9. Quadratic Surd

Let us Work Out 9.1

1 A. Question

Let us write the following numbers in the form of product of rational and irrational numbers.

$\sqrt{175}$

Answer

Given:

√175

It can be written as:

√(5×5×7)

= 5√7

Hence, √175 can be written in the form of product of rational and irrational numbers as : 5√7

Where 5 is a rational number and √7 is an irrational number.

1 B. Question

Let us write the following numbers in the form of product of rational and irrational numbers.

$\sqrt{112}$

Answer

Given:

√112

It can be written as:

√(2×2×2×2×7)

= 4√7

Hence, √112 can be written in the form of product of rational and irrational numbers as : 4√7

Where 4 is a rational number and $\sqrt{7}$ is an irrational number.

1 C. Question

Let us write the following numbers in the form of product of rational and irrational numbers.

$\sqrt{108}$

Answer

Given:

$\sqrt{108}$

It can be written as:

$\sqrt{(3\times3\times3\times2\times2)}$

$= 6\sqrt{3}$

Hence, $\sqrt{108}$ can be written in the form of product of rational and irrational numbers as : $6\sqrt{3}$

Where 6 is a rational number and $\sqrt{3}$ is an irrational number.

1 D. Question

Let us write the following numbers in the form of product of rational and irrational numbers.

$\sqrt{125}$

Answer

Given:

$\sqrt{125}$

It can be written as:

$\sqrt{(5\times5\times5)}$

$= 5\sqrt{5}$

Hence, $\sqrt{125}$ can be written in the form of product of rational and irrational numbers as : $5\sqrt{5}$

Where 5 is a rational number and $\sqrt{5}$ is an irrational number.

1 E. Question

Let us write the following numbers in the form of product of rational and irrational numbers.

$5\sqrt{119}$

Answer

Given:

5√119

It can be written as:

5√(7×17)

= 5√119

Hence, 5√119 can be written in the form of product of rational and irrational numbers as : 5√119

Where 5 is a rational number and √119 is an irrational number.

2. Question

Let us prove that, $\sqrt{108} - \sqrt{75} = \sqrt{3}$

Answer

Given:

√108-√75 = √3

Take L.H.S. = √108-√75

It can be written as:

√(3×3×3×2×2)-√(3×5×5)

= 6√3-5√3

= √3 = R.H.S.

Hence, L.H.S. = R.H.S.

3. Question

Let us show that, $\sqrt{98} + \sqrt{8} - 2\sqrt{32} = \sqrt{2}$

Answer

Given:

√98 + √8-2√32 = √2

Take L.H.S. = √98 + √8-2√32

It can be written as:

$\sqrt{(2\times7\times7)} + \sqrt{(2\times2\times2)} - 2\sqrt{(2\times2\times2\times2\times2)}$

$= 7\sqrt{2} + 2\sqrt{2} - 8\sqrt{2}$

$= 9\sqrt{2} - 8\sqrt{2}$

$= \sqrt{2}$ = R.H.S.

Hence, L.H.S. = R.H.S.

4. Question

Let us show that, $3\sqrt{48} - 4\sqrt{75} + \sqrt{192} = 0$

Answer

Given:

$3\sqrt{48} - 4\sqrt{75} + \sqrt{192} = 0$

Take L.H.S. = $3\sqrt{48} - 4\sqrt{75} + \sqrt{192}$

It can be written as:

$3\sqrt{(2\times2\times2\times2\times3)} - 4\sqrt{(3\times5\times5)} + \sqrt{(2\times2\times2\times2\times2\times2\times3)}$

$= 12\sqrt{3} - 20\sqrt{3} + 8\sqrt{3}$

$= -8\sqrt{3} + 8\sqrt{3}$

$= 0$ = R.H.S.

Hence, L.H.S. = R.H.S.

5. Question

Let us simplify :

$\sqrt{12} + \sqrt{18} + \sqrt{27} - \sqrt{32}$

Answer

Given:

$\sqrt{12} + \sqrt{18} + \sqrt{27} - \sqrt{32}$

It can be written as:

$\sqrt{(2\times2\times3)} + \sqrt{(3\times3\times2)} + \sqrt{(3\times3\times3)} - \sqrt{(2\times2\times2\times2\times2)}$

$= 2\sqrt{3} + 3\sqrt{2} + 3\sqrt{3} - 4\sqrt{2}$

$= 5\sqrt{3} - 1\sqrt{2}$

6 A. Question

Let us write what should be added with $\sqrt{5}+\sqrt{3}$ to get the sum $2\sqrt{5}$.

Answer

Given:

$\sqrt{5} + \sqrt{3}$

Let the term added to $\sqrt{5} + \sqrt{3}$ is x :

It can be written as:

$\sqrt{5} + \sqrt{3} + x = 2\sqrt{5}$

$\Rightarrow x = 2\sqrt{5}-\sqrt{5}-\sqrt{3}$

$\Rightarrow x = \sqrt{5}-\sqrt{3}$

Hence, we need to add $\sqrt{5}-\sqrt{3}$ to get $2\sqrt{5}$

6 B. Question

Let us write what should be subtracted from $7-\sqrt{3}$ to get $3+\sqrt{3}$

Answer

Given:

$7-\sqrt{3}$

Let the term subtracted from $7-\sqrt{3}$ is x :

It can be written as:

$7-\sqrt{3}-x = 3 + \sqrt{3}$

$\Rightarrow x = 7-\sqrt{3}-3-\sqrt{3}$

$\Rightarrow x = 4-2\sqrt{3}$

Hence, we need to subtract $4-2\sqrt{3}$ to get $3 + \sqrt{3}$

6 C. Question

Let us write the sum of $2+\sqrt{3}, \sqrt{3}+\sqrt{5}$ and $2+\sqrt{7}$.

Answer

Given:

$2 + \sqrt{3}, \sqrt{3} + \sqrt{5}, 2 + \sqrt{7}$

Addition of all is:

It can be written as:

2 + √3 + √3 + √5 + 2 + √7

= 4 + 2√3 + √5 + √7

Hence, result is 4 + 2√3 + √5 + √7

6 D. Question

Let us subtract $\left(-5+3\sqrt{11}\right)$ from $\left(10-\sqrt{11}\right)$ and let us write the value of subtraction.

Answer

Given:

10-√11

To subtracted -5 + 3√11 is :

It can be written as:

10-√11-(-5 + 3√11)

= 10-√11 + 5-3√11

= 15-4√11

Hence, the result is 15-4√11

6 E. Question

Let us subtract $\left(5+\sqrt{2}+\sqrt{7}\right)$ from the sum of $\left(-5+\sqrt{7}\right)$ and $\left(\sqrt{7}+\sqrt{2}\right)$ and find value of subtraction.

Answer

Given: 5 + √2 + √7, - 5 + √7 and √7 + √2

Sum of – 5 + √7 and √7 + √2 is:

- 5 + √7 + √7 + √2

= - 5 + 2√7 + √2 ....(1)

subtract 5 + √2 + √7 from (1)

We get,

- 5 + 2√7 + √2-(5 + √2 + √7)

$= -5 + 2\sqrt{7} + \sqrt{2} - 5 - \sqrt{2} - \sqrt{7}$

$= -10 - \sqrt{7}$

6 F. Question

I write two quadratic surds whose sum is a rational number.

Answer

Given: two quadratic surds

A quadratic surd is an expression containing square roots, such that number under square root is a rational number and is not a perfect square.

Let we take quadratic surds as:

$7 - \sqrt{3}$ and $7 + \sqrt{3}$

Sum of these surds will be:

$7 - \sqrt{3} + 7 + \sqrt{3}$

$= 14$

Where 14 is a rational number.

Let us Work Out 9.2

1 A. Question

Let us find the product of $3\frac{1}{2}$ and $\sqrt{3}$

Answer

Given:

$3\frac{1}{2}$ and $\sqrt{3}$

It can be written as:

$\frac{7}{2}$ and $\sqrt{3}$

Product of given values :

$\frac{7}{2} \times \sqrt{3}$

$= \frac{7\sqrt{3}}{2}$

Hence, the product of

$3\frac{1}{2}$ and $\sqrt{3}$

Is $\frac{7\sqrt{3}}{2}$.

1 B. Question

Let us write what should be multiplied with $2\sqrt{2}$ to get the product 4.

Answer

Given:

$2\sqrt{2}$

Let the term multiplied to $2\sqrt{2}$ is x :

It can be written as:

$2\sqrt{2} \times x = 4$

$\Rightarrow 2\sqrt{2} \times x = 2 \times 2$

$\Rightarrow 2\sqrt{2} \times x = 2 \times \sqrt{2} \times \sqrt{2}$

$\Rightarrow x = \sqrt{2}$

Hence, we need to multiply $\sqrt{2}$ to get 4.

1 C. Question

Let us calculate the product of $3\sqrt{5}$ and $5\sqrt{3}$

Answer

Given:

$3\sqrt{5}$ and $5\sqrt{3}$

It can be written as:

$3\sqrt{5}$ and $5\sqrt{3}$

Product of given values :

$3\sqrt{5} \times 5\sqrt{3}$

$= 15\sqrt{15}$

Hence, the product of

$3\sqrt{5}$ and $5\sqrt{3}$

Is $15\sqrt{15}$.

1 D. Question

If $\sqrt{6} \times \sqrt{15} = x\sqrt{10},$ then let us write by calculating the value of x.

Answer

Given:

$\sqrt{6} \times \sqrt{15} = x\sqrt{10}$

It can be written as:

$\sqrt{2 \times 3} \times \sqrt{3 \times 5} = x\sqrt{10}$

$\sqrt{2 \times 3 \times 3 \times 5} = x\sqrt{10}$

$\Rightarrow 3\sqrt{10} = x\sqrt{10}$

$\Rightarrow x = 3.$

Hence, the value of x = 3.

1 E. Question

If $\left(\sqrt{5} + \sqrt{3}\right)\left(\sqrt{5} - \sqrt{3}\right) = 25 - x^2$ be an equation, then let us write by calculating the value of x.

Answer

Given:

$(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3}) = 25 - x^2$

It can be written as:

$(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3}) = 25 - x^2$

$= \sqrt{5}.\sqrt{5} - \sqrt{3}.\sqrt{5} + \sqrt{5}.\sqrt{3} - \sqrt{3}.\sqrt{3} = 25 - x^2$

$\Rightarrow 5 - 3 = 25 - x^2$

$\Rightarrow 2 = 25 - x^2$

$\Rightarrow -23 = -x^2$

$\Rightarrow x^2 = 23$

$\Rightarrow x = \sqrt{23}$

Hence, the value of x = $\sqrt{23}$.

2 A. Question

Let us calculate the product :

$\sqrt{7} \times \sqrt{14}$

Answer

Given:

$\sqrt{7}$ and $\sqrt{14}$

It can be written as:

$\sqrt{7}$ and $\sqrt{7 \times 2}$

Product of given values :

$\sqrt{7} \times \sqrt{7 \times 2}$

$= \sqrt{7 \times 7 \times 2}$

$= 7\sqrt{2}$

Hence, the product of $\sqrt{7}$ and $\sqrt{14}$ is $7\sqrt{2}$.

2 B. Question

Let us calculate the product :

$\sqrt{12} \times 2\sqrt{3}$

Answer

Given:

$\sqrt{12}$ and $2\sqrt{3}$

It can be written as:

$\sqrt{2 \times 2 \times 3}$ and $2\sqrt{3}$

Product of given values :

$\sqrt{2 \times 2 \times 3} \times 2\sqrt{3}$

$= 2\sqrt{2 \times 2 \times 3 \times 3}$

$= 2 \times 2 \times 3$

$= 12$

Hence, the product of $\sqrt{12}$ and $2\sqrt{3}$ is 12.

2 C. Question

Let us calculate the product :

$$\sqrt{5} \times \sqrt{15} \times \sqrt{3}$$

Answer

Given:

$$\sqrt{5} \times \sqrt{15} \times \sqrt{3}$$

It can be written as:

$$\sqrt{5} \times \sqrt{3 \times 5} \times \sqrt{3}$$

Product of given values :

$$\sqrt{5} \times \sqrt{3 \times 5} \times \sqrt{3}$$

$$= \sqrt{5 \times 3 \times 5 \times 3}$$

$= 5 \times 3$

$= 15$

Hence, the product of $\sqrt{5} \times \sqrt{15} \times \sqrt{3}$ is 15.

2 D. Question

Let us calculate the product:

$$\sqrt{2}\left(3 + \sqrt{5}\right)$$

Answer

Given:

$$\sqrt{2}(3 + \sqrt{5})$$

It can be written as:

$$3\sqrt{2} + \sqrt{2} \times \sqrt{5}$$

Product of given values :

$3\sqrt{2} + \sqrt{10}$

Hence, the product of $\sqrt{5} \times \sqrt{15} \times \sqrt{3}$ is 15.

2 E. Question

Let us calculate the product :

$$\left(\sqrt{2}+\sqrt{3}\right)\left(\sqrt{2}-\sqrt{3}\right)$$

Answer

Given:

$(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})$

It can be written as:

$(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})$

$= \sqrt{2}.\sqrt{2} - \sqrt{3}.\sqrt{2} + \sqrt{2}.\sqrt{3} - \sqrt{3}.\sqrt{3}$

= 2 – 3

= -1

Hence, the product of $(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})$ is -1.

2 F. Question

Let us calculate the product :

$$\left(2\sqrt{3}+3\sqrt{2}\right)\left(4\sqrt{2}+\sqrt{5}\right)$$

Answer

Given:

$(2\sqrt{3} + 3\sqrt{2})(4\sqrt{2} + \sqrt{5})$

It can be written as:

$(2\sqrt{3} + 3\sqrt{2})(4\sqrt{2} + \sqrt{5})$

$= 2\sqrt{3}.4\sqrt{2} + 2\sqrt{3}.\sqrt{5} + 3\sqrt{2}.4\sqrt{3} + 3\sqrt{2}.\sqrt{5}$

$= 8\sqrt{6} + 2\sqrt{15} + 12\sqrt{6} + 3\sqrt{10}$

$= 2\sqrt{15} + 20\sqrt{6} + 3\sqrt{10}$

Hence, the product of

$(2\sqrt{3} + 3\sqrt{2})(4\sqrt{2} + \sqrt{5})$ is $2\sqrt{15} + 20\sqrt{6} + 3\sqrt{10}$.

2 G. Question

Let us calculate the product :

$$\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)\left(2-\sqrt{3}\right)\left(4+2\sqrt{3}\right)$$

Answer

Given:

$(\sqrt{3} + 1)(\sqrt{3} - 1)(2 - \sqrt{3})(4 + 2\sqrt{3})$

It can be written as:

$\{(\sqrt{3} + 1)(\sqrt{3} - 1)\}\{(2 - \sqrt{3})(4 + 2\sqrt{3})\}$

$= \{(\sqrt{3}.\sqrt{3}) - \sqrt{3} + \sqrt{3} - (1.1)\}\{(2.4) + 2.2\sqrt{3} - 4\sqrt{3} - \sqrt{3}.2\sqrt{3}\}$

$= \{3 - 1\}\{8 - 2\}$

= 2× 6

= 12

Hence, the product of $(\sqrt{3} + 1)(\sqrt{3} - 1)(2 - \sqrt{3})(4 + 2\sqrt{3})$ is 12.

3 A. Question

If √x is the rationalising factor of √5, let us write by calculating what is the smallest value of x (where x is an integer)

Answer

Given: $\sqrt{5}$

As we know, rationalization factor is the factor which make the given irrational number as a rational number.

Hence, smallest rational factor of $\sqrt{5}$ is $\sqrt{5}$.

As, $\sqrt{5} \times \sqrt{5} = 5$

Which is a rational number.

3 B. Question

Let us calculate the value of $3\sqrt{2} \div 3$

Answer

Given:

$3\sqrt{2} \div 3$

$= \frac{3\sqrt{2}}{3}$

$= \sqrt{2}$

Hence, the value of $3\sqrt{2} \div 3$ is $\sqrt{2}$.

3 C. Question

Let us write which smallest factor should we multiply the denominator to rationalise the denominator of $7 \div \sqrt{48}$

Answer

Given:

$7 \div \sqrt{48}$

It can be written as:

$= \frac{7}{\sqrt{48}}$

$= \frac{7}{\sqrt{2 \times 2 \times 2 \times 2 \times 3}}$

$= \frac{7}{2\sqrt{3}}$

So, to make it rationalize we must multiply the denominator by $\sqrt{3}$.

Hence, we get,

$= \frac{7}{2\sqrt{3}} \times \frac{1}{\sqrt{3}}$

$= \frac{7}{6}$

3 D. Question

Let us calculate the rationalising factor of $\left(\sqrt{5}+2\right)$ which is also its conjugate surd.

Answer

Given: $(\sqrt{5} + 2)$

Its conjugate surd will be :

$(\sqrt{5} - 2)$

Which is also its rationalising factor.

3 E. Question

If $\left(\sqrt{5}+\sqrt{2}\right) \div \sqrt{7} = \frac{1}{7}\left(\sqrt{35}+a\right)$, Let us calculate the value of a.

Answer

Given:

$$(\sqrt{5} + \sqrt{2}) \div \sqrt{7} = \frac{1}{7}(\sqrt{35} + a)$$

It can be written as:

$$\frac{(\sqrt{5} + \sqrt{2})}{\sqrt{7}} = \frac{1}{7}(\sqrt{35} + a)$$

$$\Rightarrow \frac{(\sqrt{5} + \sqrt{2})}{\sqrt{7}} = \frac{\sqrt{7 \times 5}}{7} + \frac{a}{7}$$

$$\Rightarrow \frac{\sqrt{5}}{\sqrt{7}} + \frac{\sqrt{2}}{\sqrt{7}} = \frac{\sqrt{5}}{\sqrt{7}} + \frac{a}{7}$$

$$\Rightarrow \frac{\sqrt{2}}{\sqrt{7}} = \frac{a}{7}$$

$$\Rightarrow a = \frac{\sqrt{2}}{\sqrt{7}} \times 7$$

$$\Rightarrow a = \sqrt{2} \times \sqrt{7}$$

$$\Rightarrow a = \sqrt{14}$$

3 F. Question

Let us write a rationalizing factor of the denominator of $\frac{5}{\sqrt{3}-2}$, which is not its conjugate surd.

Answer

Given: $\frac{5}{\sqrt{3}-2}$ Conjugate surd for the given expression will be (√3 + 2), So we have to find a factor other than this

Rationalize with $4 + 2\sqrt{3}$

We get,

$$\frac{5}{\sqrt{3}-2} \times \frac{4 + 2\sqrt{3}}{4 + 2\sqrt{3}}$$

$$= \frac{20 + 10\sqrt{3}}{4\sqrt{3} + 2.3 - 4\sqrt{3} - 8}$$

$$= \frac{20 + 10\sqrt{3}}{4\sqrt{3} + 6 - 4\sqrt{3} - 8}$$

$$= \frac{10(2 + \sqrt{3})}{-2}$$

$$= -5(2 + \sqrt{3})$$

So, $4 + 2\sqrt{3}$ will be a rationalizing factor to the given expression without being its conjugation surd

4. Question

Let us write the conjugate surds of mixed quadratic surds $\left(9-4\sqrt{5}\right)$ and $\left(-2-\sqrt{7}\right)$

Answer

Given: $(9 - 4\sqrt{5})$ and $(-2 - \sqrt{7})$

Its conjugate surd will be :

$(9 + 4\sqrt{5})$ and $(-2 + \sqrt{7})$

5. Question

Let us write two conjugate surds of each mixed quadratic surds of the followings.

i. $\sqrt{5}+\sqrt{2}$

ii. $13+\sqrt{6}$

iii. $\sqrt{8}-3$

iv. $\sqrt{17}-\sqrt{15}$

Answer

(i) Given: $(\sqrt{5}+\sqrt{2})$

Its conjugate surds will be :

$(\sqrt{5}-\sqrt{2})$ and $(-\sqrt{5}+\sqrt{2})$

(ii) Given: $(13+\sqrt{6})$

Its conjugate surds will be :

$(13-\sqrt{6})$ and $(-13+\sqrt{6})$

(iii) Given: $(\sqrt{8}-3)$

Its conjugate surds will be :

$(\sqrt{8}+3)$ and $(-\sqrt{8}-3)$

(iv) Given: $(\sqrt{17}-\sqrt{15})$

Its conjugate surds will be :

$(\sqrt{17}+\sqrt{15})$ and $(-\sqrt{17}-\sqrt{15})$

6 A. Question

Let us rationalizing the denominators of the following surds.

$$\frac{2\sqrt{3}+3\sqrt{2}}{\sqrt{6}}$$

Answer

Given: $\frac{2\sqrt{3}+3\sqrt{2}}{\sqrt{6}}$

Rationalize with $\sqrt{6}$

We get,

$$\frac{2\sqrt{3}+3\sqrt{2}}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}$$

$$=\frac{\sqrt{6}(2\sqrt{3}+3\sqrt{2})}{6}$$

$$= \frac{(2\sqrt{18} + 3\sqrt{12})}{6}$$

$$= \left(\frac{2\sqrt{18}}{6} + \frac{3\sqrt{12}}{6}\right)$$

$$= \left(\frac{\sqrt{2 \times 3 \times 3}}{3} + \frac{\sqrt{2 \times 2 \times 3}}{2}\right)$$

$$= \sqrt{2} + \sqrt{3}$$

6 B. Question

Let us rationalizing the denominators of the following surds.

$$\frac{\sqrt{2} - 1 + \sqrt{6}}{\sqrt{5}}$$

Answer

Given: $\frac{\sqrt{2} - 1 + \sqrt{6}}{\sqrt{5}}$

Rationalize with $\sqrt{5}$

We get,

$$\frac{\sqrt{2} - 1 + \sqrt{6}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{\sqrt{5}(\sqrt{2} - 1 + \sqrt{6})}{5}$$

$$= \frac{(\sqrt{10} - \sqrt{5} + \sqrt{30})}{6}$$

$$= \left(\frac{\sqrt{10}}{6} - \frac{\sqrt{5}}{6} + \frac{\sqrt{30}}{6}\right)$$

$$= \left(\frac{\sqrt{2 \times 5}}{\sqrt{2 \times 3}} - \frac{\sqrt{5}}{6} + \frac{\sqrt{6 \times 5}}{\sqrt{6 \times 6}}\right)$$

$$= \left(\frac{\sqrt{5}}{\sqrt{3}} - \frac{\sqrt{5}}{6} + \frac{\sqrt{5}}{\sqrt{6}}\right)$$

6 C. Question

Let us rationalizing the denominators of the following surds.

$$\frac{\sqrt{3}+1}{\sqrt{3}-1}$$

Answer

Given: $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

Rationalize with $\sqrt{3} + 1$

We get,

$$\frac{\sqrt{3} + 1}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$= \frac{(\sqrt{3} + 1)^2}{(\sqrt{3})^2 - (1)^2}$$

$$= \frac{3 + 1 + 2\sqrt{3}}{3 - 1}$$

$$= \frac{4 + 2\sqrt{3}}{2}$$

$$= \frac{2(2 + \sqrt{3})}{2}$$

$$= 2 + \sqrt{3}$$

6 D. Question

Let us rationalizing the denominators of the following surds.

$$\frac{3+\sqrt{5}}{\sqrt{7}-\sqrt{3}}$$

Answer

Given: $\frac{3+\sqrt{5}}{\sqrt{7}-\sqrt{3}}$

Rationalize with $\sqrt{7} + \sqrt{3}$

We get,

$$\frac{3 + \sqrt{5}}{\sqrt{7} - \sqrt{3}} \times \frac{\sqrt{7} + \sqrt{3}}{\sqrt{7} + \sqrt{3}}$$

$$= \frac{3.\sqrt{7} + 3.\sqrt{3} + \sqrt{5}.\sqrt{7} + \sqrt{5}.\sqrt{3}}{(\sqrt{7})^2 - (\sqrt{3})^2}$$

$$= \frac{\sqrt{7}(3 + \sqrt{5}) + \sqrt{5}(3 + \sqrt{5})}{7 - 3}$$

$$= \frac{(\sqrt{7} + \sqrt{5})(3 + \sqrt{5})}{4}$$

6 E. Question

Let us rationalizing the denominators of the following surds.

$$\frac{3\sqrt{2} + 1}{2\sqrt{5} - 1}$$

Answer

Given: $\frac{3\sqrt{2} + 1}{2\sqrt{5} - 1}$

Rationalize with $2\sqrt{5} + 1$

We get,

$$\frac{3\sqrt{2} + 1}{2\sqrt{5} - 1} \times \frac{2\sqrt{5} + 1}{2\sqrt{5} + 1}$$

$$= \frac{3\sqrt{2}.2\sqrt{5} + 3.\sqrt{2} + 2\sqrt{5} + 1}{(2\sqrt{5})^2 - (1)^2}$$

$$= \frac{6\sqrt{10} + 3.\sqrt{2} + 2\sqrt{5} + 1}{20 - 1}$$

$$= \frac{6\sqrt{10} + 3.\sqrt{2} + 2\sqrt{5} + 1}{19}$$

6 F. Question

Let us rationalizing the denominators of the following surds.

$$\frac{3\sqrt{2} + 2\sqrt{3}}{3\sqrt{2} - 2\sqrt{3}}$$

Answer

Given: $\frac{3\sqrt{2} + 2\sqrt{3}}{3\sqrt{2} - 2\sqrt{3}}$

Rationalize with $3\sqrt{2} + 2\sqrt{3}$

We get,

$$\frac{3\sqrt{2} + 2\sqrt{3}}{3\sqrt{2} - 2\sqrt{3}} \times \frac{3\sqrt{2} + 2\sqrt{3}}{3\sqrt{2} + 2\sqrt{3}}$$

$$= \frac{(3\sqrt{2} + 2\sqrt{3})^2}{(3\sqrt{2})^2 - (2\sqrt{3})^2}$$

$$= \frac{18 + 12 + 12\sqrt{6}}{18 - 12}$$

$$= \frac{30 + 12\sqrt{6}}{6}$$

$$= \frac{30}{6} + \frac{12\sqrt{6}}{6}$$

$$= 5 + 2\sqrt{6}$$

7 A. Question

Let us divide first by second and rationalize the divisor.

$$3\sqrt{2} + \sqrt{5}, \sqrt{2} + 1$$

Answer

Given: $3\sqrt{2} + \sqrt{5}$ and $\sqrt{2} + 1$

Acc. To condition:

$$\frac{\sqrt{2} + 1}{3\sqrt{2} + \sqrt{5}}$$

Rationalize with $3\sqrt{2} - \sqrt{5}$

We get,

$$\frac{\sqrt{2} + 1}{3\sqrt{2} + \sqrt{5}} \times \frac{3\sqrt{2} - \sqrt{5}}{3\sqrt{2} - \sqrt{5}}$$

$$= \frac{3\sqrt{2}.\sqrt{2} - \sqrt{2}.\sqrt{5} + 3\sqrt{2} - \sqrt{5}}{(3\sqrt{2})^2 - (\sqrt{5})^2}$$

$$= \frac{6 - \sqrt{10} + 3\sqrt{2} - \sqrt{5}}{18 - 5}$$

$$= \frac{6 + 3\sqrt{2} - \sqrt{5}(\sqrt{2} + 1)}{13}$$

7 B. Question

Let us divide first by second and rationalize the divisor.

$2\sqrt{3}-\sqrt{2}, \sqrt{2}-\sqrt{3}$

Answer

Given: $2\sqrt{3}-\sqrt{2}$ and $\sqrt{2}-\sqrt{3}$

Acc. To condition:

$$\frac{\sqrt{2}-\sqrt{3}}{2\sqrt{3}-\sqrt{2}}$$

Rationalize with $2\sqrt{3}+\sqrt{2}$

We get,

$$\frac{\sqrt{2}-\sqrt{3}}{2\sqrt{3}-\sqrt{2}} \times \frac{2\sqrt{3}+\sqrt{2}}{2\sqrt{3}+\sqrt{2}}$$

$$= \frac{2\sqrt{3}.\sqrt{2}+\sqrt{2}.\sqrt{2}-2\sqrt{3}.\sqrt{3}-\sqrt{3}\sqrt{2}}{(2\sqrt{3})^2-(\sqrt{2})^2}$$

$$= \frac{2\sqrt{6}+2-6-\sqrt{6}}{12-2}$$

$$= \frac{-4-\sqrt{6}}{10}$$

$$= -\left(\frac{4+\sqrt{6}}{10}\right)$$

7 C. Question

Let us divide first by second and rationalize the divisor.

$3+\sqrt{6}, \sqrt{3}+\sqrt{2}$

Answer

Given: $3+\sqrt{6}$ and $\sqrt{3}+\sqrt{2}$

Acc. To condition:

$$\frac{\sqrt{3}+\sqrt{2}}{3+\sqrt{6}}$$

Rationalize with $3-\sqrt{6}$We get,

$$\frac{\sqrt{3}\ +\ \sqrt{2}}{3\ +\ \sqrt{6}} \times \frac{3-\sqrt{6}}{3-\sqrt{6}}$$

$$= = \frac{3\sqrt{3}-\sqrt{3}.\sqrt{6}\ +\ 3\sqrt{2}-\sqrt{6}\sqrt{2}}{(3)^2-(\sqrt{6})^2}$$

$$\frac{3\sqrt{3}-3\sqrt{2}\ +\ 3\sqrt{2}-2\sqrt{3}}{9-6}$$

$$= \frac{\sqrt{3}}{3}$$

8 A. Question

Let us find the value of

$$\frac{2\sqrt{5}+1}{\sqrt{5}+1} - \frac{4\sqrt{5}-1}{\sqrt{5}-1}$$

Answer

Given: $\frac{2\sqrt{5}+1}{\sqrt{5}+1} - \frac{4\sqrt{5}-1}{\sqrt{5}-1}$

After rationalization we get,

We get,

$$\left\{\frac{2\sqrt{5}\ +\ 1}{\sqrt{5}\ +\ 1} \times \frac{\sqrt{5}-1}{\sqrt{5}-1}\right\} - \left\{\frac{4\sqrt{5}-1}{\sqrt{5}-1} \times \frac{\sqrt{5}\ +\ 1}{\sqrt{5}\ +\ 1}\right\}$$

$$= \left\{\frac{2\sqrt{5}.\sqrt{5}\ +\ \sqrt{5}-2\sqrt{5}-1}{(\sqrt{5})^2-(1)^2}\right\} - \left\{\frac{4\sqrt{5}.\sqrt{5}-\sqrt{5}\ +\ 4\sqrt{5}-1}{(\sqrt{5})^2-(1)^2}\right\}$$

$$= \left\{\frac{10\ +\ 3\sqrt{5}-1}{5-1}\right\} - \left\{\frac{20\ +\ 3\sqrt{5}-1}{5-1}\right\}$$

$$= \left\{\frac{10\ +\ 3\sqrt{5}-1}{4}\right\} - \left\{\frac{20\ +\ 3\sqrt{5}-1}{4}\right\}$$

$$= \left\{\frac{10\ +\ 3\sqrt{5}-1-20-3\sqrt{5}\ +\ 1}{4}\right\}$$

$$= \left\{\frac{-10}{4}\right\}$$

8 B. Question

Let us find the value of

$$\frac{8+3\sqrt{2}}{3+\sqrt{5}}-\frac{8-3\sqrt{2}}{3-\sqrt{5}}$$

Answer

Given: $\frac{8+3\sqrt{2}}{3+\sqrt{5}}-\frac{8-3\sqrt{2}}{3-\sqrt{5}}$

After rationalization we get,

We get,

$$\left\{\frac{8+3\sqrt{2}}{3+\sqrt{5}}\times\frac{3-\sqrt{5}}{3-\sqrt{5}}\right\}-\left\{\frac{8-3\sqrt{2}}{3-\sqrt{5}}\times\frac{3+\sqrt{5}}{3+\sqrt{5}}\right\}$$

$$=\left\{\frac{24-8\sqrt{5}+9\sqrt{2}-3\sqrt{10}}{(3)^2-(\sqrt{5})^2}\right\}-\left\{\frac{24+8\sqrt{5}-9\sqrt{2}-3\sqrt{10}}{(3)^2-(\sqrt{5})^2}\right\}$$

$$=\left\{\frac{24-8\sqrt{5}+9\sqrt{2}-3\sqrt{10}}{9-5}\right\}-\left\{\frac{24+8\sqrt{5}-9\sqrt{2}-3\sqrt{10}}{9-5}\right\}$$

$$=\left\{\frac{24-8\sqrt{5}+9\sqrt{2}-3\sqrt{10}}{4}\right\}-\left\{\frac{24+8\sqrt{5}-9\sqrt{2}-3\sqrt{10}}{4}\right\}$$

$$=\left\{\frac{24-8\sqrt{5}+9\sqrt{2}-3\sqrt{10}-24-8\sqrt{5}+9\sqrt{2}+3\sqrt{10}}{4}\right\}$$

$$=\left\{\frac{-16\sqrt{5}+18\sqrt{2}}{4}\right\}$$

$$=\left\{\frac{-8\sqrt{5}+9\sqrt{2}}{2}\right\}$$

Let us Work Out 9.3

1 A. Question

If $m+\frac{1}{m}=\sqrt{3}$, let us calculate simplified of i. $m^2+\frac{1}{m^2}$ and ii. $m^3+\frac{1}{m^3}$

Answer

Formula used.

$(a+b)^2 = a^2 + b^2 + 2ab$

$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

Let a = m and b = $\frac{1}{m}$

$(a + b)^2 = a^2 + b^2 + 2ab$

$(m + \frac{1}{m})^2 = m^2 + (\frac{1}{m})^2 + 2 \times m \times \frac{1}{m}$

$(\sqrt{3})^2 = m^2 + \frac{1}{m^2} + 2 \times 1$

$3 = m^2 + \frac{1}{m^2} + 2$

$m^2 + \frac{1}{m^2} = 3 - 2 = 1$

$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$(m + \frac{1}{m})^3 = m^3 + (\frac{1}{m})^3 + 2 \times m \times \frac{1}{m} \times (m + \frac{1}{m})$

$(\sqrt{3})^3 = m^3 + \frac{1}{m^3} + 2 \times 1 \times (\sqrt{3})$

$3\sqrt{3} = m^3 + \frac{1}{m^3} + 2\sqrt{3}$

$m^3 + \frac{1}{m^3} = 3\sqrt{3} - 2\sqrt{3} = \sqrt{3}[3 - 2]$

$= \sqrt{3}$

1 B. Question

Let us show that,

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} - \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = 2\sqrt{15}$$

Answer

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} - \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$\frac{(\sqrt{5}+\sqrt{3})^2 + (\sqrt{5}-\sqrt{3})^2}{\sqrt{5}-\sqrt{3} \times \sqrt{5}+\sqrt{3}}$$

$$\frac{(\sqrt{5})^2 + (\sqrt{3})^2 + 2\sqrt{5} \times \sqrt{3} - (\sqrt{5})^2 - (\sqrt{3})^2 + 2\sqrt{5} \times \sqrt{3}}{(\sqrt{5})^2 - (\sqrt{3})^2}$$

$$\frac{2\sqrt{5} \times \sqrt{3} + 2\sqrt{5} \times \sqrt{3}}{5-3}$$

$\frac{4\sqrt{15}}{2} = 2\sqrt{15}$

2 A. Question

Let us simplify

$$\frac{\sqrt{2}\left(2+\sqrt{3}\right)}{\sqrt{3}\left(\sqrt{3}+1\right)} - \frac{\sqrt{2}\left(2-\sqrt{3}\right)}{\sqrt{3}\left(\sqrt{3}-1\right)}$$

Answer

$$\frac{\sqrt{2}}{\sqrt{3}} \times [\frac{2 + \sqrt{3}}{\sqrt{3} + 1} - \frac{2 - \sqrt{3}}{\sqrt{3} - 1}]$$

$$\frac{\sqrt{2}}{\sqrt{3}} \times [\frac{(2 + \sqrt{3}) \times (\sqrt{3}-1) - (2-\sqrt{3}) \times (\sqrt{3} + 1)}{(\sqrt{3} + 1) \times (\sqrt{3}-1)}]$$

$$\frac{\sqrt{2}}{\sqrt{3}} \times [\frac{(2\sqrt{3}-2 + 3-\sqrt{3}) - (2\sqrt{3} + 2-3-\sqrt{3})}{(\sqrt{3})^2 - (1)^2}]$$

$$\frac{\sqrt{2}}{\sqrt{3}} \times [\frac{(1 + \sqrt{3}) - (\sqrt{3}-1)}{3-1}]$$

$$\frac{\sqrt{2}}{\sqrt{3}} \times \frac{2}{2} = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

2 B. Question

Let us simplify

$$\frac{3\sqrt{7}}{\sqrt{5}+\sqrt{2}} - \frac{5\sqrt{5}}{\sqrt{2}+\sqrt{7}} + \frac{2\sqrt{2}}{\sqrt{7}+\sqrt{5}}$$

Answer

Simplifying part 1

$$\frac{3\sqrt{7}}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{3\sqrt{7} \times [\sqrt{5}-\sqrt{2}]}{(\sqrt{5})^2-(\sqrt{2})^2} = \frac{3\sqrt{7} \times [\sqrt{5}-\sqrt{2}]}{5-2} = \sqrt{35} - \sqrt{14}$$

Simplifying part 2

$$\frac{5\sqrt{5}}{\sqrt{2}+\sqrt{7}} \times \frac{\sqrt{7}-\sqrt{2}}{\sqrt{7}-\sqrt{2}} = \frac{5\sqrt{5} \times [\sqrt{7}-\sqrt{2}]}{(\sqrt{7})^2-(\sqrt{2})^2} = \frac{5\sqrt{5} \times [\sqrt{7}-\sqrt{2}]}{7-2} = \sqrt{35} - \sqrt{10}$$

Simplifying part 3

$$\frac{2\sqrt{2}}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} = \frac{2\sqrt{2}\times[\sqrt{7}-\sqrt{5}]}{(\sqrt{7})^2-(\sqrt{5})^2} = \frac{2\sqrt{2}\times[\sqrt{7}-\sqrt{5}]}{7-5} = \sqrt{14} - \sqrt{10}$$

Putting values we get;

$[\sqrt{35} - \sqrt{14}] - [\sqrt{35} - \sqrt{10}] + [\sqrt{14} - \sqrt{10}]$

$\sqrt{35} - \sqrt{14} - \sqrt{35} + \sqrt{10} + \sqrt{14} - \sqrt{10}$

$= 0$

2 C. Question

Let us simplify

$$\frac{4\sqrt{3}}{2-\sqrt{2}} - \frac{30}{4\sqrt{3}-\sqrt{18}} - \frac{\sqrt{18}}{3-\sqrt{12}}$$

Answer

$$\frac{4\sqrt{3}}{2-\sqrt{2}} - \frac{30}{4\sqrt{3}-\sqrt{18}} - \frac{\sqrt{18}}{3-\sqrt{12}}$$

Simplifying 1^{st} part by rationalizing the expression by multiplying and dividing by $2 + \sqrt{2}$

$$\frac{4\sqrt{3}}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}}$$

$$= \frac{4\sqrt{3}\times(2+\sqrt{2})}{2^2-(\sqrt{2})^2} = \frac{4\sqrt{3}\times(2+\sqrt{2})}{4-2} = 2\sqrt{3} \times (2 + \sqrt{2}) = 4\sqrt{3} + 2\sqrt{6}$$

Simplifying 2^{nd} part by rationalizing the expression by multiplying and dividing by $4\sqrt{3} + \sqrt{18}$

$$\frac{30}{4\sqrt{3}-\sqrt{18}} \times \frac{4\sqrt{3}+\sqrt{18}}{4\sqrt{3}+\sqrt{18}}$$

$$= \frac{30\times(4\sqrt{3}+\sqrt{18})}{(4\sqrt{3})^2-(\sqrt{18})^2} = \frac{30\times(4\sqrt{3}-\sqrt{18})}{48-18} = 4\sqrt{3} - \sqrt{18}$$

Simplifying 3^{rd} part by rationalizing the expression by multiplying and dividing by $3 + \sqrt{12}$

$$\frac{\sqrt{18}}{3-\sqrt{12}} \times \frac{3+\sqrt{12}}{3+\sqrt{12}}$$

$$= \frac{3\sqrt{2}\times(3+\sqrt{12})}{3^2-(\sqrt{12})^2} = \frac{3\sqrt{2}\times(3+\sqrt{12})}{9-12} = -\sqrt{2} \times (3 + \sqrt{12})$$

Putting all values we get;

$4\sqrt{3} + 2\sqrt{6} - (4\sqrt{3} - \sqrt{18}) + (3\sqrt{2} + \sqrt{24})$

$4\sqrt{3} + \sqrt{6 \times 2^2} - (4\sqrt{3} - \sqrt{18}) + (\sqrt{2 \times 3^2} + \sqrt{24})$

$4\sqrt{3} + \sqrt{24} - (4\sqrt{3} - \sqrt{18}) + (\sqrt{18} + \sqrt{24})$

$= 2\sqrt{24} = 4\sqrt{6}$

Hence, $\frac{4\sqrt{3}}{2-\sqrt{2}} - \frac{30}{4\sqrt{3}-\sqrt{18}} - \frac{\sqrt{18}}{3-\sqrt{12}} = 4\sqrt{6}$

2 D. Question

Let us simplify

$$\frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$$

Answer

Simplifying 1st part

$$\frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} \times \frac{\sqrt{3}-\sqrt{6}}{\sqrt{3}-\sqrt{6}}$$

$$\frac{3\sqrt{2} \times (\sqrt{3}-\sqrt{6})}{(\sqrt{3})^2-(\sqrt{6})^2} = \frac{3\sqrt{2} \times (\sqrt{3}-\sqrt{6})}{3-6} = -\sqrt{6} + \sqrt{12}$$

Simplifying 2nd part

$$\frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} \times \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$$

$$\frac{4\sqrt{3} \times (\sqrt{6}-\sqrt{2})}{(\sqrt{6})^2-(\sqrt{2})^2} = \frac{4\sqrt{3} \times (\sqrt{6}-\sqrt{2})}{6-2} = \sqrt{18}-\sqrt{6}$$

Simplifying 3rd part

$$\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$

$$\frac{\sqrt{6} \times (\sqrt{2}-\sqrt{3})}{(\sqrt{2})^2-(\sqrt{3})^2} = \frac{(\sqrt{12}-\sqrt{18})}{2-3} = -\sqrt{12} + \sqrt{18}$$

Putting all values we get;

$(-\sqrt{6} + \sqrt{12}) - (\sqrt{18} - \sqrt{6}) + (-\sqrt{12} + \sqrt{18})$

$-\sqrt{6} + \sqrt{12} - \sqrt{18} + \sqrt{6} - \sqrt{12} + \sqrt{18}$

= 0

3. Question

If x = 2, y = 3 and z = 6, let us write the calculating the value of

$$\frac{3\sqrt{x}}{\sqrt{y}+\sqrt{z}}-\frac{4\sqrt{y}}{\sqrt{z}+\sqrt{x}}+\frac{\sqrt{z}}{\sqrt{x}+\sqrt{y}}$$

Answer

$$\frac{3\sqrt{x}}{\sqrt{y}\ +\ \sqrt{z}}-\frac{4\sqrt{y}}{\sqrt{z}\ +\ \sqrt{x}}+\frac{\sqrt{z}}{\sqrt{x}\ +\ \sqrt{y}}$$

Putting value x = 2, y = 3, z = 6;

$$\frac{3\sqrt{2}}{\sqrt{3}\ +\ \sqrt{6}}-\frac{4\sqrt{3}}{\sqrt{6}\ +\ \sqrt{2}}+\frac{\sqrt{6}}{\sqrt{2}\ +\ \sqrt{3}}$$

Simplifying 1^{st} part

$$\frac{3\sqrt{2}}{\sqrt{3}\ +\ \sqrt{6}}\times\frac{\sqrt{3}-\sqrt{6}}{\sqrt{3}-\sqrt{6}}$$

$$\frac{3\sqrt{2}\times(\sqrt{3}-\sqrt{6})}{(\sqrt{3})^2-(\sqrt{6})^2}=\frac{3\sqrt{2}\times(\sqrt{3}-\sqrt{6})}{3-6}= -\sqrt{6}+\sqrt{12}$$

Simplifying 2^{nd} part

$$\frac{4\sqrt{3}}{\sqrt{6}\ +\ \sqrt{2}}\times\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$$

$$\frac{4\sqrt{3}\times(\sqrt{6}-\sqrt{2})}{(\sqrt{6})^2-(\sqrt{2})^2}=\frac{4\sqrt{3}\times(\sqrt{6}-\sqrt{2})}{6-2}= \sqrt{18}-\sqrt{6}$$

Simplifying 3^{rd} part

$$\frac{\sqrt{6}}{\sqrt{2}\ +\ \sqrt{3}}\times\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$

$$\frac{\sqrt{6}\times(\sqrt{2}-\sqrt{3})}{(\sqrt{2})^2-(\sqrt{3})^2}=\frac{(\sqrt{12}-\sqrt{18})}{2-3}= -\sqrt{12}+\sqrt{18}$$

Putting all values we get;

$(-\sqrt{6} + \sqrt{12}) – (\sqrt{18} - \sqrt{6}) + (-\sqrt{12} + \sqrt{18})$

$-\sqrt{6} + \sqrt{12} – \sqrt{18} + \sqrt{6} – \sqrt{12} + \sqrt{18}$

= 0

4 A. Question

If $x = \sqrt{7} + \sqrt{6}$, let us calculate simplified value of

$$x - \frac{1}{x}$$

Answer

x = √7 + √6

then;

$$\frac{1}{x} = \frac{1}{\sqrt{7} + \sqrt{6}}$$

By simplifying

$$\frac{1}{x} = \frac{1}{\sqrt{7} + \sqrt{6}} \times \frac{\sqrt{7} - \sqrt{6}}{\sqrt{7} - \sqrt{6}}$$

$$\frac{1}{x} = \frac{\sqrt{7} - \sqrt{6}}{(\sqrt{7} - \sqrt{6}) \times (\sqrt{7} + \sqrt{6})}$$

$$\frac{1}{x} = \frac{\sqrt{7} - \sqrt{6}}{(\sqrt{7})^2 - (\sqrt{6})^2}$$

$$= \frac{\sqrt{7} - \sqrt{6}}{7 - 6}$$

= √7 – √6

Hence;

$x - \frac{1}{x} = (\sqrt{7} + \sqrt{6}) - (\sqrt{7} - \sqrt{6})$

= 2√6

4 B. Question

If $x = \sqrt{7} + \sqrt{6}$, let us calculate simplified value of

$$x + \frac{1}{x}$$

Answer

x = √7 + √6

then;

$$\frac{1}{x} = \frac{1}{\sqrt{7} + \sqrt{6}}$$

By simplifying

$\frac{1}{x} = \frac{1}{\sqrt{7}+\sqrt{6}} \times \frac{\sqrt{7}-\sqrt{6}}{\sqrt{7}-\sqrt{6}}$

$\frac{1}{x} = \frac{\sqrt{7}-\sqrt{6}}{(\sqrt{7}-\sqrt{6}) \times (\sqrt{7}+\sqrt{6})}$

$\frac{1}{x} = \frac{\sqrt{7}-\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2}$

$= \frac{\sqrt{7}-\sqrt{6}}{7-6}$

$= \sqrt{7} - \sqrt{6}$

Hence;

$x + \frac{1}{x} = (\sqrt{7} + \sqrt{6}) + (\sqrt{7} - \sqrt{6})$

$= 2\sqrt{7}$

4 C. Question

If $x = \sqrt{7} + \sqrt{6},$ let us calculate simplified value of

$$x^2 + \frac{1}{x^2}$$

Answer

Formula used.

$(a + b)^2 = a^2 + b^2 + 2ab$

$x + \frac{1}{x} = 2\sqrt{7}$

$(a + b)^2 = a^2 + b^2 + 2ab$

Put $a = x$ and $b = \frac{1}{x}$

$(x + \frac{1}{x})^2 = x^2 + [\frac{1}{x}]^2 + 2 \times x \times \frac{1}{x}$

$(2\sqrt{7})^2 = x^2 + \frac{1}{x^2} + 2$

$4 \times 7 = x^2 + \frac{1}{x^2} + 2$

$28 = x^2 + \frac{1}{x^2} + 2$

$x^2 + \frac{1}{x^2} = 28 - 2 = 26$

4 D. Question

If $x = \sqrt{7} + \sqrt{6}$, let us calculate simplified value of

$$x^3 + \frac{1}{x^3}$$

Answer

Formula used.

$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

If

$x + \frac{1}{x} = 2\sqrt{7}$

$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

Put a = x and b = $\frac{1}{x}$

$(x + \frac{1}{x})^3 = x^3 + (\frac{1}{x})^3 + 3 \times x \times \frac{1}{x} \times (x + \frac{1}{x})$

$(2\sqrt{7})^3 = x^3 + \frac{1}{x^3} + 3 \times (2\sqrt{7})$

$56\sqrt{7} = x^3 + \frac{1}{x^3} + 6\sqrt{7}$

$x^3 + \frac{1}{x^3} = 56\sqrt{7} - 6\sqrt{7}$

$x^3 + \frac{1}{x^3} = \sqrt{7}\,[56 - 6]$

$= 50\sqrt{7}$

5. Question

Let us simplify :

$$\frac{x + \sqrt{x^2 - 1}}{x - \sqrt{x^2 - 1}} + \frac{x - \sqrt{x^2 - 1}}{x + \sqrt{x^2 - 1}}$$

If the simplified value is 14, let us write by calculating the value of x.

Answer

$$\frac{x + \sqrt{x^2 - 1}}{x - \sqrt{x^2 - 1}} + \frac{x - \sqrt{x^2 - 1}}{x + \sqrt{x^2 - 1}}$$

$$\frac{\left(x + \sqrt{x^2-1}\right)^2 + \left(x - \sqrt{x^2-1}\right)^2}{\left(x - \sqrt{x^2-1}\right) \times \left(x - \sqrt{x^2-1}\right)}$$

$$\frac{x^2 + \left(\sqrt{x^2-1}\right)^2 + 2x\sqrt{x^2-1} + x^2 + \left(\sqrt{x^2-1}\right)^2 - 2x\sqrt{x^2-1}}{(x)^2 - \left(\sqrt{x^2-1}\right)^2}$$

$$\frac{2x^2 + 2\left(\sqrt{x^2-1}\right)^2}{x^2 - x^2 + 1}$$

$$\frac{2x^2 + 2x^2 - 2}{1}$$

$4x^2 - 2$

If $4x^2 - 2 = 14$

$4x^2 = 14 + 2 = 16$

$x^2 = \frac{16}{4} = 4$

$x = \sqrt{4} = \pm 2$

6 A. Question

If $a = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $b = \frac{\sqrt{5}-1}{\sqrt{5}+1}$, let us calculate the followings :

$$\frac{a^2 + ab + b^2}{a^2 - ab + b^2}$$

Answer

$a + b = \frac{\sqrt{5}+1}{\sqrt{5}-1} + \frac{\sqrt{5}-1}{\sqrt{5}+1}$

$a + b = \frac{\left(\sqrt{5}+1\right)^2 + \left(\sqrt{5}-1\right)^2}{\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)} = \frac{\left(5+1+2\sqrt{5}\right) + \left(5+1-2\sqrt{5}\right)}{\left(\sqrt{5}\right)^2 - 1^2} = \frac{12}{4} = 3$

a-b $= \frac{\sqrt{5}+1}{\sqrt{5}-1} - \frac{\sqrt{5}-1}{\sqrt{5}+1}$

a-b $= \frac{\left(\sqrt{5}+1\right)^2 - \left(\sqrt{5}-1\right)^2}{\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)} = \frac{\left(5+1+2\sqrt{5}\right) - \left(5+1-2\sqrt{5}\right)}{\left(\sqrt{5}\right)^2 - 1^2} = \frac{4\sqrt{5}}{4} = \sqrt{5}$

$ab = \frac{\sqrt{5}+1}{\sqrt{5}-1} \times \frac{\sqrt{5}-1}{\sqrt{5}+1} = 1$

$\frac{a^2 + ab + b^2}{a^2 - ab + b^2} = \frac{a^2 + ab + b^2 + (ab - ab)}{a^2 - ab + b^2 + (ab - ab)} = \frac{a^2 + 2ab + b^2 - ab}{a^2 - 2ab + b^2 + ab} = \frac{(a+b)^2 - ab}{(a-b)^2 + ab}$

Putting values we get;

$$\frac{(3)^2-1}{(\sqrt{5})^2+1}=\frac{9-1}{5+1}=\frac{8}{6}=\frac{4}{3}$$

6 B. Question

If $a=\frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $b=\frac{\sqrt{5}-1}{\sqrt{5}+1}$, let us calculate the followings :

$$\frac{(a-b)^3}{(a+b)^3}$$

Answer

$$a+b=\frac{\sqrt{5}+1}{\sqrt{5}-1}+\frac{\sqrt{5}-1}{\sqrt{5}+1}$$

$$a+b=\frac{(\sqrt{5}+1)^2+(\sqrt{5}-1)^2}{(\sqrt{5}-1)(\sqrt{5}+1)}=\frac{(5+1+2\sqrt{5})+(5+1-2\sqrt{5})}{(\sqrt{5})^2-1^2}=\frac{12}{4}=3$$

$$a-b=\frac{\sqrt{5}+1}{\sqrt{5}-1}-\frac{\sqrt{5}-1}{\sqrt{5}+1}$$

$$a-b=\frac{(\sqrt{5}+1)^2-(\sqrt{5}-1)^2}{(\sqrt{5}-1)(\sqrt{5}+1)}=\frac{(5+1+2\sqrt{5})-(5+1-2\sqrt{5})}{(\sqrt{5})^2-1^2}=\frac{4\sqrt{5}}{4}=\sqrt{5}$$

$$\frac{(a-b)^3}{(a+b)^3}=\frac{(\sqrt{5})^3}{(3)^3}=\frac{5\sqrt{5}}{27}$$

6 C. Question

If $a=\frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $b=\frac{\sqrt{5}-1}{\sqrt{5}+1}$, let us calculate the followings :

$$\frac{3a^2+5ab+3b^2}{3a^2-5ab+3b^2}$$

Answer

$$a+b=\frac{\sqrt{5}+1}{\sqrt{5}-1}+\frac{\sqrt{5}-1}{\sqrt{5}+1}$$

$$a+b=\frac{(\sqrt{5}+1)^2+(\sqrt{5}-1)^2}{(\sqrt{5}-1)(\sqrt{5}+1)}=\frac{(5+1+2\sqrt{5})+(5+1-2\sqrt{5})}{(\sqrt{5})^2-1^2}=\frac{12}{4}=3$$

$$a-b=\frac{\sqrt{5}+1}{\sqrt{5}-1}-\frac{\sqrt{5}-1}{\sqrt{5}+1}$$

a-b = $\frac{(\sqrt{5}+1)^2-(\sqrt{5}-1)^2}{(\sqrt{5}-1)(\sqrt{5}+1)} = \frac{(5+1+2\sqrt{5})-(5+1-2\sqrt{5})}{(\sqrt{5})^2-1^2} = \frac{4\sqrt{5}}{4} = \sqrt{5}$

ab = $\frac{\sqrt{5}+1}{\sqrt{5}-1} \times \frac{\sqrt{5}-1}{\sqrt{5}+1} = 1$

$\frac{3a^2+5ab+3b^2}{3a^2-5ab+3b^2} = \frac{3a^2+5ab+3b^2+(ab-ab)}{3a^2-5ab+3b^2+(ab-ab)} = \frac{3a^2+6ab+3b^2-ab}{3a^2-6ab+3b^2+ab} = \frac{3\times(a+b)^2-ab}{3\times(a-b)^2+ab}$

Putting values we get;

$\frac{3\times(3)^2-1}{3\times(\sqrt{5})^2+1} = \frac{27-1}{15+1} = \frac{26}{16} = \frac{13}{8}$

6 D. Question

If $a = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $b = \frac{\sqrt{5}-1}{\sqrt{5}+1}$, let us calculate the followings :

$\frac{a^3+b^3}{a^3-b^3}$

Answer

a + b = $\frac{\sqrt{5}+1}{\sqrt{5}-1} + \frac{\sqrt{5}-1}{\sqrt{5}+1}$

a + b = $\frac{(\sqrt{5}+1)^2+(\sqrt{5}-1)^2}{(\sqrt{5}-1)(\sqrt{5}+1)} = \frac{(5+1+2\sqrt{5})+(5+1-2\sqrt{5})}{(\sqrt{5})^2-1^2} = \frac{12}{4} = 3$

a-b = $\frac{\sqrt{5}+1}{\sqrt{5}-1} - \frac{\sqrt{5}-1}{\sqrt{5}+1}$

a-b = $\frac{(\sqrt{5}+1)^2-(\sqrt{5}-1)^2}{(\sqrt{5}-1)(\sqrt{5}+1)} = \frac{(5+1+2\sqrt{5})-(5+1-2\sqrt{5})}{(\sqrt{5})^2-1^2} = \frac{4\sqrt{5}}{4} = \sqrt{5}$

ab = $\frac{\sqrt{5}+1}{\sqrt{5}-1} \times \frac{\sqrt{5}-1}{\sqrt{5}+1} = 1$

$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

$= (3)^3 - 3 \times 1 \times 3$

= 27 – 9 = 18

$(a-b)^3 = a^3-b^3-3ab(a-b)$

$a^3-b^3 = (a-b)^3 + 3ab(a-b)$

$= (\sqrt{5})^3 + 3 \times 1 \times (\sqrt{5})$

= $5\sqrt{5} + 3\sqrt{5}$

= $\sqrt{5}$ [5 + 3]

= $8\sqrt{5}$

$$\frac{(a + b)^3}{(a - b)^3} = \frac{18}{8\sqrt{5}} = \frac{9}{4\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{9\sqrt{5}}{20}$$

7 A1. Question

If $x = 2 + \sqrt{3}, y = 2 - \sqrt{3},$ let us calculate the simplified value of

$$x - \frac{1}{x}$$

Answer

If x = 2 + $\sqrt{3}$

Then;

$$\frac{1}{x} = \frac{1}{2 + \sqrt{3}}$$

Simplifying it we get;

$$\frac{1}{x} = \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$$

$$\frac{1}{x} = \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$$

$x - \frac{1}{x}$ = 2 + $\sqrt{3}$ – [2 - $\sqrt{3}$]

= $2\sqrt{3}$

7 A2. Question

If $x = 2 + \sqrt{3}, y = 2 - \sqrt{3},$ let us calculate the simplified value of

$$y^2 + \frac{1}{y^2}$$

Answer

If y = 2-$\sqrt{3}$

Then;

$$\frac{1}{y} = \frac{1}{2-\sqrt{3}}$$

Simplifying it we get;

$$\frac{1}{y} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$\frac{1}{y} = \frac{2+\sqrt{3}}{(2)^2-(\sqrt{3})^2} = \frac{2+\sqrt{3}}{4-3} = 2 + \sqrt{3}$

$y + \frac{1}{y} = 2 - \sqrt{3} + [2 + \sqrt{3}] = 4$

$(y + \frac{1}{y})^2 = y^2 + [\frac{1}{y}]^2 + 2 \times y \times \frac{1}{y}$

$(4)^2 = y^2 + [\frac{1}{y}]^2 + 2$

$y^2 + [\frac{1}{y}]^2 = 16 - 2 = 14$

7 A3. Question

If $x = 2+\sqrt{3}, y = 2-\sqrt{3},$ let us calculate the simplified value of

$$x^3 - \frac{1}{x^3}$$

Answer

Formula used.

$(a-b)^3 = a^3-b^3-3ab(a-b)$

If $x = 2 + \sqrt{3}$

Then;

$$\frac{1}{x} = \frac{1}{2+\sqrt{3}}$$

Simplifying it we get;

$$\frac{1}{x} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$\frac{1}{x} = \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$

$x - \frac{1}{x} = 2 + \sqrt{3} - [2 - \sqrt{3}]$

$= 2\sqrt{3}$

$(x - \frac{1}{x})^3 = x^3 - [\frac{1}{x}]^3 - 3 \times x \times \frac{1}{x} \times (x - \frac{1}{x})$

$(x - \frac{1}{x})^3 = x^3 - \frac{1}{x^3} - 3 \times 1 \times (x - \frac{1}{x})$

$(2\sqrt{3})^3 = x^3 - \frac{1}{x^3} - 3 \times (2\sqrt{3})$

$x^3 - \frac{1}{x^3} = 24\sqrt{3} + 6\sqrt{3}$

$x^3 - \frac{1}{x^3} = 30\sqrt{3}$

7 A4. Question

If $x = 2 + \sqrt{3}, y = 2 - \sqrt{3},$ let us calculate the simplified value of

$$xy + \frac{1}{xy}$$

Answer

$x = 2 + \sqrt{3}$

$y = 2 - \sqrt{3}$

$xy = (2 + \sqrt{3}) \times (2 - \sqrt{3})$

$xy = (2)^2 - (\sqrt{3})^2$

$= 4 - 3$

$= 1$

$xy + \frac{1}{xy} = 1 + \frac{1}{1} = 1 + 1 = 2$

7 B. Question

If $x = 2 + \sqrt{3}, y = 2 - \sqrt{3},$ let us calculate the simplified value of

$$3x^2 - 5xy + 3y^2$$

Answer

Formula used.

$(a - b)^2 = a^2 - 2ab + b^2$

$3x^2 - 5xy + 3y^2$

Add and subtract xy to the equation.

$3x^2 - 5xy + 3y^2$ [+ xy – xy]

$3x^2 - 6xy + 3y^2 + xy$

$3[x^2 - 2xy + y^2] + xy$

$3[x-y]^2 + xy$

$x = 2 + \sqrt{3}$

$y = 2 - \sqrt{3}$

$xy = (2 + \sqrt{3}) \times (2-\sqrt{3})$

$xy = (2)^2 - (\sqrt{3})^2$

$= 4 - 3$

$= 1$

$x - y = 2 + \sqrt{3} - [2-\sqrt{3}]$

$x - y = 2\sqrt{3}$

Putting the values we get;

$3[2\sqrt{3}]^2 + 1$

$3[12] + 1$

$36 + 1 = 37$

8. Question

If $x = \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}}$ and xy = 1, let us show that $\frac{x^2+xy+y^2}{x^2-xy+y^2} = \frac{12}{11}$

Answer

Formula used.

$(a - b)^2 = a^2 - 2ab + b^2$

$x = \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}}$

$xy = 1$

$y = \frac{1}{x}$

$$y = \frac{1}{\frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}}} = \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}}$$

$$x + y = \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}} + \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}}$$

$$= \frac{(\sqrt{7}+\sqrt{3})^2 + (\sqrt{7}-\sqrt{3})^2}{(\sqrt{7}-\sqrt{3}) \times (\sqrt{7}+\sqrt{3})}$$

$$= \frac{(7+3+2\sqrt{21}) + (7+3-2\sqrt{21})}{(\sqrt{7})^2 - (\sqrt{3})^2}$$

$$= \frac{20}{4} = 5$$

$$x - y = \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}} - \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}}$$

$$= \frac{(\sqrt{7}+\sqrt{3})^2 - (\sqrt{7}-\sqrt{3})^2}{(\sqrt{7}-\sqrt{3}) \times (\sqrt{7}+\sqrt{3})}$$

$$= \frac{(7+3+2\sqrt{21}) - (7+3-2\sqrt{21})}{(\sqrt{7})^2 - (\sqrt{3})^2}$$

$$= \frac{4\sqrt{21}}{4} = \sqrt{21}$$

$$\frac{x^2 + xy + y^2}{x^2 - xy + y^2}$$

Add and Subtract xy both on numerator and denominator

$$\frac{x^2 + xy + y^2 + [xy - xy]}{x^2 - xy + y^2 + [xy - xy]} = \frac{(x+y)^2 - xy}{(x-y)^2 + xy}$$

Putting values we get;

$$\frac{(5)^2 - 1}{(\sqrt{21})^2 + 1} = \frac{25-1}{21+1} = \frac{24}{22} = \frac{12}{11}$$

Hence proved.

9. Question

Let us write which one is greater of $(\sqrt{7}+1)$ and $(\sqrt{5}+\sqrt{3})$

Answer

Formula used.

$(a - b)^2 = a^2 - 2ab + b^2$

1^{st} value is $\sqrt{7} + 1$

Its square is

$(\sqrt{7} + 1)^2 = (\sqrt{7})^2 + 1^2 + 2 \times 1 \times \sqrt{7} = 7 + 1 + 2\sqrt{7} = 8 + 2\sqrt{7}$

2^{nd} value is $\sqrt{5} + \sqrt{3}$

Its square is

$(\sqrt{5} + \sqrt{3})^2 = (\sqrt{5})^2 + (\sqrt{3})^2 + 2 \times \sqrt{3} \times \sqrt{5} = 5 + 3 + 2\sqrt{15} = 8 + 2\sqrt{15}$

If $7<15$

Then $\sqrt{7} < \sqrt{15}$

Then $8 + \sqrt{7} < 8 + \sqrt{15}$

Then $\sqrt{(8 + \sqrt{7})} < \sqrt{(8 + \sqrt{15})}$

$\therefore \sqrt{7} + 1 < \sqrt{5} + \sqrt{3}$

10 A1. Question

If $x = 2 + \sqrt{3}$, the value of $x + \frac{1}{x}$ is

A. 2

B. $2\sqrt{3}$

C. 4

D. $2 - \sqrt{3}$

Answer

If $x = 2 + \sqrt{3}$

Then;

$$\frac{1}{x} = \frac{1}{2 + \sqrt{3}}$$

Simplifying it we get;

$$\frac{1}{x} = \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$$

$$\frac{1}{x} = \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$$

$x + \frac{1}{x} = 2 + \sqrt{3} + [2 - \sqrt{3}]$

= 4

10 A2. Question

If $p + q = \sqrt{13}$ and $p - q = \sqrt{5}$, then the value of pq is

A. 2

B. 18

C. 9

D. 8

Answer

$p + q = \sqrt{13}$

$p = \sqrt{13} - q$

$p - q = \sqrt{5}$

$(\sqrt{13} - q) - q = \sqrt{5}$

$2q = \sqrt{13} - \sqrt{5}$

$q = \frac{\sqrt{13} - \sqrt{5}}{2}$

$p = \sqrt{13} - q = \sqrt{13} - \frac{\sqrt{13} - \sqrt{5}}{2} = \frac{\sqrt{13} + \sqrt{5}}{2}$

$pq = \frac{\sqrt{13} + \sqrt{5}}{2} \times \frac{\sqrt{13} - \sqrt{5}}{2} = \frac{(\sqrt{13})^2 - (\sqrt{5})^2}{2 \times 2} = \frac{8}{4} = 2$

10 A3. Question

If $a + b = \sqrt{5}$ and $a - b = \sqrt{3}$, the value of $(a^2 + b^2)$ is

A. 8

B. 4

C. 2

D. 1

Answer

$a + b = \sqrt{5}$

$a = \sqrt{5} - b$

$a - b = \sqrt{3}$

$(\sqrt{5} - b) - b = \sqrt{3}$

$2b = \sqrt{5} - \sqrt{3}$

$b = \frac{\sqrt{5} - \sqrt{3}}{2}$

$a = \sqrt{5} - b = \sqrt{5} - \frac{\sqrt{5} - \sqrt{3}}{2} = \frac{\sqrt{5} + \sqrt{3}}{2}$

$ab = \frac{\sqrt{5} + \sqrt{3}}{2} \times \frac{\sqrt{5} - \sqrt{3}}{2} = \frac{(\sqrt{5})^2 - (\sqrt{3})^2}{2 \times 2} = \frac{2}{4} = \frac{1}{2}$

$(a + b)^2 = a^2 + b^2 + 2ab$

$(\sqrt{5})^2 = a^2 + b^2 + 2 \times \frac{1}{2}$

$a^2 + b^2 = 5 - 1 = 4$

10 A4. Question

If we subtract $\sqrt{5}$ from $\sqrt{125}$, the value is

A. $\sqrt{80}$

B. $\sqrt{120}$

C. $\sqrt{100}$

D. None of this

Answer

$\sqrt{125} = \sqrt{(5 \times 5 \times 5)} = 5\sqrt{5}$

$\sqrt{125} - \sqrt{5}$

$= 5\sqrt{5} - \sqrt{5}$

$= \sqrt{5}\,[5-1]$

$= 4\sqrt{5}$

$= \sqrt{((4 \times 4) \times 5)}$

$= \sqrt{80}$

10 A5. Question

The product of $\left(5 - \sqrt{3}\right)\left(\sqrt{3} - 1\right)\left(5 + \sqrt{3}\right)\left(\sqrt{3} + 1\right)$ is

A. 22

B. 44

C. 2

D. 11

Answer

$(5-\sqrt{3})(\sqrt{3}-1)(5 + \sqrt{3})(\sqrt{3} + 1)$

$(5-\sqrt{3})(5 + \sqrt{3})(\sqrt{3}-1)(\sqrt{3} + 1)$

$(5^2 – (\sqrt{3})^2)((\sqrt{3})^2 – 1^2)$

$(25 – 3)(3 – 1)$

$22 \times 2 = 44$

10 B. Question

Let us write whether the following statements are true or false :

i. $\sqrt{75}$ and $\sqrt{147}$ are similar surds

ii. $\sqrt{\pi}$ is a quadratic surd.

Answer

(i) True.

$\sqrt{75} = \sqrt{(5 \times 5 \times 3)} = 5\sqrt{3}$

$\sqrt{147} = \sqrt{(7 \times 7 \times 3)} = 7\sqrt{3}$

$\sqrt{3}$ is common on both surds

(ii) False

π itself is an irrational number

hence;

Square root of π is not a surd.

10 C. Question

Let us fill up the blank:

i. $5\sqrt{11}$ a ________ number (rational/ irrational)

ii. Conjugate surd of $\left(\sqrt{3} - 5\right)$ is _______.

iii. If the product and sum of two quadratic surds is a rational number, then the surds are ________ surds.

Answer

(a) Irrational

As √11 is irrational number

Multiplying it with 5

Also get irrational number

(b) √3 + 5

Conjugate surds are surds which are having same terms but having different symbol(+ to- and – to +) in between both the terms.

(c) Conjugate Surds

While product of conjugate surds

They get square to become rational number

While sum of conjugate surds

They get cancel to become rational number

11 A. Question

If x = 3 + 2√2 let us write the value of $x + \frac{1}{x}$.

Answer

If x = 3 + 2√2

Then;

$$\frac{1}{x} = \frac{1}{3 + 2\sqrt{2}}$$

Simplifying it we get;

$$\frac{1}{x} = \frac{1}{3 + 2\sqrt{2}} \times \frac{3 - 2\sqrt{2}}{3 - 2\sqrt{2}}$$

$\frac{1}{x} = \frac{3-2\sqrt{2}}{(3)^2-(2\sqrt{2})^2} = \frac{3-2\sqrt{2}}{9-8}$ = 3-2√2

$x + \frac{1}{x}$ = 3 + 2√2 + [3 – 2√2]

= 6

11 B. Question

Let us write which one is greater of $\sqrt{15} + \sqrt{3}$ and $\sqrt{10} + \sqrt{8}$

Answer

1^{st} value is $\sqrt{15} + \sqrt{3}$

Its square is

$(\sqrt{15} + \sqrt{3})^2 = (\sqrt{15})^2 + (\sqrt{3})^2 + 2 \times \sqrt{15} \times \sqrt{3} = 15 + 3 + 2\sqrt{45} = 18 + 2\sqrt{45}$

2^{nd} value is $\sqrt{10} + \sqrt{8}$

Its square is

$(\sqrt{10} + \sqrt{8})^2 = (\sqrt{10})^2 + (\sqrt{8})^2 + 2 \times \sqrt{10} \times \sqrt{8} = 10 + 8 + 2\sqrt{80} = 18 + 2\sqrt{80}$

If $45<80$

Then $\sqrt{45} < \sqrt{80}$

Then $18 + \sqrt{45} < 18 + \sqrt{80}$

Then $\sqrt{(18 + \sqrt{45})} < \sqrt{(18 + \sqrt{80})}$

$\therefore \sqrt{15} + \sqrt{3} < \sqrt{10} + \sqrt{8}$

11 C. Question

Let us write two mixed quadratic surds of which product is a rational number.

Answer

Two mixed surds are

$5\sqrt{6}$ and $7\sqrt{6}$

Multiplying both

$5\sqrt{6} \times 7\sqrt{6}$

$35 \times 6 = 210$

Which is a rational number.

11 D. Question

Let us write what should be subtracted from $\sqrt{72}$ to get $\sqrt{32}$

Answer

Let the number be x

$\sqrt{72} - x = \sqrt{32}$

$6\sqrt{2} - x = 4\sqrt{2}$

$x = 6\sqrt{2} - 4\sqrt{2}$

$= \sqrt{2}\,[6 - 4]$

$= 2\sqrt{2}$

11 E. Question

Let us write simplified value of $\frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}}$

Answer

Simplifying part 1

$$\frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{(\sqrt{2})^2-1^2} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1$$

Simplifying part 2

$$\frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2} = \frac{\sqrt{3}-\sqrt{2}}{3-2} = \sqrt{3}-\sqrt{2}$$

Simplifying part 3

$$\frac{1}{\sqrt{4}+\sqrt{3}} \times \frac{\sqrt{4}-\sqrt{3}}{\sqrt{4}-\sqrt{3}} = \frac{\sqrt{4}-\sqrt{3}}{(\sqrt{4})^2-(\sqrt{3})^2} = \frac{\sqrt{4}-\sqrt{3}}{4-3} = \sqrt{4}-\sqrt{3}$$

Adding all we get;

$\sqrt{2}-1 + \sqrt{3}-\sqrt{2} + \sqrt{4}-\sqrt{3}$

$= \sqrt{4} - 1$

$= 2 - 1 = 1$