Probability

- **Axiometic Approach of Probability**

- Probability (of an event E) $= \dfrac{\text{no. of favourable outcomes}}{\text{total no. of outcomes}}$

  $= P(E)$

  $$\boxed{0 \le P(E) \le 1}$$

- Outcomes = Results.

- Random Experiment: an experiment which has two or more well defined outcomes, but we can not Predict the next outcome in advance.

  e.g. Tossing a fair coin

  Throwing a fair Die

- Sample Space: set of all possible outcomes in a random experiment.

  e.g. Random Experiment → Tossing a Die.

  Sample Space (S) → $S = \{1, 2, 3, 4, 5, 6\}$

- Event: what we want

  (E) (subset of sample space)

  e.g. Random Experiment → Tossing a Die

  Sample Space $S = \{1, 2, 3, 4, 5, 6\}$

  $\underbrace{\text{We want PRIME NUMBERS}}_{\text{Event (E)}}$ $\quad E = \{2, 3, 5\}$

  $$P(\text{Prime}) = P(E) = \frac{3 \leftarrow \{2,3,5\}}{6 \leftarrow \{1,2,3,4,5,6\}}$$

  $= \frac{1}{2} = 0.5$

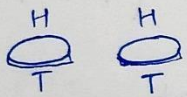
Some Important Sample Spaces

- **Tossing a fair Coin**

$S = \{H, T\}$ = Sample space

no. of elements in $S = n(S) = 2 = 2^1$

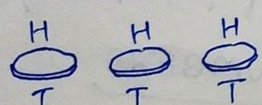
- **Tossing 2 Coins** / (Tossing a Coin Twice)



$S = \{HH, HT, TH, TT\}$

$n(S) = 4 = 2^2$

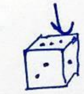
- **Tossing 3 Coins** / (Tossing a Coin Thrice)



$S = \{HHH, HHT, HTH, THH, HTT,$
$THT, TTH, TTT\}$

$n(S) = 8 = 2^3$

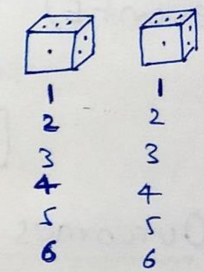
- **Roll a Die** (Throw a Die)



$S = \{1, 2, 3, 4, 5, 6\}$

$n(S) = 6$

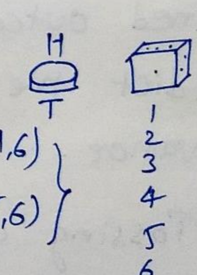
- **Roll 2 Dice** (Roll a Die twice)



$$S = \begin{Bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,5) & (4,4) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{Bmatrix}$$

$n(S) = 36$

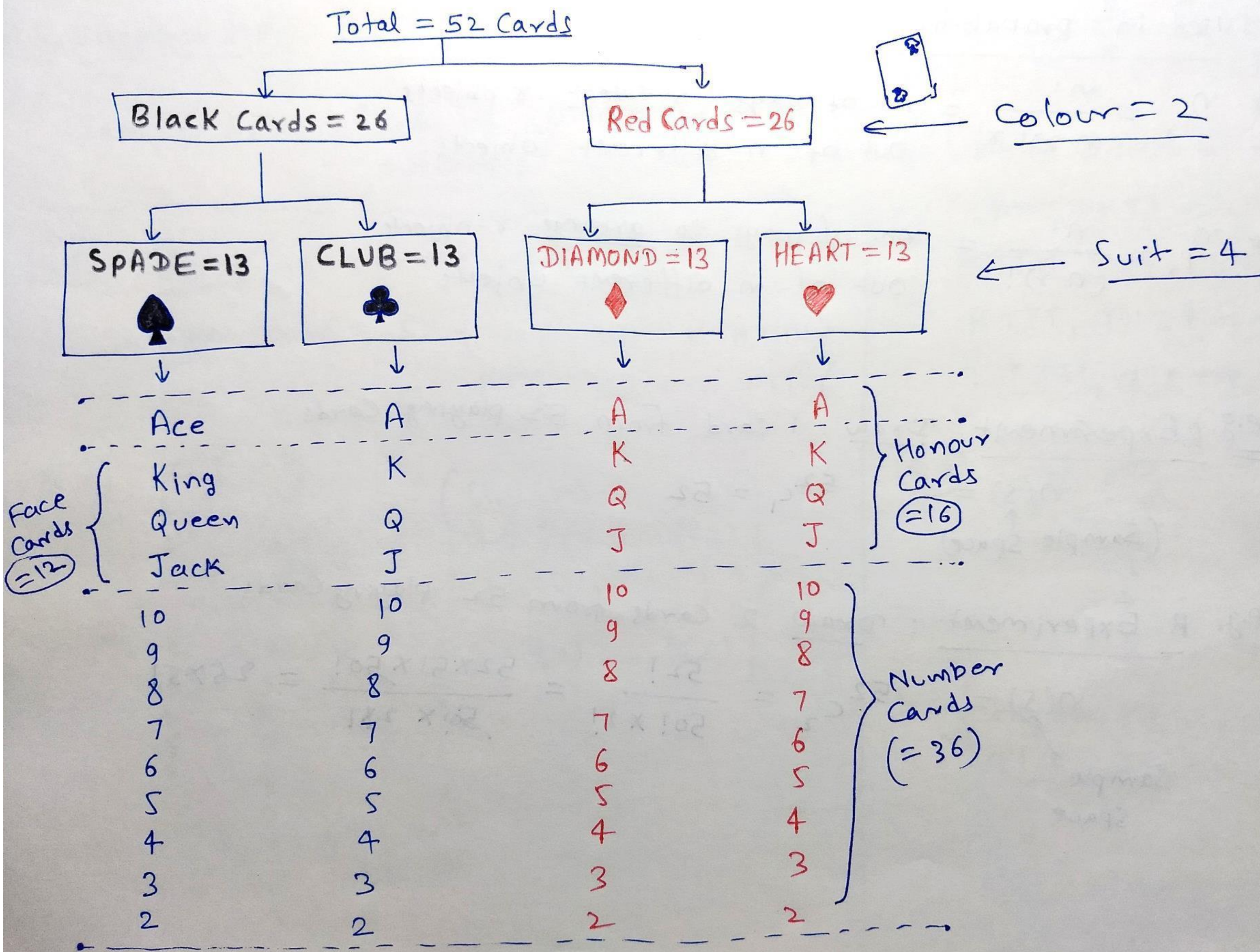
- **Toss a Coin & Roll a Die**



$$S = \begin{Bmatrix} (H,1) & (H,2) & (H,3) & (H,4) & (H,5) & (H,6) \\ (T,1) & (T,2) & (T,3) & (T,4) & (T,5) & (T,6) \end{Bmatrix}$$

$n(S) = 2 \times 6 = 12$

A Well Shuffled Deck of 52-Playing Cards



Some other important toots used in probability.

★ ${}^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$ = no. of ways to select r objects out of n-different objects.

★ ${}^{n}P_{r} = \frac{n!}{(n-r)!}$ = no. of ways to arrange r-objects out of n-different objects

e.g. R. Experiment: Draw 1 card from 52 playing cards

$n(S) = {}^{52}C_1 = 52$

(Sample space)

e.g. R. Experiment: Draw 2 cards from 52 playing cards

$n(S) = {}^{52}C_2 = \frac{52!}{50! \times 2!} = \frac{52 \times 51 \times \cancel{50!}}{\cancel{50!} \times 2 \times 1} = 26 \times 51$

Sample space

Exercise 14.1

Sample Space : "Set of all possible outcomes"

Q.1 A coin → 3 times toss

H H H / T T T

Sample space $= S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

Q.2

1st time	2nd time
1	1
2	2
3	3
4	4
5	5
6	6

$$S = \left\{ \begin{array}{cccc} (1,1) & (1,2) & \ldots\ldots & (1,6) \\ (2,1) & (2,2) & \ldots & (2,6) \\ \vdots & \vdots & & \vdots \\ (6,1) & (6,2) & \ldots & (6,6) \end{array} \right\}$$

$n(S) = 36$

Q.3 H H H H / T T T T

$S = \{HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, THHT, HTTH, THTH, TTHH, HTTT, THTT, TTHT, TTTH, TTTT\}$

Q.4 H / T ; 1, 2, 3, 4, 5, 6

$S = \{H1, H2, H3, \ldots H6, T1, T2, T3, \ldots T6\}$

Q.5

H / T $\rightarrow$ 1, 2, 3, 4, 5, 6

$S = \{ T, H1, H2, H3, H4, H5, H6 \}$

Q.6

X	Y
$2B \rightarrow B_1, B_2$	$1B \rightarrow B_3$
$2G \rightarrow G_1, G_2$	$3G \rightarrow G_3, G_4, G_5$

I $\rightarrow$ Room ✓

II $\rightarrow$ person. ✓

$$S = \{ XB_1, XB_2, XG_1, XG_2, YB_3, YG_3, YG_4, YG_5 \}$$

Q.7

Red (R) White (W) Blue (B)

$$S = \{ R1, R2, R3 \ldots R6, \; W1, W2, W3 \ldots W6, \; B1, B2, B3, \ldots B6 \}$$

Q.8

(i) B $\rightarrow$ Boy

G $\rightarrow$ Girl

$S = \{ BB, BG, GB, GG \}$

(ii) $S = \{ 0, 1, 2 \}$

number of Girls

$\downarrow$

0, 1, 2, 3, ...

Q.9

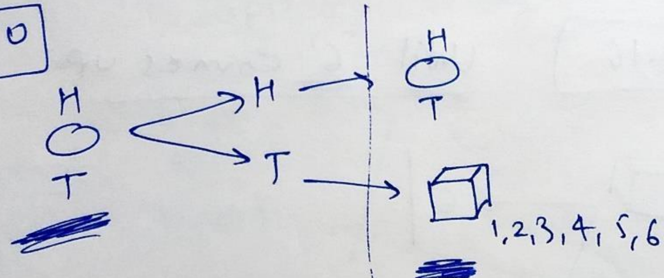
R WW W

Bag

$S = \{ RW, WR, WW \}$

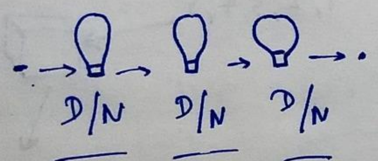
(without replacement)

Q.10



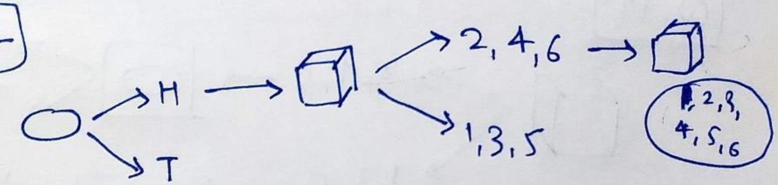
$S = \{HH, HT, T1, T2, T3, T4, T5, T6\}$

Q.11



$S = \{DDD, DDN, DND, NDD, DNN, NDN, NND, NNN\}$

Q.12



$S = \{T, H1, H3, H5, (H,2,1), (H,2,2) \ldots (H,2,6), (H,4,1), (H,4,2) \ldots (H,4,6), (H,6,1), (H,6,2), \ldots (H,6,6)\}$

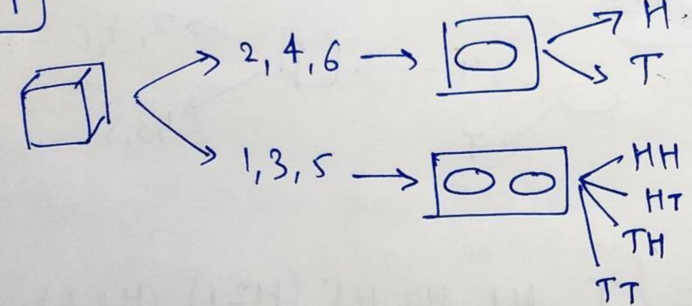
Q.13

1, 2, 3, 4

without replacement

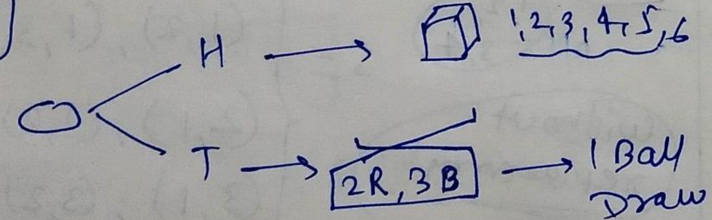
$S = \{(1,2), (1,3), (1,4), (2,1), (2,3), (2,4), (3,1), (3,2), (3,4), (4,1), (4,2), (4,3)\}$

Q.14



$S = \{2H, 2T, 4H, 4T, 6H, 6T,$
$1HH, 1HT, 1TH, 1TT$
$3HH, 3HT, 3TH, 3TT$
$5HH, 5HT, 5TH, 5TT\}$

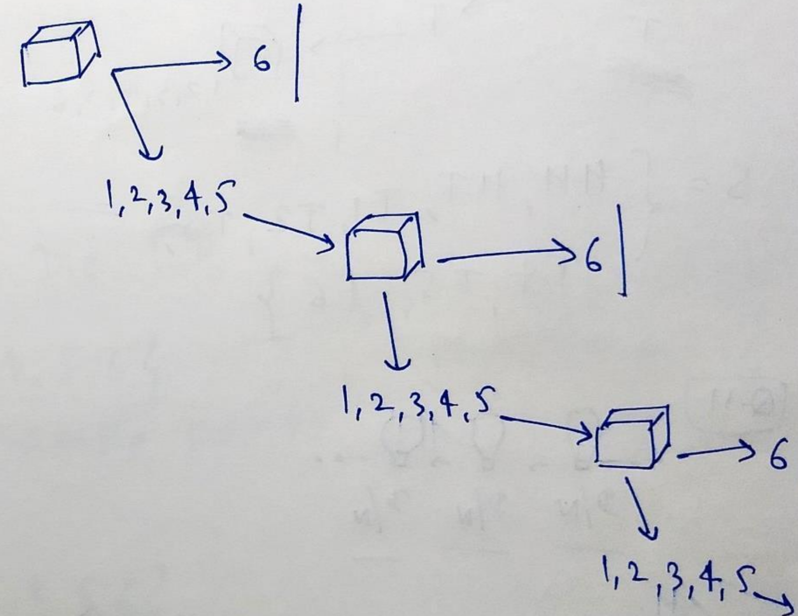
Q.15



$S = \{H1, H2, H3, H4, H5, H6,$
$TR_1, TR_2, TB_1, TB_2, TB_3\}$

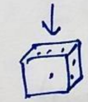
Q.16

Until '6' comes up.



$S = \{6,$
$(1,6), (2,6), (3,6), (4,6), (5,6),$
$(1,1,6), (1,2,6), (1,3,6) \ldots (1,5,6),$
$(2,1,6), (2,2,6), (2,3,6) \ldots (2,5,6)$
$\ldots\}$

Event: 'what we want' / subset of sample space (S)
(E)

e.g. Random Experiment: Rolling a Die 

Sample Space = $S = \{\underbrace{1, 2, 3, 4, 5, 6}_{\text{sample points}}\}$ $\rightsquigarrow$ (No. of sample points = 6)

Event: we want prime No.

$E = \{2, 3, 5\} \subset \{1, 2, 3, 4, 5, 6\}$

$\boxed{E \subseteq S}$ $P(E) = P(\text{Prime}) = \frac{3}{6} = \frac{1}{2}$

Note: $0 \leq P(E) \leq 1$

$P(E) = 1 \Rightarrow$ Chance = 100% $\Rightarrow$ E = Sure Event

$P(F) = 0 \Rightarrow$ Chance = 0% $\Rightarrow$ F = Impossible Event

Different Types of Events: Probability ↔ Set

① Impossible Event: $P(E) = 0 = P(\phi)$ ϕ = empty set = { }

② Sure Event: $P(E) = 1$

③ Simple Event: Event which has only one sample point.

Start to link Set & Probability

e.g. Tossing a coin. → $S = \{H, T\}$

Event 'E' = getting Head

Simple event. → $E = \{H\}$

④ Compound Event

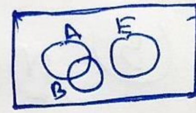
(more than one sample point)

e.g. Experiment → Rolling a Die

$S = \{1, 2, 3, 4, 5, 6\}$

Event E = Even no. = $\{2, 4, 6\}$

Algebra of Events.



→ U = Sample Space = S

Event → set

① Complementary Event

Event → E

Complementary Event $= E' = \overline{E} = E^c$

↳ not 'E'

$$\boxed{E' = S - E}$$

e.g. Roll a Die

$S = \{1, 2, 3, 4, 5, 6\}$

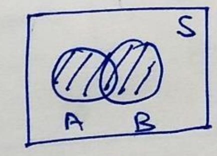
E = no. less than 3

$E = \{1, 2\}$

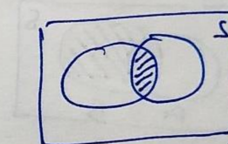
$E' = \{3, 4, 5, 6\}$

② Event 'A' or 'B' = $A \cup B$

③ Event 'A' and 'B' = $A \cap B$



$A \cup B$



$A \cap B$ (Common Part)

e.g. Rolling a Die

$S = \{1, 2, 3, 4, 5, 6\}$

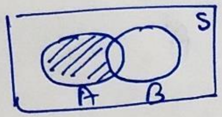
E = no. less than '3' $= \{1, 2\}$

F = odd no. $= \{1, 3, 5\}$

$E \cup F$	$E \cap F$
$\{1, 2, 3, 5\}$	$\{1\}$
$P(E \cup F) = \frac{4}{6}$	$P(E \cap F) = \frac{1}{6}$

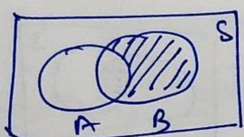
④ Event 'A but not B'

$A - B = A \cap B'$



Event 'B but not A' $= B - A$

$= B \cap A'$



e.g. Roll a Die 

$S = \{1, 2, 3, 4, 5, 6\}$

$E =$ No. less than $3 = \{1, 2\}$

$F =$ odd no. $= \{1, 3, 5\}$

↳ Find prob. of getting odd no. but not less than '3'. $\Rightarrow F - E = \{1,3,5\} - \{1,2\}$

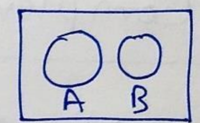
$= \{3, 5\}$

$P(F-E) = \frac{2}{6}$

⑤ Mutually Exclusive Events.

Disjoint Sets

which have nothing in common.



$A \cap B = \{\ \} = \phi$

e.g. Rolling a Die 

$S = \{1, 2, 3, 4, 5, 6\}$

$E = \{1, 2\}$ $\quad H = \{6\}$

$F = \{1, 3, 5\}$

E & F → not mutually exclusive Events.

E & H → MEE ✓

F & H → MEE ✓

(6) Exhaustive Events:

$E_1, E_2, \ldots, E_n$ are said to be Exhaustive Events if atleast one of them necessarily occurs whenever experiment is performed.

mathematically.

$$E_1 \cup E_2 \cup E_3 \cup \cdots \cup E_n = \underbrace{S}_{\text{Sample space}}$$

(7) Mutually Exclusive & Exhaustive Events:

E & F $\to$ $E \cap F = \phi$, $E \cup F = S$

A & B $\to$ $A \cap B = \phi$, $A \cup B = S$

e.g. Rolling a Die. 

$S = \{1, 2, 3, 4, 5, 6\}$

$E = \{1, 3, 5\}$ $\quad$ $A = \{2, 3\}$

$F = \{2, 4, 6\}$ $\quad$ $B = \{1, 4, 5, 6\}$

$C = \{5\}$

E & F $\to$ $E \cup F = \{1, 2, 3, 4, 5, 6\} = S$

A & B $\to$ $A \cup B = \{1, 2, 3, 4, 5, 6\} = S$

E & A $\to$ $E \cup A = \{1, 2, 3, 5\} \neq S$

E, F, A $\to$ $E \cup F \cup A = \{1, 2, 3, 4, 5, 6\} = S$

Exercise. 14.2

Sure Event

Impossible Event

Simple Event

Compound Event

Complementary Event

$A \cup B$ 'A or B'

$A \cap B$ 'A & B'

$A - B$ 'A but not B'

Mutually Exclusive Events.

Exhaustive Events.

Q.1

$S = \{1, 2, 3, 4, 5, 6\}$

$E = \{4\}$ $F = \{2, 4, 6\}$

Empty $\{ \}$

Mutually Exclusive Events : $E \cap F = \phi$

$E \cap F = \{4\} \cap \{2, 4, 6\} = \{4\} \neq \phi$

Common

E & F are not MEE

Q.2

Die

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 2, 3, 4, 5, 6\}$

$B = \{ \} = \phi$

$C = \{3, 6\}$

$D = \{1, 2, 3\}$

$E = \{6\}$

not F

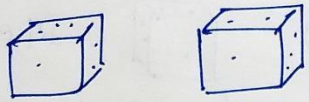
$F = \{3, 4, 5, 6\}$, $F' = \{1, 2\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

$A \cap B = \phi = \{ \}$

$A - C = \{1, 2, 3, 4, 5, 6\} - \{3, 6\}$
$= \{1, 2, 4, 5\}$

$E \cap F' = \{6\} \cap \{1, 2\}$
$= \phi = \{ \}$

Q.3 

$$S = \left\{ \begin{matrix} (1,1) & (1,2) & \cdots & (1,6) \\ (2,1) & (2,2) & \cdots & (2,6) \\ \vdots & \vdots & & \vdots \\ \underline{(6,1)} & \underline{(6,2)} & \cdots & \underline{(6,6)} \end{matrix} \right\}$$

A : sum is greater than 8
↓
9, 10, 11, 12

$A = \{ (6,3), (5,4), (4,5), (3,6)$
$(6,4), (5,5), (4,6),$
$(6,5), (5,6)$
$(6,6) \}$

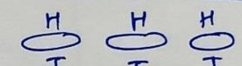
B: 2 occurs on either Die.

$B = \left\{ \begin{matrix} (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (1,2), \quad (3,2), (4,2), (5,2), (6,2) \end{matrix} \right\}$

~~Q.4~~ C : Sum at least 7
& multiple of 3.

Sum → 7, 8, 9, 10, 11, 12

$C = \left\{ \begin{matrix} (6,3), (5,4), (4,5), (3,6) \\ (6,6) \end{matrix} \right\}$

Q.4 

$S = \{ HHH, HHT, HTH, THH, HTT,$
$THT, TTH, TTT \}$

A : 3 H's $= \{HHH\}$

B : 2 H's & 1 T $= \{HHT, HTH, THH\}$

C : 3 T's $= \{TTT\}$

D : H on the first coin $= \{HHH, HHT, HTH, HTT\}$

(i) Mutually Exclusive Events $E \cap F = \phi$

Disjoint

A & B, | A & C, | B & C | C & D

$A \cap B = \phi$

(ii) Simple events — only one element

↓

A, C

(iii) Compound Events — more than one ~~elem~~ elements

B, D

Q.5 Self

Q.6

$$S = \begin{Bmatrix} (1,1) & (1,2) & \cdots & (1,6) \\ (2,1) & (2,2) & \cdots & (2,6) \\ \vdots & \vdots & & \vdots \\ (6,1) & (6,2) & \cdots & (6,6) \end{Bmatrix}$$

A = even no. of first die

B = odd no. of first die

C = Sum ≤ 5 → Sum → 5, 4, 3, 2

$$A = \begin{Bmatrix} (2,1), (2,2) \cdots (2,6) \\ (4,1), (4,2) \cdots (4,6) \\ (6,1), (6,2) \cdots (6,6) \end{Bmatrix}$$

$$B = \begin{Bmatrix} (1,1) & (1,2) & \cdots & (1,6) \\ (3,1) & (3,2) & \cdots & (3,6) \\ (5,1), & (5,2). & \cdots & (5,6) \end{Bmatrix}$$

$$C = \begin{Bmatrix} (4,1), (3,2), (2,3), (1,4) \\ (3,1), (2,2), (1,3), (1,2), (2,1), (1,1) \end{Bmatrix}$$

Q. 7.

(i) A & B are mutually exclusive events. (True)

(ii) A & B are mutually exclusive & exhaustive events.

$A \cap B = \phi$, $A \cup B = S$

(rows 1, 3, 5) (rows 2, 4, 6)

(True)

(iii) $A = B'$ True

(iv) A & C → mutually exclusive

$A \cap C \neq \phi$ False

(V) A & B' are mutually exclusive. False

$A \cap B' \neq \phi$ By (iii) part

$A \cap A \neq \phi$

$A \neq \phi$

(VI) A', B', C are mutually exclusive & ~~exclusi~~ Exhaustive

False

~~$A' \cap B' \cap C \neq \phi$~~

~~$\Rightarrow B \cap A \cap C = \phi$~~

only for 2

$A = B'$

$A' = (B')'$

$A' = B$

$A' \cap B' = B \cap A = \phi$

$A' \cap C = B \cap C \neq \phi$ Not MEE

$B' \cap C$

Probability : Axiometic Approach to Probability

- Probability of the Event (E)

$$= P(E) = \frac{\text{no. of outcomes favourable to } E}{\text{total possible outcomes}}$$

- Conditions:

S (sample Space)

(i)	$0 \le P(E_1) \le 1$
(ii)	$P(E_1) + P(E_2) + P(E_3) \dots + P(E_n) = P(S) = 1$

$S = \{a_1, a_2, a_3, \dots, a_n\}$

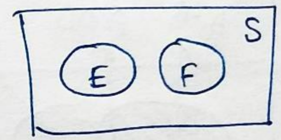
$E_1 = \{a_1\}$ Simple Event

$E_2 = \{a_2\}$ simple —

$\vdots$

$E_n = \{a_n\}$

- Mutally Exclusive Events: E & F

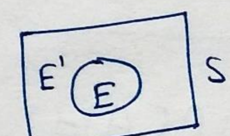


$E \cap F = \phi = \{\ \}$

$P(E \cap F) = 0$

P(both E & F) = 0

- Complementary Events.

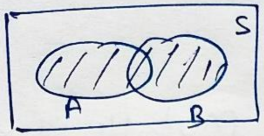


$E \cup E' = S$

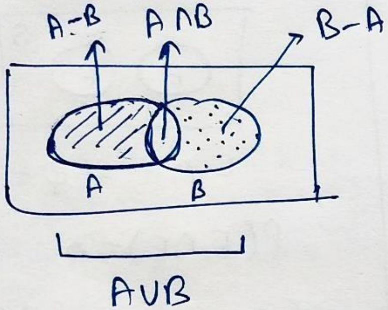
$P(E \cup E') = P(S) = 1$

$\Rightarrow P(E) + P(E') = 1$

$$\boxed{P(E') = 1 - P(E)}$$

- 

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- 

$$P(A \cup B) = P(A-B) + P(A \cap B) + P(B-A)$$

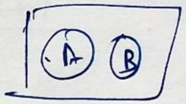
$$P(A) = \underline{P(A-B)} + P(A \cap B)$$

$$P(A \cap B') = P(A-B) = P(A) - P(A \cap B)$$

$$P(A' \cap B) = P(B-A) = P(B) - P(A \cap B)$$

- If A & B are mutually exclusive events then:

$A \cap B = \phi$



$$P(A \cup B) = P(A) + P(B) - \underline{P(A \cap B)}$$

0

$$\boxed{P(A \cup B) = P(A) + P(B)}$$

- **De Morgan's Law:**

$$\Rightarrow P((A \cup B)') = P(A' \cap B')$$

$$= 1 - P(A \cup B)$$

Question

$$\Rightarrow P((A \cap B)') = P(A' \cup B')$$

$$= 1 - P(A \cap B)$$

Exercise - 14.3

Q.1 Sample Space = $S = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7\}$

(a) $0 \le P(\omega_i) \le 1$ ✓

$P(\omega_1) + P(\omega_2) \cdots + P(\omega_7)$

$\sum_{i=1}^{7} P(\omega_i) = 0.1 + 0.01 + 0.05 + 0.03 + 0.01 + 0.2 + 0.6$

$= 1.00$

$\sum P(\omega_i) = 1$ ✓

Valid

(b) $\frac{1}{7}$ $\frac{1}{7}$ $\frac{1}{7}$ $\frac{1}{7}$ $\frac{1}{7}$ $\frac{1}{7}$ $\frac{1}{7}$

$0 \le \underbrace{P(\omega_i)}_{\frac{1}{7}} \le 1$ ✓

Valid.

$\sum_{i=1}^{7} P(\omega_i) = 7 \times \frac{1}{7} = 1$ ✓

(c) 0.1 0.2 0.3 0.4 0.5 0.6 0.

$0 \le P(\omega_i) \le 1$ ✓

$\sum_{i=1}^{7} P(\omega_i) = 2.8 \ne 1$ ✗

not valid

(d) −0.1 0.2 0.3 0.4 −0.2 0.1 0.3

Negative

$P(\omega_i) = -0.1$ Not possible.

Not valid.

(e) $\frac{1}{14}$ $\frac{2}{14}$ $\frac{3}{14}$ $\frac{4}{14}$ $\frac{5}{14}$ $\frac{6}{14}$ $\frac{15}{14}$

$\frac{15}{14} > 1$ $\quad P(\omega_7) > 1$

$0 \le P(\omega_i) \le 1$ ✗

Not valid.

Q.2

H/T H/T

$S = \{HH, HT, TH, TT\}$

P(atleast one tail)

$= \frac{3}{4}$ ← HT, TH, TT

Q.3 a die 

$S = \{1, 2, 3, 4, 5, 6\}$ ← total

(i) $P(\text{Prime no.}) = \frac{3}{6}$ ← 2,3,5 ; 6 ← total

$= \frac{1}{2}$

(ii) $P(\text{number} \ge 3) = \frac{4}{6}$ ← 3,4,5,6 ; 6 ← total

$= \frac{2}{3}$

(iii) $P(\text{no.} \le 1) = \frac{1}{6}$ ← $\{1\}$

(iv) $P(\text{no.} > 6) = \frac{0}{6}$ ← no. $\ngtr 6$

$= 0$

impossible event

(v) $P(\text{no.} < 6) = \frac{5}{6}$ ← $\{1, 2, 3, 4, 5\}$

Q.4 A card is selected from 52 cards.

any one of the 52 cards can come here.

(i) Sample space = Set of all Cards.

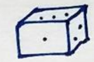
$n(S) = 52$

(ii) $P(\text{ace of spade}) = \dfrac{1}{52} \leftarrow \text{total}$

(iii) $P(\text{ace}) = \dfrac{4}{52} = \dfrac{1}{13}$

$P(\text{Black card}) = \dfrac{26}{52} = \dfrac{1}{2}$

Q.5

Coin	Die
	
1 / 6	1, 2, 3, 4, 5, 6

$\text{Value}_1 + \text{Value}_2$

(i) Sum = 3

$S = \begin{Bmatrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{Bmatrix}$

$P(\text{sum} = 3) = \dfrac{1 \leftarrow (1,2)}{12} \leftarrow \text{total}$

(ii) Sum = 12

$P(\text{Sum} = 12) = \dfrac{1 \leftarrow (6,6)}{12 \leftarrow \text{total.}}$

Q.6 4 Men , 6 Women

$\underbrace{\text{4 Men , 6 Women}}_{\text{1 person}}$

$$P(\text{woman}) = \frac{6 \leftarrow \text{women}}{10 \leftarrow \text{total}}$$

$$= \frac{3}{5}.$$

Q.7

H H H H

T T T T

S = { HHHH, HHHT, HHTH, HTHH, THHH,
HHTT, HTHT, THHT, HTTH, THTH,
TTHH, HTTT, THTT, TTHT, TTTH,
TTTT }

H → +1 ₹

T → −1.5 ₹

Different types of amounts

HHHH → 4

HHHT → 1.5
1 1 1 −1.5

HHTT → −1
1 1 −1.5 −1.5
2 −3

HTTT → −3.5
1 − 1.5×3

TTTT → −6
−(1.5) × 4

Different amounts
= {4, 1.5, −1, −3.5, −6}

$$P(\text{amt} = 4) = \frac{1 \leftarrow \text{HHHH}}{16}$$

$P(1.5) = \frac{4}{16} = \frac{1}{4}$

HHHT
HHTH
HTHH
THHH

$P(-1) = \frac{6}{16} = \frac{3}{8}$

(HHTT
⋮)

$P(-3.5) = \frac{4}{16} = \frac{1}{4}$

(HTTT, ...)

$P(-6) = \frac{1}{16}$

TTTT

Q.8 H/T H/T H/T

Sample Space $= S = \{$HHH, HHT, HTH, THH, HTT, THT, TTH, TTT$\}$

$n(S) = 8$

(i) $P(3H) = \frac{1}{8}$ ← HHH

(ii) $P(2H) = \frac{3}{8}$

(iii) $P(\text{atleast } 2H) = \frac{\overset{2H}{3} + \overset{3H}{1}}{8} = \frac{4}{8} = \frac{1}{2}$

atleast 2H ↓ 2H या 3H

(iv) $P(\text{atmost } 2H) = \frac{\overset{0H}{1} + \overset{1H}{3} + \overset{2H}{3}}{8} = \frac{7}{8}$

atmost 2H ↓ 0H, 1H, 2H

0H ↓ TTT

(V) $P(\text{no head}) = \frac{1}{8}$ ← TTT

(VI) $P(3T) = \frac{1}{8}$ ← TTT

(VII) $P(\text{exactly } 2T) = \frac{3}{8}$ ← TTH, HTT, THT

(VIII) $P(\text{No tail}) = \frac{1}{8}$ ← HHH

(IX) $P(\text{atmost } 2T) = \frac{1+3+3}{8} = \frac{7}{8}$

0T (HHH), 1T, 2T

Q.9 Event 'A'

$P(A) = \frac{2}{11}$

$P(\text{not } A) = P(A') = 1 - \frac{2}{11}$ (total, A)

(complementary event)

$= \frac{9}{11}$

Q.10 ASSASSINATION

Vowels → {A, A, A, I, I, O} → 6

Consonants → {S, S, S, S, N, T, N} → 7

total = 13

$P(\text{vowel}) = \frac{6}{13}$ ✓

$P(\text{consonant}) = \frac{7}{13}$ ✓

$\boxed{Q.11}$

Box

1, 2, 3, . . .
. . . , 20 → 6 no.

"Committee has special 6 numbers" ← Prize

→ 2, 7, 8, 10, 13, 15

Total No. of ways to select 6 numbers out of $20 = {}^{20}C_6$

$$= \frac{20!}{14! \times 6!} = \frac{\cancel{20} \times 19 \times \cancel{18} \times 17 \times \cancel{16} \times 15 \times \cancel{14!}}{\cancel{14!}\ \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1}$$

$= 19 \times 17 \times 8 \times 15$

$= 38760$

To win the prize money a person has one way to select only those special numbers.

Favourable cases = 1

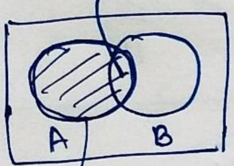
$$P = \frac{1}{38760} \begin{matrix} \leftarrow \text{win} \\ \leftarrow \text{total} \end{matrix}$$

Q.12 Consistently defined → Yes ? / No ?

(i) $P(A) = 0.5$

$P(B) = 0.7$

$P(A \cap B) = 0.6$ ← common.



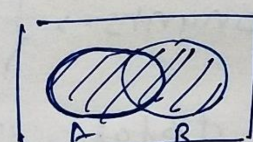
0.5

$P(A) < P(A \cap B)$

∴ Not Valid.

(ii) $P(A) = 0.5$

$P(B) = 0.4$



$P(A \cup B) = 0.8$

Yes

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.5 + 0.4 - 0.8 = 0.1$ ✓

Q.13 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\boxed{P(A \cup B) + P(A \cap B) = P(A) + P(B)}$$

	$P(A)$	$P(B)$	$P(A \cap B)$	$P(A \cup B)$
(i)	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{15}$	$\boxed{\frac{7}{15}}$
(ii)	0.35	$\boxed{0.5}$	0.25	0.6
(iii)	0.5	0.35	$\boxed{0.25}$	0.6

$\underbrace{\qquad}_{+}$ = $\underbrace{\qquad}_{+}$

$\frac{1}{3} + \frac{1}{5} = \frac{1}{15} + x$

$\frac{8}{15} = \frac{1}{15} + x$

Q.14 $P(A) = \frac{3}{5}$, $P(B) = \frac{1}{5}$

$P(A \text{ or } B)$ Mutually Exclusive events.

$\downarrow$

$P(A \cup B)$ $\boxed{P(A \cap B) = 0}$

$\downarrow$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{3}{5} + \frac{1}{5}$ 0

$= \frac{4}{5}$

Q.15 $P(E) = \frac{1}{4}$

$P(F) = \frac{1}{2}$

$P(E \text{ and } F) = \frac{1}{8} = P(E \cap F)$

(i) $P(E \text{ or } F) = P(E \cup F)$

$= P(E) + P(F) - P(E \cap F)$

$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{2+4-1}{8}$

$P(E \cup F) = 5/8$

(ii) $P(\text{not } E \text{ and } \text{not } F)$

$= P(E' \cap F')$

$= P[(E \cup F)']$ De Mor. Law

$= 1 - P(E \cup F)$

$= 1 - 5/8 = \frac{3}{8}$

Q.17 $P(A) = 0.42$

$P(B) = 0.48$

$P(A \text{ and } B) = 0.16 = P(A \cap B)$

(i) $P(\text{not } A) = P(A')$

$= 1 - P(A)$

$= 1 - 0.42 = 0.58$

(ii) $P(\text{not } B) = P(B')$

$= 1 - P(B)$

$= 1 - 0.48 = 0.52$

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \text{ or } B)$

$= 0.42 + 0.48 - 0.16$

$= 0.90 - 0.16$

$= 0.74$

Q.16 $P(\text{not } E \text{ or not } F) = 0.25$

Mutually exclusive Events. → Yes? $P(E \cap F) = 0$

No? $P(E \cap F) \neq 0$

$P(E' \cup F') = 0.25$

$\Rightarrow P(E \cap F)' = 0.25$

$\Rightarrow 1 - P(E \cap F) = 0.25$

$\Rightarrow 1 - 0.25 = P(E \cap F)$

$\Rightarrow \underbrace{P(E \cap F) = 0.75 \neq 0}$

Not M.E.E.

Q.18

40 % $\longrightarrow$ Maths (M) $\rightarrow P(M) = 0.4$

30 % $\longrightarrow$ Bio (B) $\longrightarrow P(B) = 0.3$

10 % $\longrightarrow$ Both $(B \cap M) \rightarrow P(B \cap M) = 0.1$

$$P(M \text{ or } B) = P(M \cup B) = P(M) + P(B) - P(M \cap B)$$

$$P(M \cup B) = 0.4 + 0.3 - 0.1$$

$$= 0.6$$

Q.19

$P(E_1) = 0.8$

$P(E_2) = 0.7$

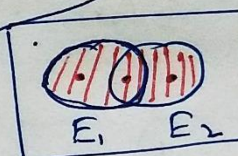
कम से कम

P(passing atleast one of them) = 0.95

1 2

P(Both) = ?

$P(E_1 \cap E_2) = ?$



$E_1 \cup E_2$

$$P(E_1 \cup E_2) = 0.95$$

$$\Rightarrow P(E_1) + P(E_2) - P(E_1 \cap E_2) = 0.95$$

$$\Rightarrow 0.8 + 0.7 - \underline{P(E_1 \cap E_2)} = 0.95$$

$$\Rightarrow \boxed{P(E_1 \cap E_2) = 0.55}$$

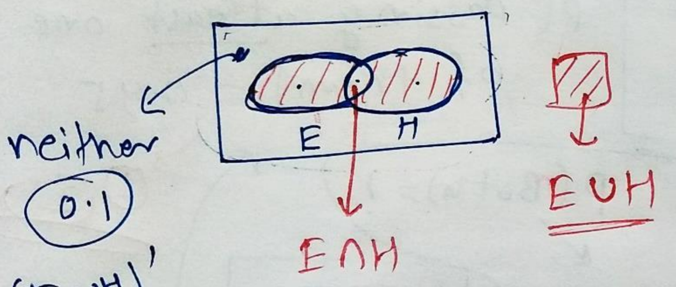
Q. 20

$P(E \cap H) = 0.5$

$P(\text{neither}) = \underline{0.1 = P((E \cup H)')}$

$P(E) = 0.75$

$P(H) = ?$



$P((E \cup H)') = 0.1$

$\Rightarrow 1 - P(E \cup H) = 0.1$

$\Rightarrow 1 - 0.1 = P(E \cup H)$

$\Rightarrow P(E \cup H) = 0.9$

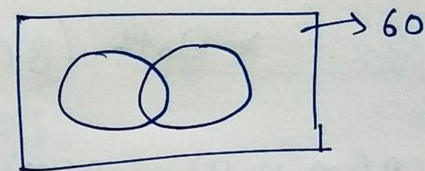
$\Rightarrow P(E) + P(H) - P(E \cap H) = 0.9$

$\Rightarrow 0.75 + \underline{P(H)} - 0.5 = 0.9$

$\Rightarrow 0.25 + \underline{P(H)} = 0.9$

$\Rightarrow \boxed{P(H) = 0.65}$

(21)



NSS $\rightarrow$ S

NCC $\rightarrow$ C

total students $= 60 = n(T)$

$n(S) = 32, \; n(C) = 30$

$n(S \cap C) = 24$

$P(S) = \dfrac{n(S)}{n(T)} = \dfrac{32}{60}$

$P(C) = \dfrac{n(C)}{n(T)} = \dfrac{30}{60}$

$P(S \cap C) = \dfrac{n(S \cap C)}{n(T)} = \dfrac{24}{60}$

(i) Prob. of NCC or NSS

$$P(C \cup S) = P(C) + P(S) - P(C \cap S)$$

$$= \frac{30}{60} + \frac{32}{60} - \frac{24}{60} = \frac{38}{60}$$

(ii) Prob. of neither NCC nor NSS $\quad \underline{P(C' \cap S')} = \underline{P((C \cup S)')}$

$$\underline{P((C \cup S)')} = 1 - P(C \cup S)$$

$$= 1 - \frac{38}{60} = \frac{22}{60}$$

(iii) Prob of NSS but not NCC.

$$P(S \cap C') = P(S - C)$$

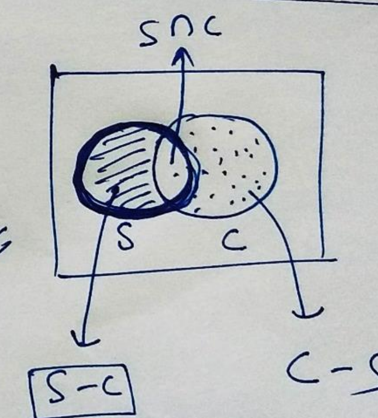
$$= P(S) - P(S \cap C)$$

$$= \frac{32}{60} - \frac{24}{60}$$

$$= 8/60$$

?

$$P(S) = P(S-C) + P(S \cap C)$$



Miscellaneous Exercise - 14.4

- $^nC_r = \frac{n!}{(n-r)!\,r!}$ = (no. of ways to select r objects out of n- different objects.)

- Multiplication Rule of Counting

Project $\rightarrow m \times n$ No. of ways

$\overbrace{\boxed{m} \text{ and } \boxed{n}}^{\text{Project}}$

Step -I ✓ Step. II ✓

$P = \frac{\text{fav.}}{\text{total}}$

Q.1 10 red, 20 Blue, 30 Green $\rightarrow$ 5 Balls

Total no. of Cases = $^{60}C_5$ ← total

(i) all are blue.

5 → Blue ← total Blue 20

fav. Cases = $^{20}C_5$

$$\boxed{P(\text{all blue}) = \frac{^{20}C_5}{^{60}C_5}}$$

(ii) atleast one green.

5 Balls

total

5 O , 0 G ×
4 O , 1 G
3 O , 2 G ✓
2 O , 3 G
1 O , 4 G
0 O , 5 G

P(atleast one green)

= 1 − P(no green)

total prob.

0 Green

$= 1 - \dfrac{{}^{30}C_5}{{}^{60}C_5}$

from Red → 10

& Blue → 20

← total

Q.2

52 Playing Card → 4 cards

3 Diamond & 1 Spade

Total no. of ways = ${}^{52}C_4$ ✓

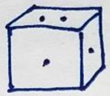
3 Diamond & 1 Spade

total ♢ → ${}^{13}C_3$ × ${}^{13}C_1$ (13 → ♠)

Fav. Cases = $({}^{13}C_3 \times {}^{13}C_1)$ ✓

$$P(3D \,\&\, 1S) = \frac{{}^{13}C_3 \times {}^{13}C_1}{{}^{52}C_4}$$

Q.3 Die (पासा)



$\{\underline{1,1}, \underline{2,2,2}, \underline{3}\}$

(i) $P(2) = \dfrac{3}{6}$ ← fav. 2 ← total

$= \dfrac{1}{2}$

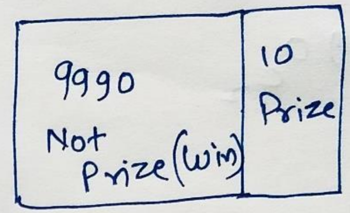
(ii) $P(1 \text{ or } 3) = \dfrac{3}{6} = \dfrac{1}{2}$

$\{1,1\}$ $\{3\}$

(iii) $P(\text{not } 3) = \dfrac{5}{6}$

$\{1,1,2,2,2\}$

Q.4 total tickets ← 10000



$^nC_r = \dfrac{n!}{(n-r)!\,r!}$

$^nC_1 = n$

P(not winning) = ?

(a) if we ~~by~~ buy 1 ticket

Total = $^{10000}C_1 = 10000$

favourable (not Prize) = $^{9990}C_1 = 9990$

$P = \dfrac{999\cancel{0}}{1000\cancel{0}} = \dfrac{999}{1000}$

(b) if we buy 2 tickets

total = $^{10000}C_2$

fav. = $^{9990}C_2$

$$P(\text{not winning}) = \frac{^{9990}C_2}{^{10000}C_2}$$ ✓

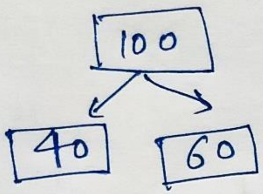
(c) if we ~~b~~ buy 10 tickets.

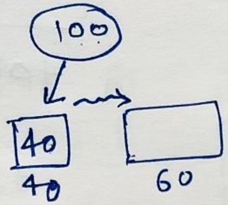
total = $^{10000}C_{10}$

fav. = $^{9990}C_{10}$

$$P(\text{not winning}) = \frac{^{9990}C_{10}}{^{10000}C_{10}}$$ ✓

Q.5



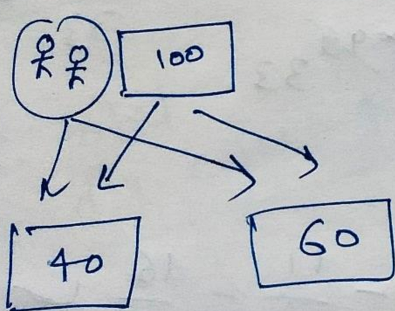


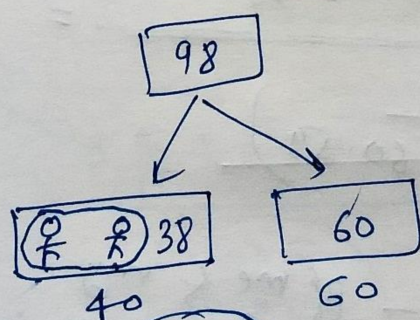
total $= {}^{100}C_{40}$ या ${}^{100}C_{60} = \frac{100!}{40!\,60!}$

selection

$\boxed{\text{Property } {}^{n}C_{r} = {}^{n}C_{n-r}}$

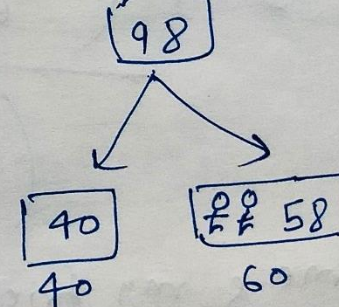
(i) me & my friend are in Same Section.





(+)

or
या



${}^{98}C_{38} = {}^{98}C_{60} + {}^{98}C_{40}$ = fav. Cases

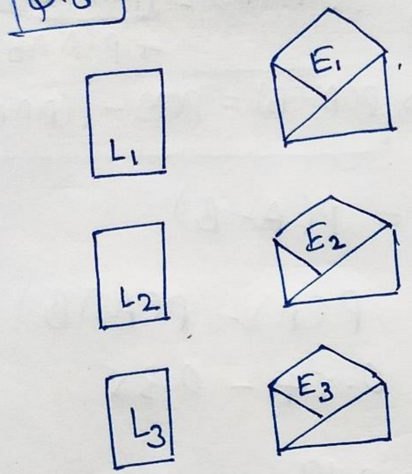
P(me & my friend are is same section) $= \dfrac{{}^{98}C_{38} + {}^{98}C_{40}}{{}^{100}C_{40}} = \dfrac{\left(\dfrac{98!}{38!\,60!}\right) + \left(\dfrac{98!}{40!\,58!}\right)}{\left[\dfrac{100!}{40!\,60!}\right]}$

$$= \frac{\left[\dfrac{98!}{38!\,60.59.58!}\right] + \left[\dfrac{98!}{40.39.38!\,58!}\right]}{\left[\dfrac{100\times 99\times 98!}{40.39.38!\;60.59.58!}\right]} = \frac{\cancel{\dfrac{98!}{38!\,58!}}\left[\dfrac{1}{60.59} + \dfrac{1}{40.39}\right]}{\cancel{\dfrac{98!}{38!\,58!}}\cdot\left[\dfrac{100\times 99}{40.39.60.59}\right]}$$

$$= \frac{\left(\dfrac{40.39 + 60.59}{\cancel{60.59.\,40.39}}\right)}{\left(\dfrac{100\times 99}{\cancel{40.39.60.59}}\right)} = \frac{\overset{17}{\cancel{5100}}}{\underset{33}{\cancel{100\times 99}}} = \frac{17}{33}$$

(b) P(me & my friend) go in different section $= 1 - P\left(\begin{array}{c}\text{me \& my}\\ \text{friend go in}\\ \text{same section}\end{array}\right) = 1 - \dfrac{17}{33} = \dfrac{16}{33}$

Q.6



1 letter → 1 enevelope

Total randomly fill $= 3 \times 2 \times 1 = 6$

Answer.

$P = \dfrac{4 \leftarrow \text{fav.}}{6 \leftarrow \text{total}}$

$= \dfrac{2}{3}$ ✓

Favourable Cases.

atleast one letter into proper envelope

कम से कम

1 letter → Proper Envelope

$L_1 \to E_1$, $L_2 \to E_3$, $L_3 \to E_2$ या $L_2 \to E_2$ या $L_3 \to E_3$

3 Cases.

2 letters → Proper Envelope

या

3 letter

$L_1 \to E_1$
$L_2 \to E_2$
$L_3 \to E_3$

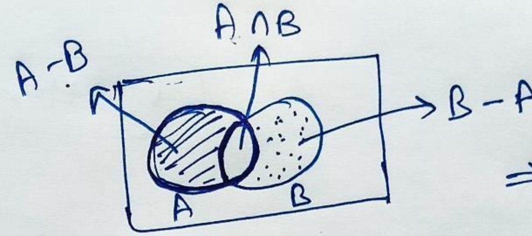
1 - case

$\therefore$ total fav. cases = 4

Q.7 $P(A) = 0.54$

$P(B) = 0.69$

$P(A \cap B) = 0.35$



$\therefore P(A) = \boxed{P(A-B)} + P(A \cap B)$

$\Rightarrow \boxed{P(A-B) = P(A) - P(A \cap B)}$

(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.54 + 0.69 - 0.35$

$= 0.54 + 0.34 = 0.88$

(ii) $P[A' \cap B'] = P[(A \cup B)']$

(De-Morgan's Law)

$= 1 - P(A \cup B)$

$= 1 - 0.88$

$= 0.12$

(iii) $P(A \cap B') = P(A-B)$

$= P(A) - P(A \cap B)$

$= 0.54 - 0.35$

$= 0.19$

(iv) $P(B \cap A') = P(B-A)$

$= P(B) - P(A \cap B)$

$= 0.69 - 0.35$

$= 0.34$

Q.8

either — or —

$P(\underbrace{\text{male}}_{A} \cup \underbrace{\text{over 35 years}}_{B}) = P(\text{male}) + P(\text{over 35 years})$

$- P(\text{male} \cap \text{over 35 years})$

$P(\text{male}) = \dfrac{3}{5}$ ← male / ← total

$P(\text{over 35 years}) = \dfrac{2}{5}$

$P(\text{male} \cap \text{over 35 years}) = \dfrac{1}{5}$

↓

both

put

$= \dfrac{3}{5} + \dfrac{2}{5} - \dfrac{1}{5}$

$= \dfrac{4}{5}$

$\boxed{Q.9}$ (i) Digits are repeated

4 digit > 5000

0/1/3/5/7

total

5 _ _ _

No. of ways → 5 . 5 . 5 = 5.5.5 − 1 (5000 वाला)

= 125 − 1

= 124

0/1/3/5/7

7 _ _ _

No. of ways → 5 . 5 . 5 = 5.5.5

= 125

total = 249

P(No. divisible by 5) = $\frac{\text{fav.}}{\text{total}} = \frac{99}{249} = \frac{33}{83}$

fav. cases → No. divisible by 5 → 0/5 last Digit

0/1/3/5/7   0/5   (5000 वाला)

5			

5 5 2

= 5.5.2 − 1

= 49

0/5

7			

5 . 5 . 2 = 50

total fav. = 99

Q.9 (ii) The repitition of digits is not allowed.

4 digit > 5000

0 | 1 | 3 | 5 | 7

total.

0 | 1 | 3 | 7

5 _ _ _

No. of ways → 4 . 3 . 2 = 24

0 | 1 | 3 | 5

7 _ _ _

No. of ways → 4 . 3 . 2 = 24

total = 48

$$P\left(\text{No. is divisible by } 5\right) = \frac{\text{fav.}}{\text{total}} = \frac{\cancel{18}^{\,3}}{\cancel{48}_{\,8}} = \frac{3}{8}$$

Repitition is not allowed

fav. cases → No. is divisible by 5 → last Digit 0/5

1/3/7 0

5	_	_	_
	3	2	1

= 6

0/5

7	_	_	_
	3	2	2

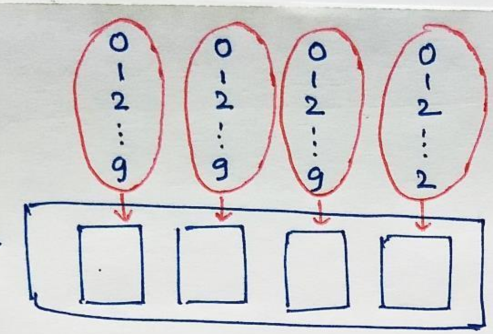
= 12

1/3/Remaining

total fav. = 18

Q.10

Lock →



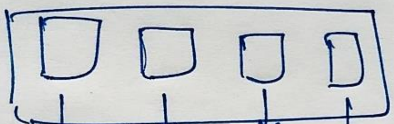
No repeats

$$P\left(\begin{array}{c}\text{get the right} \\ \text{sequence to} \\ \text{open the lock}\end{array}\right) = \frac{1}{5040}$$

fav.

total

Total =



$10 \times 9 \times 8 \times 7$

0|1|2|...|9

$$\text{total} = 10 \times 9 \times 8 \times 7$$
$$= 10 \times 9 \times 56$$
$$= 5040$$

Note: To open a lock, we always have only one code (password).

$\therefore$ $\boxed{\text{No. of favourable cases} = 1}$