CBSE Class 11 Mathematics

Important Questions

Chapter 12

Introduction to Three Dimensional Geometry

1 Marks Questions

1. Name the octants in which the following lie. (5,2,3)

Ans. I

2. Name the octants in which the following lie. (-5,4,3)

Ans. II

3. Find the image of (-2,3,4) in the y z plane

Ans. (2, 3, 4)

4. Find the image of (5,2,-7) in the xy plane

Ans. (5, 2, 7)

5. A point lie on X –axis what are co ordinate of the point

Ans. $(a, 0, 0)$

6. Write the name of plane in which x axis and y - axis taken together.

Ans. XY Plane

7. The point $(4, -3, -6)$ lie in which octants

Ans. $VIII$

8. The point $(2, 0, 8)$ **lie in which plane**

Ans. XZ

9. A point is in the XZ plane. What is the value of y co-ordinates?

Ans. Zero

10. What is the coordinates of XY plane

Ans. $(x, y, 0)$

11. The point $(-4, 2, 5)$ **lie in which octants.**

Ans. II

12. The distance from origin to point (a, b, c) **is:**

Ans. $\sqrt{a^2 + b^2 + c^2}$

CBSE Class 12 Mathematics

Important Questions

Chapter 12

Introduction to Three Dimensional Geometry

4 Marks Questions

1.Given that P(3,2,-4), Q(5,4,-6) and R(9,8,-10) are collinear. Find the ratio in which Q divides PR

Ans. Suppose Q divides PR in the ratio λ :1. Then coordinator of Q are

$$\left(\frac{9\lambda+3}{\lambda+1}, \frac{8\lambda+2}{\lambda+1}, \frac{-10\lambda-4}{\lambda+1}\right)$$

But, coordinates of Q are (5,4,-6). Therefore

$$\frac{9\lambda+3}{\lambda+1}=5, \frac{8\lambda+2}{\lambda+1}=4, \frac{-10\lambda-4}{\lambda+1}=6$$

These three equations give

$$\lambda=\frac{1}{2}.$$

So Q divides PR in the ratio $\frac{1}{2}:1$ or 1:2

2. Determine the points in xy plane which is equidistant from these point A (2,0,3) B(0,3,2) and C(0,0,1)

Ans. We know that Z- coordinate of every point on xy-plane is zero. So, let $P(x, y, 0)$ be a point in xy-plane such that PA=PB=PC

Now, PA=PB

$\Rightarrow PA^2 = PB^2$

$$\Rightarrow (x-2)^2 + (y-0)^2 + (0-3)^2 = (x-0)^2 + (y-3)^2 + (0-2)^2$$

$$\Rightarrow 4x - 6y = 0 \text{ or } 2x - 3y = 0 ....(i)$$

$PB = PC$

$$\Rightarrow PB^2 = PC^2$$

$$\Rightarrow (x-0)^2 + (y-3)^2 + (0-2)^2 = (x-0)^2 + (y-0)^2 + (0-1)^2$$

$$\Rightarrow -6y + 12 = 0 \Rightarrow y = 2.........(ii)$$

Putting $y = 2$ in (i) we obtain $x = 3$

Hence the required points (3,2,0).

3. Find the locus of the point which is equidistant from the point A(0,2,3) and B(2,-2,1)

Ans. Let $P(x, y, z)$ be any point which is equidistant from A(0,2,3) and B(2,-2,1). Then

PA=PB

$\Rightarrow PA^2 = PB^2$

$$\Rightarrow \sqrt{(x-0)^2 + (y-2)^2 + (2-3)^2} = \sqrt{(x-2)^2 + (y+2)^2 + (z-1)^2}$$

$$\Rightarrow 4x - 8y - 42 + 4 = 0 \text{ or } x - 2y - 2 + 1 = 0$$

4. Show that the points A(0,1,2) B(2,-1,3) and C(1,-3,1) are vertices of an isosceles right angled triangle.

Ans. We have

$$AB = \sqrt{(2-0)^2 + (-1-1)^2 (+3-2)^2} = \sqrt{4+4+1} = 3$$

$$BC=\sqrt{(1-2)^2+(-3+1)^2+(1-3)^2}=\sqrt{1+4+4}=3$$

And $CA=\sqrt{(1-0)^2+(-3-1)^2+(1-2)^2}=\sqrt{1+16+1}=3\sqrt{2}$

Clearly AB=BC and $AB^2+BC^2=AC^2$

Hence, triangle ABC is an isosceles right angled triangle.

5. Using section formula, prove that the three points A(-2,3,5), B(1,2,3), and C(7,0,-1) are collinear.

Ans. Suppose the given points are collinear and C divides AB in the ratio $\lambda:1$.

Then coordinates of C are

$$\left(\frac{\lambda-2}{\lambda+1},\frac{2\lambda+3}{\lambda+1},\frac{3\lambda+5}{\lambda+1}\right)$$

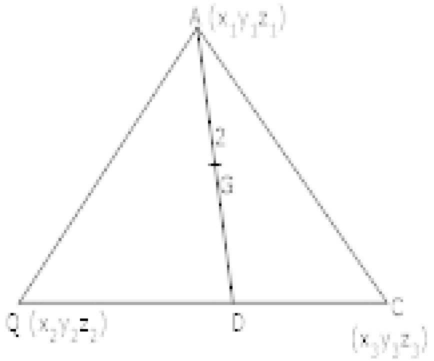
But, coordinates of C are (3,0,-1) from each of there equations, we get $\lambda=\frac{3}{2}$

Since each of there equation give the same value of V. therefore, the given points are collinear and C divides AB externally in the ratio 3:2.

6. Show that coordinator of the centroid of triangle with vertices A($x_1 y_1 z_1$), B($x_2 y_2 z_2$), and C($x_3 y_3 z_3$) is $\left[\frac{x_1+y_1+z_1}{3},\frac{y_1+y_2+y_3}{3},\frac{z_1+z_2+z_3}{3}\right]$

Ans. Let D be the mid point of AC. Then

Coordinates of D are $\left(\frac{x_2+x_3}{2},\frac{y_2+y_3}{2},\frac{z_2+z_3}{2}\right)$.



Let G be the centroid of ΔABC. Then G, divides AD in the ratio 2:1. So coordinates of D are

$$\left(\frac{1.x_1 + 2\frac{(x_2+x_3)}{2}}{1+2}, \frac{1.y_1 + 2\left(\frac{y_2+y_3}{2}\right)}{1+2}, \frac{1.z_1 + 2\left(\frac{z_2+z_3}{2}\right)}{1+2} \right)$$

i.e. $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3} \right)$

7. Prove by distance formula that the points $A(1,2,3)$, $B(-1,-1,-1)$ and $C(3,5,7)$ are collinear.

Ans. Distance

$$|AB| = \sqrt{(-1-1)^2 + (-1-2)^2 + (-1-3)^2} = \sqrt{4+9+16} = \sqrt{29}$$

Distance

$$|BC| = \sqrt{(3+1)^2 + (5+1)^2 + (7+1)^2} = \sqrt{16+36+64} = 2\sqrt{29}$$

Distance

$$|AC| = \sqrt{(3-1)^2 + (5-2)^2 + (7-3)^2} = \sqrt{4+9+16} = \sqrt{29}$$

$$\therefore \ |BC| = |AB| + |Ac|$$

$\therefore$ The paints A.B.C. are collinear.

8. Find the co ordinate of the point which divides the join of $P(2,-1,4)$ **and** $Q(4,3,2)$ **in the ratio** $2:5$ (i) **internally** (ii) **externally**

Ans. Let paint $R(x,y,z)$ be the required paint.

(i) For internal division

$$x=\frac{2\times4+5\times2}{2+5}=\frac{8+10}{7}=\frac{18}{7}$$

$$y=\frac{2\times3+5\times-1}{2+5}=\frac{6-5}{7}=\frac{1}{7}$$

$$z=\frac{2\times2+5\times4}{2+5}=\frac{4+20}{7}=\frac{24}{7}$$

$\therefore$ Required paint $R\left(\frac{18}{7},\frac{1}{7},\frac{24}{7}\right)$

(ii) For external division.

$$x=\frac{2\times4-5\times2}{2-5}=\frac{8-10}{-3}=\frac{-2}{-3}=\frac{2}{3}$$

$$y=\frac{2\times3-5\times-1}{2-5}=\frac{6+5}{-3}=\frac{11}{-3}$$

$$z=\frac{2\times2-5\times4}{2-5}=\frac{4-20}{-3}=\frac{-16}{-3}=\frac{16}{3}$$

$\therefore$ Required point $R\left(\frac{2}{3},\frac{-11}{3},\frac{16}{3}\right)$

9. Find the co ordinate of a point equidistant from the four points

$0(0,0,0)$, $A(a,0,0)$, $B(0,b,0)$ **and** $C(0,0,c)$

Ans. Let $P(x,y,z)$ be the required point

According to condition

$OP = PA = PB = PC$

Now $OP = PA$

$\Rightarrow OP^2 = PA^2$

$\Rightarrow x^2 + y^2 + z^2 = (x-a)^2 + (y-0)^2 + (z-0)^2$

$\Rightarrow x^2 + y^2 + z^2 = x^2 - 2ax + a^2 + y^2 + z^2$

$2ax = a^2$

$\therefore x = \frac{a}{2}$

Similarly $OP = PB$

$\Rightarrow y = \frac{b}{2}$

$A(x,y,z,)$ $B(x_2,y_2,z_2)$ and $C(x_3,y_3,z_3)$ D,E and F are mid points of side $BC, CA,$ and AB respectively,

Then $\frac{x_1 + x_2}{2} = -1$

$x_1 + x_2 = -2......(1)$

$\frac{y_1 + y_2}{2} = 1$

$$y_1 + y_2 = 2 \ldots\ldots (2)$$

$$\frac{z_1 + z_2}{2} = -4$$

$$z_1 + z_2 = -8 \ldots\ldots (3)$$

$$\frac{x_2 + x_3}{2} = 1$$

$$x_2 + x_3 = 2 \ldots\ldots (4)$$

$$\frac{y_2 + y_3}{2} = 2$$

$$y2 + y3 = 4 \ldots\ldots (5)$$

$$\frac{z_2 + z_3}{2} = -3$$

$$z_2 + z_3 = -6 \ldots\ldots (6)$$

$$\frac{x_1 + x_3}{2} = 3$$

$$x_1 + x_3 = 6 \ldots\ldots (7)$$

$$\frac{y_1 + y_3}{2} = 0$$

$$y_1 + y_3 = 0 \ldots\ldots (8)$$

$$\frac{z_1 + z_3}{2} = 1$$

$$z_1 + z_3 = 2 \ldots\ldots (9)$$

Adding eq (1),(4) and (7) we get

$2\left(x_1+x_2+x_3\right)=-2+2+6$

Adding eq. (2),(5) and (8)

$2\left(y_1+y_2+y_3\right)=6$

$y_1+y_2+y_3=3......(11)$

And $OP=PC$

$\Rightarrow z=\frac{c}{2}$

Hence co-ordinate of $P\left(\frac{a}{2},\frac{b}{2},\frac{c}{2}\right)$

10. Find the ratio in which the join the $A\left(2,1,5\right)$ and $B\left(3,4,3\right)$ is divided by the plane $2x+2y-2z=1$ Also find the co-ordinate of the point of division

Ans. Suppose plane $2x+2y-2z=1$ divides $A\left(2,1,5\right)$ and $B\left(3,4,5\right)$ in the ratio $\lambda:1$ at pain C

Then co-ordinate of paint C

$\left(\frac{3\lambda+2}{\lambda+1},\frac{4\lambda+1}{\lambda+1},\frac{3\lambda+5}{\lambda+1}\right)$

$\because$ Point C lies on the plane $2x+2y-2z=1$

$\therefore$ Points C must satisfy the equation of plane

$2\left(\frac{3\lambda+2}{\lambda+1}\right)+2\left(\frac{4\lambda+1}{\lambda+1}\right)-2\left(\frac{3\lambda+5}{\lambda+1}\right)=1$

$\Rightarrow 8\lambda-4=\lambda+1$

$\Rightarrow \lambda = \frac{5}{7}$

$\therefore$ Required ratio 5:7

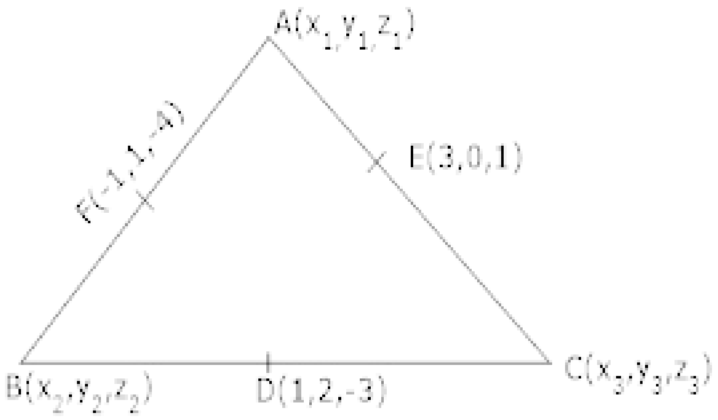
11. Find the centroid of a triangle, mid points of whose sides are $(1,2,-3)$, $(3,0,1)$ and $(-1,1,-4)$

Ans. Suppose co-ordinate of vertices of $\Delta\, ABC$ are

Adding eq. (3), (6) and (9)

$2(z_1 + z_2 + z_3) = -8 - 6 + 2$

$z_1 + z_2 + z_3 = -6 ......(12)$



Co-ordinate of centroid

$x = \frac{x_1 + x_2 + x_3}{3} = \frac{3}{3} = 1$

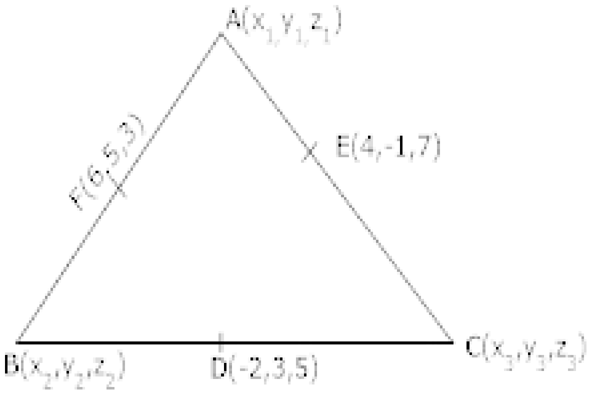
$y = \frac{y_1 + y_2 + y_3}{3} = \frac{3}{3} = 1$

$z = \frac{z_1 + z_2 + z_3}{3} = \frac{-6}{3} = -2$

$(1,\ 1, -2)$

12. The mid points of the sides of a ΔABC are given by $(-2,3,5)$, $(4,-1,7)$ and $(6,5,3)$ find the co ordinate of A, B and C

Ans. Suppose co-ordinate of point A,B,C, are (x_1,y_1,z_1), (x_2,y_2,z_2) and (x_3,y_3,z_3) respectively let D,E and F are mid points of side BC,CA and AB respectively



$$\therefore \frac{x_1+x_2}{2}=6$$

$$x_1+x_2=12 ......(1)$$

$$\frac{y_1+y_2}{2}=5$$

$$y_1+y_2=10 ......(2)$$

$$\frac{z_1+z_2}{2}=3$$

$$z_1+z_2=6 ......(3)$$

$$\frac{x_2+x_3}{2}=-2$$

$$x_2+x_3=-4 ......(4)$$

$$\frac{y_2+y_3}{2}=3$$

$$y_2+y_3=6 ......(5)$$

$$\frac{z_1+z_2}{2}=5$$

$$z_1+z_2=10\ldots\ldots(6)$$

$$\frac{x_1+x_3}{2}=4$$

$$x_1+x_3=8\ldots\ldots(7)$$

$$\frac{y_1+y_3}{2}=-1$$

$$y_1+y_3=-2\ldots\ldots(8)$$

$$\frac{z_1+z_3}{z}=7$$

$$z_1+z_3=14\ldots\ldots(9)$$

Adding eq. (1), (4) and (7)

$$2(x_1+x_2+x_3)=12-4+8$$

$$x_1+x_2+x_3=\frac{\cancel{16}}{\cancel{2}}=8\ldots\ldots(10)$$

Similarly $y_1+y_2+y_3=7\ldots\ldots(11)$

$$z_1+z_2+z_3=15\ldots\ldots(12)$$

Subtracting eq. (1), (4) and (7) from (10)

$$x_3=-4,\quad x_1=12,\quad x_2=0$$

Now subtracting eq. (2), (5) and (8) from (11)

$$y_3=-3,\quad y_1=1,\quad y_2=9$$

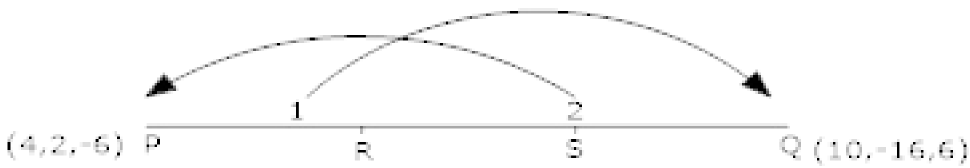
Similarly $z_3=9,\quad z_1=5,\quad z_2=1$

$\therefore$ co-ordinate of point A, B and C are

$A(12, 0, -4)$, $\quad B(1, 9, -3)$, and $C(5, 1, 9)$

13. Find the co-ordinates of the points which trisects the line segment PQ formed by joining the point $P(4, 2, -6)$ and $Q(10, -16, 6)$

Ans. Let R and S be the points of trisection of the segment PO. Then



$\therefore PR = RS = SQ$

$\Rightarrow 2PR = RQ$

$\Rightarrow \dfrac{PQ}{RQ} = \dfrac{1}{2}$

$\therefore$ R divides PQ in the ratio 1:2

$\therefore$ Co-ordinates of point

$$R\left[\frac{1(10)+2\times 4}{1+2}, \frac{1(-16)+2\times 2}{1+2}, \frac{1\times 6+2(-6)}{1+2}\right]$$

$= R(6, -4, -2)$

Similarly $PS = 2SQ$

$\Rightarrow \dfrac{PS}{SQ} = \dfrac{2}{1}$

$\therefore$ S divider PQ in the ratio 2:1

$\therefore$ co-ordinates of point S

$$\left[\frac{2(10)+1(4)}{1+2},\frac{2(-16)+1(2)}{1+2},\frac{2(6)+1(-6)}{1+2}\right]$$

$\therefore S(8,-10,2)$

14. Show that the point $P(1,2,3)$, $Q(-1,-2,-1)$, $R(2,3,2)$ and $S(4,7,6)$ taken in order form the vertices of a parallelogram. Do these form a rectangle?

Ans. Mid point of PR is $\left(\frac{1+2}{2},\frac{2+3}{2},\frac{3+2}{2}\right)$

i.e. $\left(\frac{3}{2},\frac{5}{2},\frac{5}{2}\right)$

also mid point of QS is $\left(\frac{-1+4}{2},\frac{-2+7}{2},\frac{-1+6}{2}\right)$

i.e. $\left(\frac{3}{2},\frac{5}{2},\frac{5}{2}\right)$

Then PR and QS have same mid points.

$\therefore$ PR and QS bisect each other. It is a Parallelogram.

Now $PR=\sqrt{(2-1)^2+(3-2)^2+(2-3)^2}=\sqrt{3}$ and

$QS=\sqrt{(4+1)^2+(7+2)^2+(6+1)^2}=\sqrt{155}$

$\therefore PR\neq QS$ diagonals an not equal

$\therefore PQRS$ are not rectangle.

15. A point R with x co-ordinates 4 lies on the line segment joining the points $P(2,-3,4)$ and $Q(8,0,10)$ find the co-ordinates of the point R

Ans. Let the point. R divides the line segment joining the point P and Q in the ratio $\lambda:1$, Then co-ordinates of Point R

$$\left[\frac{8\lambda+2}{\lambda+1}, \frac{-3}{\lambda+1}, \frac{10\lambda+4}{\lambda+1}\right]$$

The x co-ordinates of point R is 4

$$\Rightarrow \frac{8\lambda+2}{\lambda+1}=4 \quad , \lambda=\frac{1}{2}$$

$\therefore$ co-ordinates of point R

$$\left[4, \frac{-3}{\frac{1}{2}+1}, \frac{10\times\frac{1}{2}+4}{\frac{1}{2}+1}\right] \qquad \text{i.e.} (4, -2, 6)$$

16. If the points $P(1, 0, -6)$, $Q(-3, P, q)$ and $R(-5, 9, 6)$ are collinear, find the values of P and q

Ans. Given points

$P(1, 0, -6)$, $Q(-3, P, q)$ and $R(-5, 9, 6)$ are collinear

Let point Q divider PR in the ratio K:1

$\therefore$ co-ordinates of point $P\left(\frac{1-5K}{K+1}, \frac{0+9K}{K+1}, \frac{-6+6K}{K+1}\right)$

$Q(-3, P, q)$

$$\frac{1-5K}{K+1}=-3$$

$$1-5K=-3K-3$$

$$-2K=-4$$

$$K=\frac{-4}{-2}$$

$$K=2$$

$\therefore$ the value of P and q are 6 and 2.

17. Three consecutive vertices of a parallelogram ABCD are $A(3,-1,2)$, $B(1,2,-4)$ and $C(-1,1,2)$ find forth vertex D

Ans. Given vertices of 11gm ABCD

$A(3,-1,2)$, $B(1,2,-4)$, $C(-1,1,2)$

Suppose co-or dine of forth vertex $D(x,y,z)$

Mid point of $AC\left(\frac{3-1}{2},\frac{-1+1}{2},\frac{2+2}{2}\right)$

$=(1,0,2)$

Mid point of $BD\left(\frac{x+1}{2},\frac{y+2}{2},\frac{-4+z}{2}\right)$

Mid point of AC = mid point of BD

$$\frac{x+1}{2}=1\Rightarrow x=1$$

$$\frac{y+2}{2}=0\Rightarrow y=-2$$

$$\frac{-4+z}{2}=2 \Rightarrow z=8$$

Co-ordinates of point $D(1,-2,8)$

18. If A and B be the points $(3,4,5)$ and $(-1,3,7)$ respectively. Find the eq. of the set points P such that $PA^2+PB^2=K^2$ where K is a constant

Ans. Let co-ordinates of point P be

(x,y,z)

$PA^2=(x-3)^2+(y-4)^2+(z-5)^2$

$=x^2-6x+9+y^2-8y+16+z^2-10z+25$

$=x^2+y^2+z^2-6x-8y-10z+50$

$PB^2=(x+1)^2+(y-3)^2+(z-7)^2$

$=x^2+2x+1+y^2-6y+9+z^2-14+49$

$=x^2+y^2+z^2+2x-6y-14z+59$

$PA^2+PB^2=K^2$

$2(x^2+y^2+z^2)-4x-14y-24z+109=K^2$

$$x^2+y^2+z^2-2x-7y-12z=\frac{K^2-109}{2}$$

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Important Questions
Chapter 12
Introduction to Three Dimensional Geometry

6 Marks Questions

1. Prove that the lines joining the vertices of a tetrahedron to the centroids of the opposite faces are concurrent.

Ans. Let ABCD be tetrahedron such that the coordinates of its vertices are $A\left(x_1, y_1, z_1\right)$, $B\left(x_2, y_2, z_2\right)$, $C\left(x_3, y_3, z_3\right)$ and $D\left(x_4, y_4, z_4\right)$

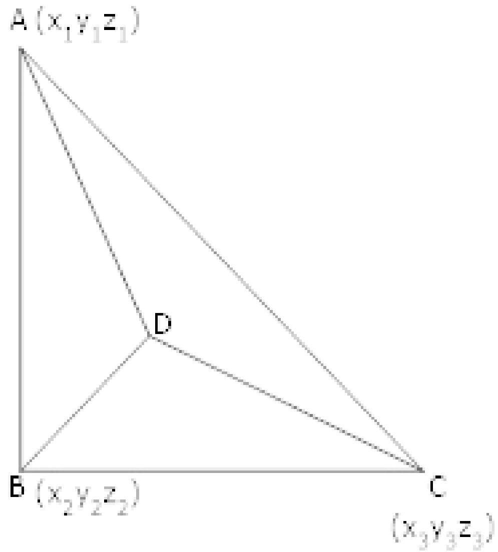
The coordinates of the centroids of faces ABC, DAB, DBC and DCA respectively

$$G_1\left[\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right]$$

$$G_2\left[\frac{x_1+x_2+x_4}{3}, \frac{y_1+y_2+y_4}{3}, \frac{z_1+z_2+z_4}{3}\right]$$

$$G_3\left[\frac{x_2+x_3+x_4}{3}, \frac{y_2+y_3+y_4}{3}, \frac{z_2+z_3+z_4}{3}\right]$$

$$G_4\left[\frac{x_4+x_3+x_1}{3}, \frac{y_4+y_3+y_1}{3}, \frac{z_4+z_3+z_1}{3}\right]$$



Now, coordinates of point G dividing DG1 in the ratio 3:1 are

$$\left[\frac{1.x_4+3\left(\frac{x_1+x_2+x_3}{3}\right)}{1+3}, \frac{1.y_4+3\left(\frac{y_1+y_2+y_3}{3}\right)}{1+3}, \frac{1.z_4+3\left(\frac{z_1+z_2+z_3}{3}\right)}{1+3}\right]$$

$$=\left[\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}, \frac{z_1+z_2+z_3+z_4}{4}\right]$$

Similarly the point dividing CG2, AG3 and BG4 in the ratio 3:1 has the same coordinates.

Hence the point $G\left[\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}, \frac{z_1+z_2+z_3+z_4}{4}\right]$ is common to DG1, CG2, AG3 and BG4.

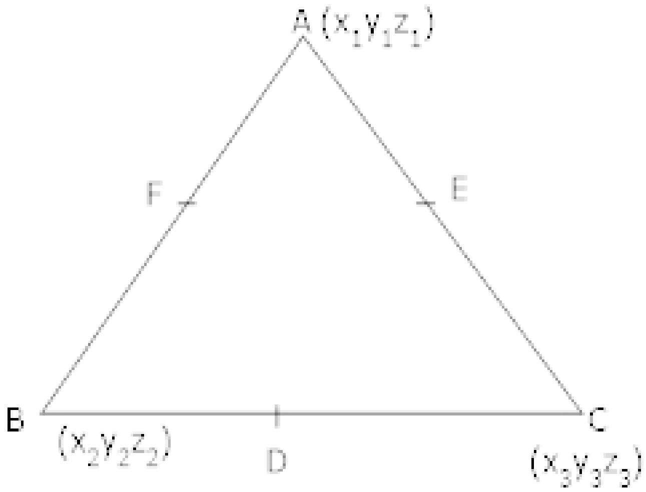
Hence they are concurrent.

2. The mid points of the sides of a triangle are (1,5,-1), (0,4,-2) and (2,3,4). Find its vertices.

Ans. Suppose vertices of ΔABC are $A(x_1 y_1 z_1)$, $B(x_2 y_2 z_2)$ and $C(x_3 y_3 z_3)$ respectively

Given coordinates of mid point of side BC, CA, and AB respectively are D(1,5,-1), E(0,4,-2) and

F(2,3,4)



$$\therefore \frac{x_2+x_3}{2}=1 \quad \frac{y_2+y_3}{2}=5 \quad \frac{z_2+z_3}{2}=-1$$

$$x_2+x_3=2......(i)$$

$$\frac{x_1+x_3}{2}=0$$

$$y_2+y_3=10......(ii)$$

$$\frac{y_1+y_3}{2}=4$$

$$z_2+z_3=2......(iii)$$

$$\frac{z_1+z_3}{2}=-2$$

$$x_1+x_3=0.......(iv)$$

$$\frac{x_1+x_2}{2}=2$$

$$y_1+y_2=8.......(v)$$

$$\frac{y_1+y_2}{2}=3$$

$z_1 + z_3 = -4 ....(vi)$

$\frac{z_1 + z_2}{2} = 4$

$x_1 + x_2 = 4 ....(vii)$

$y_1 + y_2 = 6 ....(viii)$

$z_1 + z_2 = 8 ....(ix)$

Adding eq. $(i), (iv), \& (vii)$

$2(x_1 + x_2 + x_3) = 6$

$x_1 + x_2 + x_3 = 3 ....(\mathrm{x})$

Subtracting eq. $(i), (iv), \& (vii)$ from (x) we get

$x_1 = 1, \ x_2 = 3, \ x_3 = -1$

Similarly, adding eq. $(ii), (v)$ and $(viii)$

$y_1 + y_2 + y_3 = 12 ......(\mathrm{xi})$

Subtracting eq. $(ii), (v)$ and $(viii)$ from (xi)

$y_1 = 2, \ y_2 = 4, \ y_3 = 6$

Similarly $z_1 + z_2 + z_3 = 3$

$z_1 = 1, \ z_2 = 7, \ z_3 = -5$

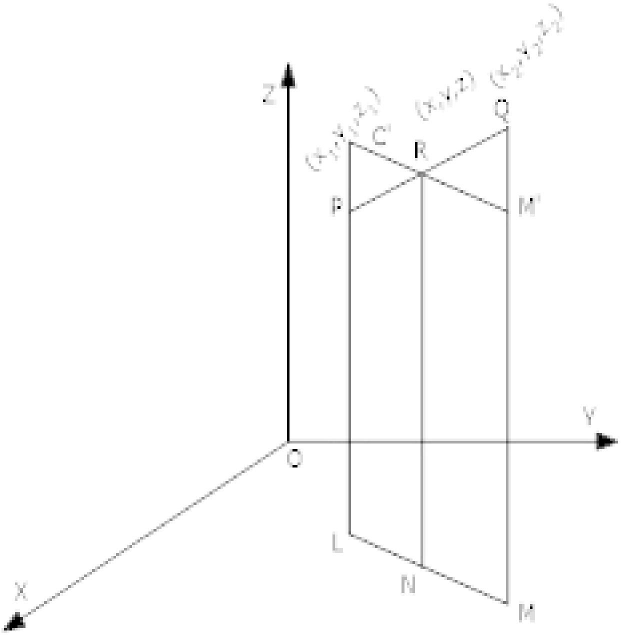
$\therefore$ Coordinates of vertices of ΔABC are A(1,3,-1), B(2,4,6) and C(1,7,-5)

3. Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be two points in space find co ordinate of point R

which divides P and Q in the ratio $m_1 : m_2$ by geometrically

Ans. Let co-ordinate of Point R be (x, y, z) which divider line segment joining the point $P\ Q$ in the ratio $m_1 : m_2$

Clearly $\Delta PRL' \sim \Delta QRM'$ $\left[By\ AA \text{ sinilsrity}\right]$



$$\therefore \frac{PL'}{MQ'} = \frac{PR}{RQ}$$

$$\Rightarrow \frac{LL' - LP}{MQ - MM'} = \frac{m_1}{m_2}$$

$$\Rightarrow \frac{NR - LP}{MQ - NR} = \frac{m_1}{m_2} \qquad \left[\begin{array}{l} \because LL' = NR \\ \text{and } MM' = NR \end{array}\right]$$

$$\Rightarrow \frac{z - z_1}{z_2 - z} = \frac{m_1}{m_2}$$

$$\Rightarrow z = \frac{m_{1:} z_2 + m_2 z_1}{m_1 + m_2}$$

Similarly $x=\frac{m_1x_2+m_2x_1}{m_1+m_2}$ and

$$y=\frac{m_1y_2+m_2y_1}{m_1+m_2}$$

4. Show that the plane $ax+by+cz+d=0$ divides the line joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio $\frac{ax_1+by_1+cz_1+d}{ax_2+by_2+cz_2+d}$ s

Ans. Suppose the plane $ax+by+cz+d=0$ divides the line joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio $\lambda : 1$

$$\therefore\ x=\frac{\lambda x_2+x_1}{\lambda+1},\quad y=\frac{\lambda y_2+y_1}{\lambda+1},\quad z=\frac{\lambda z_2+z_1}{\lambda+1}$$

$\because$ Plane $ax+by+cz+d=0$ Passing through (x, y, z)

$$\therefore\ Q\frac{(\lambda x_2+x_1)}{\lambda+1}+b\frac{(\lambda y_2+y_1)}{\lambda+1}+c\frac{(\lambda z_2+z_1)}{\lambda+1}+d=0$$

$$a(\lambda x_2+x_1)+b(\lambda y_2+y_1)+c(\lambda z_2+z_1)+d(\lambda+1)=0$$

$$\lambda(ax_2+by_2+cz_2+d)+(ax_1+by_1+cz_1+d)=0$$

$$\lambda=-\frac{(ax_1+by_1+cz_1+d)}{(ax_2+by_2+cz_2+d)}$$

Hence Proved.

5. Prove that the points $0(0,0,0)$, $A(2,0,0)$, $B(1,\sqrt{3},0)$, and $C\left(1,\frac{1}{\sqrt{3}},\frac{2\sqrt{2}}{\sqrt{3}}\right)$ are the vertices of a regular tetrahedron.

Ans. To prove O, A, B, C are vertices of regular tetrahedron.

We have to show that

$|OA| = |OB| = |OC| = |AB| = |BC| = |CA|$

$|OA| = \sqrt{(0-2)^2 + 0^2 + 0^2} = 2$ unit

$|OB| = \sqrt{(0-1)^2 + \left(0-\sqrt{3}\right)^2 + 0^2} = \sqrt{1+3} = \sqrt{4} = 2$ unit

$|OC| = \sqrt{(0-1)^2 + \left(0-\frac{1}{\sqrt{3}}\right) + \left(0-\frac{2\sqrt{2}}{3}\right)^2}$

$= \sqrt{1 + \frac{1}{3} + \frac{8}{3}}$

$= \sqrt{\frac{12}{3}} = \sqrt{4} = 2$ unit

$|AB| = \sqrt{(2-1)^2 + \left(0-\sqrt{3}\right)^2 + (10-0)^2} = \sqrt{1+3+0}$

$= \sqrt{4} = 2$ unit

$|BC| = \sqrt{(1-1)^2 + \left(\sqrt{3}-\frac{1}{\sqrt{3}}\right)^2 + \left(0-\frac{2\sqrt{2}}{\sqrt{3}}\right)^2}$

$= \sqrt{0 + \left(\frac{2}{\sqrt{3}}\right)^2 + \frac{8}{3}}$

$= \sqrt{\frac{12}{3}} = 2$ unit

|CA| = $\sqrt{(1-2)^2+\left(\frac{1}{\sqrt{3}}-0\right)^2+\left(\frac{2\sqrt{2}}{\sqrt{3}}-0\right)^2}$

$= \sqrt{1+\frac{1}{3}+\frac{8}{3}}$

$= \sqrt{\frac{12}{3}} = 2$ unit

∴ |AB| = |BC| = |CA| = |OA| = |OB| = |OC| = 2 unit

∴ O, A, B, C are vertices of a regular tetrahedron.

6. If A and B are the points $(-2, 2, 3)$ and $(-1, 4, -3)$ respectively, then find the locus of P such that 3|PA| = 2|PB|

Ans. Given points $A(-2, 2, 3)$ and $B(-1, 4, -3)$

Supper co-ordinates of point $P(x, y, z)$

|PA| = $\sqrt{(x+2)^2+(y-2)^2+(2-3)^2}$

|PA| = $\sqrt{x^2+y^2+z^2+4x-4y-6z+17}$

|PB| = $\sqrt{(x+1)^2+(y-4)^2+(z+3)^2}$

|PB| = $\sqrt{x2+y2+z2+2x-8y+6z+26}$

∵ 3|PA| = 2|PB|

9 PA2=4 PB2

$9\left(x^2+y^2+z^2+4x-4y-6z+17\right)=4\left(x^2+y^2+z^2+2x-8y+6z+26\right)$

$5x^2+5y^2+5z^2+28x-4y-30z+49=0$