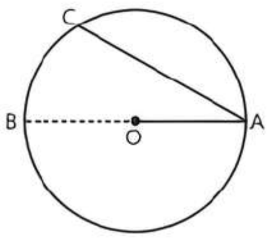
# 9 Circles

## Fastrack Revision

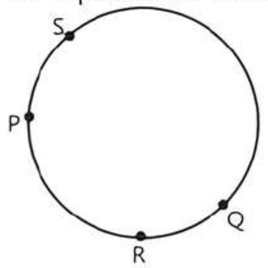
- **Circle:** A collection of all points in a plane which are at a constant distance from a fixed point. The fixed point is called the **centre** of the circle and the constant distance is called the **radius**.

  In figure, O is the centre, OA is the radius and AB is the **diameter** of the circle.

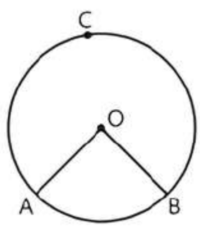



- **Chord:** A line segment joining any two points on the circle. In figure, AC is the chord.
- **Diameter:** Longest chord of the circle that passes through the centre of the circle.
- **Circumference:** Length of the boundary of a circle.
- **Arc:** Any part of the circumference of a circle.

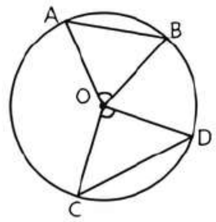
  In figure, PRQ is minor arc represented as $\overset{\frown}{PRQ}$ and PSQ is major arc represented as $\overset{\frown}{PSQ}$.




- **Semi-circle:** Parts of a circle that are divided by its diameter.
- **Segment:** The region between a chord and either of its arcs (major or minor). The segment formed with a minor arc is called **minor segment** and that formed with a major arc is called **major segment.**
- **Sector:** The region enclosed by an arc and the two radii joining the centre to the end points of the arc. The sector corresponding to minor arc is called **minor sector** *i.e.*, AOB and that corresponding to major arc is called **major sector**. *i.e.*, AOBCA.

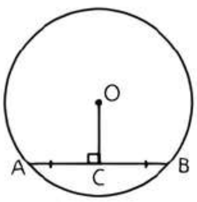



- **Angle Subtended by a Chord at a Point:**
  1. Equal chords of a circle subtend equal angles at the centre. *i.e.*, $\angle AOB = \angle COD$.

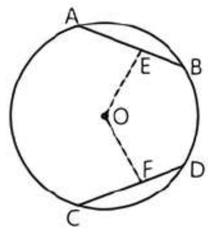



  2. If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal. *i.e.*, AB = CD.

- **Perpendicular from Centre to the Chord:**
  1. The perpendicular from the centre of a circle to a chord bisects the chord. *i.e.*, AC = BC
  2. The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord. *i.e.*, $OC \perp AB$.

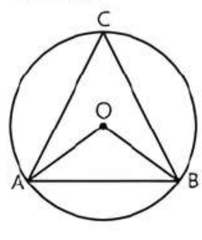



- **Distance of Equal Chords from the Centre:**
  1. Equal chords of a circle (or congruent circles) are equidistant from the centre. *i.e.*, OE = OF.
  2. Chords equidistant from the centre of a circle are equal in length. *i.e.*, AB = CD.




- **Angle Subtended by an Arc of a Circle:**
  1. The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

     *i.e.*, $\angle AOB = 2\angle ACB$.

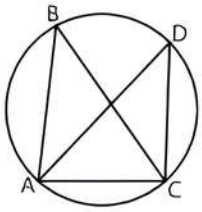



*OR*

The angle subtended by an arc at any point on the remaining part of the circle is half the angle subtended by it at the centre of circle.

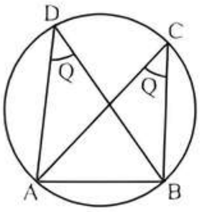
*i.e.,* $\angle ACB = \frac{1}{2}\angle AOB$

2. Angles in the same segment of a circle are equal.
   *i.e.,* $\angle ABC = \angle ADC$.

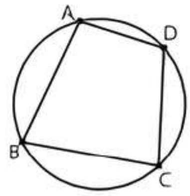



3. Angle in a semi-circle is a right angle.

- **Concylic :** If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, then four points lie on a circle. Hence it is said to be concylic.




- **Cyclic Quadrilaterals:** A quadrilateral, ABCD is said to be a cyclic quadrilateral, if all the four vertices A, B, C and D are concyclic (*i.e.,* all four vertices lie on a circle).




1. The sum of either pair of opposite angles of a cyclic quadrilateral is 180°.
   *i.e.,* $\angle A + \angle C = 180°$ or $\angle B + \angle D = 180°$
2. If the sum of a pair of opposite angles of a quadrilateral is 180°, then quadrilateral is cyclic.

**Knowledge BOOSTER**

1. *Circles having same centre are called concentric circles.*
2. *Two circles are said to be congruent if and only if either of them can be superposed on the other so as to cover it exactly or if and only if their radii are equal.*
3. *An infinite number of circles can be drawn through a given point.*
4. *The perpendicular bisector of a chord always passes through the centre of a circle.*

# Practice Exercise

## Multiple Choice Questions

**Q 1. Which of the following statements is true for the longest chord of a circle?**

a. It is two times of radius
b. It is equal to radius
c. It is two times of diameter
d. It is never equal to diameter

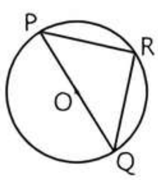
**Q 2. When two chords of a circle bisect each other, then which of the following statements is true?**

a. Both chords are perpendicular to each other
b. Both are diameter of the circle
c. Both chords are unequal
d. Both chords are parallel to each other

**Q 3. The line joining the centre of a circle to the mid-point of a chord is always:**

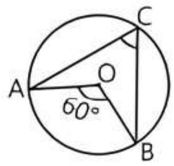
a. perpendicular to the chord
b. parallel to the chord
c. equal to the chord
d. equal to radius

**Q 4. In the figure, O is the centre of the circle and PR = QR. What is the measure of ∠PQR?**



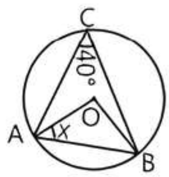

a. 60° b. 110° c. 75° d. 45°

**Q 5. In the figure, if O is the centre of a circle, then the measure of ∠ACB is:**



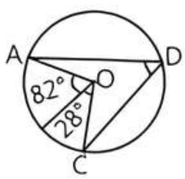

a. 30° b. 100° c. 40° d. 60°

**Q 6. In the figure, if O is the centre of the circle, then the measure of $x$ is:**




a. 40° b. 80° c. 50° d. 110°

**Q 7. In the figure, if O is the centre of the circle, then what is the measure of ∠ADC?**

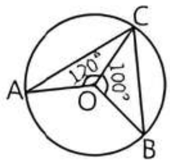



a. 55° b. 60° c. 90° d. 110°

**Q 8. The length of a chord in a circle of radius 15 cm is 24 cm. The distance of this chord from the centre of circle is:**

a. 5 cm b. 7 cm
c. 9 cm d. 10 cm

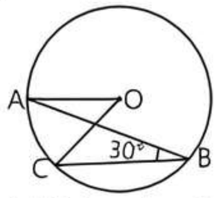
**Q 9.** In the figure, O is the centre of the circle. What is the measure of $\angle ACB$?




a. 45° b. 60° c. 70° d. 90°

**Q 10.** In the figure, if $\angle ABC = 30°$, then $\angle AOC$ is equal to:

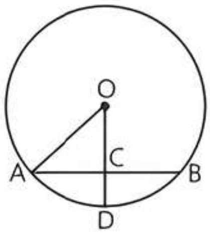
a. 30°
b. 60°
c. 15°
d. 40°




**Q 11.** AD is a diameter of a circle and AB is a chord. If AD = 34 cm, AB = 30 cm, the distance of AB from the centre of the circle is:

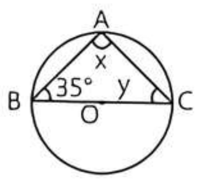
a. 17 cm b. 15 cm c. 4 cm d. 8 cm

**Q 12.** In the following figure, if OA = 5 cm, AB = 8 cm and OD is perpendicular to AB, then OC is equal to:




a. 2 cm b. 3 cm c. 4 cm d. 5 cm

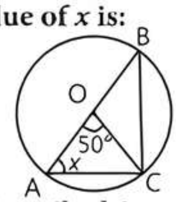
**Q 13.** In the given figure, O is the centre of the circle. For what values of $x$ and $y$, chord BC will pass through the centre of circle where points A, B and C are on the circle?




a. $x = 80°, y = 40°$ b. $x = 90°, y = 55°$
c. $x = 70°, y = 50°$ d. $x = 60°, y = 40°$

**Q 14.** In the given figure, AB is diameter, $\angle AOC = 50°$ and $\angle A + \angle B + \angle C = 180°$. The value of $x$ is:

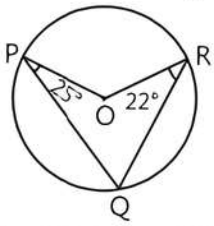
a. 70°
b. 60°
c. 65°
d. 75°




**Q 15.** If an equilateral triangle PQR is inscribed in a circle with centre O, then $\angle QOR$ is equal to:

a. 70° b. 60° c. 90° d. 120°

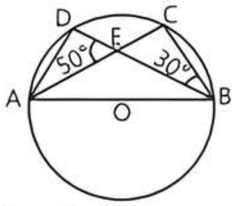
**Q 16.** In the given figure, O is the centre of the circle, $\angle OPQ = 25°$ and $\angle ORQ = 22°$. The values of $\angle PQR$ and $\angle POR$ are respectively:



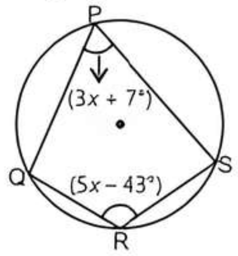

a. 46°, 92° b. 47°, 94° c. 45°, 90° d. 48°, 96°

**Q 17.** In the given figure, O is the centre of the circle, $\angle CBE = 30°$ and $\angle DEA = 50°$. The measure of $\angle ADB$ is:

a. 120°
b. 110°
c. 100°
d. 130°

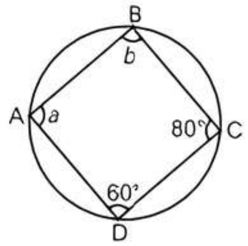



**Q 18.** In the following figure, the value of $x$ is:




a. 25° b. 27° c. 24° d. 28°

**Q 19.** If four points A, B, C and D lie on a circle, then the value of $a + b$ is equal to:




a. 230° b. 220° c. 225° d. 240°

**Q 20.** A quadrilateral PQRS is inscribed in a circle such that PQ is a diameter and $\angle PSR = 110°$. Then, $\angle QPR$ is equal to:

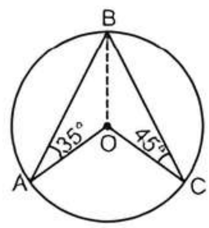
a. 25° b. 30° c. 20° d. 28°

## Assertion & Reason Type Questions

**Directions (Q.Nos. 21-25):** *In the following questions, a statement of Assertion (A) is followed by a statement of a Reason (R). Choose the correct choice as:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
c. Assertion (A) is true but Reason (R) is false.
d. Assertion (A) is false but Reason (R) is true.

**Q 21.** Assertion (A): If O is the centre of a circle and A, B and C are three points on a circle such that $\angle OAB = 35°$ and $\angle OCB = 45°$, then $\angle AOC = 160°$.

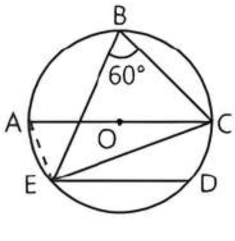



Reason (R): Angle subtended by an arc of a circle at the centre of the circle is double the angle subtended by an arc on the circumference.

**Q 22.** Assertion (A): Two diameters of a circle intersects each other at right angles. Then the quadrilateral formed by joining their end-points is a square.

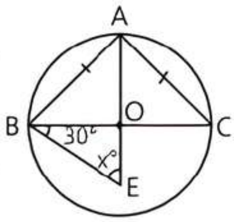
Reason (R): Equal chords subtend equal angles at the centre.

**Q 23.** Assertion (A): Chord ED is parallel to the diameter AC of the circle. If ∠ CBE = 60°, then ∠ ACE is 30°.



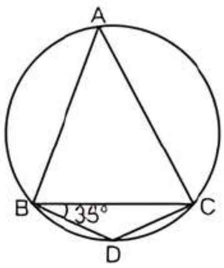

Reason (R): Same segment of a circle do not make equal angles at the circumference.

**Q 24.** Assertion (A): In the figure, E is any point in the interior of the circle with centre O. Chord AB is equal to chord AC. If ∠ OBE = 30°, the value of $x$ is 60°.




Reason (R): Equal chords subtend equal angles at the centre.

**Q 25.** Assertion (A): In the adjoining figure, BD = DC and ∠DBC = 35°, then the measure of ∠BAC is 70°.




Reason (R): The sum of opposite angles of a cyclic quadrilateral is 180°.

## Fill in the Blanks Type Questions

**Q 26.** A point, whose distance from the centre of a circle is greater than its radius lies in ………… of the circle.

**Q 27.** The longest chord of a circle is a ………… of the circle.

**Q 28.** An arc is a ………… when its ends are the ends of a diameter.

**Q 29.** AD is a diameter of a circle and AB is a chord. If AD = 34 cm, AB = 30 cm, then BD = …………

**Q 30.** If the sum of a pair of opposite angles of a quadrilateral is 180°, then quadrilateral is ………… .

## True/False Type Questions

**Q 31.** Line segment joining the centre to any point on the circle is a radius of the circle.

**Q 32.** Two chords AB and CD of a circle each are at distances 5 cm from the centre. Then AB = CD.

**Q 33.** The angle subtended by an arc at the centre is half the angle subtended by it at any point on the remaining part of the circle.

**Q 34.** ABCD is a cyclic quadrilateral such that ∠A = 90°, ∠B = 70°, ∠C = 95° and ∠D = 105°.

**Q 35.** In figure, if AOB is a diameter and ∠ADC = 100°, then ∠CBA = 80°.

## Solutions

**1.** (a) The longest chord of a circle is equal to the diameter of a circle *i.e.*, two times of radius.

**2.** (b) Both are diameter of the circle.

**3.** (a) Perpendicular to the chord.

**4.** (d) Given, PQ is a diameter of a circle.

The diameter of a circle subtends a right angle to the circumference of a circle.

∴ ∠ PRQ = 90°

In Δ PRQ, PR = RQ [Given]

⇒ ∠ PQR = ∠ QPR (angles opposite to equal sides of a triangle are equal)

Using angle sum property of a triangle,

∠ PRQ + ∠ PQR + ∠ QPR = 180°

⇒ 90° + ∠ PQR + ∠ PQR = 180° [∵ ∠ PQR = ∠ QPR]

⇒ 2 ∠ PQR = 90° ⇒ ∠ PQR = 45°

**5.** (a) Given, ∠AOB = 60°

The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

∴ ∠ AOB = 2 × ∠ ACB

⇒ $\angle ACB = \frac{1}{2} \times 60° = 30°$

**6.** (c) Given, ∠ ACB = 40°

**TiP**

*The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.*

∴ ∠ AOB = 2 ∠ ACB

⇒ ∠ AOB = 2 × 40° = 80°

In Δ AOB,

OA = OB [Radii of a circle]

$\Rightarrow \quad \angle ABO = \angle BAO$ [Angles opposite to equal sides of a triangle are equal]

Using angle sum property of a triangle.

$\angle AOB + \angle ABO + \angle BAO = 180°$

$\Rightarrow \quad 80° + x + x = 180°$

$\Rightarrow \quad 2x = 100° \quad \Rightarrow \quad x = 50°$

**7.** (a) In the given figure,

$\angle AOC = 82° + 28° = 110°$

The angle subtended by an arc at the remaining part of the circle is half the angle subtended by it at the centre of the circle.

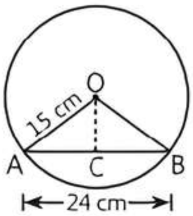
$\therefore \angle ADC = \frac{1}{2} \angle AOC$

$= \frac{1}{2} \times 110° = 55°$

**8.** (c) Given, AB = 24 cm and OA = 15 cm

**TR!CK**

*The perpendicular drawn from centre of circle to the chord bisects the chord.*




Here, AB = 2 AC

$\Rightarrow \quad AC = \frac{1}{2} \times 24$

$= 12$ cm

In right angled triangle OAC,

$OC = \sqrt{(OA)^2 - (AC)^2}$ [By Pythagoras theorem]

$= \sqrt{(15)^2 - (12)^2} = \sqrt{225 - 144}$

$= \sqrt{81} = 9$ cm

**9.** (c) Given, $\angle AOC = 120°$ and $\angle BOC = 100°$

$\therefore \angle AOB = \angle AOC + \angle BOC$

$= 120° + 100° = 220°$

$\therefore$ Reflex $\angle AOB = 360° - 220° = 140°$

The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$\therefore \angle ACB = \frac{1}{2} \times$ Reflex $\angle AOB$

$= \frac{1}{2} \times 140° = 70°$

**10.** (b) Given,

$\angle ABC = 30°$

The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

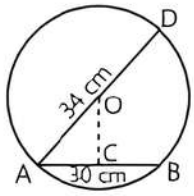
$\therefore \quad \angle AOC = 2\angle ABC$

$= 2 \times 30° = 60°$

**11.** (d) Given,

AD = 34 cm,

and AB = 30 cm




$\Rightarrow \quad AO = \frac{1}{2} AD = \frac{1}{2} \times 34$

$= 17$ cm

**TR!CK**

*The line drawn from centre of circle to the chord bisects the chord.*

$\therefore \quad AC = \frac{1}{2} AB = \frac{1}{2} \times 30 = 15$ cm

In right angled $\Delta$ AOC,

$OC = \sqrt{(AO)^2 - (AC)^2}$ [By Pythagoras theorem]

$= \sqrt{(17)^2 - (15)^2} = \sqrt{289 - 225}$

$= \sqrt{64} = 8$ cm

**12.** (b) Given,

OA = 5 cm, AB = 8 cm

**TiP**

*The perpendicular drawn from centre of circle to the chord bisects the chord.*

$\therefore \quad AC = \frac{1}{2} AB = \frac{1}{2} \times 8 = 4$ cm

In right angled $\Delta$ AOC,

$OC = \sqrt{(AO)^2 - (AC)^2}$

[By Pythagoras theorem]

$= \sqrt{(5)^2 - (4)^2} = \sqrt{25 - 16} = \sqrt{9} = 3$ cm

**13.** (b) Since, BC is a diameter of a circle. Therefore, BC subtends right angle at the point A.

**TiP**

*Diameter subtends a right angle to the circumference of a circle.*

Therefore $x = 90°$

In $\Delta$ ABC,

$\angle A + \angle B + \angle C = 180°$

[$\because$ Angles sum property of a triangle]

$\Rightarrow \quad 90° + 35° + y = 180°$

$\Rightarrow \quad y = 55°$

$\therefore \quad x = 90°$ and $y = 55°$

**14.** (c)

**TR!CK**

*Diameter of a circle subtends right angle to the circumference of a circle.*

In $\Delta$ ACB,

$\angle C = 90°$

[$\because$ Angle C is subtend by the diameter]

Now, $\angle ABC = \frac{1}{2} \angle AOC$

[∵ Angle subtended by an arc at the remaining part of the circle is half the angle subtended by it at the centre of the circle]

$= \frac{1}{2} \times 50° = 25°$

In $\Delta$ ABC,

$\angle ABC + \angle BCA + \angle CAB = 180°$ [Given]

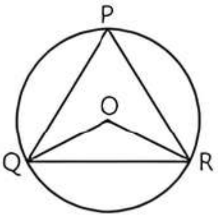
$\Rightarrow \quad 25° + 90° + x = 180°$

$\Rightarrow \quad x = 180° - 115° = 65°$

**15.** (d) Given, PQR is an equilateral triangle inscribed in a circle.

Therefore

$\angle QPR = \angle PQR = \angle PRQ = 60°$



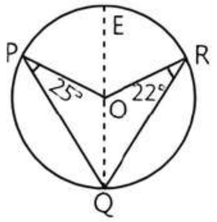

**TiP**

*The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.*

So, $\angle QOR = 2 \angle QPR = 2 \times 60° = 120°$

**16.** (b) Given, $\angle OPQ = 25°$ and $\angle ORQ = 22°$

Draw a line passing through Q and O, which intersect the circle at point E.




In the figure, OP, OQ and OR are radii of a circle.

In $\Delta$ OPQ,

OP = OQ [Radii of a circle]

$\Rightarrow \quad \angle OQP = \angle OPQ$

[Angles opposite to equal sides of a triangle are equal]

$= 25°$

Similarly in $\Delta$ OQR,

$\angle OQR = \angle ORQ = 22°$

$\therefore \quad \angle PQR = \angle OQP + \angle OQR$

$= 25° + 22° = 47°$

and $\quad \angle POR = 2\angle PQR$

[angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle]

$= 2 \times 47° = 94°$

**17.** (c) Given, $\angle CBE = 30°$ and $\angle DEA = 50°$

Here $\angle BEC = \angle DEA$

[Vertically opposite angles]

$= 50°$

In $\Delta$ BEC,

$\angle B + \angle E + \angle C = 180°$

[By angle sum property of a triangle]

$30° + 50° + \angle C = 180°$

$\therefore \quad \angle C = 100°$

Since, chord AB subtends equal angles at the circumference of a circle.

$\therefore \quad \angle D = \angle C = 100°$

**18.** (b)

**TiP**

*The sum of pair of opposite angles of a cyclic quadrilateral is 180°.*

Here $\quad \angle P + \angle R = 180°$

$\therefore (3x + 7°) + (5x - 43°) = 180°$

$\Rightarrow 8x = 180° + 36° \quad \Rightarrow \quad x = \frac{216°}{8}$

$\Rightarrow \quad x = 27°$

**19.** (b)

**TiP**

*The sum of pair of opposite angles of a cyclic quadrilateral is 180°.*

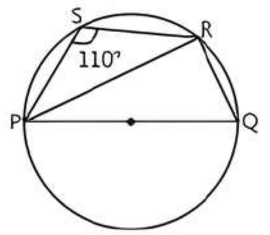
Here, $\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$

$\therefore a + 80° = 180°$ and $b + 60° = 180°$

$\Rightarrow \quad a = 100°$ and $b = 120°$

$\therefore \quad a + b = 100° + 120° = 220°$

**20.** (c) Given $\angle PSR = 110°$




Since, points P, Q, R and S are lie on a circle.

Therefore, quadrilateral PQRS is a cyclic.

$\therefore \angle S + \angle Q = 180° \quad \Rightarrow \quad 110° + \angle Q = 180°$

$\Rightarrow \quad \angle Q = 70°$

Since, PQ is a diameter of a circle, therefore

$\angle PRQ = 90°$

In $\Delta$ PQR,

$\angle QPR + \angle PRQ + \angle Q = 180°$

$\Rightarrow \quad \angle QPR + 90° + 70° = 180°$

$\Rightarrow \quad \angle QPR = 20°$

**21.** (a) **Assertion (A):** Given, $\angle OAB = 35°$ and $\angle OCB = 45°$

In $\Delta$ OAB,

OA = OB [Radii of a circle]

$\Rightarrow \quad \angle ABO = \angle BAO$ (angles opposite to equal sides of a triangle are equal)

$= 35°$

Similarly, $\angle OBC = \angle BCO = 45°$

$\therefore \quad \angle ABC = \angle ABO + \angle OBC$
$= 35° + 45° = 80°$

**TiP**
*Angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.*

$\therefore \quad \angle AOC = 2 \times \angle ABC$
$= 2 \times 80° = 160°$

So, Assertion (A) is true.

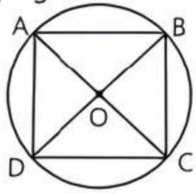
**Reason (R):** It is also true that angle subtended by an arc of a circle at the centre of circle is double the angle subtended by an arc on the circumference.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

**22.** (b) **Assertion (A):** Let AC and BD be two perpendicular diameters of a circle with centre O.

Here, $\angle ABC = 90°$ and $\angle ADC = 90°$

[angle in semi-circle is a right-angle]




Also, $\angle BAD = 90°$ and $\angle BCD = 90°$

In $\Delta$ AOB and $\Delta$ AOD,

AO = AO [Common]

$\angle AOB = \angle AOD$ [Given AO $\perp$ BD]

BO = OD [Radii of a circle]

$\therefore$ $\Delta AOB \cong \Delta AOD$ [By SAS congruence rule]

$\Rightarrow$ AB = AD [By CPCT]

Similarly, AD = DC, DC = BC, BC = AB

$\therefore$ AB = BC = CD = DA

Also, each angle of a quadrilateral is 90°.

Hence, ABCD is a square.

So, Assertion (A) is true.

**Reason (R):** It is also true that equal chords subtend equal angles at the centre.

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

**23.** (c) **Assertion (A):** In a given figure EC is a chord of a circle.

Since, $\angle EBC$ and $\angle EAC$ are the same segments of a circle.

$\therefore \angle EAC = \angle EBC = 60°$

Since, AC is the diameter of the circle and the angle in semi-circle is a right angle *i.e.,* $\angle AEC = 90°$.

Now in $\Delta$ ACE, use angles sum property of a triangle.

$\angle EAC + \angle AEC + \angle ACE = 180°$

$\Rightarrow 60° + 90° + \angle ACE = 180°$

$\Rightarrow \quad \angle ACE = 30°$

So, Assertion (A) is true.

**Reason (R):** It is false, because same segment of a circle makes equal angles at the circumference.

Hence, Assertion (A) is true but Reason (R) is false.

**24.** (a) **Assertion (A):** Given, AB = AC

**TiP**
*Equal chords subtend equal angles at the centre.*

$\Rightarrow \quad \angle AOB = \angle AOC$

In $\Delta$AOB and $\Delta$AOC,

AB = AC [Given]

OB = OC [Radii]

OA = OA [Common]

$\therefore$ $\Delta AOB \cong \Delta AOC$ [By SSS congruence criterion]

$\Rightarrow \angle AOB = \angle AOC = 90°$

$\Rightarrow$ OA $\perp$ BC

In $\Delta$ OBE,

$\angle OBE + \angle BOE + \angle BEO = 180°$

$\Rightarrow \quad 30° + 90° + x = 180°$

$\therefore \quad x = 60°$

So, Assertion (A) is true.

**Reason (R) :** It is true that equal chords subtend equal angles at the centre.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

**25.** (a) **Assertion (A):**

In $\Delta$ BDC,

BD = DC [Given]

$\Rightarrow \angle BCD = \angle DBC$ [Angles opposite to equal sides of a triangle are equal]

$= 35°$

Using angle sum property of a triangle,

$\angle CBD + \angle BCD + \angle BDC = 180°$

$\Rightarrow \quad 35° + 35° + \angle BDC = 180°$

$\Rightarrow \quad 70° + \angle BDC = 180°$

$\Rightarrow \quad \angle BDC = 110°$

Since, ABCD is a cyclic quadrilateral

**TiP**
*The sum of pair of opposite angles of a cyclic quadrilateral is 180°.*

Here $\angle BAC + \angle BDC = 180°$

$\therefore \quad \angle BAC + 110° = 180°$

$\Rightarrow \quad \angle BAC = 70°$

So, Assertion (A) is true.

**Reason (R):** It is true to say that the sum of opposite angles of a cyclic quadrilateral is 180°.

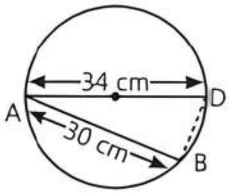
Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

**26.** exterior

**27.** diameter

**28.** semi-circle

29. Since, AD is a diameter of a circle. Therefore AD subtends right angled at the circumference of a circle.




*i.e.*, $\angle ABD = 90°$

In right angled $\Delta ABD$, use Pythagoras theorem,

$$BD = \sqrt{(AD)^2 - (AB)^2}$$
$$= \sqrt{(34)^2 - (30)^2}$$
$$= \sqrt{1156 - 900} = \sqrt{256} = 16 \text{ cm}$$

30. cyclic

31. True 32. True 33. False 34. False

**TiP**

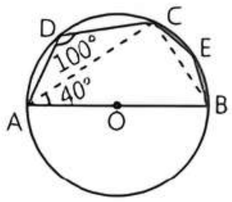
*In a cyclic quadrilateral, the sum of opposite angles is* 180°.

Now, $\angle A + \angle C = 90° + 95° = 185° \neq 180°$

and $\angle B + \angle D = 70° + 105° = 175° \neq 180°$

Here, we see that the sum of opposite angles is not equal to 180°. So, it is not cyclic quadrilateral.

35. True,




Join CA and CB.

Since, ABCD is a cyclic quadrilateral.

**TIP**

*Sum of opposite angles of cyclic quadrilateral is* 180°.

$\therefore \quad \angle ADC + \angle CBA = 180°$

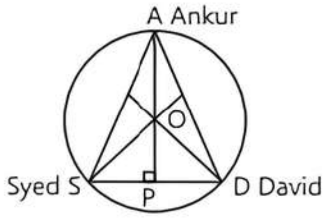
$\Rightarrow \angle CBA = 180° - 100° = 80°$

## Case Study Based Questions

## Case Study 1

A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are standing at equal distance on its boundary each having a toy telephone in his hands to talk each other.

In mathematically; A, S and D are the position of Ankur, Syed and David are sitting at equal distance and O be the position of the centroid of a triangle.




*On the basis of the above information, solve the following questions:*

**Q 1. The length of AP is:**

a. 20 m b. 30 m

c. 15 m d. 25 m

**Q 2. The distance between any two boys is:**

a. $20\sqrt{3}$ m b. $10\sqrt{3}$ m

c. $15\sqrt{3}$ m d. 20 m

**Q 3. The angle between AS and SD is:**

a. 65° b. 60° c. 70° d. 75°

**Q 4. $\angle$SOD is equal to:**

a. 70° b. 60° c. 120° d. 80°

**Q 5. $\angle$OSP is equal to:**

a. 50° b. 60° c. 30° d. 70°

## Solutions

1. (b) Let Ankur, Syed and David be represented by A, S and D respectively.

Let $AS = SD = AD = 2a$ m

Since, O is the centroid, so it divides a median in the ratio 2 : 1.

So, $\dfrac{AO}{OP} = \dfrac{2}{1}$

$\Rightarrow \dfrac{20}{OP} = \dfrac{2}{1}$ [$\because$ AO = Radius = 20 m]

$\Rightarrow$ OP = 10 m

$\therefore$ AP = AO + OP = 20 + 10 = 30 m

So, option (b) is correct.

2. (a) In right angled $\Delta APS$, by Pythagoras theorem,

$AS^2 = AP^2 + PS^2$

$\Rightarrow (2a)^2 = (30)^2 + a^2 \Rightarrow 4a^2 - a^2 = 900$

$\Rightarrow 3a^2 = 900 \Rightarrow a^2 = 300$

$\Rightarrow a = 10\sqrt{3}$ m

So, $2a = 2 \times 10\sqrt{3} = 20\sqrt{3}$ m

Hence, the distance between any two boys is $20\sqrt{3}$ m.

So, option (a) is correct.

3. (b) Since, all three boys are standing at equal distance. Therefore, $\Delta$ ASD is an equilateral triangle.

**TIP**

*In an equilateral triangle, all three internal angles are equal i.e., equal to 60°.*

So, angle between AS and SD is 60°.

So, option (b) is correct.

**4.** (c) As we know that, the intersection point of altitudes is at the centre of circumcentre of circle.

The angle subtended by an arc at the centre is double the angle subtended by it any point on the remaining part of the circle.

$\therefore \quad \angle SOD = 2\angle SAD$

$= 2 \times 60°$

$[\because \angle SAD = \angle ASD = \angle ADS = 60°]$

$= 120°$

So, option (c) is correct.

**5.** (c) Since, $\angle SOD = 120°$

$\therefore \quad \angle SOP = \frac{1}{2} \times 120° = 60° \quad [\because \text{AP is a median}]$

Now, In $\Delta OSP$,

$\angle SOP + \angle OSP + \angle OPS = 180°$

$\Rightarrow \quad 60° + \angle OSP + 90° = 180° \quad [\because AP \perp SD]$

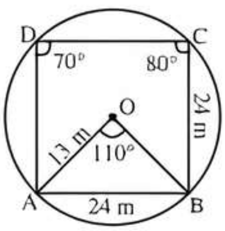
$\Rightarrow \quad \angle OSP = 180° - 150° = 30°$

So, option (c) is correct.

## Case Study 2

I have a circular garden in the outside of the city. Muncipality decided to put some benches, so that the people can sit their and can have some fresh air.

In the given figure, there are two benches of same colour A and B, which are placed at a distance AB = 24 m. Similarly two other benches of same colour C and D are also placed at a same distance of 24 m from each other.




*On the basis of the above information, solve the following questions:*

**Q 1. The measure of ∠BOC is:**

a. 120° b. 55° c. 110° d. 60°

**Q 2. The measure of ∠ABC is:**

a. 130° b. 120° c. 105° d. 110°

**Q 3. The perpendicular distance from centre O to the chord AB is:**

a. 6 m b. 5 m c. 10 m d. 12 m

**Q 4. The value of angle ∠BAO is:**

a. 60° b. 35° c. 70° d. 80°

**Q 5. The measure of ∠DAO is:**

a. 70° b. 65° c. 80° d. 85°

### Solutions

**1.** (c) Given, $\angle AOB = 110°$

In the given figure, AB and BC are of equal chords. Therefore, they subtend equal angles at the centre.

$\therefore \quad \angle BOC = \angle AOB = 110°$

So, option (c) is correct.

**2.** (d) Since, points A, B, C and D lie on the circle. Therefore, it forms a cyclic quadrilateral.

**TIP**

*The pair of opposite angles of a cyclic quadrilateral is 180°.*

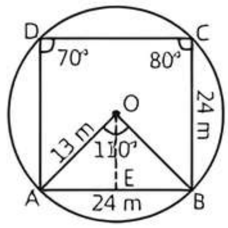
$\Rightarrow \quad \angle ABC + \angle ADC = 180°$

$\Rightarrow \quad \angle ABC + 70° = 180°$

$\therefore \quad \angle ABC = 110°$

So, option (d) is correct.

**3.** (b) Draw a perpendicular line from centre O to the chord AB, which bisects the chord.




$\therefore \quad AE = BE = 12 \text{ m}$

In right angle $\Delta OEA$ at E, use Pythagoras theorem

$OE = \sqrt{(OA)^2 - (AE)^2}$

$= \sqrt{(13)^2 - (12)^2} = \sqrt{169 - 144}$

$= \sqrt{25} = 5 \text{ m}$

So, option (b) is correct.

**4.** (b) In $\Delta AOB$,

$OA = OB \quad$ [Radii of a circle]

$\Rightarrow \quad \angle ABO = \angle BAO$

[Angles opposite to equal sides are equal]

Use angle, sum property of a $\Delta AOB$,

$\angle ABO + \angle BAO + \angle AOB = 180°$

$\Rightarrow \quad \angle BAO + \angle BAO + 110° = 180°$

$\Rightarrow \quad 2\angle BAO = 70°$

$\therefore \quad \angle BAO = 35°$

So, option (b) is correct.

**5.** (b) Since, A and C are opposite points of a cyclic quadrilateral. Therefore

$\angle BAD + \angle DCB = 180°$

$\Rightarrow \quad \angle BAD = 180° - 80° = 100°$

$\Rightarrow \quad \angle BAO + \angle DAO = 100°$

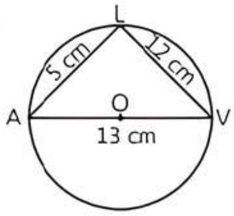
$\Rightarrow \quad 35° + \angle DAO = 100°$

$\Rightarrow \quad \angle DAO = 65°$

So, option (b) is correct.

## Case Study 3

Three boys Anshul, Vivek and Lalit are playing a game by standing on the boundary of a circle of diameter 13 cm with centre O. Anshul throws a ball to Vivek, Vivek to Lalit and Lalit to Anshul and so on. The distance between Anshul and Lalit is 5 cm and between Vivek and Lalit is 12 cm.




*On the basis of the given information, solve the following questions:*

**Q 1. Measure ∠ ALV:**

a. 60° b. 90° c. 50° d. 40°

**Q 2. The length of the longest chord is:**

a. 12 cm b. 5 cm c. 13 cm d. 16 cm

**Q 3. If ∠ LAV = 50°, then ∠ AVL is:**

a. 140° b. 50° c. 40° d. 100°

**Q 4. The area covered by these three persons is:**

a. 20 $cm^2$ b. 30 $cm^2$ c. 25 $cm^2$ d. 35 $cm^2$

**Q 5. Equal chords of a circle subtends equal angle at:**

a. centre
b. end point of diameter
c. other than centre
d. None of the above

### Solutions

1. (b) Given, AV is a diameter of a circle. So, angle subtended by diameter to the circumference of the circle is 90°.

$\therefore \quad \angle ALV = 90°$

So, option (b) is correct.

2. (c) The length of the longest chord in a circle is equal to the length of diameter.

∴ Length of longest chord = length of diameter = 13 cm

So, option (c) is correct.

3. (c) In Δ AVL,

$\angle LAV + \angle AVL + \angle ALV = 180°$ [by angle sum property of a triangle]

$\Rightarrow \quad 50° + \angle AVL + 90° = 180° \quad [\because \angle ALV = 90°]$

$\therefore \quad \angle AVL = 40°$

So, option (c) is correct.

4. (b) The area covered by these three persons is equal to the area of Δ AVL.

$\therefore$ Area of Δ AVL $= \frac{1}{2} \times AL \times LV$

$= \frac{1}{2} \times 5 \times 12 = 30\ cm^2$

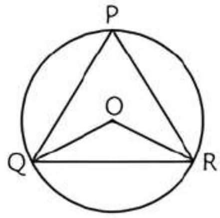
So, option (b) is correct.

5. (a) Equal chords of a circle subtends equal angle at the centre.

So, option (a) is correct.

## Case Study 4

Government of India is working regularly for the growth of handicapped persons. For this three STD booths situated at point P, Q and R are as shown in the figure, which are operated by handicapped persons. These three booths are equidistant from each other as shown in the figure.




*On the basis of the above information, solve the following questions:*

**Q 1. Which type of Δ PQR in the given figure?**

**Q 2. Measure angle ∠ QOR.**

**Q 3. Find the value of ∠ OQR.**

**Q 4. Is it true that points P, Q and R lie on the circle?**

### Solutions

1. Given P, Q and R are equidistant. It means their distances are equal.

**TiP**

*In an equilateral triangle, length of all three sides are equal.*

So, Δ PQR is an equilateral triangle.

2. Since, Δ PQR is an equilateral triangle.

$\therefore \quad \angle PQR = \angle PRQ = \angle QPR = 60°$

The angle subtended by an arc at the centre is double the angle subtended by it any point on the remaining part of the circle.

$\therefore \angle QOR = 2\angle QPR = 2 \times 60° = 120°$

3. In Δ OQR,

$OQ = OR$ [Radii of a circle]

$\Rightarrow \quad \angle ORQ = \angle OQR$ [Angles opposite to equal sides of a triangle are equal]

Using angle sum property of a triangle,

$\angle OQR + \angle ORQ + \angle QOR = 180°$

$\Rightarrow \quad \angle OQR + \angle OQR + 120° = 180°$

$\Rightarrow \quad 2\angle OQR = 60°$

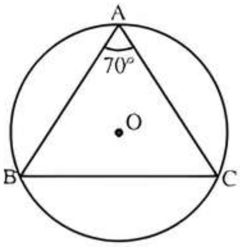
$\therefore \quad \angle OQR = 30°$

4. Yes, it is true that points P, Q and R lie on the circle.

## Case Study 5

Narendra Modi Stadium also known as Motera Stadium is situated in Ahmedabad, Gujarat. Currently it is the largest stadium in the world with a capacity of 1,32,000 viewers. In the field there are 2 ends known as bowler end and striker end.

Geometrically, O is the centre of a circle, $\angle BAC = 70^\circ$, which is shown in figure.




*On the basis of the above information, solve the following questions:*

**Q 1. Find the angle subtended by the chord BC at the centre.**

**Q 2. Find $\angle OBC + \angle OCB$.**

**Q 3. If OB = 5 cm and BC = 6 cm, then find the distance from centre O to the chord BC.**

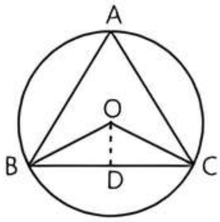
**Q 4. If point D lies on the circle between B and C, then find $\angle BDC$.**

## Solutions

1. Given, $\angle BAC = 70^\circ$
   The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
   $\therefore \quad \angle BOC = 2\angle BAC$
   $= 2 \times 70^\circ = 140^\circ$

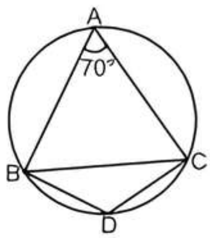
2. In $\Delta$ OBC, use angle sum property of a triangle,
   $\angle OBC + \angle OCB + \angle BOC = 180^\circ$
   $\Rightarrow \quad \angle OBC + \angle OCB + 140^\circ = 180^\circ$
   $\Rightarrow \quad \angle OBC + \angle OCB = 40^\circ$

3. As we know that, perpendicular drawn from centre O of circle to the chord BC bisects the chord.
   $\therefore$ BD $= \frac{1}{2}$ BC $= \frac{1}{2} \times 6 = 3$ cm
   In right angled $\Delta$ ODB, use Pythagoras theorem




   OD $= \sqrt{(OB)^2 - (BD)^2}$
   $= \sqrt{(5)^2 - (3)^2} = \sqrt{25 - 9} = \sqrt{16} = 4$ cm

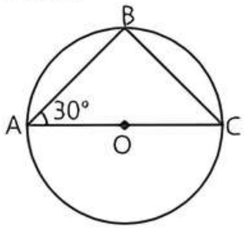
4. Here, we see that four points A, B, C and D lie on a circle, so it forms a cyclic quadrilateral.
   $\therefore \quad \angle BAC + \angle BDC = 180^\circ$
   $\Rightarrow \quad 70^\circ + \angle BDC = 180^\circ$
   $\therefore \quad \angle BDC = 110^\circ$



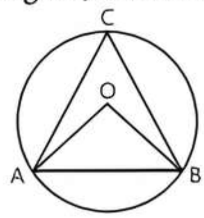

## Very Short Answer Type Questions

**Q 1. If PQ is a chord of a circle with radius $r$ units and R is a point on the circle such that $\angle PRQ = 90^\circ$, then find the length of PQ.**

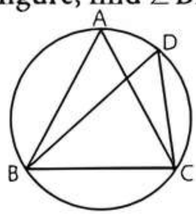
**Q 2. If O is the centre of a circle and $\angle OAB = 30^\circ$, then find $\angle ACB$.**



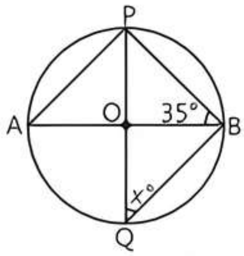

**Q 3. In the given figure, if $\angle AOB = 80^\circ$, find $\angle ACB$.**



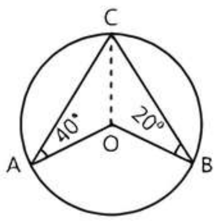

**Q 4. In the given figure, find $\angle BDC$ if $\angle BAC = 72^\circ$.**



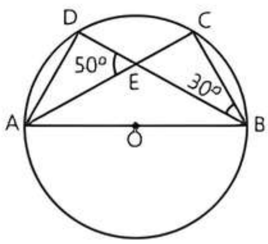

**Q 5. In the given figure, O is the centre of a circle. Find the value of $x$.**



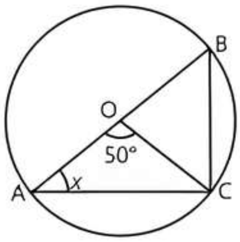

**Q 6. In the given figure, O is the centre of the circle. Find $\angle AOB$.**



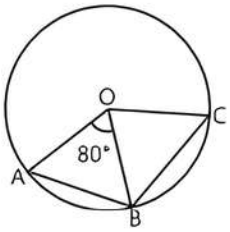

**Q 7. In the given figure, O is the centre of the circle, $\angle CBE = 30^\circ$ and $\angle DEA = 50^\circ$. Find the angle $\angle DAE$.**



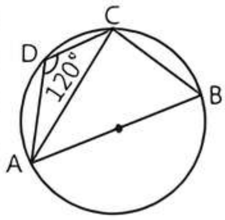
**Q 8.** In the given figure, AB is diameter, $\angle AOC = 50°$. Find the value of $x$.




**Q 9.** In the figure, O is the centre of the circle. If AB = BC and $\angle AOB = 80°$, then find $\angle OBC$.

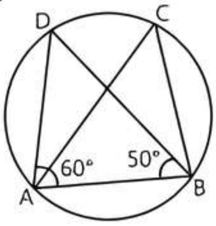



**Q 10.** In the given figure, ABCD is a cyclic quadrilateral whose side AB is a diameter of the circle through A, B, C and D. If $\angle ADC = 120°$, find $\angle ABC$.

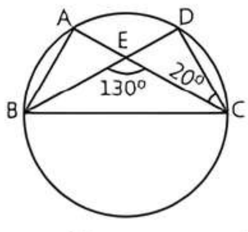



## Short Answer Type-I Questions

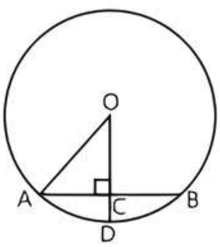
**Q 1.** In the figure, if $\angle DAB = 60°$, $\angle ABD = 50°$, then find $\angle ACB$.




**Q 2.** In the figure, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that $\angle BEC = 130°$ and $\angle ECD = 20°$. Find $\angle BAC$.

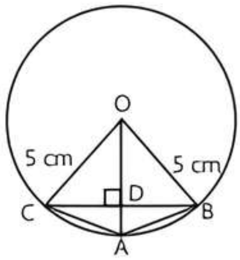



**Q 3.** In the given figure, if OA = 5 cm, AB = 8 cm and OD is perpendicular to AB, then find the length of CD.




**Q 4.** A chord of length 10 cm is at a distance of 12 cm from the centre of a circle. Find the radius of the circle.

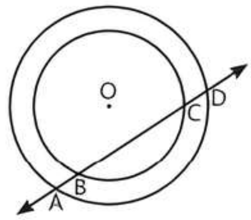
**Q 5.** In the given figure, chords AB = AC = 6 cm. Find the length of BC, if radius is 5 cm.




**Q 6.** Two chords AB and CD of a circle are parallel and a line $l$ is the perpendicular bisector of AB. Show that $l$ bisects CD.

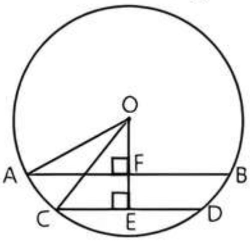
**Q 7.** Prove that the line joining the mid-points of two parallel chords of a circle passes through the centre of the circle.

**Q 8.** If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD.

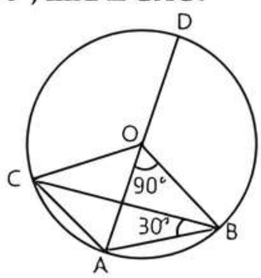



**Q 9.** Prove that equal chords of a circle subtend equal angles at the centre.

**Q 10.** In the given figure, OE ⊥ CD, OF ⊥ AB, AB || CD, AB = 48 cm, CD = 20 cm, radius OA = 26 cm. Find the length of EF.

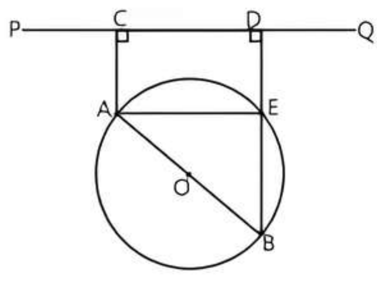



**Q 11.** In the given figure, $\angle AOB = 90°$ and $\angle ABC = 30°$, find $\angle CAO$.

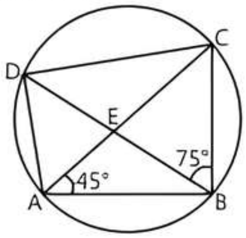



**Q 12.** If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

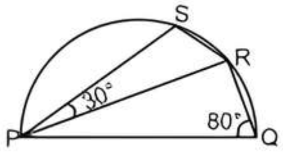
**Q 13.** In the given figure, AB is a diameter of the circle with centre O. If AC and BD are perpendiculars on a line PQ and BD meets the circle at E, then prove that AC = ED.



**Q 14.** In the given figure, if $\angle DBC = 75°$ and $\angle BAC = 45°$, then find $\angle DCB$.

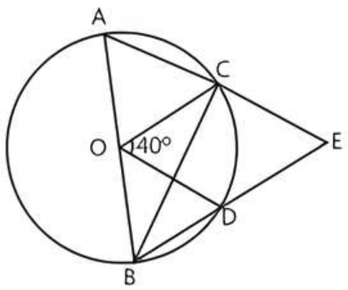



**Q 15.** In the following figure, R and S are the points on the semi-circle inscribed on PQ as diameter $\angle PQR = 80°$ and $\angle RPS = 30°$. Find the value of $\angle QRS$.



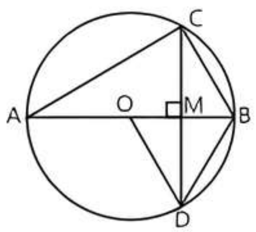

## Short Answer Type-II Questions

**Q 1.** In the given figure, AB is a diameter of the circle with centre O, AC and BD produced meet at E and $\angle COD = 40°$. Calculate $\angle CED$.

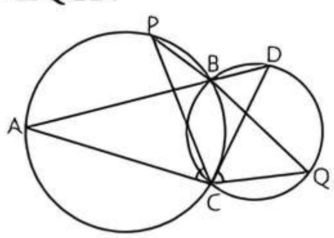



**Q 2.** Two equal chords AB and CD of a circle when produced, intersect at a point P. Prove that PB = PD.

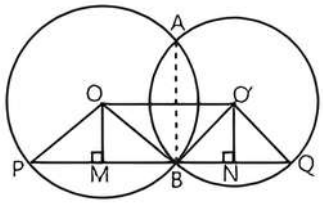
**Q 3.** In the adjoining figure, O is the centre of the circle, BD = OD and $CD \perp AB$. Find $\angle CAB$.



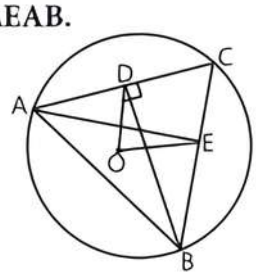

**Q 4.** Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see figure). Prove that $\angle ACP = \angle QCD$.




**Q 5.** Two circles with centres O and O′ intersect at two points A and B. A line PQ is drawn parallel to OO′ through A (or B) intersecting the circles at P and Q. Prove that PQ = 2OO′.




**Q 6.** In the given figure, O is the centre of the circle, $OD \perp AC$, $OE \perp BC$ and OD = OE. Show that $\Delta DBA \cong \Delta EAB$.



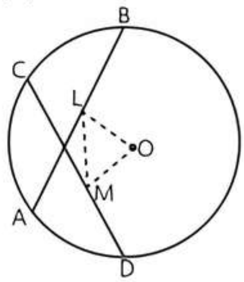

**Q 7.** Two chords AB and AC of a circle are equal. Prove that the centre of the circle lies on the angle bisector of $\angle BAC$.

**Q 8.** If two chords of a circle are equally inclined to the diameter through their point of intersection, prove that the chords are equal.

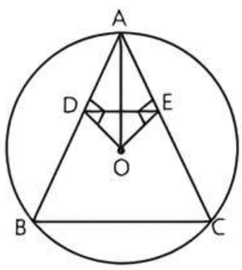
**Q 9.** In the given figure, L and M are the mid-points of two equal chords AB and CD of a circle.

Prove that: (i) $\angle OLM = \angle OML$

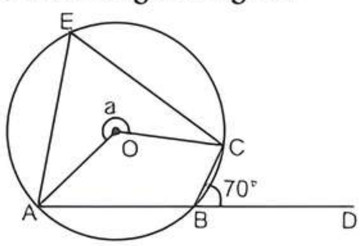
(ii) $\angle ALM = \angle CML$




**Q 10.** In the given figure, AB and AC are two chords of circle whose centre is O. If $OD \perp AB$, $OE \perp AC$ and AO bisects $\angle DAE$, prove that $\Delta ADE$ is an isosceles triangle and $\angle ACB = \angle ABC$.



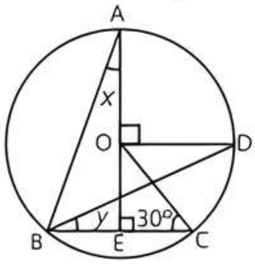

**Q 11.** If O is the centre of the circle, then find the value of $a$ in the given figure.



## Long Answer Type Questions

**Q 1.** If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.

**Q 2.** In the given figure, O is the centre of the circle, $\angle BCO = 30°$. Find $x$ and $y$.




**Q 3.** Bisectors of angles A, B and C of a $\triangle$ ABC intersect its circumcircle at D, E and F respectively. Prove that the angles D, E and F are $90°-\frac{1}{2}A, 90°-\frac{1}{2}B$ and $90°-\frac{1}{2}C$.

**Q 4.** Prove that the line joining the mid-points of two equal chords of a circle subtends equal angles with chords.

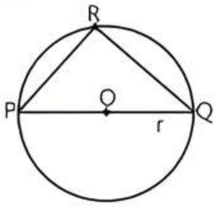
**Q 5.** Show that if two chords of a circle bisect each other, they must be diameters of the circle.

**Q 6.** Prove that the quadrilateral formed by the internal angle bisectors of any quadrilateral is cyclic.

## Solutions

### Very Short Answer Type Questions

**1.** Since, PQ is a chord of a circle and R is a point on the circle such that $\angle PRQ = 90°$, therefore the arc PRQ is a semi-circle. So, PQ will be a diameter of a circle.




$\therefore$ Length of PQ = 2 × radius = $2 \times r = 2r$

**COMMON ERR(!)R**

*Some of the students do not apply the concept that if chord subtends a right angle, then the chord becomes a diameter of a circle. Adequate practice is required for these type of questions.*

**2.**

**TR!CK**

*Diameter subtends a right angle to the circumference of a circle.*

Here, $\angle ABC = 90°$ and $\angle BAC = 30°$

In $\triangle$ ABC, use angle sum property of a triangle

$$\angle CAB + \angle ABC + \angle ACB = 180°$$

$\Rightarrow \quad 30° + 90° + \angle ACB = 180°$

$\therefore \quad \angle ACB = 60°$

**3.** Given, $\angle AOB = 80°$

$\angle AOB = 2\angle ACB$

or $\quad \angle ACB = \frac{1}{2}\angle AOB$

[Angle subtended by an arc at the centre is double the angle subtended by it at any other point on the remaining part of the circle].

$\therefore \quad \angle ACB = \frac{1}{2} \times 80° = 40°$

**4.** Given, $\angle BAC = 72°$

$\therefore \quad \angle BDC = \angle BAC = 72°$

[Angles in the same segment of a circle are equal]

Hence, $\angle BDC = 72°$.

**5.** In $\triangle$APB,

$\angle APB + \angle PAB + \angle ABP = 180°$

[Angle sum property of a triangle]

$\Rightarrow \quad 90° + \angle PAB + 35° = 180°$

[$\because$ $\angle APB = 90°$, Angle in a semi-circle]

$\Rightarrow \quad \angle PAB + 125° = 180°$

$\Rightarrow \angle PAB = 180° - 125° = 55°$

**TR!CK**

*Angles in the same segment of a circle are equal.*

$\therefore \quad \angle PQB = \angle PAB = 55°$

Hence, the value of $x$ is 55°.

**6.** In $\triangle$ AOC,

OA = OC [Radii of a circle]

$\Rightarrow \quad \angle ACO = \angle OAC$ [Angles opposite to equal sides of a triangle are equal]

$= 40°$

Similarly,

$\angle OCB = 20°$

$\therefore \quad \angle ACB = \angle ACO + \angle BCO$

$= 40° + 20° = 60°$

The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$\therefore \quad \angle AOB = 2 \times \angle ACB$

$= 2 \times 60° = 120°$

**7.** In the given figure,

$\angle BEC = \angle DEA$ [Vertically opposite angles]

$= 50°$

In $\triangle$ BEC,

$\angle E + \angle B + \angle C = 180°$

$\Rightarrow \quad 50° + 30° + \angle C = 180°$

[Angle sum property of a triangle]

$\therefore \quad \angle C = 100°$

Since, angles in same segment of a circle are equal

$\therefore \quad \angle D = \angle C = 100°$

Now In $\triangle$AED,

$\angle ADE + \angle AED + \angle DAE = 180°$

$\Rightarrow \quad 100° + 50° + \angle DAE = 180°$

$\therefore \quad \angle DAE = 180° - 150° = 30°$

8. Given, AB is a diameter of a circle. Therefore, AB subtends a right angle at the circumference of a circle.

$\therefore \quad \angle ACB = 90°$

Since, angle subtended by any point on the remaining circle is half the angle subtended by it at centre of the circle.

$\therefore \quad \angle ABC = \frac{1}{2} \times \angle AOC$

$= \frac{1}{2} \times 50° = 25°$

In $\Delta$ ABC,

$\angle A + \angle B + \angle C = 180°$

[By angle sum property of a triangle]

$\Rightarrow \quad x + 25° + 90° = 180°$

$\Rightarrow \quad x = 180° - 115° = 65°$

9. Given, AB = BC and $\angle AOB = 80°$

**TR!CK**

*Equal chords of a circle subtends equal angles at the centre.*

$\therefore \quad \angle BOC = \angle AOB = 80°$

$\because \quad OC = OB$ [Radii of circle]

$\therefore \quad \angle OBC = \angle OCB$

[$\because$ Angles opposite to equal sides of a $\Delta$ are equal]

$\therefore$ In $\Delta$ BOC,

$\angle BOC + \angle OBC + \angle OCB = 180°$

[By angle sum property of triangle]

$\Rightarrow 80° + \angle OBC + \angle OBC = 180°$

$\Rightarrow 2\angle OBC = 180° - 80° = 100°$

$\therefore \quad \angle OBC = 50°$

10. Given, ABCD is a cyclic quadrilateral.

Therefore, $\angle ADC + \angle ABC = 180°$

$\Rightarrow \quad 120° + \angle ABC = 180°$

$\Rightarrow \quad \angle ABC = 180° - 120°$

$\therefore \quad \angle ABC = 60°$

## Short Answer Type-I Questions

1. In $\Delta ADB$, $\angle ABD + \angle ADB + \angle BAD = 180°$

[By angle sum property of a triangle]

$\therefore \quad 50° + \angle ADB + 60° = 180°$

$\Rightarrow \quad \angle ADB = 180° - (50° + 60°) = 70°$

$\therefore \quad \angle ACB = \angle ADB = 70°$

[Angles in the same segment]

Hence, $\angle ACB = 70°$

2. $\angle BEC + \angle DEC = 180°$ [Linear pair]

$\Rightarrow \quad 130° + \angle DEC = 180°$

$\Rightarrow \quad \angle DEC = 180° - 130° = 50°$

Now, in $\Delta$ DEC,

$\angle DEC + \angle DCE + \angle CDE = 180°$

[By angle sum property of a triangle]

$\Rightarrow \quad 50° + 20° + \angle CDE = 180°$

$\therefore \quad \angle CDE = 180° - 70° = 110°$

Also, $\angle BAC = \angle CDB$

[Angles in same segment are equal]

$= 110°$ [$\because \angle CDB = \angle CDE = 110°$]

Hence, $\angle BAC = 110°$

3. Given, AB = 8 cm

$\therefore \quad AC = \frac{1}{2} AB = \frac{1}{2} \times 8 = 4$ cm

[Perpendicular drawn from centre bisects the chord

OA = 5 cm [Given]

In right angled $\Delta$OCA, by Pythagoras theorem,

$OA^2 = OC^2 + AC^2$

$\Rightarrow \quad 5^2 = OC^2 + 4^2$

$\Rightarrow \quad 25 = OC^2 + 16$

$\Rightarrow \quad OC^2 = 25 - 16 = 9$

$\Rightarrow \quad OC = \sqrt{9} = 3$ cm

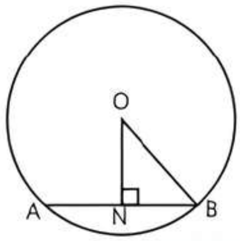
So, $CD = OD - OC = 5 - 3 = 2$ cm [$\because$ OA = OD]

Hence, the length of CD is 2 cm.

4. Given, AB = 10 cm, ON = 12 cm.

Also, ON $\perp$ AB and AN = BN

[$\because$ Perpendicular drawn from the centre bisects the chord]




In right angled $\Delta$ONB, $OB^2 = ON^2 + NB^2$

[By Pythagoras theorem]

$\therefore \quad OB^2 = 12^2 + 5^2 \left(\because BN = \frac{AB}{2} = \frac{10}{2} = 5 \text{ cm}\right)$

$= 144 + 25 = 169$

$\therefore \quad OB = 13$ cm

Hence, the radius of the circle is 13 cm.

5. Let $OD = x \quad \Rightarrow \quad AD = 5 - x$

[$\because$ OA = OB = OC = 5 cm]

In right angled $\Delta$ ODC, use Pythagoras theorem,

$OC^2 = OD^2 + CD^2$

$\Rightarrow \quad (5)^2 = (x)^2 + CD^2$

$\Rightarrow \quad CD^2 = 25 - x^2$ ...(1)

In right angled $\Delta$ ACD,

$AC^2 = AD^2 + CD^2$ [By Pythagoras theorem]

$(6)^2 = (5 - x)^2 + CD^2$

$\Rightarrow \quad CD^2 = 36 - (25 + x^2 - 10x)$

$= 11 + 10x - x^2 \quad ...(2)$

From eqs. (1) and (2), we get

$25 - x^2 = 11 + 10x - x^2$

$\Rightarrow \quad 10x = 14$

$\therefore \quad x = 1.4$ cm

From eq. (1), we get

$CD^2 = 25 - (1.4)^2$

$= 25 - 1.96 = 23.04$

$\therefore \quad CD = 4.8$ cm

$\because$ Perpendicular drawn from the centre O to the chord BC bisect it.

$\therefore \quad BC = 2 \times CD$

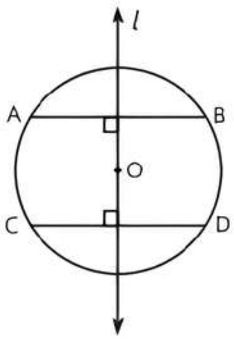
$= 2 \times 4.8 = 9.6$ cm

**6.**

**TR!CK**

*The perpendicular bisector of any chord of a circle always passes through its centre.*

Here, $l$ is the perpendicular bisector of AB, *i.e.*, passes through the centre O of the circle.

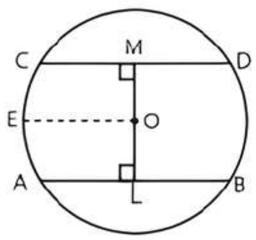



Since, AB || CD and $l \perp AB$

$\Rightarrow \quad l \perp CD$

Also, $l$ passes through the centre O, so $l$ also bisects the chord CD. **Hence proved**

**7.** Let M and L are the mid-points of two parallel chords CD and AB respectively of given circle C(O, r).




Draw OE || AB || CD

Since, the line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord.

$\therefore \quad OL \perp AB$ and $OM \perp CD$

Since, $OL \perp AB$ and AB || EO

So, $\angle EOL + \angle OLA = 180°$ [Co-interior angles]

$\Rightarrow \quad \angle EOL + 90° = 180°$

$\Rightarrow \quad \angle EOL = 180° - 90°$

$\therefore \quad \angle EOL = 90°$

Similarly, $OM \perp CD$ and CD || EO

So, $\angle EOM + \angle CMO = 180°$ [Co-interior angles]

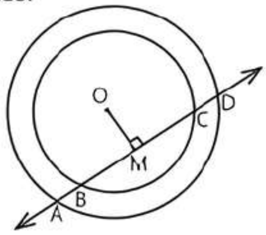
$\Rightarrow \quad \angle EOM + 90° = 180°$

$\therefore \quad \angle EOM = 180° - 90° = 90°$

Now, $\angle EOL + \angle EOM = 90° + 90° = 180°$

Hence, LOM is a straight line.

**8.** Given, a line AD that intersects two concentric circles at A, B, C and D, where O is the centre of these circles.




Draw $OM \perp AD$.

$\because$ AD is the chord of larger circle.

$\therefore \quad AM = DM \quad ...(1)$ [OM bisects the chord]

BC is the chord of smaller circle.

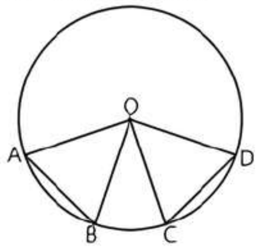
$\therefore \quad BM = CM \quad ...(2)$ [OM bisects the chord]

Subtracting eq. (2) from eq. (1), we get

$AM - BM = DM - CM$

$\therefore \quad AB = CD$ **Hence proved**

**9. Given:** A circle with centre O. AB and CD are two equal chords of the circle which subtend $\angle AOB$ and $\angle COD$ at the centre O.




**To Prove:** $\angle AOB = \angle COD$

**Proof:** In $\Delta AOB$ and $\Delta COD$,

$OA = OC$ [Radii of same circle]

$OB = OD$ [Radii of same circle]

$AB = CD$ [Given]

$\therefore \quad \Delta AOB \cong \Delta COD$ [by SSS congruence rule]

Then, $\angle AOB = \angle COD$ [By CPCT]

**Hence proved**

**10.** Given, $OE \perp CD$, $OF \perp AB$, AB = 48 cm, CD = 20 cm and OA = 26 cm.

**T!P**

*The perpendicular drawn from centre of circle to the chord bisect the chord.*

Here, E and F are the mid-points of CD and AB respectively.

Therefore, $CE = \frac{1}{2} CD = \frac{1}{2} \times 20 = 10$ cm

and $AF = \frac{1}{2} AB = \frac{1}{2} \times 48 = 24$ cm

In right angled $\Delta$ OFA,

$$OF = \sqrt{(OA)^2 - (AF)^2}$$

[Use Pythagoras theorem]

$$= \sqrt{(26)^2 - (24)^2}$$

$$= \sqrt{676 - 576} = \sqrt{100} = 10 \text{ cm}$$

In right angled $\Delta$ OEC,

$$OE = \sqrt{(OC)^2 - (CE)^2}$$

[$\because$ OA = OC = Radii]

$$= \sqrt{(26)^2 - (10)^2} = \sqrt{676 - 100}$$

$$= \sqrt{576} = 24 \text{ cm}$$

$\therefore$ Required length EF = OE – OF

= 24 – 10 = 14 cm

**11.**

**TR!CK**

*The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.*

$\therefore \quad \angle AOB = 2\angle ACB$

$\Rightarrow \quad \angle ACB = \frac{1}{2}\angle AOB$

$\therefore \quad \angle ACB = \frac{1}{2} \times 90° = 45°$

Now, $\quad \angle COA = 2\angle CBA$

$= 2 \times 30° = 60°$

$\because \angle COD + \angle COA = 180°$ [Linear pair]

$\therefore \quad \angle COD = 180° - \angle COA$

$= 180° - 60° = 120°$

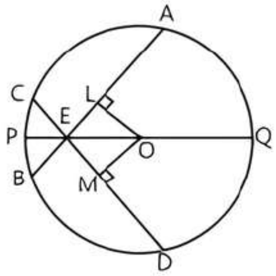
Again using the theorem, the angle subtended by an arc at the centre is double the angle subtended by it any point on the remaining part of the circle.

$\therefore \quad \angle COD = 2\angle CAO$

$\Rightarrow \quad \angle CAO = \frac{1}{2}\angle COD$

$\therefore \quad \angle CAO = \frac{1}{2} \times 120° = 60°$

**12.** Given, AB and CD are two equal chords of a circle which meets at E within the circle and a line PQ joining the point of intersection to the centre.




Draw OL $\perp$ AB and OM $\perp$ CD.

In $\Delta$OLE and $\Delta$OME,

OL = OM (equal chords are equidistant)

OE = OE [Common]

$\angle OLE = \angle OME$ [Each 90°]

$\therefore \quad \Delta OLE \cong \Delta OME$

(by RHS congruence rule)

Thus, $\angle LEO = \angle MEO$ [By CPCT]

So, the line joining the point of intersection to the centre makes equal angles with the chords.

**Hence proved**

**13.** **Given:** AB is a diameter of the circle with centre O. AC $\perp$ PQ, BD $\perp$ PQ and BD meets the circle at E.

**To Prove:** AC = ED

**Proof:** $\angle AEB = 90° = \angle AED$

[Angle in a semi-circle]

$\angle EAC + \angle ACD + \angle CDE + \angle AED = 360°$

[Sum of angles of a quadrilateral]

$\angle EAC + 90° + 90° + 90° = 360°$

$\therefore \quad \angle EAC = 360° - 270° = 90°$

Since, each angle of quadrilateral EACD is 90°.

$\therefore$ EACD is a rectangle.

Hence, AC = ED. **Hence proved**

**14.** Given, $\angle DBC = 75°$ and $\angle BAC = 45°$

**T!P**

*Angles in the same segment of a circle are equal.*

Here, consider CD is a chord. Therefore,

$\angle CAD = \angle CBD$

$= 75°$

Now, $\quad \angle DAB = \angle CAD + \angle CAB$

$= 75° + 45° = 120°$

Since, ABCD is a cyclic quadrilateral.

Therefore,

$\angle DAB + \angle DCB = 180°$

$\Rightarrow 120° + \angle DCB = 180°$

$\therefore \quad \angle DCB = 60°$

**15.** Since, $\Delta$PQR is a right angled triangle, right angled at R. Use angle sum property of a triangle.

$\therefore \angle RPQ + \angle PQR + \angle QRP = 180°$

$\Rightarrow \quad \angle RPQ + 80° + 90° = 180°$

$\therefore \quad \angle RPQ = 10°$

Since, PQRS is a cyclic quadrilateral.

Therefore, $\angle SPQ + \angle QRS = 180°$

$\Rightarrow (\angle SPR + \angle RPQ) + \angle QRS = 180°$

$\Rightarrow \quad 30° + 10° + \angle QRS = 180°$

$\therefore \quad \angle QRS = 140°$

## Short Answer Type-II Questions

1. Since, $\angle ACB + \angle BCE = 180°$ [Linear pair]

$\Rightarrow$ $90° + \angle BCE = 180°$

[$\because \angle ACB$ is in a semi-circle]

$\Rightarrow \angle BCE = 180° - 90° = 90°$

**TR!CK**

*Angle subtended by an arc at any point on the remaining part of the circle is half the angle subtended by it at the centre of circle.*

Here, $\angle DBC = \frac{1}{2}\angle COD$

$\Rightarrow$ $\angle DBC = \frac{1}{2} \times 40° = 20°$

Now, in $\Delta EBC$,

$\angle CEB + \angle EBC + \angle BCE = 180°$

[Angle sum property of a triangle]

$\Rightarrow$ $\angle CEB + 20° + 90° = 180°$

[$\because \angle EBC = \angle DBC = 20°$]

$\Rightarrow$ $\angle CEB + 110° = 180°$

$\Rightarrow$ $\angle CEB = 180° - 110°$

$\Rightarrow$ $\angle CEB = 70° = \angle CED$

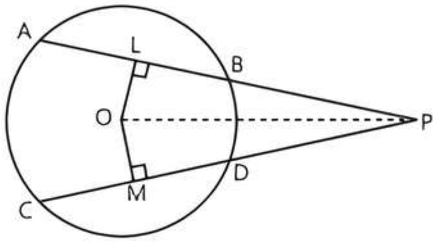
[$\because \angle CEB = \angle CED$]

Hence, $\angle CED = 70°$.

2. **Given:** AB = CD, which intersect at P, when produced.

**To Prove:** PB = PD

**Construction:** Join OP. Draw $OM \perp CD$ and $OL \perp AB$.




**Proof:** In $\Delta OLP$ and $\Delta OMP$,

OL = OM

(equal chords are equidistant from the centre)

$\angle OLP = \angle OMP$ [Each 90°]

OP = OP [Common]

$\therefore$ $\Delta OLP \cong \Delta OMP$ [By RHS congruence]

Thus, LP = MP [By CPCT] ...(1)

But AB = CD [Given]

$\Rightarrow$ $\frac{1}{2}AB = \frac{1}{2}CD$

**T!P**

*Perpendicular drawn from centre of a circle to the chord, bisect the chord.*

$\Rightarrow$ BL = DM ...(2)

Subtracting eq. (2) from eq. (1), we get

LP – BL = MP – DM

$\therefore$ PB = PD **Hence proved**

3. In $\Delta OBD$,

BD = OD = OB

[Given OD = OB radii of the semi-circle]

$\Rightarrow$ $\Delta OBD$ is an equilateral triangle.

So, $\angle BOD = \angle OBD = \angle ODB = 60°$

In $\Delta BMC$ and $\Delta BMD$,

CM = MD [OM bisects chord CD]

$\angle CMB = \angle DMB$ [Each 90°]

MB = MB [Common]

$\therefore$ $\Delta BMC \cong \Delta BMD$

[By SAS congruence rule]

Thus, $\angle MBC = \angle MBD$ [By CPCT]

But $\angle MBD = \angle OBD = 60°$

$\therefore$ $\angle MBC = 60°$

Now, in $\Delta ACB$,

$\angle CAB + \angle ACB + \angle ABC = 180°$

[Aangle sum property of a triangle]

$\Rightarrow$ $\angle CAB + 90° + 60° = 180°$

[$\because \angle ACB$ is in semi-circle]

$\Rightarrow$ $\angle CAB = 180° - 150° = 30°$

Hence, $\angle CAB = 30°$

4. **Given:** Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively.

**To Prove :** $\angle ACP = \angle QCD$

**Proof:** $\angle ACP = \angle ABP$ ...(1)

[Angles in the same segment]

$\angle QCD = \angle QBD$ ...(2)

[Angles in the same segment]

But, $\angle ABP = \angle QBD$ ...(3)

[Vertically opposite angles]

From eqs. (1), (2) and (3), we get

$\angle ACP = \angle QCD$ **Hence proved**

5. **Given:** Two circles having centres O and O′ intersect at points A and B and PQ || OO′.

**To Prove:** PQ = 2OO′

**Construction:** Join OP, O′Q, $OM \perp PB$ and $O'N \perp BQ$.

**Proof:** In $\Delta OPB$, PM = BM ...(1)

[Perpendicular drawn from the centre bisects the chord]

Similarly, In $\Delta O'BQ$,

NQ = BN ...(2)

Adding eqs. (1) and (2), we get

PM + NQ = BM + BN

$\Rightarrow$ PM + NQ + BM + BN = 2 (BM + BN)

[Adding BM + BN to both sides]

$\Rightarrow$ PQ = 2OO′ [$\because$ BM + BN = OO′]

**Hence proved**

6. **Given:** O is the centre of the circle, $OD \perp AC$, $OE \perp BC$ and OD = OE.

**To Prove:** $\Delta DBA \cong \Delta EAB$.

**Proof:** Since, OD = OE

$\therefore$ $AC = BC$

[Chords equidistant from the centre are equal]

In $\Delta ACE$ and $\Delta BCD$,

$AC = BC$ [Proved above]

$\angle C = \angle C$ [Common]

$CE = CD$ $\left[\frac{1}{2}BC = \frac{1}{2}AC\right]$

$\therefore$ $\Delta ACE \cong \Delta BCD$ [By SAS congruence rule]

Thus, $AE = BD$ [By CPCT]

Now, In $\Delta DBA$ and $\Delta EAB$,

$BD = AE$ [Proved Above)

$DA = EB$ $\left[\frac{1}{2}AC = \frac{1}{2}BC\right]$

$AB = AB$ [Common]

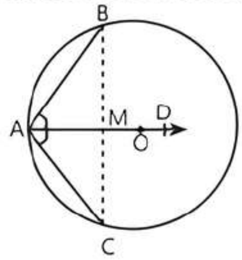
$\therefore$ $\Delta DBA \cong \Delta EAB$ [By SSS congruence rule]

**Hence proved**

7. **Given:** AB, AC are two equal chords of a circle and AD is bisector of $\angle BAC$.

**To Prove:** O is a point on AD.

**Construction:** Join BC meeting AD at M.




**Proof:** In $\Delta BAM$ and $\Delta CAM$, we have

$AB = AC$ [Given]

$\angle BAM = \angle CAM$ [$\because$ AD is bisector of $\angle BAC$]

$AM = AM$ [Common]

$\therefore$ $\Delta BAM \cong \Delta CAM$ [By SAS congruence rule]

$\Rightarrow$ $BM = CM$ and $\angle BMA = \angle CMA$ (by CPCT)

$\therefore \angle BMA = \angle CMA$

Therefore, they are right angled triangles.

$\therefore$ AD is the perpendicular bisector of chord BC.

**T!P**

*Perpendicular bisector of chord always passes through the centre of the circle.*

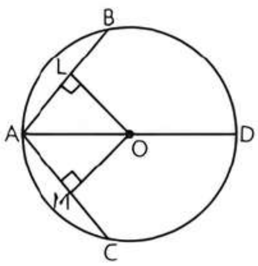
Thus, O lies on AD. **Hence proved**

8. **Given:** Two chords AB and AC of a circle and AOD is the diameter such that

$\angle OAB = \angle OAC$

**To Prove:** $AB = AC$

**Construction:** Draw $OL \perp AB$ and $OM \perp AC$




**Proof:** In $\Delta OLA$ and $\Delta OMA$, we have

$\angle OLA = \angle OMA$ [Each 90°]

$OA = OA$ [Common]

$\angle OAL = \angle OAM$ [Given]

$\therefore$ $\Delta OLA \cong \Delta OMA$ [By ASA congruence rule]

$AL = AM$ (By CPCT)

$\therefore$ $AB = AC$ ($\because$ $AL = AM$, when L and M will be extended they will touch the circle at two different points and the distance from those point to L and M will be equal) **Hence proved**

9. **Proof:** Since, L is the mid-point of chord AB, we have $OL \perp AB$.

Again, M is the mid-point of chord CD, we have

$OM \perp CD$

$\therefore$ $OM = OL$ [$\because$ mid-points of equal chord of a circle are equidistant from the centre]

$\angle OLM = \angle OML$

[$\because$ opposite angles of equal sides are also equal]

$\therefore \angle OLA = 90°$

$\Rightarrow \angle OLM + \angle ALM = 90°$

$\Rightarrow \angle ALM = 90° - \angle OLM$

$\Rightarrow \angle ALM = 90° - \angle OML$ [$\because \angle OLM = \angle OML$]

Similarly,

$\angle CML = 90° - \angle OML$

$\therefore$ $\angle ALM = \angle CML$ **Hence proved**

10. **Given:** AB and AC are two chords of circle with centre O, $OD \perp AB$, $OE \perp AC$ and AO bisects $\angle DAE$.

**To Prove:** $\Delta ADE$ is an isosceles triangle and $\angle ABC = \angle ACB$.

**Proof:** In $\Delta AOD$ and $\Delta AOE$,

$\angle OAD = \angle OAE$ [$\because$ AO is bisector]

$\angle ADO = \angle AEO = 90°$ [Given]

$AO = AO$ [Common]

$\therefore$ $\Delta AOD \cong \Delta AOE$

[By AAS congruence rule]

Thus, $AD = AE$ [By CPCT]

$\therefore$ $\Delta ADE$ is an isosceles triangle.

Again, $OD = OE$ [By CPCT]

$\Rightarrow$ $AB = AC$

[Chords equidistant from the centre are equal]

$\therefore$ $\angle ACB = \angle ABC$

[Angles opposite to equal sides of a triangle are equal] **Hence proved**

11. In the given figure, ABD is a straight line.

Therefore,

$\angle ABC + \angle CBD = 180°$ [By linear pair]

$\Rightarrow$ $\angle ABC + 70° = 180°$

$\therefore$ $\angle ABC = 110°$

Also, AECB is a cyclic quadrilateral.

$\therefore$ $\angle AEC + \angle ABC = 180°$

$\Rightarrow$ $\angle AEC + 110° = 180°$

$\therefore$ $\angle AEC = 70°$

**T!P**

*The angle subtended by an arc on the centre is twice the angle subtended by an arc on circle.*

$\therefore$ $\angle AOC = 2\angle AEC$

$= 2 \times 70°$

$= 140°$

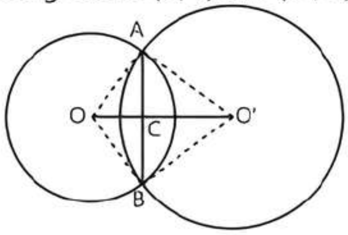
$\Rightarrow$ Reflex $\angle AOC = 360° - \angle AOC$

$\Rightarrow \quad a = 360° - 140°$

$= 220°$

## Long Answer Type Questions

1. **Given:** AB is the common chord of two intersecting circles $(O, r)$ and $(O', r')$.




**To Prove:** Centres of both circles lie on the perpendicular bisector of chord AB, *i.e.*, AB is bisected at right angle by OO'.

**Construction:** Join AO, BO, AO' and BO'.

**Proof:** In $\Delta AOO'$ and $\Delta BOO'$,

$AO = OB$ [Radii of the circle $(O, r)$]

$AO' = BO'$ (Radii of the circle $(O', r')$)

$OO' = OO'$ [Common]

$\therefore \Delta AOO' \cong \Delta BOO'$ (by SSS congruence rule)

Thus, $\angle AOO' = \angle BOO'$ [By CPCT]

Now, in $\Delta AOC$ and $\Delta BOC$,

$AO = BO$ [Radii of the circle $(O, r)$]

$\angle AOC = \angle BOC$ [$\because \angle AOO' = \angle BOO'$]

$OC = OC$ [Common]

$\therefore \Delta AOC \cong \Delta BOC$ [by SAS congruence rule]

Thus, $AC = BC$ and $\angle ACO = \angle BCO$ ...(1) [By CPCT]

$\Rightarrow \angle ACO + \angle BCO = 180°$ ...(2) [Linear pair]

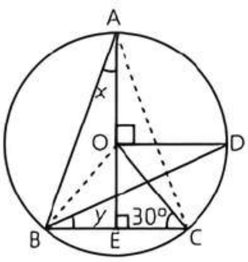
$\Rightarrow \angle ACO = \angle BCO = 90°$ [Grom eqs. (1) and (2)]

So, OO' lie on the perpendicular bisector of AB.

**Hence proved**

2. Given, $\angle BCO = 30°$

Join OB and AC.




In $\Delta BOC$, $OB = OC$

(radii of the same circle)

$\Rightarrow \angle OCB = \angle OBC$

(angles opposite to equal sides are equal)

$\Rightarrow \quad 30° = \angle OBC$

$\Rightarrow \angle OBC = 30°$

In $\Delta OBC$,

$\angle BOC + \angle OBC + \angle OCB = 180°$

[Angle sum property of a triangle]

$\Rightarrow \quad \angle BOC + 30° + 30° = 180°$

$\Rightarrow \quad \angle BOC = 180° - 60°$

$\Rightarrow \quad \angle BOC = 120°$

**TR!CK**

*Angle subtended by an arc at any point on the remaining part of the circle is half the angle subtended by it at the centre of circle.*

$\because \quad \angle BAC = \frac{1}{2} \angle BOC$

$\therefore \quad \angle BAC = \frac{1}{2} \times 120° = 60°$

Also, $\quad \angle BAE = \angle CAE = 30°$

$\therefore \quad x = 30°$ [$\because$ AE bisects $\angle A$]

$\because \quad \angle AOD = 90°$

$\therefore \quad \angle DOE = 180° - 90° = 90°$

$\Rightarrow \quad \angle DOC + \angle COE = 90°$

$\Rightarrow \quad \angle DOC + (180° - 120°) = 90°$

[$\because$ In $\Delta OEC$, $\angle COE = 180° - (90° + 30°)$]

$\Rightarrow \angle DOC = 90° - 60° \quad \therefore \quad \angle DOC = 30°$

**T!P**

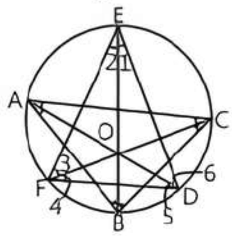
*Angle subtended by an arc of a circle at any point on the remaining part of the circle is half the angle subtended by it at the centre of circle.*

$\therefore \quad \angle DBC = \frac{1}{2} \angle DOC$

$\Rightarrow \quad y = \frac{1}{2} \times 30° \quad \Rightarrow \quad y = 15°$

Hence, the value of $x$ is 30° and that of $y$ is 15°.

3. **Given:** $\Delta ABC$ circumscribes a circle. AD, BE, CF are bisectors of $\angle A$, $\angle B$, $\angle C$ respectively.




**Construction:** Join DE, EF and FD.

**Proof:** We know that angles in the same segment are equal.

$\therefore \quad \angle 5 = \frac{\angle C}{2}$ and $\angle 6 = \frac{\angle B}{2}$ ...(1)

$\angle 1 = \frac{\angle A}{2}$ and $\angle 2 = \frac{\angle C}{2}$ ...(2)

$\angle 4 = \frac{\angle A}{2}$ and $\angle 3 = \frac{\angle B}{2}$ ...(3)

From eq. (1), we get

$\angle 5 + \angle 6 = \frac{\angle C}{2} + \frac{\angle B}{2}$

$\Rightarrow \quad \angle D = \frac{\angle C}{2} + \frac{\angle B}{2}$ [$\because \angle 5 + \angle 6 = \angle D$] ...(4)

But $\angle A + \angle B + \angle C = 180°$

[Angle sum property of a triangle]

$\Rightarrow \quad \angle B + \angle C = 180° - \angle A$

$\Rightarrow \quad \frac{\angle B}{2} + \frac{\angle C}{2} = 90° - \frac{\angle A}{2}$

$\therefore$ Eq. (4) becomes, $\angle D = 90° - \frac{\angle A}{2}$

Similarly, from eqs. (2) and (3), we can prove that

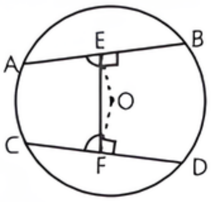
$\angle E = 90° - \frac{\angle B}{2}$ and $\angle F = 90° - \frac{\angle C}{2}$

**Hence proved**

**4. Given:** Two equal chords AB and CD of a circle which have E and F as their mid-points respectively.

**To Prove:** $\angle AEF = \angle CFE$

and $\angle BEF = \angle DFE$




**Construction:** Join OE and OF

**Proof:** We know that the line joining the centre of a circle to the mid-points of the chord is perpendicular to the chord.

$\therefore$ $OE \perp AB$ and $OF \perp CD$

**T!P**

*Equal chords are equidistant from the centre.*

$\because$ AB and CD are equal chords, therefore $OE = OF$.

Now, in $\triangle OEF$, we have

$OF = OE$

$\therefore$ $\angle OEF = \angle OFE$

[$\because$ Opposite angles of equal sides are also equal.]

$90° - \angle OEF = 90° - \angle OFE$

$\angle AEF = \angle CFE$ [$\because$ $OE \perp AB$, $OF \perp CD$]

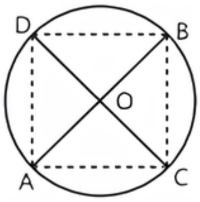
Also, $180° - \angle AEF = 180° - \angle CFE$

$\angle BEF = \angle DFE$ [By linear pair]

**Hence proved**

**5. Given:** AB and CD are two chords of a circle intersecting at O such that OA = OB and OC = OD

**To Prove:** AB and CD are diameters of the circle.




**Construction:** Join AC, AD and BC, BD

**Proof:** In $\triangle AOC$ and $\triangle BOD$, we have

$OA = OB$ [Given]

$OC = OD$ [Given]

$\angle AOC = \angle BOD$

[Vertically opposite angles]

$\therefore$ $\triangle AOC \cong \triangle BOD$ [by SAS congruence rule]

$AC = BD$ [By CPCT]

$\overset{\frown}{AC} = \overset{\frown}{BD}$ ...(1)

In $\triangle AOD$ and $\triangle BOC$, we have

$OA = OB$ [Given]

$OD = OC$ [Given]

$\angle AOD = \angle BOC$ [Vertically opposite angles]

$\therefore$ $\triangle AOD \cong \triangle BOC$ [by SAS congruence rule]

$AD = BC$ [By CPCT]

$\overset{\frown}{AD} = \overset{\frown}{BC}$ ...(2)

Adding eqs. (1) and (2), we get

$\overset{\frown}{AC} + \overset{\frown}{AD} = \overset{\frown}{BD} + \overset{\frown}{BC}$

$\overset{\frown}{CAD} = \overset{\frown}{CBD}$

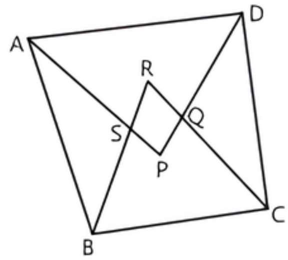
CD is dividing the circle into two semi-circles.

$\therefore$ CD is the diameter.

Similarly, AB is also the diameter.

**Hence proved**

**6.** Let ABCD be a quadrilateral. Then angles bisectors of A, B, C and D are AP, BR, CR and DP, when we join these bisectors, they form quadrilateral PQRS.




**To prove:** PQRS is a cyclic quadrilateral.

*i.e.,* $\angle S + \angle Q = 180°$

or $\angle P + \angle R = 180°$

**Proof:** In $\triangle ABS$, use angle sum property of a triangle

$\angle SAB + \angle ABS + \angle ASB = 180°$

$\Rightarrow$ $\angle ASB = 180° - (\angle SAB + \angle ABS)$

$= 180° - \frac{1}{2}(2\angle SAB + 2\angle ABS)$

$= 180° - \frac{1}{2}(\angle A + \angle B)$

[$\because$ AP and BR are the bisectors of $\angle A$ and $\angle B$]

$\therefore$ $\angle RSP = \angle ASB$ [Vertically opposite angles]

$= 180° - \frac{1}{2}(\angle A + \angle B)$ ...(1)

Similarly, $\angle RQP = \angle CQD$

$= 180° - (\angle QCD + \angle QDC)$

$= 180° - \frac{1}{2}(\angle C + \angle D)$ ...(2)

Adding eqs. (1) and (2), we get

$\angle RSP + \angle RQP = 180° - \frac{1}{2}(\angle A + \angle B)$

$+ 180° - \frac{1}{2}(\angle C + \angle D)$

$= 360° - \frac{1}{2}(\angle A + \angle B + \angle C + \angle D)$

$= 360° - \frac{1}{2} \times 360°$

$= 360° - 180° = 180°$

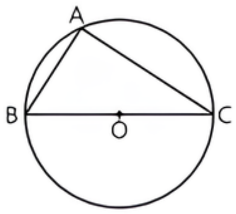
It implies that pair of opposite angles is 180°.

Hence, quadrilateral PQRS form a cyclic quadrilateral.

# Chapter Test

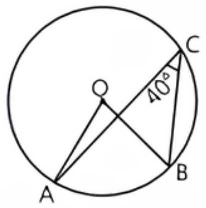
## Multiple Choice Questions

**Q 1.** In the given figure, BOC is a diameter of a circle and AB = AC. Then $\angle$ ABC is equal to:




a. 45° b. 40° c. 60° d. 90°

**Q 2.** In the given figure, O is the centre of a circle and $\angle$ ACB = 40°. The $\angle$ AOB is equal to:




a. 80° b. 70° c. 90° d. 100°

## Assertion and Reason Type Questions

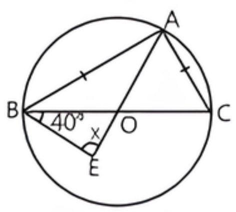
**Directions (Q. Nos. 3-4):** *In the following questions, a statement of Assertion (A) is followed by a statement of a Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
c. Assertion (A) is true but Reason (R) is false.
d. Assertion (A) is false but Reason (R) is true.

**Q 3.** Assertion (A): The length of a chord which is at a distance of 5 cm from the centre of a circle of radius 13 cm is 24 cm.

Reason (R): The perpendicular from centre of a circle to a chord bisects the chord.

**Q 4.** Assertion (A): In the figure, E is any point in the interior of the circle with centre O. Chord AB = AC. If $\angle$ OBE = 40° the value of $x$ is 50°.




Reason (R): Equal chords subtends equal angles at the centre.

## Fill in the Blanks

**Q 5.** Segment of a circle is the region between an arc and the related .................. of the circle.

**Q 6.** A diameter of a circle is a chord that passes through the .................. of that circle.

## True/False

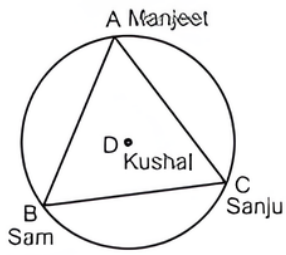
**Q 7.** Equal chords subtends equal angles at the centre.

**Q 8.** If the sum of a pair of opposite angles of a quadrilateral is 180°, then quadrilateral is cyclic.

## Case Study Based Questions

**Q 9.** There are 4 friends Sanju, Manjeet, Sam and Kushal. They all live in the same colony. The colony is circular in shape such that Manjeet, Sam and Sanju houses are at the boundary of the circle as shown in figure and are equidistant from other friend. Kushal house is at the centre of the colony.

*On the basis of the above information, solve the following questions:*




(i) What is the type of $\Delta$ ABC?

(ii) Find the angle between line segments AB and BC.

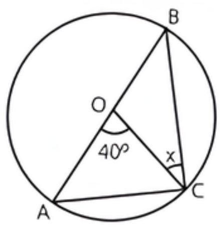
*OR*

If the side of a triangle is 5 cm, what is the area covered by Sam, Sanju and Manjeet?

(iii) Write the measure of $\angle$BDC.

## Very Short Answer Type Questions

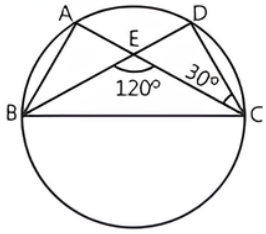
**Q 10.** In the given figure, AB is diameter, $\angle$ AOC = 40°. Find the value of $x$.



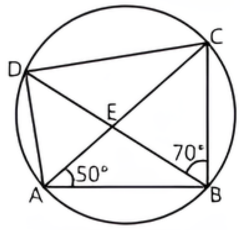
**Q 11.** If an equilateral triangle PQR is inscribed in a circle with centre O, then find $\angle QOR$.

## Short Answer Type-I Questions

**Q 12.** In the figure, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that $\angle BEC = 120°$ and $\angle ECD = 30°$. Find $\angle BAC$.

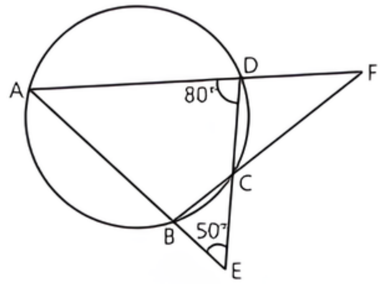



**Q 13.** In the given figure, if $\angle DBC = 70°$ and $\angle BAC = 50°$, then find $\angle BCD$.

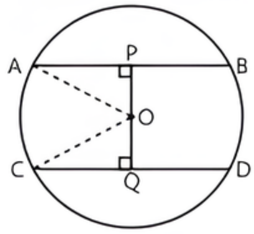



## Short Answer Type-II Questions

**Q 14.** In the given figure, sides AB and DC of a cyclic quadrilateral ABCD are produced to meet at E. Sides AD and BC are produced to meet at F. If $\angle ADC = 80°$ and $\angle BEC = 50°$, then find $\angle BAD$ and $\angle CFD$.




**Q 15.** In the given figure, AB and CD are two parallel chords of a circle with centre O and radius 5 cm such that AB = 8 cm and CD = 6 cm. If $OP \perp AB$ and $OQ \perp CD$, determine the length PQ.




## Long Answer Type Question

**Q 16.** In the adjoining figure, P is any point on the chord BC of a circle such that AB = AP. Prove that CP = CQ.