$ There will be 3 real BA points.

*** Original method →**

$$s^4 + 8s^3 + 36s^2 + 80s + K = 0$$

$$K = -s^4 - 8s^3 - 36s^2 - 80s$$

$$\frac{dK}{ds} = -4s^3 - 24s^2 - 72s - 80 = 0$$

$$4s^3 + 24s^2 + 72s + 80 = 0$$

$$s = -2 \; ; \; -2 \pm 2.45j$$

Note:- To check the validity of complex BA points use angle condⁿ.

$$G(s)H(s)\Big|_{s=-2\pm 2.45j} = \frac{K}{(-2+2.45j)(2+j2.45)(0+6.45j)(0-1.55j)}$$

$$\underline{|G(s)\cdot H(s)}\Big|_{s=-2+j2.45} = \frac{0^\circ}{(130^\circ)(50^\circ)(90^\circ)(-90^\circ)} = -180^\circ$$

(7.)

s^4	1	36	K
s^3	8	80	0
s^2	26	K	0
s^1	$\frac{2080-8K}{26}$	0	0
s^0	K	0	0

$$\frac{2080-8K}{26} > 0 \quad , \quad K > 0$$

$$K < 260$$

$$\boxed{0 < K < 260}$$

$$K_{mar} = 260$$

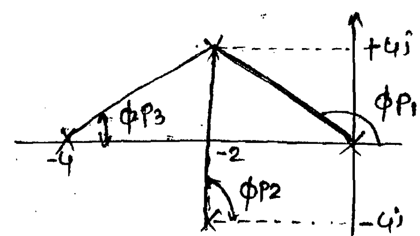
$$A(s) = 26s^2 + K = 0$$

$$26s^2 + 260 = 0$$

$$\boxed{s = \pm 3.16j}$$

$$\boxed{\begin{array}{l} y = \tan 45 \times 2 = 2 \\ = \pm j2 \end{array}}$$

(8.) Angle of departure →



$\phi_{P_1} = 180^\circ - \tan^{-1}\left(\frac{4-0}{0-(-2)}\right)$

$= 116.6^\circ$

$\phi_{P_2} = 90^\circ$

$\phi_{P_3} = \tan^{-1}\left[\frac{4-0}{-2-(-4)}\right] = 63.4^\circ$

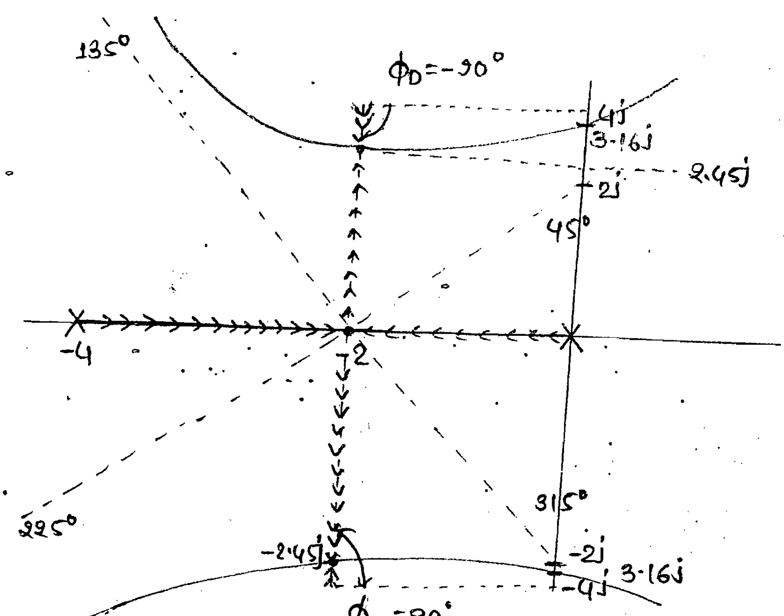
$\phi = \Sigma\phi_z - \Sigma\phi_P$

$= 0 - [116.6 + 90 + 63.4]$

$\phi = -270^\circ$

$\phi_D = 180^\circ + \phi = 180^\circ - 270^\circ$

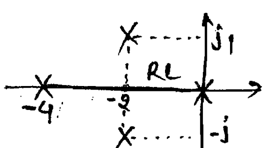
$\boxed{\phi_D = -90^\circ}$



(9/68) $G(s).H(s) = \frac{K}{s(s+4)(s^2+4s+5)}$

(2.) $P=4$; $Z=0$, $P-Z=4$

(3.)



(4.) $\theta_1 = 45^\circ$, $\theta_2 = 135^\circ$, $\theta_3 = 225^\circ$, $\theta_4 = 315^\circ$

(5.) $\dfrac{0+(-2)+(-2)+(-4)-0}{4} = -2$

(6.) BA points →

$$s^4 + 8s^3 + 21s^2 + 20s + K = 0$$

$$K = -s^4 - 8s^3 - 21s^2 - 20s$$

$$\frac{dK}{ds} = 0.$$

$$-4s^3 - 24s^2 - 42s - 20 = 0$$

$$4s^3 + 24s^2 + 42s + 20 = 0$$

$$\boxed{s = -0.78,\ -2,\ -3.22}$$

(7.)

s^4	1	21	K
s^3	8	20	0
s^2	18.5	K	0
s^1	$\frac{370-8K}{18.5}$	0	0
s^0	K	0	0

$\frac{370-8K}{18.5} > 0$, $K < 46.25$

$$\boxed{0 < K < 46.25}$$

$$K = K_{mar} = 46.25$$

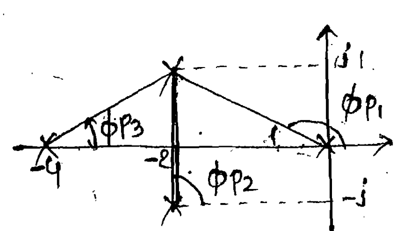
$$A(s) = 18.5s^2 + K = 0$$

$$18.5s^2 + 46.25 = 0$$

$$\boxed{s = \pm j1.58}$$

$$y = \tan^{-1}45^\circ \times 2 = 2 = \pm 2j$$

(8.) Angle of departure →



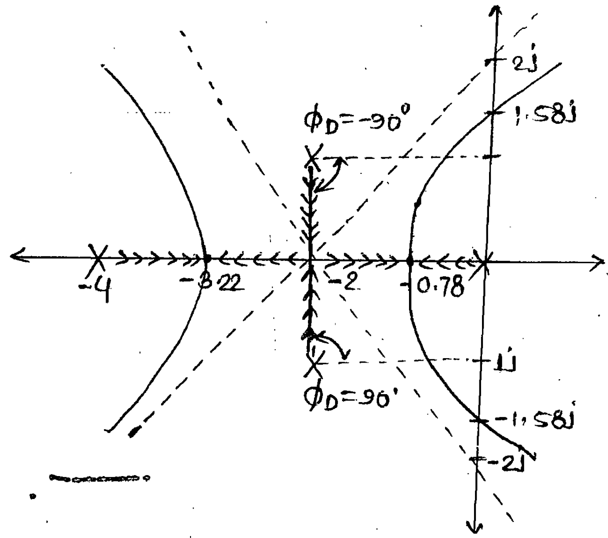
$$\phi p_1 = 180^\circ - \tan^{-1}\left(\frac{1-0}{0-(-2)}\right) = 153.5^\circ$$

$$\phi p_2 = 90^\circ;\quad \phi p_3 = \tan^{-1}\left(\frac{1-0}{-2-(-4)}\right) = 26.5^\circ$$

$$\phi = 0^\circ - (153.5 + 90^\circ + 26.5) = -270^\circ$$

$$\phi_D = 180^\circ + \phi = 180 - 270^\circ = -90$$

$$\boxed{\phi_D = -90^\circ}$$

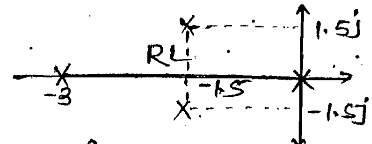


conv (1.1) 69

$$G(s) = \frac{K}{s(s+3)(s^2+3s+4.5)}$$

(2.) $P=4, \; Z=0, \; P-Z=4$

(3.)



(4.) $\theta_1 = 45^0, \; \theta_2 = 135^0, \; \theta_3 = 225^0, \; \theta_4 = 315^0$

(5.) $$\frac{0+(-1.5)+(-1.5)+(-3)-0}{4} = -1.5$$

(6.) BA points :-

$$s^4 + 6s^3 + 13.5s^2 + 13.5s + K = 0$$

$$K = -s^4 - 6s^3 - 13.5s^2 - 13.5s$$

$$\frac{dK}{ds} = -4s^3 - 18s^2 - 27s - 13.5 = 0$$

$$4s^3 + 18s^2 + 27s + 13.5 = 0$$

$$\boxed{s = -1.5, -1.5, -1.5}$$

(7.)

s^4	1	13.5	K
s^3	6	13.5	0
s^2	11.25	K	0
s^1	$\frac{151.87-6K}{11.25}$	0	0
s^0	K	0	0

$$\frac{151.87-6K}{11.25} > 0$$

$$K < 25.3$$

$$\boxed{0 < K < 25.3}$$

at $K = K_{max} = 25.3$

$A(s) = 11.25s^2 + K = 0$

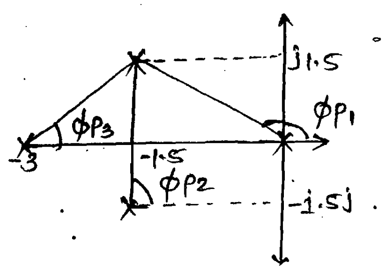
$11.25s^2 + 25.3 = 0$

$\boxed{s = \pm j1.5}$

$y = \tan^{-1} 45^\circ \times 1.5 = 1.5$

$= \pm 1.5j$

(8.) Angle of departure →



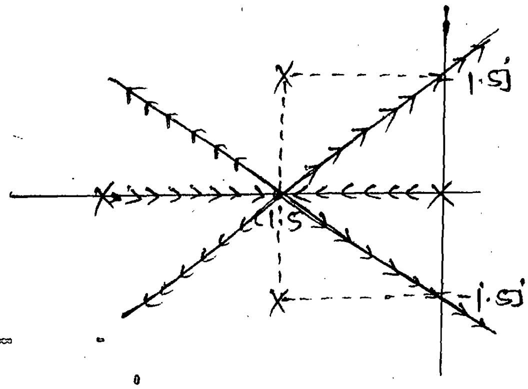
$\phi P_1 = 180^\circ - \tan^{-1}\left(\frac{1.5-0}{0-(-1.5)}\right) = 135^\circ$

$\phi P_2 = 90^\circ$, $\phi P_3 = \tan^{-1}\left(\frac{1.5-0}{-1.5-(-3)}\right) = 45^\circ$

$\phi = 0^\circ - (135^\circ + 90 + 45) = -270^\circ$

$\phi_D = 180^\circ - 270^\circ = -90^\circ$

$\boxed{\phi_D = -90^\circ}$



$\boxed{\frac{\text{con.(2)}}{69}}$

$$s(s+4)(s^2+2s+2)+K(s+1)=0$$

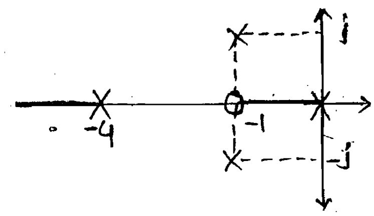
$$1+\frac{K(s+1)}{s(s+4)(s^2+2s+2)}=0$$

$$1+G(s)\cdot H(s)=0$$

$$G(s)\cdot H(s)=\frac{K(s+1)}{s(s+4)(s^2+2s+2)}$$

(2.) $P=4$; $Z=1$, $P-Z=3$

(3.)



(4.) $\theta_1=60^\circ$, $\theta_2=180^\circ$, $\theta_3=300^\circ$

(5.) $$\frac{0+(-1)+(-1)+(-4)-(-1)}{3}=-1.6$$

(6.) BA points:— Nill.

(7.) $s^4+6s^3+10s^2+(K+8)s+K=0$

s^4	1	10	K
s^3	6	$(K+8)$	0
s^2	$\frac{52-K}{6}$	K	0
s^1	$\frac{\frac{(52-K)(K+8)}{6}-6K}{\frac{52-K}{6}}$	0	0
s^0	K	0	0

$$(52-K)(8+K)-36K=0$$

$$K^2-8K-416=0$$

$$K=24.78,\ -16.75$$

$$\boxed{K_{mar}=24.78}$$

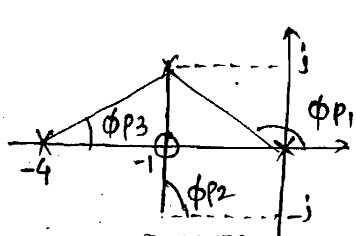
$$A(s)=\left(\frac{52-24.78}{6}\right)s^2+24.78=0$$

$$\boxed{s=\pm j2.34}$$

$$y=\tan 60^\circ\times 1.6=\sqrt{3}\times 1.6=2.77$$

$$=\pm 2.77j$$

(8.)



$$\phi P_1=180^\circ-\tan^{-1}\left(\frac{1-0}{0-(-1)}\right)=135^\circ$$

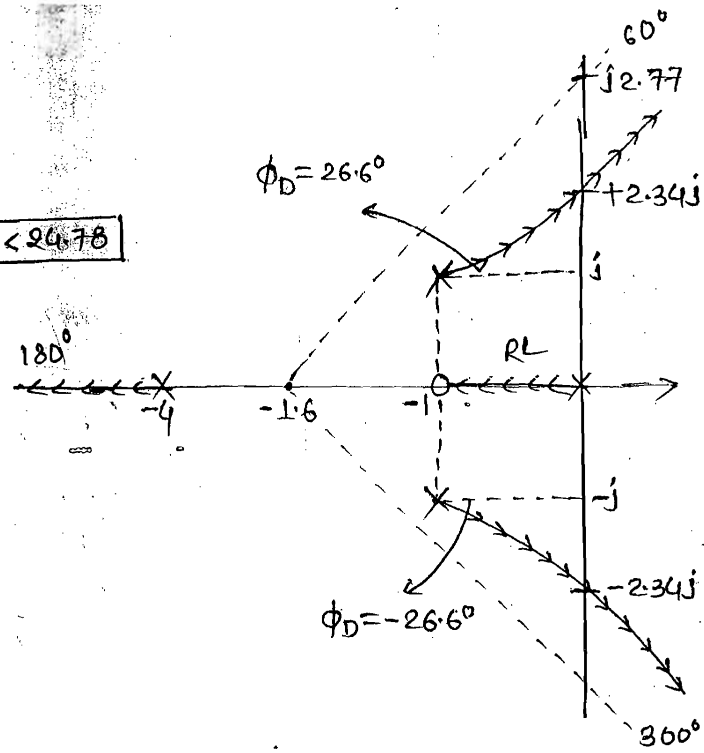
$$\phi P_2=90^\circ;\ \phi P_2=90^\circ$$

$$\phi P_3=\tan^{-1}\left(\frac{1-0}{-1-(-4)}\right)=18.4^\circ$$

$$\phi=90^\circ-(135^\circ+90^\circ+18.4)=-153.4^\circ$$

$$\phi_D=180-153.4^\circ=26.6^\circ$$

$\boxed{0<K<24.78}$

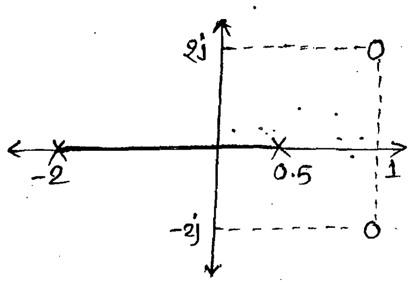


Q(3)/69

$$G(s) = \frac{K(s^2-2s+5)}{(s+2)(s-0.5)}$$

(2.) $P = 2,\ Z = 2,\ P-Z = 0$

(3.)



(6.) <u>BA. point→</u>

$$(s+2)(s-0.5) + K(s^2-2s+5) = 0$$

$$s^2+1.5s-1 + K(s^2-2s+5) = 0$$

$$K = \frac{-s^2-1.5s+1}{s^2-2s+5}$$

$$\frac{dK}{ds} = 0$$

$$\Rightarrow \quad 3.5s^2-12s-5.5=0$$

$$s = -0.4,\ 3.8$$

(7.) $s^2(1+K) + s(1.5-2K) + (5K-1) = 0$

$$\begin{array}{c|cc} s^2 & (1+K) & 5K-1 \\ s^1 & (1.5-2K) & 0 \\ s^0 & 5K-1 & 0 \end{array}$$

$1.5 - 2K = 0$

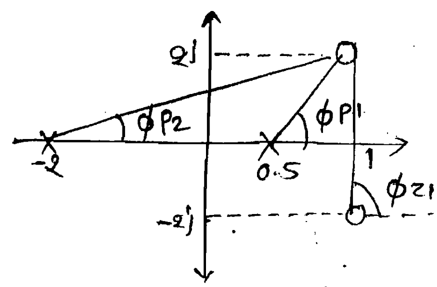
$K_{mar} = 0.75$

$A(s) = (1+K)s^2 + (5K-1) = 0$

$(1+0.75)s^2 + (5\times 0.75 - 1) = 0$

$\boxed{s = \pm j1.25}$

(8.) Angle of arrival $\rightarrow$



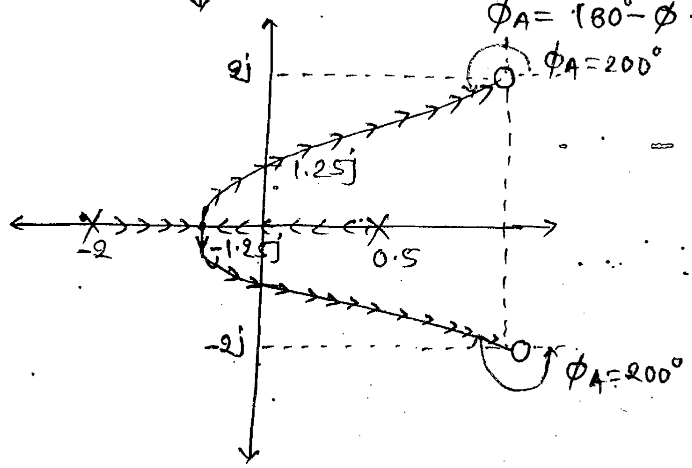
$\phi z_1 = 90^\circ$

$\phi P_1 = \tan^{-1}\left(\frac{2-0}{1-0.5}\right) = 76^\circ$

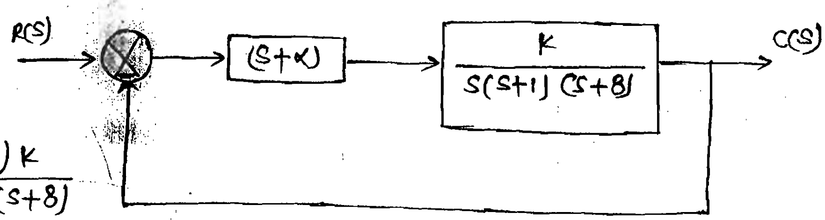
$\phi P_2 = \tan^{-1}\left(\frac{2-0}{1-(-2)}\right) = 34^\circ$

$\phi = 90^\circ - (76+34) = -20^\circ$

$\phi_A = 180^\circ - \phi = 180 - (-20) = 200^\circ$



* Root contour →



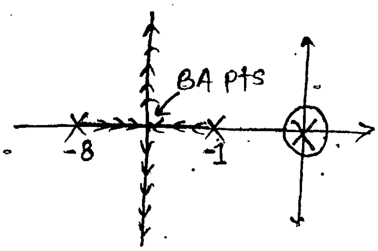
$G(S) = \frac{K(S+\alpha)K}{S(S+1)(S+8)}$

* These are multiple RL diagrams obtained by varying multiple parameter in a TF drawn on same s-plane.

Case(i) →

$\alpha = 0$

$G(S) = \frac{K\cdot \cancel{S}}{\cancel{S}(S+1)(S+8)}$



Case(2) →

$1 + \frac{K(S+\alpha)}{S(S+1)(S+8)} = 0$

$S(S+1)(S+8) + K(S+\alpha) = 0$

$S(S+1)(S+8) + KS + K\alpha = 0$

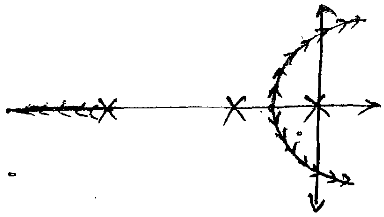
$1 + \frac{K\alpha}{S(S+1)(S+8)+KS} = 0$

$1 + G(S)\cdot H(S) = 0$

$G(S)\cdot H(S) = \frac{K\alpha}{S(S+1)(S+8)+KS}$

if value of K is not given then K=1

$G(S)\cdot H(S) = \frac{\alpha}{S(S+1)(S+8)+S} = \frac{\alpha}{S(S^2+9S+9)}$



Q.→ Find BA. points for k=10 ?

Soln →

$$G(s).H(s) = \frac{10\alpha}{s(s+1)(s+8)+10s}$$

let $10\alpha = k'$

$$= \frac{k'}{s(s+1)(s+8)+10s}$$

$$= \frac{k'}{s(s^2+9s+18)}$$

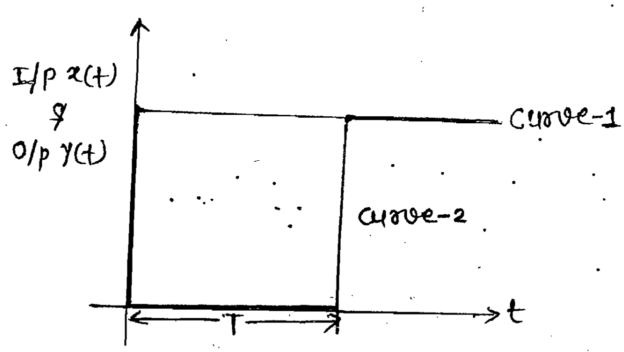
$$1 + \frac{k'}{s(s^2+9s+18)} = 0$$

$$s^3+9s^2+18s+k' = 0$$

$$k' = -s^3-9s^2-18s$$

$$\frac{dk'}{ds} = -3s^2-18s-18 = 0$$

* Analysis of sys. having DEAD TIME (or) TRANSPORTATION LAG →



For curve-(1) ·
O/p $y(t) = $ I/p $x(t)$

For curve-(2)
O/p $y(t) = x(t-T)$

$$Y(s) = e^{-Ts} X(s)$$

$$\boxed{\frac{Y(s)}{X(s)} = e^{-Ts}}$$

Time domain approximation → (Time domain analysis, RH, RL plots)

$$y(t) = x(t-T) = x(t) - T\dot{x}(t) + \frac{T^2}{2!}\ddot{x}(t) - \frac{T^3}{3!}\dddot{x}(t) + \cdots\cdots$$

$$y(t) = x(t) - T\dot{x}(t)$$

$$Y(s) = X(s)(1-Ts) \quad \& \quad Y(s) = X(s)\cdot e^{-Ts}$$

$$\boxed{e^{-Ts} \simeq (1-Ts)}$$

Ex:- $G(s) = \dfrac{Ke^{-s}}{s(s+3)} = \dfrac{K(1-s)}{s(s+3)}$

* Dead time is one of the forms of non-linearity & is approximated as zero in RHS of s-plane.
* TF having poles (or) zeros in RHS of s-plane are known as non-min^m phase f^n.

Non min^m phase f^n →

$$\boxed{\angle F(s)\Big|_{\omega\to\infty} \neq -(P-Z)\cdot 90^\circ}$$

Eg:- $G(s) = \dfrac{Ke^{-s}}{s(s+3)} = \cancel{\dfrac{K(s-3)}{}}\ \dfrac{K(1-s)}{s(s+3)} = \dfrac{K(1-s)}{(s)\,3\left(1+\frac{s}{3}\right)}$

$$G(j\omega) = \frac{\left(\frac{K}{3}+j0\right)(1-j\omega)}{(0+j\omega)\left(1+\frac{j\omega}{3}\right)}$$

$$\angle G(j\omega) = \frac{(0^\circ)(-\tan^{-1}\omega)}{(90^\circ)\left(\tan^{-1}\frac{\omega}{3}\right)} = -90^\circ - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{3}$$

$$\angle G(j\omega)\Big|_{\omega=\infty} = -90^\circ - 90 - 90 = -270^\circ \neq -(P-Z)90^\circ$$

* TF having poles & zeros in the LHS of s-plane are known as min^m phase f^n. LTI TF should be min^m phase f^n.

min^m phase f^n →

$$\boxed{\angle F(s)\Big|_{\omega\to\infty} = -(P-Z)\,90^\circ}$$

Eg:- $G(s) = \frac{K(1+s)}{s(s+3)} = \frac{(K/3)(1+s)}{s(1+\frac{s}{3})}$

$$G(j\omega) = \frac{(\frac{K}{3}+j0)(1+j\omega)}{(0+j\omega)(1+\frac{j\omega}{3})}$$

$$\angle G(j\omega) = \frac{(0^\circ)\,[\tan^{-1}(\omega)]}{(90^\circ)\,(\tan^{-1}\frac{\omega}{3})} = -90^\circ + \tan^{-1}\omega - \tan^{-1}\frac{\omega}{3}$$

$$\angle G(j\omega)\big|_{\omega=\infty} = -90 + 90^\circ - 90^\circ = -90$$

$$\boxed{-90^\circ = -(P-Z)90^\circ}$$

* since s indicates time & can't be -ve (1-s) factor should be expressed as -(s-1) in time domain methods.

$$G(s) = \frac{Ke^{-s}}{s(s+3)} = \frac{K(1-s)}{s(s+3)} = \frac{-K(s-1)}{s(s+3)}$$

* Complemetry R-L (CRL) or Inverse RL (IRL) →

$$G(s)H(s) = 1+j0$$

(1.) Angle condn

$$\angle G(s)\cdot H(s) = 0^\circ = \pm(2q)180^\circ$$

(2.) mag. condn

$$|G(s)\cdot H(s)| = 1$$

* construction Rule of CRL →

Rule(1). The CRL is symmetrical about real axis.

Rule(2) Same as RL.

Rule(3) A point on real axis is said to be on CRL is to the right side of this point the sum of open loop poles & zeros is even.

Rule(4) Angle of asymptodes →

$$\boxed{\theta = \frac{(2q)180}{P-Z}}$$

where, $q = 0, 1, 2, \ldots$

Rule (5) Centroid → same as RL

Rule (6) BA points → same as RL

Rule (7) Intersection of CRL with $j\omega$ axis → same as RL

Rule (8) Angle of departure & arrival →

$$\phi_D = 0° + \phi$$

$$\phi_A = 0° - \phi$$

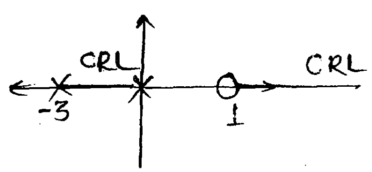
where;

$$\phi = \Sigma\phi_Z - \Sigma\phi_P$$

Que.→ $G(s) = \dfrac{Ke^{-s}}{s(s+3)} = \dfrac{K(1-s)}{s(s+3)}$

Soln→ (2.) $P = 2$; $Z = 1$, $P - Z = 1$

(3.)



(6.) BA points:-

$$1 + \frac{K(1-s)}{s(s+3)} = 0$$

$$s(s+3) + K(1-s) = 0$$

$$K = \frac{-s^2 - 3s}{1-s}$$

$$\frac{dK}{ds} = 0$$

$$\frac{(1-s)(-2s-3) - [(-s^2-3s)(-1)]}{(1-s)^2} = 0$$

$$s^2 - 2s - 3 = 0$$

$$s = \frac{2 \pm \sqrt{4+12}}{2}$$

$$s = 1 \pm 2 = -1, 3$$

(7.) $s^2 + s(3-K) + K = 0$

s^2	1	K
s	$3-K$	0
s^0	K	0

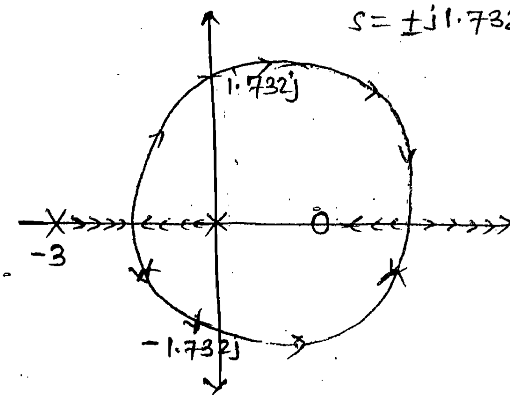
$3-K=0$

$K_{mar}=3$

$A(s)=s^2+K=0$

$s^2+3=0$

$s=\pm j1.732$



$\frac{1}{67}$

$$G(s)=\frac{K(s+a)}{s^2(s+b)}$$

an (s) (c)

$1+G(s)=0$

$$1+\frac{K(s+a)}{s^2(s+b)}=0$$

$s^3+bs^2+Ks+aK=0$

s^3	1	K
s^2	b	aK
s^1	$\frac{bK-aK}{b}$	0
s^0	aK	0

(i) $aK>0$

$K>0$

(ii) $\frac{bK-aK}{b}>0$

$K(b-a)>0$

$\boxed{K>0}$

$\frac{bK-aK}{b}=0$

$K(b-a)=0$

$K=K_{mar}=0$

$A(s)=bs^2+aK=0$

$bs^2+0=0$

$\boxed{s=0}$

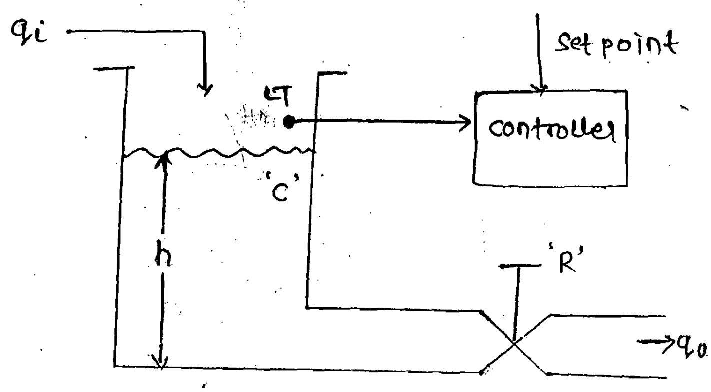
$\frac{10}{69}$

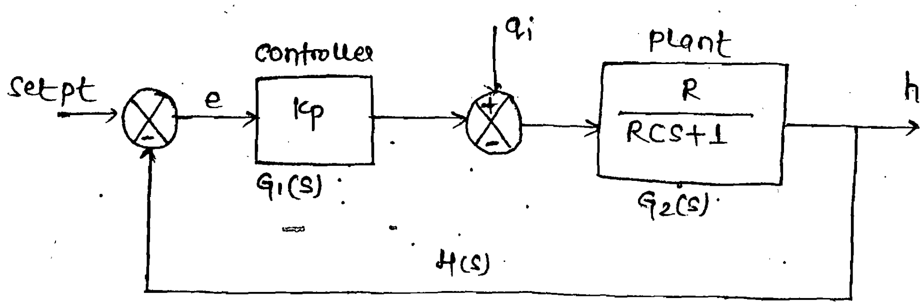
$0\le K<1 \rightarrow$ overdamped (1, 3) ans. (c)

$K>5$. overdamped

DATE-21/11/14

INDUSTRIAL CONTROLLER





$$|e_{ss}| = \lim_{S\to 0} \frac{S \cdot Q_i(S) \cdot G_2(S)}{1+G_1(S)G_2(S)}$$

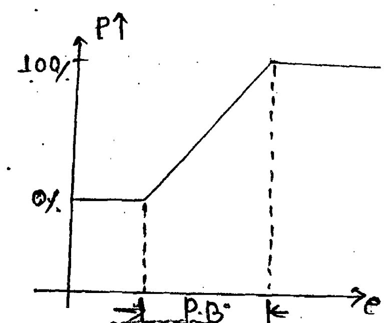
Proportional mode →

$p \propto e$

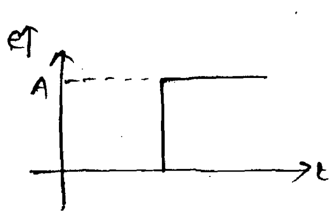
$p = K_p e$; K_p = proportional gain

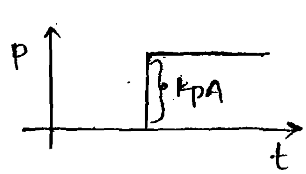
$P(S) = K_p E(S)$

$E(S) \rightarrow \boxed{K_p} \rightarrow P(S)$



$$\boxed{\text{Proportional Band (PB)} = \frac{100}{K_p}}$$





$p = K_p e \quad (e = A)$

$p = K_p A$

$$|e_{ss}| = \lim_{S\to 0} \frac{S \cdot \frac{A}{S} \cdot \frac{R}{RCS+1}}{1 + \frac{RK_p}{RCS+1}}$$

$$\boxed{|e_{ss}| = \frac{AR}{1+RK_p}}$$ Offset

$$\boxed{\text{Offset} \propto \frac{1}{K_p}}$$

* It is a natural extension of ON/OFF controller.
* The band of error where every value of error has unique value of controller o/p is known as proportional band.
* The disadvantage of this controller is it exibits a permanant residual error known as offset.

20/63

$P = K_p e$

$100 = K_p \times 1$

$K_p = 100$

$PB = \frac{100}{K_p} = \frac{100}{100} = 1$

$\% PB = 1 \times 100\% = 100\%$

$100\%\ PB = 100V$

$20\%\ PB = ?$

$\frac{20}{100} \times 100 = 20V$

$\boxed{1V, 20V}$

(2) Integral mode →

$\frac{dp}{dt} \propto e$

$\frac{dP}{dt} = K_I e$ (K_I = Integral scaling)

$p = K_I \int e\,dt$

$P(s) = \frac{K_I}{s} E(s)$

$$E(s) \longrightarrow \boxed{\frac{K_I}{s}} \longrightarrow P(s)$$

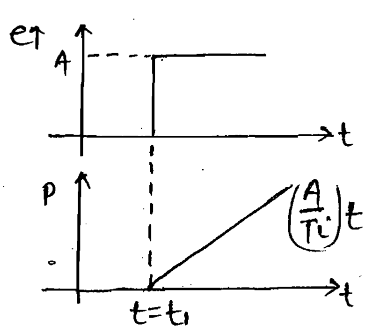
Defining "RESET TIME!"

$T_i = \frac{1}{K_I}$

$P = \frac{1}{T_i} \int e\,dt$

$P(s) = \frac{1}{T_i s} . E(s)$

$$E(s) \longrightarrow \boxed{\frac{1}{T_i s}} \longrightarrow P(s)$$



$P = \frac{1}{T_i} \int A\,dt \quad (e = A)$

$P = \left(\frac{A}{T_i}\right) t$

Let $e = \sin\omega t$

$P = \frac{1}{T_i} \int \sin\omega t\,dt$

$P = \frac{1}{\omega T_i} (-\cos\omega t)$

$P = \frac{1}{\omega T_i} \sin(\omega t - \frac{\pi}{2})$

$$|e_{ss}| = \lim_{s \to 0} \frac{\cancel{s} \cdot \frac{A}{\cancel{s}} \times \frac{R}{RCs+1}}{1 + \frac{R}{T_i s (T+s)}}$$

$$\boxed{|e_{ss}| = \frac{AR}{1+\infty} = 0}$$

* The disadvantage of integral controller is its responce to errors is slow. However it is capable of eliminating the error completely in the sys.

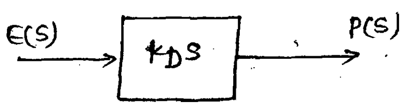
3) Derivative (OR) Rate mode →

$$P \propto \frac{de}{dt}$$

$$P = K_D \frac{de}{dt}$$

K_D = Rate constant

$$P(s) = K_D s \cdot E(s)$$

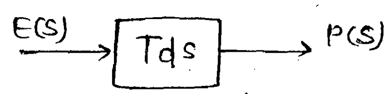


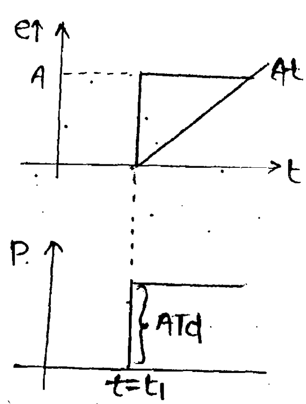
Defining "RATE TIME"

$$T_d = K_D$$

$$P = T_d \frac{de}{dt}$$

$$P(s) = T_d s \cdot E(s)$$





$$P = T_d \frac{d}{dt}(A) \qquad (e = A)$$

$$P = 0$$

$$P = T_d \frac{d}{dt}(At) \qquad (e = At)$$

$$P = T_d A$$

$$|e_{ss}| = \lim_{s \to 0} \frac{s \cdot \frac{A}{s^2} \cdot \frac{R}{RCs+1}}{1 + T_d s \cdot \frac{R}{RCs+1}}$$

$$|e_{ss}| = \lim_{s \to 0} \frac{\frac{AR}{RCs+1}}{\frac{s + s^2 T_d R}{RCs+1}}$$

$$|e_{ss}| = \frac{AR}{0} = \infty$$

$$\boxed{|e_{ss}| = \infty}$$

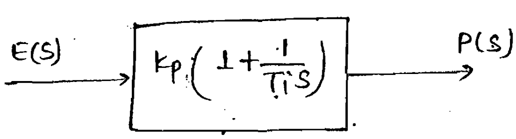
* The disadvantage of this controller is it can't respond to sudden error.
* It is also called as anticipatary controller because it sends a control signal in anticipation of error.
* This anticipatry nature may result in large instability in the system.

* Composite controller mode →

(1) P+I mode →

$$P = K_p e + \frac{K_p}{T_i}\int e\,dt$$

$$P(S) = \left[K_p\left(1+\frac{1}{T_i S}\right)\right] E(S)$$



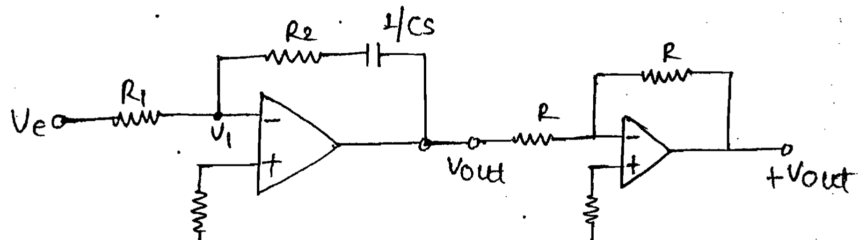
Effect on transient state

Let $e = \sin\omega t$

$$P = K_p \sin\omega t + \frac{K_p}{T_i}\int \sin\omega t\,dt$$

$$P = K_p \sin\omega t + \left(\frac{-K_p}{\omega T_i}\right)\cos\omega t$$

$$P = \sqrt{K_p^2 + \left(\frac{K_p}{\omega T_i}\right)^2}\,\sin\left(\omega t - \tan^{-1}\frac{1}{\omega T_i}\right)$$



$$\frac{V_e - V_1}{R_1} = \frac{V_1 - V_{out}}{\frac{R_2 CS + 1}{CS}}$$

$\because V_1 = 0$

$$\frac{V_e(R_2 CS + 1)}{R_1 CS} = -V_{out}$$

$$-V_{out} = \frac{V_e R_2 \cancel{CS}}{R_1 \cancel{CS}} + \frac{V_e}{R_1 CS}$$

$$V_{out} = \frac{R_2}{R_1} V_e + \frac{R_2}{R_1}\frac{1}{R_2 C}\int V_e\,dt$$

$$\boxed{K_p = \frac{R_2}{R_1}, \quad T_i = R_2 C}$$

* It is capable of improving steady state responce c/s of the sys. i.e. elimination of steady state error b/n i/p & o/p.
* The integral controller eliminates offset of proportional controller.
* It is also known as proportional + Reset controller because the rate of change of controller o/p can be reset by changing the value of reset time Ti.
* For sinusoidal i/p the phase of the controller o/p lags by $\tan^{-1}\left(\frac{1}{\omega T_i}\right)$. Hence it is similar to phase lag compensator.
* It terms of filtering property it acts as low pass filter.
* The P+I controller increases the type & order of system by one.

<u>Effect on performance specification→</u>

1.) It increases rise time.

2.) It reduces BW.

3.) It reduces the stability of the system.

4.) It increases the damping ratio & hence reduces the peak overshoot.

5.) It eliminate steady state error.

2.) P+D mode→

$$P = K_p e + K_p T_d \frac{de}{dt}$$

$$P(s) = \left[K_p(1+T_d s)\right] E(s)$$

$$E(s) \longrightarrow \boxed{K_p(1+T_d s)} \longrightarrow P(s)$$

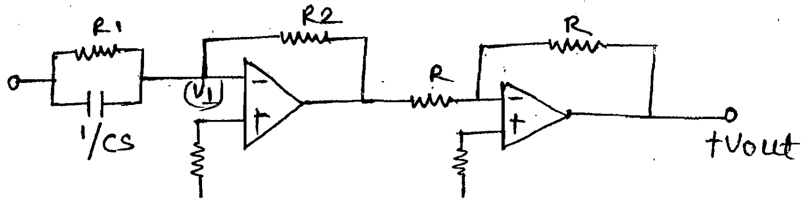
<u>Effect on transient state</u>

Let $e = \sin\omega t$

$$P = K_p \sin\omega t + K_p T_d \frac{d}{dt}\sin\omega t$$

$$P = K_p \sin\omega t + \omega K_p T_d \cos\omega t$$

$$P = \sqrt{K_p^2 + (\omega K_p T_d)^2}\cdot \sin\left(\omega t + \tan^{-1}\omega T_d\right)$$



$$\frac{\frac{Ve - V_1}{\frac{R_1}{R_1CS+1}}}{} = \frac{V_1 - Vout}{R_2}$$

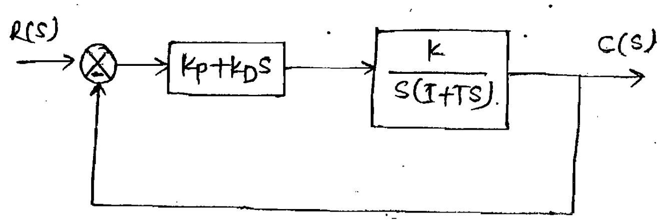
$$\boxed{Kp = \frac{R_2}{R_1}\ ;\ Td = R_1C}$$

$$-Vout = \frac{Ve(R_1CS+1) \cdot R_2}{R_1}$$

$$-Vout = \frac{R_2}{R_1} Ve + \frac{R_2}{R_1} \cdot R_1 CS Ve$$

$$+Vout = \frac{R_2}{V_1} ve + \frac{R_2}{R_1} \cdot R_1 C \cdot \frac{dve}{dt}$$

Chapter (3.)
Conv. (4.)



(1.) without P+D controller →

$$G(s) = \frac{K}{S(1+TS)}$$

Type-1/order-2

with P+D controller

$$G(S) = \frac{K(Kp + K_D S)}{S(1+TS)}$$

Type-1/order-2

(2.) with P-controller →

$$G(S) = \frac{KKp}{S(1+TS)}$$

$$1 + \frac{KKp}{S(1+TS)} = 0$$

$$S(1+TS) + KKp = 0$$

$$TS^2 + S + KKp = 0$$

$$S^2 + \frac{S}{T} + \frac{K \cdot Kp}{T} = 0$$

$$\boxed{\omega_n = \sqrt{\frac{KKp}{T}}\ r/s}$$

$$2\xi\sqrt{\frac{KKp}{T}} = \frac{1}{T}$$

$$\boxed{\xi = \frac{1}{2\sqrt{KKpT}}}$$

(III) With P+D controller →

$$G(s) = \frac{K(K_P + K_D s)}{s(1+Ts)}$$

$$1 + \frac{K(K_P + K_D s)}{s(1+Ts)} = 0$$

$$Ts^2 + s + K_D s + KK_P = 0$$

$$s^2 + \frac{s(1+K_D)}{T} + \frac{KK_P}{T} = 0$$

$$\boxed{\omega_n = \sqrt{\frac{KK_P}{T}}\ r/s}$$

$\cancel{2\xi\omega_n}$

$$2\xi\sqrt{\frac{KK_P}{T}} = \frac{1+KK_D}{T}$$

$$\boxed{\xi = \frac{1+KK_D}{2\sqrt{KK_P T}}}$$

(IV) e_{ss} | unit Ramp i/p →

$$e_{ss} = \lim_{s\to 0} \frac{s \cdot \frac{1}{s^2}}{1 + \frac{K(K_P + K_D s)}{s(1+Ts)}}$$

$$\boxed{e_{ss} = \frac{1}{KK_P}}$$

* It is capable of improving the transient state c/s of the system only. i.e. it improves the speed of responce of sys.
* For sinusoidal i/p the phase of controller O/p leads by $\tan^{-1}\omega T_d$. Hence it is similar to phase lead compensator.
* In terms of filtering property it acts as HPF.
* The P+D controller does not affect the type & order of the system.

* effect on performance specification →

(1.) Reduces the Rise time.

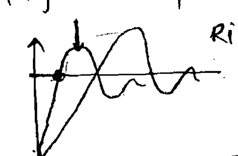
(2.) Increases the BW.

(3.) It amplifies noise & hence reduces $\frac{S}{N}$ ratio.

(4.) It increases the stability of the sys.

(5.) It increases the damping ratio ξ & Hence reduces peak overshoot.

Rise time & peak overshoot ↓.



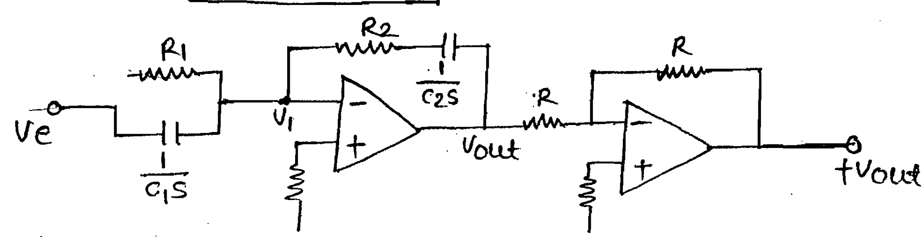
14/62 (d.) 15/63 (c.) 16/63 (c.) 18/63 (c.) 8/77 (b.) 9/77 (a.)

(3.) PID mode →

$$p = k_p e + \frac{k_p}{T_i}\int e\,dt + k_p T_d \frac{de}{dt}$$

$$P(s) = \left[k_p\left(1 + \frac{1}{T_i s} + T_d s\right)\right] E(s)$$

$$E(s) \longrightarrow \boxed{k_p\left(1+\frac{1}{T_i s}+T_d s\right)} \longrightarrow P(s)$$



$$\Rightarrow \frac{V_e - V_1}{\frac{R_1}{R_1C_1s+1}} = \frac{V_1 - V_{out}}{\frac{R_2C_2s+1}{C_2 s}}$$

$$\Rightarrow -V_{out} = \frac{\cancel{V_e(R_1C_1}s+1)(R_2C_2s+1)}{R_1C_2s}$$

$$\Rightarrow -V_{out} = \frac{V_e(\cancel{R_1}C_1R_2\cancel{C_2}s^{\cancel{2}})}{\cancel{R_1}\cancel{C_2}\cancel{s}} + \frac{V_e\cancel{s}(R_1C_1+R_2C_2)}{R_1C_2\cancel{s}} + \frac{V_e}{R_1C_2s}$$

$$\Rightarrow -V_{out} = V_e\left[\frac{R_1C_1}{R_1C_2} + \frac{R_2C_2}{R_1C_2}\right] + \frac{V_e}{R_1C_2s} + R_2C_1sV_e$$

$$\Rightarrow +V_{out} = \frac{R_2}{R_1}v_e + \frac{R_2}{R_1}\cdot\frac{1}{R_2C_2}\int v_e\,dt + \frac{R_2}{R_1}\cdot R_1C_1\frac{dv_e}{dt}$$

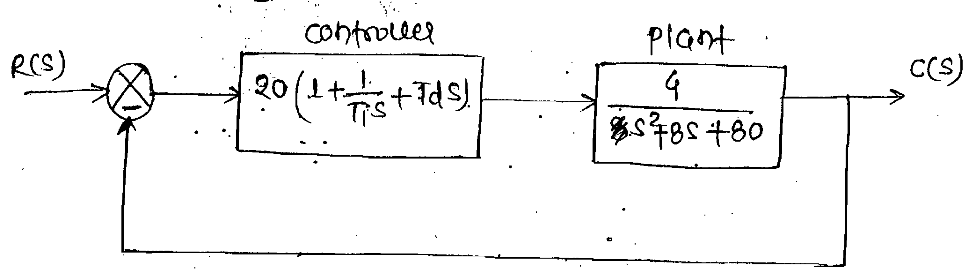
$$\boxed{k_p = \frac{R_2}{R_1}\ ;\ T_i = R_2C_2 \quad T_d = R_1C_1}$$

* It improves both transient state & steady state responce c/s.
* It is similar to lag lead compensator.
* In terms of filtering property it acts as band reject filter.

*** Effect on performance specification →**

(1.) It reduces rise time.
2.) It increases BW.
3.) It amplifies noise & Hence reduces S/N ratio.
4.) It increases stability of the sys.
5) It increases damping ratio & Hence reduces peak overshoot.
6.) It eliminates steady state error b/n i/p & o/p.
7.) The PID controller increases type & order of sys. by 1.

1/78 $\theta_c(s) = \left\{20\left[1+\frac{1}{T_i s}+T_d s\right]\right\} E(s)$



(a.) $G(s) = \frac{4\times 20(1+T_d s)}{s^2+8s+80}$

(T_i is ∞ because given)

$1+G(s) = 0$

$1+\frac{80(1+T_d s)}{s^2+8s+80} = 0$

$s^2+8s+80+80(1+T_d s) = 0$

$s^2+s(8+80T_d)+160 = 0$

$\omega_n = \sqrt{160} = 12.64\ r/s$

$2\xi\times 12.64 = 8+80T_d$

$\xi = 1$ (given)

$2\times 1\times 12.64 = 8+80T_d$

$\boxed{T_d = 0.2s}$

(b.) $G(s) = \frac{20(T_i s+20+4T_i s^2)4}{T_i s(s^2+8s+40)}$

$1+G(s) = 0$

$T_i s^3+8T_i s^2+80T_i s+80T_i s+80+16T_i s^2 = 0$

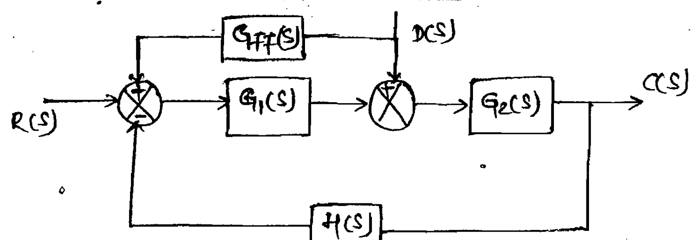
$T_i s^3+24T_i s^2+160T_i s+80 = 0$

$s^3+24s^2+160s+\frac{80}{T_i} = 0$

$\frac{80}{T_i} = 24\times 160$

$\boxed{T_i = 0.02s}$

* Feed Forward Compensation →



$$\left.\frac{C(s)}{R(s)}\right|_{D(s)=0} = \frac{G_1(s)\cdot G_2(s)}{1+G_1(s)\cdot G_2(s)\cdot H(s)}$$

$$\left.\frac{C(s)}{R(s)}\right|_{R(s)=0} \quad \frac{G_2(s)+G_{ff}(s)\cdot G_1(s)\cdot G_2(s)}{1+G_1(s)\cdot G_2(s)H(s)}$$

$$C(s) = \frac{R(s)\cdot[G_1(s)G_2(s)] + D(s)[G_2(s)+G_{ff}(s)G_1(s)G_2(s)]}{1+G_1(s)G_2(s)H(s)}$$

To eliminate the effect of the disturbances in the sys., the condn for feed forward controller is

$$\boxed{G_{ff}(s) = \frac{-1}{G_1(s)}}$$

$\frac{Q(B)}{Chap(Q)}$.

$$G_1(s) = \frac{k(s+a)(s+c)}{(s+b)(s+d)} \quad ; \quad G_2(s)=1$$

$$G_c(s) = \frac{1}{G_1(s)} = \frac{(s+b)(s+d)}{k(s+a)(s+c)}$$

$$\boxed{G_c(s) = \frac{(s+b)(s+d)}{k(s+a)(s+c)}}$$