Chapter 1

Rational and Irrational Numbers

Exercise1.1

Question 1.

Insert a rational number between and $\frac{2}{9}$ and $\frac{3}{8}$ arrange in descending order:

Solution :

Given :

Rational numbers: $\frac{2}{9}$ and $\frac{3}{8}$

Let us rationalize the numbers,

By taking LCM for denominators 9 and 8 which is 72.

$\frac{2}{9} = \frac{(2\times8)}{9\times8} = \frac{16}{72}$

$\frac{3}{8} = \frac{3\times9}{8\times9} = \frac{27}{72}$

Since ,

$$\frac{16}{72} < \frac{27}{72}$$

So, $\frac{2}{9} < \frac{3}{8}$

The rational number between $\frac{2}{9}\, and\, \frac{3}{8}\, is$

$= \frac{\frac{2}{9}+\frac{3}{8}}{2}$

$= \frac{\frac{(2\times8)+(3\times9)}{72}}{2}$

$= \frac{16+27}{72\times2}$

$= \frac{43}{144}$

Hence , $\frac{3}{8} > \frac{43}{144} > \frac{2}{9}$

The descending order of the numbers is $\frac{3}{8} > \frac{43}{144} > \frac{2}{9}$

Question 2.

Insert two rational numbers between $\frac{1}{3}$ and $\frac{1}{4}$ and arrange in ascending order:

Solution:

Given

The rational numbers $\frac{1}{3}\ and\ \frac{1}{4}$

By taking LCM and rationalizing, we get

$= \frac{\frac{1}{3}+\frac{1}{4}}{2}$

$= \frac{\frac{4+3}{12}}{2}$

$= \frac{7}{12\times2}$

$= \frac{7}{24}$

Now let us find the rational number between $\frac{1}{4}$ *and* $\frac{7}{24}$

By taking LCM and rationalizing, we get

$= \frac{\frac{1}{4}+\frac{7}{24}}{2}$

$= \frac{\frac{6+7}{24}}{2}$

$= \frac{13}{24\times 2}$

$= \frac{13}{48}$

So,

The two rational numbers between $\frac{1}{3}$ *and* $\frac{1}{4}$ are

$\frac{7}{24}$ *and* $\frac{13}{48}$

Hence we, know that, $\frac{1}{3} > \frac{7}{24} > \frac{13}{48} > \frac{1}{4}$

The ascending order is as following : $\frac{1}{4}, \frac{13}{48}, \frac{7}{24}, \frac{1}{3}$

Question 3.

Insert two rational numbers between $-\frac{1}{3}$ and $-\frac{1}{2}$ and arrange in ascending order.

Solution:

Given :

The rational numbers - $-\frac{1}{3}$ and - $\frac{1}{2}$

By taking LCM and rationalizing, we get

$= \frac{\frac{-1}{3}+\frac{-1}{2}}{2}$

$= \frac{\frac{-2-3}{6}}{2}$

$= \frac{-5}{6\times2}$

$= \frac{-5}{12}$

So, the rational number between $\frac{-1}{3}$ and $\frac{-1}{2}$ is $\frac{-5}{12}$

$$\frac{-1}{3} > \frac{-5}{12} > \frac{-1}{2}$$

Now, let us find the rational number between $\frac{-1}{3}$ *and* $\frac{-5}{12}$

By taking LCM and rationalizing , we get

$= \frac{\frac{-1}{3}+\frac{-5}{12}}{2}$

$= \frac{\frac{-4-5}{12}}{2}$

$= \frac{-9}{12\times2}$

$= \frac{-9}{24}$

$= \frac{-3}{8}$

So, the rational number between $\frac{1}{3}$ *and* $\frac{5}{12}$ *is* $\frac{3}{8}$

$\frac{-1}{3} > \frac{-3}{8} > \frac{-5}{12}$

Hence, the two rational numbers between $\frac{1}{3}$ $and - \frac{1}{2}$

$\frac{-1}{3} > \frac{-3}{8} > \frac{-5}{12} >$ - $\frac{1}{2}$

The ascending is a follows:

- $\frac{1}{2}$, $\frac{-5}{12}$, $\frac{-3}{8}$, - $\frac{-1}{3}$

Question 4.

Insert three rational numbers between $\frac{1}{3}$ and $\frac{4}{5}$ and arrange in descending order.

Solution

Given:

The rational numbers $\frac{1}{3}$ $and \frac{4}{5}$

By taking LCM and rationalizing, we get

$$= \frac{\frac{1}{3}+\frac{4}{5}}{2}$$

$$= \frac{\frac{5+12}{15}}{2}$$

$$= \frac{17}{15\times2}$$

$$= \frac{17}{30}$$

So, the rational number between $\frac{1}{3}$ $and \frac{4}{5}$ $is \frac{17}{30}$

$\frac{1}{3} < \frac{17}{30} < \frac{4}{5}$

Now , let us find the rational numbers between $\frac{1}{3}$ and $\frac{17}{30}$

By taking LCM and rationalizing, we get

$= \frac{\frac{1}{3}+\frac{17}{30}}{2}$

$= \frac{\frac{10+17}{30}}{2}$

$= \frac{27}{30\times 2}$

$= \frac{27}{60}$

So, the rational number between $\frac{1}{3}$ and $\frac{17}{30}$ is $\frac{27}{60}$

$\frac{1}{3} < \frac{27}{60} < \frac{17}{30}$

Now, let us find the rational numbers between $\frac{17}{30}$ $and \frac{4}{5}$

By taking LCM and rationalizing , we get

$= \frac{\frac{17}{30}+\frac{4}{5}}{2}$

$= \frac{\frac{17+24}{30}}{2}$

$= \frac{41}{30\times 2}$

$= \frac{41}{60}$

So, the rational number between $\frac{17}{30}$ and $\frac{4}{5}$ is $\frac{41}{60}$

$\frac{17}{30} < \frac{41}{60} < \frac{4}{5}$

Hence, the three rational numbers between $\frac{1}{3}$ *and* $\frac{4}{5}$ are

$\frac{1}{3} < \frac{27}{60} < \frac{17}{30} < \frac{41}{60} < \frac{4}{5}$

The descending order is as follows: $\frac{4}{5}, \frac{41}{60}, \frac{17}{30}, \frac{27}{60}, \frac{1}{3}$

Question 5.

Insert three rational numbers between 4 and 4.5

Solution:

Given :

The rational numbers 4 and 4.5

By rationalizing, we get

$= \frac{(4 + 4.5)}{2}$

$= \frac{8.5}{2}$

= 4.25

So, the rational number between 4 and 4.5 is 4.25

4 < 4.25 < 4.5

Now, let us find the rational number between 4 and 4.25

By rationalizing , we get

$$= \frac{(4 + 4.5)}{2}$$

$$= \frac{8.5}{2}$$

= 4.125

So, the rational number between 4 and 4.25 is 4.125

$4 < 4.125 < 4.25$

Now, let us find the rational number between 4 and 4.125

By rationalizing , we get

$$= \frac{(4 + 4.125)}{2}$$

$$= \frac{8.125}{2}$$

= 4.0625

So, the rational number between 4 and 4.125 is 4.0625

$4 < 4.0625 < 4.125$

Hence, the rational numbers between 4 and 4.5 are

$4 < 4.0625 < 4.125 < 4.25 < 4.5$

The three rational numbers between 4 and 4.5

4.0625, 4.125, 4.25

Question 6.

Find six rational numbers between 3 and 4.

Solution:

Given :

The rational number 3 and 4

So let us find the six rational numbers between 3 and 4,

First rational number between 3 and 4 is

$= \frac{(3+4)}{2}$

$= \frac{7}{2}$

Second rational number between 3 and $\frac{7}{2}$ is

$= \frac{3+\frac{7}{2}}{2}$

$= \frac{(6+7)}{(2\times 2)}$ [By taking 2 LCM]

$= \frac{13}{4}$

Third rational number between $\frac{7}{2}$ *and* 4 *is*

$= \frac{\frac{7}{2}+4}{2}$

$= \frac{(7+8)}{(2\times 2)}$ [By taking 2 as LCM]

$= \frac{15}{4}$

Fourth rational number between 3 and $\frac{13}{4}$ is

$= \frac{3+\frac{13}{4}}{2}$

$= \frac{(12+13)}{(4 \times 2)}$ [By taking 4 as LCM]

$= \frac{25}{8}$

Find rational number between $\frac{13}{4}$ *and* $\frac{7}{2}$ *is*

$= \frac{\left(\frac{13}{4}+\frac{7}{2}\right)}{2}$

$= \frac{\left(\frac{13+14}{4}\right)}{2}$ [By taking 4 as LCM]

$= \frac{(13+14)}{(4 \times 2)}$

$= \frac{27}{8}$

Sixth rational number between $\frac{7}{2}$ *and* $\frac{15}{4}$ is

$= \frac{\left(\frac{7}{2}+\frac{15}{4}\right)}{2}$

$= \frac{\left(\frac{14+15}{4}\right)}{2}$ [By taking 4 as LCM]

$= \frac{(14+15)}{(4 \times 2)}$

$= \frac{29}{8}$

Hence, the six rational numbers between 3 and 4 are

$\frac{25}{8}, \frac{13}{4}, \frac{27}{8}, \frac{7}{2}, \frac{29}{8}, \frac{15}{4}$

7. find five rational numbers between $\frac{3}{5}$ *and* $\frac{4}{5}$

Solution:

Given:

The rational numbers $\frac{3}{5}$ *and* $\frac{4}{5}$

Now, let us find the five rational numbers between $\frac{3}{5}$ *and* $\frac{4}{5}$

So we need to multiply both numerator and denominator with 5+1 = 6 we get,

$$\frac{3}{5} = \frac{(3 \times 6)}{(5 \times 6)} = \frac{18}{30}$$

$$\frac{4}{5} = \frac{(4 \times 6)}{(5 \times 6)} = \frac{24}{30}$$

Now, we have $\frac{18}{30} < \frac{19}{30} < \frac{20}{30} < \frac{21}{30} < \frac{22}{30} < \frac{23}{30} < \frac{24}{30}$

Hence , the five rational numbers between $\frac{3}{5}$ *and* $\frac{4}{5}$ *are*

$\frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}$

Question 8.

Find ten rational numbers between $-\frac{2}{5}$ *and* $\frac{1}{7}$

Solution :

Given :

The rational numbers $\frac{-2}{5}$ *and* $\frac{1}{7}$

By taking LCM for 5 and 7 which is 35

So, $\frac{-2}{5} = \frac{(-2 \times 7)}{(5 \times 7)} = \frac{-14}{35}$

$$\frac{1}{7} = \frac{(1 \times 5)}{(7 \times 5)} = \frac{5}{35}$$

Now, we can insert any10 numbers between $\frac{14}{35}$ *and* $\frac{5}{35}$

i.e. $\frac{-13}{35}, \frac{-12}{35}, \frac{-11}{35}, \frac{-10}{35}, \frac{-9}{35}, \frac{-8}{35}, \frac{-7}{35}, \frac{-6}{35}, \frac{-5}{35}, \frac{-4}{35}, \frac{-3}{35}, \frac{-2}{35}, \frac{-1}{35},$
$\frac{1}{35}, \frac{2}{35}, \frac{3}{35}, \frac{4}{35}$

hence, the ten rational numbers between $\frac{2}{5}$ *and* $\frac{1}{7}$ *are*

$\frac{-6}{35}, \frac{-5}{35}, \frac{-4}{35}, \frac{-3}{35}, \frac{-2}{35}, \frac{-1}{35}, \frac{1}{35}, \frac{2}{35}, \frac{3}{35}, \frac{4}{35}$

Question 9.

Find six rational numbers between $\frac{1}{2}$ *and* $\frac{2}{3}$

Solution:

Given :

The rational numbers $\frac{1}{2}$ *and* $\frac{2}{3}$

To make the denominators similar let us take LCM for 2 and 3 which is 6

$$\frac{1}{2} = \frac{(1 \times 3)}{(2 \times 3)} = \frac{3}{6}$$

$$\frac{2}{3} = \frac{(2 \times 2)}{(3 \times 2)} = \frac{4}{6}$$

Now , we need to insert six rational numbers, so multiply both numerator and denominator by 6 +1 = 7

$$\frac{3}{6} = \frac{(3 \times 7)}{(6 \times 7)} = \frac{21}{42}$$

$$\frac{4}{6} = \frac{(4 \times 7)}{(6 \times 7)} = \frac{28}{42}$$

We know that, $\frac{21}{42} < \frac{22}{42} < \frac{23}{42} < \frac{24}{42} < \frac{25}{42} < \frac{26}{42} < \frac{27}{42} < \frac{28}{42}$

Hence, the six rational numbers between $\frac{1}{2}$ and $\frac{2}{3}$ are
$\frac{22}{42}, \frac{23}{42}, \frac{24}{42}, \frac{25}{42}, \frac{26}{42}, \frac{27}{42}$

Exercise 1.2

Question 1.

Prove that, $\sqrt{5}$ is an irrational number.

Solution:

Let us consider $\sqrt{5}$ be a rational, number, then

$\sqrt{5} = \frac{p}{q}$, where ‘p’ and ‘q’ are integer , q $\neq$ 0 and p,q have no common factors (except 1) .

So,

$5 = \frac{p^2}{q^2}$

$p^2 = 5q^2$........ (1)

As we know, ‘5’ divides $5q^2$, so, ‘5’ divides p^2 as well . hence, ‘5’ is prime.

So 5 divides p

Now, let p = 5k where k is an integer

Square on both sides, we get

$P^2 = 25k^2$

$5q^2 = 25k^2$ [since, $p^2 = 5q^2$, from equation (1)]

$Q^2 = 5k^2$

As we know, ‘5’ divides $5k^2$, so, ‘5’ divides q^2 as well. But ‘5’ is prime.

So 5 divides q

Thus, p and q have a common factor 5. This statement contradicts that ‘p’ and ‘q’ has no common factors (except1) .

We can say that, $\sqrt{5}$ is not an rational number.

$\sqrt{5}$ is an irrational number.

Hence proved.

Question 2.

Prove that, $\sqrt{7}$ be a rational number, then

$\sqrt{7} = \frac{p}{q}$ where ‘p’ and ‘q’ are integers $q \neq 0$ and p,q have no common factors (except 1).

So,

$7 = \frac{p^2}{q^2}$

$P^2 = 7q^2$ ......(1)

As we know, ‘7’ divides so, ‘7’ divides p^2 as well. Hence, ‘7’ is prime.

So 7 divides p

Now, let p = 7k , where ‘k’ is an integer ‘square on both sides, we get

$P^2 = 49k^2$

$7q^2 = 49k^2$ [since , $p^2 = 7q^2$, from equation (1)]

$Q^2 = 7k^2$

As we know , ‘7’ divides $7k^2$, so 7’ divides q^2 as well. But ‘7’ is prime. So 7 divides q

Thus, p and q have a common factor 7 . this statement contradicts that ‘p’ and ‘q’ has no common factors (except 1).

We can say that, $\sqrt{7}$ is not a rational number.

$\sqrt{7}$ is an irrational number.

Hence proved.

Question 3.

Prove that $\sqrt{6}$ is an irrational number.

Solution:

Let us consider $\sqrt{6}$ be a rational number, then

$\sqrt{6} = \frac{p}{q}$ where ‘p’ and ‘q’ are integer $q \neq 0$ and p, q have no common factors (except 1).

So,

$6 = \frac{p^2}{q^2}$

$P^2 = 6q^2$ .....(1)

As we know, ‘2’ divides $6q^2$, so ‘2’ divides p^2 as well. Hence , ‘2’ is prime.

So 2 divides p

Now, let p = 2k, where ‘k’ is an integer

Square on both sides, we get

$P^2 = 4k^2$

$6q^2 = 4k^2$ [Since, $p^2 = 6q^2$, from equation (1)]

$3q^2 = 2k^2$

As we know, ‘2’ divides $2k^2$, so ‘2’ divides $3q^2$ as well.

‘2’ should either 3 or divide q^2 .

but ‘2’ does not divide 3. ‘2’ divides q^2 so ‘2’ is prime.

So 2 divides q

Thus, p and q have a common factor 2. This statement contradicts that ‘p’ and ‘q’ has no common factors (except 1) .

We can say that, $\sqrt{6}$ is not a rational number.

$\sqrt{6}$ is an irrational number.

Hence proved.

Question 4.

Prove that $\frac{1}{\sqrt{11}}$ is an irrational number.

Solution:

Let us consider $\frac{1}{\sqrt{11}}$ be a rational number, then

$\frac{1}{\sqrt{11}} = \frac{p}{q}$ where ‘p’ and ‘q’ are integers $q \neq 0$ and p,q have no common factors (except 1.)

So,

$$\frac{1}{11} = \frac{p^2}{q^2}$$

$Q^2 = 11p^2$ ..... (1)

As we know, ‘11’ divides $11p^2$, so ‘11’ divides q^2 as well. Hence, ‘11’ is prime.

So 11 divides q

Now, let q = 11k , where ‘k’ is an integer

Square on both sides, we get

$Q^2 = 121k^2$

$11p^2 = 121k^2$ [since, $q^2 = 11p^2$, from equation (1)]

$P^2 = 11k^2$

As we know. ‘11’ divides $11k^2$, so ‘11’ divides p^2 as well. But ‘11’ is prime.

So 11 divides p

Thus, p and q have a common factor. 11. This statement contradicts that ‘p’ and ‘q’ has no common factor (except) .

We can say that, $\frac{1}{\sqrt{11}}$ is not a rational number.

$\frac{1}{\sqrt{11}}$ is an irrational number.

Hence proved.

Question 5.

Prove that $\sqrt{2}$ is an irrational number. Hence show that 3 - $\sqrt{2}$ is an irrational .

Solution:

Let us consider $\sqrt{2}$ is an irrational number. Then

$\sqrt{2}\,\frac{p}{q}$ where 'p' and 'q' are integers , $q \neq 0$ and p,q have no common

Factors (except 1)

So,

$2 = \frac{p^2}{q^2}$

$P^2 = 2q^2$.......(1)

As we know, '2' divides $2q^2$, so '2' divides p^2 as well. Hence , '2' is

prime.

So 2 divides p

Now, let p = 2k , where 'k' is an integer

Square on both sides , we get

$P^2 = 4k^2$

$2q^2 = 4k^2$ [Since, $p^2 = 2q^2$, from equation (1)]

$Q^2 = 2k^2$

As we know, '2' divides $2k^2$, so '2' divides q^2 as well . but '2' is prime.

So 2 divides q

Thus, p and q have a common factor 2. This statement contradicts that

'p' and 'q' has no common factors (except 1.)

We can say that. $\sqrt{2}$ is not a rational number.

$\sqrt{2}$ is not a rational number.

Now, let us assume3 - $\sqrt{2}$ be a rational number, 'r'

So, 3 - $\sqrt{2}$ = r

3 – r = $\sqrt{2}$

We know that, 'r' is rational, '3 – r' is rational, so $\sqrt{2}$ is also rational .this contradicts the statement that $\sqrt{2}$ is irrational .

So , 3 - $\sqrt{2}$ is irrational number.

Hence proved.

Question 6.

Prove that, $\sqrt{3}$ is an irrational number . hence show that $\frac{2}{5} \times \sqrt{3}$

Is an irrational number.

Solution:

Let us consider $\sqrt{3}$ be a rational number, then

$\sqrt{3} = \frac{p}{q}$ where 'p' and 'q' are integers, $q \neq 0$ and p, q have no common factors (except 1).

So,

$3 = \frac{p^2}{q^2}$

$P^2 = 3q^2$ ..... (1)

As we know, '3' divides $3q^2$, so '3' divides p^2 as well. Hence, '3' is prime.

So 3 divides p

Now, let p = 3k, where 'k ' is an integer

Square on both sides, we get

$P^2 = 9k^2$

$3q^2 = 9k^2$ [since, $p^2 = 3q^2$, from equation (1)]

$Q^2 = 3k^2$

As we know, '3' divides $3k^2$, so '3' divides q^2 as well. But '3' is prime.

So 3 divides q

Thus, p and q have a common factor, 3 this statement contradicts that 'p' and 'q' has no common factors (except 1)

We can say that, $\sqrt{3}$ is not a rational number.

$\sqrt{3}$ is an irrational number.

Now, let us assume $\left(\frac{2}{5}\right)\sqrt{3}$ be a rational number, 'r'

So, $\left(\frac{2}{5}\right)\sqrt{3}$

$\frac{5r}{2} = \sqrt{3}$

We know that , 'r' is rational , $\frac{5r}{2}$ is rational ,so '$\sqrt{3}$ is also rational

This contradicts the statement that $\sqrt{3}$ is irrational.

So, $\left(\frac{2}{5}\right)\sqrt{3}$ is irrational number.

Hence proved.

Question 7.

Prove that $\sqrt{5}$ be a rational number, then $\sqrt{5}$ is an irrational number. Hence, show that -3 + $2\sqrt{5}$ is an irrational number.

Solution:

Let us consider $\sqrt{5}$ be a rational number, then

$\sqrt{5} = \frac{p}{q}$ where 'p' and 'q' are integers, q $\neq$ 0 and p, q have no common factors (except 1).

So,

$5 = \frac{p^2}{q^2}$

$P^2 = 5q^2$.....(1)

As we know, '5' divides $5q^2$, so '5' divides p^2 as well. Hence , '5' is prime.

So 5 divides p

Now, let p = 5k , where 'k' is an integer

Square on both sides, we get

$P^2 = 25k^2$

$5q^2 = 25k^2$ [since , $p^2 = 5q^2$, from equation (1)]

$Q^2 = 5k^2$

As we know, ‘5’ divides $5k^2$, so ‘5’ divides q^2 as well. But ‘5’ is prime. So 5 divides q

Thus, p and q have a common factor 5. This statement contradicts ‘p’ and ‘q’ has no common factors (except 1)

We can say that, $\sqrt{5}$ is not a rational number.

$\sqrt{5}$ is an irrational number.

Now, let us assume -3 + $2\sqrt{5}$ be a rational number , ‘r’

So, -3 + $2\sqrt{5}$ = r

$-3 – r = 2\sqrt{5}$

$\frac{(-3 - r)}{2} = \sqrt{5}$

We know that , ‘r’ rational $\frac{(-3-r)}{r}$ is rational , so ‘$\sqrt{5}$’ is also rational .

This contradicts the statement that $\sqrt{5}$ is irrational .

So, -3 + $2\sqrt{5}$ is irrational number.

Hence proved.

Question 8.

(i) $5 + \sqrt{2}$

(ii) $3 - 5\sqrt{3}$

(iii) $2\sqrt{3} - 7$

(iv) $\sqrt{2} + \sqrt{5}$

Solution:

(i) $5 + \sqrt{2}$

Now, let us assume $5 + \sqrt{2}$ be a rational number, ‘r’

So, $5 + \sqrt{2} = r$

$R - 5 = \sqrt{2}$

We know that, ‘r’ is rational , ‘r – 5 ‘ is rational so ‘$\sqrt{2}$ is also rational. This contradicts the statement that $\sqrt{2}$ is irrational.

So, $5 + \sqrt{2}$ is irrational number .

(ii) $3 - 5\sqrt{3}$

Now, let us assume $3 - 5\sqrt{3}$ be a rational number, ‘r’

So, $3 - 5\sqrt{3} = r$

$3 - r = 5\sqrt{3}$

$\frac{(3-r)}{5} = \sqrt{3}$

We know that, ‘r’ is rational, $\frac{(3-r)}{5}$ is rational, so ‘ $\sqrt{3}$ is also rational. This contradicts the statement that $\sqrt{3}$ is irrational.

So, $3 - 5\sqrt{3}$ is irrational number.

(iii) $2\sqrt{3} - 7$

Now, let us assume $2\sqrt{3} - 7$ be a rational number , ‘r’

So, $2\sqrt{3}$ - 7 = r

$2\sqrt{3}$ = r + 7

$\sqrt{3} = \frac{r+7}{2}$

We know that, 'r' is rational, $\frac{r+7}{2}$ is rational, so '$\sqrt{3}$' is also rational.

This contradicts the statement that $\sqrt{3}$ is irrational.

So, $2\sqrt{3}$ - 7 is irrational number.

(iv) $\sqrt{2} + \sqrt{5}$

Now, let us assume $\sqrt{2} + \sqrt{5}$ be a rational number , 'r'

So $\sqrt{2} + \sqrt{5}$ = r

$\sqrt{5}$ = r - $\sqrt{2}$

Square on both sides,

($\sqrt{5}$) = (r - $\sqrt{2}$)^2

5 = $r^2 + (\sqrt{2})^2 - 2r\sqrt{2}$

5 = $r^2 + 2 - 2\sqrt{2}r$

5 – 2 = $r^2 - 2\sqrt{2r}$

$R^2 - 3 = 2\sqrt{2}r$

$\frac{r^2 - 3}{2r} = \sqrt{2}$

We know that , 'r' is rational , $\frac{r^2 - 3}{2r}$ is rational, so '$\sqrt{2}$' is also rational. This contradicts the statement that $\sqrt{2}$ is irrational. So, $\sqrt{2} + \sqrt{5}$ is irrational number.

Exercise 1.3

Question 1.

Locate $\sqrt{10}$ and $\sqrt{17}$ on the amber line.

Solution:

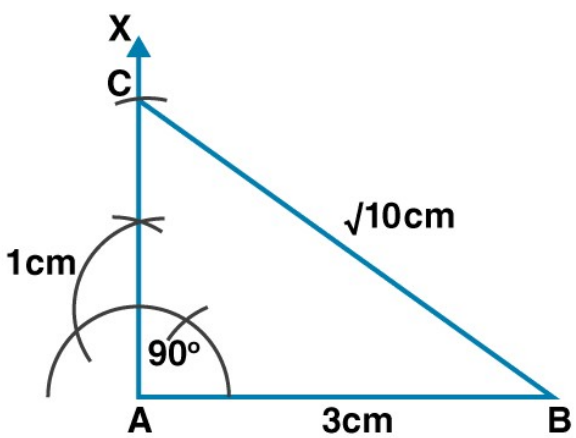
$\sqrt{10}$

$\sqrt{10} = \sqrt{(9 + 1)} = \sqrt{((3)^2 + 1^2)}$

Now let us construct:

- Draw a line segment AB = 3cm.
- At point A, draw a perpendicular AX and cut off AC = 1cm.
- Join BC.

BC= $\sqrt{10}$ cm



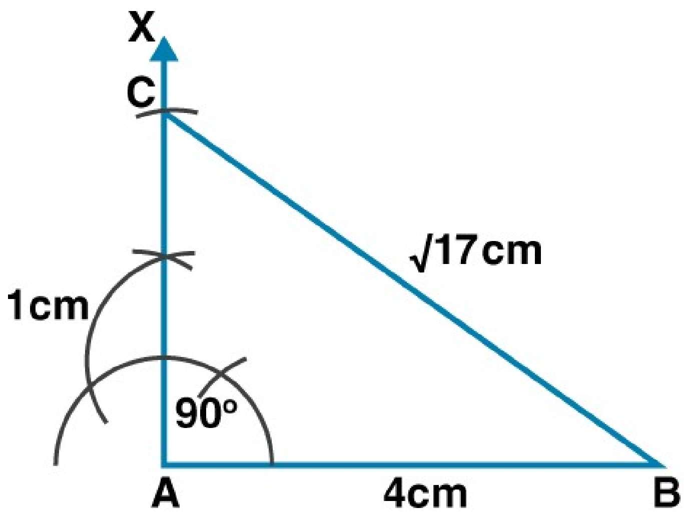
$\sqrt{17}$

$\sqrt{17} = \sqrt{(16 + 1)} = \sqrt{((4)^2 + 1^2)}$

Now let us construct:

- Draw a line segment AB= 4cm.
- At point A , draw a perpendicular AX and cut off AC = 1cm.
- Join BC.

BC = $\sqrt{17}$ cm



Question 2

Write the decimal expansion of each of the following numbers and say what kind of decimal expansion each has:

(i) $\frac{36}{100}$

(ii) $4\frac{1}{8}$

(iii) $\frac{2}{9}$

(iv) $\frac{2}{11}$

(v) $\frac{3}{13}$

(vi) $\frac{329}{400}$

Solution:

(i) $\frac{36}{100}$

```
         0 0. 3 6 0
1 0 0 | 3 6.. 0 0 0
     -  0
        3 6
     -    0
        3 6 0
        3 0 0
          6 0 0
         -6 0 0
              0 0
            -   0
                0
```

$\frac{36}{100} = 0.36$

It is a terminating decimal.

(ii) $4\frac{1}{8}$

$4\frac{1}{8} = \frac{(4 \times 8 + 1)}{8} = \frac{33}{8}$

```
     0  4.  1  2  5
8  | 3  3.  0  0  0
 -   0
     3  3
 -   3  2
        1  0
   -       8
           2  0
      -    1  6
```

```
      4   0
  -   4   0
      ------
          0
```

$\frac{33}{8} = 4.125$

It is a terminating decimal.

(iii) $\frac{2}{9}$

```
     0.  2   2   2
9  | 2.  0   0   0
-    0
     ----
     2   0
-    1   8
     --------
         2   0
     -   1   8
         --------
             2   0
         -   1   8
             -------
                 2
```

$\frac{2}{9} = 0.222$

It is a non-terminating recurring decimal.

(iv) $\frac{2}{11}$

```
      0.  1   8   1
11 | 2.   0   0   0
-     0
     ----
      2   0
-     1   1
     --------
          9   0
      -   8   8
         --------
              2   0
          -   1   1
             --------
                  9
```

$\frac{2}{11} = 0.181$

It is a non-terminating recurring decimal.

(v) $\frac{3}{13}$

```
       0.  2   3   0   7   6   9   2  3   0  7
1  3 | 3.  0   0   0   0   0   0   0  0   0  0
   -   0
      ----
       3   0
   -   2   6
      --------
           4   0
       -   3   9
          ---------
               1   0
           -       0
              -------------
               1   0   0
           -       9   1
                  ---------
                       9   0
                   -   7   8
                      ---------
                       1   2   0
                   -   1   1   7
                      -----------
```

30
-26
40
-39
10
-0
100
-91
9

$\frac{3}{13} = 0.2317692307$

It is a non- terminating recurring decimal.

(vi) $\frac{329}{400}$

```
        0 0 0. 8 2 2 5
4 0 0 | 3 2 9. 0 0 0 0
      - 0
        3 2
          0
        3 2 9 0
      - 3 2 0 0
            9 0 0
          - 8 0 0
            1 0 0 0
              8 0 0
              2 0 0 0
            - 2 0 0 0
                    0
```

$\frac{329}{400} = 0.8225$

It is a terminating decimal

Question 3.
Without actually performing the king division, state whether the following rational numbers will have a terminating decimal expansion or a non – terminating repeating decimal expansion:

(i) $\frac{13}{3125}$

(ii) $\frac{17}{8}$

(iii) $\frac{23}{75}$

(iv) $\frac{6}{15}$

(v) $\frac{1258}{625}$

(vi) $\frac{77}{210}$

Solution:
We know that, if the denominator of a fraction has only 2 or 5 or both factors, it is a terminating decimal otherwise it is non-terminating repeating decimals.

(i) $\frac{13}{3125}$

5	3125
5	625
5	125
5	25
5	5
	1

$3125 = 5 \times 5 \times 5 \times 5 \times 5$

Prime factor of 3125 = 5, 5, 5, 5, 5, 5 [i.e in the form of 2^n , 5^n]
It is a terminating decimal.

(ii) $\frac{17}{8}$

2	8
2	4
2	2
	1

$8 = 2 \times 2 \times 2$

Prime factor of 8 = 2, 2, 2 [i.e in the form of 2^n , 5^n] it is a terminating decimal.

(iii) $\frac{23}{75}$

3	75
5	25
5	5
	1

$75 = 3 \times 5 \times 5$

Prime factor or 75 = 3, 5, 5
It is a non- terminating repeating decimal.

(iv) $\frac{6}{15}$
Let us divide both numerator and denominator by 3

$$\frac{6}{15} = \frac{(6 \div 3)}{(15 \div 3)}$$

$$= \frac{2}{5}$$

Since the denominator is 5.

It is a terminating decimal.

```
        0. 4  0
1  5 | 6. 0  0
   -   0
       6  0
   -   6  0
          0  0
      -      0
             0
```

$\frac{6}{15} = 0.4$

(v) $\frac{1258}{625}$

5	625
5	125
5	25
5	5
	1

$625 = 5 \times 5 \times 5 \times 5$

Prime factor of 625 = 5, 5, 5, 5 [i.e in the form of 2^n , 5^n]
It is a terminating decimal.

(vi) $\frac{77}{210}$
Let us divide both numerator and denominator by 7

$$\frac{77}{210} = \frac{(77 \div 7)}{(210 \div 7)}$$
$$= \frac{11}{30}$$

2	30
3	15
5	5
	1

$30 = 2 \times 3 \times 5$

Prime factor 30 = 2, 3, 5

It is a non – terminating repeating decimal.

Question 4.

Without actually performing the long division, find if $\frac{987}{10500}$ will have terminating or non – terminating repeating decimal expansion.

Give reasons for your answer.

Solution:

Given:

The fraction $\frac{987}{10500}$

Let us divide numerator and denominator by 21, we get

$$\frac{987}{10500} = \frac{(987 \div 21)}{(10500 \div 21)}$$

$$= \frac{47}{500}$$

So,

The prime factors for denominator $500 = 2 \times 2 \times 5 \times 5 \times 5$

Since it is of the form : 2^n , 5^n

Hence it is a terminating decimal.

Question 5.

Write the decimal expansions of the following numbers which have Terminating decimal expansions:

Solution :

(i) $\frac{17}{8}$

(ii) $\frac{13}{3125}$

(iii) $\frac{7}{80}$

(iv) $\frac{6}{15}$

(v) $2^2 \times \frac{7}{5^4}$

(vi) $\frac{237}{1500}$

Solution:

(i) $\frac{17}{8}$

2	8
2	4
2	2
	1

Denominator, 8 = $2 \times 2 \ \times 2$
= 2^3

It is a terminating decimal.
When we divide $\frac{17}{8}$, we get

	0	2.	1	2	5	0
8	1	7.	0	0	0	0
-	0					
	1	7				
-	1	6				
		1	0			
	-		8			
			2	0		
			1	6		
				4	0	
			-	4	0	
					0	0
				-		0
						0

$\frac{17}{8} = 2.125$

(ii) $\frac{13}{3125}$

5	3125
5	625
5	125
5	25
5	5
	1

$3125 = 5 \times 5 \times 5 \times 5 \times 5$

Prime factor of 3125 = 5, 5, 5, 5, 5 [i. e in the form of 2^n , 5^n]
It is a terminating decimal.

When we divide $\frac{13}{3125}$, we get

$$
\begin{array}{r|l}
 & 0\;0.\;0\;0\;4\;1\;6 \\ \hline
3125 & 1\;3.\;0\;0\;0\;0\;0 \\
 & -\;0 \\ \cline{2-2}
 & 1\;3 \\
 & -\;\;\;0 \\ \cline{2-2}
 & 1\;3\;0 \\
 & -\;\;\;\;\;0 \\ \cline{2-2}
 & 1\;3\;0\;0 \\
 & \;\;\;\;\;\;\;\;0 \\ \cline{2-2}
 & 1\;3\;0\;0\;0 \\
 & -\;1\;2\;5\;0\;0 \\ \cline{2-2}
 & \;\;\;\;5\;0\;0\;0 \\
 & -\;3\;1\;2\;5 \\ \cline{2-2}
 & \;\;\;\;1\;8\;7\;5\;0 \\
 & -\;1\;8\;7\;5\;0 \\ \cline{2-2}
 & \;\;\;\;\;\;\;\;\;\;\;\;0 \\ \cline{2-2}
\end{array}
$$

$\frac{13}{3125} = 0.00416$

(iii) $\frac{7}{80}$

2	80
2	40
2	20
2	10
5	5
	1

$80 = 2 \times 2 \times 2 \times 2 \times 2$

Prime factor of 80 = 2^4, 5^1 [i.e in the form of 2^n , 5^n]

It is a terminating decimal.

When we divide $\frac{7}{80}$ we, get

		0.	0	8	7	5
8	0	7.	0	0	0	0
	-	0				
		7	0			
	-		0			
		7	0	0		
	-	6	4	0		
			6	0	0	
		-	5	6	0	
				4	0	0
			-	4	0	0
						0

$\frac{7}{80} = 0.0875$

(iv) $\frac{6}{15}$

Let us divide both numerator and denominator by 3, we get

$\frac{6}{15} = \frac{(6 \div 3)}{(15 \div 3)}$

$= \frac{2}{5}$

Since the denominator is 5,
It is terminating decimal.

		0.	4	0
1	5	6.	0	0
	-	0		
		6	0	
	-	6	0	
			0	0
		-		0
				0

$\frac{6}{15} = 0.4$

(v) $\frac{2^2 \times 7}{5^4}$

We know that the denominator is 5^4 it is a terminating decimal.

$$\frac{2^2 \times 7}{5^4} = \frac{(2 \times 2 \times 7)}{(5 \times 5 \times 5 \times 5)}$$

$$= \frac{28}{625}$$

			0	0.	0	4	4	8
6	2	5	2	8.	0	0	0	0
		-	0					
			2	8				
		-		0				
			2	8	0			
		-			0			
			2	8	0	0		
		-	2	5	0	0		
				3	0	0	0	
			-	2	5	0	0	
					5	0	0	0
				-	5	0	0	0
								0

$\frac{28}{625} = 0.0448$

It is a terminating decimal.

(vi) $\frac{237}{1500}$

Let us divide both numerator and denominator by 3, we get

$\frac{237}{1500} = \frac{(237 \div 3)}{(1500 \div 3)}$

$= \frac{79}{500}$

Since the denominator is 500,
Its factors are, $500 = 2 \times 2 \times 5 \times 5 \times 5$

$= 2^2 \times 5^3$

It is terminating decimal.

			0	0.	1	5	8
5	0	0	7	9.	0	0	0
		-	0				
			7	9			
		-		0			
			7	9	0		
		-	5	0	0		
			2	9	0	0	
		-	2	5	0	0	
				4	0	0	0
			-	4	0	0	0
							0

$\frac{237}{1500} = \frac{79}{500} = 0.1518$

Write the denominator of the rational number $\frac{257}{5000}$ in the form $2^m \times 5^n$
Where m , n is non – negative integers. Hence, write its decimal
Expansion on without actually division.

Solution:
Given :

The fraction = $\frac{257}{500}$

Since the denominator is 5000,
The factors for 5000 are:

2	5000
2	2500
2	1250
5	625
5	125
5	25
5	5
	1

$5000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$
$= 2^3 \times 5^4$
$\frac{257}{5000} = \frac{257}{2^3 \times 5^4}$

It is a terminating decimal.
So,
Let us multiply both numerator and denominator by 2, we get

$\frac{257}{5000} = \frac{(257 \times 2)}{(5000 \times 2)}$
$= \frac{514}{10000}$

$= 0.0514$

Question 7
Write the decimal expansion of $\frac{1}{7}$. hence, write the decimal Expression of ? $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, and\ \frac{6}{7}$

Solution:
Given :
The fraction : $\frac{1}{7}$

```
     0. 1 4 2 8 5 7 1 4 2 8 5 7
7 | 1. 0 0 0 0 0 0 0 0 0 0 0 0
-    0
    ----
     1 0
   -   7
    ------
       3 0
   -   2 8
       ----
         2 0
      -  1 4
         -----
           6 0

           5 6
           -----
             4 0
          -  3 5
             ----
               5 0
            -  4 9
               -----
                 1 0
              -    7
                 -----
                   3 0
              -    2 8
                   -----
                     2 0
                  -  1 4
                     -----
                       6 0
                    -  5 6
                       -----
                         4 0
                      -  3 5
                         -----
                           5 0
                        -  4 9
                           -----
                               1
                           -----
```

$\frac{1}{7} = 0.142857142857$

Since it is recurring ,

$= 0.\overline{142857}$

Now,

$\frac{2}{7} = 2 \times \left(\frac{1}{7}\right)$

$= 2 \times 0.\overline{142857}$

$= 0.\overline{285714}$

$\frac{3}{7} = 3 \times \left(\frac{1}{7}\right)$

$= 3 \times 0.\overline{142857}$

$= 0.\overline{428571}$

$\frac{4}{7} = 4 \times \left(\frac{1}{7}\right)$

$= 4 \times 0.\overline{142857}$

$= 0.\overline{714285}$

$\frac{6}{7} = 6 \times \left(\frac{1}{7}\right)$

$= 6 \times 0.\overline{142857}$

$= 0.\overline{857142}$

Question 8.

Express the following numbers in the form $\frac{p}{q}$. where p and q are both integers and $q \neq 0$;

(i) $0.\overline{3}$

(ii) $5.\overline{2}$

(iii) 0.404040.....

(iv) $0.4\overline{7}$

(v) $0.1\overline{34}$

(vi) $0.\overline{001}$

Solution:

(i) $0.\overline{3}$

Let x = $0.\overline{3}$ = 0.3333....

Since there is one repeating digit after the decimal point,
Multiplying by 10 on both sides, we get
10x = 3.3333.....
Now, subtract both the values,

9x = 3
$X = \frac{3}{9}$

$= \frac{1}{3}$

$0.\overline{3} = \frac{1}{3}$

(ii) 5.2

Let x = $5..\overline{2}$ = 5.2222....
Since there is one repeating digit after the decimal point,
Multiplying by 10 on both sides, we get
10x = 52.2222....
Now, subtract both the values,
9x = 52 = 52 – 5
9x = 47
$X = \frac{47}{9}$
$5..\overline{2} = \frac{47}{9}$

(iii) 0.404040.....
Let x = 0.404040
Since there is two repeating digit after the decimal point,
Multiplying by 100 on both sides, we get

$100x = 40.404040....$

Now, subtract both the values,

$99x = 40$

$X = \frac{40}{99}$ $0.404040.... = \frac{40}{99}$

(iv) $0.4\overline{7}$

Let $x = 0 . 4\overline{7}$ $= 0.47777.....$

Since there is one non – repeating digit after the decimal point,

Multiplying by 10 on both sides, we get

$10x = 4.7777$

Since there is one repeating digit after the decimal point,

Multiplying by 10 on both sides, we get

$100x = 47.7777$

Now, subtract both the values,

$90x = 47 – 4$

$90x = 43$

$X = \frac{43}{90}$

$0.4\overline{7} = \frac{43}{90}$

(v) $0.1\overline{34}$

Let $x = 0.1\overline{34} = 0.13434343....$

Since there is one non- repeating digit after the decimal point,

Multiplying by 10 on both sides, we get

$10x = 1.343434$

Since there is two repeating digit after the decimal point,

Multiplying by 100 on both sides, we get

$1000x = 134.343434$

Now, subtract both the values,

$990x = 133$

$X = \frac{133}{990}$

$0.1\overline{34} = \frac{133}{990}$

(vi) $0.\overline{001}$

Let x = $0.\overline{001}$ = 0.001001001...
Since There is three repeating digit after the decimal point,
Multiplying by 1000 on both sides, we get
1000x = 1.001001
Now, subtract both the values,
999x = 1
$X = \frac{1}{999}$

$0.\overline{001} = \frac{1}{999}$

Question 9 .
Classify the following numbers as rational or irrational:
(i) $\sqrt{23}$
(ii) $\sqrt{225}$
(iii) 0. 3796
(iv) 7.478478
(v) 1.101001000100001....
(vi) 345. 0. $\overline{456}$

Solution:
(i) $\sqrt{23}$

Since , 23 is not a perfect square,
$\sqrt{23}$ is an irrational number.

(ii) $\sqrt{225}$

$\sqrt{225} = \sqrt{(15)^2} = 15$
Since , 225 is a perfect square,
$\sqrt{225}$ = is a rational number.

(iii) 0.3796

$0.3796 = \frac{3796}{1000}$
Since , the decimal expansion is terminating decimal.
0.3796 is a rational number.

(iv) 7.478478

Let x = 7.478478

Since there is three repeating digit after the decimal point,

Multiplying by 1000 on both sides, we get

1000x = 7478.478478....

Now, subtract both the values,

999x = 7478 – 7

999x = 7471

$X = \frac{7471}{999}$

$7.478478 = \frac{7471}{999}$

Hence, it is neither terminating nor non – terminating or non – repeating decimal.
7.478478 is an irrational number.

(v) 1.101001000100001....

Since number of zero's between two consecutive ones are increasing.
So it is non – terminating or non – repeating decimal.
1.101001000100001... is an irrational number.

(vi) $345.0\overline{456}$

Let x = 345. 0456456

Multiplying by 10 on both sides, we get

10x = 3450. 456456

Since there is three repeating digit after the decimal point,

Multiplying by 1000 on both sides, we get

1000x = 3450456.456456....

Now, subtract both the values,

10000x – 10x = 3450456 – 345

9990x = 3450111

$X = \frac{3450111}{9990}$

Since, it is non- terminating repeating decimal.

$345.0\overline{456}$ is a rational number.

Question 10.
Insert.... following .

(i) One irrational number between $\frac{1}{3}$ and $\frac{1}{2}$

(ii) One irrational number between $\frac{2}{5}$ and $\frac{1}{2}$

(iii) One irrational number between 0 and 0.1

Solution:
(i) One irrational number between $\frac{1}{3}$ and $\frac{1}{2}$

```
       0.  3   3   3
3   | 1.   0   0   0
-     0
     1   0
-        9
         1   0
   -         9
             1
```

$\frac{1}{3} = 0.333$

```
       0.   5
2   | 1.    0
-     0
      1     0
-     1     0
            0
```

$$\frac{1}{2} = 0.5$$

So there are infinite irrational numbers between $\frac{1}{3}$ and $\frac{1}{2}$

One irrational number among them can be 0.4040040004....

(ii) One irrational number between $\frac{2}{5}$ *and* $\frac{1}{2}$

```
      -  0. 4
 + 5 | - 2. 0
   -     0
         2  0
      -  2  0
            0
```

$\frac{-2}{5}$ = -0.4

```
    0.  5
 2 |1.  0
 -  0
    1   0
 -  1   0
        0
```

$\frac{1}{2}$ = 0.5

So there are infinite irrational numbers between $\frac{2}{5}$ *and* $\frac{1}{2}$
One irrational number among them can be 0.1010010001....

(iii) One irrational number between 0 and 0.1

There are infinite irrational numbers between 0 and 1.
One irrational number among them can be 0.06006000600006....

Question 11.

Insert two irrational numbers between 2 and 3.

Solution:

2 is expressed as $\sqrt{4}$

And 3 is expressed as $\sqrt{9}$

So, two irrational numbers between 2 and 3 or $\sqrt{4}$ and $\sqrt{9}$ are , $\sqrt{5}$ $\sqrt{6}$

Question 12.
Write two irrational numbers between $\frac{4}{9}$ *and* $\frac{7}{11}$.

Solution :

$\frac{4}{9}$ *is expressed as* 0.4444......

$\frac{7}{11}$ is expressed as 0.636363....

So, two irrational numbers between $\frac{4}{9}$ *and* $\frac{7}{11}$ are 0.4040040004....and 0.6060060006...

Question 13.
Find one rational number between $\sqrt{2}$ and $\sqrt{3}$

Solution:
$\sqrt{2}$ and $\sqrt{3}$ is 1.5

Question 14.

Find two rational numbers between $\sqrt{12}$ and $\sqrt{15}$

Solution:

$\sqrt{12} = \sqrt{(4 \times 3)} = 2\sqrt{3}$

Since, 12 < 12.25 < 12.96 < 15

So, $\sqrt{12} < \sqrt{12.25} < \sqrt{12.96} < \sqrt{15}$

Hence, two rational numbers between $\sqrt{12}$ *and* $\sqrt{15}$ are

[$\sqrt{12.25}$, $\sqrt{12.96}$] or [$\sqrt{3.5}$, $\sqrt{3.6}$] .

Question 15.

Insert irrational numbers between $\sqrt{5}$ and $\sqrt{7}$.

Solution:

Since , 5 < 6 < 7

So, irrational number between $\sqrt{5}$ and $\sqrt{7}$ is $\sqrt{6}$

Question 16.

Insert two irrational numbers between $\sqrt{3}$ and $\sqrt{7}$

Solution:

Since, $3 < 4 < 5 < 6 < 7$

So,

$\sqrt{3} < \sqrt{4} < \sqrt{5} < \sqrt{6} < \sqrt{7}$

But $\sqrt{4} = 2$, which is a rational number .

So,

Two irrational numbers between $\sqrt{3}$ and $\sqrt{7}$ are $\sqrt{5}$ and $\sqrt{6}$

Exercise 1.4

Question 1.

Simplify the following :

(i) $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$

(ii) $3\sqrt{3} + 2\sqrt{27} + \frac{7}{\sqrt{3}}$

(iii) $6\sqrt{5} \times 2\sqrt{5}$

(iv) $8\sqrt{15} \div 2\sqrt{3}$

(v) $\frac{\sqrt{24}}{8} + \frac{\sqrt{54}}{9}$

(vi) $\frac{3}{\sqrt{8}} + \frac{1}{\sqrt{2}}$

Solution:

(i) $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$

Let us simplify the expression ,

$\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$

$= \sqrt{9 \times 5} - 3\sqrt{4 \times 5} + 4\sqrt{5}$

$= 3\sqrt{5} - 3 \times 2\sqrt{5} + 4\sqrt{5}$

$= 3\sqrt{5} - 6\sqrt{5} + 4\sqrt{5}$

$= \sqrt{5}$

(ii) $3\sqrt{3} + 2\sqrt{27} + \frac{7}{\sqrt{3}}$

Let us simplify the expression,

$3\sqrt{3} + 2\sqrt{27} + \frac{7}{\sqrt{3}}$

$3\sqrt{3} + 2\sqrt{(9 \times 3)} + \frac{7\sqrt{3}}{(\sqrt{3} \times \sqrt{3})}$ (By rationalizing)

$= 3\sqrt{3} + (2 \times 3)\sqrt{3} + \frac{7\sqrt{3}}{3}$

$= 3\sqrt{3} + 6\sqrt{3} + \left(\frac{7}{3}\right)\sqrt{3}$

$= \sqrt{3}\left(3 + 6 + \frac{7}{3}\right)$

$= \sqrt{3}\left(9 + \frac{7}{3}\right)$

$= \sqrt{3}\,\frac{(27+7)}{3}$

$= \frac{34}{3}\sqrt{3}$

(iii) $6\sqrt{5} \times 2\sqrt{5}$

Let us simplify the expression,

$6\sqrt{5} + 2\sqrt{5}$

$= 12 \times 5$

$= 60$

(iv) $8\sqrt{15} \div 2\sqrt{3}$

Let us simplify the expression,

$8\sqrt{15} \div 2\sqrt{3}$

$= \frac{(8\sqrt{5}\sqrt{3})}{2\sqrt{3}}$

$= 4\sqrt{5}$

(v) $\frac{\sqrt{24}}{8} + \frac{\sqrt{54}}{9}$

Let us simplify the expression,

$\frac{\sqrt{24}}{8} + \frac{\sqrt{54}}{9}$

$= \frac{\sqrt{(4\times 6)}}{8} + \frac{\sqrt{(9\times 6)}}{9}$

$= \frac{2\sqrt{6}}{8} + \frac{3\sqrt{6}}{9}$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{6}}{3}$

By taking LCM

$= \frac{3\sqrt{6} + 4\sqrt{6}}{12}$

$= \frac{7\sqrt{6}}{12}$

(vi) $\frac{3}{\sqrt{8}} + \frac{1}{\sqrt{2}}$

Let us simplify the expression,

$\frac{3}{\sqrt{8}} + \frac{1}{\sqrt{2}}$

$= \frac{3}{2}\sqrt{2} + \frac{1}{\sqrt{2}}$

By taking LCM

$= \frac{(3+2)}{2\sqrt{2}}$

$= \frac{5}{2\sqrt{2}}$

By rationalizing,

$= \frac{5\sqrt{2}}{2\sqrt{2} \times 2\sqrt{2}}$

$= \frac{5\sqrt{2}}{2\times 2}$

$= \frac{5\sqrt{2}}{4}$

Question 2.

Simplify the following :

(i) $(5 + \sqrt{7})(2 + \sqrt{5})$

(ii) $(5 + \sqrt{5})(5 - \sqrt{5})$

(iii) $(\sqrt{5} + \sqrt{2})^2$

(iv) $(\sqrt{3} + \sqrt{7})^2$

(v) $(\sqrt{2} + \sqrt{3})(\sqrt{5} + \sqrt{7})$

(vi) $(4 + \sqrt{5})(\sqrt{3} - \sqrt{7})$

Solution:

(i) $(5 + \sqrt{7})(2 + \sqrt{5})$

Let us simplify the expression,
$= 5(2 + \sqrt{5}) + \sqrt{7}(2 + \sqrt{5})$

$= 10 + 5\sqrt{5} + 2\sqrt{7} + \sqrt{35}$

(ii) $(5 + \sqrt{5}) (5 - \sqrt{5})$

Let us simplify the expression,
By using the formula,

$(a)^2 – (b)^2 = (a + b) (a – b)$

So,
$= (5)^2 – (\sqrt{5})^2$

$= 25 – 5$

$= 20$

(iii) $\left(\sqrt{5} + \sqrt{2}\right)^2$

Let us simplify the expression,
By using the formula,

$(a + b)^2 = a^2 + b^2 + 2ab$

$\left(\sqrt{5} + \sqrt{2}\right)^2 = (5)^2 + (\sqrt{2})^2 + 2\sqrt{5}\sqrt{2}$

$= 5 + 2 + 2 \sqrt{10}$

$= 7 + 2\sqrt{10}$

(iv) $(\sqrt{3}-\sqrt{7})^2$

Let us simplify the expression,
By using the formula,

$(a-b)^2 = a^2 + b^2 - 2ab$

$(\sqrt{3}-\sqrt{7})^2 = (\sqrt{3})^2 + (\sqrt{7})^2 - 2\sqrt{3}\sqrt{7}$

$= 3 + 7 - 2\sqrt{21}$

$= 10 - 2\sqrt{21}$

(v) $(\sqrt{2}+\sqrt{3})(\sqrt{5}+\sqrt{7})$

Let us simplify the expression,

$= \sqrt{2}(\sqrt{5}+\sqrt{7})\sqrt{3}(\sqrt{5}+\sqrt{7})$

$= \sqrt{2}\times\sqrt{5}\times\sqrt{2}\times\sqrt{7}\times\sqrt{3}\times\sqrt{5}\times\sqrt{3}\times\sqrt{7}$

$= \sqrt{10}+\sqrt{14}+\sqrt{15}+\sqrt{21}$

(vi) $(4+\sqrt{5})(\sqrt{3}-\sqrt{7})$

Let us simplify the expression,

$= 4(\sqrt{3}-\sqrt{7}) + \sqrt{5}(\sqrt{3}-\sqrt{7})$

$= 4\sqrt{3} - 4\sqrt{7} + \sqrt{15} - \sqrt{35}$

Question 3.

If $\sqrt{2}$ = 1.414 , then find the value of

(i) $\sqrt{8} + \sqrt{50} + \sqrt{72} + \sqrt{98}$

(ii) $3\sqrt{32}$ - $2\sqrt{50}$ + $4\sqrt{128}$ - $20\sqrt{18}$

Solution:

(i) $\sqrt{8} + \sqrt{50} + \sqrt{72} + \sqrt{98}$

Let us simplify the expression,

$\sqrt{8} + \sqrt{50} + \sqrt{72} + \sqrt{98}$

= $\sqrt{2 \times 4} + \sqrt{2 \times 25} + \sqrt{2 \times 36} + \sqrt{2 \times 49}$

= $\sqrt{2}\sqrt{4} + \sqrt{2}\sqrt{25} + \sqrt{2}\sqrt{36} + \sqrt{2}\sqrt{49}$

= $20\sqrt{2}$

= 20×1.141

= 28.28

(ii) $3\sqrt{32}$ - $2\sqrt{50}$ + $4\sqrt{128}$ - $20\sqrt{18}$

Let us simplify the expression,

$3\sqrt{32}$ - $2\sqrt{50}$ + $4\sqrt{128}$ - $20\sqrt{18}$

$3\sqrt{16 \times 2}$ - $2\sqrt{25 \times 2}$ + $4\sqrt{64 \times 2}$ - $20\sqrt{9 \times 2}$

= $3\sqrt{16}\sqrt{2} - 2\sqrt{25}\sqrt{2} + 4\sqrt{64}\sqrt{2} - 20\sqrt{9}\sqrt{2}$

$= 3.4\sqrt{2} - 2.5\sqrt{2} + 4.8\sqrt{2} - 20.3\sqrt{2}$

$= 12\sqrt{2} - 10\sqrt{2} + 32\sqrt{2} - 60\sqrt{2}$

$= (12 – 10 + 32 – 60) \sqrt{2}$

$= -26 \sqrt{2}$

$= -26 \times 1.414$

$= - 36 . 764$

Question 4.
If $\sqrt{3} = 1.732$, then find the value of

(i) $\sqrt{27} + \sqrt{75} + \sqrt{108} - \sqrt{243}$

(ii) $5\sqrt{12} - 3\sqrt{48} + 6 \sqrt{75} + 7\sqrt{108}$

Solution:
(i) $\sqrt{27} + \sqrt{75} + \sqrt{108} - \sqrt{243}$

Let us simplify the expression,

$\sqrt{27} + \sqrt{75} + \sqrt{108} - \sqrt{243}$

$= \sqrt{9 \times 3} + \sqrt{25 \times 3} + \sqrt{36 \times 3} - \sqrt{81 \times 3}$

$= \sqrt{9}\sqrt{3} + \sqrt{25}\sqrt{3} + \sqrt{36}\sqrt{3} – \sqrt{81}\sqrt{3}$

$= 3\sqrt{3} + 5\sqrt{3} + 6\sqrt{3} – 9\sqrt{3}$

$= (3 + 5 + 6 – 9)$

$= 5\sqrt{3}$

$= 5 \times 1.732$

$= 8.660$

(ii) $5\sqrt{12} - 3\sqrt{48} + 6\sqrt{75} + 7\sqrt{108}$

Let us simplify the expression,

$5\sqrt{12} - 3\sqrt{48} + 6\sqrt{75} + 7\sqrt{108}$

$= 5\sqrt{(4 \times 3)} - 3\sqrt{(16 \times 3)} + 6\sqrt{(25 \times 3)} + 7\sqrt{36 \times 3}$

$= 5\sqrt{4}\sqrt{3} - 3\sqrt{16}\sqrt{3} + 6\sqrt{25}\sqrt{3} + 7\sqrt{36}\sqrt{3}$

$= 5.2\sqrt{3} - 3.4\sqrt{3} + 6.5\sqrt{3} + 7.6\sqrt{3}$

$= 10\sqrt{3} - 12\sqrt{3} + 30\sqrt{3} + 42\sqrt{3}$

$= (10 - 12 + 30 + 42)\sqrt{3}$

$= 70\sqrt{3}$

$= 70 \times 1.732$

$= 121.24$

Question 5.

State which of the following are rational or irrational decimals.

(i) $\sqrt{\left(\frac{4}{9}\right)}, \frac{-3}{70}, \sqrt{\left(\frac{7}{25}\right)}, \sqrt{\left(\frac{16}{5}\right)}$

(ii) $-\sqrt{\left(\frac{2}{49}\right)}, \frac{3}{200}, \sqrt{\left(\frac{25}{3}\right)}, -\sqrt{\left(\frac{49}{16}\right)}$

Solution:

(i) $\sqrt{\left(\frac{4}{9}\right)}, \frac{-3}{70}, \sqrt{\left(\frac{7}{25}\right)}, \sqrt{\left(\frac{16}{5}\right)}$

$\sqrt{\left(\frac{4}{9}\right)} = \frac{2}{3}$

$\frac{-3}{70} = \frac{-3}{70}$

$\sqrt{\left(\frac{7}{25}\right)} = \frac{\sqrt{7}}{5}$

$\sqrt{\left(\frac{16}{5}\right)} = \frac{4}{\sqrt{5}}$

So,

$\frac{\sqrt{7}}{5}$ *and* $\frac{4}{\sqrt{5}}$ are irrational decimals.

$\frac{2}{3}$ *and* $\frac{-3}{70}$ are rational decimals.

(ii) $-\sqrt{\left(\frac{2}{49}\right)}, \frac{3}{200}, \sqrt{\left(\frac{25}{3}\right)}, -\sqrt{\left(\frac{49}{16}\right)}$

$-\sqrt{\left(\frac{2}{49}\right)} = -\frac{\sqrt{2}}{7}$

$\frac{3}{200} = \frac{3}{200}$

$\sqrt{\left(\frac{25}{3}\right)} = \frac{5}{\sqrt{3}}$

$-\sqrt{\left(\frac{49}{16}\right)} = \frac{-7}{4}$

So,

- $\frac{\sqrt{2}}{7}$ *and* $\frac{5}{\sqrt{3}}$ are irrational decimals.

$\frac{3}{200}$ *and* $\frac{-7}{4}$ are rational decimals .

Question 6.

State which of the following are rational or irrational decimals.

(i) $-3\sqrt{2}$

(ii) $\sqrt{\frac{256}{81}}$

(iii) $\sqrt{\frac{27}{16}}$

(iv) $\sqrt{\frac{5}{36}}$

Solution:

(i) $-3\sqrt{2}$

We know that $\sqrt{2}$ is an irrational number.

So, $-3\sqrt{2}$ will also be irrational number.

(ii) $\sqrt{\frac{256}{81}}$

$\sqrt{\frac{256}{81}} = \frac{16}{9} = \frac{4}{3}$

It is a rational number.

(iii) $\sqrt{\frac{27}{16}}$

$\sqrt{\frac{27}{16}} = \sqrt{9 \times 3 \times 16} = 3 \times 4\sqrt{3} = 12\sqrt{3}$

It is an irrational number.

(iv) $\sqrt{\frac{5}{36}}$

$\sqrt{\frac{5}{36}} = \frac{\sqrt{5}}{6}$

It is an irrational number.

Question 7.
State which of the following are irrational numbers.

(i) $3 - \sqrt{\frac{7}{25}}$

(ii) $-\frac{2}{3} + 3\sqrt{2}$

(iii) $\frac{3}{\sqrt{3}}$

(iv) $\frac{-2}{7}\,3\sqrt{5}$

(v) (2 - $\sqrt{3}$) (2 + $\sqrt{3}$)

(vi) $(3 + \sqrt{5})^2$

(vii) $\left(\frac{2}{5}\,\sqrt{7}\right)^2$

(viii) $(3 - \sqrt{6})^2$

Solution:

(i) 3 - $\sqrt{\frac{7}{25}}$

Let us simplify,

$3 - \sqrt{\frac{7}{25}} = 3 - \sqrt{\frac{7}{25}}$

$3 - \frac{\sqrt{7}}{5}$

Hence, 3 - $\frac{\sqrt{7}}{5}$ is an irrational number.

(ii) - $\frac{2}{3}$ + $3\sqrt{2}$

Let us simplify,

$- \frac{2}{3} + 3\sqrt{2} \;=\; - \frac{2}{3} + 2^{\frac{1}{3}}$

Since, 2 is not a perfect cube.
Hence it is an irrational number.

(iii) $\frac{3}{\sqrt{3}}$

Let us simplify,

By rationalizing. We get

$\frac{3}{\sqrt{3}} = \frac{3\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$

$= \frac{3\sqrt{3}}{3}$

$= \sqrt{3}$

Hence, $\frac{3}{\sqrt{3}}$ is an irrational number.

(iv) $\frac{-2}{7}3\sqrt{5}$

Let us simplify,

$\frac{-2}{7}3\sqrt{5} = \frac{-2}{7}5^{\frac{1}{3}}$

Since, 5 is not a perfect cube.
Hence it is an irrational number.

(v) (2 - $\sqrt{3}$) (2 + $\sqrt{3}$)

Let us simplify,

By using the formula,

(a + b) (a – b) = $(a)^2$ $(b)^2$

$(2 - \sqrt{3})(2 + \sqrt{3}) = 3^2 + (\sqrt{5})^2 + 2.3\sqrt{5}$

$= 9 + 5 + 6\sqrt{5}$

$= 14 + 6\sqrt{5}$

Hence, it is an irrational number.

(vii) $(\frac{2}{5}\sqrt{7})^2 = (\frac{2}{5}\sqrt{7})^2 \times (\frac{2}{5}\sqrt{7})^2$

$= \frac{4}{25} \times 7$

$= \frac{28}{25}$

Hence it is a rational number.

(viii) $(3 - \sqrt{6})^2$

Let us simplify,

By using $(a - b)^2 = a^2 + b^2 - 2ab$

$(3 - \sqrt{6})^2 = 3^2 + (\sqrt{6})^2 - 2.3\sqrt{6}$

$= 9 + 6 - 6\sqrt{6}$

$= 15 - 6\sqrt{6}$

Hence it is an irrational number.

Question 8.

Prove the following are irrational numbers.

(i) $3\sqrt{2}$

(ii) $\sqrt[3]{3}$

(iii) $\sqrt[4]{5}$

Solution:

(i) $3\sqrt{2}$

We know that $3\sqrt{2} = 2^{\frac{1}{3}}$

Let us consider $2^{\frac{1}{3}} = \frac{p}{q}$, where p, q are integers, q > 0 .

P and q have no common factors (except 1).

So,

$2^{\frac{1}{3}} = \frac{p}{q}$

$2 = \frac{p^3}{q^3}$

$P^3 = 2q^3$ ...... (1)

We know that, 2 divides $2q^3$ then 2 divides p^3

So, 2 divides p

Now, let us consider p = 2k, where k is an integer

Substitute the value of p in (1) , we get

$P^3 = 2q^3$

$(2k)^3 = 2q^3$

$8k^3 = 2q^3$

$4k^3 = q^3$

We know that, 2 divides $4k^3$ then 2 divides q^3

So, 2 divides q

Thus p and q have a common factor ‘2’

This contradicts the statement, p and q have no common factor (except 1) .

Hence, $3\sqrt{2}$ is an irrational number.

(ii) $\sqrt[3]{3}$

We know that $\sqrt[3]{3} = 3^{\frac{1}{3}}$

Let us consider $3^{\frac{1}{3}} = \frac{p}{q}$, where p, q are integers, q > 0

P and q have no common factors (except 1.)

So,

$3^{\frac{1}{3}} = \frac{p}{q}$

$3 = \frac{p^3}{q^3}$

$P^3 = 3q^3$..... (1)

We know that , 3 divides $3q^3$ then 3 divides p^3

So, 3 divides p

Now, let us consider p = 3k, where k is an integer

Substitute the value of p in (1) , we get

$P^2 = 3q^3$

$(3k)^3 = 3q^3$

$9k^3 = 3q^3$

$3k^3 = q^3$

We know that , 3 divides $9k^3$ then 3 divides q^3

So, 3 divides q

Thus p and q have a common factor '3'

This contradicts the statement, p and q have no common factor (expect 1).

Hence, $\sqrt[3]{3}$ is an irrational number.

(iii) $\sqrt[4]{5}$

We know that $\sqrt[4]{5} = 5^{\frac{1}{4}}$

Let us consider $5^{\frac{1}{4}} = \frac{p}{q}$, where p, q are integers, q > 0.

P and q have no common factors (except 1)

So,

$5^{\frac{1}{4}} = \frac{p}{q}$

$5 = \frac{p^4}{q^4}$

$P^4 = 5q^4$..... (1)

We know that, 5 divides $5q^4$ then 5 divides p^4

So, 5 divides p

Now, let us consider p = 5k, where k is an integer

Substitute the value of p in (1) , we get

$P^4 = 5q^4$

$(5k)^4 = 5q^4$

$625k^4 = 5q^4$

$125\ k^4 = q^4$

We know that, 5 divides $125k^4$ then 5 divides q^4

So, 5 divides q

Thus p and q have a common factor '5'

This contradicts the statement, p and q have no common factor (except 1).

Hence, $\sqrt[4]{5}$ is an irrational number.

Question 9.

Find the greatest and the smallest real numbers.

(i) $2\sqrt{3}$, $\frac{3}{\sqrt{2}}$, -7 , $\sqrt{15}$

(ii) -3 $\sqrt{2}$, $\frac{9}{\sqrt{5}}$, -4 , $\frac{4}{3}\sqrt{5}$, $\frac{3}{2}\sqrt{3}$

Solution:

(i) $2\sqrt{3}$, $\frac{3}{\sqrt{2}}$, -7 , $\sqrt{15}$

Let us simplify each fraction

$2\sqrt{3} = \sqrt{(4 \times 3)} = \sqrt{12}$

$\frac{3}{\sqrt{2}} = \frac{3\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = 3\,\frac{\sqrt{2}}{2} = \sqrt{\left(\frac{9}{4}\right) \times 2} = \sqrt{\frac{9}{2}} = \sqrt{4.5}$

$-\sqrt{7} = -\sqrt{7}$

$\sqrt{15} = \sqrt{15}$

So,

The greatest real number = $\sqrt{15}$

Smallest real number = - $\sqrt{7}$

(ii) -3 $\sqrt{2}$, $\frac{9}{\sqrt{5}}$, -4 , $\frac{4}{3}\sqrt{5}$, $\frac{3}{2}\sqrt{3}$

Let us simplify each fraction.

-3$\sqrt{2}$ = -$\sqrt{(9 \times 2)}$ = -$\sqrt{18}$

$\frac{9}{\sqrt{5}} = \frac{(9\times\sqrt{5})}{(\sqrt{5}\times\sqrt{5})} = \frac{9\sqrt{5}}{5} = \sqrt{\left(\frac{81}{25}\right)\times 5} = \sqrt{\frac{81}{5}} = \sqrt{16.2}$

-4 = -$\sqrt{16}$

$\frac{4}{3}\sqrt{5} = \sqrt{\left(\frac{16}{9}\right)\times 5} = \sqrt{\frac{80}{9}} = \sqrt{8.88} = \sqrt{8.8}$

$\frac{3}{2}\sqrt{3} = \sqrt{\frac{9}{4}}\times 3 = \sqrt{\frac{27}{4}} = \sqrt{6.25}$

So,

The greatest real number = 9 $\sqrt{5}$

Smallest real number = -3$\sqrt{2}$

Question 10.

Write in ascending order.

(i) 3$\sqrt{2}$ = $\sqrt{9 \times 2}$ = $\sqrt{18}$

2$\sqrt{3}$ = $\sqrt{4 \times 3}$ = $\sqrt{12}$

$\sqrt{15}$ = $\sqrt{15}$

4 = $\sqrt{16}$

Now, let us arrange in ascending order

$\sqrt{12}, \sqrt{15}, \sqrt{16}, \sqrt{18}$

So, $2\sqrt{3}, \sqrt{15}, 4, 3\sqrt{2}$

(ii) $3\sqrt{2}, 2\sqrt{8}, 4, \sqrt{50}, 4\sqrt{3}$

$3\sqrt{2} = \sqrt{9 \times 2} = \sqrt{18}$

$2\sqrt{8} = \sqrt{4 \times 8} = \sqrt{32}$

$4 = \sqrt{16}$

$\sqrt{50} = \sqrt{50}$

$4\sqrt{3} = \sqrt{16 \times 3} = \sqrt{48}$

Now, let us arrange in ascending order.

$\sqrt{16}\ \sqrt{18}\ \sqrt{32}\ \sqrt{48}\ \sqrt{50}$

So, $4, 3\sqrt{2}, 4\sqrt{3}, \sqrt{50}$

Question 11.

Write in descending order:

(i) $\frac{9}{\sqrt{2}}, \frac{3}{2}\sqrt{5}, 4\sqrt{3}, 3\sqrt{\frac{6}{5}}$

(ii) $\frac{5}{\sqrt{3}}, \frac{7}{3}\sqrt{2} - \sqrt{3}, 3\sqrt{5}, 2\sqrt{7}$

Solution:

$\frac{9}{\sqrt{2}}, \frac{3}{2}\sqrt{5}, 4\sqrt{3}, 3\sqrt{\frac{6}{5}}$

$\frac{9}{\sqrt{2}} = \frac{9\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \frac{9\sqrt{2}}{2} = \sqrt{\frac{81}{4}} \times 2 = \sqrt{\frac{81}{2}} = \sqrt{40.5}$

$\frac{3}{2}\sqrt{5} = \sqrt{\frac{9}{4}} \times 5 = \sqrt{\frac{45}{4}} = \sqrt{11.25}$

$4\sqrt{3} = \sqrt{(16 \times 3)} = \sqrt{48}$

$3\sqrt{\frac{6}{5}} = \sqrt{\frac{(9\times6)}{5}} = \sqrt{\frac{54}{5}} = \sqrt{10.8}$

Now, let us arrange in descending order

$\sqrt{48}, \sqrt{40.5}, \sqrt{11.25}, \sqrt{10.8}$

So,

$4\sqrt{3}, \frac{9}{\sqrt{2}}, \frac{3}{2}\sqrt{5}, 3\sqrt{\frac{6}{5}}$

(ii) $\frac{5}{\sqrt{3}}, \frac{7}{3}\sqrt{2}, -\sqrt{3}, 3\sqrt{5}, 2\sqrt{7}$

$\frac{5}{\sqrt{3}} = \sqrt{\frac{25}{3}} = \sqrt{8.33}$

$\frac{7}{3}\sqrt{2} = \sqrt{\frac{49}{9}} \times 2 = \sqrt{\frac{98}{9}} = \sqrt{10.88}$

$-\sqrt{3} = -\sqrt{3}$

$3\sqrt{5} = \sqrt{(9 \times 5)} = \sqrt{45}$

$2\sqrt{7} = \sqrt{(4 \times 7)} = \sqrt{28}$

Now, let us arrange in descending order

$\sqrt{45}, \sqrt{28}, \sqrt{10.88}.., \sqrt{8.33}.., -\sqrt{3}$

So,

$3\sqrt{5}$, $2\sqrt{7}$, $\frac{7}{3}\sqrt{2}$, $\frac{5}{\sqrt{3}}$, $-\sqrt{3}$

Question 12.

Arrange in ascending order.

$\sqrt[3]{2}\ \sqrt{3}\ \sqrt[6]{5}$

Solution:

Here we can express the given expressions as:

$\sqrt[3]{2} = 2^{\frac{1}{3}}$

$\sqrt{3} = 3^{\frac{1}{2}}$

$\sqrt[6]{5} = 5^{\frac{1}{6}}$

Let us make the roots common so,

$2^{\frac{1}{3}} = 2^{(2\times\frac{1}{2}\times\frac{1}{3})} = 4^{\frac{1}{6}}$

$3^{\frac{1}{2}} = 3^{\left(3\times\frac{1}{3}\times\frac{1}{2}\right)} = 27^{\frac{1}{6}}$

$5^{\frac{1}{6}} = 5^{\frac{1}{6}}$

Now, let us arrange in ascending order,

$4^{\frac{1}{6}}$, $5^{\frac{1}{6}}$, $27^{\frac{1}{6}}$

So,

$2^{\frac{1}{3}}$, $5^{\frac{1}{6}}$, $3^{\frac{1}{2}}$

So,

$\sqrt[3]{2}$, $\sqrt[6]{5}$, $\sqrt{3}$

Exercise 1.5

Question 1 .

Rationalize the following :

(i) $\frac{3}{4}\sqrt{5}$

(ii) $\frac{5\sqrt{7}}{\sqrt{3}}$

(iii) $\frac{3}{4-\sqrt{7}}$

(iv) $\frac{17}{3\sqrt{2}}+1$

(v) $\frac{16}{\sqrt{41}-5}$

(vi) $\frac{1}{\sqrt{7}-\sqrt{6}}$

(vii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

(viii) $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}$

Solution:

(i) $\frac{3}{4}\sqrt{5}$

Let us rationalize,

$$\frac{3}{4}\sqrt{5} = \frac{3\times\sqrt{5}}{4\sqrt{5}\times\sqrt{5}}$$

$$= \frac{3\sqrt{5}}{4\times5}$$

$$= \frac{3\sqrt{5}}{20}$$

(ii) $\frac{5\sqrt{7}}{\sqrt{3}}$

Let us rationalize,

$$\frac{5\sqrt{7}}{\sqrt{3}} = \frac{5\sqrt{7}\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$$

$$= \frac{5\sqrt{21}}{3}$$

(iii) $\frac{3}{4-\sqrt{7}}$

Let us rationalize,

$$\frac{3}{4-\sqrt{7}} = \frac{3\times 4+\sqrt{7}}{(4-\sqrt{7})\times(4+\sqrt{7})}$$

$$= \frac{3(4+\sqrt{7})}{(4^2-(\sqrt{7})^2}$$

$$= \frac{3(4+\sqrt{7})}{(16-7)}$$

$$= \frac{3(4+\sqrt{7})}{9}$$

$$= \frac{(4+\sqrt{7})}{3}$$

(iv) $\frac{17}{3\sqrt{2}} + 1$

Let us rationalize,

$$\frac{17}{3\sqrt{2}} + 1 = \frac{17(3\sqrt{2}-1)}{(3\sqrt{2}+1)(3\sqrt{2}-1)}$$

$$= \frac{17(3\sqrt{2}-1)}{(3\sqrt{2})^2-1^2}$$

$$= \frac{17(3\sqrt{2}-1)}{9.2-1}$$

$$= \frac{17(3\sqrt{2}-1)}{18-1}$$

$$= \frac{17(3\sqrt{2}-1)}{17}$$

$$= (3\sqrt{2}-1)$$

(v) $\frac{\mathbf{16}}{\sqrt{\mathbf{41}}-\mathbf{5}}$

Let us rationalize,

$$\frac{16}{\sqrt{41}-5} = \frac{16(\sqrt{41}+5)}{(\sqrt{41}-5)(\sqrt{41}+5)}$$

$$= \frac{16(\sqrt{41}+5)}{(\sqrt{41})^2-5^2}$$

$$= \frac{16(\sqrt{41}+5)}{41-25}$$

$$= \frac{16(\sqrt{41}+5}{16}$$

$$= (\sqrt{41}+5)$$

(vi) $\frac{\mathbf{1}}{\sqrt{\mathbf{7}}-\sqrt{\mathbf{6}}}$

Let us rationalize,

$$\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{1(\sqrt{7}+\sqrt{6})}{(\sqrt{7}-\sqrt{6})\ (\sqrt{7}+\sqrt{6})}$$

$$= \frac{(\sqrt{7}+\sqrt{6})}{(\sqrt{7})^2-(\sqrt{6})^2}$$

$$= \frac{\sqrt{7}+\sqrt{6}}{7-6}$$

$$= \frac{\sqrt{7}+\sqrt{6}}{1}$$

$$= (\sqrt{7}+\sqrt{6})$$

(vii) $\frac{\mathbf{1}}{\sqrt{\mathbf{5}}+\sqrt{\mathbf{2}}}$

Let us rationalize,

$$\frac{1}{\sqrt{5}+\sqrt{2}} = \frac{1(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}$$

$$= \frac{(\sqrt{5}-\sqrt{2})}{[(\sqrt{5})^2-(\sqrt{2})^2]}$$

$= \frac{\sqrt{5}-\sqrt{2}}{5-2}$

$= \frac{\sqrt{5}-\sqrt{2}}{3}$

$= \frac{\sqrt{5}-\sqrt{2}}{3}$

(viii) $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}$

Let us rationalize,

$\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \frac{(\sqrt{2}+\sqrt{3})\ (\sqrt{2}+\sqrt{3})}{(\sqrt{2}-\sqrt{3})\ (\sqrt{2}+\sqrt{3})}$

$= \frac{(\sqrt{2}+\sqrt{3})^2}{(\sqrt{2})^2-(\sqrt{3})^2}$

$= \frac{2+3+2\sqrt{2}\sqrt{3}}{2-3}$

$= \frac{5+2\sqrt{6}}{1}$

$= -(5 + 2\sqrt{6})$

Question 2.

Simplify:

(i) $\frac{(7 + 3\sqrt{5})}{(7 - 3\sqrt{5})}$

(ii) $\frac{(3 - 2\sqrt{2})}{(3 + 2\sqrt{2})}$

(iii) $\frac{(5 - 3\sqrt{14})}{(7 + 2\sqrt{14})}$

Solution:

(i) $\frac{(7 + 3\sqrt{5})}{(7 - 3\sqrt{5})}$

Let us rationalize the denominator, we get

$$\frac{(7 + 3\sqrt{5})}{(7 - 3\sqrt{5})} = \frac{[(7 + 3\sqrt{5})\ (7 + 3\sqrt{5})]}{[(7 - 3\sqrt{5})\ (7 + 3\sqrt{5})]}$$

$$= \frac{(7+3\sqrt{5})^2}{7^2-(3\sqrt{5})^2}$$

$$= \frac{7^2+3(\sqrt{5})^2+2.7.3\sqrt{5}}{49-9.5}$$

$$= \frac{49+9.5+42\sqrt{5}}{49-45}$$

$$= \frac{[49 + 45 + 42\sqrt{5}]}{[4]}$$

$$= \frac{[94 + 42\sqrt{5}]}{4}$$

$$= \frac{2[47 + 21\sqrt{5}]}{4}$$

$$= \frac{[47 + 21\sqrt{5}]}{2}$$

(ii) $\frac{(3 - 2\sqrt{2})}{(3 + 2\sqrt{2})}$

Let us rationalize the denominator, we get

$$\frac{(3 - 2\sqrt{2})}{(3 + 2\sqrt{2})} = \frac{[(3 - 2\sqrt{2})\ (3 - 2\sqrt{2})]}{[(3 + 2\sqrt{2})\ (3 - 2\sqrt{2})]}$$

$$= \frac{(3-2\sqrt{2}\ )^2}{(3^2- 2\sqrt{2}\ )^2}$$

$$= \frac{[3^2+ (2\sqrt{2})^2- 2.3.2\sqrt{2}\ ]}{[9-4.2]}$$

$$= \frac{[9 + 4.2 - 12\sqrt{2}]}{[9 - 8]}$$

$$= \frac{[9 + 8 - 12\sqrt{2}]}{1}$$

$= 17 - 12\sqrt{2}$

(iii) $\frac{(5 - 3\sqrt{14})}{(7 + 2\sqrt{14})}$

Let us rationalize the denominator, we get

$$\frac{(5 - 3\sqrt{14})}{(7 + 2\sqrt{14})} = \frac{[(5 - 3\sqrt{14})(7 - 2\sqrt{14})]}{[(7 + 2\sqrt{14})(7 - 2\sqrt{14})]}$$

$$= \frac{[5(7 - 2\sqrt{14}) - 3\sqrt{14}(7 - 2\sqrt{14})]}{[7^2 - (2\sqrt{14}]^2}$$

$$= \frac{[35 - 10\sqrt{14} - 21\sqrt{14} + 6.14]}{[49 - 4.14]}$$

$$= \frac{[35 - 31\sqrt{14} + 84]}{[49 - 56]}$$

$$= \frac{[119 - 31\sqrt{14}]}{[-7]}$$

$$= \frac{-[119 - 31\sqrt{14}]}{7}$$

$$= \frac{[31\sqrt{14} - 119]}{7}$$

Question 3.

Simplify:

$$\left(\frac{7\sqrt{3}}{\sqrt{10} + \sqrt{3}}\right) - \left(\frac{2\sqrt{5}}{(\sqrt{6} + \sqrt{5})}\right) - \left(\frac{3\sqrt{2}}{(\sqrt{15} + 3\sqrt{2})}\right)$$

Solution:

Let us simplify individually,

$$\left(\frac{7\sqrt{3}}{\sqrt{10} + \sqrt{3}}\right)$$

Let us rationalize the denominator,

$$\left(\frac{7\sqrt{3}}{\sqrt{10} + \sqrt{3}}\right) = \frac{[7\sqrt{3}(\sqrt{10} - \sqrt{3})]}{[(\sqrt{10} + \sqrt{3})(\sqrt{10} - \sqrt{3})]}$$

$$= \frac{[7\sqrt{3}.\sqrt{10} - 7\sqrt{3}.\sqrt{3}]}{[(\sqrt{10})^2 - (\sqrt{3})^2}$$

$= \dfrac{[7\sqrt{30} - 7.3]}{[10 - 3]}$

$= \dfrac{7[\sqrt{30} - 3]}{7}$

$= \sqrt{30} - 3$

Now

$\left(\dfrac{2\sqrt{5}}{(\sqrt{6} + \sqrt{5})}\right)$

Let us rationalize the denominator, we get

$\left(\dfrac{2\sqrt{5}}{(\sqrt{6} + \sqrt{5})}\right) = \dfrac{[2\sqrt{5}\,(\sqrt{6} - \sqrt{5})]}{[(\sqrt{6}+ \sqrt{5})\,(\sqrt{6}- \sqrt{5})]}$

$= \dfrac{[2\sqrt{5}.\sqrt{6} - 2\sqrt{5}.\sqrt{5}]}{[(\sqrt{6})^2 - (\sqrt{5})^2]}$

$= \dfrac{[2\sqrt{30}- 2.5]}{[6 - 5]}$

$= \dfrac{[2\sqrt{30} - 10]}{1}$

$= 2\sqrt{30} - 10$

Now,

$\left(\dfrac{3\sqrt{2}}{(\sqrt{15} + 3\sqrt{2})}\right)$

Let us rationalize the denominator, we get

$\left(\dfrac{3\sqrt{2}}{(\sqrt{15} + 3\sqrt{2})}\right) = \dfrac{[3\sqrt{2}\,(\sqrt{15}- 3\sqrt{2})]}{[(\sqrt{15}+ 3\sqrt{2})\,(\sqrt{15}- 3\sqrt{2})]}$

$= \dfrac{[3\sqrt{2}.\sqrt{15}- 3\sqrt{2}.3\sqrt{2}]}{[(\sqrt{15})^2 - (3\sqrt{2})^2}$

$= \dfrac{[3\sqrt{30}- 9.2]}{[15 - 9.2]}$

$= \dfrac{[3\sqrt{30} - 18]}{[15 - 18]}$

$= \dfrac{3[\sqrt{30}- 6]}{[-3]}$

$= \frac{[\sqrt{30} - 6]}{-1}$

$= 6 - \sqrt{30}$

So, according to the question let us substitute the obtained values,

$\left(\frac{7\sqrt{3}}{\sqrt{10} + \sqrt{3}}\right) - \left(\frac{2\sqrt{5}}{(\sqrt{6} + \sqrt{5})}\right) - \left(\frac{3\sqrt{2}}{(\sqrt{15} + 3\sqrt{2})}\right)$

$= (\sqrt{30} - 3) - (2\sqrt{30} - 10) - (6 - \sqrt{30})$
$= \sqrt{30} - 3 - 2\sqrt{30} + 10 - 6 + \sqrt{30}$
$= 2\sqrt{30} - 2\sqrt{30} - 3 + 10 - 6$
$= 1$

Question 4.

Simplify:

$\left[\frac{1}{\sqrt{4}+\sqrt{5}}\right] + \left[\frac{1}{\sqrt{5}+\sqrt{6}}\right] + \left[\frac{1}{\sqrt{6}+\sqrt{7}}\right] + \left[\frac{1}{\sqrt{7}+\sqrt{8}}\right] + \left[\frac{1}{\sqrt{8}+\sqrt{9}}\right]$

Solution:
let us simplify individually,
$\left[\frac{1}{\sqrt{4}+\sqrt{5}}\right]$
Rationalize the denominator, we get

$\left[\frac{1}{\sqrt{4}+\sqrt{5}}\right] = \left[\frac{\frac{1}{\sqrt{4}-\sqrt{5}}}{(\sqrt{4}+\sqrt{5})(\sqrt{4}-\sqrt{5})}\right]$
$= \frac{(\sqrt{4}-\sqrt{5})}{(\sqrt{4})^2 - (\sqrt{5})^2}$

$= \left[\frac{(\sqrt{4}-\sqrt{5})}{(4-5)}\right]$

$$= \left[\frac{(\sqrt{4}-\sqrt{5})}{-1}\right]$$
$$= -\ (\sqrt{4} - \sqrt{5})$$
Now,
$$\left[\frac{1}{\sqrt{5}+\sqrt{6}}\right]$$
Rationalize the denominator, we get
$$\left[\frac{1}{\sqrt{5}+\sqrt{6}}\right] = \left[\frac{\frac{1}{\sqrt{5}-\sqrt{6}}}{(\sqrt{5}+\sqrt{6})(\sqrt{5}-\sqrt{6})}\right]$$
$$= \left[\frac{\sqrt{5}-\sqrt{6}}{(\sqrt{5})^2(\sqrt{6})^2}\right]$$
$$= \left[\frac{(\sqrt{5}-\sqrt{6})}{(5-6)}\right]$$
$$= \left[\frac{(\sqrt{5}-\sqrt{6})}{-1}\right]$$
$$= -\ (\sqrt{5} - \sqrt{6})$$
Now,
$$\left[\frac{1}{\sqrt{6}+\sqrt{7}}\right]$$
Rationalize the denominator, we get

$$\left[\frac{1}{\sqrt{6}+\sqrt{7}}\right] = \left[\frac{\frac{1}{\sqrt{6}-\sqrt{7}}}{(\sqrt{6}+\sqrt{7})(\sqrt{6}-\sqrt{7})}\right]$$
$$= \left[\frac{(\sqrt{6}-\sqrt{7})}{(\sqrt{6})^2-(\sqrt{7})^2}\right]$$
$$= \left[\frac{(\sqrt{6}-\sqrt{7})}{(6-7)}\right]$$
$$= \left[\frac{(\sqrt{6}-\sqrt{7})}{-1}\right]$$
$$= -(\sqrt{6} - \sqrt{7})$$

Now ,
$$\left[\frac{1}{(\sqrt{7}-\sqrt{8})}\right]$$
Rationalize the denominator , we get

$\left[\frac{1}{(\sqrt{7}-\sqrt{8})}\right] = \left[\frac{\frac{1}{(\sqrt{7}-\sqrt{8})}}{(\sqrt{7}+\sqrt{8})(\sqrt{7}-\sqrt{8})}\right]$

$= \left[\frac{(\sqrt{7}-\sqrt{8})}{(\sqrt{7})^2-(\sqrt{8})^2}\right]$

$= \left[\frac{(\sqrt{7}-\sqrt{8})}{(7-8)}\right]$

$= \left[\frac{(\sqrt{7}-\sqrt{8})}{-1}\right]$

$= -\left(\sqrt{7}-\sqrt{8}\right)$

Now,

$\left[\frac{1}{\sqrt{8}+\sqrt{9}}\right]$

Rationalize the denominator, we get

$\left[\frac{1}{\sqrt{8}+\sqrt{9}}\right] = \left[\frac{\frac{1}{\sqrt{8}-\sqrt{9}}}{(\sqrt{8}+\sqrt{9})(\sqrt{8}-\sqrt{9})}\right]$

$= \frac{(\sqrt{8}-\sqrt{9})}{(\sqrt{8})^2-(\sqrt{9})^2}$

$= \left[\frac{\sqrt{8}-\sqrt{9}}{8-9}\right]$

$= \left[\frac{(\sqrt{8}-\sqrt{9})}{-1}\right]$

$= -\left(\sqrt{8}-\sqrt{9}\right)$

So, according to the question let us substitute the obtained values,

$\left[\frac{1}{\sqrt{4}+\sqrt{5}}\right] + \left[\frac{1}{\sqrt{5}+\sqrt{6}}\right] + \left[\frac{1}{\sqrt{6}+\sqrt{7}}\right] + \left[\frac{1}{\sqrt{7}+\sqrt{8}}\right] + \left[\frac{1}{\sqrt{8}+\sqrt{9}}\right]$

$= -(\sqrt{4}-\sqrt{5}) + -(\sqrt{5}-\sqrt{6}) + -(\sqrt{6}-\sqrt{7}) + -(\sqrt{7}-\sqrt{8}) + -(\sqrt{8}-\sqrt{9})$

$= -\sqrt{4}+\sqrt{5}-\sqrt{5}+\sqrt{6}-\sqrt{6}+\sqrt{7}-\sqrt{7}+\sqrt{8}-\sqrt{8}+\sqrt{9}$

$= -\sqrt{4}+\sqrt{5}$

$= -2 + 3$

$= 1$

Question 5.

Given , Find the value of a and b, if

(i) $\frac{[3-\sqrt{5}]}{[3+2\sqrt{5}]}$ = -1 $\frac{19}{11}$ + a$\sqrt{5}$

(ii) $\frac{[\sqrt{2}+\sqrt{3}]}{[3\sqrt{2}-2\sqrt{3}]}$ = a - b$\sqrt{6}$

(iii) $\frac{[7+\sqrt{5}]}{[7-\sqrt{5}]} - \frac{[7-\sqrt{5}]}{[7+\sqrt{5}]}$ = a + $\frac{7}{11}$ $b\sqrt{5}$

Solution:

(i) $\frac{[3-\sqrt{5}]}{[3+2\sqrt{5}]}$ = -1 $\frac{19}{11}$ + a$\sqrt{5}$

Let us consider LHS

$\frac{[3-\sqrt{5}]}{[3+2\sqrt{5}]}$

Rationalize the denominator,

$$\frac{[\mathbf{3-\sqrt{5}}]}{[\mathbf{3+2\sqrt{5}}]} = \frac{\frac{[\mathbf{3-\sqrt{5}}]}{[\mathbf{3-2\sqrt{5}}]}}{[(3+2\sqrt{5})(3-2\sqrt{5})}$$

$$= \left[\frac{3(3-2\sqrt{5})-\sqrt{5}\,(3-2\sqrt{5})}{3^2-(2\sqrt{5})^2}\right]$$

$$= \frac{[9-6\sqrt{5}-3\sqrt{5}+2.5]}{[9-4.5]}$$

$$= \frac{[9-6\sqrt{5}-3\sqrt{5}+10]}{[9-20]}$$

$$= \frac{[19-9\sqrt{5}]}{-11}$$

$= -\frac{19}{11} + \frac{9\sqrt{5}}{11}$

So when comparing with RHS

$-\frac{19}{11} + \frac{9\sqrt{5}}{11} = -\frac{19}{11} + a\sqrt{5}$

Hence, value of a $= \frac{9}{11}$

(ii) $\frac{[\sqrt{2}+\sqrt{3}]}{[3\sqrt{2}-2\sqrt{3}]}$ **= a – b**$\sqrt{6}$

Let us consider LHS

$\frac{[\sqrt{2}+\sqrt{3}]}{[3\sqrt{2}-2\sqrt{3}]}$

Rationalize the denominator,

$\frac{[\sqrt{2}+\sqrt{3}]}{[3\sqrt{2}-2\sqrt{3}]} = \left[\frac{(\sqrt{2}+\sqrt{3})(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2}-2\sqrt{3})(3\sqrt{2}+2\sqrt{3})}\right]$

$= \frac{[\sqrt{2}(3\sqrt{2}+2\sqrt{3})+\sqrt{3}(3\sqrt{2}+2\sqrt{3})]}{[(3\sqrt{2})^2-(2\sqrt{3})^2]}$

$= \frac{[3.2+2\sqrt{2}\sqrt{3}+ 3\sqrt{2}\sqrt{3}+2.3]}{[9.2-4.3]}$

$= \frac{[6+2\sqrt{6}+3\sqrt{6}+6]}{[18-12]}$

$= \frac{[12+5\sqrt{6}]}{6}$

$= \frac{12}{6} + \frac{5\sqrt{6}}{6}$

$= 2 + \frac{5\sqrt{6}}{6}$

$= 2 - \left(-\frac{5\sqrt{6}}{6}\right)$

So when comparing with RHS

$2 - (- \frac{5\sqrt{6}}{6}) = a - b\sqrt{6}$

Hence, value of a = 2 and b = $-\frac{5}{6}$

(iii) $\left[\frac{7+\sqrt{5}}{7-\sqrt{5}}\right] - \left[\frac{7-\sqrt{5}}{7+\sqrt{5}}\right] = \mathbf{a} + \frac{7}{11}\mathbf{b}\sqrt{5}$

$\frac{[7+\sqrt{5}]}{[7-\sqrt{5}]} - \frac{[7-\sqrt{5}]}{[7+\sqrt{5}]} = a + \frac{7}{11}b\sqrt{5}$

Let us consider LHS

Since there are two terms, let us solve individually

$\frac{[7+\sqrt{5}]}{[7-\sqrt{5}]}$

Rationalize the denominator,

$\frac{[7+\sqrt{5}]}{[7-\sqrt{5}]} = \frac{[(7+\sqrt{5})(7+\sqrt{5})]}{[(7-\sqrt{5})(7+\sqrt{5})]}$

$= \frac{[(7+\sqrt{5})]^2}{[7^2-(\sqrt{5})^2]}$

$= \frac{[7^2+(\sqrt{5})^2+2.7.\sqrt{5}]}{[49-5]}$

$= \frac{[49+5+14\sqrt{5}]}{[44]}$

$= \frac{[54+14\sqrt{5}]}{44}$

Now,

$$\frac{[7-\sqrt{5}]}{[7+\sqrt{5}]}$$

Rationalize the denominator,

$$\frac{[7-\sqrt{5}]}{[7+\sqrt{5}]} = \frac{(7-\sqrt{5})\,(7-\sqrt{5})}{(7+\sqrt{5})(7-\sqrt{5})}$$

$$= \left[\frac{(7-\sqrt{5})^2}{\left(7^2-(\sqrt{5})^2\right)}\right]$$

$$= \frac{[7^2+(\sqrt{5})^2-2.7.\sqrt{5}]}{[49-5]}$$

$$= \frac{[49+5-14\sqrt{5}]}{[44]}$$

$$= \frac{[54-14\sqrt{5}]}{44}$$

So, according to the question

$$\frac{[7+\sqrt{5}]}{[7-\sqrt{5}]} - \frac{[7-\sqrt{5}]}{[7+\sqrt{5}]}$$

By substituting the obtained values,

$$= \frac{[54+14\sqrt{5}]}{44} - \frac{[54-14\sqrt{5}]}{44}$$

$$= \frac{[54+14\sqrt{5}-54+14\sqrt{5}]}{44}$$

$$= \frac{28\sqrt{5}}{44}$$

$$= \frac{7\sqrt{5}}{11}$$

So when comparing with RHS

$$\frac{7\sqrt{5}}{11} = a + \frac{7}{11} b\sqrt{5}$$

Hence, value of a = 0 and b = 1

Question 6.

Simplify:

$$\frac{[7+3\sqrt{5}]}{[3+\sqrt{5}]} - \frac{[7-3\sqrt{5}]}{[3-\sqrt{5}]} = p + q\sqrt{5}$$

Solution:

Let us consider LHS

Since there are two terms, let us solve individually

$$\frac{[7+3\sqrt{5}]}{[3+\sqrt{5}]}$$

Rationalize the denominator ,

$$\frac{[7+3\sqrt{5}]}{[3+\sqrt{5}]} = \frac{(7+3\sqrt{5})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$$

$$= \frac{[7(3-\sqrt{5})+3\sqrt{5}(3-\sqrt{5})]}{[3^2-(\sqrt{5})^2]}$$

$$= \frac{[21-7\sqrt{5}+9\sqrt{5}-3.5]}{[9-5]}$$

$$= \frac{[21+2\sqrt{5}-15]}{4}$$

$$= \frac{6+2\sqrt{5}}{4}$$

$$= \frac{2[3+\sqrt{5}]}{4}$$

$$= \frac{[3+\sqrt{5}]}{2}$$

Rationalize the denominator,

$$\frac{[7-3\sqrt{5}]}{[3-\sqrt{5}]} = \frac{[(7-3\sqrt{5})(3+\sqrt{5})]}{[(3-\sqrt{5})(3+\sqrt{5})]}$$

$$= \frac{[7(3+\sqrt{5}) - 3\sqrt{5}(3+\sqrt{5})]}{[3^2 - (\sqrt{5})^2]}$$

$$= \frac{[21+7\sqrt{5} - 9\sqrt{5} - 3.5]}{[9-5]}$$

$$= \frac{[21-2\sqrt{5} - 15]}{4}$$

$$= \frac{[6-2\sqrt{5}]}{4}$$

$$= \frac{2[3-\sqrt{5}]}{4}$$

$$= \frac{[3-\sqrt{5}]}{2}$$

So, according to the question

$$\frac{[7+3\sqrt{5}]}{[3+\sqrt{5}]} - \frac{[7-3\sqrt{5}]}{[3-\sqrt{5}]}$$

By substituting the obtained values,

$$= \frac{[3+\sqrt{5}]}{2} - \frac{[3-\sqrt{5}]}{2}$$

$= \frac{[3+\sqrt{5}-3+\sqrt{5}]}{2}$

$= \frac{[2\sqrt{5}]}{2}$

$= \sqrt{5}$

So when comparing with RHS

$\sqrt{5} = p + q\sqrt{5}$

Hence, value of $p = 0$ and $q = 1$

Question 7.

If $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$ find

(i) $\frac{\sqrt{2}}{(2+\sqrt{2})}$

(ii) $\frac{1}{(\sqrt{3}+\sqrt{2})}$

Solution:

(i) $\frac{\sqrt{2}}{(2+\sqrt{2})}$

By rationalizing the denominator,

$\frac{\sqrt{2}}{(2+\sqrt{2})} = \frac{[\sqrt{2}(2-\sqrt{2})]}{[(2+\sqrt{2})(2-\sqrt{2})]}$

$= \frac{[2\sqrt{2}-2]}{[2^2-(\sqrt{2})^2]}$

$= \frac{[2\sqrt{2}-2]}{[4-2]}$

$= \frac{2[\sqrt{2}-1]}{2}$

$= \sqrt{2}$ - 1

= 1.414 – 1

= 0.414

(ii) $\frac{\mathbf{1}}{(\sqrt{\mathbf{3}} + \sqrt{\mathbf{2}})}$

By rationalizing the denominator,

$\frac{1}{(\sqrt{\mathbf{3}} + \sqrt{\mathbf{2}})} = \frac{[1(\sqrt{3}-\sqrt{2})]}{[\sqrt{3}+\sqrt{2}][\sqrt{3}-\sqrt{2}]}$

$= \frac{[(\sqrt{3}-\sqrt{2})]}{[(\sqrt{3})^2-(\sqrt{2})^2]}$

$= \left[\frac{(\sqrt{3}-\sqrt{2})}{3-2}\right]$

$= \left(\frac{(\sqrt{3}-\sqrt{2})}{1}\right)$

= 1.732 – 1.414

= 0.318

Question 8.

If a = 2 + $\sqrt{3}$, find $\frac{1}{a}$, (a - $\frac{1}{a}$)

Solution :

Given:

a = 2 + $\sqrt{3}$

so,

$\frac{1}{a} = \frac{1}{2+\sqrt{3}}$

By rationalizing the denominator,

$\frac{1}{(2+\sqrt{3})} = \frac{1(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$

$= \frac{[(2-\sqrt{3})]}{[2^2-(\sqrt{3})^2]}$

$= \frac{[(2-\sqrt{3})]}{[4-3]}$

$= (2 - \sqrt{3})$

Then ,

A - $\frac{1}{a} = 2 + \sqrt{3} - (2 - \sqrt{3})$

$= 2 + \sqrt{3} - 2 + \sqrt{3}$

$= 2\sqrt{3}$

Question 9.

Solve:

If x = 1 - $\sqrt{2}$, find $\frac{1}{x}$, $(x - \frac{1}{x})^4$

Solution:

Given:

X = 1 - $\sqrt{2}$

So,

$\frac{1}{x} = \frac{1}{(1-\sqrt{2})}$

By rationalizing the denominator,

$\frac{1}{(1-\sqrt{2})} = \frac{[1(1+\sqrt{2})]}{[(1-\sqrt{2})(1+\sqrt{2})]}$

$= \left[\frac{(1+\sqrt{2})}{1^2-(\sqrt{2})^2}\right]$

$= \left[\frac{(1+\sqrt{2})}{1-2}\right]$

$= \frac{(1+\sqrt{2})}{-1}$

$= -(1 + \sqrt{2})$

Then ,

$(x - \frac{1}{x})^4 = [1 - \sqrt{2} - (-1 - \sqrt{2})]^4$

$= [1 - \sqrt{2} + 1 + \sqrt{2}]^2$

$= 2^4$

$= 16$

Question : 10.

If $x = 5 - 2\sqrt{6}$, find $\frac{1}{x}$, $(x^2 - \frac{1}{x^2})$

Solution:

Given :

$X = 5 - 2\sqrt{6}$

So,

$\frac{1}{x} = \frac{1}{(5 - 2\sqrt{6})}$

By rationalizing the denominator,

$\frac{1}{(5 - 2\sqrt{6})} = \frac{[1(5+2\sqrt{6})]}{[(5-2\sqrt{6})(5+2\sqrt{6})]}$

$= \frac{[(5+2\sqrt{6})]}{[5^2-(2\sqrt{6})^2]}$

$= \frac{[(5+2\sqrt{6})]}{[25-4.6]}$

$= \frac{[(5+2\sqrt{6})]}{[25-24]}$

$(5 + 2\sqrt{6})$

Then ,

$X + \frac{1}{x} = 5 - 2\sqrt{6} + (5 + 2\sqrt{6})$

= 10

Square on both sides we get

$\left(x + \frac{1}{x}\right)^2 = 10^2$

$x^2 + \frac{1}{x}^2 + 2x.\frac{1}{x} = 100$

$x^2 + \frac{1}{x}^2 + 2 = 100$

$x^2 + \frac{1}{x}^2 = 100 - 2$

= 98

Question 11.

If $p = \frac{(2-\sqrt{5})}{(2+\sqrt{5})}$ **and** $q = \frac{(2+\sqrt{5})}{(2-\sqrt{5})}$, find the values of

(i) p + q

(ii) p – q

(iii) $p^2 + q^2$

(iv) $p^2 - q^2$

Solution:

Given :

$$P = \frac{(2-\sqrt{5})}{(2+\sqrt{5})} \text{ and } q = \frac{(2+\sqrt{5})}{(2-\sqrt{5})}$$

(i) p + q

$$\frac{(2-\sqrt{5})}{(2+\sqrt{5})} + \frac{(2+\sqrt{5})}{(2-\sqrt{5})}$$

So by rationalizing the denominator, we get

$$= \left[\frac{(2-\sqrt{5})^2 + (2+\sqrt{5})^2}{2^2-(\sqrt{5})^2}\right]$$

$$= \frac{[4+5-4\sqrt{5}+4+5+4\sqrt{5}]}{[4-5]}$$

$$= \frac{18}{-1}$$

= -18

(ii) p – q

$$\frac{(2-\sqrt{5})}{(2+\sqrt{5})} - \frac{(2+\sqrt{5})}{(2-\sqrt{5})}$$

So by rationalizing the denominator, we get

$= \frac{(2-\sqrt{5})^2 - (2+\sqrt{5})^2}{(2^2 - (\sqrt{5})^2)}$

$= \frac{[4+5-4\sqrt{5} - (4+5+4\sqrt{5})]}{4-5}$

$= \frac{[9-4\sqrt{5} - 9-4\sqrt{5}]}{-1}$

$= \frac{-8\sqrt{5}}{-1}$

$= 8\sqrt{5}$

(iii) $p^2 + q^2$

We know that $(p + q)^2 = p^2 + q^2 + 2pq$

So,

$P^2 + q^2 = (p + q)^2 - 2pq$

$Pq = \left[\frac{(2-\sqrt{5})}{(2+\sqrt{5})}\right] \times \left[\frac{(2+\sqrt{5})}{(2-\sqrt{5})}\right]$

$= 1$

$P + q = -18$

So,

$P^2 + q^2 = (p + q)^2 - 2pq$

$= (-18)^2 - 2(1)$

$= 324 - 2$

$= 322$

(iv) $p^2 - q^2$

We know that, $p^2 - q^2 = (p + q) (p - q)$

So, by substituting the values

$P^2 - q^2 = (p + q) (p - q)$

$= (-18) (8\sqrt{5})$

$= -144 \sqrt{5}$

Question 12.

If $x = \frac{(\sqrt{2} - 1)}{(\sqrt{2} + 1)}$ **and** $\mathbf{y = \frac{(\sqrt{2} + 1)}{(\sqrt{2} - 1)}}$ **find**

(i) x + y

(ii) xy

Solution:

Given :

X= $\frac{(\sqrt{2} - 1)}{(\sqrt{2} + 1)}$ and $y = \frac{(\sqrt{2} + 1)}{(\sqrt{2} - 1)}$

(i) x + y

$= \left[\frac{(\sqrt{2} - 1)}{(\sqrt{2} + 1)}\right] + \left[\frac{(\sqrt{2} + 1)}{(\sqrt{2} - 1)}\right]$

By rationalizing the denominator

$= \left[\frac{(\sqrt{2} - 1)^2 + (\sqrt{2} + 1)^2}{(\sqrt{2})^2 - 1^2}\right]$

$$= \frac{[2+1-2\sqrt{2}+ 2+1+2\sqrt{2}]}{[2-1]}$$

$$= \frac{6}{1}$$

$= 6$

(ii) xy

$$\left[\frac{(\sqrt{2} - 1)}{(\sqrt{2}+ 1)}\right] \times \left[\frac{(\sqrt{2} + 1)}{(\sqrt{2}- 1)}\right]$$

$= 1$