Ratio & Proportion

RATIO

Ratio is strictly a mathematical term to compare two similar quantities expressed in the same units.

The ratio of two terms 'x' and 'y' is denoted by x : y.

In general, the ratio of a number x to a number y is defined as the quotient of the numbers x and y.

COMPARISON OF TWO OR MORE RATIOS

Two or more ratios may be compared by reducing the equivalent fractions to a common denominator and then comparing the magnitudes of their numerator. Thus, suppose 2 : 5, 4 : 3 and 4 : 5

are three ratios to be compared then the fractions $\dfrac{2}{5}, \dfrac{4}{3}$ and $\dfrac{4}{5}$

are reduced to equivalent fractions with a common denominator. For this, the denominator of each is changed to 15 equal to the L.C.M. their denominators Hence the given ratios are expressed

$\dfrac{6}{15}, \dfrac{20}{15}$ and $\dfrac{12}{15}$ or 2 : 5, 4 : 3, 4 : 5 according to magnitude.

EXAMPLE 1. **Which of the ratios 2 : 3 and 5 : 9 is greater ?**

Sol. In the form of fractions, the given ratios are $\dfrac{2}{3}$ and $\dfrac{5}{9}$,

Reducing them to fractions with a common denominator they are written as $\dfrac{6}{9}$ and $\dfrac{5}{9}$.

Hence the greater ratio is $\dfrac{6}{9}$ or 2 : 3.

EXAMPLE 2. **Are the ratios 3 to 4 and 6:8 equal ?**

Sol. The ratios are equal if 3/4 = 6/8.

These are equal if their cross products are equal; that is, if 3 × 8 = 4 × 6. Since both of these products equal 24, the answer is yes, the ratios are equal.

Remember to be careful! Order matters!

A ratio of 1 : 7 is not the same as a ratio of 7:1.

✎ REMEMBER

★ In the ratio of two quantities the two quantities must be of the same kind and in same unit.

★ The ratio is a pure number, i.e., without any unit of measurement.

★ The ratio would stay unaltered even if both the numerator and the denominator are multiplied or divided by the same number.

COMPOUND RATIO

Ratios are compounded by multiplying together the numerators for a new denominator and the denominator for a new denominator.

The compound ratio of a : b and c : d is $\dfrac{a \times c}{b \times d}$, i.e., ac : bd.

EXAMPLE 3. **Find the compound ratio of the four ratios :**
4 : 5, 15 : 13, 26 : 3 and 6 : 17

Sol. The required ratio $= \dfrac{4 \times 15 \times 26 \times 6}{5 \times 13 \times 3 \times 17} = \dfrac{48}{17}$

or 48 : 17

☞ **Shortcut Approach**

↝ The duplicate ratio of x : y is $x^2 : y^2$.

The triplicate ratio of x : y is $x^3 : y^3$.

The subduplicate ratio of x : y is $\sqrt{x} : \sqrt{y}$.

The subtriplicate ratio of x : y is $\sqrt[3]{x} : \sqrt[3]{y}$.

Reciprocal ratio of a : b is $\dfrac{1}{a} : \dfrac{1}{b}$ or b : a

↝ **Inverse ratio**

Inverse ratio of x : y is y : x.

PROPERTIES OF RATIOS

1. If $\dfrac{a}{b} = \dfrac{c}{d}$ then $\dfrac{b}{a} = \dfrac{d}{c}$, i.e., the inverse ratios of two equal ratios are equal. The property is called **Invertendo**.

2. If $\dfrac{a}{b} = \dfrac{c}{d}$ then $\dfrac{a}{c} = \dfrac{b}{d}$, i.e., the ratio of antecedents and consequents of two equal ratios are equal. This property is called **Alternendo**.

3. If $\dfrac{a}{b} = \dfrac{c}{d}$, then $\dfrac{a+b}{b} = \dfrac{c+d}{d}$. This property is called **Componendo**.

4. If $\dfrac{a}{b} = \dfrac{c}{d}$, then $\dfrac{a-b}{b} = \dfrac{c-d}{d}$. This property is called **Dividendo**.

5. If $\dfrac{a}{b} = \dfrac{c}{d}$, then $\dfrac{a+b}{a-b} = \dfrac{c+d}{c-d}$. This property is called **Componendo and Dividendo.**

6. If $\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{e}{f} = $ .... Then,

Each ratio $= \dfrac{\text{sum of Numerators}}{\text{sum of Denominators}}$

i.e. $\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{a+c+e+...}{b+d+f+...}$

7. If we have two equations containing three unknowns as

$a_1x + b_1y + c_1z = 0$ and ... (i)

$a_2x + b_2y + c_2z = 0$ ... (ii)

then, the values of x, y and z cannot be resolved without having a third equation.

However, in the absence of a third equation, we can find the ratio x : y : z.

This will be given by

$b_1c_2 - b_2c_1 : c_1a_2 - c_2a_1 : a_1b_2 - a_2b_1$.

☞ **Shortcut Approach**

To divide a given quantity into a given ratio.

Suppose any given quantity a, is to be divided in the ratio m : n.

Let one part of the given quantity be x then the other part will be a − x.

∴ $\dfrac{x}{a-x} = \dfrac{m}{n}$ or $nx = ma - mx$ or $(m+n)x = ma$

∴ one part is $\dfrac{ma}{m+n}$ and the other part will be

$a - \dfrac{ma}{m+n} = \dfrac{na}{m+n}$

EXAMPLE 4. **Divide 70 in the ratio 3 : 7.**

Sol. Let one part be x

then the other part $= 70 - x$

∴ $\dfrac{x}{70-x} = \dfrac{3}{7}$ or $7x = 210 - 3x$

or $x = 21$ and $70 - x = 49$

Hence the two required parts of 70 are 21 and 49.

EXAMPLE 5. **What is the least integer which when subtracted from both the numerator and denominator of $\dfrac{60}{70}$ will give a ratio equal to $\dfrac{16}{21}$?**

Sol. Let x be the required integer. Then,

$\dfrac{60-x}{70-x} = \dfrac{16}{21}$

$\Rightarrow 1260 - 21x = 1120 - 16x$

$\Rightarrow 5x = 140 \Rightarrow x = 28.$

EXAMPLE 6. **If $\dfrac{x}{y} = \dfrac{4}{5}$, find the value of $\dfrac{3x+4y}{4x+3y}$.**

Sol. $\dfrac{3x+4y}{4x+3y} = \dfrac{\dfrac{3x}{y}+4}{\dfrac{4x}{y}+3} = \dfrac{3 \times \dfrac{4}{5}+4}{4 \times \dfrac{4}{5}+3} = \dfrac{32}{31}$.

EXAMPLE 7. **Find the value of $\dfrac{x+a}{x-a} + \dfrac{x+b}{x-b}$, if $x = \dfrac{2ab}{a+b}$.**

Sol. $x = \dfrac{2ab}{a+b} \Rightarrow \dfrac{x}{a} = \dfrac{2b}{a+b}$

By componendo – dividendo,

$\dfrac{x+a}{x-a} = \dfrac{3b+a}{b-a}$

Similarly, $\dfrac{x}{b} = \dfrac{2a}{a+b}$

$\Rightarrow \dfrac{x+b}{x-b} = \dfrac{3a+b}{a-b}$

∴ $\dfrac{x+a}{x-a} + \dfrac{x+b}{x-b} = \dfrac{3b+a}{b-a} + \dfrac{3a+b}{a-b}$

$= \dfrac{-(3b+a)}{a-b} + \dfrac{3a+b}{a-b} = \dfrac{2a-2b}{a-b} = 2$.

EXAMPLE 8. **Divide ₹ 581 among A, B and C such that four time A's share is equal to 5 times B's share which is equal to seven times C's share.**

Sol. 4 times A's share = 5 times B's share

= 7 times C's share.

$\dfrac{\text{A's share}}{35} = \dfrac{\text{B's share}}{28} = \dfrac{\text{C's share}}{20}$

[Dividing by L.C.M. of 4, 5 and 7 i.e. 140]

∴ A : B : C = 35 : 28 : 20

∴ Share of A $= \dfrac{35}{35+28+20} \times 581 = ₹ 245.$

Share of B $= \dfrac{28}{83} \times 581 = ₹ 196.$

Share of C $= \dfrac{20}{83} \times 581 = ₹ 140.$

☞ **Shortcut Approach**

(i) If A : B = a : b and B : C = m : n, then A : B : C = am : mb : nb and A : C = am : bn

(ii) If A : B = a : b, B : C = c : d and C : D = e : f, then A : B : C : D = ace : bce : bde : bdf

EXAMPLE 9. **The ratio of A : B = 1 : 3, B : C = 2 : 5 and C : D = 2 : 3. Find the value of A : B : C : D.**

Sol. A : B = 1 : 3, B : C = 2 : 5, C : D = 2 : 3

A : B : C : D = $(1 \times 2 \times 2) : (3 \times 2 \times 2) : (3 \times 5 \times 2) : (3 \times 5 \times 3)$

$= 4 : 12 : 30 : 45$

☞ **Shortcut Approach**

Two numbers are in ratio a : b and x is subtracted from the numbers, then the ratio becomes c : d. The two numbers will be $\dfrac{xa(d-c)}{ad-bc}$ and $\dfrac{xb(d-c)}{ad-bc}$, respectively.

EXAMPLE 10. Two numbers are in the ratio 3 : 5. If 9 is subtracted from each, the ratio becomes 12 : 23. Find the greater number.

Sol. Here a = 3, b = 5, c = 12, d = 23 and x = 9

Then, 1st number $= \dfrac{xb(d-c)}{ad-bc} = \dfrac{9 \times 3(23-12)}{3 \times 23 - 5 \times 12}$

$= \dfrac{27 \times 11}{69-60} = \dfrac{297}{9} = 33$

2nd number $= \dfrac{xb(d-c)}{ad-bc} = \dfrac{9 \times 5(23-12)}{3 \times 23 - 5 \times 12}$

$= \dfrac{45 \times 11}{69-60} = \dfrac{45 \times 11}{9} = 55$

☞ *Shortcut Approach*

In any 2-dimensional figures, if the corresponding sides are in the ratio x : y, then their areas are in the ratio $x^2 : y^2$.

EXAMPLE 11. The ratio of the radius of two circles is 2 : 5. Find the ratio of their areas.

Sol. Ratio of their areas $= 2^2 : 5^2 = 4 : 25$

☞ *Shortcut Approach*

↦ In any two 3-dimensional similar figures, if the corresponding sides are in the ratio x : y, then their volumes are in the ratio $x^3 : y^3$.

↦ If the ratio between two numbers is a : b and if each number is increased by x, the ratio becomes c : d. Then, two numbers are given as $\dfrac{xa(c-d)}{ad-bc}$ and $\dfrac{xb(c-d)}{ad-bc}$

EXAMPLE 12. The ratio between two numbers is 3 : 4. If each number be increased by 2, the ratio becomes 7 : 9. Find the number.

Sol. Numbers are $\dfrac{2 \times 3\,(7-9)}{3 \times 9 - 4 \times 7}$ and $\dfrac{2 \times 4\,(7-9)}{3 \times 9 - 4 \times 7}$ or 12 and 16.

☞ *Shortcut Approach*

If the sum of two numbers is A and their difference is a, then the ratio of numbers is given by A + a : A – a.

EXAMPLE 13. The sum of two numbers is 60 and their difference is 6. What is the ratio of the two numbers ?

Sol. The required ratio of the numbers

$= \dfrac{60+6}{60-6} = \dfrac{66}{54} = \dfrac{11}{9}$ or 11 : 9

EXAMPLE 14. Three persons A, B, C whose salaries together amount to ₹14400, spend 80, 85 and 75 per cent of their salaries respectively. If their savings are in the ratio 8 : 9 : 20, find their respective salaries.

Sol. A, B and C spend 80 %, 85 % and 75 % respectively of their salaries.

⇒ A, B and C save 20 %, 15 % and 25 % respectively of their salaries.

So, 20 % of A's salary : 15 % of B's salary : 25 % of C's salary = 8 : 9 : 20

⇒ $\dfrac{1}{5}$ of A's salary : $\dfrac{3}{20}$ of B's salary :

$\dfrac{1}{4}$ of C's salary = 8 : 9 : 20   ....(i)

Now $\dfrac{\dfrac{1}{5} \text{ of A's salary}}{\dfrac{3}{20} \text{ of B's salary}} = \dfrac{8}{9}$

⇒ $\dfrac{\text{A's salary}}{\text{B's salary}} = \dfrac{3}{20} \times 8 \times \dfrac{5}{9} = \dfrac{2}{3}$

∴ A's salary : B's salary = 2 : 3   ...(ii)
Similarly, B's salary : C's salary = 3 : 4   ...(iii)
From (ii) and (iii)
A's salary : B's salary : C's salary = 2 : 3 : 4.

∴ A's salary $= \dfrac{2}{2+3+4} \times 14400 = ₹\,3200$

B's salary $= \dfrac{3}{2+3+4} \times 14400 = ₹\,4800$

C's salary $= \dfrac{4}{2+3+4} \times 14400 = ₹\,6400$.

Removal and Replacement

☞ *Shortcut Approach*

(i) Let a vessel contains Q unit of mixture of ingredients A and B. From this, R unit of mixture is taken out and replaced by an equal amount of ingredient B only.
If this process is repeated n times, then after n operations

$\dfrac{\text{Quantity of A left}}{\text{Quantity of A originally present}} = \left(1 - \dfrac{R}{Q}\right)^n$

and Quantity of B left = Q – Quantity of A Left

EXAMPLE 15. A container contains 40 litres of milk. From this container, 4 litres of milk was taken out and replaced by water. This process was repeated further two times. How much milk is now contained by the container ?

Sol.

	Milk	Water
To start with	40 litres	
After 1st operation	36 litres	4 litres
After 2nd operation	$36 - \dfrac{4}{40} \times 36$	$4 - \dfrac{4}{40} \times 4 + 4$
	= 32.4 litres	= 4 – 0.4 + 4
		= 7.6 litres

After 3rd operation

$32.4 - \dfrac{4}{40} \times 32.4$ $7.6 - \dfrac{4}{40} \times 7.6 + 4$

$= 32.4 - 3.24$ $= 7.6 - 0.76 + 4$

$= 29.16$ $= 10.84$

∴ The quantity of milk in the container is 29.16 litres.

SHORTCUT METHOD

Quantity of milk in container : $40\left(1 - \dfrac{4}{40}\right)^3 = 29.16$ litres

---

☞ Shortcut Approach

In a container, milk and water are present in the ratio a : b. If x L of water is added to this mixture, the ratio becomes a : c. Then,

Quantity of milk in original mixture $= \dfrac{ax}{c-b} L$

and quantity of water in original mixture $= \dfrac{bx}{c-b} L$

EXAMPLE 16. **In a container, milk and water are present in the ratio 7 : 5. If 15 L water is added to this mixture, the ratio of milk and water becomes 7 : 8. Find the quantity of water in the new mixture.**

Sol. Let the quantity of milk and water in initial mixture be 7x and 5x L.

Then, according to the question,

$$\dfrac{7x}{5x + 15} = \dfrac{7}{8} \Rightarrow 7x \times 8 = 7(5x + 15)$$

$\Rightarrow \quad 56x = 35x + 105 \Rightarrow 56x - 35x = 105$

$\Rightarrow \quad 21x = 105 \Rightarrow x = \dfrac{105}{21} = 5$

∴ Quantity of water in initial mixture $= 5 \times 5 = 25$ L

and quantity of water in new mixture $= 25 + 15 = 40$ L

SHORTCUT METHOD

Here, a = 7, b = 5, c = 8 and x = 15 L

According to the formula,

Quantity of water in original mixture $= \dfrac{bx}{c-b} = \dfrac{5 \times 15}{8-5} = 25$ L

∴ Quantity of water in new mixture $= 25 + 15 = 40$ L

☞ Shortcut Approach

A container has milk and water in the ratio a : b, a second container has milk and water in the ratio c : d. If both the mixtures are emptied into a third container, then the ratio of milk of water in third container is given by

$$\left(\dfrac{a}{a+b} + \dfrac{c}{c+d}\right) : \left(\dfrac{b}{a+b} + \dfrac{d}{c+d}\right)$$

EXAMPLE 17. **2 containers have milk and water in the ratio 2 : 1 and 3 : 1, respectively. If both containers are emptied into a bigger container, then find the ratio of milk of water in bigger container?**

Sol. Given, ratio of milk and water in 1st container = 2 : 1

∴ Quantity of milk in 1st container $= \dfrac{2}{3}$

and quantity of water in 1st container $= \dfrac{1}{3}$

Similarly, ratio of milk and water in 2nd container = 3 : 1

∴ Quantity of milk in 2nd container $= \dfrac{3}{4}$

and quantity of water in second container $= \dfrac{1}{4}$

Now, after pouring both mixture in one container

Quantity of milk $= \dfrac{2}{3} + \dfrac{3}{4} = \dfrac{17}{12}$

and quantity of water $= \dfrac{1}{4} + \dfrac{1}{3} = \dfrac{7}{12}$

Hence, required ratio = 17 : 7

SHORTCUT METHOD

Here, a = 2, b = 1, c = 3, d = 1

Ratio of milk of water in bigger container

$$= \left(\dfrac{a}{a+b} + \dfrac{c}{c+d}\right) : \left(\dfrac{b}{a+b} + \dfrac{d}{c+d}\right)$$

$$= \left(\dfrac{2}{2+1} + \dfrac{3}{3+1}\right) : \left(\dfrac{1}{2+1} + \dfrac{1}{3+1}\right) = \dfrac{17}{12} : \dfrac{7}{12} = 17 : 7$$

PROPORTION

When two ratios are equal, the four quantities composing them are said to be in proportion.

If $\dfrac{a}{b} = \dfrac{c}{d}$, then a, b, c, d are in proportions.

This is expressed by saying that 'a' is to 'b' as 'c' is to 'd' and the proportion is written as

a : b :: c : d   or   a : b = c : d

The terms a and d are called the extremes while the terms b and c are called the means.

✎ REMEMBER

★ If four quantities are in proportion, the product of the extremes is equal to the product of the means.

Let a, b, c, d be in proportion, then

$\dfrac{a}{b} = \dfrac{c}{d} \Rightarrow ad = bc$.

★ If three quantities a, b and c are in continued proportion, then a : b = b : c

∴ $ac = b^2$

b is called mean proportional.

EXAMPLE 18. Find the mean proportional between 3 and 75.

Sol. Let x be the required mean proportional. Then,

$3 : x :: x : 75$

∴ $x = \sqrt{3 \times 75} = 15$

EXAMPLE 19. **What must be added to each of the four numbers 10, 18, 22, 38 so that they become in proportion ?**

Sol. Let the number to be added to each of the four numbers be x

By the given condition, we get

$(10 + x) : (18 + x) : : (22 + x) : (38 + x)$

⇒ $(10 + x)(38 + x) = (18 + x)(22 + x)$

⇒ $380 + 48x + x^2 = 396 + 40x + x^2$

Cancelling x^2 from both sides, we get

$380 + 48x = 396 + 40x$

⇒ $48x - 40x = 396 - 380$

⇒ $8x = 16 \Rightarrow x = \dfrac{16}{8} = 2$

Therefore, 2 should be added to each of the four given numbe₹

EXAMPLE 20. **Find the fourth proportional to**
$$p^2 - pq + q^2, p^3 + q^3, p - q$$

Sol. Let x be the fourth proportional

∴ $(p^2 - pq + q^2) : (p^3 + q^3) = (p - q) : x$

⇒ $(p^2 - pq + q^2) \times x = (p^3 + q^3)(p - q)$

∴ $x = \dfrac{(p^3 + q^3)(p - q)}{(p^2 - pq - q^2)}$

⇒ $x = \dfrac{(p + q)(p^2 - pq + q^2)(p - q)}{(p^2 - pq - q^2)}$

⇒ $x = (p + q)(p - q) = p^2 - q^2$

∴ The required fourth proportional is $p^2 - q^2$

EXAMPLE 21. **Find third proportional to $a^2 - b^2$ and $a + b$.**

Sol. Let x be the required third proportional

Then $a^2 - b^2 : a + b = a + b : x$

∴ $(a^2 - b^2) x = (a + b)(a + b)$

∴ $x = \dfrac{(a + b)(a + b)}{a^2 - b^2} = \dfrac{a + b}{a - b}$

DIRECT PROPORTION

If on the increase of one quantity, the other quantity increases to the same extent or on the decrease of one, the other decreases to the same extent, then we say that the given two quantities are directly proportional. If A and B are directly proportional then we denote it by A ∝ B.

Some Examples :
1. Work done ∝ number of men
2. Cost ∝ number of Articles
3. Work ∝ wages
4. Working hour of a machine ∝ fuel consumed
5. Speed ∝ distance to be covered

INDIRECT PROPORTION (OR INVERSE PROPORTION)

If on the increase of one quantity, the other quantity decreases to the same extent or vice versa, then we say that the given two quantities are indirectly proportional. If A and B are indirectly proportional then we denote it by $A \propto \dfrac{1}{B}$.

Also, $A = \dfrac{k}{B}$ (k is a constant)

⇒ $AB = k$

If b_1, b_2 are the values of B corresponding to the values a_1, a_2 of A respectively, then

$$a_1 b_1 = a_2 b_2$$

Some Examples :
1. More men, less time
2. Less men, more time
3. More speed, less taken time to be covered distance

EXAMPLE 22. **A garrison of 3300 men had provision for 32 days, when given at the rate of 850 gm per head. At the end of 7 days, a reinforcement arrived and it was found that the provision would last 17 days more, when given at the rate of 825 gm per head. What was the strength of the reinforcement ?**

(a) 1500	(b) 1700
(c) 1800	(d) 1600

Sol. (b) There is a provision for 2805 × 32 kg for 3300 men for 32 days @ 850 gm per head per day.

In 7 days, 3300 men consumed

$\dfrac{2805 \times 32}{32} \times 7 = 2805 \times 7$ kg.

Let the strength of the reinforcement arrived after 7 days be x.

∴ (3300 + x) men had provision of 2805 × 25 kg for 17 days @ 825 gm per head per day, i.e.

∴ $\dfrac{(3300 + x) \times 825 \times 17}{1000} = 2805 \times 25$

⇒ $(3300 + x) = \dfrac{1000 \times 2805 \times 25}{825 \times 17} = 5000$

⇒ $x = 1700$

∴ Strength of the reinforcement arrived after 7 days = 1700.

RULE OF THREE

In a problem on simple proportion, usually three terms are given and we have to find the fourth term, which we can solve by using Rule of three. In such problems, two of given terms are of same kind and the third term is of same kind as the required fourth term. First of all we have to find whether given problem is a case of direct proportion or indirect proportion.

For this, write the given quantities under their respective headings and then mark the arrow in increasing direction. If both arrows are in same direction then the relation between them is direct otherwise it is indirect or inverse proportion. Proportion will be made by either head to tail or tail to head.

The complete procedure can be understand by the examples.

EXAMPLE **23. A man completes 5/8 of a job in 10 days. At this rate, how many more days will it take him to finish the job ?**

 (a) 5   (b) 6

 (c) 7   (d) $7\dfrac{1}{2}$

Sol. (b) Work done $= \dfrac{5}{8}$. Balance work $= \left(1 - \dfrac{5}{8}\right) = \dfrac{3}{8}$.

 Less work, Less days (Direct Proportion)
 Let the required number of days be x. Then,

Work	days
↑ 5/8	10 ↑
3/8	x

 Then, $\dfrac{5}{8} : \dfrac{3}{8} :: 10 : x \Rightarrow \dfrac{5}{8} \times x = \dfrac{3}{8} \times 10$

 $\Rightarrow x = \left(\dfrac{3}{8} \times 10 \times \dfrac{8}{5}\right) = 6$.

EXAMPLE **24. A fort had provision of food for 150 men for 45 days. After 10 days, 25 men left the fort. The number of days for which the remaining food will last, is :**

 (a) $29\dfrac{1}{5}$ (b) $37\dfrac{1}{4}$

 (c) 42   (d) 54

Sol. (c) After 10 days : 150 men had food for 35 days.
 Suppose 125 men had food for x days. Now,
 Less men, More days (Indirect Proportion)
 Then,

men	days
↑ 150	35 ↓
125	x ↓

 $\therefore 125 : 150 :: 35 : x \Rightarrow 125 \times x = 150 \times 35$

 $\Rightarrow x = \dfrac{150 \times 35}{125} \Rightarrow x = 42.$

 Hence, the remaining food will last for 42 days.

EXAMPLE **25. If the cost of printing a book of 320 leaves with 21 lines on each page and on an average 11 words in each line is ₹ 19, find the cost of printing a book with 297 leaves, 28 lines on each page and 10 words in each line.**

 (a) ₹ $22\dfrac{3}{8}$ (b) ₹ $20\dfrac{3}{8}$

 (c) ₹ $21\dfrac{3}{8}$ (d) ₹ $21\dfrac{3}{4}$

Sol. (c) **Less leaves, less cost** **(Direct Proportion)**
   More lines, more cost **(Direct Proportion)**
   Less words, less cost **(Direct Proportion)**

 leaves 320 : 297 ⎫
 lines 21 : 28 ⎬ :: 19 : x
 words 11 : 10 ⎭

 $\therefore 320 \times 21 \times 11 \times x = 297 \times 28 \times 10 \times 19$

 $\Rightarrow x = \dfrac{171}{8} = 21\dfrac{3}{8}$

PARTNERSHIP

A partnership is an association of two or more persons who invest their money in order to carry on a certain business.

A partner who manages the business is called the **working partner** and the one who simply invests the money is called the **sleeping partner.**

Partnership is of two kinds :

(i) Simple   (ii) Compound.

Simple partnership :

If the capitals is of the partners are invested for the same period, the partnership is called simple.

Compound partnership :

If the capitals of the partners are invested for different lengths of time, the partnership is called compound.

☞ **Shortcut Approach**

If the period of investment is the same for each partner, then the profit or loss is divided in the ratio of their investments.

If A and B are partners in a business, then

$$\frac{\text{Investment of A}}{\text{Investment of B}} = \frac{\text{Profit of A}}{\text{Profit of B}} = \frac{\text{Loss of A}}{\text{Loss of B}}$$

If A, B and C are partners in a business, then
Investment of A : Investment of B : Investment of C

   = Profit of A : Profit of B : Profit of C, or
   = Loss of A : Loss of B : Loss of C

EXAMPLE **26. Three partner Rahul, Puneet and Chandan invest ₹ 1600, ₹ 1800 and ₹ 2300 respectively in a business. How should they divide a profit of ₹ 399 ?**

Sol. Profit is to be divided in the ratio 16 : 18 : 23

 Rahul's share of profit $= \dfrac{16}{16 + 18 + 23} \times 399$

      $= \dfrac{16}{57} \times 399 = ₹ 112$

 Puneet's share of profit $= \dfrac{18}{57} \times 399 = ₹ 126$

 Chandan's share of profit $= \dfrac{23}{57} \times 399 = ₹ 161$

EXAMPLE **27. A and B invested in the ratio 3 : 2 in a business. If 5% of the total profit goes to charity and A's share is ₹ 855, find the total profit.**

Sol. Let the total profit be ₹ 100.
 Then, ₹ 5 goes to charity.
 Now, ₹ 95 is divided in the ratio 3 : 2.

 $\therefore$ A's share $= \dfrac{95}{3 + 2} \times 3 = ₹ 57$

But A's actual share is ₹ 855.

$\therefore$ Actual total profit $= 855\left(\dfrac{100}{57}\right) = ₹ 1500$

☞ **Shortcut Approach**

When the amount of capital invested by different partners is same (say ₹ x) for differend time periods, $t_1, t_2, t_3, ...$, then Ratio of profit/loss = Ratio of time period for which the capital is invested

$$P_1 : P_2 : P_3 : ... = t_1 : t_2 : t_3 : ...$$

EXAMPLE **28. A, B and C start a business with investment of ₹ 50000 each. A remians in partnershi for 9 months, B for 6 months and C for 12 months. Then, find the ratio of their profits.**

Sol. Ratio of profit of A, B and C will be in the ratio of time period of investment

So, A's profit : B's Profit : C's profit = 9 : 6 : 12 = 3 : 2 : 4

MONTHLY EQUIVALENT INVESTMENT

It is the product of the capital invested and the period for which it is invested.

If the period of investment is different, then the profit or loss is divided in the ratio of their Monthly Equivalent Investment.

$$\dfrac{\text{Monthly Equivalent Investment of A}}{\text{Monthly Equivalent Investment of B}}$$

$$= \dfrac{\text{Profit of A}}{\text{Profit of B}} \text{ or } \dfrac{\text{Loss of A}}{\text{Loss of B}}$$

i.e., $\dfrac{\text{Investment of A} \times \text{Period of Investment of A}}{\text{Investment of B} \times \text{Period of Investment of B}}$

$$= \dfrac{\text{Profit of A}}{\text{Profit of B}} \text{ or } \dfrac{\text{Loss of A}}{\text{Loss of B}}$$

☞ **Shortcut Approach**

If A, B and C are partners in a business, then

Monthly Equivalent Investment of A : Monthly Equivalent Investment of B : Monthly Equivalent Investment of C

= Profit of A : Profit of B : Profit of C.

= Loss of A : Loss of B : Loss of C.

EXAMPLE **29. A and B start a business. A invests ₹ 600 more than B for 4 months and B for 5 months. A's share is ₹ 48 more than that of B, out of a total profit of ₹ 528. Find the capital contributed by each.**

Sol. B's profit $= \dfrac{528 - 48}{2} = ₹ 240$

A's profit $= 528 - 240 = ₹ 288$

$\dfrac{\text{A's capital} \times 4}{\text{B's capital} \times 5} = \dfrac{288}{240} = \dfrac{6}{5}$

$\therefore$ $\dfrac{\text{A's capital}}{\text{B's capital}} = \dfrac{6}{5} \times \dfrac{5}{4} = \dfrac{3}{2}$

$\Rightarrow \dfrac{\text{B's capital} + 600}{\text{B's capital}} = \dfrac{3}{2}$

$\Rightarrow$ B's capital = ₹ 1200 and A's capital = ₹ 1800

EXAMPLE **30. Three persons A, B, C rent the grazing of a park for ₹ 570. A puts in 126 oxen in the park for 3 months, B puts in 162 oxen for 5 months and C puts in 216 oxen for 4 months. What part of the rent should each person pay ?**

Sol. Monthly equivalent rent of A = $126 \times 3 = 378$

Monthly equivalent rent of B = $162 \times 5 = 810$

Monthly equivalent rent of C = $216 \times 4 = 864$

$\therefore$ Rent is to be divided in the ratio

$378 : 810 : 864$, ie. $7 : 15 : 16$

$\therefore$ A would have to pay $\dfrac{7}{7 + 15 + 16}$ of the rent

$$= \dfrac{7}{38} \text{ of the rent } = \dfrac{7}{38} \times 570 = ₹ 105$$

$\therefore$ B would have to pay $\dfrac{15}{38}$ of the rent $= \dfrac{15}{38} \times 570 = ₹ 225$

and C would have to pay $\dfrac{16}{38}$, i.e. $\dfrac{8}{19}$ of the rent

$$= \dfrac{8}{19} \times 570 = ₹ 240$$

☞ **Shortcut Approach**

When capital invested by the partners is givne as $X_1, X_2, X_3, ...$ for different time period $t_1, t_2, t_3,....$ in a business, then

Ratio of their profits $P_1 : P_2 : P_3 : .... = X_1 t_1 : X_2 t_2 : X_3 t_3 : ....$

EXAMPLE **31. A starts a business with ₹ 4000 and B joins the business 4 months later with an investment of ₹ 5000. After 1 yr. they earn a profit of ₹ 22000. Find the share of A and B.**

Sol. A's share : B's share

$= 4000 \times 12 : 5000 \times (12 - 4) = 4 \times 12 : 5 \times 8 = 6 : 5$

Now, let the share of A = 6x, and the share of B = 5x

According to the question, $6x + 5x = 22000 \Rightarrow 11x = 22000$

$\therefore x = ₹ 2000$

Share of A = 6x = $6 \times 2000 = ₹ 12000$,

and share of B = 5x = $5 \times 2000 = ₹ 10000$

SHORTCUT METHOD

A's share : B's share = $4000 \times 12 : 5000 \times 8 = 6 : 5$

Now, A's share $= \dfrac{6}{6 + 5} \times 22000 = ₹ 12000$

and B's share $= \dfrac{5}{6 + 5} \times 22000 = ₹ 10000$

☞ **Shortcut Approach**

If $P_1 : P_2 : P_3 : ....$ is the ratio of profit and $t_1 : t_2 : t_3 : ...$ is the ratio of time periods, then ratio of invesements is given by

$$\dfrac{P_1}{t_1} : \dfrac{P_2}{t_2} : \dfrac{P_3}{t_3} : .....$$

EXAMPLE 32. A, B and C each does certain investments for time periods in the ratio of 5 : 6 : 8. At the end of the business terms, they received the profit in the ratio of 5 : 3 : 12. Find the ratio of investments of A, B and C.

Sol. Here, $t_1 : t_2 : t_3 = 5 : 6 : 8$ and $P_1 : P_2 : P_3 = 5 : 3 : 12$

$$\text{Required ratio} = \frac{P_1}{t_1} : \frac{P_2}{t_2} : \frac{P_3}{t_3} = \frac{5}{5} : \frac{3}{6} : \frac{12}{8} = 1 : \frac{1}{2} : \frac{3}{2} = 2 : 1 : 3$$

MIXTURE

Simple Mixture : When two different ingredients are mixed together, it is known as a simple mixture.

Compound Mixture : When two or more simple mixtures are mixed together to form another mixture, it is known as a compound mixture.

Alligation : Alligation is nothing but a faster technique of solving problems based on the weighted average situation as applied to the case of two groups being mixed together.

The word 'Alligation' literally means 'linking'.

ALLIGATION RULE

It states that when different quantities of the same or different ingredients of different costs are mixed together to produce a mixture of a mean cost, the ratio of their quantities is inversely proportional to the difference in their cost from the mean cost.

$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{\text{Price of Dearer} - \text{Mean Price}}{\text{Mean Price} - \text{Price of Cheaper}}$$

Graphical representation of Alligation Rule :
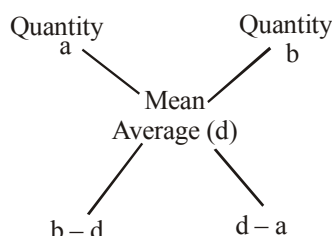
$$\frac{\text{Quantity of a}}{\text{Quantity of b}} = \frac{b-d}{d-a}$$

Applications of Alligation Rule :

(i) To find the mean value of a mixture when the prices of two or more ingredients, which are mixed together and the proportion in which they are mixed are given.

(ii) To find the proportion in which the ingredients at given prices must be mixed to produce a mixture at a given price.

EXAMPLE 33. In what proportion must sugar at ₹ 13.40 per kg be mixed with sugar at ₹ 13.65 per kg, so that the mixture be worth ₹ 13.20 a kg ?

Sol.
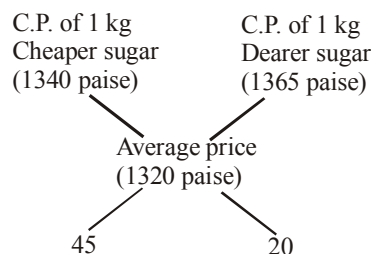
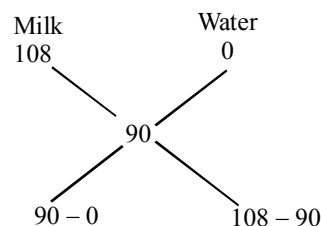
$$\frac{\text{Quantity of cheaper sugar}}{\text{Quantity of dearer sugar}} = \frac{45}{20} = \frac{9}{4}$$

∴ They must be mixed in the ratio 9 : 4.

EXAMPLE 34. A mixture of a certain quantity of milk with 16 litres of water is worth 90 P per litre. If pure milk be worth ₹ 1.08 per litre, how much milk is there in the mixture ?

Sol. The mean value is 90P and the price of water is 0 P.

By the Alligation Rule, milk and water are in the ratio of 5 : 1.

∴ Quantity of milk in the mixture $= 5 \times 16 = 80$ litres.

☞ **Shortcut Approach**

Price of the Mixture :

When quantities Q_i of ingredients M_i's with the cost C_i's are mixed then cost of the mixture C_m is given by

$$C_m = \frac{\sum C_i Q_i}{\sum Q_i}$$

EXAMPLE 35. 5 kg of rice of ₹ 6 per kg is mixed with 4 kg of rice to get a mixture costing ₹ 7 ker kg. Find the price of the costlier rice.

Sol. Let the price of the costlier rice be ₹ x.
By direct formula,

$$7 = \frac{6 \times 5 + 4 \times x}{9}$$

$\Rightarrow 63 - 30 = 4x \Rightarrow 4x = 33$

$\Rightarrow x = \frac{33}{4} = 8.25$

STRAIGHT LINE APPROACH OF ALLIGATION

Let Q_1 and Q_2 be the two quantities, and n_1 and n_2 are the number of elements present in the two quantities respectively,
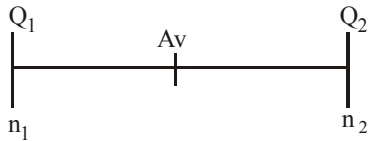
where Av is the average of the new group formed then

n_1 corresponds to $Q_2 - Av$, n_2 corresponds to $Av - Q_1$ and $(n_1 + n_2)$ corresponds to $Q_2 - Q_1$.

Let us consider the previous example.

EXAMPLE **36. 5 kg of rice at ₹ 6 per kg is mixed with 4 kg of rice to get a mixture costing ₹ 7 per kg. Find the price of the costlier rice.**

Sol. Using straight line method,
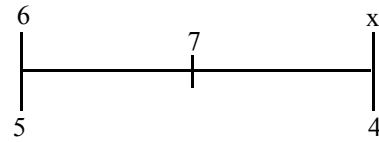
4 corresponds to $7 - 6$ and 5 corresponds to $x - 7$.

i.e. $4 \to 1$

$\quad 5 \to 1.25$

Hence, $x - 7 = 1.25$

$\Rightarrow \ x = 8.25$

EXERCISE

1. Salaries of A, B and C were in the ratio of 3 : 5 : 7 respectively. If their salaries were increased by 50%, 60% and 50% respectively, what will be the new ratio of their respective salaries?
(a) 3 : 6 : 7   (b) 4 : 5 : 7
(c) 4 : 5 : 8   (d) Data inadequate
(e) None of these

2. If 40% of a number is equal to two-thirds of another number, what is the ratio of the first number to the second?
(a) 7 : 3   (b) 3 : 7
(c) 2 : 5   (d) 5 : 3
(e) None of these

3. Radha started a business, investing ₹ 75,000. After 3 months, Sunidhi joined her with an amount of ₹ 1,25,000 and after another six months Neha joined them with an amount of ₹ 1,50,000. Profit earned at the end of three years from when Radha started the business should be distributed in what ratio among Radha, Sunidhi and Neha respectively?
(a) 36 : 55 : 54   (b) 18 : 28 : 27
(c) 35 : 54 : 55   (d) Cannot be determined
(e) None of these

4. What should come in place of the question mark(?) in the following equation? $\frac{28}{?} = \frac{?}{112}$
(a) 70   (b) 56
(c) 48   (d) 64
(e) None of these

5. An empty fuel tank to a car was filled with A type of petrol. When the tank was half empty, it was filled with B type of petrol. Again when the tank was half empty, it was filled with A type of petrol. When the tank was half empty again, it was filled with B type of petrol. At this time, what was the percentage of A type of petrol in the tank?
(a) 50%   (b) 40%
(c) 33.5%   (d) 37.5%
(e) None of these

6. The ratio of A's and B's salary is 9 : 4. If A's salary is increased by 15%, then his total salary becomes ₹ 5175. What is the salary of B?
(a) ₹ 2,000   (b) ₹ 4,000
(c) ₹ 4,500   (d) ₹ 2,500
(e) None of these

7. Three friends *A, B* and *C* started a business by investing a sum of money in the ratio of 5 : 7 : 6. After 6 months *C* withdraws half of his capital. If the sum invested by '*A*' is ₹ 40,000, out of a total annual profit of ₹ 33,000, *C*'s share will be
(a) ₹ 9,000   (b) ₹ 12,000
(c) ₹ 11,000   (d) ₹ 10,000
(e) None of these

8. Seats for Maths, Physics and Biology are in the ratio of 5 : 7 : 8 respectively. There is a proposal to increase these seats by 40%, 50% and 75% respectively. What will be the respective ratio of increased seats?
(a) 2 : 3 : 4   (b) 6 : 7 : 8
(c) 6 : 8 : 9   (d) Cannot be determined
(e) None of these

9. Mr Sharad started a business investing ₹ 50000. Four months later Mr Praveen joined the business by investing ₹ 90000. If the profit in the business at the end of the year was ₹ 22000 how much amount would Mr Praveen have received as the profit?
(a) ₹ 16000   (b) ₹ 14000
(c) ₹ 12000   (d) ₹ 11000
(e) None of these

10. The ratio of Gomati's and Rashmi's ages is 3 : 5 respectively. After ten years this ratio will become 2 : 3. What is Rashmi's age in years?
(a) 50   (b) 40
(c) 60   (d) Cannot be determined
(e) None of these

11. Salaries of Rajesh and Sunil are in the ratio of 2 : 3. If the salary of each one is increased by ₹ 4000 the new ratio becomes 40 : 57. What is Sunil's present salary?
(a) ₹ 17000   (b) ₹ 20000
(c) ₹ 25500   (d) Csannot be determined
(e) None of these

12. The numbers of students speaking English and Hindi are in the ratio of 4:5. If the number of students speaking English increased by 35% and that speaking Hindi increased by 20%, what would be the new respective ratio?
(a) 19 : 20   (b) 7 : 8
(c) 8 : 9   (d) Cannot be determined
(e) None of these

13. Abhijit started a business investing ₹ 70000. Anuja joined him after six months with an amount of ₹ 105000 and Sunil joined them with ₹ 1.4 lakhs after another six months. The amount of profit earned should be distributed in what ratio among Abhijit, Anuja and Sunil respectively, three years after Abhijit started the business?
(a) 42 : 45 : 56   (b) 7 : 6 : 10
(c) 12 : 15 : 16   (d) Cannot be determined
(e) None of these

14. The ratio of males and females in a city is 7 : 8 and the percentage of children among males and females is 25% and 20% respectively. If the number of adult females in the city is 156800 what is the total population?
(a) 245000   (b) 367500
(c) 196000   (d) 171500
(e) None of these

15. Hariprasad and Madhusudan started a business, investing sums in the ratio of 2 : 3. If Hariprasad had invested an additional amount of ₹ 10,000 the ratio of Hariprasad's investment to Madhusudan's investment would have been 3 : 2. What was the amount invested by Hariprasad?
(a) ₹ 8000   (b) ₹ 12000
(c) ₹ 9000   (c) Data inadequate
(e) None of these

16. The ratio of the present ages of a son and his father is 1 : 5 and that of his mother and father is 4 : 5. After 2 years the ratio of the age of the son to that of his mother becomes 3 : 10. What is the present age of the father?
    (a) 30 years   (b) 28 years
    (c) 37 years   (d) Data inadequate
    (e) None of these

17. The ratio of the number of students appearing for examination in the year 1998 in the states *A, B* and *C* was 3 : 5 : 6. Next year if the number of students in these states increases by 20%, 10% and 20% respectively, the ratio in states *A* and *C* would be 1 : 2. What was the number of students who appeared for the examination in the state *A* in 1998?
    (a) 7200   (b) 6000
    (c) 7500   (d) Data inadequate
    (e) None of these

18. A man spends ₹ 1810 for buying bedsheets at ₹ 200 each and pillows at ₹ 70 each. What will be the ratio of bedsheets to pillows when maximum number of bedsheets are bought?
    (a) 3:8   (b) 8:3
    (c) 9:1   (d) 1:9
    (e) None of these

19. Mr Shivkumar started a business, investing ₹ 25000 in 1996. In 1997 he invested an additional amount of ₹ 10000 and Mr Rakesh joined him with an amount of ₹ 35000. In 1998, Mr Shivkumar invested another additional amount of ₹ 10000 and Mr Suresh joined them with an amount of ₹ 35000. What will be Rakesh's share in the profit of ₹ 150000 earned at the end of three years from the start of the business in 1996?
    (a) ₹ 70000   (b) ₹ 50000
    (c) ₹ 45000   (d) ₹ 75000
    (e) None of these

20. Incomes of two companies A and B are in the ratio of 5 : 8. Had the income of company A been more by ₹ 25 lakh, the ratio of their incomes would have been 5 : 4. What is the income of company B?
    (a) ₹ 80 lakh   (b) ₹ 50 lakh
    (c) ₹ 40 lakh   (d) ₹ 60 lakh
    (e) None of these

21. The ratio of number of students studying Arts, Commerce and Science in a College is 3 : 5 : 8. What is the new ratio of the number of students studying Arts, Commerce and Science respectively if there is an increase of 20%, 40% and 25% in the number of students studying Arts, Commerce and Science?
    (a) 18:35:50   (b) 3:10:10
    (c) 4:8:5   (d) 32:35:25
    (e) None of these

22. Abhishek started a business investing ₹ 50,000. After one year he invested another ₹ 30,000 and Sudin also joined him with a capital of ₹ 70,000. If the profit earned in three years from the starting of business was ₹ 87,500, then find the share of Sudin in the profit.
    (a) ₹ 37,500   (b) ₹ 32,500
    (c) ₹ 38,281   (d) ₹ 52,500
    (e) None of these

23. Weights of two friends Ram and Shyam are in the ratio of 4 : 5. Ram's weight increases by 10% and the total weight of Ram and Shyam together becomes 82.8 kg, with an increase of 15%. By what per cent did the weight of Shyam increase?
    (a) 12.5%   (b) 17.5%
    (c) 19%   (d) 21%
    (e) None of these

24. When 50% of one number is added to a second number, the second number increases to its four-thirds. What is the ratio between the first number and the second number?
    (a) 3 : 2   (b) 3 : 4
    (c) 2 : 3   (d) Data inadequate
    (e) None of these

25. The ratio of present ages of Nisha and Shilpa is 7:8 respectively. Four years hence this ratio becomes 9:10 respectively. What is Nisha's present age in years?
    (a) 18   (b) 14
    (c) 17   (d) Data inadequate
    (e) None of these

26. When a number is added to another number the total becomes $33\frac{1}{3}$ per cent of the second number. What is the ratio between the first and the second number?
    (a) 3 : 7   (b) 7 : 4
    (c) 7 : 3   (d) Data inadequate
    (e) None of these

27. The ratio between the present ages of *P* and *Q* is 5 : 8. After four years, the ratio between their ages will be 2 : 3. What is *Q*'s age at present?
    (a) 36 years   (b) 20 years
    (c) 24 years   (d) Data inadequate
    (e) None of these

28. Jaydeep purchased 25 kg of rice at the rate of ₹ 16.50 per kg and 35 kg of rice at the rate of ₹ 24.50 per kg. He mixed the two and sold the mixture. Approximately, at what price per kg did he sell the mixture to make 25 per cent profit?
    (a) ₹ 26.50   (b) ₹ 27.50
    (c) ₹ 28.50   (d) ₹ 30.00
    (e) ₹ 29.00

29. In 1 kg mixture of sand and iron, 20% is iron. How much sand should be added so that the proportion of iron becomes 10%?
    (a) 1 kg   (b) 200 gms
    (c) 800 gms   (d) 1.8 kg
    (e) None of these

30. The ratio of *P*'s and *Q*'s ages is 5 : 7. If the difference between the present age of *Q* and the age of *P* six years hence is 2 then what is the total of present ages of *P* and *Q*?
    (a) 52 years   (b) 48 years
    (c) 56 years   (d) Data inadequate
    (e) None of these

31. There is a ratio of 5 : 4 between two numbers. If forty percent of the first number is 12 then what would be the 50 percent of the second number?
    (a) 12   (b) 24
    (c) 18   (d) Data inadequate
    (e) None of the above

32. An amount of money is to be distributed among P, Q and R in the ratio of 5 : 8 : 12 respectively. If the total share of Q and R is four times that of P, what is definitely P's share?

(a) ₹ 3,000   (b) ₹ 5,000
(c) ₹ 8,000   (d) Data inadequate
(e) None of these

33. When 30 per cent of a number is added to another number the second number increases to its 140 per cent. What is the ratio between the first and the second number?
(a) 3 : 4   (b) 4 : 3
(c) 3 : 2   (d) Data inadequate
(e) None of these

34. If 25% of a number is subtracted from a second number the second number reduces to its five-sixths. What is the ratio between the first number and the second number?
(a) 2 : 3   (b) 3 : 2
(c) 1 : 3   (d) Data inadequate
(e) None of these

35. Two friends *P & Q* started a business investing amounts in the ratio of 5 : 6. R joined them after six months investing an amount equal to that of *Q*'s amount. At the end of the year 20% profit was earned which was equal to ₹ 98,000. What was the amount invested by R?
(a) ₹ 2,10,000   (b) ₹ 1,05,000
(c) ₹ 1,75,000   (d) Data inadequate
(e) None of these

36. One year ago the ratio of Yamini's and Gamini's ages was 6 : 7 respectively. Four years hence this ratio would become 7 : 8. How old is Gamini?
(a) 35 years   (b) 30 years
(c) 31 years   (d) Cannot be determined
(e) None of these

37. Ratio of present age of P and Q is 7 : 3. After four years their ages are in the ratio of 2 : 1. What is the present age of P ?
(a) 24 years   (b) 28 years
(c) 32 years   (d) Data inadequate
(e) None of these

38. If 40 per cent of a number is added to an other number then it becomes 125 per cent of itself. What will be the ratio of first and second numbers?
(a) 8 : 5   (b) 5 : 7
(c) 5 : 8   (d) Data inadequate
(e) None of these

39. An amount of money is to be divided among *P, Q* and *R* in the ratio of 4 : 9 : 16. If *R* gets 4 times more than *P*, what is *Q*'s share in it?
(a) ₹ 1,800   (b) ₹ 2,700
(c) ₹ 3,600   (d) Data inadequate
(e) None of these

40. Jagtap purchases 30 kg of wheat at the rate of ₹ 11.50 per kg and 20 kg of wheat at the rate of ₹ 14.25 per kg. He mixed the two and sold the mixture. **Approximately** at what price per kg should he sell the mixture to make 30 per cent profit?
(a) ₹ 16.30   (b) ₹ 18.20
(c) ₹ 15.60   (d) ₹ 14.80
(e) ₹ 15.40

41. Mr. Gangadhar, Mr. Ramesh and Mr. Shridhar together earned ₹ 19800. The ratio of earnings between Mr. Gangadhar and Mr. Ramesh is 2 : 1 while that between Mr. Ramesh and Mr. Shridhar is 3 : 2. How much did Mr. Ramesh earn?
(a) ₹ 3600   (b) ₹ 5400
(c) ₹ 1800   (d) ₹ 6300
(e) None of these

42. Mr. Kutty has only hens and sheep. If the total number of their heads is 38 and the total number of legs is 100 then what is the ratio between the numbers of hens and sheep?
(a) 2 : 1   (b) 1 : 2
(c) 6 : 13   (d) 13 : 6
(e) None of these

43. If A : B : C = 2 : 3 : 4, then $\frac{A}{B} : \frac{B}{C} : \frac{C}{A}$ is equal to :
(a) 4 : 9 : 16   (b) 8 : 9 : 12
(c) 8 : 9 : 16   (d) 8 : 9 : 24
(e) None of these

44. A sum of money is to be distributed among A, B, C, D in the proportion of 5 : 2 : 4 : 3. If C gets ₹ 1000 more than D, what is B's share?
(a) ₹ 500   (b) ₹ 1500
(c) ₹ 2000   (d) ₹ 1400
(e) None of these

45. The sum of three numbers is 98. If the ratio of the first to the second is 2 : 3 and that of the second to the third is 5 : 8, then the second number is :
(a) 20   (b) 30
(c) 38   (d) 48
(e) None of these

46. The ratio of number of ladies to gents at a party was 1 : 2, but when 2 ladies and 2 gents left, the ratio became 1 : 3. How many people were originally present at the party?
(a) 6   (b) 9
(c) 12   (d) 10
(e) None of these

47. A man divides his property so that his son's share to his wife's and the wife's share to his daughter are both in the ratio 3 : 1. If the daughter gets ₹ 10,000 less than the son, find the total worth of the property.
(a) ₹ 16,200   (b) ₹ 16,250
(c) ₹ 16,500   (d) ₹ 15,300
(e) None of these

48. A bag contains an equal number of one rupee, 50 paise and 25 paise coins respectively. If the total value is ₹ 35, how many coins of each type are there?
(a) 20 coins   (b) 30 coins
(c) 28 coins   (d) 25 coins
(e) None of these

49. The salaries of A,B,C are in the ratio 2 : 3 : 5. If the increments of 15%, 10% and 20% are allowed respectively in their salaries, then what will be the new ratio of their salaries?
(a) 3 : 3 : 10   (b) 10 : 11 : 20
(c) 23 : 33 : 60   (d) Cannot be determined
(e) None of these

50. In an express train, the passengers travelling in A.C. sleeper class, First class and Sleeper class are in the ratio 1 : 2 : 7, and rate for each class is in the ratio 5 : 4 : 2. If the total income from this train is ₹ 54, 000, find the income of Indian Railways from A.C. sleeper class.
(a) ₹ 12,000   (b) ₹ 20,000
(c) ₹ 22,000   (d) ₹ 10,000
(e) None of these

51. What is the ratio whose terms differ by 40 and the measure

    of which is $\frac{2}{7}$?
    (a) 16 : 56   (b) 14 : 56
    (c) 15 : 56   (d) 16 : 72
    (e) None of these

52. The average age of three boys is 25 years and their ages are
    in the proportion 3 : 5 : 7. The age of the youngest boy is:
    (a) 21 years   (b) 18 years
    (c) 15 years   (d) 9 years
    (e) None of these

53. A photograph measuring $2\frac{1}{2}'' \times 1\frac{7}{8}''$ is to be enlarged so

    that the length will be 4". How many inches will the enlarged
    breadth be?

    (a) $1\frac{1}{2}$ (b) $2\frac{1}{8}$

    (c) 3   (d) $3\frac{3}{8}$

    (e) None of these

54. In a partnership, A invests $\frac{1}{6}$ of the capital for $\frac{1}{6}$ of the time,

    B invests $\frac{1}{3}$ of the capital for $\frac{1}{3}$ of the time and C, the rest of

    the capital for whole time. Find A's share of the total profit
    of ₹ 2,300.
    (a) ₹ 100   (b) ₹ 200
    (c) ₹ 300   (d) ₹ 400
    (e) None of these

55. A, B and C start a business each investing ₹ 20,000. After 5
    months A withdrew ₹ 5000, B withdrew ₹ 4000 and C invests
    ₹ 6000 more. At the end of the year, a total profit of ₹ 69,900
    was recorded. Find the share of B.
    (a) ₹ 20,000   (b) ₹ 21,200
    (c) ₹ 28,200   (d) ₹ 20,500
    (e) None of these

56. A is a working partner and B is a sleeping partner in a
    business. A puts in ₹ 50,000 and B ₹ 60,000. A gets 12.5% of
    the profit for managing the business, and the rest is divided
    in proportion to their capitals. Find the share of A in profit
    of ₹ 8800.
    (a) ₹ 3500   (b) ₹ 4600
    (c) ₹ 5400   (d) ₹ 4800
    (e) None of these

57. A began business with ₹ 12500 and is joined afterwards by
    B with ₹ 37500. When did B join, if the profits at the end of
    the year are divided equally?
    (a) 8 months   (b) 9 months
    (c) 10 months   (d) 7 months
    (e) None of these

58. A began business with ₹ 45,000 and was later joined by B
    with ₹ 54,000. When did B join if the profit at the end of the
    year were divided in the ratio 2 : 1?

    (a) 5 months after   (b) 10 months after
    (c) 7 months after   (d) 12 months after
    (e) None of these

59. A and B enter into partnership with capitals in the ratio 3 : 4.
    At the end of 10 months A withdraws, and the profits now
    are divided in the ratio of 5 : 6. Find how long B remained in
    the business?
    (a) 9 months   (b) 8 months
    (c) 6 months   (d) 7 months
    (e) None of these

60. A and B invest ₹ 3,000 and ₹ 4,000 in a business. A receives
    ₹ 10 per month out of the profit as a remuneration for running
    the business and the rest of profit is divided in proportion
    to the investments. If in a year 'A' totally receives ₹ 390,
    what does B receive?
    (a) ₹ 375   (b) ₹ 360
    (c) ₹ 350   (d) ₹ 260
    (e) None of these

61. A started a business with ₹ 4500 and another person B
    joined after some period with ₹ 3000. Determine this period
    after B joined the business if the profit at the end of the year
    is divided in the ratio 2 : 1
    (a) After 3 months   (b) After 4 months

    (c) After 6 months   (d) After $2\frac{1}{2}$ months

    (e) None of these

62. A and B entered into a partnership with capitals in the ratio

    of 4 : 5. After 3 months, A withdrew $\frac{1}{4}$ of his capital and B

    withdrew $\frac{1}{5}$ of his capital. The gain at the end of 10 months

    was ₹ 760. Find the profit of B.
    (a) ₹ 450   (b) ₹ 430
    (c) ₹ 410   (d) ₹ 340
    (e) None of these

63. A and B rent a pasture for 10 months; A puts in 80 cows for
    7 months. How many can B put in for the remaining 3 months,
    if he pays half as much again as A?
    (a) 120   (b) 180
    (c) 200   (d) 280
    (e) None of these

64. In a partnership between X and Y, X's capital is $\frac{2}{5}$ of total

    and is invested for $\frac{2}{3}$ year. If his share of the profit is $\frac{4}{7}$ of

    the total, for how long is Y's capital in the business?

    (a) 1 year   (b) $\frac{1}{8}$ years

    (c) $\frac{1}{3}$ years   (d) $\frac{1}{4}$ years

    (e) None of these

65. X and Y put in ₹ 3,000 and ₹ 4,000 respectively into a business. X reinvests into the business his share of the first year's profit of ₹ 2,100 whereas Y does not reinvest. In what ratio should they share the second year's profit?
   - (a) 39 : 40
   - (b) 3 : 4
   - (c) 3 : 7
   - (d) 40 : 79
   - (e) None of these

66. Gold is 19 times as heavy as water and copper is 9 times heavy. In what ratio must these metals be mixed so that the mixture may be 15 times as heavy as water?
   - (a) 2 : 3
   - (b) 3 : 2
   - (c) 1 : 3
   - (d) 2 : 1
   - (e) None of these

67. Six litres of a 20% solution of alcohol in water are mixed with 4 litres of a 60% solution of alcohol in water. The % alcoholic strength of the mixture is
   - (a) 80
   - (b) 40
   - (c) 36
   - (d) 48
   - (e) None of these

68. A merchant lent out ₹ 1,000 in two parts, one at 8% and the other at 10% interest. The yearly average comes out to be 9.2%. Find the amount lent in two parts.
   - (a) ₹ 400, ₹ 600
   - (b) ₹ 500, ₹ 500
   - (c) ₹ 300, ₹ 700
   - (d) cannot be determined
   - (e) None of these

69. One litre of water was mixed to 3 litres of sugar solution containing 4% of sugar. What is the percentage of sugar in the solution?
   - (a) 3
   - (b) 4
   - (c) 6
   - (d) Insuffficient data
   - (e) None of these

70. How much water must be added to 60 litres of milk at $1\frac{1}{2}$ litres for ₹ 20 so as to have a mixture worth ₹ $10\frac{2}{3}$ a litre?

   - (a) 10 litres
   - (b) 12 litres
   - (c) 15 litres
   - (d) 18 litres
   - (e) None of these

71. How many kg of salt at 42 paise per kg must a man mix with 25 kg of salt at 24 paise per kg so that he may, on selling the mixture at 40 paise per kg gain 25% on the outlay?
   - (a) 15 kg
   - (b) 18 kg
   - (c) 20 kg
   - (d) 24 kg
   - (e) None of these

72. A trader mixes 80 kg of tea at ₹15 per kg with 20 kg of tea at cost price of ₹ 20 per kg. In order to earn a profit of 25%, what should be the sale price of the mixed tea?
   - (a) ₹ 23.75
   - (b) ₹ 22
   - (c) ₹ 20
   - (d) ₹ 19.20
   - (e) None of these

73. A company blends two varieties of tea from two different tea gardens, one variety costing ₹ 20 per kg and other ₹ 25 per kg, in the ratio 5 : 4. He sells the blended tea at ₹ 23 per kg. Find his profit percent :
   - (a) 5% profit
   - (b) 3.5% loss
   - (c) 3.5% profit
   - (d) No profit, no loss
   - (e) None of these

74. Alcohol cost ₹ 3.50 per litre and kerosene oil cost ₹ 2.50 per litre. In what proportion these should be mixed so that the resulting mixture may be ₹ 2.75 per litre?
   - (a) 2 : 5
   - (b) 1 : 3
   - (c) 4 : 7
   - (d) 2 : 3
   - (e) None of these

75. Pure milk costs ₹ 3.60 per litre. A milkman adds water to 25 litres of pure milk and sells the mixture at ₹ 3 per litre. How many litres of water does he add?
   - (a) 2 litres
   - (b) 5 litres
   - (c) 7 litres
   - (d) 11 litres
   - (e) None of these

ANSWER KEY

1	(e)	9	(c)	17	(d)	25	(b)	33	(b)	41	(b)	49	(c)	57	(a)	65	(a)	73	(c)
2	(d)	10	(a)	18	(b)	26	(c)	34	(a)	42	(d)	50	(d)	58	(c)	66	(b)	74	(b)
3	(a)	11	(e)	19	(b)	27	(e)	35	(b)	43	(d)	51	(a)	59	(a)	67	(c)	75	(b)
4	(b)	12	(e)	20	(c)	28	(a)	36	(e)	44	(c)	52	(c)	60	(b)	68	(a)		
5	(d)	13	(c)	21	(a)	29	(a)	37	(b)	45	(b)	53	(c)	61	(a)	69	(a)		
6	(a)	14	(b)	22	(e)	30	(d)	38	(e)	46	(c)	54	(a)	62	(b)	70	(c)		
7	(a)	15	(a)	23	(c)	31	(a)	39	(d)	47	(b)	55	(b)	63	(d)	71	(c)		
8	(a)	16	(e)	24	(c)	32	(d)	40	(a)	48	(a)	56	(b)	64	(c)	72	(c)		

Hints & Explanations

1. (e) Suppose the salaries of A, B and C were 300k, 500k and 700k respectively.
   After increment salary of
   A = 300k + 50% of 300k = 450k
   B = 500k + 60% of 500k = 800k
   C = 700k + 50% of 700k = 1050k
   Hence, new ratio of the respective salaries of A, B and C = 450k : 800k : 1050k = 9 : 16 : 21

2. (d) Suppose the first number is x and the second number y.

   Therefore, 40% of $x = \dfrac{2}{3}$ of y

   ∴ $\dfrac{x}{y} = \dfrac{2}{3} \times \dfrac{100}{40} = \dfrac{5}{3}$

3. (a) Ratio of their profits (Radha's : Sunidhi's : Neha's)
   $= 75 \times 36 : 125 \times 33 : 150 \times 27$
   $= 3 \times 36 : 5 \times 33 : 6 \times 27$
   $= 3 \times 12 : 5 \times 11 : 6 \times 9$
   $= 36 : 55 : 54$

4. (b) $\dfrac{28}{?} = \dfrac{?}{112}$ ∴ $? = \sqrt{28 \times 112} = 56$

5. (d)

	Petrol **A**	Petrol **B**
I:	A	0
II:	$\dfrac{A}{2}$	$\dfrac{B}{2}$
III:	$\dfrac{A}{4} + \dfrac{A}{2}$	$\dfrac{B}{4}$
IV:	$\dfrac{A}{8} + \dfrac{A}{4}$	$\dfrac{B}{8} + \dfrac{B}{2}$

   Now, amount of petrol A $= \dfrac{A}{4} + \dfrac{A}{8} = \dfrac{3A}{8}$

   ∴ required % $= \dfrac{3A}{8 \times A} \times 100 = 37.50\%$

6. (a) Let the salaries of A and B be 9x and 4x.

   $9x \times \dfrac{115}{100} = 5175$

   ∴ x = 500
   ∴ salary of B = 500 × 4 = ₹ 2000

7. (a) Sum invested by A, B and C is
   $5 \times 12 : 7 \times 12 : 6 \times 6 + 3 \times 6$
   or, 60 : 84 : 54   or,   10 : 14 : 9

   ∴ Share of C = $\dfrac{9}{33} \times 33,000 = ₹9,000$

8. (a) Reqd ratio $5 \times \dfrac{140}{100} : 7 \times \dfrac{150}{100} : 8 \times \dfrac{175}{100}$
   $= 5 \times 140 : 7 \times 150 : 8 \times 175 = 2 : 3 : 4$

9. (c) Ratio of their investment
   $= 50000 \times 12 : 90000 \times 8 = 5 : 6$

   ∴ Amount received by Praveen $= \dfrac{6}{11} \times 22,000$

   $= ₹ 12000$

10. (a) After 10 years, the ratio of Gomati and Rashmi's ages is 2 : 3. We can also write 2 : 3 as 4 : 6. Now, difference in the ratio is I in both the cases. Therefore, $1 = 10$
    ∴ $5 \to 5 \times 10 = 50$ years.

11. (e) Let the salaries of Rajesh and Sunil be ₹ 2x and ₹ 3x respectively.

    Then, $\dfrac{2x + 4000}{3x + 4000} = \dfrac{40}{57}$
    or, $114x + 228000 = 120x + 160000$
    or, $6x = 68000$
    or, $3x = ₹ 34000$

12. (e) Reqd ratio $= \dfrac{4 \times 135}{5 \times 120} = 9 : 10$

13. (c) Ratio of their investments
    $= 70 \times 36 : 105 \times 30 : 140 \times 24 = 12 : 15 : 16$

14. (b) Number of females $= 156800 \times \dfrac{100}{80} = 196000$

    ∴ Number of males $= \dfrac{7}{8} \times 196000 = 171500$

    ∴ Total population $= 196000 + 171500 = 367500$

15. (a) Let the initial investments of Hariprasad and Madhusudan be 2x and 3x, respectively.
    From the question,

    $\dfrac{2x + 10000}{3x} = \dfrac{3}{2}$

    or, $4x + 20000 = 9x$
    ∴ $x = 4000$
    ∴ Amount invested by Hariprasad $= 2x = ₹ 8000$

16. (e) $\dfrac{S}{F} = \dfrac{1}{5} \Rightarrow F = 5S$

    $\dfrac{M}{F} = \dfrac{4}{5} \Rightarrow M = \dfrac{4}{5} F$

    $\dfrac{S + 2}{M + 2} = \dfrac{3}{10}$

    $\Rightarrow$ $10S + 20 = 30M + 6 = 3 \times \dfrac{4}{5} \times 5S + 6 = 12S + 6$

    ∴ $2S = 14 \Rightarrow S = 7$ years
    ∴ $F = 5S = 35$ years

17. (d) Let the number of students appearing for examination in the year 1998 in the states *A, B* and *C* be 3*x*, 5*x* and 6*x* respectively.
According to the question,

$$\frac{3x \times \frac{120}{100}}{6x \times \frac{120}{100}} = \frac{1}{2}$$

Hence data inadequate.

18. (b) The man can't purchase more than 8 bedsheets
∴ cost of 8 bedsheets = 8 × 200 = ₹ 1600
Remaining amount = 1810 – 1600 = ₹ 210
And in ₹ 210, the man can purchase

$$\frac{210}{70} = 3 \text{ pillows}$$

Reqd ratio = 8 : 3

19. (b) Ratio of their investments = 25,000 × 1 + 35000 × 1 + 45000 × 1 : 35000 × 2 : 35000 × 1 = 3 : 2 : 1.

∴ Rakesh's share = $\frac{2}{6} \times 150000 = ₹ 50000$

20. (c) Let the incomes of two companies A and B be 5*x* and 8*x* respectively.
From the question,

$$\frac{5x + 25}{8x} = \frac{5}{4} \Rightarrow 20x + 100 = 40x \therefore x = 5$$

∴ Income of company B = 8*x* = ₹ 40 lakh

21. (a) Suppose the number of students studying Arts, Commerce and Science be 3*x*, 5*x* and 8*x* respectively. When their numbers are increased by 20%, 40% and 25% respectively the new ratio becomes
3*x* × 120% : 5*x* × 140% :
8*x* × 125% = 36 : 70 : 100
= 18 : 35 : 50

22. (e) Ratio of Abhishek and Sudin for one month
= (50,000 × 36) + (30,000 × 24) : (70,000 × 24)
= (18,00,000 + 7,20,000) : 16,80,000 = 3 : 2
Hence share of Sudin in the profit earned from the business.

$$= \frac{87,500}{(3+2)} \times 2 = ₹ 35,000.$$

23. (c) Let the weights of Ram and Shyam be 4*x* and 5*x*. Now, according to question,

$$\frac{4x \times 110}{100} + \text{Shyam's new wt} = 82.8 \qquad .... (i)$$

and $(4x + 5x) = \frac{9x \times 115}{100} = 82.8 \qquad .... (ii)$

From (ii), *x* = 8
Putting in (i), we get
Shyam's new wt = (82.8 – 35.2) = 47.6

% increase in Shyam's wt = $\left(\frac{47.6 - 40}{40} \times 100\right) = 19\%$

24. (c) Let the numbers be *y* and *x* respectively

$$x + 50\% \text{ of } y = \frac{4x}{3} \text{ or, } \frac{y}{2} = \frac{4x}{3} - x$$

or, $\frac{y}{2} = \frac{x}{3}$ or, $\frac{y}{x} = \frac{2}{3}$

25. (b)   Nisha   Shilpa
        7   8
   4   9   10

Present age of Nisha = $\frac{4 \times (10 - 9)}{9 \times 8 - 10 \times 7} \times 7$

                    = 14 years

26. (c) Let the first and second numbers be *x* and *y* respectively.

Then, $x + y = \frac{10}{3} y$

or, $x = \frac{7}{3} y$

∴ *x* : *y* = 7 : 3

27. (e) $\frac{P}{Q} = \frac{5}{8}$ or $P = \frac{5Q}{8}$ ....(i)

$$\frac{P + 4}{Q + 4} = \frac{2}{3}$$

or, 3P + 12 = 2Q + 8   or, 2Q – 3P = 4   ... (ii)
Putting value of P from eq. (i),

$$2Q - 3 \times \frac{5}{8} Q = 4 \Rightarrow Q = 32$$

28. (a) CP = 25 × 16.50 + 35 × 24.50 = ₹ 1270

$$SP = 1270 \times \frac{125}{100} = ₹ 1587.50$$

Price per kg = $\frac{1587.50}{60} \approx 26.50$

29. (a) In 1 kg mixture quantity of iron = 200 gm
Let *x* gm sand should be added, then
10% of (1000 + *x*) = 200
∴ *x* = 1000 gm = 1 kg

30. (d) $\frac{P}{Q} = \frac{5}{7}$

or, $Q = \frac{7}{5} P$
Case I : $Q - (P + 6) = 2$
or, $Q = P + 8$

∴ $\frac{7}{5} P = P + 8$

or, $7P = 5P + 40$

∴ $P = \frac{40}{2} = 20$ and $Q = \frac{7}{5} \times 20 = 28$

∴ $P + Q = 20 + 28 = 48$
Case II : $(P + 6) - Q = 2$

or, $P + 6 - \frac{7}{5} P = 2$

or, $P = -10$ and $Q = 14$
∴ $P + Q = 10 + 14 = 24$ years

31. (a) $\dfrac{a}{b} = \dfrac{5}{4}$, $b = \dfrac{4}{5}a$ Given (40% of a =) $\dfrac{2}{5}a = 12$

$\therefore$ $a = 5 \times 6$ and $b = \dfrac{4}{5} \times 5 \times 6 = 24$

$\therefore$ 50% of $b = \dfrac{24}{2} = 12$

32. (d) $P : Q : R = 5 : 8 : 12$

$\dfrac{\text{Total share of } Q \text{ and } R}{\text{share of } P} = \dfrac{8 + 12}{5} = \dfrac{20}{5} = 4$

So, we see that no new information has been given in the question and P's share can't be determined.

33. (b) Let the first and the second numbers be x and y respect then

$y + 30\%$ of $x = 140\%$ of y

or, $y + 0.3x = 1.4y$

or, $0.3x = 0.4y$

$\therefore$ $x : y = 0.4 : 0.3 = 4 : 3$

34. (a) Let the first and second number be x and y respectively.

$x - y \times \dfrac{25}{100} = y \times \dfrac{5}{6}$

or, $y - \dfrac{x}{4} = \dfrac{5}{6}y$

or, $\dfrac{1}{6}y = \dfrac{x}{4}$

$\therefore$ $x : y = 2 : 3$

35. (b) Ratio for amount invested by P, Q & R

$= 5x \times 12 : 6x \times 12 : 6x \times 6$

$= 60x : 72x : 36x$

$= 5x : 6x : 3x$

Profit $= 98000 = 20\%$ of T

where, T = Total amount

$T = ₹490000$

Amount received by

$R = \dfrac{3x}{3x + 6x + 5x}(490000)$

$= ₹105000$

36. (e) $Y : G = 6 : 7$; One year before

$Y : G = 7 : 8$; After 4 years

Now, $1 \rightarrow 5$ $\therefore 7 \rightarrow 35$

Therefore, the age of Gamini now is $35 + 1 = 36$ years

37. (b) $\dfrac{P}{Q} = \dfrac{7}{3} \Rightarrow Q = \dfrac{3}{7}P$ ....(i)

also, $2Q + 8 = P + 4$

or, $2Q - P = -4$ ...(ii)

Combining equations (i) and (ii), we get

$P = 28$ years

38. (e) $40\dfrac{x}{100} + y = \dfrac{125x}{100} \Rightarrow \dfrac{x}{y} = \dfrac{20}{17}$

39. (d) Here, neither the total amount nor the individual amount is given. So the share of Q cannot be determined.

40. (a) $CP = 30 \times 11.50 + 20 \times 14.25 = ₹\ 630$

$SP = 630 \times \dfrac{130}{100} \times \dfrac{1}{50} \approx ₹16.30$

41. (b) Ratio of investment $= 6 : 3 : 2$

$\therefore$ Share of Mr. Ramesh $= \dfrac{3}{11} \times 19800 = ₹\,5400$

42. (d) Let the total number of hens and sheep be x and y respectively.

i.e., $x + y = 38$ and $2x + 4y = 100$ $\therefore$ Ratio $= 13 : 6$

43. (d) Let A = 2x, B = 3x and C = 4x. Then,

$\dfrac{A}{B} = \dfrac{2x}{3x} = \dfrac{2}{3}$, $\dfrac{B}{C} = \dfrac{3x}{4x} = \dfrac{3}{4}$ and $\dfrac{C}{A} = \dfrac{4x}{2x} = \dfrac{2}{1}$

$\Rightarrow \dfrac{A}{B} : \dfrac{B}{C} : \dfrac{C}{A} = \dfrac{2}{3} : \dfrac{3}{4} : \dfrac{2}{1} = 8 : 9 : 24.$

44. (c) Let the shares of A, B, C and D be ₹ 5x, ₹ 2x, ₹ 4x, ₹ 3x respectively.

Then, $4x - 3x = 1000 \Rightarrow x = 1000$

$\therefore$ B's Share $= ₹ 2x = ₹ 2000$

45. (b) A : B = 2 : 3 = 2 × 5 : 3 × 5 = 10 : 15

and B : C = 5 : 8 = 5 × 3 : 8 × 3 = 15 : 24

Therefore, A : B : C = 10 : 15 : 24

$\therefore$ A : B : C = 10 : 15 : 24

Let the number be 10x, 15x and 24x.

Then, $10x + 15x + 24x = 98$

or $49x = 98$ or $x = 2$

$\Rightarrow$ Second number = 15x = 15 × 2 = 30

46. (c) Let number of ladies = x

then, number of gents = 2x

Now, $\dfrac{x - 2}{2x - 2} = \dfrac{1}{3} \Rightarrow 3x - 6 = 2x - 2$

$\Rightarrow x = 4$

$\therefore$ Total number of people originally present

$= 4 + 8 = 12$

47. (b) Let Son's share = ₹ S;

Daughter's share = ₹ D;

and Wife's share = ₹ W.

Also, S : W = W : D = 3 : 1

$\therefore$ S : W : D = 9 : 3 : 1

then S = 9x , D = x

and $9x - x = 10,000 \Rightarrow x = ₹ 1250$

$\therefore$ Total worth of the property $= (9 + 3 + 1) x = 13x$

$= 13 \times 1250 = ₹ 16,250$

48. (a) Let number of each type of coin = x. Then,

$1 \times x + .50 \times x + .25 x = 35$

$\Rightarrow 1.75x = 35 \Rightarrow x = 20$ coins

49. (c) Let A = 2k, B = 3k and C = 5k.

A's new salary $= \dfrac{115}{100}$ of $2k = \left(\dfrac{115}{100} \times 2k\right) = \dfrac{23}{10}k$

B's new salary $= \dfrac{110}{100}$ of $3k = \left(\dfrac{110}{100} \times 3k\right) = \dfrac{33}{10}k$

C's new salary $= \dfrac{120}{100}$ of $5k = \left(\dfrac{120}{100} \times 5k\right) = 6k$

∴ New ratio $= \dfrac{23k}{10} : \dfrac{33k}{10} : 6k = 23 : 33 : 60.$

50. (d) Let number of passengers $= x, 2x, 7x$
and Rate $= 5y, 4y, 2y$
Now, since income $=$ Rate $\times$ Number of passengers
∴ Income $= 5xy, 8xy, 14xy$
∴ Income in ratio $= 5 : 8 : 14$

∴ Income from A.C. sleeper class $= \dfrac{5}{5+8+14} \times 54,000$

$= ₹\,10,000$

51. (a) Let the ratio be $x : (x+40)$. Then,

$\dfrac{x}{(x+40)} = \dfrac{2}{7} \Rightarrow 7x = 2x + 80 \Rightarrow x = 16.$

∴ Required ratio $= 16 : 56.$

52. (c) Total age of 3 boys $= (25 \times 3)$ years $= 75$ years. Ratio of their ages $= 3 : 5 : 7$.

Age of the youngest $= \left(75 \times \dfrac{3}{15}\right)$ years $= 15$ years.

53. (c) Let enlarged breadth be x inches. Then,

$\dfrac{5}{2} : 4 :: \dfrac{15}{8} : x$

$\Rightarrow \dfrac{5}{2}x = 4 \times \dfrac{15}{8} \Rightarrow x = 3$ inches

54. (a) Remaining capital $= 1 - \left(\dfrac{1}{6} + \dfrac{1}{3}\right) = \dfrac{1}{2}$

Ratio of their profit

$= \dfrac{1}{6} \times \left[\dfrac{1}{6} \times 12\right] : \dfrac{1}{3} \times \left[\dfrac{1}{3} \times 12\right] : \dfrac{1}{2} \times 12$

$= \dfrac{1}{3} : \dfrac{4}{3} : 6 = 1 : 4 : 18$

∴ A's share $= \dfrac{1}{1+4+18} \times 2300 = ₹\,100$

55. (b) Ratio of the capitals of A, B and C
$= 20000 \times 5 + 15000 \times 7 : 20000 \times 5 + 16000 \times 7 : 20000 \times 5 + 26000 \times 7$
$= 205000 : 212000 : 282000 = 205 : 212 : 282.$

B's share $= ₹\left(69900 \times \dfrac{212}{699}\right) = ₹\,21200;$

56. (b) The amount A gets for managing
$= 12.5\%$ of $₹\,8800 = ₹\,1100$
Remaining profit $= ₹\,8800 - ₹\,1100 = ₹\,7700$
This is to be divided in the ratio $5 : 6$.
Share of A $= 5/11$ of $₹\,7700 = ₹\,3500$
$\Rightarrow$ Total share of A $= ₹\,3500 + ₹\,1100 = ₹\,4600.$

57. (a) Let B join after x months of the start of the business so that B's money is invested for $(12-x)$ months.
∴ Profit ratio is $12 \times 12500 : (12-x) \times 37500$
or $12 : 3(12-x)$
Since profit is equally divided so
$12 = 3(12-x)$ or $x = 8$. Thus B joined after 8 months.

58. (c) Let B join after 'x' month of the start of the business.
$\Rightarrow (45,000 \times 12) : 54,000 \times (12-x) = 2 : 1$
∴ $(45,000 \times 12) \times 1 = 54,000 \times (12-x) \times 2$
$\Rightarrow x = 7$

59. (a) Initially A's investment $= 3x$ and B's investment $= 4x$
Let B remain in the business for 'n' months.
$\Rightarrow 3x \times 10 : 4x \times n = 5 : 6$
∴ $3x \times 10 \times 6 = 4x \times n \times 5 \Rightarrow n = 9$

60. (b) In a year, for A, total amount as a remuneration
$= 10 \times 12 = ₹\,120$
∴ Amount of A's profit $= 390 - 120 = ₹\,270$
Ratio of investment $= 3 : 4$
Let total profit $= ₹\,x$
Then, B's profit $= ₹\,(x - 270)$

∴ $\dfrac{3}{3+4} \times x = 270 \Rightarrow x = 630$

∴ B's profit $= 630 - 270 = ₹\,360$

61. (a) Let B joined after x months.
Then, $4500 \times 12 : 3000(12-x) = 2 : 1$
Ratio of their investments

$= \dfrac{4500 \times 12}{3000(12-x)} = \dfrac{2}{1} \Rightarrow x = 3$

62. (b) Let A's capital $= ₹\,4x$ and B's capital $= ₹\,5x$
∴ Ratio of profit

$= 4x \times 3 + \dfrac{3}{4}(4x) \times 7 : 5x \times 3 + \dfrac{4}{5}(5x) \times 7$

$= 33 : 43$

∴ Profit of B $= \dfrac{43}{33+43} \times 760 = ₹\,430$

63. (d) Let B puts $= x$ cows

then amount paid by $B = \dfrac{3}{2} \times$ amount paid by A.

∴ $\dfrac{80 \times 7}{x \times 3} = \dfrac{\text{amount paid by A}}{3/2 \times \text{amount paid by A}}$

$\Rightarrow x = \dfrac{80 \times 7 \times 3}{3 \times 2} = 280$ cows

64. (c) $\because$ X's capital $= \dfrac{2}{5}$ of total

$\therefore$ Y's capital $= 1 - \dfrac{2}{5} = \dfrac{3}{5}$ of total

Let Y invested capital for t years.

$\therefore$ Ratio of profit $= \dfrac{2}{5} \times \dfrac{2}{3} : \dfrac{3}{5} \times t$

$= \dfrac{4}{15} : \dfrac{3t}{5}$ … (i)

Share of Y's profit $= 1 - \dfrac{4}{7} = \dfrac{3}{7}$ of the total

Actual ratio of profit $= 4 : 3$

$\therefore$ By (i), $\dfrac{4/15}{3t/5} = \dfrac{4}{3} \Rightarrow t = \dfrac{1}{3}$ year.

65. (a) For first year, ratio of profit $= 3 : 4$

X's profit of first year $= \dfrac{3}{7} \times 2100 = ₹900$

Now, for second year,
ratio of profit $= 3000 \times 12 + 900 \times 12 : 4000 \times 12$
$= 46800 : 48000 = 39 : 40$

66. (b) By the rule of alligation, we have

Gold   Copper
19 times   9 times

15 times

$15 - 9 = 6$ $19 - 15 = 4$

$\therefore$ Required ratio $= \dfrac{6}{4} = 3 : 2$

67. (c) % alcoholic strength in mixture $= \dfrac{6 \times 20 + 4 \times 60}{6 + 4} = 36\%$

68. (a) $\dfrac{\text{Quantity lent at } 8\%}{\text{Quantity lent at } 10\%} = \dfrac{10 - 9.2}{9.2 - 8} = \dfrac{0.8}{1.2} = \dfrac{2}{3}$

$\therefore$ Quantity of money lent at 8%

$= \dfrac{2}{2 + 3} \times 1000 = ₹400$

and quantity of money lent at 10%

$= \dfrac{3}{2 + 3} \times 1000 = ₹600$

69. (a) New % of sugar in $(3 + 1)$ litre solution

$= \dfrac{0.04 \times 3}{(3 + 1)} = 0.03 = 3\%$

70. (c) C.P. of 1 litre of milk $= ₹\left(20 \times \dfrac{2}{3}\right) = ₹\dfrac{40}{3}$.

C.P. of 1 litre   C.P. of 1 litre
of water   of milk

0 Mean price $\left(₹\dfrac{32}{3}\right)$ $₹\dfrac{40}{3}$

$\left(\dfrac{40}{3} - \dfrac{32}{3}\right) = \dfrac{8}{3}$ $\left(\dfrac{32}{3} - 0\right) = \dfrac{32}{3}$

$\therefore$ Ratio of water and milk $= \dfrac{8}{3} : \dfrac{32}{3} = 8 : 32 = 1 : 4$.

$\therefore$ Quantity of water to be added to 60 litres of milk

$= \left(\dfrac{1}{4} \times 60\right)$ litres $= 15$ litres.

71. (c) Here, cost price of mixture $= 40 \times \dfrac{100}{100 + 25} = 32$ paise

$\therefore \dfrac{q_1}{q_2} = \dfrac{32 - 24}{42 - 32} = \dfrac{8}{10} = \dfrac{4}{5}$

and hence $q_1 = \dfrac{4}{5} \times 25 = 20$ kg

72. (c) C.P. of mixture $= \dfrac{80 \times 15 + 20 \times 20}{80 + 20} = ₹16$

$\therefore$ S.P. $= \dfrac{(100 + 25)}{100} \times 16 = ₹20$

73. (c) Let the quantity of two varieties of tea be 5x kg and 4x kg, respectively.
Now, SP $= 23 \times 9x = 207x$
and CP $= 20 \times 5x + 25 \times 4x = 200x$

Profit % $= \dfrac{7x}{200x} \times 100 = 3.5\%$

74. (b) By the rule of alligation, we have

Cost of Alcohol   Cost of Kerosene Oil
₹ 3.50   ₹ 2.50

cost of mixture ₹ 2.75

$2.75 - 2.50 = 0.25$ $3.30 - 2.75 = 0.75$

$\therefore$ Required ratio $= \dfrac{0.25}{0.75} = \dfrac{1}{3}$ i.e. $1 : 3$

75. (b) In mixture,

$\dfrac{\text{Quantity of pure milk}}{\text{Quantity of water}} = \dfrac{3 - 0}{3.6 - 3} = \dfrac{3}{0.6} = \dfrac{5}{1}$

Since in every 5 litres of milk, he adds 1 litre of water.
$\therefore$ in every 25 litres of milk, he adds 5 litres of water.

❖❖❖