Chapter 1

Kinematics and Dynamics of Mechanisms

CHAPTER HIGHLIGHTS

- *Mechanisms, Machines and Various Inversions of Mechanisms*
- *Kinematic Link or Kinematic Element and Kinematic Chain*
- *Relation between Kinematic Chain and Mechanism*
- *Planar Mechanism (or Planar Linkage) and Planar Kinematic Chain*
- *The Degree of Freedom of Space of a Rigid Body*
- *Simple and Compound Mechanisms*
- *Inversions of Slider Crank Chain*
- *Approximate Straight Line Motion Mechanisms*
- *Modified Scott - Russel Mechanism*
- *Grasshopper and Tchebicheff's Mechanism*
- *Engine Indicators*
- *Velocity and Acceleration Analysis of Mechanisms*
- *Velocity Analysis of a 4-bar Linkage*
- *Aaronhold Kennedy's Theorem*
- *Centrodes and Axodes*

Mechanisms, Machines and Various Inversions of Mechanisms

Introduction

The branch of Mechanical Engineering that deals with the study of relative motions of various components of a machine and the static and dynamic forces which act on them, is called the 'Theory of Machines'.

Kinematic Analysis deals with the analysis of position, velocity and acceleration of those components. It does not consider the forces that cause the motion but only deals with the geometry of motion.

Dynamic Analysis deals with the static and dynamic force analysis on those components.

A material body which does not deform under the action of any magnitude of force is called a **Rigid Body**. The distance between any two points on a rigid body always remains constant. Practically no body is rigid. Hence, a rigid body is an ideal concept.

If the deformation of a body is negligible when a range of force is applied on it, such a body is called a **Resistant Body**. Such bodies can be treated as rigid bodies when transmitting forces within the range where their deformation is negligible. A resistant body which can transmit motion and/or force can be a **flexible** body like belt, chain, rope or even fluid.

Mechanisms are used for transmission of motion. They may or may not be used for transmission of forces also. For example, a typewriting mechanism is used for transmitting motion only and not force. However, a slider-crank mechanism transmits both motion and force. There is relative motion between various components of a mechanism. Clock and mini-drafter are also examples of mechanisms which transmit only motion.

Machines are made up of one or more mechanisms. They are used for transmitting motion, force and also to convert some available energy (usually mechanical energy) into useful work. Thus, we need machines to do some useful work. All machines are mechanisms but all mechanisms need not be machines. Hence, there is a relative motion between components of a machine.

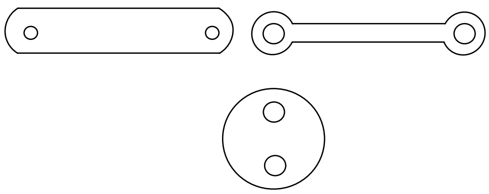
A **structure** is a system of rigid bodies (or resistant bodies) connected together in such a way that they can transmit force without any relative motion between the components and without doing any useful work. For example, roof truss, railway bridge, etc.

Basic Concepts and Definitions

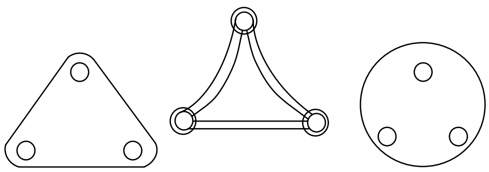
Kinematic Link or Kinematic Element

If a rigid (or resistant) body A is connected to another rigid (or resistant) body B, such that a relative motion can occur between bodies A and B after they are connected, then each of A and B is called as kinematic link or kinematic element. The connection between A and B is called a **joint**. Resistant bodies which are flexible, when used as links, are called as **Flexible Links**. For example, a fluid like water can be used for transmitting compressive forces, i.e. a fluid link. A belt can be used for transmitting tensile forces. So, belt is an example of flexible link. A kinematic link provides connections to other links by at least two joints.

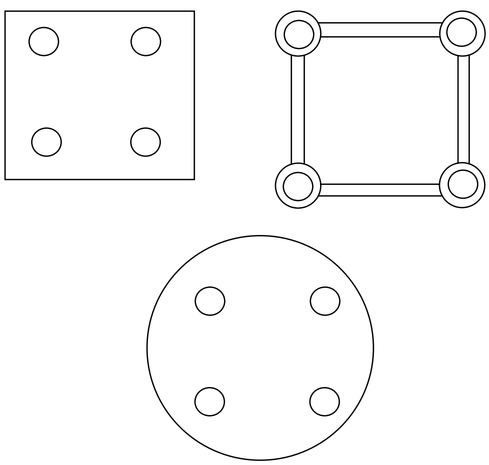
A **binary link** is a link which is connected to other links at two points as shown below.



A ternary link is a link which is connected to other links at three points as shown below.

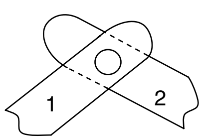


A quarternary link is a link which is connected to other links at four points as shown below.

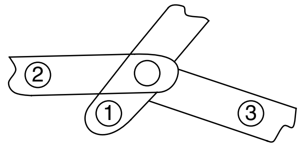


Kinematic Joint is a connection between two or more links which permits relative motion between the links and also physically produces some constraint (or constraints) to their relative motion, i.e. the relative motion between the links is predictable and can occur only is a particular manner.

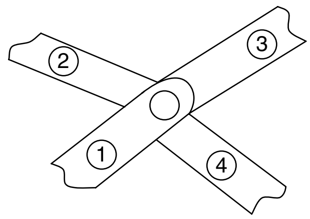
When two links are connected at a joint to form a kinematic joint, it is called a **binary joint** and the **joint order is 1**.



When three links are connected at a joint to form a kinematic joint, it is called a **ternary joint** and the **joint order is 2** (i.e. this joint is equivalent to two binary joints)



When four links are connected at a joint to form a kinematic joint, it is called a **quarternary joint** and the **joint order is 3** (i.e. this joint is equivalent to three binary joints)



NOTE

If n number of links are connected at a kinematic joint, it is equivalent to $(n - 1)$ binary joints.

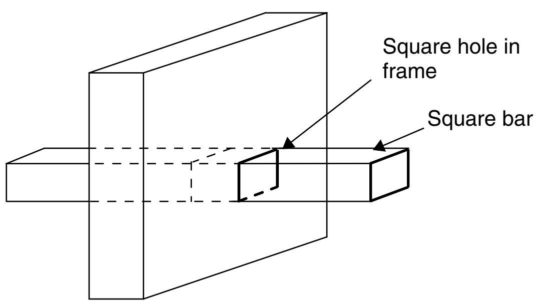
Types of Constrained Motion

Motion between two elements of a kinematic pair can be classified as:

1. Completely constrained motion
2. Successfully constrained motion and
3. Incompletely constrained motion.

If the **relative motion** between two elements of kinematic pair takes place in a particular manner or is in a definite direction, irrespective of the direction of force applied on them, such a motion is known as **completely constrained motion**.

For example, a square bar inserted inside a square hole in a frame can only slide along the square hole. Thus, the relative motion between the rod and frame is completely constrained.



If the relative motion between two elements of a kinematic pair **can take place in more than one direction** and **depends upon the direction of force applied on them**, such type of motion is called **incompletely constrained motion**. For example, a cylindrical shaft in a journal bearing can either slide on the bearing or turn in the bearing depending upon the direction of the applied force. Hence, it is an **incompletely constrained motion**. However, if the cylindrical shaft is **provided with collars** so that it cannot slide on the bearing, then it can only rotate in the bearing and the motion becomes completely constrained.

If the **relative** motion between two elements of a kinematic pair can possibly take place in more than one direction but an external force like gravity or spring force, etc. makes them to have relative motion only in one direction, then such type of motion is called **successfully constrained motion**. If the external force is removed, the motion between the elements becomes incompletely constrained.

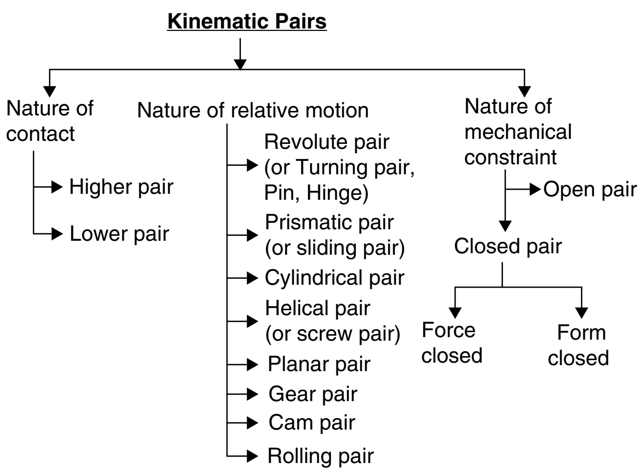
e.g. A shaft in a foot step bearing can only rotate in the bearing, if an axial load (either weight or spring force) is applied along the shaft. Thus, the shaft and bearing become a successfully constrained pair.

If the weight or spring force acting along the shaft is removed, the shaft can either rotate or slide in the foot-step bearing and hence the motion becomes incompletely constrained.

Other examples of successfully constrained motions are cam and follower, piston in a cylinder of internal combustion engine, etc.

Classification of Kinematic Pairs

Kinematic pairs can be classified (a) According to the nature of contact (b) According to nature of relative motion and (c) According to nature of mechanical constraint.



Classification of joints according to nature of contact are

(i) Lower Pair and (ii) Higher Pair

A joint for which the contact between the two kinematic elements is along a **surface or area** is called a **lower pair**;

E.g. All pairs of 4-bar linkage
All pairs of slider crank mechanism
A shaft in a journal bearing
A square bar in a square hole of a frame
A nut turning on a screw

A joint for which the contact between the two kinematic elements is along **a point or a line** is called a **higher pair**.

E.g. Cam and follower pair
Gear pair (two mating gears)
A disc or wheel rolling on a surface

Classification of joints according to the nature of relative motion are (i) Revolute pair (or Turning Pair) (ii) Prismatic Pair (or Sliding pair) (iii) Helical Pair (or Screw Pair) (iv) Cylindrical Pair (v) Spherical Pair (vi) Planar Pair (vii) Gear Pair (viii) Cam Pair and (ix) Rolling Pair

Revolute Pair (or **Turning Pair**) is a joint in which the relative motion of the links joined together is pure rotation about the joint. These links form lower pair also. Symbol for this joint is *R* (or *T*)

E.g. A hinged joint or a pin-joint
A shaft with collars in a journal bearing.

Prismatic Pair (or Sliding Pair) is a joint in which the relative motion of the links joined together is pure sliding. These links also form a lower pair. Symbol for this joint is *P*.

E.g. A piston sliding inside the cylinder of IC engine.
A square bar sliding in a square hole of a frame.

Helical Pair (or Screw Pair) is a joint in which the mating links have screw shaped (or helical) surfaces such that their relative motion is both rotational and translational, and both motions are interdependent, i.e. a particular amount of rotation of a link results in a proportional amount of translation as well. These are also lower pairs and the symbol for this joint is *H*.

E.g. Lead screw and nut of lathe.

Cylindrical Pair is a joint in which the mating links have cylindrical surfaces such that their relative motion is both rotational and translational and these motions are independent of each other, i.e. the rotational motion of a link is independent of its translational motion.

These are also lower pairs and the symbol for this joint is *C*.

E.g. A cylindrical shaft in a journal bearing.

Spherical Pair is a joint in which the mating links have spherical shape at the joint and relative motion between them is purely rotational about three mutually perpendicular axes (about *X*, *Y* and *Z* axis) without any translation. These joints are also lower pairs. Their symbol is *S*.

E.g. A ball and socket joint (as in the rear-view mirror of cars)

Planar Pair is a joint in which the links mate along plane surface (say *XY* plane) and the relative motion between them is translational (which can be resolved along *X*-direction and *Y*-direction) and/or rotational about an axis perpendicular to the plane (say *Z* axis). The translation along *x* and *y* direction and rotation along *y* direction are independent of each other. These are also lower pairs. Their symbol is *E*.

E.g. A flat block sliding and/or turning on another flat surface.

Hence, there are six (6) lower pairs of joints. All the other pairs of joints are higher pairs (having line contact or point contact). Gear pairs, Cam pairs, Rolling pairs etc are examples of higher pairs.

A rolling pair is made of two links out of which one link rolls over the other link. The relative motion between the links is pure rotation and pure translation.

If one link does **pure rolling** relative to the other, then the rotational motion is dependent on translation and vice versa. If one link slips and rolls (i.e. impure rolling or rolling with slip) relative to the other link, then the **translation and rotation are independent of each other**. A rolling pair is a higher pair.

Classification of joints according to nature of mechanical constraint are (i) **Open Pair** (or Unclosed Pair) and (ii) **Closed Pair**

When contact between kinematic elements is maintained only for some possible positions, such type of a joint is called **open pair** (or unclosed pair)

E.g. Cam and follower

When contact between kinematic elements is maintained for all possible positions, such type of joint is called **closed pair**. It can further be classified as

1. **Form closed pair**, which is a joint in which the contact between kinematic elements is maintained due to geometry. Removal of the contact between the elements requires physical destruction of at least one of the members.
2. **Force closed pair**, which is a joint in which the contact between kinematic elements is maintained by some external force (e.g. spring force, gravity force, etc).
   E.g. Cam and follower

It must be noted that all lower pairs are closed pairs.

Kinematic Chain

When various rigid (or resistant) links are connected in such a way that the first link is connected to the second, the second link to the third and so on, the configuration obtained is called an **Open Chain**. When the last link of the open chain is connected to the first link, we obtain a **closed configuration of links** known as **Closed Chain**.

Various possibilities of relative motion between the links exist in a closed chain.

1. If no relative motion is possible between the links of a closed chain, then it is either a **statically determinate structure or statically indeterminate structure (or** redundant structure/super structure)
2. If a relative motion is possible between the various links of a closed chain, then it is either **kinematic chain** (constrained chain) or an **unconstrained chain**.

A closed chain is a **kinematic chain** when the relative motion between the links **is completely constrained or successfully constrained** (i.e. the relative motion between the links takes place in a particular direction or particular manner, irrespective of the direction of applied force).

A closed chain is an **unconstrained chain** if the relative motion between the links takes place in a random manner. How do we know whether a closed chain is a structure, unconstrained chain or a kinematic chain? There are two relations to determine this and both the relations are equivalent. The **first relation** is

$$L = 2P - 4$$

where L = number of links and P = number of lower pairs of joints.

The **second relation** is

$$2J = 3L - 4$$

where J = number of **binary joints** and L = number of links.

In both relations,

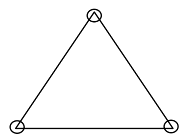
If LHS > RHS, **chain is locked** (i.e. chain is redundant or a frame or structure)

If LHS = RHS, it is **kinematic chain** (completely constrained) and can be converted into a mechanism by fixing any one link.

If LHS < RHS, it is **an unconstrained chain**.

Solved Examples

Example 1: Determine whether the chain shown below is a kinematic chain.



Solution:

Using relation 1,

Here $L = 3$ and $P = 3$

($\because$ 3 binary joints)

We have $2P - 4 = 2 \times 3 - 4 = 2$

$$\therefore L > 2P - 4$$

LHS > RHS $\Rightarrow$ It is a structure

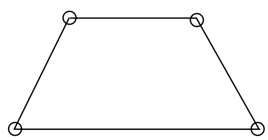
Using relation 2, $J = 3$

$$\Rightarrow 2J = 2 \times 3 = 6$$

$$3L - 4 = 3 \times 3 - 4 = 5$$

$$2J > (3L - 4) \Rightarrow \text{structure}$$

Example 2: Determine whether the chain shown below is a kinematic chain.



Solution:

Using relation 1

$L = 4$ and $P = 4$

$$\therefore 2P - 4 = 2 \times 4 - 4 = 4$$

$\therefore L = (2P - 4) \Rightarrow$ kinematic chain

Using relation 2

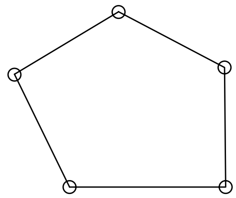
$$J = 4;\ 3L - 4 = 3 \times 4 - 4 = 8,\ 2J = 8$$

$$2J = 3L - 4 \Rightarrow \text{kinematic chain}$$

NOTE

Hence, a kinematic chain requires minimum four (4) links

Example 3: Determine whether the chain shown below is a kinematic chain.



Solution:

Using relation 1

$L = 5, P = 5$

$\therefore\ 2P - 4 = 2 \times 5 - 4 = 6$

$\therefore\ L < (2P - 4) \Rightarrow$ unconstrained chain

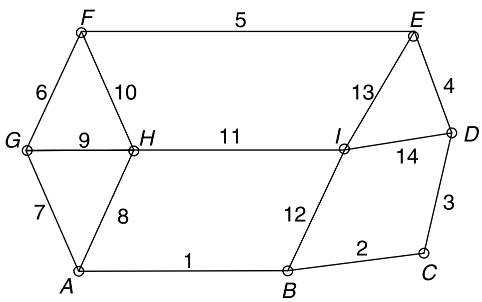
Using relation 2

$J = 5, L = 5, 2J = 2 \times 5 = 10$

$3L - 4 = 3 \times 5 - 4 = 11$

$2J < (3L - 4) \Rightarrow$ unconstrained chain.

Example 4:



The number of binary, ternary and quarternary joints in the above chain are ________, ____________ and ___________ respectively. Fill up the blanks What is the equivalent number of binary joints in the chain?

Solution:

C(connects link 2 and 3) is a binary joint.

Joint A (1, 7, 8), B(1, 2, 12), D(3, 4, 14), E(4, 5, 13), F(5, 6, 10), G(6, 7, 9) are ternary joints $\Rightarrow$ Total 6 joints.

Joint H(8, 9, 10, 11) and I(11, 12, 13, 14) are quaternary joints $\Rightarrow$ Total 2 joints.

Hence, there is 1 binary, 6 ternary and 2 quarternary joints .

J = Equivalent number of binary joints

$= 1 + 6 \times (3 - 1) + 2 \times (4 - 1)$

$= 1 + 12 + 6 = 19$

($\because$ 1 ternary joint = 2 binary joints and 1 quarternary joint = 3 binary joints)

Example 5: Is the chain shown in Example 4 a kinematic chain?

Solution:

The equation $2J = (3L - 4)$ shall be used to determine the nature of closed chains having different types of links and different types of joints. If chain is made up of only binary links and different types of joints, both equations can be used.

Here $L = 14$ (All binary links)

$3L - 4 = 3 \times 14 - 4 = 38$

$2J = 2 \times 19$ (J is calculated already in Example 4)

$= 38$

$\therefore\ 2J = 3L - 4$

$\Rightarrow$ It is a kinematic chain.

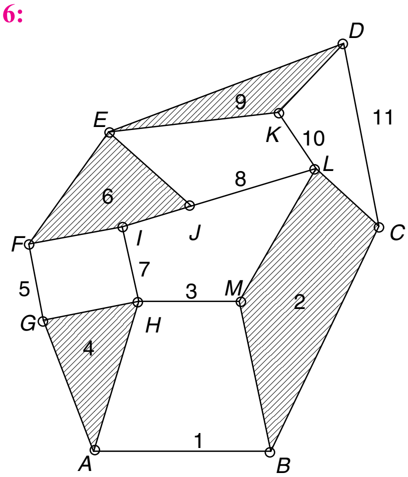
Using relation 1

$L = 14, P = 9(A, B, C, D, E, F, G, H, I)$

$2P - 4 = 2 \times 9 - 4 = 14$

$\therefore\ L = 2P - 4 \Rightarrow$ It is a kinematic chain.

Example 6:



In the chain shown in figure, the shaded links are of single piece. The closed chain

(A) is a structure

(B) is a kinematic chain

(C) is an unconstrained chain

(D) can never be a structure

Solution:

L = Total number of links

= 11 (marked in figure)

No. of binary joints ($A, B, M, C, D, E, J, I, F, G, K$) = 11 nos

No. of ternary joints (H, L) = 2

$= 2 \times (3 - 1)$

= 4 binary joints

$\therefore\ J$ = Total number of binary joints

$= 11 + 4$

= 15; Ternary and Quarternary links are used.

Hence, use $2J = (3L - 4)$ equation to check.

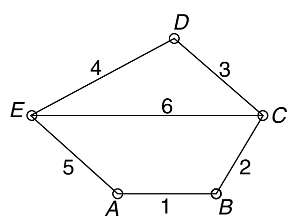
$\Rightarrow 2J = 2 \times 15 = 30,$

$(3L - 4) = 3 \times 11 - 4$

$= 33 - 4 = 29$

$\Rightarrow 2J > (3L - 4) \Rightarrow$ structure

Example 7:



The closed chain shown in figure is

(A) a locked chain

(B) an unconstrained chain

(C) a kinematic chain
(D) not a closed chain

Solution:
Using relation 1
$L = 6$ (all binary links)
No. of binary joints (A, B, D) = 3
No. of ternary joints (C, E) = 2
$= 2 \times (3 - 1) = 4$ binary joints
$\therefore J$ = equivalent number of binary joints
$= 3 + 4 = 7$
$(3L - 4) = (3 \times 6 - 4) = 18 - 4 = 14$
$2J = 2 \times 7 = 14$
$2J = (3L - 4) \Rightarrow$ kinematic chain
Using relation 2
$L = 6$, $P = 5$ (A, B, C, D, E)
= no. of pairs
$(2P - 4) = (2 \times 5) - 4 = 6$
$\therefore L = (2P - 4) \Rightarrow$ kinematic chain.

Relation between Kinematic Chain and Mechanism

When **one of the links** of a kinematic chain is fixed, it is called a **mechanism**. The fixed link is called **frame** (or **ground link**)

A mechanism which contains only lower pairs is called a **linkage**. For example, 4-bar linkage which is made of 4 binary links with 4 revolute (turning) pairs and one link fixed.

By choosing different links of a kinematic chain as the fixed link, different mechanisms can be generated. Each mechanism so generated is called as **Inversion** of the kinematic chain.

For example, the slider-crank chain has four links, having three revolute pairs and one prismatic (sliding) pair. By fixing different links of slider-crank chain, four inversions of slider-crank chain can be obtained. Each inversion is a different mechanism. **If a kinematic chain has n links**, then **n inversions** of the chain **are obtained** by fixing different links, one at a time. **Some inversions may give the same type of mechanism**.

For example, in a 4-bar linkage, four inversions are obtained by fixing different links one at a time. These inversions are double-crank mechanism, crank-rocker mechanism, rocker-rocker mechanism and again crank-rocker mechanism. So, though there are four inversions, effectively there are only three mechanisms! This is discussed later.

It must be noted that in the process of inversion, the relative motions of the links of the mechanism generated remain unchanged.

Spatial Mechanism (or Spatial Linkage) and Spatial kinematic Chains

If the motion of the elements (or links) of a kinematic chain or mechanism (or linkage) is in three-dimensional space (3-D), then it is a spatial kinematic chain or spatial mechanism (or linkage). It may contain any of the six lower pairs viz Revolute (R), Prismatic (P), Helical (H), Cylindrical (C), Spherical (S) and Planar (E)

Planar Mechanism (or Planar Linkage) and Planar Kinematic Chain

If the motion of the elements (or links) of a kinematic chain or mechanism (or linkage) is on a plane or parallel planes, then it is a planar kinematic chain or planar mechanism (or linkage). It may contain revolute (R) and prismatic (P) joints.

The Degree of Freedom of Space of a Rigid Body

The number of independent parameters that must be specified to define the position of a rigid body in that space is called as **the degree of freedom of space** of that body. It is the number of independent motions that the body can have in that space.

For a 3-D link (**spatial link**), the degree of freedom is 6 (which is 3 translations along X, Y and Z directions and 3 rotations about X, Y and Z axes)

For a 2-D link (**Planar link**), the degree of freedom is 3 (which is 2 translation along X and Y directions and 1 rotation about Z axis).

Degree of freedom is also known as **'mobility'**

The Degree of Freedom of a Joint

When one rigid link (or resistant link) is connected to another at a joint, the joint imposes some restraints on the relative motion of the links. Thus, certain degree of freedom which the links had (before connecting together) is lost due to connection. The number of independent parameters required to determine the relative position of one rigid body with respect to the other that is connected by the joint is called **the degree of freedom of the joint**.

For joints connecting spatial (3D) links, degree of freedom (dof) = 6 − number of restraints

For joints connecting planar (2D) links, degree of freedom (dof) = 3 − number of restraints.

The number of restraints can never be zero for the joint because in such a case, the links are disconnected.

For spatial (3D) joints, the number of restraints cannot be six (6) because the joint becomes rigid and no relative motion is possible for the links connected at the joint. Similarly, for planar (2D) joints, the number of restraints cannot be three (3).

The degree of freedom for various types of common joints are given below.

Type of joint	Symbol	Joint DOF	Relative motion	
			Translational	Rotational
Revolute or Turning pair (Hinge, Pin)	R	1	0	1
Prismatic (sliding)	P	1	1	0
Cylindrical	C	2	1	1
Spherical	S	3	0	3
Plane	E	3	2	1

For Rolling Pair, Gear Pair and Cam Pair, the degree of freedom will be two (2), if rolling and sliding occurs; if pure rolling (no slip) only occurs, degree of freedom will be one (1).

The Degree of Freedom of Mechanisms

The number of independent parameters that should be specified to define the position of every link in the mechanism is called as the degree of freedom (dof) of the mechanism. It is usually denoted as F.

1. Kutzback's expression for degree of freedom of 3D (space) mechanism is given by

   $F = 6(L - 1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - P_5$,

   where

   F = degree of freedom (dof)
   L = Total number of links in the mechanism
   P_1 = number of pairs (or joints) having one dof (e.g. Revolute pair, Prismatic Pair etc)
   P_2 = number of pairs (or joints) having two dof (e.g. Cylindrical Pair, Rolling Pair etc)
   P_3 = number of pairs (or joints) having three dof (e.g. spherical pair)
   P_4 = number of pairs (or joints) having four dof
   P_5 = number of pairs (or joints) having five dof

   As one link is fixed in a mechanism, effective number of links becomes $(L - 1)$ and hence the term $6(L - 1)$ represents the total degree of freedom of links before connections (or joints) are made. The type of joint introduces restraints in the mechanism and hence total degree of freedom gets reduced as per the expression given.
2. Kutzback criteria for degree of freedom of 2D (Planar) mechanism is given by

   $F = 3(L - 1) - 2P_1 - P_2$,

   where

   L = number of links in the mechanism
   P_1 = number of pairs (or joints) having one dof.
   P_2 = number of pairs (or joints) having two dof

In **planar mechanisms**, P_1 represents the number of binary joints (j) and P_2 represents the **higher pairs** (h) (e.g. rolling pair, gear pair, cam pair etc having rolling with slip etc), the above equation can also be written as

$F = 3(L - 1) - 2J - h$ for planar mechanism

For planar mechanism that have degree of freedom $F = 1$ and no higher pair (i.e. $J = 0$), Grubler modified the above equation as follows:

$1 = 3(L - 1) - 2J - 0$
$3L - 2J - 4 = 0$

$\Rightarrow J = \frac{3}{2}L + 2$ is the **Grubler's criteria** for planar mechanisms with degree of freedom $F = 1$ and having no higher pairs ($h = 0$, means no joints with two degree of freedom)

If $F = 1$ and $h \neq 0$ (I,.e there are some two degree of freedom joints) in a planar mechanism, then Grubler's criteria will become

$$j + \frac{h}{2} = \frac{3}{2}L + 2$$

Degree of freedom

$$F = 0$$

$\Rightarrow$ statically determinate structure or frame

$$F = 1$$

$\Rightarrow$ mechanism which is completely constrained

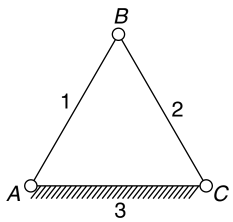
$$F > 1$$

$\Rightarrow$ unconstrained (i.e. more than one input needed for a desired output)

$$F < 0$$

$\Rightarrow$ statically indeterminate structure or super structure (i.e. it has some redundant members)

Example 8:



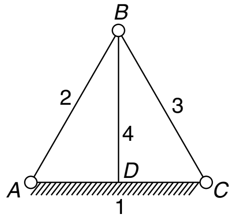
What is the degree of freedom for the above linkage?

Solution:

$L = 3$ (= number of links)
$P_1 = 3$ (= number of binary joints)
$P_2 = 0$
($\because$ no higher pair or do $f = 2$ joints)
$F = 3(L - 1) - 2P_1 - P_2$
$= 3(3 - 1) - 2 \times 3 - 0$
$= 6 - 6 - 0$
$= 0 \Rightarrow$ Degree of freedom = 0

$\therefore$ This is a statically determinate structure $\rightarrow$ the forces transmitted by each member can be analyzed.

Example 9:



What is the degree of freedom of planar frame shown in figure?

Solution:

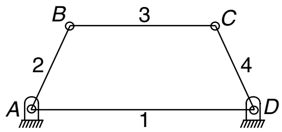
$L = 4$
Number of binary joints (A, D, C) = 3; B is a ternary joint = 2 binary joints
$\therefore P_1$ = Total number of binary joints = 3 + 2 = 5
$F = 3(L - 1) - 2P_1$

$= 3(4 - 1) - 2 \times 5 = 9 - 10 = -1$

$\therefore F = -1 (< 0)$

Hence, the frame is a statically indeterminate frame i.e. there are redundant members in the frame.

Example 10:



What is the degree of freedom for the above frame?

Solution:

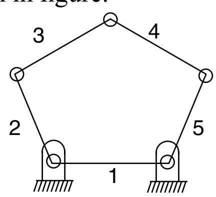
$L = 4$; $P_1 = 4(A, B, C$ and $D)$; $P_2 = 0$

$F = 3(L - 1) - 2P_1 - P_2$

$= 3(4 - 1) - 2 \times 4 - 0 = 9 - 8 = 1$

Hence, it is a completely constrained Mechanism. i.e. it needs only one input to get an output.

Example 11: Determine the degree of freedom for the planar frame shown in figure.

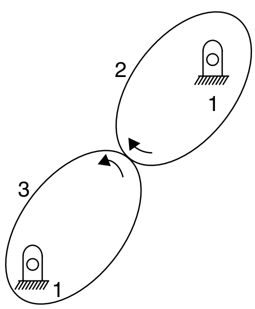


Solution:

$$L = 5;\ P_1 = 5;\ P_2 = 0$$
$$F = 3(L - 1) - 2P_1 - P_2$$
$$= 3(5 - 1) - 2 \times 5 - 0$$
$$= 12 - 10 = 2$$

Hence, it is an unconstrained mechanism. It requires two inputs to obtain an output. To locate the positions of all links with respect to fixed link 1, angle between links 1 and 2 as well as angle between links 1 and 5 need to be specified; i.e. two angles needed.

Example 12:



Point of contact of links 2 and 3 slips and rolls. The degree of freedom for the system shown is

(A) 0 (B) −1 (C) 1 (D) 2

Solution:

$L = 3$

$P_1 = 2$ (= no. of binary joints)

$P_2 = 1$ (between links 2 and 3, dof is 2 as there is rolling and slipping at point of contact)

$\therefore F = 3(L - 1) - 2P_1 - P_2$

$= 3(3 - 1) - 2 \times 2 - 1 = 6 - 4 - 1 = 1$

$\therefore F = 1$

Hence, it is a completely constrained mechanism.

Example 13: For the system shown in Example 12, if link 2 does pure rolling on link 3 (i.e. there is no slip at the point of contact), the degree of freedom for system is

(A) 0 (B) −1 (C) 1 (D) 2

Solution:

$L = 3$

$P_1 = 3$ (dof for joint between 2 and 3 is now 1, not 2, as there is no slip)

$P_2 = 0$

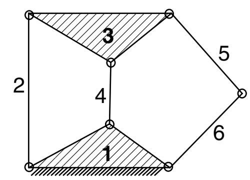
$\therefore F = 3(L - 1) - 2P_1 - P_2$

$= 3(3 - 1) - 2 \times 3 - 0 = 0$

$\therefore F = 0$

Hence, there is no relative motion possible between any link in this case and it becomes a statically determinate structure.

Example 14:



The degree of freedom for the planar system shown is

(A) 0 (B) 1 (C) −1 (D) 2

Solution:

$L = 6$; $P_1 = 7$ (= no. of binary joints);

$P_2 = 0$

$\therefore F = 3(L - 1) - 2P_1 - P_2$

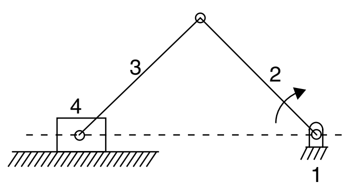
$= 3(6 - 1) - 2 \times 7 - 0$

$= 15 - 14 = 1$

$\therefore F = 1$

Hence, it is a completely constrained mechanism.

Example 15:



The degree of freedom for the system shown is

(A) 0 (B) 1 (C) −1 (D) 2

Solution:

$L = 4$; $P_1 = 4$; $P_2 = 0$

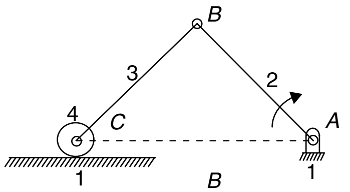
$\therefore F = 3(L - 1) - 2P_1 - P_2$

$= 3(4 - 1) - 2 \times 4 - 0 = 9 - 8 = 1$

$\therefore F = 1$

This is called slider-crank mechanism.

Example 16:



The wheel rolls and slips on the floor. The degree of freedom for the system is

(A) 2 (B) 1 (C) 0 (D) 3

Solution:

$L = 4$; $P_1 = 3$ (joints A, B and C are binary joints)

$P_2 = 1$ ($\because$ wheel rolls and slips on floor)

$\therefore F = 3(L - 1) - 2P_1 - P_2$

$= 3(4 - 1) - 2 \times 3 - 1$

$= 9 - 6 - 1 = 2$

$\therefore F = 2$.

This is an unconstrained mechanism.

Example 17: In Example 16, if the wheel does pure rolling on the floor, what is the degree of freedom of system?

(A) 2 (B) 1 (C) 0 (D) 3

Solution:

$L = 4$; $P_1 = 4$ (joints A, B, C and wheel with floor are all binary joints now)

$P_2 = 0$

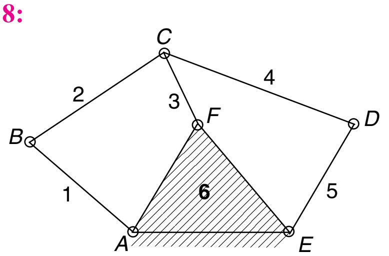
$\therefore F = 3(L - 1) - 2P_1 - P_2$

$= 3(4 - 1) - 2 \times 4 - 0$

$= 9 - 8 = 1$

$\therefore F = 1$

Example 18:



The degree of freedom for the system shown is (Link 6 is made from a single piece)

(A) 0 (B) 1 (C) 2 (D) −1

Solution:

$L = 6$ (= no. of links)

n_1 = no. of binary joints

$= 5$ (A, B, D, E, F)

n_2 = no. of ternary joints

$= 1(C)$

$= 1 \times (3 - 1)$

$= 2$ binary joints

$\therefore P_1 = n_1 + n_2 = 5 + 2 = 7$; $P_2 = 0$

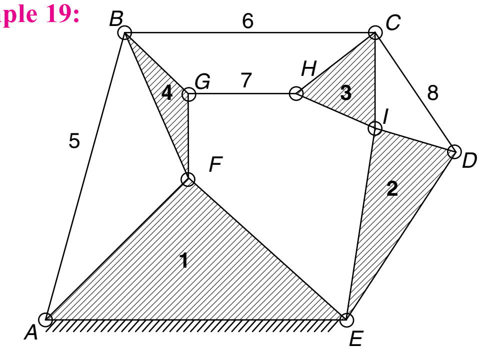
$\therefore F = 3(L - 1) - 2P_1 - P_2$

$= 3(6 - 1) - 2 \times 7 - 0$

$= 15 - 14 = 1$

$\therefore F = 1$

Example 19:



The mobility of the planar frame shown in figure is

(A) 2 (B) 1 (C) 0 (D) −1

Solution:

Number of links, $L = 8$ (4 binary and 4 ternary)

J_1 = No. of binary joints

$= 7(A, E, F, I, D, H, G)$

J_2 = No. of ternary joints

$= 2(B, C)$

$= 2 \times (3 - 1)$

$= 4$ no. of binary joints

$\therefore P_1$ = No. of binary joints $= J_1 + J_2$

$= 7 + 4 = 11$

$F = 3(L - 1) - 2P_1 - P_2$

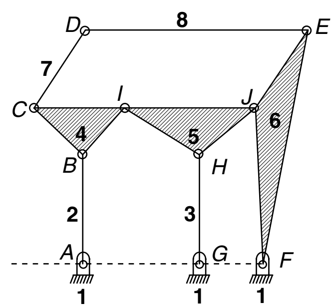
$= 3(8 - 1) - 2 \times 11 - 0$

$= 21 - 22 = -1$

$\therefore F = -1$

⇒ This is a super structure or statically indeterminate structure.

Example 20:



The mobility of planar linkage shown in figure is

(A) −1 (B) 0 (C) 1 (D) 2

Solution:

L = No. of links = 8 (4 binary and 4 ternary)

P_1 = No. of binary joints

$= 10$ (A, B, C, D, E, F, G, H, I, J)

$P_2 = 0$

Degree of freedom or mobility

$F = 3(L - 1) - 2P_1 - P_2$

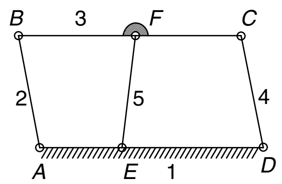
$= 3(8 - 1) - 2 \times 10 - 0$

$= 21 - 20 = 1$

$\therefore F = 1$

⇒ Linkage is a constrained mechanism

Example 21:



In the above planar frame, link 3 is made from single rod (BFC), link 2 and link 4 are parallel to each other and of same length ($AB = DC$), link 1 (AED) is fixed and length of link 5 (EF) is not equal to that of AB (or CD). The mobility of frame is

(A) −1 (B) 0 (C) 1 (D) 2

Solution:

No. of links $L = 5$

P_1 = no. of binary joints = 6

P_2 = no. of higher joints = 0

$F = 3(L - 1) - 2P_1 - P_2$

$= 3(5 - 1) - 2 \times 6 - 0$

$= 12 - 12 = 0$

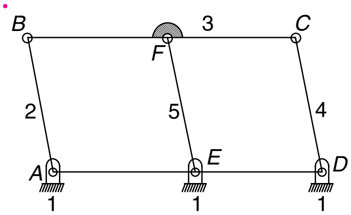
⇒ Mobility = 0

⇒ Statically determinant structure.

Failure of Kutzback Criteria

In the development of Kutzback criteria, no consideration is given to the lengths or other dimensional properties of the elements (or links). Hence, it is possible that exceptions to Kutzback criteria can be for specific cases with parallel links, equal link length or other special geometric features.

Example 22:



In the above frame, link 1 is fixed, link 3 (made from a single link) is parallel to link 1. Links 2, 5 and 4 are parallel to each other and of same lengths. The mobility of this frame is

(A) −1 (B) 0 (C) 1 (D) 2

Solution:

Problem is similar to Example 21

$L = 5; P_1 = 6, P_2 = 0$

$\therefore F = 3(L - 1) - 2P_1 - P_2$

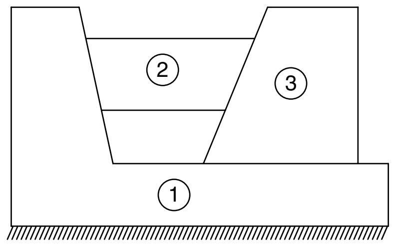
$= 3(5 - 1) - 2 \times 6 - 0 = 0$

But this is not correct. We can see that if link 2 is given an angular motion with respect to fixed link 1, link 5 and link 4 can also rotate by the same extent and link 3 can also have motion with respect to link 1. Thus, all links can have constrained relative motion.

⇒ mobility is 1 and it is a constrained mechanism.

Hence, this is a case of failure of Kutzback criteria. This mechanism is a double parallelogram mechanism.

Example 23:



All wedges in figure are smooth. The degree of freedom for the system shown in figure is

(A) 2 (B) 1 (C) 0 (D) −1

Solution:

No. of links $L = 3$

P_1 = No. of binary joint = 3; $P_2 = 0$

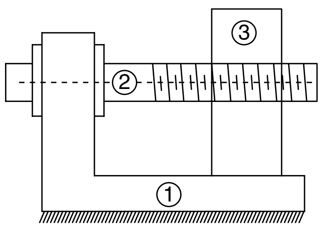
$F = 3(L - 1) - 2P_1 - P_2$

$= 3(3 - 1) - 2 \times 3 - 0 = 0$

But this is not a structure. If wedge 3 is pushed to the left, wedge 2 will move up. Hence, all links can have relative motion with respect to each other and the mobility is equal to 1.

Hence, this is also a case of failure of Kutzback criteria. This mechanism is a wedge mechanism.

Example 24:



Link 1 is fixed. Link 2 is a threaded shaft with collar which forms a revolute pair with link 1. Link 3 is a threaded block (a nut) which form a helical pair with link 2. Link 3 is free to slide smoothly over link 1. The mobility of this frame is

(A) 2 (B) 1 (C) 0 (D) −1

Solution:

No. of links $L = 3$

P_1 = No. of binary joints or lower pairs

$= 3$

P_2 = Number of higher pairs = 0

$\therefore F = 3(L - 1) - 2P_1 - P_2$

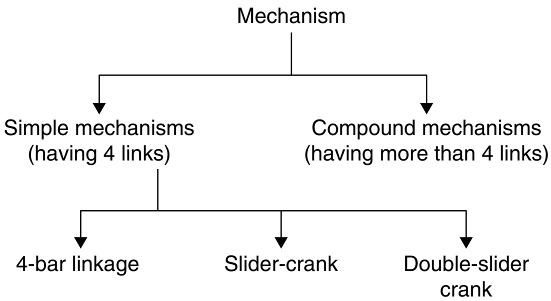
$= 3(3 - 1) - 2 \times 3 - 0 = 0$

But this is not correct because a rotational motion given to link 2 can make the block 3 (nut) more over from 1. Thus, the motion is completely constrained and $F = 1$.

Hence, this is also a case of failure of Kutzback criteria. This is a screw mechanism.

Simple and Compound Mechanisms

A mechanism having four (4) links is called a **simple mechanism**. If it is made of only lower pairs, it is called a **linkage**. A mechanism having more than four links is called a **compound mechanism**.

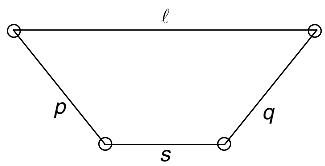


The simple mechanism can be obtained from

1. **4-bar chain**, which is made of 4 binary links and 4 revolute pairs. i.e. *R-R-R-R* linkage. It is also known as **quadric cyclic chain**.
2. **Slider-crank chain** which is made of 4 binary links, 3 revolute pairs and 1 prismatic (sliding) pair. i.e. *R-R-R-P* linkage.
3. **Double-slider-crank chain**, which is made of 4 binary links, 2 revolute pairs and 2 prismatic (sliding) pairs. i.e. *R-R-P-P* linkage.

We will now look at the inversions of each of these chains.

Inversions of 4-bar Linkage (Quadric Cyclic Chain)



A kinematic chain with four binary links and four revolute pairs is called a 4-bar kinematic chain. When any one link is fixed, it becomes a mechanism and is called as **4-bar linkage**. As there are four (4) links, there are four (4) inversions of 4-bar linkage.

The shortest link is of length s and longest link is of length ℓ, the other two links are of lengths p and q respectively. In a **4-bar linkage**, continuous relative motion between any two links is possible, only if the sum of the lengths of the shortest and longest link is less than the sum of the lengths of the other two links. This is called **Grashof's Rule**.

i.e. For continuous relative motion between two links

$$s + \ell < p + q$$

is the mathematical representation of Grashof's Rule.

If $s + \ell > p + q$, no link can move continuously and we will get only rocker – rocker (or lever-lever) mechanism even if any link is fixed.

If a 4-bar linkage fulfills Grashof's Rule, and $s + \ell < p + q$, three situations arise.

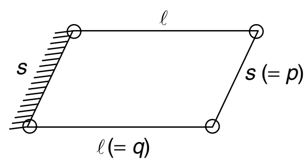
1. **$s + \ell < p + q$ and shortest link s is fixed**
   Both the links adjacent to the shortest link can rotate completely and the link opposite to the shortest link will oscillate. Hence, this gives **double-crank mechanism** (or **crank-crank mechanism**). Drag-link mechanism is an example of double-crank mechanism in which the crank lengths, are different (i.e. all four links are of different lengths, Grashof's Rule is fulfilled and shortest link is fixed). Drag-link mechanism is a quick return mechanism which makes use of 4 links and 4 revolute pairs.
2. $s + \ell < p + q$ and any link adjacent to the shortest link is fixed
   The shortest link s will be able to make full rotation and hence will be the crank. The other two movable links can only oscillate. Hence we get **crank-rocker mechanism** (or **crank-lever mechanism**). An example of this inversion is the **beam engine**.
3. $s + \ell < p + q$ and the link opposite to shortest link is fixed
   The links adjacent to the fixed link can only oscillate. The shortest link connects those two links (i.e. shortest link is the coupler) and the coupler can make full rotation. Hence, we get **rocker-rocker mechanism** (or **double – rocker mechanism**). **Watt's indicator** (which is an approximate straight line mechanism) is an example of this inversion.

   The **automobile steering gear** (Ackermann steering Gear) is also another example of this mechanism, in which the **oscillating links** (adjacent to the fixed link) **are of equal lengths**.

   If 4-bar linkage fulfill's Grashof's Rule and $s + \ell = p + q$, three situations arise.

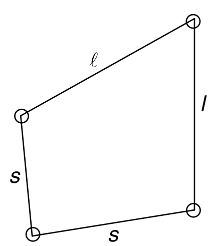
   (a) $s + \ell = p + q$ and all links are of different lengths
   Same situation as in (a), (b) and (c) above

   (b) $s + \ell = p + q$, $s = p$ and $\ell = q$, links of equal lengths are opposite to each other.



This is called **parallelogram linkage**. Whether s is fixed or ℓ is fixed, we get **double crank (crank- crank) mechanism**.
The coupling rod of locomotive is an application of this inversion.

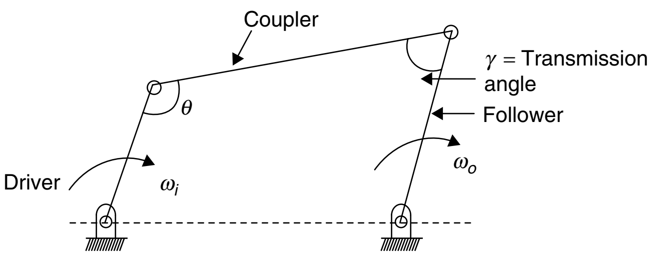
(c) $s + \ell = p + q$, $s = p$, $\ell = q$ and links of equal lengths are adjacent to each other.
This is called **deltoid linkage**.



In deltoid linkage, if **smaller link s is fixed**, we get **double-crank** (or **crank-crank** mechanism). If **longer link ℓ is fixed**, we get **crank-rocker** (or **crank-lever**) mechanism.

From the above, we see that there are effectively only three types of inversions for a 4-bar linkage which are double-crank (crank-crank) mechanism, crank-rocker (crank-lever) mechanism and rocker-rocker (Lever-lever) mechanism.

In a 4-bar linkage (or 4-bar mechanism), the input motion is given to the link called **driver** and output motion is taken from the link called **follower** (or **driven link**). The **coupler** connects the driver and follower. The fixed link is the frame.



The angle between the coupler and follower is called **transmission angle (γ)** which is usually around 45° to 50°. The angle between driver and coupler is (θ). The torque available at the follower is the output torque (T_o) and the torque given to the driver link (T_i) is the input torque.

In the ideal case, Input power

$$P_i = \text{Input torque} \times \text{angular velocity} = T_i\,\omega_i$$

Output power P_o = Output torque × angular velocity $= T_o\omega_o$

and $P_i = P_o$

$$\therefore\ T_i\omega_i = T_o\omega_o$$

$$\Rightarrow \frac{T_o}{T_i} = \frac{\omega_i}{\omega_o}$$

Mechanical Advantage (MA) is defined as the ratio of output torque to the input torque.

$$\therefore\ MA = \frac{T_o}{T_i} = \frac{\omega_i}{\omega_o} \text{ for ideal 4-bar linkage}$$

For a 4-bar linkage, it can be shown that the mechanical advantage (MA) is directly proportional to the sine of transmission angle (γ) and inversely proportional to the sine of angle between the driver and coupler (θ)

$$\text{i.e. } MA \propto \frac{\sin\gamma}{\sin\theta}$$

Mechanical efficiency

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{T_o\omega_o}{T_i\omega_i}$$

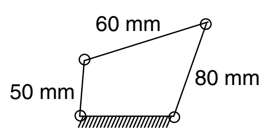
$$\Rightarrow \text{Mechanical Advantage, } MA = \frac{T_o}{T_i} = \eta\,\frac{\omega_i}{\omega_o}$$

The positions of the links for which the **angle θ between driver and coupler became 0° or 180°** are called as **toggle positions**. **In the toggle positions**, the output angular velocity (ω_o) becomes zero and hence **mechanical advantage becomes infinite**. The toggle positions are also known as **extended dead centre** and **folded dead centre** positions. If the transmission angle (γ) increases, mechanical advantage increases and vice versa. It must be noted that if the driver and follower links are interchanged, the transmission angle (γ) also changes. The driver is usually the link that can make a full rotation.

In a crank-rocker mechanism, the transmission angle (γ) will be **maximum** when the angle between the crank and frame is 180°. The transmission angle (γ) will be **minimum** when the angle between crank and frame is 0°.

Example 25: A 4-bar mechanism with all revolute pairs has link lengths ℓ_f = 25 mm, ℓ_{in} = 50 mm, ℓ_{co} = 60 mm and ℓ_{out} = 80 mm respectively. The suffices '*f*', '*in*', '*co*' and '*out*' denote the fixed link, the input link, the coupler and the output link, respectively. Which one of the following statements is true about the input and output links?
(A) Both links cannot execute full circular motion.
(B) Only the output link cannot execute full circular motion
(C) Only the input link cannot execute full circular motion
(D) Both links can execute full circular motion

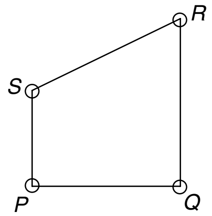
Solution:
Shortest link, s = 25 mm
Longest link, ℓ = 80 mm
Other link, r = 50 mm
Other link, q = 60 mm
$s + \ell = 25 + 80 = 105$ mm
$p + q = 50 + 60 = 110$ mm
$\therefore\ s + \ell < p + q$



⇒ Grashof's rule is fulfilled
The smallest link s = 25 mm is fixed
⇒ double-crank mechanism (crank-crank mechanism)
Hence, both input and output links rotate fully.

Example 26: A planar closed kinematic chain is formed with rigid links PQ = 1.0 m, QR = 2.0 m, RS = 1.5 m and SP = 1.7 m with all revolute joints. The link to be fixed to obtain a double rocker (rocker-rocker) mechanism is
(A) SP (B) RS
(C) QR (D) PQ

Solution:



Smallest link, $s = PA = 1.0$ m
Largest link, $\ell = QR = 2.0$ m
Other link, $p = RS = 1.5$ m
$q = SP = 1.7$ m
$s + \ell = 1 + 2 = 3.0$ m
$p + q = 1.5 + 1.7 = 3.2$ m
$\therefore s + \ell < p + q$
$\Rightarrow$ fulfills Grashof's rule

So to obtain double rocker (rocker-rocker) mechanism, the link opposite to the shortest link shall be fixed i.e. link *RS*.

Example 27: In a 4-bar linkage, s denotes the shortest link length, ℓ is the longest link length, *p* and *q* are the lengths of the other two links. At least one of the three moving links will rotate by 360°, if

(A) $s + \ell \le p + q$ (B) $s + \ell > p + q$
(C) $s + p \le \ell + q$ (D) $s + p > \ell + q$

Solution:
For at least one link to rotate fully, Grashof's rule must be fulfilled

i.e. $s + \ell \le p + q$

Example 28: For a 4-bar linkage in toggle position, the value of mechanical advantage is

(A) 0 (B) ∞ (C) 0.5 (D) 1.0

Solution:

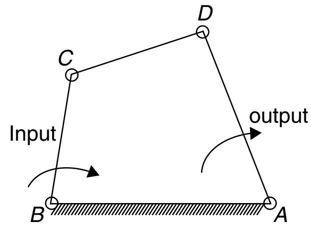
Mechanical advantage $\propto \dfrac{\sin\gamma}{\sin\theta}$, where γ = Transmission angle and θ = angle between driver and coupler
In the toggle position, $\theta = 0°$ or $180°$
$\Rightarrow \sin\theta = 0$
$\therefore$ *MA* is ∞.

Example 29: In a planar 4-bar linkage, link *AB* = 50 cm, link *BC* = 100 cm, link *CD* = 80 cm and link *DA* = 65 cm. The link to be fixed to obtain a double crank mechanism is

(A) *AB*
(B) *BC*
(C) *DA*
(D) not possible with these link lengths

Solution:
Shortest link, $s = AB = 50$ cm
Longest link, $\ell = BC = 100$ cm
Other link $p = CD = 80$ cm
$q = DA = 65$ cm
$s + \ell = 50 + 100 = 150$ cm
$p + q = 80 + 65 = 145$ cm
$\therefore s + \ell > p + q$
$\Rightarrow$ Non-Grashof 4-bar linkage means only rocker-rocker mechanism is possible by fixing any link. So double-crank mechanism not possible with the given link lengths

Example 30:

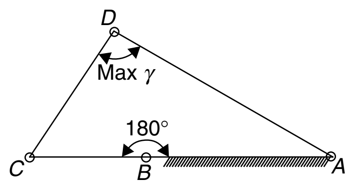


In the planar 4-bar linkage *ABCD* shown in figure, *AB* = 9 cm, *BC* = 6 cm, *CD* = 12 cm and *DA* = 10 cm. If input link is *AB*, the maximum transmission angle for the mechanism shown will be

Solution:
s = shortest link length = BC = 6 cm

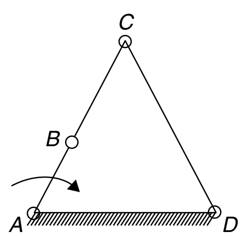
ℓ = longest link length = CD = 12 cm

other lengths $P = AB = 9$ cm,
$q = DA = 10$ cm
$s + \ell = 6 + 12 = 18$ cm
$p + q = 9 + 10 = 19$ cm
$\therefore s + \ell < p + q$
$\Rightarrow$ Grashof mechanism
$\Rightarrow$ Crank rocker mechanism
Maximum transmission angle (γ) occurs when the angle made by crank (*BC*) with fixed link (*AB*) is 180°.



$CA = CB + BA$
$= 6 + 9 = 15$ cm
from ΔCDA,
$CA^2 = CD^2 + DA^2 - 2.CD.DA.\cos\gamma$
$15^2 = 12^2 + 10^2 - 12.10.\cos\gamma$
$\cos\gamma = 0.1583$
$\gamma = 80.89°$.
$\therefore$ Maximum transmission angle
$= 80.89°$.

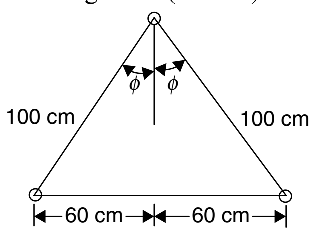
Example 31:



The link lengths are
$AB = 30$ cm
$BC = 70$ cm
$CD = 100$ cm
$DA = 120$ cm
Crank is AB

For the position shown, the transmission angle and mechanical advantage are ________ and ________ respectively. Fill up the blanks.

Solution: The angle between crank AB and coupler BC is 180°. Hence, the mechanism is in toggle position
⇒ mechanical advantage is ∞ (infinite)

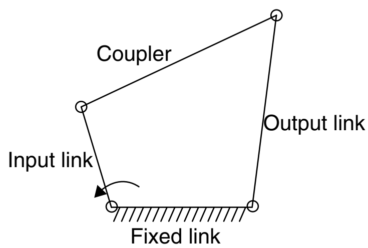


Transmission angle $\gamma = 2\phi$

$$\sin\phi = \frac{60}{100} = 0.6$$

$\Rightarrow \phi = \sin^{-1}(0.6) = 36.87°$
$\therefore\ \gamma = 2\phi = 2 \times 36.87 = 73.74°$
Hence, in the given position, the transmission angle is 73.74° and mechanical advantage is infinite.

Example 32:



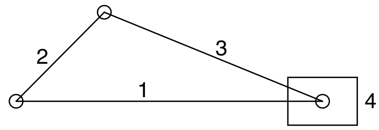
The input link, coupler link and the fixed link of a four bar linkage have lengths of 400 mm, 500 mm and 200 mm respectively. When the input link is rotated by a motor, the output link can make complete revolutions, if its length 'L' in mm satisfies the condition.

(A) $L < 300$ (B) $L > 700$
(C) $300 < L < 700$ (D) $L < 300$ or $L > 800$

Solution:
For complete rotation of a link, Grashof's Rule shall be fulfilled.
i.e. $s + \ell < p + q$
If coupler is the longest link, then
$200 + 500 < 400 + L$
$\Rightarrow 300 < L$
If output link L is the largest link, then
$200 + L < 400 + 500$
$\Rightarrow L < 700$
$\therefore\ 300 < L < 700$
The given option $300 < L < 700$ is the correct choice.

Inversions of Slider Crank Chain

Slider crank chain is a kinematic chain consisting of four binary links having three (3) revolute pairs and one prismatic (sliding) pair.

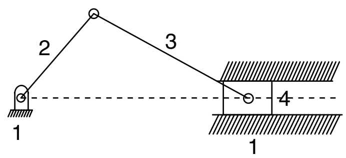


By fixing different links, we can get different mechanisms. Each mechanism is an inversion of slider crank chain.

Inversion 1 of Slider Crank Chain

Link 1 is fixed. The input can be given to link 2 (crank) and output taken from slider 4. The **reciprocating compressor** is an example of this inversion which is called **slider crank mechanism**.

In some cases, input is given at link 4 (slider) and output is taken from link 1. The **internal combustion engine** is an example of such case.

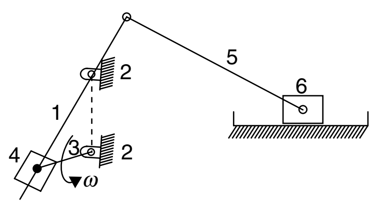


NOTE

The joint between fixed link 1 and crank 2 and the joint between coupler 3 and slider 4 lie on the same horizontal line (or vertical line). Otherwise, this mechanism will **act as a quick return mechanism**, known as **off-set slider crank mechanism**.

Inversion 2 of Slider Crank Chain

In this inversion, the link 2 of the chain is fixed and input is given to link 3 which acts as the crank. As the slider 4 moves over link 1, which is inserted through a hole in the slider, the link 1 oscillates. The oscillation of link 1 is transmitted through link 5 to the ram of tool carriage 6, which produces reciprocating motion. This arrangement is called **Whitworth Quick Return Mechanism**.



The quick return ratio (QRR) is defined as the ratio of time taken for cutting stroke (or forward stroke) (t_c) to the time taken for return stroke (t_r)

$$QRR = \frac{t_c}{t_r} = \frac{\theta_c}{\theta_r} = \frac{(2\pi - \theta_r)}{\theta_r}$$

where θ_c = angle turned by link 3 (crank) during cutting stroke

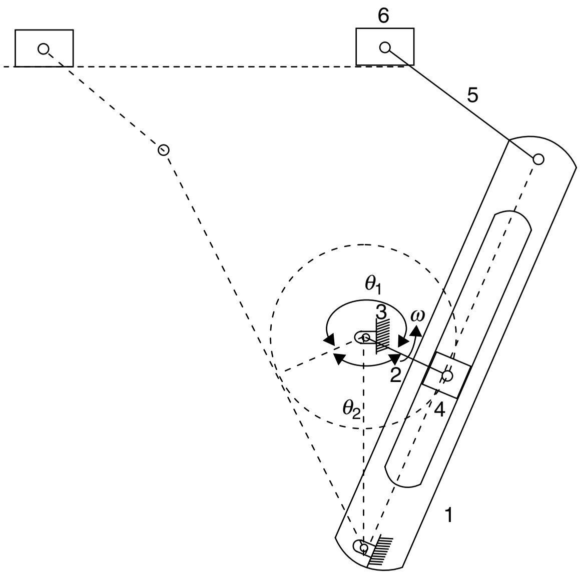
θ_r = angle turned by link 3 (crank) during return stroke

Shaper machines make use of this inversion.

Another example of the second inversion of slider-crank chain is the **rotary** (or **radial cylinders**) **engine**.

Inversion 3 of the Slider-crank Chain

The third inversion is obtained by fixing link 3 (connecting rod) of the slider-crank chain. Link 1 is a slotted link. The slider 4 slides along the slot in link 1. The link 2 (which is the crank) connects the fixed link 3 and slider 4. A constant angular velocity is given to link 2 which acts as the crank. Slotted link 1 oscillates and this motion is transmitted to the ram of tool carriage (link 6) through link 5. This arrangement acts as quick return mechanism and is called as **Crank and Slotted Lever Quick Return Mechanism**. It is used in shaper machines.



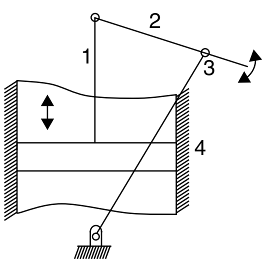
Quick Return Ratio,

$$QRR = \frac{t_c}{t_r} = \frac{\theta_1}{\theta_2} = \frac{2\pi - \theta_2}{\theta_2}$$

The Oscillating Cylinder Engine is also an example of this inversion.

Inversion 4 of the Slider-crank Chain

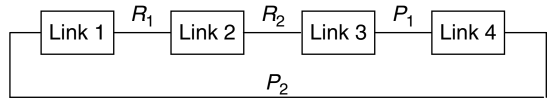
This inversion is obtained by fixing link 4 (which is the slider) of slider-crank chain. The **hand pump** and **pendulum pump** (**bull engine**) are examples.



Thus, there are 4 inversions of slider-crank chain.

Inversions of Double-Slider Crank Chain

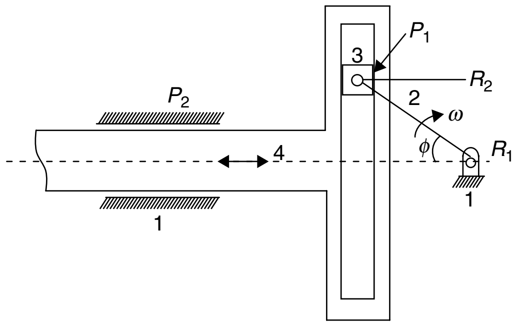
The double-slider crank chain consists of four (4) binary links, two (2) revolute pairs and two (2) prismatic (sliding) pairs. As fixing any slider will give same type of mechanism the double-slider crank chain produces effectively three different mechanisms.



The scheme drawing of double-slider crank chain with 4 links, revolute pairs R_1 and R_2 and prismatic pairs P_1 and P_2 is as shown.

Inversion 1 of Double Slider Crank Chain

Scotch Yoke mechanism is an inversion 1 of double-slider crank chain.

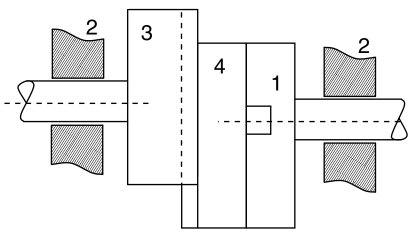


It is a mechanism used for converting simple harmonic motion (SHM) of link 4 into rotational motion of link 2 or vice versa. Link 1 is the fixed link, which is a slider similar to link 3. Link 4 is T shaped link with a slot in the vertical portion. Link 3 slides in this slot. Link 2 is the crank which rotates at constant angular velocity ω.

Inversion 2 of Double-slider Crank Chain

Oldham's coupling is an inversion 2 of double-slider crank chain. It is used for connecting two parallel shafts having an eccentricity (i.e. their axes are displaced by a small distance).

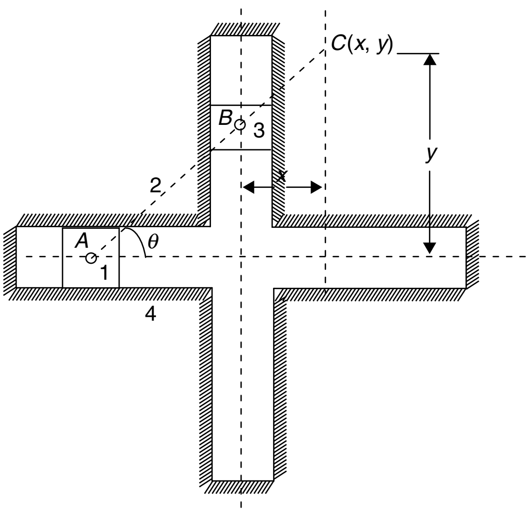
In this inversion **link 2 is fixed**. Link 3 is a shaft rotating in link 2 and there is a flat disc with a diametral slot at the end of link 3. Link 1 is another shaft (eccentric with link 2) and having flat disc with diametral slot at its end. Link 4 is a disc with a diametral tongue on each face and these tongues are perpendicular to each other. One tongue slides is the slot in link 3 and the other tongue slides in slot in link 1. The centre of link 4 will move in a circle of diameter equal to the eccentricity of shafts (Link 3 and link 1).



Inversion 3 of Double-slider Crank Chain

The elliptical trammel, which is used for drawing ellipses, is an example of this inversion. The link 4, which is a planar link, is the fixed link. It has got two mutually perpendicular slots on its plane (say along *X* and *Y* direction). Link 1, which is a slider moves in one slot (say along *X*-direction) and link 3, which is also a slider, moves in the other slot (i.e., along *Y*-direction). Link 2 connects link 1 and link 3 by revolute pair joints. A point *C* on the extension of link 2 traces an ellipse.

If length of link 2 (i.e. *AC*) is *p* and extended portion $BC = q$, then from the below figure, we get



$$p \sin\theta = y \Rightarrow \left(\frac{y}{p}\right) = \sin\theta \quad \text{and}$$

$$q \cos\theta = x \Rightarrow \left(\frac{x}{q}\right) = \cos\theta$$

$\therefore \frac{y^2}{p^2} + \frac{x^2}{q^2} = \sin^2\theta + \cos^2\theta = 1$, which is the equation of an ellipse. Thus, the path traced by point *C* on link 2 is an ellipse, with major and minor axis equal to $2p$ and $2q$ respectively.

Example 33: The mechanism in a shaping machine is
(A) a closed 4-bar chain having 4 revolute pairs
(B) a closed 6-bar chain having 6 revolute pairs
(C) a closed 4-bar chain having 2 revolute and 2 sliding pairs
(D) an inversion of single slider-crank chain

Solution:
Shaping machines use quick return mechanism which are either Whitworth quick return mechanism or Crank and Slotted Lever quick return mechanism. These are inversions of single slider-crank chain.

Example 34: The number of inversions of a slider-crank chain is
(A) 6 (B) 5 (C) 4 (D) 3

Solution:
A slider crank chain has 4 links and by fixing each link, one at a time, we get 4 different mechanisms, each of which is an inversion. Hence, a slider-crank chain has 4 inversions.

Example 35: Which of the following is an inversion of single slider-crank chain?
(A) Elliptical Trammel (B) Hand Pump
(C) Oldham's Coupling (D) Scotch Yoke

Solution:
Hand pump is an inversion of single slider-crank chain.

Example 36: Which of the following are the inversions of double slider-crank chain?
1. Oldham coupling
2. Whitworth quick return mechanism
3. Beam engine mechanism
4. Elliptical Trammel mechanism

The correct answer codes are
(A) 1 and 2 (B) 1 and 4
(C) 2, 3 and 4 (D) 1, 2 and 3

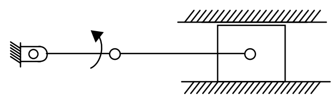
Solution:
Oldham coupling and Elliptical trammel are inversions of double slider-crank chain.

Whitworth quick return mechanism is an inversion of single slider crank chain. Beam engine mechanism is an inversion of 4-bar linkage.

Example 37: In a double slider-crank mechanism, a point on a link connecting the sliders (excluding the end points) traces a/an
(A) straight line (B) hyperbolic path
(C) elliptical path (D) parabolic path

Solution:
The point on connecting link traces an elliptical path (as in an elliptical trammel)

Example 38: For the mechanism shown in figure, the mechanical advantage for the given configuration is



(A) 0 (B) 0.5 (C) 1 (D) ∞

Solution:
In the configuration, the system is in toggle position and in toggle position, mechanical advantage is ∞ (infinite)

Example 39: Which of the following is an inversion of single slider crank chain?
(A) Beam engine
(B) Watt's indicator mechanism
(C) Elliptical Trammel
(D) Oscillating cylinder engine

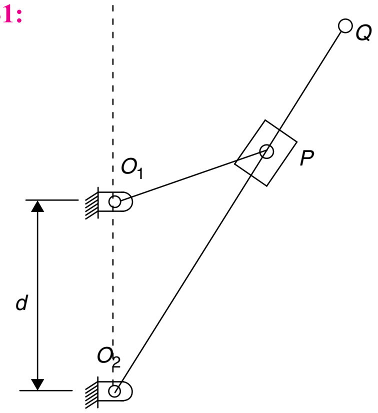
Solution:
Oscillating cylinder engine is an inversion of single slider crank chain.

Example 40: Oldham's coupling is used to connect two shafts which are

(A) intersecting (B) parallel
(C) perpendicular (D) co-axial

Solution:
Oldham's coupling is used to connect slightly offset parallel shafts.

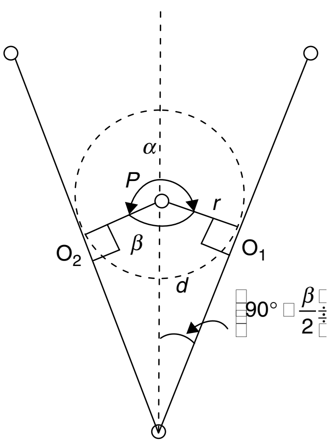
Example 41:



The quick return ratio of the quick return mechanism shown in figure is 2:1. If the radius of the crank O_1P is 175 mm, then the distance d (in mm) between the crank centre to lever pivot centre point is

(A) 247.5 mm (B) 303 mm
(C) 350 mm (D) 291 mm

Solution:



$r = 175$ mm (crank radius)

Quick return ratio (QRR) = $\frac{2}{1}$ (given)

But $QRR = \frac{\text{Time for cutting}}{\text{Time for return}} = \frac{t_1}{t_2}$

$$= \frac{\left(\frac{\alpha}{\omega}\right)}{\left(\frac{\beta}{\omega}\right)}$$

$$= \frac{\alpha}{\beta} = \frac{2\pi - \beta}{\beta} = \frac{360° - \beta}{\beta}$$

$$\therefore 2 = \frac{360° - \beta}{\beta} \Rightarrow 3\beta = 360°$$

$$\beta = \frac{360°}{3} = 120°$$

$$\frac{\beta}{2} = \frac{120}{2} = 60°$$

We have $\sin\left(90° - \frac{\beta}{2}\right) = \frac{r}{d}$

$\Rightarrow \sin(90° - 60°) = \frac{r}{d}$ or $\sin 30° = \frac{r}{d}$

$\Rightarrow d = \frac{r}{\sin 30°} = \frac{175\,\text{mm}}{0.5} = 350$ mm.

Example 42: The quick return mechanism which is an inversion of 4-bar linkage is

(A) Drag link mechanism
(B) Whitworth quick return mechanism
(C) Crank and slotted lever mechanism
(D) both (B) and (C)

Solution:
Drag link mechanism is an inversion of 4-bar linkage, which is a crank – crank mechanism with different crank lengths. It is made up of revolute pairs only.

Intermittent Motion Mechanisms

A mechanism which converts continuous motion into an intermittent motion is called an intermittent motion mechanism. Machine tools commonly use such mechanisms for indexing. **Geneva wheel mechanism** and **Ratchet and Pawl mechanism** are some common mechanisms used for intermittent motion.

In **Geneva Wheel Mechanism**, the continuous rotational motion of driver link, with a pin projecting above its surface, is to drive intermittently a driven link. The projecting pin engages with slots in the driven link. The number of slots in the driven link and the position of the slots are arranged in such a manner that the pin enters and leaves them tangentially without any impact loading during transmission of motion. The driven link makes partial rotations during engagement and disengagement of pin.

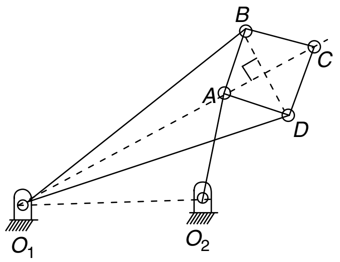
Ratchets and pawls are used to transform rotational motion or translational motion into intermittent rotation or translation.

Exact Straight Line Motion Mechanisms made up of only turning pairs

(i) **Peaucellier** mechanism and (ii) **Hart's** mechanism produce exact straight line motion and they are made up of only turning pairs. In **Peaucellier mechanism**, there are 8 binary links, 2 binary joints and 4 ternary joints.

$$\therefore P_1 = 2 + 4 \times (3 - 1) = 10$$
$$\therefore F = 3(L - 1) - 2P_1$$
$$= 3(8 - 1) - 2 \times 10$$
$$= 21 - 20 = 1$$

Hence, degree of freedom is 1.

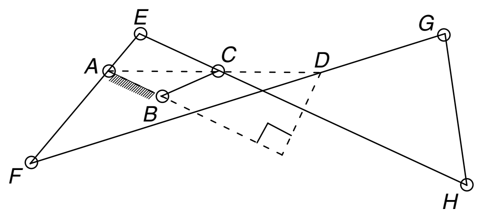


O_1O_2 is fixed link
O_2A is crank
$O_1O_2 = O_2A$
$O_1B = O_1D$
$AB = BC = CD = DA$

Point C traces an exact straight line.

The **Hart mechanism**, also known as **crossed-parallelogram mechanism**, is an application of 4-bar chain to produce exact straight line motion. It has 6 links and 7 binary joints giving degree of freedom

$$F = 3(6 - 1) - 2 \times 7 = 1$$

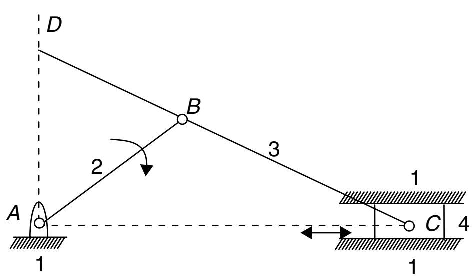


AB is fixed link
BC is crank
$EF = GH$
$EH = GF$

Points A, C and D divide links FE, HE, and FG in same ratio. Point D moves along an exact straight line, perpendicular to link AB.

Exact Straight Line Motion Mechanism Consisting of One Sliding Pair

An example of this type of mechanism is **Scott-Russel Mechanism.**



It is basically a single slider-crank mechanism, with an extension of the connecting rod. Here there are 4 binary links, 3 revolute joints and 1 prismatic (sliding) joint.

Here crank $AB = BC = BD$ (i.e. B is the midpoint of connecting rod with extension). When the slider C moves horizontally, the point D on the connecting rod generates a vertical straight line which also passes through hinge A.

The friction and wear of the sliding pair is much more than that of the turning pair. Hence, practically this mechanism does not have much value.

Approximate Straight Line Motion Mechanisms

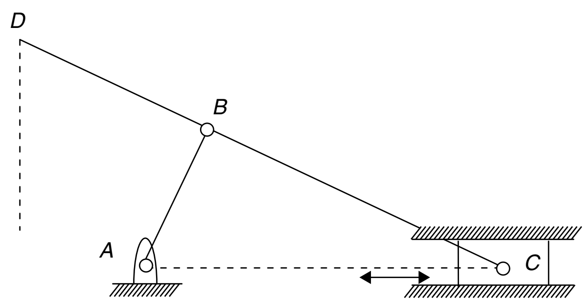
Some of the approximate straight line motion mechanisms are

1. Modified Scott-Russel Mechanism
2. Grasshopper Mechanism
3. Tchebicheff's Mechanism
4. Robert's Mechanism and
5. Watt's mechanism

These are explained below.

Modified Scott-Russel Mechanism

In the Scott-Russel mechanism which was discussed earlier, the path of point D which moved in an exact straight line (see below figure for Scott-Russel mechanism given earlier) also passes through hinge A, which is not desirable. Hence, modified Scott-Russel mechanism is used which produces an approximate straight line motion.



The joint B on the connecting rod CD is located such that

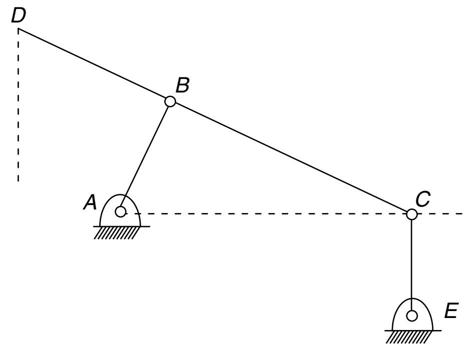
$$\frac{AB}{BC} = \frac{BC}{DC}$$
$$\Rightarrow BC^2 = AB \times DC$$

Also $AB \neq BC$ in this case

For small horizontal movements of slider C, the point D traces an approximate straight line perpendicular to AC. This is the **modified Scott-Russel mechanism**.

Grasshopper Mechanism

The **Grasshopper mechanism is a modification** of the **modified Scott-Russel mechanism**. The sliding pair at c is replaced by a turning pair, using a link *EC*, which is hinged at *E* and pinned at *C*.

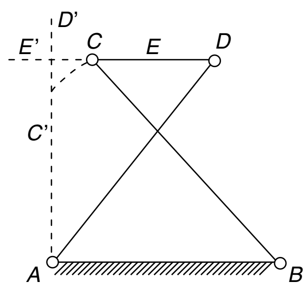


It has 4 binary links and 4 revolute pairs. In the mean position, link *EC* is perpendicular to line *AC*. If length *EC* is large enough, point *c* moves approximately in a straight line perpendicular to *EC*. Joint *B* on link *CD* is located such that

$$\frac{AB}{BC} = \frac{BC}{DC} \text{ or } BC^2 = AB \times DC$$

For small movement of joint *C*, the end *D* will move in an approximate straight line perpendicular to *AC*.

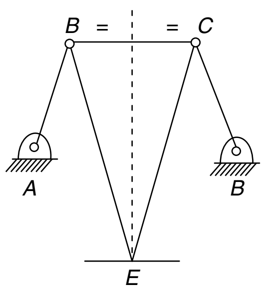
Tchebicheff's Mechanism



$$AD = BC$$

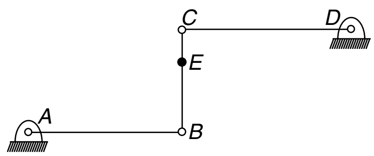
E is the midpoint of link *CD*. In the extreme position, *E* is directly above *A* or *B*. This is an inversion of 4-bar linkage in which links of equal lengths, (*AD* and *BC*) cross each other. The midpoint *E* of link *CD* traces an approximate straight line parallel to the fixed link *AB*. In the extreme positions, points *C*, *E* and *D* lie on a vertical line (either above *A* or above *B*). The length of *CD*: length of *AB*: length of *AD* is 1:2:2.5.

Robert's Mechanism



This is also an inversion of 4-bar linkage. In the mean position, it has the shape of a trapezium. Lengths of link *AB* and *CD* are equal. The coupler *BCE* is made from a single plate such that point *E* is directly below the midpoint of *BC* in the mean position. For small motion of *AB* (or *CD*), the point *E* traces an approximate straight line. Robert's mechanism has 4 binary links and 4 revolute pairs.

Watt's Straight Line Mechanism



This is an inversion of 4-bar linkage (rocker-rocker or double-lever mechanism). Link *AB* can oscillate about hinge *A* while link *DC* can oscillate about hinge *D*. In the mean position, links *AB* and *CD* are parallel and link *BC* is perpendicular to link *AB* (or link *CD*). The point *E* on link *BC* (which is the tracing point) is located such that $\frac{EB}{EC} = \frac{DC}{AB}$.

For small oscillation of links *AB* and *CD*, point *E* will trace an approximate straight line. This mechanism is used for guiding the motion of the piston of steam engine.

Engine Indicators

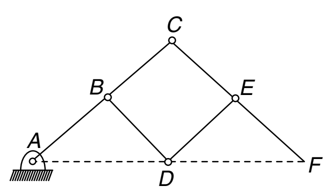
Engine indicators are devices which keep the graphical record of pressure inside the engine cylinder during the piston stroke. The straight line motion mechanisms are mostly used in the design of engine indicators. Some of the engine indicators which work on the straight line motion mechanism are

1. **Simplex Indicator** (uses the principle of pantograph)
2. **Crossby Indicator** (uses a modified form of pantograph)
3. **Thomson Indicator** (uses a straight line motion of Grasshopper type)
4. **Double McInnes Indicator** (uses a straight line motion of Grasshopper type)

Other Mechanisms Using Lower Pairs

(i) Pantograph and (ii) Toggle mechanism use lower pairs and have lot of industrial applications. The automobile steering mechanisms (Davis steering gear and Ackermann steering gear) and Universal joints are discussed in the section under 'Velocity and acceleration analysis in mechanisms'.

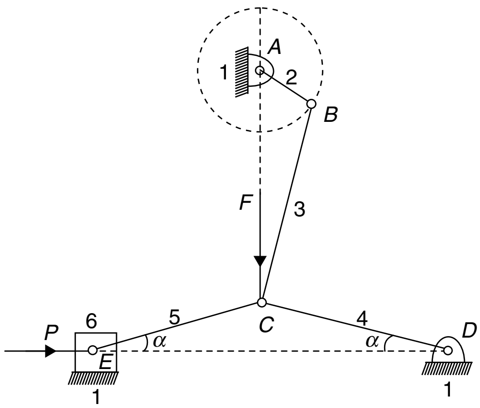
Pantograph



Links BD and CE are of same length and links DE and BC are of same length, $BCED$ forms a parallelogram. Link CB is extended upto A where it is hinged and link CE is extended upto free end F. The points A, D and F lie on the same straight line. It can be shown that point F traces the same path as that traced by point D.

In the drawing office, pantograph is used as a geometrical instrument for producing drawings to different scales (i.e. for enlargement or reduction in scales of drawings). In the workshops, pantograph is used for guiding cutting tools or torches.

Toggle Mechanism



It has six (6) binary links, 4 binary revolute joints, one (1) ternary revolute joint and one (1) prismatic (sliding) joint. Toggle mechanisms are used when large resistances are to be overcome through short distances as in riveting machines, presses, rock crushers etc. The **toggle mechanism** is shown in figure. The link AB (Link 2) is the input link or crank to which power is given. The slider 6 is the output link which has to overcome the external resistance. Links 4 and 5 (CD and CE) are of equal lengths. Considering the equilibrium condition of slider 6, it can be shown that $\tan\alpha = \frac{(F/2)}{P}$ or $F = 2P\tan\alpha$, where F is the effort and P is the resistance.

For small angles α, $F << P$

i.e. effort << resistance

Example 43: Match List-I with List-II and select the correct answer using the codes given below.

List-I	List-II
A. Pantograph	**1.** Scotch yoke mechanism
B. Single slider crank	**2.** Double lever mechanism
C. Double slider crank chain	**3.** Tchebicheff's mechanism
D. Straight line motion mechanism	**4.** Double crank
	5. Hand pump

Codes:

	A	B	C	D
(A)	4	3	5	1
(B)	2	5	1	3
(C)	2	1	5	3
(D)	4	5	2	1

Solution:

Pantograph is double lever mechanism (rocker-rocker) Handpump is an inversion of single slider crank chain. Scotch yoke mechanism is an inversion of double slider crank chain.

Tchebicheff's mechanism is an approximate straight line motion mechanism.

Example 44: Match the following:

List-I (Type of mechanism)	List-II (Motion Achieved)
P. Scott - Russel	**1.** Intermittent Mechanismmotion
Q. Geneva	**2.** Quick return mechanism motion
R. Offset slider-crank	**3.** Simple motion harmonic mechanism
S. Scotch Yoke	**4.** Straight line mechanism motion

(A) P - 2, Q - 3, R - 1, S - 4
(B) P - 3, Q - 2, R - 4, S - 1
(C) P - 4, Q - 1, R - 2, S - 3
(D) P - 4, Q - 3, R - 1, S - 2

Solution:

P. Scott-Russel mechanism-straight line motion
Q. Geneva mechanism-intermittent motion
R. Offset slider crank mechanism-Quick return mechanism
S. Scotch yoke mechanism-simple harmonic motion

Example 45: When a cylinder is located in a Vee-block, the number of degrees of freedom which are arrested is

(A) 2 (B) 4 (C) 7 (D) 8

Solution:

Before placement on Vee-block, cylinder has 6 degrees of freedom (3 translation and 3 rotation). After placement on Vee-block, the cylinder has only 2 degrees of freedom (one translation and one rotation). Hence, the degrees of freedom which are arrested is $6 - 2 = 4$.

Example 46: Match the following with respect to spatial mechanisms.

Type of joint	Motion constrained
P. Revolute	**1.** Three
Q. Cylindrical	**2.** Five
R. Spherical	**3.** Four
	4. Two
	5. Zero

	P	Q	R
(A)	1	3	1
(B)	5	4	3
(C)	2	3	1
(D)	4	5	3

Solution:

For revolute pair, degree of freedom = 1 and constrained DOF = 6 – 1 = 5

For cylindrical pair, dof = 2 and constrained dof = 6 – 2 = 4
For spherical pair, dof = 3 and constrained dof = 6 – 3 = 3.

Example 47: Match the items in column-I and column-II

Column-I	Column-II
P. Higher kinematic pair	**1.** Grubler's equation
Q. Lower kinematic pair	**2.** Line contact
R. Quick return mechanism	**3.** Euler's equation
S. Mobility of a linkage	**4.** Planer
	5. Shaper
	6. Surface Contact

(A) P – 2, Q – 6, R – 4, S – 3
(B) P – 6, Q – 2, R – 4, S – 1
(C) P – 6, Q – 2, R – 5, S – 3
(D) P – 2, Q – 6, R – 5, S – 1

Solution:
P-Higher kinematic pair-Line contact
Q-Lower kinematic pair-surface contact
R-Quick return mechanism-shaper
S-Mobility of a linkage-Grubler's equation

Example 48: The number of binary links, number of binary joints and number of ternary joints in Peaucelliar mechanism is
(A) 6, 6, 0 (B) 8, 2, 4 (C) 8, 4, 2 (D) 8, 8, 0

Solution:
The Peaucelliar mechanism has eight (8) binary links, 2 binary joints and 4 ternary joints.

Example 49: The number of degree of freedom of a planar linkage with 8 links and 9 simple revolute joints is
(A) 1 (B) 2 (C) 3 (D) 4

Solution:
$L = 8$ (= number of links)
$P_1 = 9$ (= number of simple revolute joints)

$$\therefore F = 3(L - 1) - 2P_1$$
$$= 3(8 - 1) - 2 \times 9$$
$$= 21 - 18$$
$$= 3$$

∴ Degree of freedom = 3

Example 50: The following list of statements is given.
(1) Grashoff's rule states that for a planar crank-rocker 4-bar mechanism, the sum of the shortest and longest link lengths cannot be less than the sum of the remaining two link lengths
(2) Inversions of a mechanism are created by fixing different links, one at a time.
(3) Geneva mechanism is an intermittent motion device.
(4) Grubler's criterion assumes mobility of a planar mechanism to be one.

The number of correct statements in the above list is
(A) 1 (B) 2 (C) 3 (D) 4

Solution:
Except statement 1, all other three statements are correct.

VELOCITY AND ACCELERATION ANALYSIS OF MECHANISMS

VELOCITY ANALYSIS

Introduction

The process of determining the velocities of various points of a mechanism is called **velocity analysis** of a mechanism. This is required for determining the accelerations of various points of the mechanism. If we know the velocity at any one point of the mechanism, then the velocities of the other points in the mechanism can be determined either by (i) **Relative velocity method** or (ii) by **Instantaneous Centre method**. The accuracy required and the nature of mechanism will usually decide the method to be employed for velocity analysis.

A rigid body is said to be in **pure translation**, if every point on the body **moves in parallel planes** with the **same velocity**. The motion of the body can be described by any point on the body. A rigid body is said to be in **pure rotation**, if every point on the body **moves in concentric circle** with the **same angular velocity**. In the general motion of a rigid body, it can have both translational and rotational motion, for example, a rolling body.

The velocity of a point with respect to a fixed (or stationery) point is called **absolute velocity** and the velocity of a point with respect to another moving point is called relative velocity. For a rigid link, the velocity of every point on the link along the link must be the same. This is the condition for rigidity of a link. If this condition is not met, then the link can stretch or compress which makes it non-rigid.

Let us first look at the relative velocity method of velocity analysis of mechanisms.

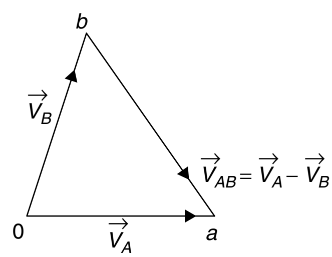
Relative Velocity Method

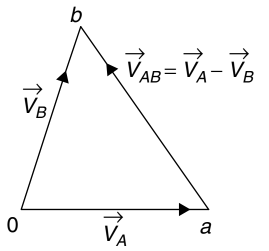
This method is a quick method for determining the angular and linear velocities in mechanisms. This can also be used for making acceleration analysis.

If a body A moves with a velocity $\vec{V}_A$ with respect to a fixed body (called ground link) and another body B moves with a velocity $\vec{V}_B$ with respect to the fixed body, then the relative velocity of A with respect to B is $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$ (i.e. the vector difference of velocity of A and velocity of B). Also the relative velocity of B with respect to A is

$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A\text{; Hence, } \vec{V}_{AB} = -\vec{V}_{BA}$$

Graphically, this is represented as shown below.





Here, $\vec{oa}$ represents $\vec{V}_A$ in magnitude and direction

$\vec{ob}$ represents $\vec{V}_B$ in magnitude and direction

$\vec{ba}$ represents $\vec{V}_{AB}$ in magnitude and direction

$\vec{ab}$ represents $\vec{V}_{BA}$ in magnitude and direction

If A and B are moving in the same direction, then

$$V_{AB} = |\vec{V}_{AB}| = |\vec{V}_A - \vec{V}_B| = V_{BA} = |\vec{V}_{BA}|$$

If A and B are moving in the opposite directions, then

$$V_{AB} = |\vec{V}_{AB}| = |V_A + V_B| = V_{BA} = |\vec{V}_{BA}|$$

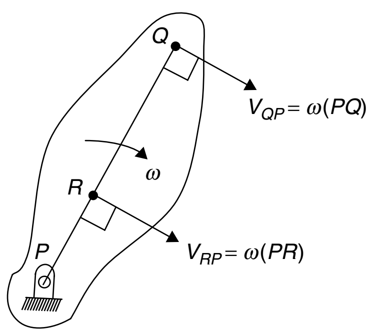
If $\vec{V}_A$ and $\vec{V}_B$ subtend an angle θ between them, then

$$V_{AB} = |\vec{V}_{AB}|$$
$$= \sqrt{V_A^2 + V_B^2 - 2V_A V_B \cos\theta}$$

It must be noted that both $\vec{V}_{AB}$ and $\vec{V}_{BA}$ are in the same plane containing $\vec{V}_A$ and $\vec{V}_B$.

Relative Velocities of Points on a Rigid Link

Consider a rigid link PQ, rotating at a constant angular velocity ω about the end P.



Velocity of point Q with respect to P is

$V_{QP} = \omega(PQ)$ and its direction is perpendicular to PQ. Similarly, velocity of point R on link with respect to P is $V_{RP} = \omega(PR)$ and its direction is perpendicular to PR.

$$\Rightarrow \frac{V_{QP}}{V_{RP}} = \frac{PQ}{PR}$$

Hence, velocity of Q relative to R is $V_{QP} - V_{RP}$

($\because$ their velocities are in the same direction)

$$= \omega(PQ) - \omega(PR)$$
$$= \omega(PQ - PR) = \omega(RQ)$$
$$\therefore V_{QR} = \omega(RQ)$$

The direction of velocity of Q relative to R is perpendicular to line RQ.

Also, $\omega = \dfrac{V_{QR}}{(RQ)}$ is the angular velocity of link about P.

Let us now consider another case. A rigid link AB is considered. Velocity of end A is $\vec{V}_A$, making an angle α with AB and velocity of end B is $\vec{V}_B$, making an angle β with AB.

How do we calculate velocity of B relative to A?

Resolve $\vec{V}_A$ along AB and normal to AB as follows:

V_A along $AB = V_A \cos\alpha$ and

V_A normal to $AB = V_A \sin\alpha$

Similarly, the velocity of B along AB is $V_B \cos\beta$ and the velocity of B normal to AB is $V_B \sin\beta$

For rigidity of rod AB, velocity of A along AB = velocity of B along AB

$\Rightarrow$ i.e. $V_A \cos\alpha = V_B \cos\beta$

$$\Rightarrow \frac{V_A}{V_B} = \frac{\cos\beta}{\cos\alpha} \text{ from rigidity of link } AB.$$

Now, the relative velocity of B with respect to A is only due to the velocity components normal to link AB.

$$\therefore V_{BA} = (V_B \sin\beta - V_A \sin\alpha)$$

(if normal components of velocities at B and A are in the same direction)

$$= (V_B \sin\beta + V_A \sin\alpha)$$

(if normal components of velocities at B and A are in the opposite directions)

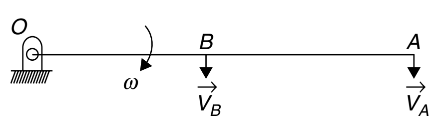
Also, $\vec{V}_{BA} = -\vec{V}_{AB}$

The angular velocity of link AB is given by

$$\omega = \frac{V_{BA}}{AB} = \frac{V_B \sin\beta \pm V_A \sin\alpha}{AB}$$

From the above explanations, the following key points emerge.

1. The ratio of magnitudes of velocities of any two points on a rotating link is the same as the ratio of the radial distances of those points from the fulcrum (or hinge) about which the link is rotating.



$$\frac{V_A}{V_B} = \frac{OA}{OB}$$

2. The angular velocity of a link about an extremity is of the same magnitude as the angular velocity about the other end.

$$\omega = \frac{V_{AB}}{AB} = \frac{V_{BA}}{AB} = \frac{V_{AO}}{OA} = \frac{V_{OA}}{AO}$$

3. The relative velocity of any two points on a rigid link is always normal to the line joining the two points.
4. The velocity of any point on a fixed link relative to any other point on the same link is always zero.

Example 51: There are two points A and B on a planar rigid body. The relative velocity of A with respect to B

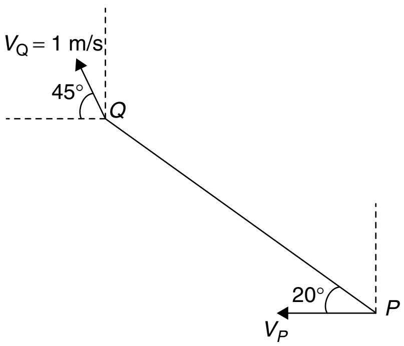
(i) should be always along AB
(ii) can be oriented along any direction
(iii) should be always perpendicular to AB
(iv) should be along BA when the body undergoes pure translation
(v) should be always zero if the rigid body is fixed.

The correct statements are

(A) (i) and (v) (B) (ii) and (v)
(C) (iii) and (v) (D) (iv) and (v)

Solution:
Statements (iii) and (v) are correct

Example 52:



A rigid link PQ is 2 m long and oriented at 20° to the horizontal as shown in figure. The magnitude and direction of velocity V_Q and the direction of velocity V_P are given. The magnitude of V_p (in m/s) at this instant is

(A) 2.14 (B) 1.89 (C) 1.21 (D) 0.96

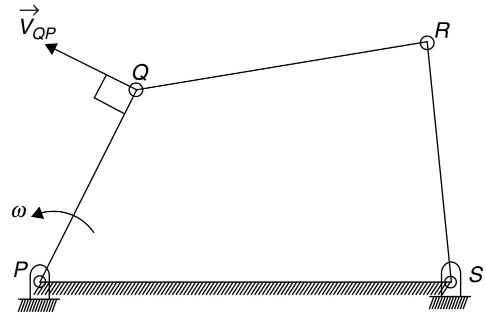
Solution:
The component of V_Q along PQ = component of V_P along PQ ($\because$ PQ is a rigid link)

$\therefore$ $V_Q \cos(45° - 20°) = V_P \cos 20°$

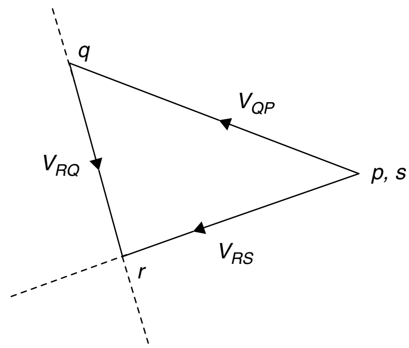
$$\Rightarrow V_P = V_Q \frac{\cos(45° - 20°)}{\cos 20°} = 1 \times \frac{\cos 25°}{\cos 20°}$$

$$= \frac{1 \times 0.9063}{0.9397} = 0.96.$$

Velocity Analysis of a 4-bar Linkage (Quadric Cycle Chain)



Space Diagram



$PQRS$ is the space diagram of the 4-bar linkage and the joints are marked as P, Q, R and S (in capital letters) in the space diagram. The link PQ is the crank, rotating at a constant angular velocity ω in the anti-clockwise direction. PS is the fixed link.

In the velocity diagram, the fixed joints P and S are marked as same point p and s (in small letters). The velocity of joint Q with respect to P is $V_{QP} = \omega(PQ)$ in a direction perpendicular to PQ and in the anti-clockwise sense. In the velocity diagram, with p as centre, a line is drawn which is perpendicular to link PQ and on that line $pq = \omega(PQ)$ is marked to get point q(which represents joint Q). Hence, pq represent V_{QP} in the velocity diagram. The velocity of R relative to Q (V_{QR}) is perpendicular to link QR and the velocity of R relative to $S(V_{RS})$ is perpendicular to link SR. Hence, in the velocity diagram, a line is drawn through point q, which is perpendicular to QR and another line is drawn through s(or p), which is perpendicular to SR. These two lines meet at point r. The velocity of joint R is represented in magnitude and direction by line sr and the relative velocity of R with respect to Q is represented by line qr.

$$\omega_0 = \text{The angular velocity of link } SR \text{ (driven link)} = \frac{sr}{(SR)}$$

(sr is measured from velocity diagram and multiplied by scale of velocity diagram, SR measured from space diagram and multiplied by scale of space diagram)

ω_c = The angular velocity of the coupler

$$QR = \frac{qr}{(QR)}$$

NOTE

The velocity diagram should be started from such a link, whose velocity is known both in magnitude and direction.

Rubbing Velocities at the Pin-Joints of 4-bar Linkage

The **product of the radius of a pin** and the **algebraic difference** between **the angular velocities of the two links that** are connected by the pin-joint, is called the **rubbing velocity at that joint**.

Let ***r* be the radius of pin** for each of the joints P, Q, R and S.

Then, the rubbing velocity at joint $P = r(\omega - 0) = r\omega$

Rubbing velocity at joint $S = r(\omega_0 - 0) = r\omega_0$

Rubbing velocity at joint $Q = r(\omega + \omega_c)$, if ω and ω_c are in opposite sense

$= r(\omega - \omega_c)$, if ω and ω_c are in same sense and $\omega > \omega_c$.

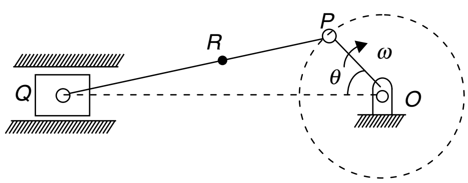
$= r(\omega_c - \omega)$, if ω and ω_c are in same sense and $\omega_c > \omega$

Rubbing velocity at joint $R = r(\omega_c + \omega_0)$, if ω_c and ω_0 are in opposite sense

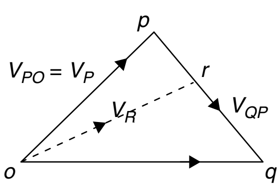
$= r(\omega_c - \omega_0)$, if ω_c and ω_0 are in same sense and $\omega_c > \omega_0$

$= r(\omega_0 - \omega_c)$, if ω_c and ω_0 are in same sense and $\omega_0 > \omega_c$.

Velocity Analysis of a Slider Crank Mechanism



Space Diagram



OP is the crank, rotating at constant angular velocity ω.

Velocity of P relative to $O = V_{PO}$

= absolute velocity of $P = V_P$

$= \omega(OP)$, perpendicular to OP

The joint O in space diagram is marked as joint o in velocity diagram and it represents the fixed end O of crank OP. In the velocity diagram draw line OP to some scale and equal to $\omega(OP)$. Then, the line op represents $\vec{V}_{PO}$ (or $\vec{V}_p$). The velocity of Q relative to P is perpendicular to the connecting rod PQ and the velocity of Q relative to O is along the line QO. Draw a line pq, passing through p and perpendicular to PQ and line oq, passing through O and parallel to OQ. These two lines intersect at point q.

Then, line oq represents velocity of Q relative to o (i.e., $\vec{V}_{QO}$ or $\vec{V}_Q$) to scale. Also line pq represents velocity of Q relative to P.

The angular velocity of connecting rod PQ is given by $\omega_c = \dfrac{pq}{PQ}$

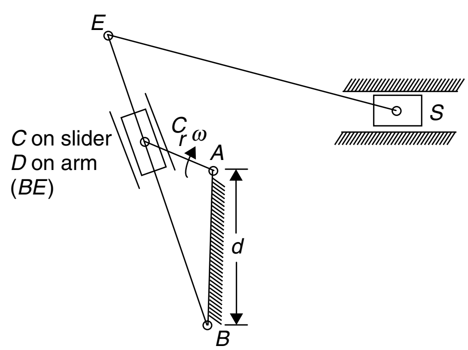
Let R be any point on connecting rod PQ. The velocity of R relative to P (i.e. V_{RP}) can be calculated as

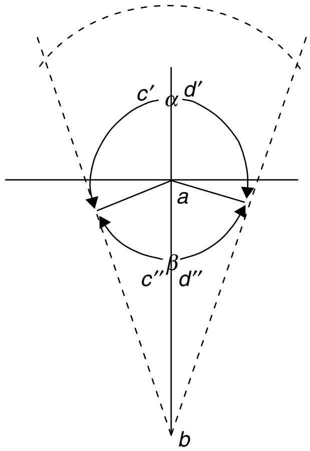
$$V_{RP} = \omega_c(PR) = \left(\frac{pq}{PQ}\right)PR \quad \left(\because \omega_c = \frac{pq}{PQ}\right)$$

$\therefore\ pr = \left(\dfrac{PR}{PQ}\right)pq$ and using this, point r can be located in the velocity diagram. The velocity of R relative to O (i.e. $V_{RO} = V_R$ = absolute velocity of point R) is given by line (or) in the velocity diagram multiplied by scale of velocity diagram.

The rubbing velocities at pins O, P and Q can all be determined in the same manner as done for quadric cyclic mechanism (4-bar linkage)

Velocity analysis of Crank and Slotted Lever Quick Return Mechanism





AB is the fixed link of length *d*. Crank *AC* of length *r* rotates at constant angular velocity ω. *BE* is the slotted lever. The angle turned by the crank during forward (cutting) stroke is α and during return stroke is β. The length of slotted arm $BE = \ell$.

During cutting stroke, let the maximum velocity of slider *S* be V_1. Then, $V_1 = (BE) \times \omega_c$, where ω_c = angular velocity of BE when V_1 occurs and it occurs when *B*, *A* and *C* are along same line.

$V_c = (AC)\omega = r\omega$, treating *C* as a point on crank *AC*. Also,
$V_c = (BC)\omega_c = (BA + AC)\ \omega_c = (d + r)\omega_c$, treating *C* as a point in lever *BE*.

$\therefore\ V_c = \omega_c(d + r) = r\omega$

$$\Rightarrow \omega_c = \frac{r\omega}{(d+r)} \text{ during cutting stroke}$$

$$\therefore\ V_1 = \omega_c(BE) = \frac{r\ell\omega}{(d+r)}$$ is the maximum velocity of slider during cutting stroke

During return stroke, let V_2 be the maximum velocity of slider and it occurs when *B*, *C* and *A* are along same line and ω_c' = angular velocity of slotted lever *BE* when V_1 occurs

We have $V_c = \omega(\text{AC}) = r\omega$ and
$V_c = \omega_c'(\text{BC}) = \omega_c'(BA - AC)$
$= \omega_c'(d - r)$
$\therefore\ V_c = \omega_c'(d - r) = \omega r$

$$\Rightarrow \omega_c' = \frac{\omega r}{(d-r)}$$

$$\therefore\ V_2 = \omega_c'(BE) = \frac{r\omega\ell}{(d-r)}$$

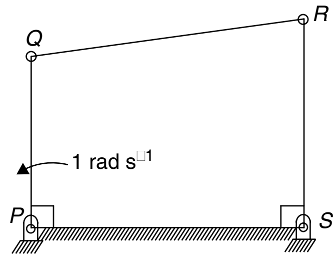
$$\therefore\ \frac{V_{\max}(\text{cutting})}{V_{\max}(\text{return})} = \frac{V_1}{V_2} = \frac{(d-r)}{(d+r)}$$

$$\text{Quick return ratio} = \frac{\text{Time of cutting}}{\text{Time of return}}$$

$$= \frac{(\alpha/\omega)}{(\beta/\omega)} = \frac{\alpha}{\beta}$$

$$= \frac{(2\pi - \beta)}{\beta}.$$

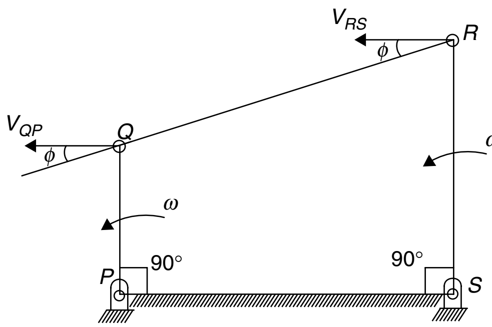
Example 53:



For the 4-bar linkage shown in figure, the angular velocity of link *PQ* is 1 rad s^{-1}. The length of link *SR* is 2.5 times the length of link *PQ*. In the configuration shown, the angular velocity of link *SR* (in rad s^{-1}) is

(A) 0.67 (B) 2.5 (C) 0.4 (D) 1.25

Solution:



$V_{QP} = \omega(PQ)$, ⊥ to *PQ*
$V_{RS} = \omega_1(SR) = \omega_1\ (2.5PQ)$
$= 2.5\omega_1(PQ)$, ⊥ to *SR*

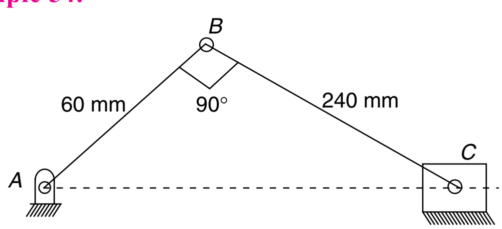
As *PQ* and *SR* are parallel, V_{QP} and V_{RS} are also parallel
But $V_{QP} \cos\phi = V_{RS}\cos\phi$ ($\because$ link *QR* is rigid)
$\Rightarrow V_{QP} = V_{RS}$

$\therefore\ \omega(PQ) = \omega_1(2.5\ PQ)$

$$\Rightarrow \omega_1 = \frac{\omega}{2.5} = \frac{1}{\left(\frac{5}{2}\right)} = \frac{2}{5} \text{ rad s}^{-1}$$

$$\therefore \text{ Angular velocity of link } SR = \frac{2}{5} \text{ rad s}^{-1}$$
$$= 0.4 \text{ rad s}^{-1}$$

Example 54:



A slider-crank mechanism with crank radius 60 mm and connecting rod length 240 mm is shown in figure. The crank is rotating with a uniform angular speed of 20 rad s^{-1} clockwise. For the given configuration, the speed (in ms^{-1}) of the slider is ___________ and speed of c with respect to B (in ms^{-1}) is ___________ . Fill up the blanks.

Solution: Velocity of B relative to A

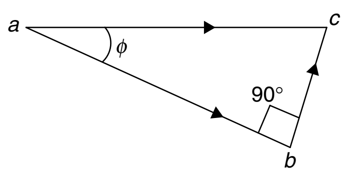
$V_{BA} = \omega_{BA}(BA)$ [BA = 60 mm = 0.06 m]

$= 20 \times 0.06$

$= 1.2$ ms^{-1}, along BC as $BC \perp AB$

$$\angle BCA = \phi \text{ and } \tan\phi = \frac{60}{240} = \frac{1}{4}$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{1}{4}\right) = 14.04°$$



$ab = V_{BA} = 1.2$ ms^{-1}

$ac = V_{CA}$ = velocity of slider

$$\frac{ab}{ac} = \frac{1.2}{V_{CA}} = \cos\phi$$

$$\Rightarrow V_{CA} = \frac{1.2}{\cos\phi} = \frac{1.2}{\cos 14.04°}$$

$$\therefore \text{Velocity of slider} = \frac{1.2}{0.9701}$$

$$= 1.237 \text{ ms}^{-1}$$

$$bc = V_{CB}$$

$$\frac{V_{CB}}{V_{BA}} = \frac{bc}{ab} = \tan\phi$$

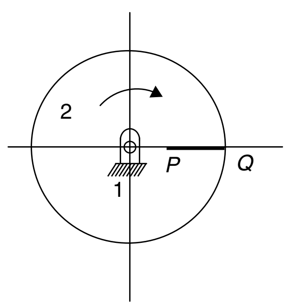
$\Rightarrow V_{CB} = V_{BA}\tan\phi = 1.2 \times \tan 14.04°$

$= 1.2 \times 0.25$

$= 0.3$ ms^{-1}

Hence, the speed of slider is 1.237 ms^{-1} and speed of C with respect to B is 0.3 ms^{-1}.

Example 55:



The speeds of two points P and Q, located along the radius of a wheel rotating at constant angular velocity as shown in figure, are 120 ms^{-1} and 200 ms^{-1} respectively. The distance between the points P and Q is 400 mm. The radius of the wheel (in mm) is ___________ and the angular velocity of wheel (in rad s^{-1}) is ___________ . Fill up the blanks.

Solution: $V_P = 120$ ms^{-1}; $V_Q = 200$ ms^{-1} in same sense as P

$\therefore V_{QP}$ = velocity of Q relative to $P = V_Q - V_P$

$= 200 - 120 = 80$ ms^{-1};

$PQ = 400$ mm $= 0.4$ m

$$\omega_{PQ} = \frac{V_{QP}}{PQ} = \frac{80\,\text{ms}^{-1}}{0.4\,\text{m}} = 200 \text{ rad s}^{-1}$$

Let radius be R mm

$$V_Q = \frac{\omega_{PQ}R}{1000}$$

$$V_P = \frac{\omega_{PQ}(R-400)}{1000}$$

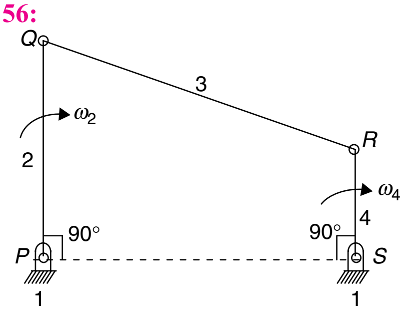
$$\therefore \frac{V_Q}{V_P} = \frac{R}{R-400} \Rightarrow \frac{200}{120} = \frac{R}{(R-400)}$$

$\Rightarrow 5R - 2000 = 3R \Rightarrow 2R = 2000$

$$R = \frac{2000}{2} = 1000 \text{ mm}$$

$\therefore$ Radius of wheel is 1000 mm and its angular velocity is 200 rad s^{-1}.

Example 56:



A 4-bar mechanism is as shown in figure. The following statements pertain to the instant considered.

(i) $\omega_2 = \omega_4$

(ii) $V_{QR} = (\omega_4 - \omega_2)QR$

(iii) Velocity diagram is a straight line

(iv) Link QR undergoes pure translation

The correct statements are

(A) (i), (ii), (iii) and (iv) (B) (i) and (ii) only

(C) (ii) and (iii) only (D) (iii) and (iv) only

Solution:

V_{QP} parallel to V_{RS} as PQ and SR are perpendicular to fixed link. As link QR is rigid, component of V_{QR} along QR is same as component of V_{RS} along QR.

$\Rightarrow V_{QP} = V_{RS}$

$\therefore PQ(\omega_2) = (SR)\omega_4$

$\Rightarrow \frac{\omega_2}{\omega_4} = \frac{SR}{PQ} \neq 1 \Rightarrow$ (i) is wrong

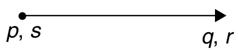
As $V_{QP} = V_{RS}$, components of V_{QP} and V_{RS} normal to PQ are same

$\Rightarrow V_{QR} = 0 \Rightarrow$ (ii) is wrong

$\therefore$ Link QR undergoes pure translation $\Rightarrow$ (iv) is correct

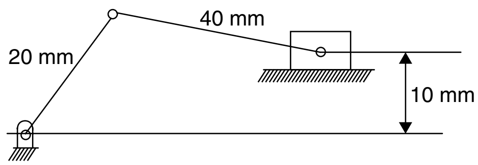
The velocity diagram is as shown below:

$\Rightarrow$ (iii) is correct



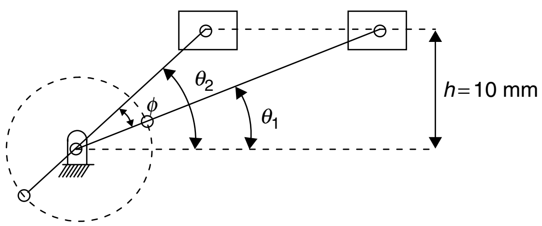
$\therefore$ statement (iii) and (iv) are correct.

Example 57:



An off-set slider crank mechanism is shown in the figure at an instant. Conventionally, the quick return ratio (QRR) is considered to be greater than one. The value of QRR is ________________ . Fill up the blank.

Solution: Two extreme positions (extended dead centre and folded dead centre positions) are shown in the figure



Off set, $h = 10$ mm

Crank radius, $r = 20$ mm

Length of connecting rod, $\ell = 40$ mm

$$\theta_1 = \sin^{-1}\left(\frac{h}{\ell + r}\right) = \sin^{-1}\frac{10}{(40+20)}$$

$$= \sin^{-1}\left(\frac{1}{6}\right) = 9.6°$$

$$\theta_2 = \sin^{-1}\left(\frac{h}{\ell - r}\right) = \sin^{-1}\left(\frac{10}{40-20}\right)$$

$$= \sin^{-1}\left(\frac{1}{2}\right) = 30°$$

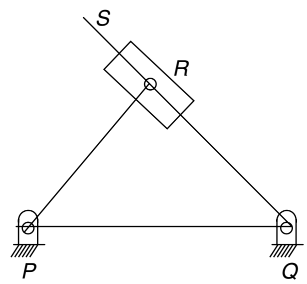
$\therefore \phi = \theta_2 - \theta_1 = 30° - 9.6° = 20.4°$

$$\text{Quick return ratio} = \frac{180° + \phi}{180° - \phi} = \frac{180° + 20.4°}{180° - 20.4°}$$

$$= \frac{200.4°}{159.6°} = 1.256$$

$\therefore$ QRR = 1.256.

Example 58:



$PR = 250$ mm

$QR = 250\sqrt{3}$ mm

$PQ = 500$ mm

For the configuration shown, the angular velocity of link PR is 10 rad s^{-1} counter clockwise. The length of links are as shown. The magnitude of the relative sliding velocity (in ms^{-1}) of slider R with respect to rigid link QS is

(A) 2.50 (B) 0.86 (C) 0 (D) 1.25

Solution:

We have $PQ^2 = 500^2$

$$QR^2 + PR^2 = 250^2 + (250\sqrt{3})^2$$
$$= 250^2 + 250^2 \times 3$$
$$= (250)^2 \times [1 + 3]$$
$$= 250^2 \times 2^2$$
$$= 500^2$$

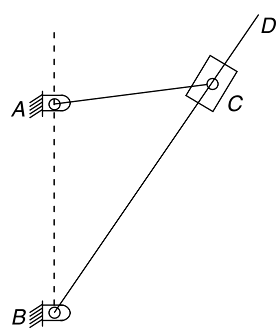
$\therefore PQ^2 = QR^2 + PR^2$

$\Rightarrow$ configuration shown is a right angled triangle and crank PR is perpendicular to lever QS. Hence, velocity of R is along QS only which is purely sliding component.

$\therefore$ velocity of slider $= PR \times \omega_{PR}$

$$= 250 \text{ mm} \times 10 \text{ rad s}^{-1}$$
$$= 2500 \text{ mm/s}$$
$$= 2.5 \text{ m/s}$$

Example 59:

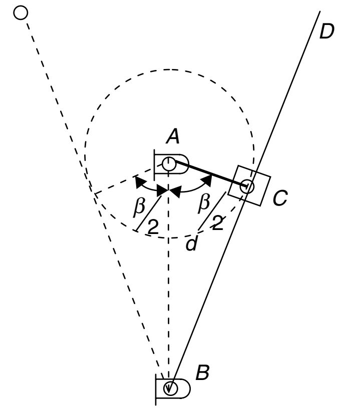


A quick return mechanism is shown. $AB = 6$ cm is the fixed link.

Crank AC = 2 cm rotates anti-clockwise with constant angular velocity. The ratio of time for forward motion to that for return motion is

(A) 1.414 (B) 1.552 (C) 1.732 (D) 3

Solution:



$$\cos\left(\frac{\beta}{2}\right) = \frac{AC}{AB} = \frac{2}{6} = \frac{1}{3}$$

$$\Rightarrow \frac{\beta}{2} = \cos^{-1}\left(\frac{1}{3}\right) = 70.53°$$

$$\therefore\ \beta = 2 \times 70.53° = 141.06°$$

$$\therefore \text{ Quick return ratio} = \frac{t_f}{t_r} = \frac{(360° - \beta)}{\beta}$$

$$= \frac{360° - 141.06°}{141.06°}$$

$$= \frac{218.94°}{141.06°} = 1.552.$$

Example 60:

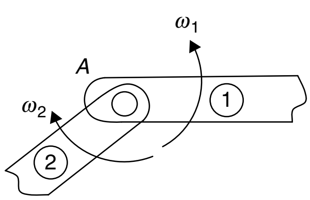


Figure shows link 1 and link 2 connected by a revolute joint A. The diameter of the pin at A is 50 mm. If link 1 is rotating at an angular velocity ω_1 = 10 rad s^{-1} and link 2 is rotating at an angular velocity ω_2 = 25 rad s^{-1} in opposite sense as shown, what is the rubbing velocity at joint A? If both links were rotating in the same sense, what will be the rubbing velocity at pin A?

Solution: For links rotating in the opposite sense, rubbing velocity

$$V = \text{pin radius} \times \text{relative angular velocity}$$

$$= \left(\frac{50}{2}\right) \text{mm} \times (\omega_1 + \omega_2)$$

$$= 25 \times (10 + 25) = 25 \times 35$$

$$= 875 \text{ mm/s} = 0.875 \text{ m/s}$$

If links are rotating in same sense, rubbing velocity

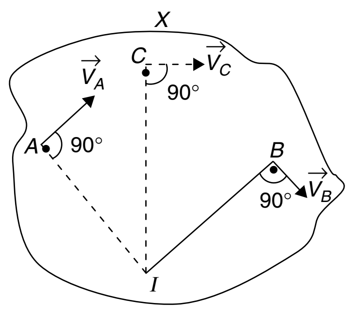
$$= r\,(\omega_2 - \omega_1)$$

$$= \frac{50}{2} \times (25 - 10) = 25 \times 15$$

$$= 375 \text{ mm/s} = 0.375 \text{ m/s}.$$

Velocity Analysis of Mechanisms by Instantaneous Centre Method (I centre)

When a rigid body is having a general planar motion (i.e. having both translational and rotational motion in a plane), at any instant, this body can be considered to be in pure rotation (with angular velocity ω) about a zero velocity point, called instantaneous centre.



Consider a rigid body X having both translational and rotational motion in a plane. At the instant shown, point A on the rigid body has velocity $\vec{V}_A$ and point B on the rigid body has velocity $\vec{V}_B$. If the body X is considered to be in pure rotation about a point at that time, that point should be lying on the radius vector at A and B i.e. along direction perpendicular to $\vec{V}_A$ and $\vec{V}_B$. If we draw a perpendicular to $\vec{V}_A$ through A and a perpendicular to $\vec{V}_B$ though B, these lines intersect at the instantaneous centre (I).

Then, the angular velocity ω of body about I is given by

$$\omega = \frac{V_A}{(IA)} = \frac{V_B}{(IB)}$$

where

IA = distance between I and A

IB = distance between I and B

Now, the velocity of any point C on the rigid body at that instant can be found as follows:

V_c = radial distance from I to C, multiplied by ω

$$= (IC)\omega$$

$$= (IC)\,\frac{V_A}{(IA)} \text{ or } (IC)\,\frac{V_B}{(IB)}$$

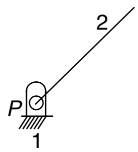
NOTE

If $\vec{V}_A$ and $\vec{V}_B$ are parallel, the instantaneous centre of body lies at infinity.

Types of Instantaneous Centres

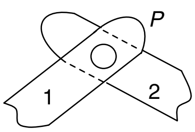
There are three types of instantaneous centres

1. **Fixed instantaneous centres**
   The centres of pin joints that remain at same place for all configurations of a mechanism are called **fixed instantaneous centres**.



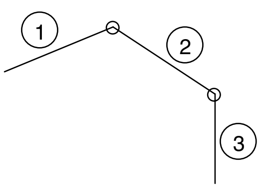
   For example, a fixed link 1 is connected to another link 2, through a revolute joint P, The centre of pin at P is a fixed instantaneous centre as joint P remains at same place for all configurations of link 2. It is called I_{12} or I_{21}.

2. **Permanent instantaneous centres**
   The centres of pin joints in a mechanism where the joints are of permanent nature but the joints also move when the mechanism moves, are called as **permanent instantaneous centres**.



   For example, when link 1 and 2 are connected through a pin joint P and both links 1 and 2 can move (so that joint P also moves), then the centre of pin at P is a **permanent instantaneous centre** (I_{12} or I_{21})

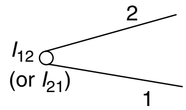
3. **Neither fixed nor permanent instantaneous centres**
   When two links are not directly joined together but have relative motion with respect to each other, then their instantaneous centre is neither fixed nor permanent but keeps on changing positions for different configuration of the mechanism. The instantaneous centres of such links are called as **neither fixed nor permanent instantaneous centres.**



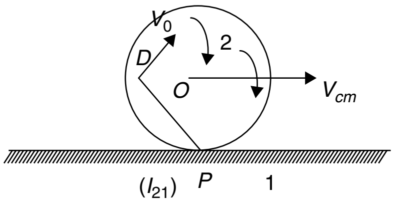
   For example, The instantaneous centre I_{13} (or I_{31}) of links 1 and 3 (of part of mechanism shown in figure) will be neither fixed nor permanent instantaneous centre.

Locations of Instantaneous Centres

1. The instantaneous centre of two links which are connected by a pin will be always at the centre of pin. This can be either fixed type or permanent type.

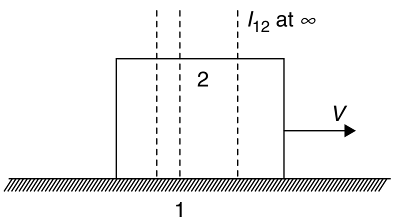


2. When one link rolls over another fixed link without slip (i.e. pure rolling), the point of contact will be the instantaneous centre for the moving link

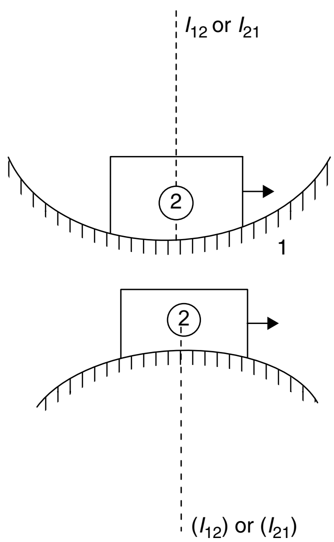


   Velocity of point D is $V_D = (PD)\omega$ where $\omega = \dfrac{V_{cm}}{PO}$, where O is the centre of mass of rolling body and V_{cm} = velocity of centre of mass

3. For object **sliding on a plane fixed surface, the instantaneous centre will be at infinity**, in a direction normal to the sliding surface.



4. For object sliding over fixed convex or concave surfaces, the instantaneous centre will be on the normal to the contact surface (ie along the radial direction)



Aaronhold Kennedy's Theorem (or Three Centres-in-line Theorem)

According to Kennedy's theorem, when three links move relative to each other, their instantaneous centres will lie on a straight line.

Number of Instantaneous Centres in a Mechanism

If there are n binary links in a mechanism, the total number of instantaneous centres (N) is given by

$$N = \frac{n(n-1)}{2}$$

Example 61: A mechanism has 5 binary links. The total number of instantaneous centres of the mechanism is ___________ . Fill up the blank.

Solution: No. of binary links $n = 5$

∴ Total number of instantaneous centres,

$$N = \frac{n(n-1)}{2}$$

$$= \frac{5\times(5-1)}{2} = 10$$

Example 62: A mechanism has 8 links, out of which 5 are binary, 2 are ternary and 1 is quaternary. The number of instantaneous centres of rotation will be

(A) 28 (B) 56 (C) 62 (D) 66

Solution:

$n_1 = 5$ (no. of binary links)

No. of ternary links, $n_2 = 2$

$= 2(3 - 1) = 4$ binary link

No. of quaternary links, $n_3 = 1$

$= 1(4 - 1) = 3$ binary links

∴ Total number of binary links,

$n = n_1 + n_2 + n_3$

$= 5 + 4 + 3 = 12$

∴ Number of instantaneous centres,

$$N = \frac{n(n-1)}{2}$$

$$= \frac{12\times(12-1)}{2} = 66$$

Centrodes and Axodes

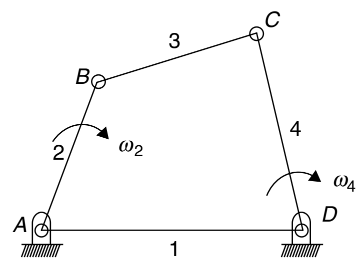
The position of instantaneous centre of a link, in general, changes with the motion of the link. The locus of the instantaneous centre of a particular link is called **centrode**. It is a line (**which is either straight or curved**). For fixed instantaneous centres, it is a point.

The line passing through the instantaneous centre and perpendicular to the plane of motion is called instantaneous axis. The position of instantaneous axis changes throughout the motion. The locus of instantaneous axis of a link for the whole motion is called **axode**. It is a **surface** (**either plane or curved**). For fixed instantaneous axis, it is a straight line.

Procedure for Locating Instantaneous Centres of Mechanism

1. Determine the number of instantaneous centres (N) using the relation $N = \frac{n(n-1)}{2}$, where n = number of binary links.
2. Tabulate the list of instantaneous centres (i.e. prepare a table)
3. By observation, locate fixed and permanent instantaneous centres
4. The remaining instantaneous centres (neither fixed nor permanent) can be located by using Kennedy's theorem and consider a set of three links.

Instantaneous Centre Analysis of 4-bar Linkage



A 4-bar linkage $ABCD$ with link AD(Link 1) fixed is shown. We have to determine the instantaneous centres. Crank AB (Link 2) is given input angular velocity ω_2. We have to determine the angular velocity of driven link $DC(=\omega_4)$ and the coupler $BC(=\omega_3)$

Step 1:

n = no. of binary links = 4

∴ Total number of instantaneous centres,

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6.$$

Step 2:

Link	1	2	3	4
Instantaneous Centres	I_{12}	I_{23}	I_{34}	-
	I_{13}	I_{24}	-	-
	I_{14}	-	-	-

Step 3:

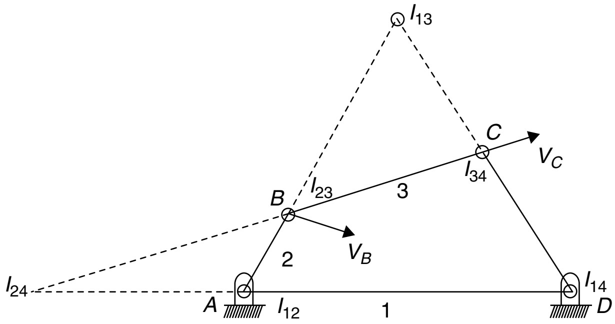
By inspection, joint A is I_{12} (same as I_{21}) and joint D is I_{14} (same as I_{41}) both are fixed instantaneous centres.

By inspection, joint B is I_{23} (same as I_{32}) and joint c is I_{34} (same as I_{43}) and both are permanent instantaneous centres.

Step 4:

We have to now find the instantaneous centres I_{13} and I_{24} which are neither fixed nor permanent instantaneous centres. For this we apply Kennedy's theorem to links 1, 2 and 3. Accordingly, I_{12}, I_{23} and I_{13} shall lie on the same straight line. Hence, extend line AB. Then, I_{13} will lie on AB extended.

Next, consider links 1, 3 and 4. According to Kennedy's theorem, I_{13}, I_{14} and I_{34} will lie on same straight line. Hence, extend line DC till it intersects AB extended at I_{13} (which is common on line AB extended and line DC extended). To determine I_{24}, consider links 1, 2 and 4 and links 2, 3 and 4. I_{24} will lie at the intersection point of DA extended and CB extended. The instantaneous centres are as shown.



Velocity of B, $V_B = (AB)\omega_2$, perpendicular to AB as B is a point on link AB.

$$V_B = (I_{12}I_{23})\omega_2$$

Also treating B as a point on link 3,

$V_B = (I_{13}B)\omega_3 = (I_{13}I_{23})\omega_3$

$\Rightarrow (I_{12}I_{23})\omega_2 = (I_{13}I_{23})\omega_3$

$$\Rightarrow \frac{\omega_2}{\omega_3} = \frac{(I_{13}I_{23})}{(I_{12}I_{23})} \Rightarrow \omega_3 = \omega_2 \frac{(I_{12}I_{23})}{(I_{13}I_{23})}$$

Similarly, velocity of c is given by

$V_c = (I_{14}I_{34})\omega_4 = (I_{13}I_{34})\omega_3$

$$\Rightarrow \omega_4 = \frac{(I_{13}I_{34})}{(I_{14}I_{34})}\omega_3$$

Thus, the angular velocities of all links can be determined. Here ω_2 = angular velocity of link 2 with respect to 1 = ω_{21}

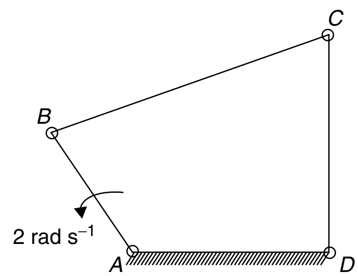
$$\omega_3 = \text{angular velocity of link 3}$$

with respect to 1 = ω_{31} and ω_4 = angular velocity of link 4 with respect to 1 = ω_{41}. Hence, we can now state the **Angular velocity ratio theorem** as follows. The angular velocity ratio of two links (say 2 and 3) relative to a third link (say 1) is inversely proportional to the distance of their common instantaneous centre (I_{23} in this case) from their respective centres of rotation (i.e. I_{12} and I_{13} in this case)

$$\therefore \frac{\omega_{21}}{\omega_{31}} = \frac{\omega_2}{\omega_3} = \frac{I_{13}I_{23}}{I_{12}I_{23}}$$

Also $\dfrac{\omega_{21}}{\omega_{41}} = \dfrac{\omega_2}{\omega_4} = \dfrac{I_{14}I_{24}}{I_{12}I_{24}}$ using angular velocity ratio theorem.

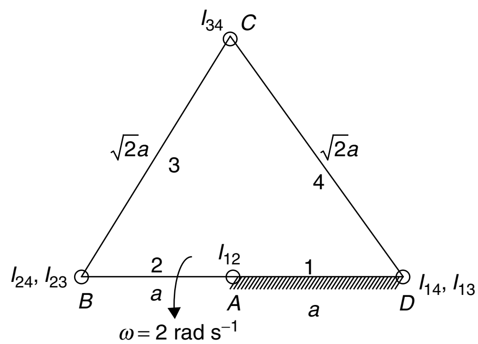
Example 63:



The input link AB of a 4-bar linkage is rotated at 2 rad s^{-1} in the counter clockwise direction as shown. The length of links are $AB = AD = a$ and $BC = DC = \sqrt{2}a$ respectively. At the instant when $\angle DAB = 180°$, the angular velocity of coupler BC in rad s^{-1} is

(A) $\dfrac{1}{\sqrt{2}}$ (B) 1 (C) $2\sqrt{2}$ (D) 4

Solution:



The configuration is shown above. I_{12}, I_{23} and I_{13} lie on same line.

Also I_{14}, I_{34} and I_{13} lie on the same line

$\Rightarrow$ joint D is both I_{14} and I_{13}

Similarly I_{23}, I_{34} and I_{24} lie on a line and I_{12}, I_{13} and I_{23} lie on a line

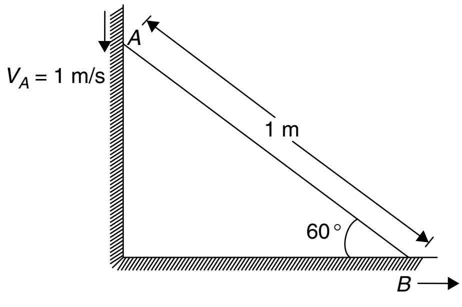
$\Rightarrow$ Joint B is both I_{24} and I_{23}

As per angular velocity ratio theorem,

$$\frac{\omega_3}{\omega_2} = \frac{\omega_{31}}{\omega_{21}} = \frac{I_{12}I_{23}}{I_{13}I_{23}} = \frac{a}{2a} = \frac{1}{2}$$

$$\Rightarrow \omega_3 = \frac{\omega_2}{\omega} = \frac{2}{2} = 1 \text{ rad s}^{-1}.$$

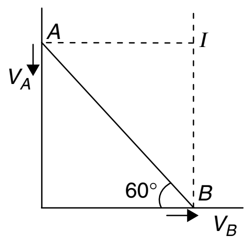
Example 64:



A rigid rod *AB* of length 1m is sliding on a smooth wall and smooth floor at a corner as shown in figure. At the instant when the rod makes an angle of 60° with the horizontal plane, the velocity of point *A* on the rod is 4 m/s. The angular velocity of the rod and velocity of end *B* at that instant are respectively

(A) 8 rad s^{-1}, $4\sqrt{3}$ ms^{-1} (B) $4\sqrt{3}$ rad s^{-1}, 8 ms^{-1}
(C) 4 rad s^{-1}, 2 ms^{-1} (D) 4 rads^{-1}, $4\sqrt{3}$ ms^{-1}

Solution:
By drawing a normal to V_A and V_B and locating the intersection point, the instantaneous centre of rotation *I* of the rod *AB* is determined.



Let the angular velocity of rod *AB* about *I* at that instant be ω.

Then, $V_A = (IA)\omega$

$$= (AB\cos 60°)\omega$$

$$= 1 \times \frac{1}{2} \times \omega$$

$$\Rightarrow V_A = \frac{\omega}{2}; V_A = 4\ ms^{-1}$$

$$\Rightarrow \omega = 2\ V_A = 2 \times 4 = 8 \text{ rad } s^{-1}$$

$$V_B = (IB)\omega = (AB\sin 60°)\omega$$

$$= 1 \times \frac{\sqrt{3}}{2} \times \omega$$

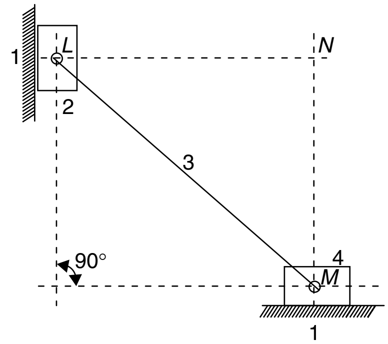
$$\Rightarrow V_B = \frac{\sqrt{3}\omega}{2}$$

$$\therefore \frac{V_B}{V_A} = \sqrt{3}$$

$$\Rightarrow V_B = \sqrt{3}V_A = 4\sqrt{3}\ ms^{-1}$$

At that instant, angular velocity of rod = 8 rad s^{-1} and velocity of end *B* is $4\sqrt{3}$ ms^{-1}

Example 65:



The figure shows a planar mechanism with single degree of freedom. The instantaneous centre I_{24} for the given configuration is located at position

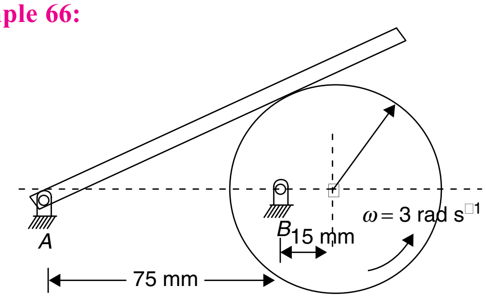
(A) L (B) M (C) N (D) ∞

Solution:
The instantaneous centre I_{14}, I_{12} and I_{24}, lie on the same line. (A. Kennedy's theorem for links 1, 2 and 4)
I_{14} is at ∞ as 4 slides on plane surface 1
I_{12} is also at ∞ as 2 slides on plane surface 1
∴ I_{24} also must be at ∞.

NOTE

Joint *M* is I_{34} and joint *L* is I_{23}. As per Kennedy's theorem, I_{34}, I_{14} and I_{13} lie on a straight line and I_{12}, I_{23} and I_{13} also lie on a straight line. Hence, I_{13} is the intersection point of line connecting I_{14}, I_{13} and I_{34} with line connecting I_{12}, I_{23} and I_{13} which is point *N*. Hence, *N* represents I_{13}.

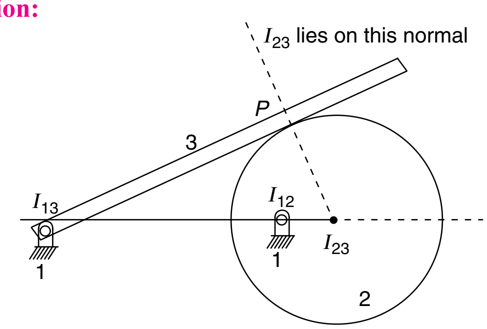
Example 66:



In the mechanism shown, if the angular velocity of the eccentric circular disc is 3 rad s^{-1}, the angular velocity in rad s^{-1} of the follower link for the instant shown in figure is

(A) 1.5 (B) 1.0 (C) 0.5 (D) 6.0

Solution:



There are 3 links (1, 2 and 3). I_{13} and I_{12} are fixed instantaneous centres. *P* is point of contact between links 2 and 3.

Hence, I_{23} lies on the normal at P(as the disc is having contact with the follower link, the velocity of disc (2) relative to follower (3) is only tangent to the follower. i.e. the disc slips on the follower ⇒ [instantaneous centre is normal to the follower at the point of contact]. As per Kennedy's theorem, I_{12}, I_{13} and I_{23} lie on the same line
⇒ the centre of disc is I_{23}.

$$\text{Now } \frac{\omega_{31}}{\omega_{21}} = \frac{\omega_3}{\omega_2}$$

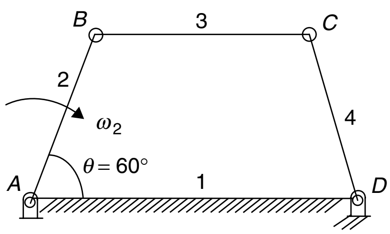
$$= \frac{I_{12}I_{23}}{I_{13}I_{23}}$$

$$= \frac{15 \text{ mm}}{(75+15) \text{ mm}} = \frac{1}{6}$$

$$\therefore \omega_3 = \frac{\omega_2}{6} = \frac{3}{6} = \frac{1}{2} \text{ rad s}^{-1} = 0.5 \text{ rad s}^{-1}$$

∴ Angular velocity of follower
= 0.5 rad s^{-1}.

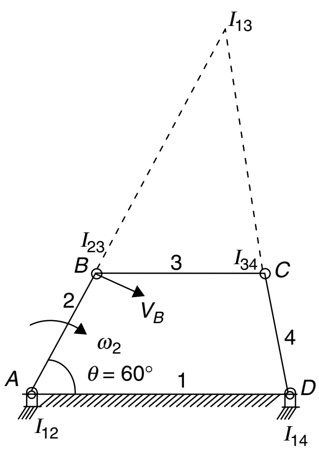
Example 67:



The angular velocity of link AB in the configuration shown is ω_2 = 12 rad s^{-1} clockwise and the magnitude of the angular velocity of link BC (= ω_3) is 8 rad s^{-1}.

The magnitude and direction of relative angular velocity of link 3 (BC) with respect to link 2 (AB) is
(A) 20 rad s^{-1} in the clockwise direction.
(B) 20 rad s^{-1} in the counter clockwise direction.
(C) 4 rad s^{-1} in the clockwise direction.
(D) 4 rad s^{-1} in the counter clockwise direction.

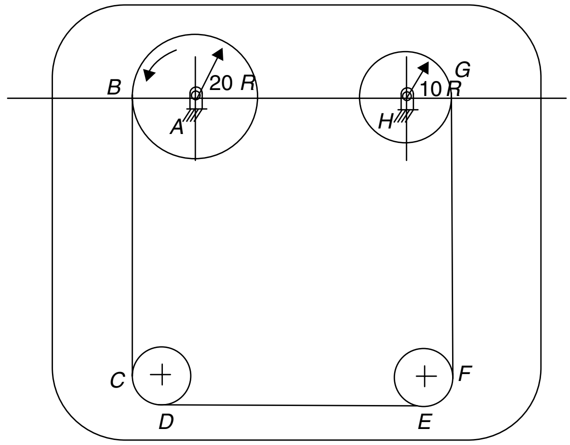
Solution:



The instantaneous centres are as marked. I_{24} is at ∞ as BC and AD are parallel. With respect to A, point B moves clockwise ⇒ $\vec{\omega}_2 = -12 \text{ rad s}^{-1}$
(clockwise is taken as negative)
with respect to I_{13}, point B moves anticlockwise
⇒ $\vec{\omega}_3 = +8 \text{ rad s}^{-1}$ (anticlockwise is taken as positive)

$\therefore \vec{\omega}_{32} = \vec{\omega}_3 - \vec{\omega}_2 = 8 - (-12)$
= + 20 rad s^{-1}
⇒ Angular velocity of link 3 with respect to link 2 is 20 rad s^{-1} in the counter clockwise direction.

Example 68:



For the audio cassette mechanism shown in figure, where is the instantaneous centre of rotation (Point P) of the two spools?
(A) Point P lies to the left of both the spools but at infinity along the line joining A and H.
(B) Point P lies in between the two spools on the line joining A and H, such that $PH = 2AP$
(C) Point P lies to the right of both the spools on the line joining A and H, such that $AH = HP$
(D) Point P lies at the intersection of the line joining B and C and the line joining B and F

Solution:
Frame is link 1 (fixed link), spool A is link 2 and spool H is link 3.
No. of instantaneous centres,

$$N = \frac{3\times(3-1)}{2} = 3$$

Point A is I_{12} and point H is I_{13}. As per Kennedy's Theorem I_{23} must lie on line joining A and H
⇒ option (d) is not correct.
For no slip, the tape must have same peripheral speed at B and G
⇒ $V_B = V_G$
⇒ $(20R)\omega_A = (10R)\omega_H$

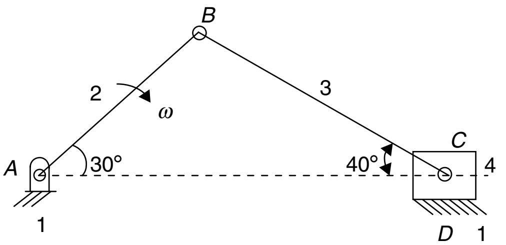
$$\Rightarrow \frac{\omega_H}{\omega_A} = \frac{20R}{10R} = 2$$

[ω_A = angular velocity of spool A
ω_H = angular velocity of spool H]
Also, both A and H rotate in the same sense (i.e. anti clockwise)
As per angular velocity ratio theorem,

$$\frac{\omega_H}{\omega_A} = \frac{\omega_3}{\omega_2} = \frac{I_{12}I_{23}}{I_{13}I_{23}} = \frac{AP}{HP} = 2$$

$\Rightarrow AP = 2HP \Rightarrow (B)$ is wrong
Also, point P cannot be to the left of both spools because in such a case AP will be less than HP $\Rightarrow$ (a) is wrong. Hence, (C) is the only correct option.

Example 69:



Mark the instantaneous centres of the slider crank mechanism shown above. $AB = 50$ mm, $\angle BAC = 30°$, $\angle BCA = 40°$

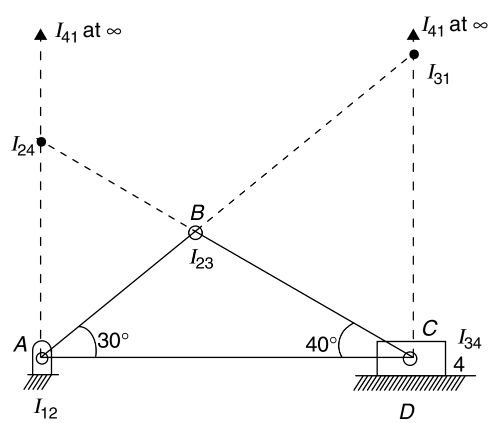
Solution: $n = 4$ binary links

$$\therefore N = \frac{n(n-1)}{2} = \frac{4 \times 3}{2} = 6$$

Link	1	2	3	4
Instantaneous centres	I_{12}	I_{23}	I_{34}	-
	I_{13}	I_{24}	-	-
	I_{14}	-	-	-

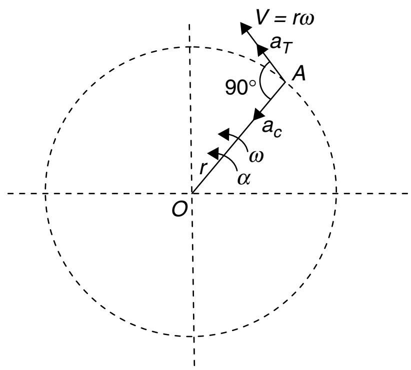
Joint A is I_{12}, joint B is I_{23} and joint C is I_{34}. As link 4 slides on link 1, I_{14} is normal to the surface and at infinity. i.e. s I_{14} at ∞ ($I_{14} = I_{41}$)

I_{13} and I_{24} are located using Kennedy's Theorem.



Acceleration Analysis in Mechanisms

For a body in motion, the change of position is called **displacement** and the time rate of displacement is called **velocity**. The time rate of change of velocity is called **acceleration**. The direction of acceleration is **along the direction of change of velocity**.



Consider a particle moving in a circular path with the centre at O and radius equal to r. At an instant of time, the particle is at point A where its position vector is $\overrightarrow{OA}$ ($= \vec{r}$) and velocity vector is $\vec{V}$ tangential to the circle at A. Let us consider that at this instant the angular velocity and angular acceleration of the particle are ω and α respectively. We can show that the velocity of particle $\vec{V} = \vec{\omega} \times \vec{r}$ and $V = \omega r$ (magnitude) and $\vec{V} \perp \vec{r}$.

The acceleration of the particle at this instant is $\vec{a} = \frac{d\vec{V}}{dt}$

i.e. $\vec{a} = \frac{d\vec{V}}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r}) = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} + \frac{d\vec{r}}{dt}$

$= (\vec{\alpha} \times \vec{r}) + (\vec{\omega} \times \vec{V})$

$$\left[\because \frac{d\vec{\omega}}{dt} = \vec{\alpha} \text{ and } \frac{d\vec{r}}{dt} = \vec{V}\right]$$

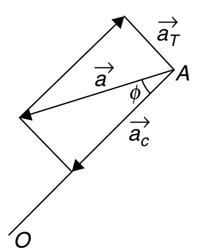
i.e. $\vec{a} = \vec{a}_T + \vec{a}_c$

where $\vec{a}_T = \vec{\alpha} \times \vec{r}$ (or $a_T = \alpha r$ as $\vec{\alpha} \perp \vec{r}$) is the **tangential component of acceleration** of the particle and $\vec{a}_c = \vec{\omega} \times \vec{v}$ (or $a_c = \omega$ variables as $\vec{\omega} \perp \vec{v}$) is the **centripetal or radial component of acceleration** of the particle.

$a_c = \omega v = \omega(\omega r) = \omega^2 r = \frac{V^2}{r}$ is the centripetal acceleration.

$\therefore a = \sqrt{{a_T}^2 + {a_c}^2} = \sqrt{\alpha^2 r^2 + r^2\omega^4}$

$\Rightarrow a = r\sqrt{\alpha^2 + \omega^4}$

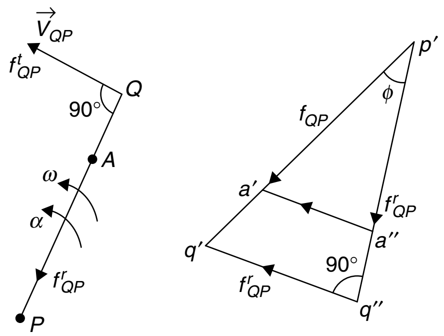


Angle made by acceleration $\vec{a}$ with $\overrightarrow{OA}$ is ϕ such that

$$\tan\phi = \frac{a_T}{a_c} = \frac{\alpha r}{\omega^2 r} = \frac{\alpha}{\omega^2}$$

The radial component of acceleration is due to angular velocity (i.e. a_c is due to ω) and tangential component of acceleration is due to angular acceleration. If there is **no angular acceleration**, there will be no change in the speed of particle and hence **no tangential acceleration**. The particle will have only centripetal acceleration in such case.

Acceleration Diagram for a Rigid Link



Consider a rigid link PQ (of length r) in which point Q is moving with respect to point P. Clearly, with respect to P, end Q will have only rotational motion with P as fixed centre. At a particular instant,

ω = angular velocity of link PQ

α = angular acceleration of link PQ

Acceleration of Q with respect to P $\left(\bar{f}_{QP}\right)$ will have two components.

1. The radial component (or centripetal component is) due to angular velocity ω. This acts along QP (or parallel to QP) and is directed from Q towards P. Its magnitude is $\omega^2 r$ $\left(\text{or } \frac{V^2}{r}\right)$

   $$\therefore\ f^r_{QP} = \omega^2 \times QP = \omega^2 r \ \ (QP = r)$$

   $$\text{or}\ \ f^r_{QP} = \frac{V^2_{QP}}{QP} = \frac{V^2_{QP}}{r}$$

2. The tangential component is due to angular acceleration (α). This acts parallel to velocity of Q and it is perpendicular to PQ. The magnitude of this component is $\alpha \times QP = \alpha r$

   $$\therefore\ f^t_{QP} = \alpha \times QP = \alpha r$$

   $\therefore$ The total acceleration of Q with respect to P is given by f_{QP} = vector sum of $f^r_{QP} + f^t_{QP}$.

NOTE

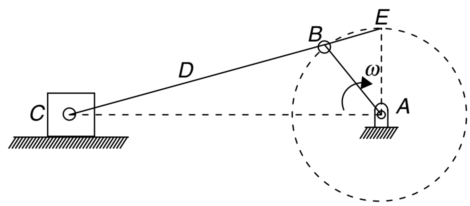
If the point Q is rotating with constant angular velocity ω with respect to P, then the angular acceleration α is zero and Q will have only centripetal acceleration.

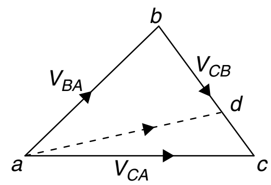
To draw the acceleration diagram, mark the fixed point (P is space diagram) as p' in the acceleration diagram. From p' draw a line parallel to QP and of magnitude $f^r_{QP} = \frac{V^2_{QP}}{QP}$ (or $\omega^2\ QP$). That point is marked as q'' in the acceleration diagram. From q'', draw a line perpendicular to $p'q'$ and on that mark $q''q' = f^t_{QP} = \alpha(QP) = \alpha r$.

Joint $p'q'$ which represents f_{QP} (i.e. acceleration of Q relative to P)

The acceleration of intermediate point on the link (say point A) can be obtained by dividing the acceleration vectors in the same ratio as the point A divides the link.

Acceleration Diagram for Single Slider Crank Mechanism



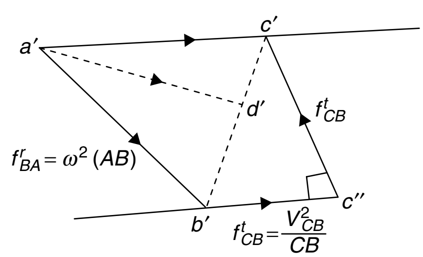


The space diagram of a single slider crank mechanism, with crank AB rotating at constant angular velocity ω, connecting rod BC and slider C is as shown.

The velocity diagram abc is drawn as described in earlier sections. $ab = V_{BA}$, $ac = V_{CA}$ and $bc = V_{CB}$ respectively, drawn to scale. bc is perpendicular to link bc, ab is perpendicular to link AB, AC is parallel to CA. Velocity of point D is marked as ad.

NOTE

If the velocity diagram is rotated through 90° in a direction opposite to ω (angular velocity of crank), AB becomes parallel to crank AB, BC becomes parallel to connecting rod and ac becomes vertical (i.e. perpendicular to CA). By observing, it can be seen that the triangle ABE shown in space diagram is similar to this rotated velocity diagram. This principle is used in **Klein's construction, which is** explained later.



Acceleration Diagram

To draw the acceleration diagram, the fixed point A in space diagram is marked as a' in acceleration diagram. Draw $a'b' = f^r_{AB} = \omega^2(AB)$ in a direction parallel to BA and mark b'. From b' draw $b'c''$ parallel to BC such that $b'c'' = \frac{f^t_{CB}}{CB} = \frac{V^2_{CB}}{CB}$ (V_{CB} is BC in velocity diagram). From c'' draw a line perpendicular to $b'c''$ and from a' draw a line parallel to AC. These two lines intersect at c'. Join $a'c'$ which gives the magnitude and direction of acceleration of slider c at that instant. $c''c'$ gives magnitude and direction of f^t_{CB}. The angular acceleration of CB at that instant is

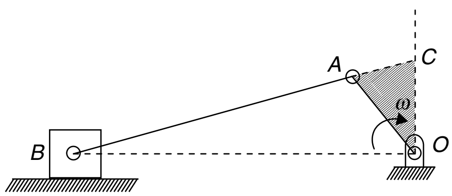
$$\alpha_{CB} = \frac{f^t_{CB}}{CB} = \frac{c''c'}{CB}$$

$b'c'$ gives the acceleration of c relative to B. On this line, we can mark d' such that $\frac{b'd'}{d'c'} = \frac{BC}{DC}$. Then, $a'd'$ gives the absolute acceleration of point D on the connecting rod.

Klein's Construction

When the **crank** of a single slider-crank mechanism is **rotating with a constant angular velocity**. (i.e. angular acceleration $\alpha = 0$ for the crank), A.W. Klein gave a method to construct the velocity and acceleration diagrams of the mechanism on the configuration diagram (space diagram) itself. This method is called Klein's construction.

OAB is the space diagram of a slider crank chain in which crank *OA* rotates at **constant angular velocity** ω.



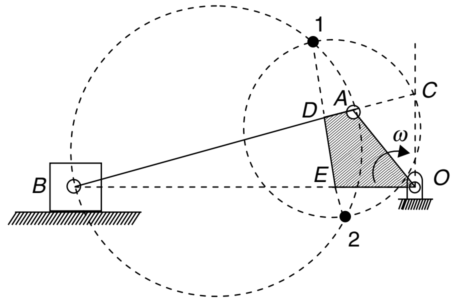
Velocity diagram construction

ΔOAC is the velocity diagram

$$\frac{V_A}{OA} = \frac{V_B}{OC} = \frac{V_{BA}}{AC} = \omega_{crank}$$

To obtain **velocity diagram**, extend the connecting rod *BA* and at hinge *O*, draw a perpendicular to the slider travel line *BO*. These two lines meet at *C*. The shaded **triangle *OAC* represents the velocity diagram of the mechanism**, rotated through 90° in the direction opposite to ω. Velocity of *B* relative to *A* is given by $V_{BA} = \omega(AC)$ and velocity of slider *B* relative to hinge *O* is $V_{BO} = \omega(OC)$. Directions of all these velocities are obtained by rotating *AC* and *OC* through 90° in the direction of ω.

To obtain the acceleration diagram, draw two circles, first with the connecting rod length *AB* as diameter and second with crank end *A* as centre and radius equal to *AC* (extension of connecting rod)



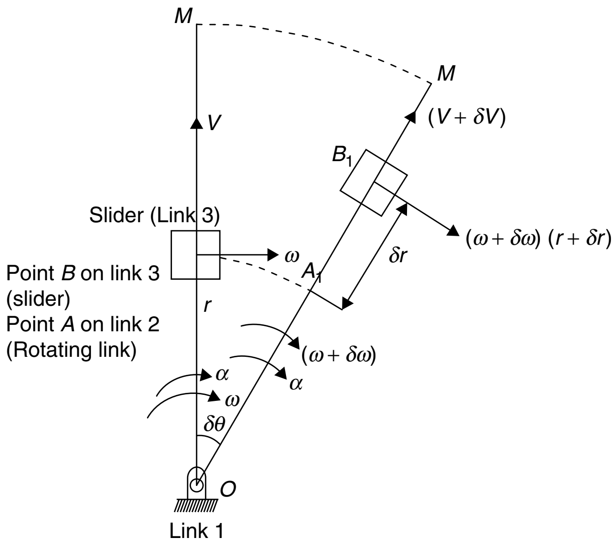
Acceleration Diagram Construction

These two circles intersect at point 1 and 2 as shown in figure. Join 1 and 2 and let this line intersect the connecting rod at point *D* and the slider travel line *BO* at point *E*. Join *DE* and *EO*. Then, **quadrilateral *OADEO*** represents **the acceleration diagram** of the mechanism.

$$\frac{f_B}{OE} = \frac{f^r_{BA}}{AD} = \frac{f_A}{OA} = \frac{f^t_{BA}}{ED} = \omega^2$$

Coriolis Acceleration Component

Coriolis component of acceleration comes into existence wherever a slider has relative motion (i.e. sliding motion) along a rotating link. Examples of a slider sliding on a rotating (or oscillating link) are found in quick return mechanisms like Whitworth quick return mechanism and Crank and slotted lever quick return mechanism, used in shaper machines (but not in Drag link mechanism, which is made of turning pairs)



Consider a rigid link *OM*, hinged at *O* and in vertical position at time $t = 0$. The link is rotating with angular velocity ω and constant angular acceleration α at this instant. There is a slider on link *OM* (Link 2) at a distance r from end *O*. This slider is sliding along the link *OM* with a velocity *V* relative to OM at that instant. The point of contact between slider and OM is B on slider and A on OM at $t = 0$. After a small interval of time Δt, the link OM has rotated through a small angle $\Delta\theta$. Point A on OM has rotated to point A_1 while point B on slider has moved to point B_1. At time $t = 0$, the tangential acceleration of point A on link OM was $f_t = r\alpha$ (for point A at $t = 0$)

$$f_t = r\alpha \text{ (for point } A \text{ at } t = 0)$$

As point B has moved a larger distance to the right (compared to point A) during the small interval δt, the tangential acceleration of B at $t = 0$ must be greater than f_t for A at $t = 0$.

$$\therefore (f_t)_B \text{ at } t = 0 > (f_t)_A \text{ at } t = 0$$

i.e. $(f_t)_B$ (at $t = 0$) $= (f_t)_A +$ additional tangential acceleration. This additional tangential acceleration of the slider on rotating link is called **Coriolis acceleration**.

We can show that at $t = 0$, for the slider,

$$f_{\text{coriolis}} = 2\vec{\omega} \times \vec{v} \ ,$$

where

$\vec{\omega} =$ angular velocity of rotating link at that instant.

$\vec{v}$ = relative velocity of slider with respect to rotating link at that instant.

$\therefore$ At $t = 0$, the actual tangential acceleration of slider with respect to O is

$$f_B^t = (\vec{\alpha} \times \vec{r}) + 2(\vec{\omega} \times \vec{v})$$

NOTES

1. The magnitude of Coriolis component acceleration is given by

   $$f_{coriolis} = 2\omega v \ \ (\because \vec{\omega} \perp \vec{v}),$$

   where

   ω = magnitude of angular velocity of rotating link and

   v = magnitude of relative velocity of slider with respect to the rotating link. **The Coriolis component of acceleration is always perpendicular to the rotating link**.

   It may add or substract to the tangential acceleration (αr).
2. $\vec{\omega}$ anticlockwise is taken positive (+) and $\vec{v}$ radialy outwards is taken as positive (+). Whether Coriolis component of acceleration will be positive (anti-clockwise sense) or negative (clockwise sense) will depend on signs of $\vec{\omega}$ and $\vec{v}$.
3. The direction of Coriolis component of acceleration is obtained by rotating the radial velocity vector $\vec{v}$ (relative velocity of slider with respect to rotating link) through 90° in the direction of rotation of the rotating link.
4. If f is the **radial outward acceleration of slider relative to the rotating link** (i.e. radial outward acceleration of point B on slider with respect to point A on rotating link shown in figure), then **the total radical acceleration of slider relative to the hinge** (i.e, radial acceleration of point B on slider with respect to hinge O shown in figure) is given by

   $$f_{BO}^r = (f - \omega^2 r), \text{ radially outward}$$

   Here, $\omega^2 r$ is the centripetal acceleration of point B on slider.
5. The total tangential acceleration of slider (Point B) with respect to hinge (Point O) is given by

   $f_{BO}^t = \alpha r \pm 2\omega V$ (+ when αr and ωv in same sense; otherwise use −)
6. The magnitude of absolute acceleration of slider (Point B) with respect to hinge (Point O) is given by

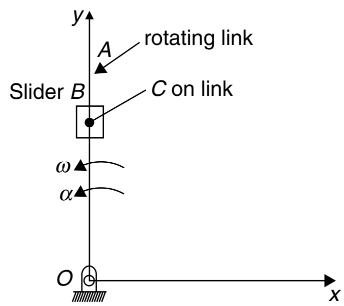
   $$f_{BO} = \sqrt{\left(f_{BO}^r\right)^2 + \left(f_{BO}^t\right)^2}$$

   $$\text{i.e. } f_{BO} = \sqrt{\left(f - \omega^2 r\right)^2 + \left(\alpha r \pm 2\omega v\right)^2}$$

   This acts at an angle ϕ with the link given by

   $$\tan\phi = \frac{f_{Bo}^t}{f_{BO}^r} = \frac{\alpha r \pm 2\omega v}{\left(f - \omega^2 r\right)}$$

Direction for questions (Example 70 to 79)



A rigid link OA is rotating in the XY plane with the hinge at origin O. At time $t = 0$, the link is along Y-axis as shown in figure and has an angular velocity of 3 rad s^{-1} counter clockwise and an angular acceleration of 5 rad s^{-2} counter clockwise. At that instant, a slider B is sliding radially outwards and the distance of slider from hinge is 3 m (i.e. OB = 3 m). The contact point of slider on the link OA is C. The velocity and acceleration of point B (on slider) at that instant with respect to point C (on rotating link) are 4 m/s and 5 m/s^2, both in the radially outward direction.

Example 70: The acceleration of point C on link relative to hinge O (i.e. f_{CO}) is of magnitude

(A) 42 m/s^2 (B) 30.89 m/s^2

(C) 37.63 m/s^2 (D) 27 m/s^2

Example 71: The acceleration of slider (Point B) relative to point C on link (i.e. f_{BC}) is of magnitude

(A) 24.33 m/s^2 (B) 39.12 m/s^2

(C) 24 m/s^2 (D) 11 m/s^2

Example 72: The acceleration of slider (Point B) relative to hinge (Point O) [i.e. f_{BO}] is of magnitude.

(A) 23 m/s^2 (B) 39 m/s^2
(C) 45.28 m/s^2 (D) 24 m/s^2

Solution:

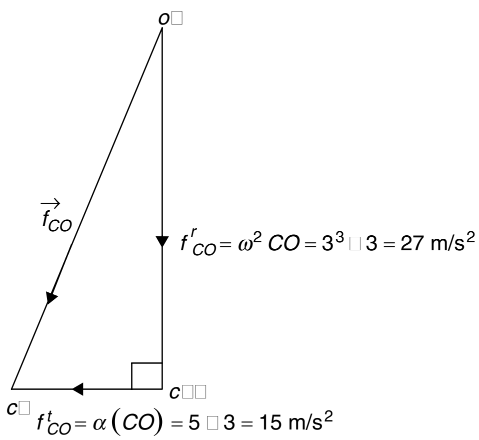
(i) (B) for Example 77,
(ii) (A) for Example 78
(iii) (C) for Example 79

Given $OB = OC = 3$ m
$\omega = 3$ rad s^{-1} (+, because counter clockwise)
$\alpha = 5$ rad s^{-2} (+, because counter clock wise)
$V = V_{BC} = 4$ m/s (+, because radially outwards)
f = radial acceleration of B with respect to C
= 5 m/s^2 (+, because radially outwards)

(i) Acceleration of point c on link with respect to O

The acceleration of c with respect to O (i.e. $\vec{f}_{CO}$) has two components. One is the radial component (centripetal component) equal to $f^r_{CO} = \omega^2(CO)$ directed from C towards O and the other is the tangential component $f^t_{CO} = \alpha(OC)$, perpendicular to OC in the anticlockwise direction

$$\therefore\ \vec{f}_{CO} = \vec{f}^r_{CO} + \vec{f}^t_{CO}$$



$$\therefore f_{CO} = \sqrt{[\omega^2(CO)]^2 + [\alpha(CO)]^2}$$

$$= \sqrt{(3^2\times 3)^2 + (5\times 3)^2} = \sqrt{27^2 + 15^2} = \sqrt{954}$$

= 30.89 m/s^2.

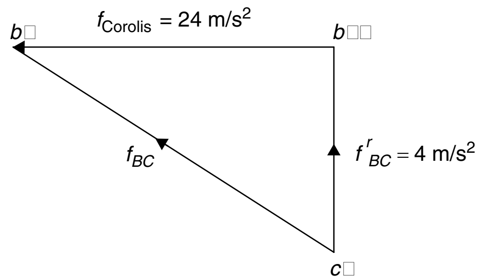
(ii) Acceleration of slider B relative to link c ($\vec{f}_{BC}$)

This has got a radial component $f^r_{BC} = f = 4$ m/s^2 (radially onward i.e. from O to C) and a tangential component (called Coriolis component of acceleration)

$\vec{f}_{\text{Coriolis}} = 2\vec{\omega}\times\vec{v}$ (+ or anticlockwise sense because both $\vec{w}$ and $\vec{v}$ are positive)

$\therefore f_{\text{Coriolis}} = 2 \times 3 \times 4 = 24$ m/s^2

$$\therefore\ \vec{f}_{BC} = \vec{f}^r_{BC} + \vec{f}^t_{BC}$$



$\therefore$ Magnitude of acceleration of B relative C,

$$f_{CB} = \sqrt{(f_{coriolis})^2 + (f^r_{BC})^2}$$

$$= \sqrt{(24)^2 + 4^2} = \sqrt{592}$$

= 24.33 m/s^2.

(iii) Acceleration of slider B relative to hinge O ($\vec{f}_{BO}$)

This has two components. The radial component is

$$f^r_{BO} = f - \omega^2(BO)$$

$= 4 - 3^2 \times 3$
$= -23$ m/s^2
$= 23$ m/s^2 from B towards O

The tangential component also includes the Coriolis component.

$$\therefore\ f^t_{BO} = \alpha(BO) + 2\omega v$$

$= (5 \times 3) + (2 \times 3 \times 4)$
$= 15 + 24$
$= 39$ m/s^2 ($\perp$ to OC and in anticlockwise sense)

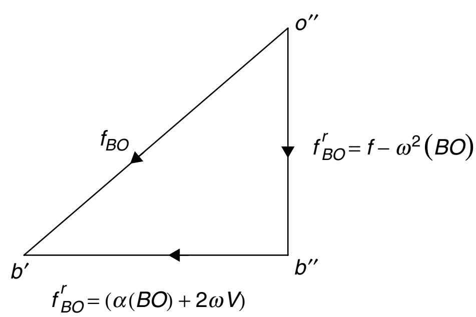
$$\vec{f}_{BO} = \vec{f}^r_{BO} + \vec{f}^t_{BO}$$

$$\therefore f_{BO} = \sqrt{(f^r_{BO})^2 + (f^t_{BO})^2}$$

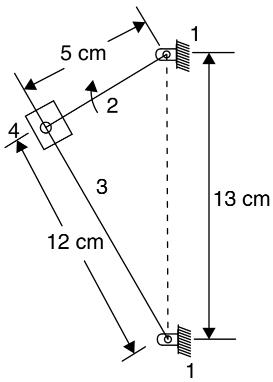
$$= \sqrt{(23)^2 + (39)^2}$$

$$= \sqrt{2050} = 45.28 \text{ m/s}^2$$

$\therefore$ Acceleration of B relative to O is 45.28 m/s^2.



Example 73:



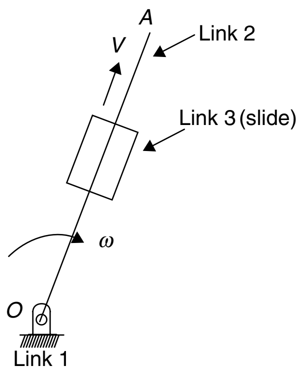
In the figure shown, link 2 rotates at an angular velocity of 3 rad s^{-1}. The magnitude of Coriolis acceleration of link 4 (with respect to link 3) is

(A) 0.45 m/s^2 (B) 0.9 m/s^2
(C) 1.8 m/s^2 (D) 0

Solution:
$13^2 = 5^2 + 12^2 \Rightarrow$ link 2 and link 3 are perpendicular to each other. Hence, at the instant shown, angular velocity of link 3 is zero (i.e. $\omega_3 = 0$) as slider 4 does not have any velocity component normal to link 3)

$$f_{coriolis} = 2\omega_3 \text{ V} = 0 \ (\because \ \omega_3 = 0)$$

Example 74:



In the figure shown, link 2 (OA) is hinged at O to fixed link 1 and rotates clockwise at a constant speed of 240 rpm. The slider (Link 3) has a relative velocity of 20 m/s radially outwards with respect to link 2. The magnitude of Coriolis component of acceleration of link 3 is

(A) 906 m/s^2 (B) 1005.3 m/s^2
(C) 1208 m/s^2 (D) 604 m/s^2

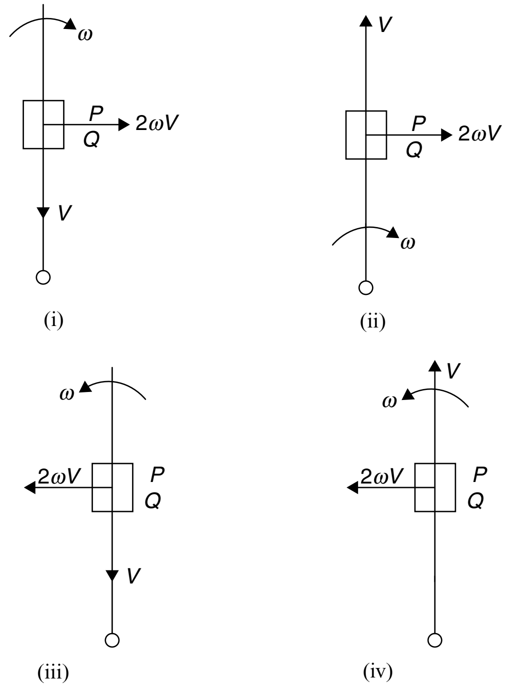
Solution:

$$N = 240 \text{ rpm} \Rightarrow \omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60}$$
$$= 8\pi \text{ rad s}^{-1}$$

$V = 20$ m/s (data)

$$\therefore f_{coriolis} = 2\omega\text{V} = 2 \times 8\pi \times 20$$
$$= 1005.3 \text{ m/s}^2$$

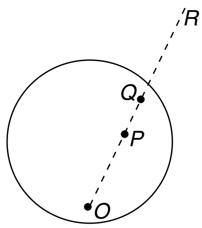
Example 75: The directions of Coriolis component of acceleration $2\omega V$ of slider P with respect to the coincident point Q is shown in Figures (i), (ii), (iii) and (iv). Which figures show the wrong direction of Coriolis acceleration?



(A) Figure (i) and Figure (iv) are wrong
(B) Figure (i) and Figure (ii) are wrong
(C) Figure (i) and Figure (iii) are wrong
(D) Figure (ii) and Figure (iii) are wrong

Solution:
To find the direction of Coriolis component of acceleration, rotate the velocity V through 90° in the direction of rotation (clockwise or anticlockwise) of the rotating link. Fig. (i) and Fig. (iii) indicate wrong directions of Coriolis component of acceleration.

Direction for questions (Examples 76 and 77)



The circular disc shown in its plan view in the figure rotates in a plane parallel to the horizontal plane about the point O at a uniform angular velocity ω. Other points P and Q are located on the line OR at distance r_P and r_Q from O respectively.

Example 76: The velocity of point Q with respect to point P is a vector of magnitude

(A) zero
(B) $\omega(r_Q - r_P)$, directed opposite to the direction of motion of Q
(C) $\omega(r_Q - r_P)$, directed in the same direction as motion of Q
(D) $\omega(r_Q - r_P)$ and directed from O to R

Solution:

$V_{QO} = \omega r_Q$, perpendicular to OB towards right
$V_{PO} = \omega r_P$, perpendicular to OP towards right
$\therefore V_{QP} = V_{QO} - V_{PO}$
$= \omega(r_Q - r_P)$, perpendicular to OB towards right

Example 77: The acceleration of point Q with respect to point P is a vector of magnitude.

(A) zero
(B) $\omega(r_Q^2 - r_P^2)$ and having same direction as direction of motion of point Q.
(C) $\omega^2(r_Q - r_P)$ and directed opposite to the direction of motion of Q
(D) $\omega^2(r_Q - r_P)$, direction from R to O.

Solution:

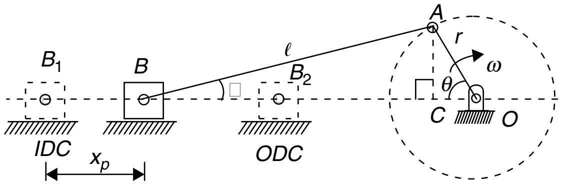
$f_{QO} = \omega^2 r_Q$ (from Q to O)
$f_{PO} = \omega^2 r_p$ (from P to Q)
$\therefore f_{QP} = \omega^2 (r_Q - r_P)$ from Q to O (same as R to O)

Dynamic Analysis of Slider Crank Mechanism

If we consider the crank of a single-slider crank mechanism (eg: steam engine, *IC* engine etc) rotating with a constant angular velocity (say ω) and not having any angular acceleration (i.e. $\alpha = 0$ for crank), then it is possible to analytically determine the displacement, velocity and acceleration of the reciprocating parts (eg piston) and also the angular velocity and angular acceleration of the connecting rod for various angular positions (θ) of the crank with the line of motion of the reciprocating part. This method is explained below.

Analytical Method For Slider-Crank Mechanism

Crank OA rotates at **constant angular velocity** ω. When the crank has rotated through an angle θ from inner dead centre (IDC) portion, the piston has been displaced by x_p, velocity of piston is v_p and the acceleration of piston is a_p



IDC = Inner Dead Centre (Top dead centre for vertical engine)
ODC = Outer Dead Centre (Bottom Dead Centre for vertical engine)
Stroke, $L = IDC - ODC = 2$ times crank radius
$= 2r$;
$\theta = 0°$ corresponds to *IDC* and $\theta = 180°$ corresponds to *ODC*
The following nomenclatures are used
r = radius of crank OA
ℓ = length of connecting rod AB
θ = angle turned by crank from IDC
x_P = displacement of piston from IDC
ϕ = angle made by connecting rod AB at point B with line of motion of piston BO.
n = ratio of connecting rod length ℓ to crank radius r

$$\text{i.e. } n = \frac{\ell}{r}$$

AC is drawn $\perp$ to BO. From ΔABC and ΔAOC,
$AC = AB\sin\phi = OA\sin\theta$

$$\Rightarrow \sin\phi = \frac{OA}{AB}\sin\theta = \frac{r}{\ell}\sin\theta = \frac{\sin\theta}{n}$$

$$\therefore \sin\phi = \frac{\sin\theta}{n}$$

$$\Rightarrow \cos\phi = \sqrt{1-\sin^2\phi} = \frac{\sqrt{n^2-\sin^2\theta}}{n} \text{ and}$$

$$\tan\phi = \frac{\sin\theta}{\sqrt{n^2-\sin^2\theta}}$$

Displacement of Piston (x_p)

We have $x_P = B_1O - BO$

$$= (\ell + r) - [\ell\cos\phi + r\cos\theta]$$
$$= \ell[1 - \cos\phi] + r[1 - \cos\theta]$$
$$= r\left[\frac{\ell}{r}(1-\cos\phi)+[1-\cos\theta]\right]$$
$$= r\left[n(1-\cos\phi)+(1-\cos\theta)\right]$$
$$= r\left[n\left(1-\frac{\sqrt{n^2-\sin^2\theta}}{n}\right)+(1-\cos\theta)\right]$$

[using value of $\cos\phi$]

$$= r\left[(n+1)-\left(\sqrt{n^2-\sin^2\theta}+\cos\theta\right)\right]$$
$$= r\left[(1-\cos\theta)+n-\sqrt{n^2-\sin^2\theta}\right]$$
$$\therefore x_P = r\left[(1-\cos\theta)+n-\sqrt{n^2-\sin^2\theta}\right]$$

NOTE

If n **is very large** (i.e. $\ell >> r$), $\sqrt{n^2-\sin^2\theta} = n$, then $x_P = r(1 - \cos\theta)$ which is the expression for simple harmonic motion. So we can conclude that **if the length of connecting rod is very large compared to the length of crank, the piston executes SHM.**

If the piston was at the inner dead centre at time $t = 0$, the angle $\theta = \omega t$. Using this value, the displacement of piston can be expressed as a function of time t as

$$x_p = r\left[(1-\cos\omega t)+n-\sqrt{n^2-\sin^2(\omega t)}\right]$$

Velocity of Piston (V_p)

The velocity of the piston is obtained by taking the time derivative of its displacement.

i.e. $V_P = \frac{dx_p}{dt} = \frac{dx_p}{d\theta}.\frac{d\theta}{dt} = \omega\frac{dx_p}{d\theta}\left(\because \frac{d\theta}{dt} = \omega\right)$

$$x_p = r\left[(1-\cos\theta)+n-\sqrt{n^2-\sin^2\theta}\right]$$

$$\therefore \frac{dx_p}{dt} = \omega\frac{dx_p}{d\theta} = \omega r\left[\sin\theta - \frac{(-2\sin\theta\cos\theta)}{2\sqrt{n^2-\sin^2\theta}}\right]$$

$$= \omega r\left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2-\sin^2\theta}}\right]$$

If n^2 is large (i.e. $\ell >> r$), $n^2 - \sin^2\theta \approx n^2$

$$\Rightarrow V_p = \frac{dx_p}{dt} = \omega r\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]$$

In the time domain,

$$V_p = \omega r\left[\sin\omega t + \frac{\sin 2\omega t}{2n}\right]$$

When n is large, $\frac{\sin 2\theta}{n}$ becomes negligible

$\Rightarrow V_P = \omega r\sin\theta$

$= \omega r\sin\omega t$, which is the velocity in *SHM*.

Acceleration of Piston (a_p)

The acceleration of the piston is obtained by taking the time derivative of the piston velocity.

$$\therefore a_P = \frac{dV_p}{dt} = \frac{dV_p}{d\theta}.\frac{d\theta}{dt}$$

$$= \frac{d}{d\theta}\left[r\omega\left(\sin\theta + \frac{\sin 2\theta}{2n}\right)\right].\frac{d\theta}{dt}$$

$$= r\omega\left[\cos\theta + \frac{2\cos 2\theta}{2n}\right]\omega\left[\because \frac{d\theta}{dt} = \omega\right]$$

$$= r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

$$\therefore a_P = r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

In the time domain,

$$a_P = r\omega^2\left[\cos\omega t + \frac{\cos 2\omega t}{n}\right]$$

The above expression for a_p are only approximate and not exact, because those expressions have been obtained by differentiating the approximate expression for velocity (when n is large). In most of the cases, $\frac{\ell}{r} = n$ is large and hence the approximate expression for a_p is quite sufficient in most cases.

For $\theta = 0°$,

$$a_P = r\omega^2\left[1+\frac{1}{n}\right];$$

For $\theta = 180°$, $a_P = r\omega^2\left(-1+\frac{1}{n}\right)$;

From ODC as the crank moves towards IDC, the motion is reversed and sign will be changed in the above expression.

$$\therefore a_P = -(r\omega^2)\left(-1+\frac{1}{n}\right)$$

$$= r\omega^2\left[1-\frac{1}{n}\right].$$

Angular Velocity of Connecting Rod (ω_c)

As ϕ is the angle made by the connecting rod *AB* with the piston movement line *BO*, $\frac{d\phi}{dt}$

Gives the angular velocity of connecting rod (ω_c).

We have $\sin\phi = \frac{\sin\theta}{n}$ $\left(\because n = \frac{\ell}{r}\right)$

Differentiating with respect to time t, we get

$$\cos\phi\left(\frac{d\phi}{dt}\right) = \frac{1}{n}\cos\theta\left(\frac{d\theta}{dt}\right)$$

$$\Rightarrow \omega_c\cos\phi = \frac{\omega\cos\theta}{n}$$

$$\therefore \omega_c = \frac{\omega\cos\theta}{n\cos\phi} = \frac{\omega\cos\theta}{n\sqrt{n^2-\sin^2\theta}}$$

$$\left(\because \cos\phi = \sqrt{n^2-\sin^2\theta}\right)$$

$$\Rightarrow \omega_c = \frac{\omega\cos\theta}{\sqrt{n^2-\sin^2\theta}}$$

Angular Acceleration of the Connecting Rod (α_C)

Angular acceleration of connecting rod,

$$\alpha_c = \frac{d\omega_c}{dt} = \frac{d\omega_c}{d\theta}.\frac{d\theta}{dt}\left[\omega_c = \frac{\omega\cos\theta}{\sqrt{n^2-\sin^2\theta}}\right]$$

$$= \frac{d}{d\theta}\left[\frac{\omega\cos\theta}{\sqrt{n^2-\sin^2\theta}}\right]\frac{d\theta}{dt}\ \left(\because \frac{d\theta}{dt} = \omega\right)$$

$$= \omega\frac{d}{d\theta}\left[\cos\theta\left(n^2-\sin^2\theta\right)^{-1/2}\right]\omega$$

$$\Rightarrow \alpha_c = -\omega^2 \sin\theta \left[\frac{(n^2 - 1)}{(n^2 - \sin^2\theta)^{3/2}} \right]$$

Depending upon the value of $\sin\theta$ (positive or negative), α_c can be positive or negative. When α_c is positive, ϕ increases with time which means crank and connecting rod have opposite sense of rotation. When α_c is negative, ϕ decreases with time which means crank and connecting rod have same sense of rotation.

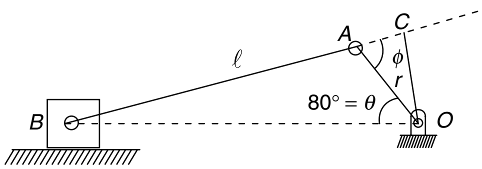
NOTES

1. If $n^2 > \sin^2\theta$, then $(n^2 - \sin^2\theta)^{3/2} \approx n^3$ and $\alpha_c = \dfrac{-\omega^2 \sin\theta [n^2 - 1]}{n^3}$
2. If $n^2 >> 1$, then $\alpha_c = \dfrac{-\omega^2 \sin\theta}{n}$

Example 78: Consider the triangle formed by the connecting rod and the crank of an IC engine as the two sides of the triangle. If the maximum area of this triangle occurs when the crank angle is 80°, the ratio of connecting rod length to crank radius is

(A) 3.14 (B) 4.26 (C) 3.73 (D) 5.67

Solution:



OAB is the triangle. OC is drawn ⊥ to connecting rod AB.

$\angle CAO = \phi$;

$OC = r\sin\phi$

$$A = \text{Area of } \Delta OAB = \frac{1}{2} \times AB \times OC$$

$$= \frac{1}{2} \times \ell \times r \sin\phi$$

$$= \frac{\ell r}{2} \sin\phi$$

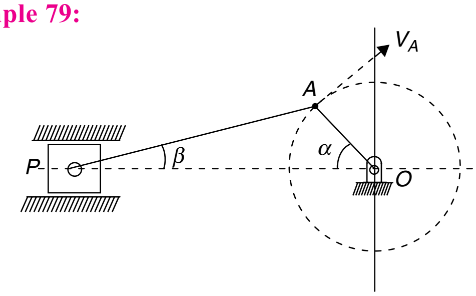
Area A will be maximum when $\sin\phi$ is maximum.

$\Rightarrow \phi = 90°$

$\theta = 80°$(given)

$$\Rightarrow \tan\theta = \frac{\ell}{r} \Rightarrow \frac{\ell}{r} = \tan 80° = 5.67.$$

Example 79:



For the slider crank mechanism shown in figure, if V_A is velocity of point A of crank in this configuration, The velocity of cross-head for the position shown is

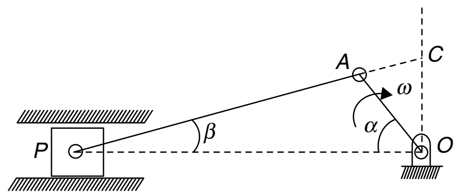
(A) $V_A \cos(90° - \overline{\alpha + \beta})\cos\beta$

(B) $V_A \cos(90° - \overline{\alpha + \beta})\sec\beta$

(C) $V_A \cos(90° - \overline{\alpha - \beta})\cos\beta$

(D) $V_A \cos(90° - \overline{\alpha - \beta})\sec\beta$

Solution:



OAC represents the velocity triangle (Klein's construction)

$$\therefore \omega = \frac{V_A}{OA} = \frac{V_P}{OC} = \frac{V_{PA}}{AC}$$

$$\Rightarrow V_P = V_A \left(\frac{OC}{OA} \right) \qquad (1)$$

From ΔOAC, $\angle OAC = \overline{\alpha + \beta}$

From ΔPCO, $\angle PCO = \angle ACO$

$= 180° - (90° + \beta) = (90° - \beta)$

Also from ΔOAC,

$$\frac{OA}{\sin\angle ACO} = \frac{OC}{\sin\angle OAC}$$

$$\Rightarrow \frac{\sin\angle OAC}{\sin\angle ACO} = \frac{OC}{OA}$$

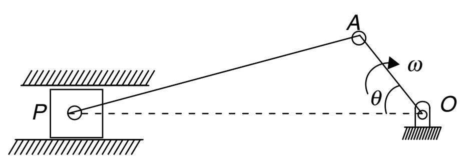
$$\Rightarrow \frac{\sin\overline{\alpha + \beta}}{\sin(90° - \beta)} = \frac{OC}{OA}$$

$$\therefore \frac{OC}{OA} = \frac{\sin\overline{\alpha + \beta}}{\cos\beta} = \frac{\cos(90° - \overline{\alpha + \beta})}{\cos\beta}$$

$$= \cos(90° - \overline{\alpha + \beta})\sec\beta$$

$$\therefore (1) \rightarrow V_P = V_A \cos(90° - \overline{\alpha + \beta})\sec\beta$$

Example 80:



For an IC engine with crank radius $= r$, connecting rod length $= \ell$ and $\dfrac{\ell}{r} = n$ is very large. The crank rotates at

constant angular velocity ω. The crank angle θ for which the velocity of piston (V_P) becomes equal in magnitude to velocity of crank end (V_A) is
(A) 0° (B) 90° (C) 45° (D) 75°

Solution:
Velocity of crank end $V_A = r\omega$
For large value of n, piston executes SHM and $V_P = r\omega \sin\theta$
When $\theta = 90°$, $\sin\theta = 1$ and $V_P = r\omega = V_A$.

Example 81: When the crank is at the angular position corresponding to inner dead centre of a horizontal steam engine, V_P and a_p are the velocity and acceleration of the piston. The $\frac{\ell}{r}$ ratio is n. Then,

(i) V_P is zero, $a_P = \omega^2 r\left[1+\frac{1}{n}\right]$

(ii) $V_P = \omega r\left[1+\frac{1}{n}\right]$, $a_p = \omega^2 r$

(iii) $V_P = \omega r$, $a_P = 0$
(iv) V_P is zero, a_p is maximum
(v) V_P is zero, a_p is minimum
(vi) V_P is maximum, a_p is zero
The correct choices is/are

(A) (i) and (v) (B) (iii) and (vi)
(C) (i) and (iv) (D) (ii) only

Solution:

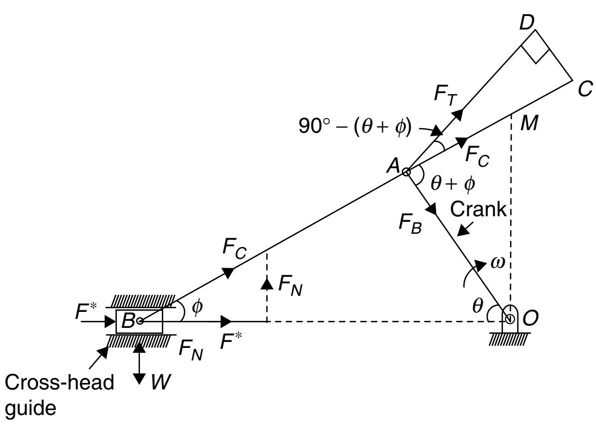
$$V_P = \omega r\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]; \text{ when } \theta = 0, V_P = 0$$

$$a_P = r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]; \text{ when } \theta = 0, a_P = r\omega^2\left[1+\frac{1}{n}\right],$$

which is the maximum value of a_p.

Analysis of Forces on Various Parts of Reciprocating Engines

Neglecting the weight of the connecting rod, the various forces acting on the parts of reciprocating engine (*IC* or steam) are shown in below figure.



The various forces on the parts are listed as follows (neglecting the weight of the connecting rod)

1. Piston Effort (F^*)
2. Force transmitted along connecting rod (F_c)
3. Side thrust on cylinder wall/Normal reaction (F_N)
4. Crank effort (F_T) and
5. Thrust on crank shaft bearings (F_B)

We have already obtained expressions for x_p, v_p, a_p, ω_c and α_c for various crank positions (θ) earlier. We will now analyse all these forces in terms of piston effort F^*, crank angle θ and connecting rod angle ϕ.

Piston Effort (F^*)

The **effective driving force** or the **net force** acting on piston (or cross-head pin) **along the line of stroke** is known as piston effort, denoted as F^*. The gas (or steam) exerts a force on the piston and this force is denoted as F_p.

$$F_P = p_1A_1 - p_2A_2$$

or $F_P = p_1A_1 - p_2(A_1 - a)$,

where
p_1 = pressure on cover end
p_2 = pressure on piston rod end
A_1 = area of cover end
A_2 = area of piston rod end
a = cross-sectional area of piston rod

This force on piston is **decreased** or **increased** by the **inertial force** (F_i) due to acceleration of the masses. The piston, which executes simple harmonic motion (*SHM*), accelerates during the first half of stroke and hence inertial force F_i opposes force on piston (F_p) during this time. However, the piston is retarding during the second half of the stroke and inertial force aids force on piston (F_p). Hence, the net force on piston or piston effort $F^* = F_p + F_i$ where F_i is negative for first half of stroke and F_i is positive for second half of stroke.

The inertia force on reciprocating parts, $F_i = -ma$, where
m = mass of reciprocating parts (piston, cross-head pins etc)
a = acceleration of piston

$$= \omega^2 r\left(\cos\theta + \frac{\cos 2\theta}{n}\right) \text{ (as already calculated earlier)}$$

If **frictional resistance** (F_R) is also taken into account, then piston effort $F^* = F_p + F_i - F_R$

NOTE

For vertical engines, the weight of reciprocating parts $W = mg$ (m = mass of reciprocating parts) also must be considered.

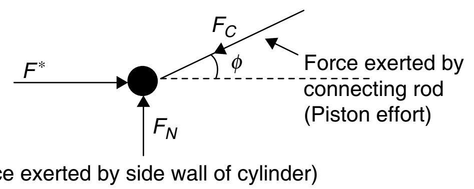
During the downward motion from Top Dead Centre to Bottom Dead Centre, the weight W assists the piston effort. When the piston moves upwards (from *BDC* to *TDC*), the weight W opposes the piston effort.

∴ For vertical engines, piston effort

$$F^* = F_p + F_i \pm W - F_R$$

Force Transmitted Through Connecting Rod (F_c)

The cross-head pin exerts a force F_c on the connecting rod, which is transmitted through the connecting rod. At the same time, the connecting rod exerts an equal and opposite force on the cross-head pin (as per Newton's third law of motion). The various forces on the cross-head pin are as shown.



For equilibrium of the cross-head pin, we get

$F_c \cos\phi = F^*$

$$\therefore F_c = \frac{F^*}{\cos\phi} = \frac{nF^*}{\sqrt{n^2 - \sin^2\theta}}$$

$\left(\because \cos\phi = \dfrac{\sqrt{n^2 - \sin^2\theta}}{n}\right.$, as established earlier)

Side Thrust on Cylinder Wall/Normal Reaction (F_N)

The normal reaction on the cylinder-wall is equal and opposite to F_N exerted by the cylinder wall on cross-head pin. We can see that

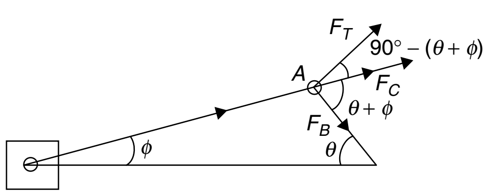
$$F_N = F_c \sin\phi = F^* \frac{\sin\phi}{\cos\phi} = F^* \tan\phi$$

As $\tan\phi = \dfrac{\sin\theta}{\sqrt{n^2 - \sin^2\theta}}$,

$$F_N = \frac{F^* \sin\theta}{\sqrt{n^2 - \sin^2\theta}}$$

Crank Effort (F_T)

The force (F_c) transmitted by the connecting rod acts on the crank pin (which connects crank and connecting rod). This force F_c can be resolved into two components, crank effort F_T, which acts tangential to the crank and F_B, which acts along the crank.



$$F_T = F_c \cos[90° - (\theta + \phi)]$$

$$= F_c \sin(\theta + \phi) \quad \left(F_C = \frac{F^*}{\cos\theta}\right)$$

$$= \frac{F^*}{\cos\phi}\sin(\theta + \phi)$$

$$\Rightarrow F_T = \frac{F^* \sin(\theta + \phi)}{\cos\phi} \text{ is the crank effort.}$$

Thrust on Crank Shaft Bearings (F_B)

The component of F_c, resolved along the crank results in a thrust F_B on the crank shaft bearing.

$$\therefore F_B = F_C \sin[90° - (\theta + \phi)]$$

$$= F_C \cos(\theta + \phi)$$

$$= \frac{F^*}{\cos\phi}\cos(\theta + \phi)$$

$\therefore F_B = \dfrac{F^* \cos(\theta + \phi)}{\cos\phi}$ is the radial thrust on crank shaft bearing.

Turning Moment (or Torque) on Crank Shaft

The product of crank-effort (F_T) and the radius of crank (r) is called the turning moment or torque on crank shaft, denoted by symbol T.

$$\therefore T = F_T r$$

$$= F^* \frac{\sin(\theta + \phi)}{\cos\phi} r$$

$$= F^* r \left[\frac{\sin\theta\cos\phi + \cos\theta\sin\phi}{\cos\phi}\right]$$

$$= F^* r[\sin\theta + \cos\theta\tan\phi]$$

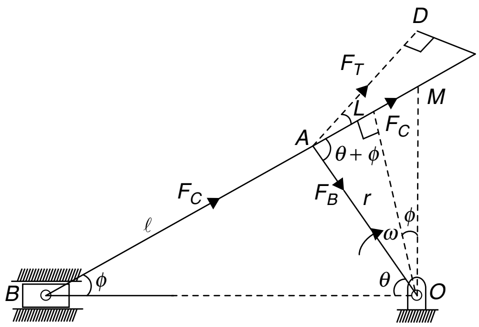
$$= F^* r\left[\sin\theta + \frac{\cos\theta\sin\theta}{\sqrt{n^2 - \sin^2\theta}}\right]$$

$$\left[\because \tan\phi = \frac{\sin\theta}{\sqrt{n^2 - \sin^2\theta}}, \text{ where } n = \frac{\ell}{r}\right]$$

$$= F^* r\left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right]$$

$$\therefore T = F_T r = F^* r \frac{\sin(\theta + \phi)}{\cos\phi}$$

$$= F^* r\left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right]$$



OL is drawn perpendicular to BA extended. From ΔOAL, we have $OL = OA\sin(\theta+\phi) = r\sin(\theta+\phi)$

Also from ΔOML, $OL = OM\cos\phi$

$\Rightarrow r\sin(\theta+\phi) = OM\cos\phi$

$$\therefore\ T = \frac{F^* r\sin(\theta+\phi)}{\cos\phi} = \frac{F^* OM\cos\phi}{\cos\phi} = F^* OM$$

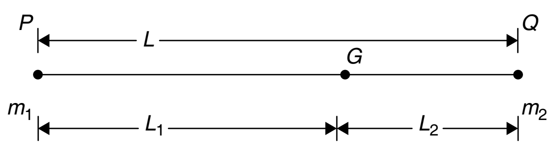
$\therefore\ T = F^* OM$, where OM is perpendicular to line of stroke BO and point M lies on the extension of connecting rod.

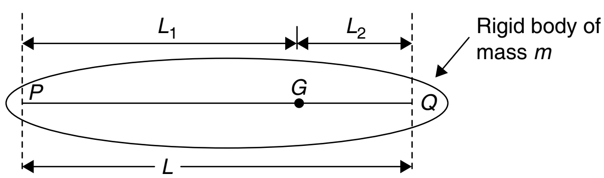
The pin connecting the smaller end of the connecting rod and the cross-head (or piston) is called as the **gudgeon pin**.

Dynamically Equivalent System

A two mass system is dynamically equivalent to a rigid body if

1. The mass of the rigid body is equal to the sum of the two masses.
2. The centre of gravity of the two masses coincides with the centre of gravity of the rigid body.
3. The total moment of inertia of the two masses about an axis passing through the centre of gravity is the same as the moment of inertia of the rigid body about the same axis passing through the C.G.





The two masses m_1 and m_2, having their CG at G, is equivalent to a rigid body of mass m, having its CG at G, if

(a) $m_1 + m_2 = m$
(b) $m_1L_1 = m_2L_2$
(c) $m_1L_1^2 + m_2L_2^2 = mk^2$, where k = radius of gyration of rigid body about an axis though G

The above equations give $m_1 = \dfrac{mL_2}{(L_1+L_2)}$,

$$m_2 = \frac{mL_1}{(L_1+L_2)} \text{ and}$$

$$k^2 = L_1L_2$$

NOTES

1. The two mass system and the rigid body should have the same acceleration. So if the position of one mass is fixed, the position of the other mass is obtained from the three equations (i), (ii) and (iii) given above.
2. If $m_1 + m_2 = m$ and the CG of m_1 and m_2 coincides with the CG of the rigid body, the third condition (of equality of moment of inertia) will not be fulfilled, if the second mass (m_2) is arbitrarily placed at some convenient position. In order to make the system dynamically equivalent to the rigid body, a correction couple is to be applied to the two-mass system, in such cases.

The value of correction couple (T_c) is given by

$T_c = mL_1(\ell - L)\alpha$, where
m = mass of the solid body
L_1 = distance of CG from the point where mass m_1 is placed
ℓ = distance between mass m_1 and m_2 placed arbitrarily
L = distance between mass m_1 and m_2 which form a true dynamically equivalent system
α = angular acceleration of rigid body

Whenever the mass of connecting rod is not negligible, the above method of equivalent dynamic system of two masses is used for analysis.

Example 82: A reciprocating engine slider crank mechanism has a crank of 100 mm length and a connecting rod of 450 mm length. Line of reciprocation of the slider passes through the centre of rotation of the crank shaft. If the total axial force on the piston is 1 kN, determine the torque (in Nm) produced on the crank shaft when the crank is 60° away from the inner dead centre position. Crank shaft is rotating at 1800 rpm. Neglect frictional losses.

Solution: crank radius, $r = 100$ mm $= 0.1$ m

Length of connecting rod, $\ell = 450$ mm $= 0.45$ m

$$\therefore\ n = \frac{\ell}{r} = \frac{0.45}{0.1} = 4.5$$

Crank angle, $\theta = 60°$ (data)

$$\sin\phi = \frac{\sin\theta}{n} = \frac{\sin 60°}{4.5}$$

$$= \frac{0.866}{4.5} = 0.1925$$

$\therefore\ \phi = \sin^{-1}0.1925 = 11.10°$

Piston effort, $F^* = 1$ kN $= 1000$ N

Torque $T = F_T r$

$$= \frac{F^* \sin(\theta+\phi)}{\cos\phi}.r$$

$$= \frac{1000\sin(60°+11.10°)}{\cos 11.10°} \times 0.1 = 96.41 \text{ Nm.}$$

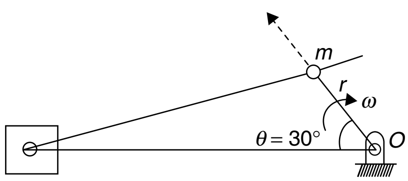
Hence, the torque produce on the crank shaft is 96.41 Nm.

Example 83: A slider crank mechanism has a slider of mass 10 kg, stroke of 0.2 m and rotates with a uniform angular velocity of 10 rad s^{-1}. The primary inertia forces of the slider are partially balanced by a revolving mass of 6 kg at the crank, placed at a distance equal to crank radius. Neglect the mass of connecting rod and crank. When the crank angle (with respect to slider axis) is 30°, the unbalanced force (in N) normal to the slider axis is ____________. Fill up the blank.

Solution: Stroke, $L = 0.2$ m

$$\Rightarrow \text{crank radius } r = \frac{L}{2} = \frac{0.2}{2} = 0.1 \text{ m}$$

$\theta = 30°$; Balancing mass $m_b = 6$ kg
$\omega = 10$ rad s^{-1}



The centrifugal force in balancing mass $F = m_b r\omega^2 = 6 \times 0.1 \times 10^2 = 60$ N

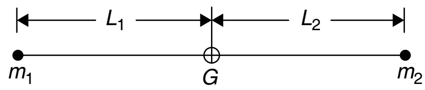
$\therefore F_N$ = unbalanced force normal to slider axis = $F \sin 30°$ = $60 \times \frac{1}{2} = 30$ N.

Example 84: In a dynamically equivalent system, a uniformly distributed mass is divided into

(A) Three point masses (B) Four point masses
(C) Two point masses (D) Infinite point masses

Solution:
Dynamically equivalent system of a rigid body is made of two point masses.

Example 85: A disc type flywheel having a mass of 12 kg and radius of gyration 0.3 m is replaced in a single cylinder engine by a system of dynamically equivalent concentrated masses m_1 and m_2 rotating about the flywheel as shown in figure. If distance L_1 is 0.2 m, then the distance L_2 is (G = centre of gravity of system and also of the flywheel)

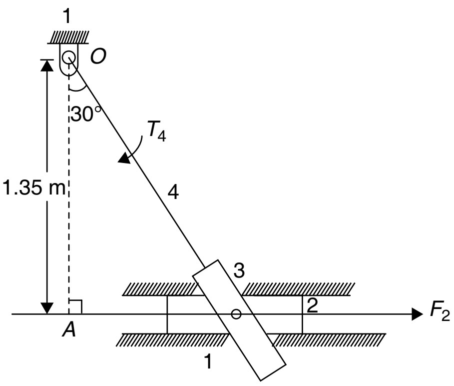


(A) 0.3 m (B) 0.2 m (C) 0.35 m (D) 0.45 m

Solution:
$k = 0.3$ m (radius of gyration of flywheel)
For equivalent dynamic system,
$L_1L_2 = k^2$

$$\Rightarrow L_2 = \frac{k^2}{L_1} = \frac{(0.3)^2}{0.2} = 0.45 \text{ m}$$

Example 86:



In the mechanism shown, a force F_2 is applied on link 2 to overcome a torque T_4 = 12500 Nm acting on link 4. Neglecting friction, gravity and inertia forces, the value of required force F_2(in N) is

(A) 4934.21 N (B) 9259.26 N
(C) 6250 N (D) 8317.31 N

Solution:
The net torque on system is zero.
i.e. $\Sigma T = 0$
$\therefore F_2(\text{AO}) = T_4$
$\Rightarrow F_2 \times 1.35 \text{ m} = 12500$ Nm

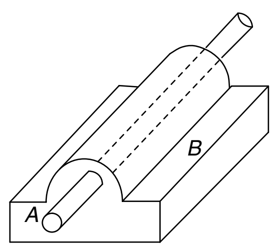
$$\Rightarrow F_2 = \frac{12500 \text{ Nm}}{1.35 \text{ m}} = 9259.26 \text{ N}$$

Exercises

Practice Problems I

1. For the given statements:
 I. Mating spur gear teeth is an example of higher pair.
 II. A revolute joint is an example of lower pair.
 Indicate the correct answer.
 (A) Both I and II are false.
 (B) Both I and II are true.
 (C) I is true and II is false.
 (D) I is false and II is true.

2.



A round bar A passes through a cylindrical hole in B as shown in figure. The following statements are given:

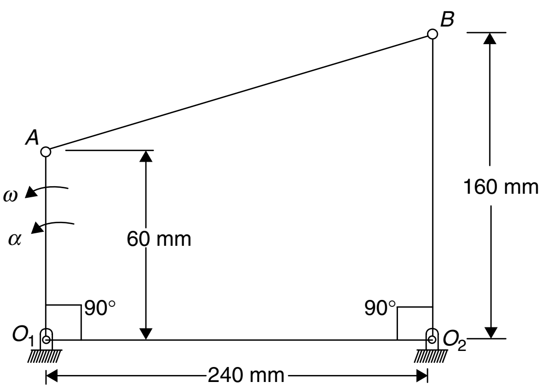
(1) The two links A and B form a kinematic pair
(2) The pair is completely constrained
(3) The pair has incompletely constrained
(4) The pair is successfully constrained.

Correct statements are:

(A) 1 and 2 only (B) 1 and 3 only
(C) 1 and 4 only (D) 1 only

3. The number of degrees of freedom of a planar linkage with 9 links and 10 simple revolute joints is ________.

Direction for questions 4 and 5:



An instantaneous configuration of a 4-bar mechanism, whose plane is horizontal, is shown in the figure. At this instant, the angular velocity and angular acceleration of link O_1A are $\omega = 8$ rad s^{-1} and $\alpha = 0$ respectively. The link O_1A is balanced so that its centre of mass falls at O_1.

4. Which kind of 4-bar mechanism is O_1ABO_2?
(A) Double crank mechanism
(B) Crank-rocker mechanism
(C) Double-rocker mechanism
(D) Parallelogram mechanism

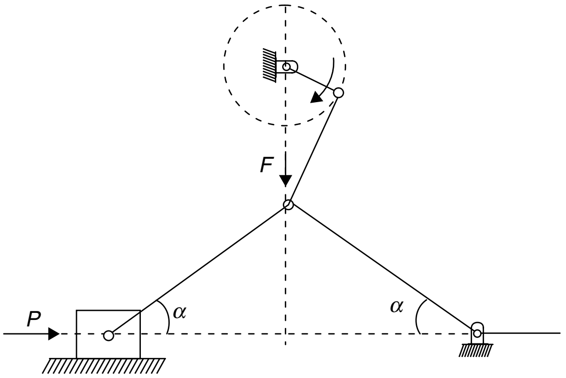
5. At the instant considered, what is the magnitude of angular velocity of O_2B?
(A) 1 rad s^{-1} (B) 3 rad s^{-1}
(C) 8 rad s^{-1} (D) 64/3 rad s^{-1}

6. In a kinematic chain, a quarternary joint is equivalent to
(A) One binary joint (B) Two binary joints
(C) Three binary joints (D) Four binary joints

7. Which mechanism produces intermittent rotary motion from continuous rotary motion?
(A) Whitworth mechanism
(B) Scotch Yoke mechanism
(C) Geneva mechanism
(D) Elliptical trammel

8. In an elliptical trammel, the length of the link connecting the two sliders is 100 mm and the tracing pen is placed on 150 mm extension of this link. The major and minor axes of the ellipse traced by the mechanism would be respectively
(A) 300 mm and 200 mm
(B) 250 mm and 100 mm
(C) 500 mm and 200 mm
(D) 500 mm and 300 mm

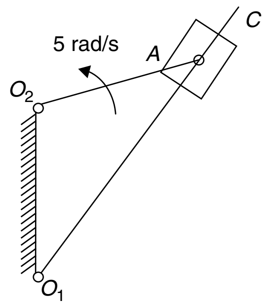
9.



With reference to the mechanism shown in figure, the relation between effort F and resistance P is

(A) $F = \frac{P}{2}\tan\alpha$ (B) $F = P\tan\alpha$
(C) $P = 2F\tan\alpha$ (D) $F = 2P\tan\alpha$

Direction for questions 10 and 11:



The crank and slotted lever quick return motion mechanism is shown in figure. The lengths of links O_1O_2, O_1C and O_2A are respectively 10 cm, 20 cm and 5 cm. Link O_2A rotates at constant angular velocity of 5 rad s^{-1} in the counter clockwise direction.

10. The quick return ratio of the mechanism is
(A) 3.0 (B) 2.75 (C) 2.5 (D) 2.0

11. The maximum velocity of point C of link O_1C during the forward stroke is (in cm s^{-1})

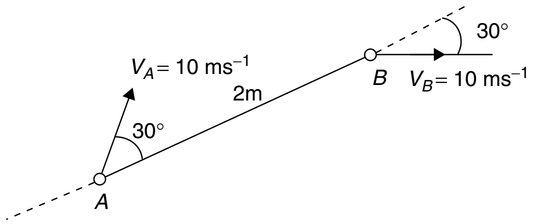
(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{1}{4}$

12. A mechanism has 6 links out of which 4 are binary, 1 is ternary and 1 is quarternary. The number of instantaneous centres of rotation will be ________

13. Oldham's coupling
(i) is an inversion of single slider-crankchain.
(ii) is an inversion of double slider-crank chain.
(iii) is used to connect two shafts whose axes are parallel but separated by a small distance.
(iv) is used to connect two shafts whose axes intersect at a small angle.

The true statements are
(A) (i) and (iii) (B) (i) and (iv)
(C) (ii) and (iii) (D) (ii) and (iv)

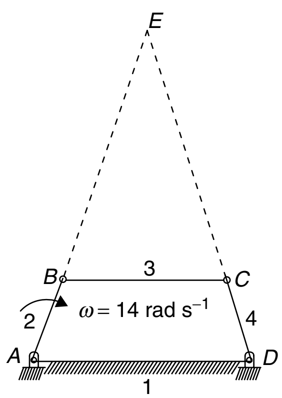
Direction for questions 14 and 15:



14. The magnitude of relative velocity of B with respect to A at that instant is
(A) zero (B) 10 m s^{-1}
(C) 17.32 m s^{-1} (D) 20 m s^{-1}

15. The angular velocity of link AB at that instant is of magnitude
(A) 20 rad s^{-1} (B) 10 rad s^{-1}
(C) 2.5 rad s^{-1} (D) 5 rad s^{-1}

Direction for questions 16 to 18:



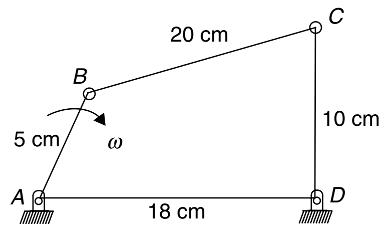
In the 4-bar linkage shown, link 1 (AD) is the fixed link. Link 2 (AB) rotates at constant angular velocity of 14 rad s^{-1}, clockwise. Link 3 (BC) and link 4 (CD) are connected by pin joint C. The radius of pin at B is 20 mm. Point E is the intersection of AB extended and DC extended. Given AB = 2 m and EB = 3.5 m.

16. Point E represents instantaneous centre
(A) I_{24} (B) I_{31} (C) I_{43} (D) I_{23}

17. The angular velocity of link 3 with respect to link 1 (ω_{31}) at that instant is
(A) 8 rad s^{-1}, counter clockwise
(B) 24.5 rad s^{-1}, clockwise
(C) 24.5 rad s^{-1}, counter clockwise
(D) 8 rad s^{-1}, clockwise

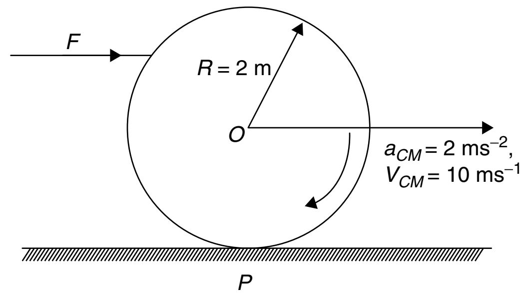
18. The rubbing velocity at the pin joint B at that instant is (in ms^{-1})
(A) 0.12 (B) 0.77 (C) 0.21 (D) 0.44

19.



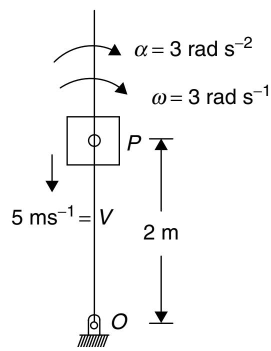
The link lengths in the figure are AB = 5 cm, BC = 20 cm, CD = 10 cm and DA = 18 cm (fixed link). AB is the crank rotating clockwise at constant angular speed of 2 rad s^{-1}. In the extended dead centre position, the transmission angle (γ) and the mechanical advantage are
(A) 31.23°, ∞ (B) 23.61°, ∞
(C) 36.68°, ∞ (D) 43.53°, 0

20.



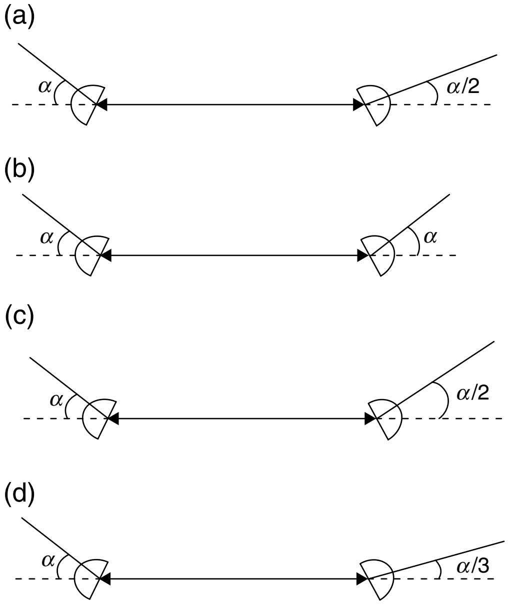
A thin uniform disc is rolling without slip on a horizontal floor under the action of a constant horizontal force F applied as shown. The radius of disc R = 2 m, the translational acceleration of centre of mass (a_{cm}) is 2 m s^{-2} and at that instant shown, the velocity of centre of mass (V_{CM}) = 10 ms^{-1}. The magnitude of the acceleration of point of contact P(in ms^{-2}) at that instant is
(A) zero (B) 50 (C) 50.16 (D) 25

21.

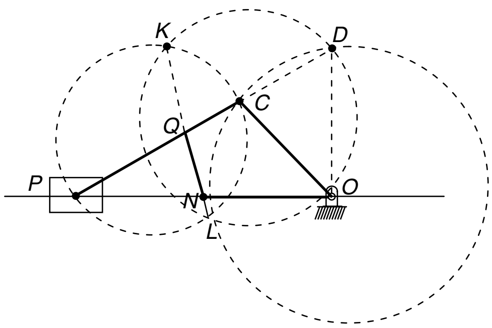


A link OA is rotating with an instantaneous angular velocity of 3 rad s^{-1} in clockwise direction at an instant when a slider P is at a distance of 2 m from hinge O and sliding towards the hinge with a velocity 5 ms^{-1} relative to link OA. The total acceleration of slider P at that instant if the angular acceleration of the link OA is 3 rad s^{-2}, is of magnitude (in ms^{-2})
(A) 30 (B) 34.99 (C) 40.25 (D) 24

22. Which one of the following figures representing Hooke's jointed inclined shaft system will result in a velocity ratio of unity?



23.



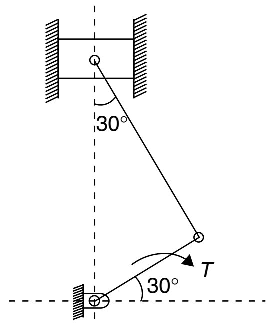
The Klein's construction for determining the acceleration of piston P is shown in the figure. When point N coinsides with crank centre O,

(A) acceleration of piston is zero and its velocity is zero
(B) acceleration of piston is maximum and its velocity is maximum
(C) acceleration of piston is maximum and its velocity is zero
(D) acceleration of piston is zero and its velocity is maximum

24. In a single slider crank mechanism, the crank rotates at 1500 rpm. The crank length is 60 mm and connecting rod length is 300 mm. When the crank has rotated through 60° from the inner dead centre position, the velocity of the piston (in m/s) and the angular velocity of the connecting rod (in rad s^{-1}) are respectively

(A) 11.45 and 19.75 (B) 3.45 and 9.63
(C) 8.98 and 15.95 (D) 5.41 and 12.44

25.

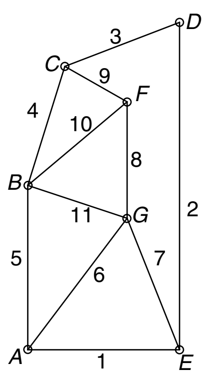


The figure shows the schematic diagram of an IC engine producing a torque $T = 50$ Nm at the given instant. The Coulomb friction coefficient between the cylinder and the piston is 0.08. If the mass of the piston is 0.45 kg and the crank radius is 0.1 m, the Colombian friction force occurring at the piston cylinder interface is

(A) 16 N (B) 20 N
(C) 18.4 N (D) 24.3 N

Practice Problems 2

1.



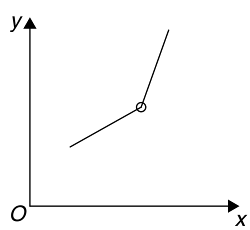
The chain shown in figure is

(A) an unconstrained chain
(B) a completely constrained chain
(C) a structure
(D) one whose nature cannot be determined

2. In Qn. No. 1, if link BG (link #11) is removed, what is the nature of the chain now?

(A) Locked chain (structure)
(B) completely constrained kinematic chain
(C) unconstrained kinematic chain
(D) cannot be determined

3.



The two-link system shown in figure is constrained to move in the XY plane (i.e. planar motion). It possesses
(A) 2 degrees of freedom
(B) 1 degree of freedom
(C) 3 degrees of freedom
(D) 6 degrees of freedom

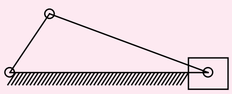
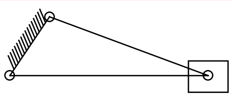
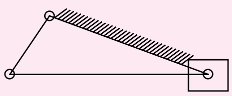
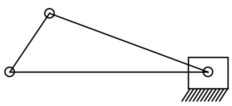
4. Match List-I with List-II and select the correct answer using the codes given below the lists.

List-I	List-II
P. 4 links, 4 turning pairs	**1.** Complete constraint
Q. 3 links, 3 turning pairs	**2.** Successful constraint
R. 5 links, 5 turning pairs	**3.** Rigid Frame
S. Foot step bearing constraint	**4.** Incomplete

Codes:

	P	Q	R	S
(A)	3	1	4	2
(B)	1	3	2	4
(C)	3	1	2	4
(D)	1	3	4	2

5. Match List-I (Kinematic Inversions) with List-II (application) and select the correct answer.

List-I	List-II
a. 	**1.** Hand Pump
b. 	**2.** Compressor
c. 	**3.** Whitworth quick return mechanism
d. 	**4.** Oscillating cylinder engine

Codes:

	A	B	C	D
(A)	1	3	4	2
(B)	2	4	3	1
(C)	2	3	4	1
(D)	1	4	3	2

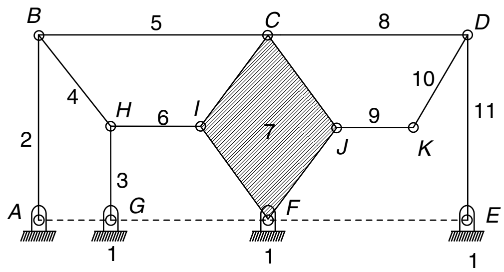
6. Match List-I with List-II and select the correct answer.

List-I (Mechanism)	List-II (Motion)
P. Hart mechanism	**1.** Quick return motion
Q. Pantograph	**2.** Copying mechanism
R. Whitworth mechanism	**3.** Exact straight line motion
S. Scotch Yoke	**4.** Simple harmonic motion
	5. Approximate straight line motion

Codes:

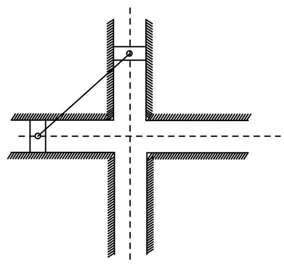
	P	Q	R	S
(A)	5	1	2	3
(B)	3	2	1	4
(C)	5	2	1	3
(D)	3	1	2	4

7. The minimum number of links in a constrained planar mechanism involving revolute pairs and two higher pairs is
(A) 3 (B) 4
(C) 5 (D) 6

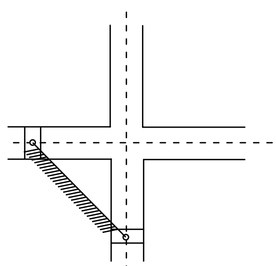
8. 

The chain shown in figure is (Link 7 is made from a single piece)
(A) statically determinate structure.
(B) statically indeterminate structure.
(C) completely constrained mechanism.
(D) incompletely constrained chain.

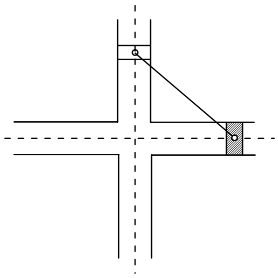
9. Three possible inversions of the double slider crank chain are shown in the figures below. The link shown in hatched is the fixed link.



Inversion 1



Inversion 2



Inversion 3

Which one of the following statements is correct?

(A) Inversion (1) is for elliptical trammel and inversion (2) is for Oldham coupling
(B) Inversion (1) is for elliptical trammel and inversion (3) is for Oldham coupling
(C) Inversion (2) is for elliptical trammel and inversion (3) is for Oldham coupling
(D) Inversion (3) is for elliptical trammel and inversion (2) is for Oldham coupling

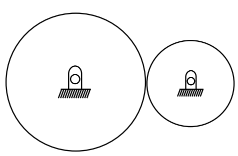
10. In an Oldham coupling, the diameter of the driving shaft is 50 mm and the diameter of the driven shaft is 40 mm. The disc with diametral tongue on each face has its centre moving in a circle of diameter 55 mm. If the driving shaft is rotating at 150 rpm, the distance between the axes of the shafts and maximum speed of sliding of the tongue of disc along the groove are respectively

(A) 72.5 mm, 0.864 m/s
(B) 55 mm, 0.864 m/s
(C) 100 mm
(D) 55 mm, 0.432 m/s

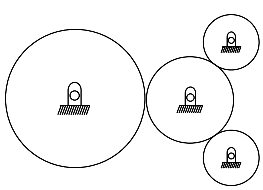
11. The driver and driven shafts connected by Hooke's joint are inclined by an angle α to each other. The angle through which the driver shaft turns is given by θ. The condition for the two shafts to have equal speed is

(A) $\cos\theta = \sin\alpha$
(B) $\sin\theta = \pm\sqrt{\tan\alpha}$
(C) $\tan\theta = \pm\sqrt{\cos\alpha}$
(D) $\cot\theta = \cos\alpha$

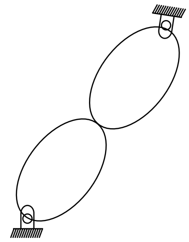
12.



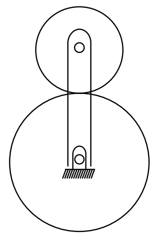
(i)



(ii)



(iii)



(iv)

Which of the mechanisms shown in the figures has/have more than single degree of freedom? (consider all gears have slipping and rolling motion)

(A) 3 and 4 (B) 2 and 3
(C) 3 only (D) 4 only

13. Consider the following statements.

(1) The degree of freedom for lower kinematic pairs is always equal to one.
(2) A ball and socket joint has 3 degrees of freedom and is higher kinematic pair.
(3) Scott − Russel mechanism is an exact straight line motion mechanism and it has three revolute pairs and one sliding pair.

Which of the above statements is/are correct?

(A) 1, 2 and 3 (B) 1 only
(C) 2 and 3 only (D) 3 only

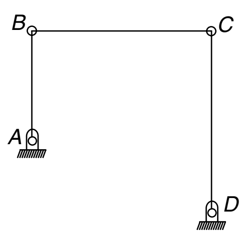
14. Instantaneous center of a body rolling with sliding on a stationary curved surface lies

(A) at the point of contact
(B) on the common normal at the point of contact
(C) on the common tangent at the point of contact
(D) at the centre of curvature of the stationary surface

15. A solid disc of radius r rolls without slipping on a horizontal floor with angular velocity ω and angular acceleration α. The magnitude of the acceleration of the point of contact on the disc is

(A) zero (B) $r\alpha$
(C) $\sqrt{(\gamma\alpha)^2 + (r\omega^2)^2}$ (D) $r\omega^2$

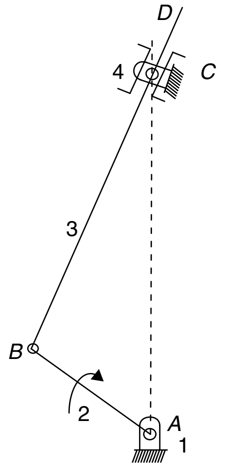
16. In the given figure, *ABCD* is a 4-bar mechanism. At the instant shown, links *AB* and *CD* are vertical and link *BC* is horizontal. *AB* is shorter than *CD* by 30 cm. *AB* is rotating at 5 rad s^{-1} and *CD* is rotating at 2 rad s^{-1}.



The length of link *AB* is

(A) 10 cm (B) 20 cm
(C) 30 cm (D) 50 cm

17.



In the mechanism *ABCD* shown in figure, link 1 is the fixed link, link 2 (crank *AB*), link 3 (rocker *BD*) and link 4 (swivel trunnion at *C*) respectively. The instantaneous centre I_{41} is at

(A) the centre of swivel trunnion
(B) the intersection of line *AB* and a perpendicular to *BD*
(C) infinity, along *AC*
(D) infinity, perpendicular to *BD*

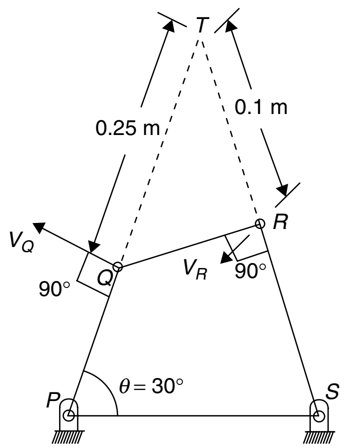
18. The number of links in a planar mechanism with only revolute joints having 21 instantaneous centres is

(A) 5 (B) 6
(C) 7 (D) 8

19. A slider sliding at 20 cm/s on a link which is rotating at 120 rpm is subjected to Coriolis acceleration of magnitude

(A) $160\,\pi^2$ cm/s^2 (B) $160\,\pi$ cm/s^2
(C) $80\,\pi$ cm/s^2 (D) $320\,\pi$ cm/s^2

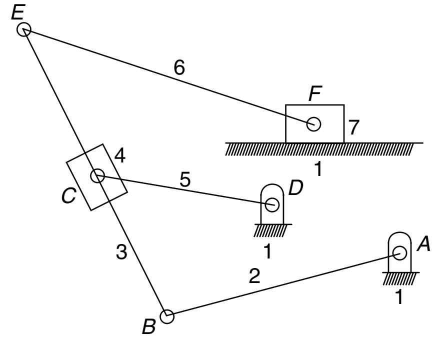
20.



A 4 bar mechanism *PQRS* is shown in the given figure. The linear velocity of point *Q* is 0.75 m/s. The magnitude of linear velocity V_R of point *R* will be

(A) 0.3 ms^{-1} (B) 0.5 ms^{-1}
(C) 0.25 ms^{-1} (D) 0.4 ms^{-1}

21.

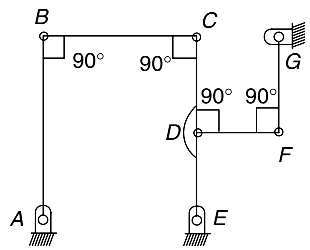


Which of the statements given below are correct?

(1) I_{34} is at ∞, perpendicular to *BE*
(2) I_{45} is at ∞, perpendicular to *BE*
(3) I_{71} is at *F*
(4) I_{45} is at *C*

(A) 1 and 2 (B) 1 and 4
(C) 2 and 3 (D) 1, 3 and 4

22.



GF = 50 mm
FD = 60 mm
ED = 50 mm
DC = 50 mm
EC = is a single link
CB = 60 mm
BA = 100 mm

For the position shown in figure, if link *GF* has momentarily an angular velocity of 2 rad s^{-1} without any angular acceleration, then the velocity and acceleration of point *B* will be

(A) 200 mm/s, 100 mm/s^2
(B) 200 mm/s, 300 mm/s^2
(C) 200 mm/s, 400 mm/s^2
(D) 100 mm/s, 100 mm/s^2

23. Given ϕ = angle through which the axis of the outer forward wheel turns,

θ = angle through which the axis of the inner forward wheel turns

w = distance between pivots of front axle and

L = wheel base

For correct steering, centre lines of the axes of the two front wheels and two rear wheels of an automobile should meet at a common point. This condition will be satisfied if

(A) $\cos\phi - \cos\theta = \frac{w}{L}$

(B) $\cot\phi - \cot\theta = \frac{w}{L}$

(C) $\cot\theta - \cot\phi = \frac{w}{L}$

(D) $\tan\theta + \tan\phi = \frac{w}{L}$

24. Examine the following statements.

(1) Davis steering gear fulfills the fundamental equation for correct steering in all positions.
(2) Ackermann steering gear fulfills the fundamental equation for correct steering in all positions.
(3) Davis steering gear is mounted on front side of front axle while Ackermann steering gear is mounted on rear side of front axle.
(4) Both Davis steering gear and Ackermann steering gear are inversions of Quadric cyclic chain.
(5) Davis steering gear is always more accurate than Ackermann steering gear.

The true statements are

(A) 1, 3 and 5 only. (B) 1, 2, 3, 4 and 5.
(C) 1, 3 and 4 only. (D) 1 and 3 only.

25.

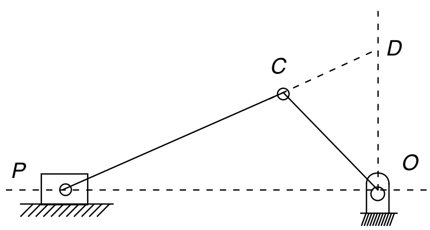
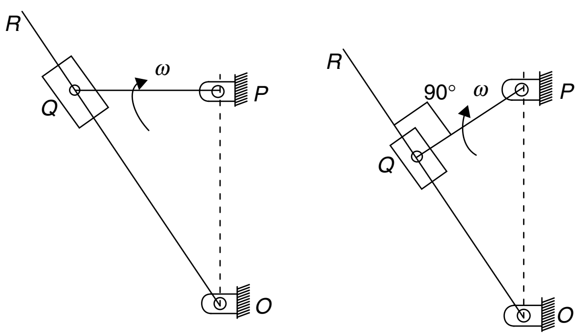


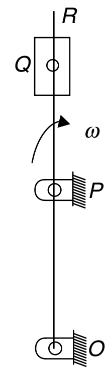
Figure shows Klein's construction for slider-crank mechanism *OCP* drawn to full scale. What velocity does *CD* represent?

(A) Velocity of the crank pin.
(B) Velocity of the piston.
(C) Velocity of piston with respect to crank pin.
(D) Angular velocity of the connecting rod.

26.



(i) (ii)



(iii) Three positions of a quick return mechanism are shown. In which of the cases does the Coriolis component of acceleration exist?

Select the correct answer using the codes given below.

(A) (i) only (B) (i) and (ii) only
(C) (i), (ii) and (iii) (D) (i) and (iii) only

27. Match List-I with List-II and select the correct answer (Notations have their usual meanings) using the codes given below the lists.

List-I	List-II
P. Law of correct steering	**1.** $f = 3(n-1) - 2j$
Q. Displacement relation of Hooke's joint	**2.** $x = R\left[(1-\cos\theta) - \left(n - \sqrt{n^2 - \sin^2\theta}\right)\right]$
R. Relation between kinematic pairs and links	**3.** $\cot\phi - \cot\theta = \frac{w}{L}$
S. Displacement equation of reciprocating engine piston	**4.** $\tan\theta = \tan\phi\cos\alpha$

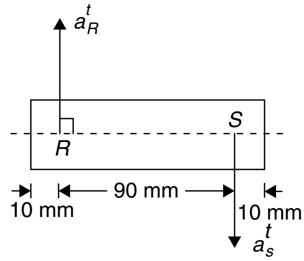
Codes:

	P	Q	R	S
(A)	1	4	3	2
(B)	1	2	3	4
(C)	3	4	1	2
(D)	3	2	1	4

28. A motor car has wheel base of 280 cm and the pivot distance of front stub axles is 140 cm. When the outer wheel has turned through 30°, the angle of the turn of the inner front wheel for correct steering will be

(A) 60° (B) $\cot^{-1} 2.33$
(C) $\cot^{-1} 1.23$ (D) 30°

29.



The figure shows a rigid body undergoing planar motion. The absolute tangential acceleration of the points *R* and *S* on the body are 150 mm/s^2 and 300 mm/s^2 respectively in the directions shown. The angular acceleration of the rigid body at that instant is

(A) 5.00 rad s^{-2} (B) 2.50 rad s^{-2}
(C) 3.33 rad s^{-2} (D) 1.66 rad s^{-2}

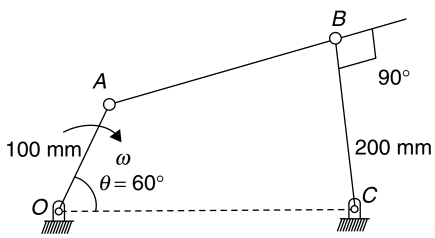
30. A single cylinder, four-stroke *IC* engine rotating at 900 rpm has crank length of 50 mm and a connecting rod length of 200 mm. If the effective reciprocating mass of the engine is 1.2 kg, the approximate magnitude of the maximum 'shaking force' created by the engine is

(A) 533 N (B) 666 N
(C) 133 N (D) None of these

31. In a reciprocating engine mechanism, the crank and connecting rod are of same length (each equal to *r* metre). The crank is rotating with a constant angular velocity of ω rad s^{-1}. At the instant when the crank is making an angle of 45° with IDC position, the angular acceleration of connecting rod is

(A) $2\omega^2 r$ (B) $\omega^2 r$ (C) $\dfrac{\omega^2}{r}$ (D) zero

Direction for questions 32 and 33:



In the 4-bar linkage shown, crank *OA* = 100 mm and rotates clockwise at a constant angular velocity of *w* rad s^{-1}. Link *OC* is the fixed horizontal link. Link *BC* = 200 mm long. At an instant when OA makes 60° with *OC* as shown in figure, $\angle OAB$ = 150° and $\angle ABC$ = 90°. The input power is given to the crank and the output power is taken from *CB* at a mechanical efficiency of 70%.

32. The mechanical advantage for the instant shown in figure is

(A) 1.9 (B) 2.4
(C) 2.8 (D) 3.2

33. If the input torque to the crank is 210 Nm, the output torque from the follower for the instant shown is

(A) 504 Nm (B) 399 Nm
(C) 672 Nm (D) 588 Nm

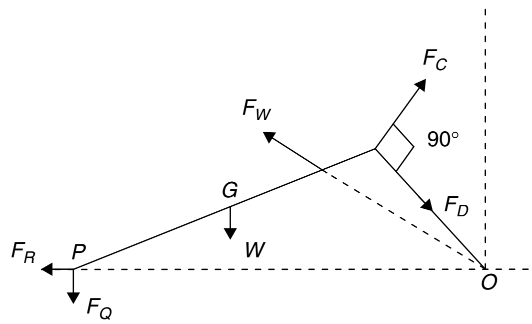
34. For a slider-crank mechanism with radius of crank *r*, length of connecting rod ℓ, obliquity ratio *n*, crank rotating at an angular velocity ω, for any angle θ of the crank, match List-I (kinematic variable) with List-II (equation) and select the correct answer using the codes given in the following lists.

List-I (Kinematic variable)	List-II (Equation)
P. Velocity of piston	**1.** $\frac{\omega}{n}\cos\theta$
Q. Acceleration of piston	**2.** $\omega^2 r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$
R. Angular Velocity of connecting rod	**3.** $-\frac{\omega^2}{n}\sin\theta$
S. Angular acceleration of connecting rod	**4.** $\omega r\left(\sin\theta + \frac{\sin 2\theta}{n}\right)$

Codes:

	P	Q	R	S
(A)	4	2	3	1
(B)	2	4	3	1
(C)	4	2	1	3
(D)	2	4	1	3

35.



With reference to engine mechanism shown in given figure, match List-I with List-II and select the correct answer.

List-I	List-II
P. F_Q	**1.** Inertia force of reciprocating mass
Q. F_R	**2.** Inertia force of connecting rod
R. F_W	**3.** Crank effort
S. F_C	**4.** Piston side thrust

Codes:

	P	Q	R	S
(A)	1	2	4	3
(B)	1	2	3	4
(C)	4	1	2	3
(D)	4	1	3	2

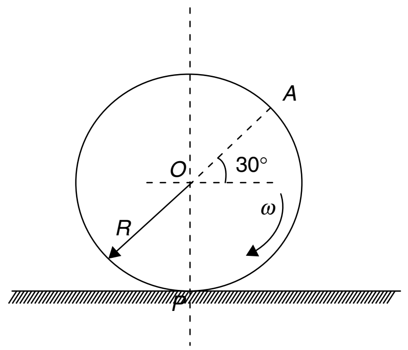
36. A connecting rod has a mass of 0.5 kg. The radius of gyration through its centre of gravity is 5 cm and its acceleration is 2×10^4 rad s^{-2}. The equivalent two mass system for the connecting rod has radius of gyration 6 cm. The correction couple of the equivalent system is

(A) 11 *Nm* (B) 9 *Nm*
(C) 6 *Nm* (D) 2 *Nm*

37. Which one of the following sets of acceleration is involved in the motion of the piston inside the cylinder of a uniformly rotating cylinder mechanism?

(A) Coriolis and radial acceleration.
(B) Radial and tangential acceleration.
(C) Coriolis and gyroscopic acceleration.
(D) Gyroscopic and tangential acceleration.

38.



A thin uniform disc of radius R metre is rolling without slipping on a horizontal surface, at a constant angular velocity ω rad s^{-1}. O is the centre of mass of disc and P is the point of contact. The point A on circumference (see figure) has a speed of (in ms^{-1})

(A) $\sqrt{2}\, R\omega$ (B) $\sqrt{3}\, R\omega$

(C) $\sqrt{5}\, R\omega$ (D) $\sqrt{\dfrac{7}{3}}\, R\omega$

39. Statement 1: Hydraulic fluid is one form of Link.

Statement 2: A link need not necessarily be a rigid body but it must be a resistant body.

Then,

(A) Statement 1 is true, statement 2 is true and statement 2 is the correct explanation for statement 1.
(B) Statement 1 is true, statement 2 is true but statement 2 is not the correct explanation for statement 1.
(C) Statement 1 is true but statement 2 is false.
(D) Both statements are false.

40. In a crank and slotted link quick return mechanism, the distance between the fixed centres is 300 mm and the radius of crank is 120 mm. If the uniform angular velocity of the crank is 10 rad s^{-1}, the quick return ratio (QRR) and the maximum angular velocity of the slotted link (in rad s^{-1}) are respectively

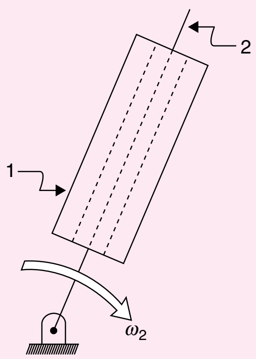
(A) 2.24, 8.33
(B) 1.33, 5.67
(C) 1.71, 6.67
(D) 2.92, 4.33

Previous Years' Questions

1. For a mechanism shown below, the mechanical advantage for the given configuration is **[2004]**

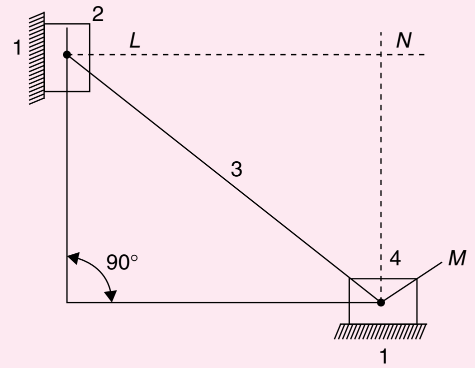
(A) 0 (B) 0.5 (C) 1.0 (D) ∞

2. In the figure shown, the relative velocity of link 1 with respect to link 2 is 12 m/sec. Link 2 rotates at a constant speed of 120 rpm. The magnitude of Coriolis component of acceleration of link 1 is **[2004]**



(A) 302 m/s^2 (B) 604 m/s^2
(C) 906 m/s^2 (D) 1208 m/s^2

3. The figure below shows a planar mechanism with single degree of freedom. The instant center 24 for the given configuration is located at a position **[2004]**



(A) L (B) M (C) N (D) ∞

4. Match the following:

Type of mechanism	Motion achieved
P. Scott–Russell mechanism	**1.** Intermittent motion
Q. Geneva mechanism	**2.** Quick return motion
R. Off set slider-crank mechanism	**3.** Simple harmonic motion
S. Scotch yoke mechanism	**4.** Straight line motion

[2004]

(A) P – 2 Q – 3 R – 1 S – 4
(B) P – 3 Q – 2 R – 4 S – 1
(C) P – 4 Q – 1 R – 2 S – 3
(D) P – 4 Q – 3 R – 1 S – 2

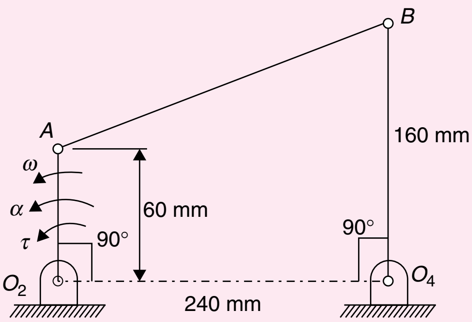
5. Match the following with respect to spatial mechanisms.

Type of joint	Degree of constraint
P. Revolute	**1.** Three
Q. Cylindrical	**2.** Five
R. Spherical	**3.** Four
	4. Two
	5. Zero

[2004]

(A) P–2 Q–3 R–3
(B) P–5 Q–4 R–3
(C) P–2 Q–3 R–1
(D) P–4 Q–5 R–3

Direction for questions 6 to 8: An instantaneous configuration of a 4-bar mechanism, whose plane is horizontal, is shown in the figure below. At this instant, the angular velocity and angular acceleration of link O_2A are ω = 8 rad s^{-1} and $\alpha = 0$, respectively, and the driving torque (τ) is zero. The link O_2A is balanced so that its center of mass falls at O_2.



6. Which kind of 4-bar mechanism is O_2ABO_4? **[2005]**
(A) Double-crank mechanism,
(B) Crank-rocker mechanism
(C) Double-rocker mechanism
(D) Parallelogram mechanism

7. At the same instant considered, what is the magnitude of the angular velocity of O_4B? **[2005]**
(A) 1 rad s^{-1} (B) 3 rad s^{-1}
(C) 8 rad s^{-1} (D) $\frac{64}{3}$ rad s^{-1}

8. At the same instant, if the component of the force at joint A along AB is 30 N, then the magnitude of the joint reaction at O_2 **[2005]**
(A) is zero
(B) is 30 N
(C) is 78 N
(D) Cannot be determined from the given data

9. For a 4-bar linkage in toggle position, the value of mechanical advantage is **[2006]**
(A) 0.0 (B) 0.5
(C) 1.0 (D) ∞

10. The number of inversions for a slider crank mechanism is **[2006]**
(A) 6 (B) 5
(C) 4 (D) 3

11. Match the items in columns I and II.

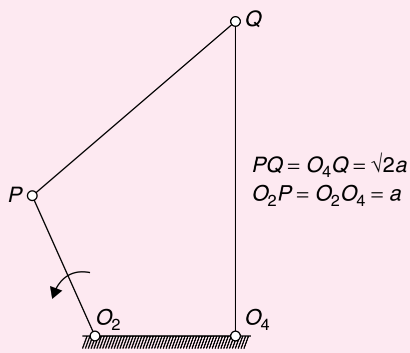
Column I	Column II
P. Higher kinematic pair	**1.** Grubler's equation
Q. Lower kinematic pair	**2.** Line contact
R. Quick return mechanism	**3.** Euler's equation
S. Mobility of a linkage	**4.** Planer
	5. Shaper
	6. Surface contact

[2006]

(A) P – 2 Q – 6 R – 4 S – 3
(B) P – 6 Q – 2 R – 4 S – 1
(C) P – 6 Q – 2 R – 5 S – 3
(D) P – 2 Q – 6 R – 5 S – 1

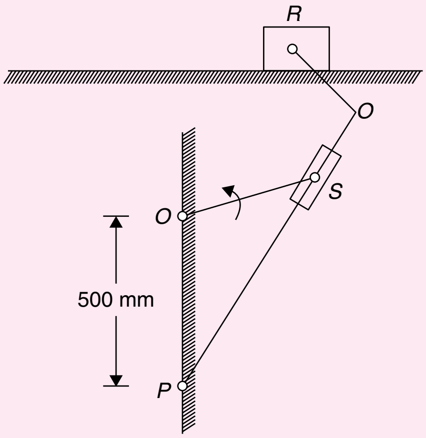
12. In a 4-bar linkage, S denotes the shortest link length, L is the longest link length, P and Q are the lengths of other two links. At least one of the three moving links will rotate by 360° if **[2006]**
(A) $S + L \leq P + Q$ (B) $S + L > P + Q$
(C) $S + P \leq L + Q$ (D) $S + P > L + Q$

13. The input link O_2P of a 4-bar linkage is rotated at 2 rad s^{-1} in counter clockwise direction as shown below. The angular velocity of the coupler PQ in rad s^{-1}, at an instant when $\angle O_4O_2P = 180°$, is **[2007]**



(A) 4 (B) $2\sqrt{2}$
(C) 1 (D) $1/\sqrt{2}$

Direction for questions 14 and 15: A quick return mechanism is shown as follows. The crank OS is driven at 2 rev/s in counter clockwise direction.



14. If the quick return ratio is 1:2, then the length of the crank in mm is **[2007]**

(A) 250 (B) $250\sqrt{3}$
(C) 500 (D) $500\sqrt{3}$

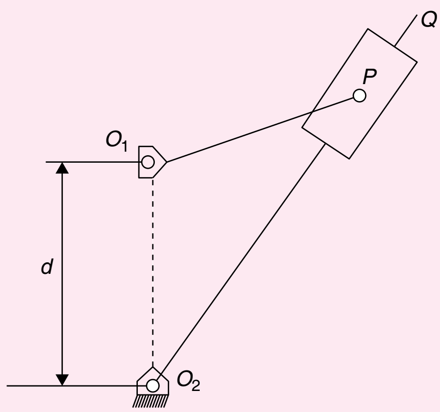
15. The angular speed of *PQ* in rev/s when the block *R* attains maximum speed during forward stroke (stroke with slower speed) is **[2007]**

(A) 1/3 (B) 2/3
(C) 2 (D) 3

16. A planar mechanism has 8 links and 10 rotary joints. The number of degrees of freedom of the mechanism, using Gruebler's criterion, is **[2008]**

(A) 0 (B) 1 (C) 2 (D) 3

17. A simple quick return mechanism is shown in the figure. The forward to return ratio of the quick return mechanism is 2 : 1. If the radius of the crank O_1P is 125 mm, then the distance '*d*' (in mm) between the crank centre to lever pivot centre point should be **[2009]**



(A) 144.3 (B) 216.5
(C) 240.0 (D) 250.0

18. Match the approaches given in the following table to perform stated kinematics/dynamics analysis of machine.

Analysis	Approach
P. Continuous relative rotation	**1.** D' Alembert's principle
Q. Velocity and acceleration	**2.** Grubler's criterion
R. Mobility	**3.** Grashoff's law
S. Dynamicstatic analysis	**4.** Kennedy's theorem

[2009]

(A) P–1, Q–2, R–3, S–4
(B) P–3, Q–4, R–2, S–1
(C) P–2, Q–3, R–4, S–1
(D) P–4, Q–2, R–1, S–3

19. Mobility of a statically indeterminate structure is **[2010]**

(A) ≤ −1 (B) 0
(C) 1 (D) ≥ 2

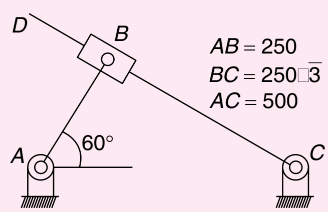
20. There are two points *P* and *Q* on a planar rigid body. The relative velocity between the two points **[2010]**

(A) should always be along *PQ*
(B) can be oriented along any direction
(C) should always be perpendicular to *PQ*
(D) should be along *QP* when the body undergoes pure translation

21. Which of the following statements is INCORRECT? **[2010]**

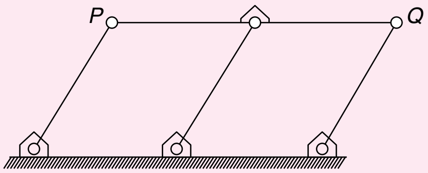
(A) Grashof's rule states that for a planar crank-rocker 4-bar mechanism, the sum of the shortest and longest link lengths cannot be less than the sum of the remaining two link lengths.
(B) Inversions of a mechanism are created by fixing different links one at a time.
(C) Geneva mechanism is an intermittent motion device
(D) Grubbler's criterion assumes mobility of a planar mechanism to be one.

22. For the configuration shown, the angular velocity of link *AB* is 10 rad s^{-1} counterclockwise. The magnitude of the relative sliding velocity (in ms^{-1}) of slider *B* with respect to rigid link *CD* is **[2010]**



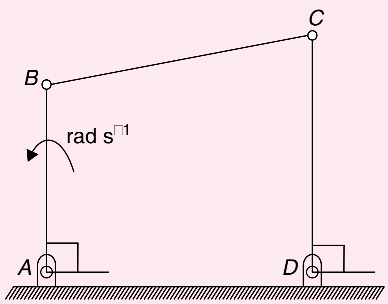
(A) 0 (B) 0.86
(C) 1.25 (D) 2.5

23. A double-parallelogram mechanism is shown in the figure. Note that *PQ* is a single link. The mobility of the mechanism is **[2011]**



(A) −1 (B) 0 (C) 1 (D) 2

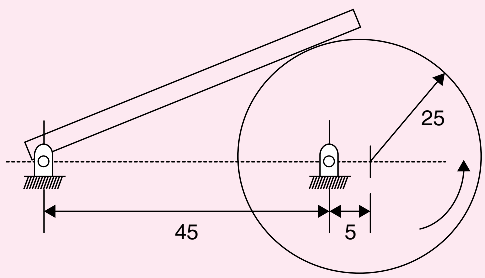
24. For the 4-bar linkage shown in the figure, the angular velocity of link *AB* is 1 rad s^{-1}. The length of link *CD* is 1.5 times the length of link *AB*. In the configuration shown, the angular velocity of link *CD* in rad s^{-1} is **[2011]**



(A) 3 (B) $\frac{3}{2}$ (C) 1 (D) $\frac{2}{3}$

25. In the mechanism given below, if the angular velocity of the eccentric circular disc is 1 rad s^{-1}, the angular velocity (rad s^{-1}) of the follower link for the instant shown in the figure is

Note: All dimensions are in mm. **[2012]**



(A) 0.05 (B) 0.1 (C) 5.0 (D) 10.0

26. A planar closed kinematic chain is formed with rigid links *PQ* = 2.0 m, *QR* = 3.0 m, *RS* = 2.5 m and *SP* = 2.7 m with all revolute joints. The link to be fixed to obtain a double rocker (rocker-rocker) mechanism is **[2013]**

(A) *PQ* (B) *QR* (C) *RS* (D) *SP*

27. A circular object of radius *r* rolls without slipping on a horizontal level floor with the center having velocity *V*. The velocity at the point of contact between the object and the floor is **[2014]**

(A) zero.
(B) *V* in the direction of motion.
(C) *V* opposite to the direction of motion.
(D) *V* vertically upward from the floor.

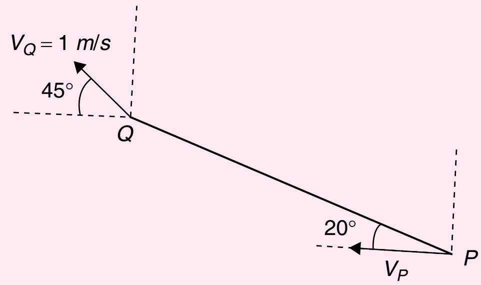
28. For the given statements:

I. Mating spur gear teeth is an example of higher pair.
II. A revolute joint is an example of lower pair.

Indicate the correct answer. **[2014]**

(A) Both I and II are false
(B) I is true and II is false
(C) I is false and II is true
(D) Both I and II are true

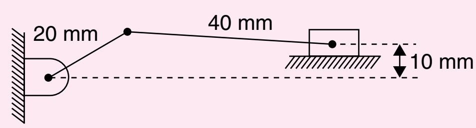
29. A rigid link *PQ* is 2 m long and oriented at 20° to the horizontal as shown in the figure. The magnitude and direction of velocity V_Q, and the direction of velocity V_P are given. The magnitude of V_P (in m/s) at this instant is **[2014]**



(A) 2.14 (B) 1.89 (C) 1.21 (D) 0.96

30. A slider crank mechanism has slider mass of 10 kg, stroke of 0.2 m and rotates with a uniform angular velocity of 10 rad s^{-1}. The primary inertia forces of the slider are partially balanced by a revolving mass of 6 kg at the crank, placed at a distance equal to crank radius. Neglect the mass of connecting road and crank. When the crank angle (with respect to slider axis) is 30°, the unbalanced force (in newton) normal to the slider axis is ______ **[2014]**

31. An offset slider–crank mechanism is shown in the figure at an instant. Conventionally, the Quick Return Ratio (*QRR*) is considered to be greater than one. The value of *QRR* is ______ **[2014]**

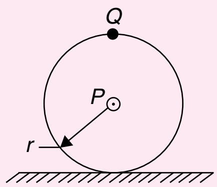


32. A 4-bar mechanism with all revolute pairs has link lengths l_F = 20 mm, l_{in} = 40 mm, l_{co} = 50 mm and l_{out} = 60 mm. The suffixes '*f*', '*in*', '*co*' and '*out*' denote the fixed link, the input link, the coupler and output link respectively. Which one of the following statements is true about the input and output links? **[2014]**

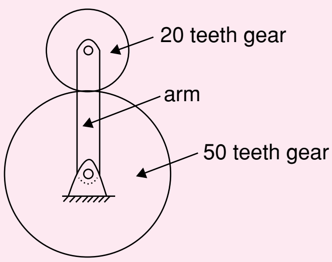
(A) Both links can execute full circular motion
(B) Both links cannot execute full circular motion
(C) Only the output link cannot execute full circular motion
(D) Only the input link cannot execute full circular motion

33. A slider-crank mechanism with crank radius 60 mm and connecting rod length 240 mm is shown in figure. The crank is rotating with a uniform angular speed of 10 rad s^{-1}, counter clockwise. For the given configuration, the speed (in m/s) of the slider is _____ **[2014]**

34. A wheel of radius r rolls without slipping on a horizontal surface shown below. If the velocity of point P is 10 m/s in the horizontal direction, the magnitude of velocity of point Q (in m/s) is _____. **[2015]**



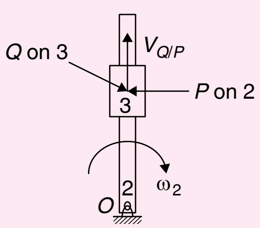
35. The number of degrees of freedom of the planetary gear train shown in the figure is: **[2015]**



(A) 0 (B) 1
(C) 2 (D) 3

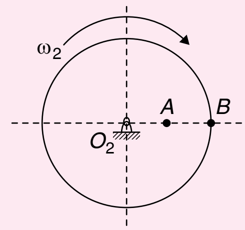
36. In a certain slider-crank mechanism, lengths of crank and connecting rod are equal. If the crank rotates with a uniform angular speed of 14 rad/s and the crank length is 300 mm, the maximum acceleration of the slider (in m/s^2) is ______. **[2015]**

37. In the figure, link 2 rotates with constant angular velocity ω_2. A slider link 3 moves outwards with a constant relative velocity $V_{Q/P}$, where Q is a point on slider 3 and P is a point on link 2. The magnitude and direction of Coriolis component of acceleration is given by: **[2015]**

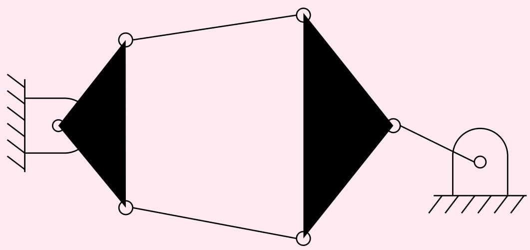


(A) $2\omega_2 V_{Q/P}$; direction of $V_{Q/P}$ rotated by 90° in the direction of ω_2.
(B) $\omega_2 V_{Q/P}$; direction of $V_{Q/P}$ rotated by 90° in the direction of ω_2.
(C) $2\omega_2 V_{Q/P}$; direction of $V_{Q/P}$ rotated by 90° opposite to the direction of ω_2.
(D) $\omega_2 V_{Q/P}$; direction of $V_{Q/P}$ rotated by 90° opposite of ω_2.

38. Figure shows a wheel rotating about O_2. Two points A and B located along the radius of wheel have speeds of 80 m/s and 140 m/s respectively. The distance between the points A and B is 300 mm. The diameter of the wheel (in mm) is ____. **[2015]**



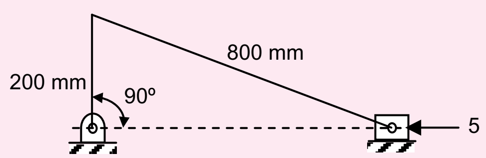
39. The number of degrees of freedom of the linkage shown in the figure is **[2015]**



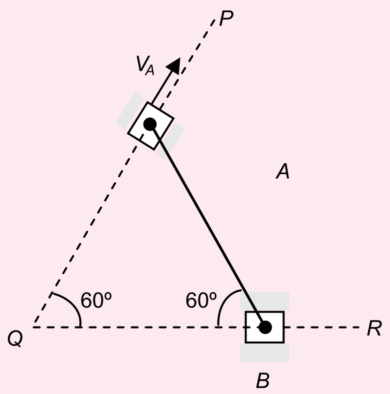
(A) −3 (B) 0
(C) 1 (D) 2

40. A car is moving on a curved horizontal road of radius 100 m with a speed of 20 m/s. The rotating masses of the engine have an angular speed of 100 rad/s in clockwise direction when viewed from the front of the car. The combined moment of inertia of the rotating masses is 10 kg-m^2. The magnitude of the gyroscopic moment (in N-m) is _______. **[2016]**

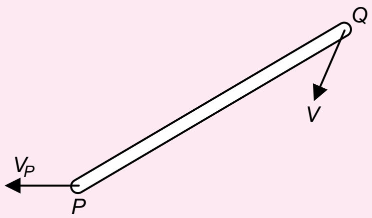
41. A slider crank mechanism with crank radius 200 mm and connecting rod length 800 mm is shown. The crank is rotating at 600 rpm in the counterclockwise direction. In the configuration shown, the crank makes an angle of 90° with the sliding direction of the slider, and a force of 5 kN is acting on the slider. Neglecting the inertia forces, the turning moment on the crank (in kN-m) is ________. **[2016]**



42. The rod AB, of length 1 m, shown in the figure is connected to two sliders at each end through pins. The sliders can slide along QP and QR. If the velocity V_A of the slider at A is 2 m/s, the velocity of the mid-point of the rod at this instant is _______ m/s. **[2016]**



43. A rigid link PQ is undergoing plane motion as shown in the figure (V_P and V_Q are non-zero). V_{QP} is the relative velocity of point Q with respect to point P. **[2016]**



Which one of the following is TRUE?

(A) V_{QP} has components along and perpendicular to PQ.

(B) V_{QP} has only one component directed from P to Q.

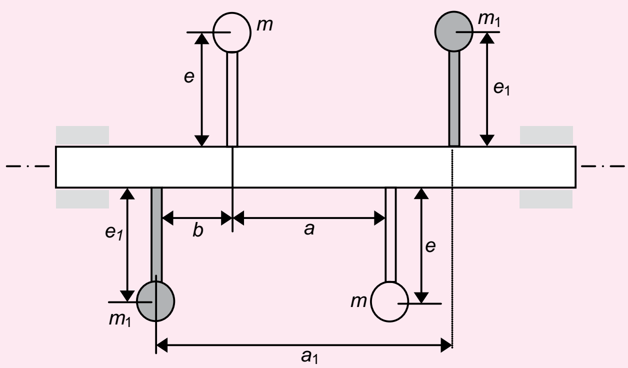
(C) V_{QP} has only one component directed from Q to P.

(D) V_{QP} has only one component perpendicular to PQ.

44. The number of degrees of freedom in a planer having n links and j simple hinge joints is:

(A) $3(n - 3) - 2j$ (B) $3(n - 1) - 2j$
(C) $3n - 2j$ (D) $2j - 3n + 4$

45. Two masses m are attached to opposite sides of a rigid rotating shaft in the vertical plane. Another pair of equal masses m_1 is attached to the opposite sides of the shaft in the vertical plane as shown in figure. Consider $m = 1$ kg, $e = 50$ mm, $e_1 = 20$ mm, $b = 0.3$ m, $a = 2$ m and $a_1 = 2.5$ m. For the system to be dynamically balanced, m_1 should be _______ kg. **[2016]**



Answer Keys

Exercises

Practice Problems 1

1. B	**2.** B	**3.** 4	**4.** B	**5.** B	**6.** C	**7.** C	**8.** D	**9.** D	**10.** D
11. A	**12.** 36	**13.** C	**14.** B	**15.** D	**16.** B	**17.** A	**18.** D	**19.** C	**20.** B
21. A	22. B	23. D	24. C	25. B					

Practice Problems 2

1. C	2. B	3. C	4. D	5. C	6. B	7. B	8. A	9. A	10. B
11. C	12. D	13. D	14. B	15. D	16. B	17. A	18. C	19. B	20. A
21. B	22. C	23. B	24. D	25. C	26. A	27. C	28. C	29. A	30. B
31. D	32. C	33. D	34. C	35. C	36. A	37. A	38. B	39. A	40. C

Previous Years' Questions

1. D	2. A	3. C	4. C	5. C	6. B	7. B	8. D	9. D	10. C
11. D	12. A	13. C	14. A	15. B	16. B	17. D	18. B	19. A	20. C
21. A	22. D	23. C	24. D	25. B	26. C	27. A	28. D	29. D	

30. 29 to 31 **31.** 1.2 to 1.3 **32.** A **33.** 0.54 to 0.68 **34.** 20 **35.** C
36. 115 to 120 **37.** A **38.** 1390 to 1410 **39.** C **40.** 200 **41.** 1 **42.** 0.95 to 1.05
43. D **44.** B **45.** 2