Linear Equations in One Variable

- An algebraic equation is an equality involving variables. In an equation, the value of expression on the left hand side (LHS) is equal to the value of expression on the right hand side (RHS).

- Linear equations that contain expressions on one side and numbers on the other can be solved by transposing terms from one side to the other or by performing same mathematical operations on both the sides.

  Example: Solve $2x - 7 = 3$

  Solution: $2x - 7 = 3$
  Transposing –7 from LHS to RHS, we obtain
  $2x = 3 + 7 = 10$
  $\Rightarrow x = 5$ [By dividing both sides by 2]

- Linear equations that contain expressions on both the sides can be solved by transposing the terms or by performing same mathematical operation on both the sides.

  Example: Solve: $2x - 4 = 3x - 2$

  Solution: Transposing $3x$ from R.H.S to L.H.S., we obtain
  $2x - 4 - 3x = -2$
  Transposing –4 from L.H.S. to R.H.S., we obtain
  $2x - 3x = -2 + 4$
  $\Rightarrow -x = 2$
  $\therefore x = -2$ [Dividing both sides by -1]

- There are certain linear equations in which the denominators of expressions on both sides are not 1. In such cases, both sides of the equation are multiplied by the LCM of the denominators on both sides.

  Example: Reduce to simpler form:
  $\frac{3x - 1}{4} + 2 = \frac{2x + 3}{6} + 7$

  Solution: LCM of 4 and 6 is 12. Therefore, multiplying both sides of the equation by 12, we obtain

$$\left(\tfrac{3x-1}{4} + 2\right) \times 12 = \left(\tfrac{2x+3}{6} + 7\right) \times 12$$

$$\Rightarrow \left(\tfrac{3x-1}{4}\right) \times 12 + 24 = \left(\tfrac{2x+3}{6}\right) \times 12 + 84$$

$$\Rightarrow 9x - 3 + 24 = 4x + 6 + 84$$

$$\Rightarrow 9x + 21 = 4x + 90$$

This is a simple linear equation which can be solved by transposing the terms.

- There are certain equations which can be reduced to linear equations.

Example: Reduce the given equation to linear equation: $\dfrac{2-4x}{3x-2} = \dfrac{3}{2}$

Solution: By cross multiplication, we obtain

$$\dfrac{2-4x}{3x-2} \times \dfrac{3}{2}$$

$$\Rightarrow (2-4x)2 = 3(3x-2)$$

$$\Rightarrow 4 - 8x = 9x - 6$$

This is a linear equation in one variable which can be solved easily.

Linear equations in one variable can be used to solve many problems.

Example: Meesha's age is 8 years more than twice her daughter's age.
Four years ago, Meesha's age was $\dfrac{7}{2}$ times her daughter's age. What is her daughter's present age?

Solution: Let Meesha's daughter's age be x years.
$\therefore$ Meesha's age = $(2x + 8)$ years

Four years ago, her daughter's age = $(x - 4)$ years
Four years ago, Meesha's age = $(2x + 8 - 4)$ years = $(2x + 4)$ years
According to the given information,

$$\left(2x + 4\right) = \tfrac{7}{2}(x - 4)$$

$$\Rightarrow 2 \times (2x + 4) = 7(x - 4)$$

$$\Rightarrow 4x + 8 = 7x - 28$$

$$\Rightarrow 4x - 7x = -28 - 8$$

$$\Rightarrow -3x = -36$$

$$\Rightarrow x = 12$$

Thus, Meesha's daughter's age is 12 years.