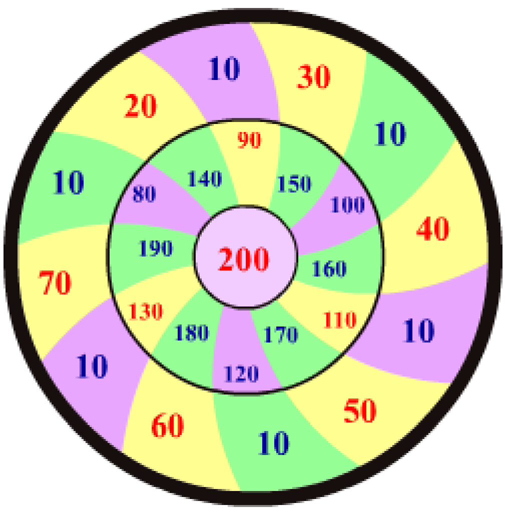
Probability

Terminology Related to Probability

Observing an Experiment

It is not always possible to tell the exact outcome of a particular action. Take, for example, a dart board.



A dart is repeatedly thrown toward the dartboard, targeting a random number in each throw. We do not know which number is targeted in a particular throw. What we do know is that there is a fixed group of numbers and each time the targeted number is one of them.

We know that the likelihood of occurrence of an unpredictable event is studied under the theory of probability. So, we can say that there is a certain probability for each number to be targeted in the above experiment.

Let us learn more about probability and the meanings of terms associated with it, for example, ‘experiment’ and ‘outcome’.

Did You Know?

The word ‘probability’ has evolved from the Latin word ‘probabilitas’, which can be considered to have the same meaning as the word ‘probity’. In olden days in Europe, ‘probity’ was a measure of authority of a witness in a legal case, and it often correlated with the nobility of the witness.

The modern meaning of probability, however, focuses on the statistical observation of the likelihood of occurrence of an event.

Know More

Probability is widely applicable in daily life and in researches pertaining to different fields. It is an important factor in the diverse worlds of share market, philosophy, artificial intelligence or machine learning, statistics, etc. All gambling is based on probability. In gambling, one considers all possibilities and then tries to predict a result that is most likely to happen. The concept of probability is perhaps the most interesting topic to discuss in mathematics.

Terms Related to Probability

Experiment: When an operation is planned and done under controlled conditions, it is known as an experiment. For example, tossing a coin, throwing a die, drawing a card from a pack of playing cards without seeing, etc., are all experiments. A chance experiment is one in which the result is unknown or not predetermined.

Outcomes: Different results obtained in an experiment are known as outcomes. For example, on tossing a coin, if the result is a head, then the outcome is a head; if the result is a tail, then the outcome is a tail.

Random: An experiment is random if it is done without any conscious decision. For example, drawing a card from a well-shuffled pack of playing cards is a random experiment if it is done without seeing the card or figuring it out by touching.

Trial: A trial is an action or an experiment that results in one or several outcomes. For example, if a coin is tossed five times, then each toss of the coin is called a trial.

Sample space: The set of all possible outcomes of an experiment is called the sample space. It is denoted by the English letter 'S' or Greek letter 'Ω' (omega). In the experiment of tossing a coin, there are only two possible outcomes—a head (H) and a tail (T).

∴ Sample space (S) = {H, T}

Event: The event of an experiment is one or more outcomes of the experiment. For example, tossing a coin and getting a head or a tail is an event. Throwing a die and getting a face marked with an odd number (i.e., 1, 3 or 5) or an even number (2, 4 or 6) is also an event.

Know More

Initially, the word 'probable' meant the same as the word 'approvable' and was used in the same sense to support or approve of opinions and actions. Any action described as ‘probable’ was considered the most likely and sensible action to be taken by a rational and sensible person.

Whiz Kid

Equally Likely: If each outcome of an experiment has the same probability of occurring, then the outcomes are said to be equally likely outcomes.

Know Your Scientist

Girolamo Cardano (1501–1576) was a great Italian mathematician, physicist, astrologer and gambler. His interest in gambling led him to do more research on the concept of probability and formulate its rules. He was often short of money and kept himself solvent through his gambling skills. He was also a very good chess player. He wrote a book named *Liber de Ludo Aleae*. In this book about games of chance, he propounded the basic concepts of probability.

Solved Examples

Example 1: **A fair die is thrown. What is the sample space of this experiment?**

Solution:

When a die is thrown, we can have six outcomes, namely, 1, 2, 3, 4, 5 and 6.

We know that sample space is the collection of all possible outcomes of an experiment.

∴ Sample space (S) = {1, 2, 3, 4, 5, 6}

Example 2: **Which of the following are experiments?**

i)Tossing a coin

ii)Rolling a six-sided die

iii)Getting a head on a tossed coin

Solution:

Tossing a coin and rolling a six-sided die are experiments, while getting a head on a tossed coin is the outcome of an experiment.

Medium

Example 1: **What is the sample space when two coins are tossed together?**

Solution:

When two coins are tossed together, we can get four possible outcomes. These are as follows:

i)A head (H) on one coin and a tail (T) on the other

ii)A head (H) on one coin and a head (H) on the other

iii)A tail (T) on one coin and a head (H) on the other

iv)A tail (T) on one coin and a tail (T) on the other

∴ Sample space (S) = {HT, HH, TH, TT}

Experimental Probability

An Experiment with a Die

When a single six-sided die is rolled, we get a single outcome every time. Every outcome can be either an odd number or an even number. In an experiment, a single six-sided die is rolled seven times. The seven outcomes are listed in the following table.

Trial	1	2	3	4	5	6	7
Outcomes	Even number	Odd number	Odd number	Even number	Even number	Even number	Odd number

What do you observe? Is there any pattern in the outcomes? There is no pattern. So, we cannot predict the outcome of rolling the die for an eighth time. However, we can calculate the probability of getting an odd or even number by observing the outcomes of the previous seven rolls of the die.

In this lesson, we will learn to calculate the probability of occurrence of an event in an experiment. We will also solve problems related to the same.

Theoretical Probability

If we divide the number of ways in which a favourable event can occur by the total number of outcomes, then we get the theoretical probability of occurrence of that particular event.

The formula of theoretical probability is as follows:

$$\text{Theoretical probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

For example, with respect to a single roll of a six-sided die, we have:

Number of favourable outcomes for getting 6 = 1

Total number of outcomes = 6(∵ Any number from 1−6 can be obtained)

$$\therefore \text{Theoretical probability of getting 6 on rolling a die} = \frac{1}{6}$$

Similarly,

Number of favourable outcomes for getting an even number = 3 (i.e., 2, 4 or 6)

Total number of outcomes = 6(i.e., 1, 2, 3, 4, 5 and 6)

$$\therefore \text{Theoretical probability of getting an even number on rolling a die} = \frac{3}{6} = \frac{1}{2}$$

In this manner, we can find the theoretical probability of occurrence of any event.

Experimental probability

The probability of an event ascertained by observing the outcomes of an experiment is known as **empirical or experimental probability**.The formula for finding the experimental probability of an event (E) is as follows:

$$\mathbf{P}(E) = \frac{\textbf{Number of trials in which the event happened}}{\textbf{Total number of trials}}$$

Let us once again consider the observation table made at the beginning. In this experiment, we got an odd number 3 times and an even number 4 times on rolling the die a total of 7 times.

Let the probability of getting an odd number be P(E_1) and that of getting an even number be P(E_2). Then, we get:

$$P(E_1) = \frac{3}{7} \text{ and } P(E_2) = \frac{4}{7}$$

In this manner, we can find the experimental probability of any event in an experiment.

Did You Know?

- The probability of an event always lies between 0 and 1.
- The sum of the probabilities of all events in a single experiment is always 1, i.e., $P(E_1) + P(E_2) + \ldots + P(E_n) = 1$, where $E_1, E_2, \ldots, E_n$ are n events in a single experiment.

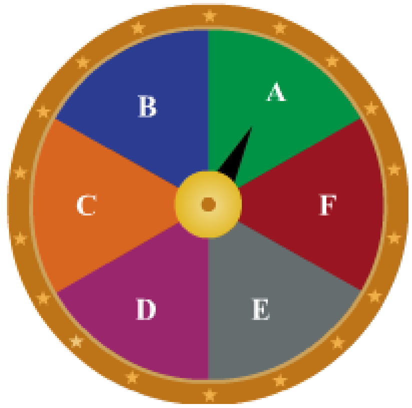
- **Did You Know?**
- **Applications of probability**
- 1.Life expectancy: It is the prediction of the age of individuals on the basis of the ages of their ancestors or the ages of people belonging to similar groups in the past. This prediction is used as a guideline by financial advisers to help their clients in planning for their retirement years.
- 2.Casino games: Casino owners always consider the concept of probability to ensure that they don't lose money in the business. The odds are always in favour of casino owners. Smart gamblers who know to use probability in casino games try to defy these odds.

- **Did You Know?**
- The concept of probability is applied in different contexts, for example, in risk assessment and in trading in financial markets. Governments apply methods of probability in environmental regulation. This is known as pathway analysis.

Solved Examples

Easy

Example 1: **The given figure shows a wheel with six letters of the English alphabet written in six sectors of equal area.**



The follwing table shows the results of spinning the wheel ten times.

Trials	1	2	3	4	5	6	7	8	9	10
Outcomes	A	E	D	C	A	B	F	C	A	F

What is the probability of the most favorable outcome?

Solution:

It can be observed from the table that the most favorable outcome is 'A' as it is obtained the most number of times (3) in the experiment.

We know that the experimental probability of an event is given as:

$$\mathbf{P}(E) = \frac{\textbf{Number of trials in which the event happened}}{\textbf{Total number of trials}}$$

$$\therefore \text{Required probability} = \frac{\text{Number of trials in which 'A' is obtained}}{\text{Total number of trials}} = \frac{3}{10}$$

Thus, the probability of the most favourable outcome is $\frac{3}{10}$.

Example 2: In a pack of 5000 bulbs, 250 bulbs are defective. Find the probability that a bulb chosen at random will be non-defective.

Solution:

It is given that:

Total number of bulbs = 5000

Number of defective bulbs = 250

∴ Number of non-defective bulbs = 5000 − 250 = 4750

So, probability of choosing a non-defective bulb = $\dfrac{\text{Number of non-defective bulbs}}{\text{Total number of bulbs}}$

$$= \frac{4750}{5000}$$
$$= \frac{19}{20}$$

Medium

Example 1: **A die is thrown 100 times. The faces marked with numbers 1, 2, 3, 4, 5 and 6 are observed 20, 15, 25, 10, 10 and 20 times respectively. Find the probability of getting each of these events.**

Solution:

Total number of trials = 100

Let E_1, E_2, E_3, E_4, E_5 and E_6 be the respective events of getting the faces marked with numbers 1, 2, 3, 4, 5 and 6.

Number of outcomes for the face marked '1' = 20

Number of outcomes for the face marked '2' = 15

Number of outcomes for the face marked '3' = 25

Number of outcomes for the face marked '4' = 10

Number of outcomes for the face marked '5' = 10

Number of outcomes for the face marked '6' = 20

We know that:

Probability of an event (E) = $\dfrac{\text{Number of outcomes of the event}}{\text{Number of trials}}$

So, we have:

$$P(E_1) = \frac{20}{100} = \frac{1}{5}$$

$$P(E_2) = \frac{15}{100} = \frac{3}{20}$$

$$P(E_3) = \frac{25}{100} = \frac{1}{4}$$

$$P(E_4) = \frac{10}{100} = \frac{1}{10}$$

$$P(E_5) = \frac{10}{100} = \frac{1}{10}$$

$$P(E_6) = \frac{20}{100} = \frac{1}{5}$$

Example 2: **In a survey conducted by a leading newspaper, 1000 families with two children were selected at random and the following data was recorded.**

Number of boys in a family	2	1	0
Number of families	200	700	100

If we choose one of these families at random, then what is the probability that the chosen family has

i)1 boy.

ii)2 boys.

iii)no boy.

Solution:

Let E_1, E_2 and E_3 denote the respective events of a family having one boy, two boys and no boy.

We know that:

Probability of an event (E) = $\dfrac{\text{Number of outcomes of the event}}{\text{Number of trials}}$

Using this formula, we can calculate the probabilities of events E_1, E_2 and E_3.

i) $P(E_1) = \dfrac{\text{Number of families having 1 boy}}{\text{Total number of families}}$

$= \dfrac{700}{1000}$

$= \dfrac{7}{10}$

ii) $P(E_2) = \dfrac{\text{Number of families having 2 boys}}{\text{Total number of families}}$

$= \dfrac{200}{1000}$

$= \dfrac{1}{5}$

iii) $P(E_3) = \dfrac{\text{Number of families having no boy}}{\text{Total number of families}}$

$= \dfrac{100}{1000}$

$= \dfrac{1}{10}$

Hard

Example 1: **The monthly salary ranges of 100 workers of a company are given in the following table.**

Salary ranges (in Rs)	**Number of workers in each range**
0–1000	10
1000–2000	15
2000–3000	25
3000–4000	35
4000–5000	15

If a worker is chosen at random, then find the probability of selecting a worker who earns

i)above Rs 3000 per month.

ii)below Rs 2000 per month.

iii)between Rs 24000 and Rs 48000 per annum.

Solution:

i)Total number of workers = 100

Number of workers who earn above Rs 3000 per month = 35 + 15 = 50

Thus, the probability of selecting a worker who earns above Rs 3000 per month is given as:

$$\frac{\text{Number of workers earning above Rs 3000 per month}}{\text{Total number of workers}}$$
$$= \frac{50}{100}$$
$$= \frac{1}{2}$$

ii)Number of workers who earn below Rs 2000 per month = 10 + 15 = 25

Thus, the probability of selecting a worker who earns below Rs 2000 per month is given as:

$$\frac{\text{Number of workers earning below Rs 2000 per month}}{\text{Total number of workers}}$$
$$= \frac{25}{100}$$
$$= \frac{1}{4}$$

iii)Annual salary of the workers = Salaries of 12 months

Now, the monthly salary of workers getting an annual salary of Rs 24000 is given as:

$$\frac{\text{Annual salary}}{12}$$
$$= \text{Rs}\left(\frac{24000}{12}\right)$$
$$= \text{Rs } 2000$$

Similarly, the monthly salary of workers getting an annual salary of Rs 48000 is given as:

$$\frac{\text{Annual salary}}{12}$$
$$= \text{Rs}\left(\frac{48000}{12}\right)$$
$$= \text{Rs } 4000$$

Number of workers earning between Rs 24000 and Rs 48000 per year

= Number of workers earning between Rs 2000 and Rs 4000 per month

= 25 + 35

= 60

Thus, the probability of selecting a worker who earns between Rs 24000 and Rs 48000 per annum is given as:

$$\frac{\text{Number of workers earning between Rs 2000 and Rs 4000 per month}}{\text{Total number of workers}}$$
$$= \frac{60}{100}$$
$$= \frac{3}{5}$$

Example 2: **There are 150 telephone numbers on each page of a telephone directory. The frequency distribution of the unit-place digits in the telephone numbers on a particular page is shown.**

Digit:0123456789

Frequency:15221215171216151412

A number is chosen at random. Find the probability that the digit at the unit's place is

i)6.

ii)a non-zero multiple of 3.

iii)a non-zero even number.

Solution:

i)Total number of telephone numbers = 150

It is given that the digit '6' occurs 16 times at the unit's place.

∴ Probability that the digit at the unit's place is 6 = $\frac{16}{150} = 0.1067$

ii)A non-zero multiple of 3 means 3, 6 or 9.

Number of telephone numbers in which the unit's digit is 3, 6 or 9 = 15 + 16 + 12 = 43

∴ Probability that the digit at the unit's place is a non-zero multiple of 3 = $\frac{43}{150} = 0.286$

iii)A non-zero even number means 2, 4, 6 or 8.

Number of telephone numbers in which the unit's digit is 2, 4, 6 or 8 = 12 + 17 + 16 + 14

= 59

∴ Probability that the digit at the unit's place is a non-zero even number = $\frac{59}{150} = 0.393$