Chapter 13
Surface area and volumes

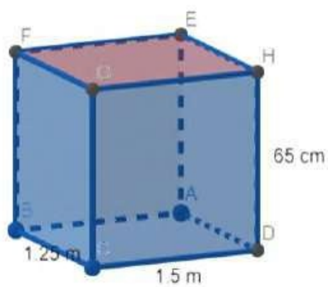
Exercise 13.1

Question :1 A plastic box 1.5 m long, 1.25 m wide and 65 cm deep is to be made. It is to be open at the top. Ignoring the thickness of the plastic sheet, determine:

(i) The area of the sheet required for making the box.

(ii) The cost of sheet for it, if a sheet measuring $1m^2$ costs Rs 20

Answer:



(i) Box is to be open at top (Pink color in the above diagram shows that the box is open)

Method 1:

Lateral Surface Area = Area of the four sides of the cuboid (Area of four rectangles on the sides)

= 2 (length x height) + 2(width x height)

Area of sheet required = lateral Surface Area + Area of the base

= 2 lh + 2 bh + lb

$= [2 \times 1.5 \times 0.65 + 2 \times 1.25 \times 0.65 + 1.5 \times 1.25]\ m^2$

$= (1.95 + 1.625 + 1.875)\ m^2$

$= 5.45\ m^2$

(ii) Cost of sheet per m^2 area = Rs 20

Cost of sheet of $5.45\ m^2$ area = Rs (5.45×20)

= Rs 109

Method 2:

(i) Area of Sheet Required = Total Surface Area - Area of top

Area of Sheet Required = $2(lb + bh + hl) – lb$

Area of sheet Required = $2 \times 1.5 \times 0.65 + 2 \times 1.25 \times 0.65 + 2 \times 1.5 \times 1.25 - 1.5 \times 1.25$

Area of Sheet Required = $5.45\ m^2$

(ii) Cost of sheet per m^2 area = Rs 20

Cost of sheet of $5.45\ m^2$ area = Rs (5.45×20)

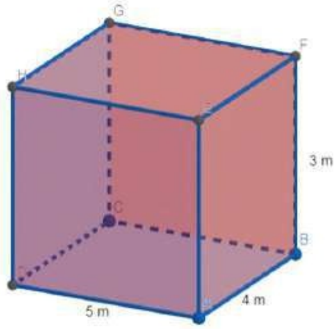
= Rs 109

Hence, the cost of sheet is Rs. 109.

Question :2 The length, breadth and height of a room are 5 m, 4 m and 3 m respectively. Find the cost of white washing the walls of the room and the ceiling at the rate of Rs 7.50 per m^2.

Solution: Given: Length = 5m, Breadth = 4m, and Height = 3m

Diagram:



It can be observed that four walls and the ceiling of the room are to be whitewashed.

The floor of the room is not to be white-washed

The area to be white-washed = Area of walls + Area of the ceiling of the room

= 2lh + 2bh + lb

= $[2 \times 5 \times 3 + 2 \times 4 \times 3 + 5 \times 4]\ m^2$

= $(30 + 24 + 20)\ m^2$

= 74 m^2

Cost of white-washing per m^2 area = Rs 7.50

Cost of white-washing 74 m^2 area = Rs (74×7.50)

= Rs 555

Question: 3 The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of Rs 10 per m^2 is Rs 15000, find the height of the hall.

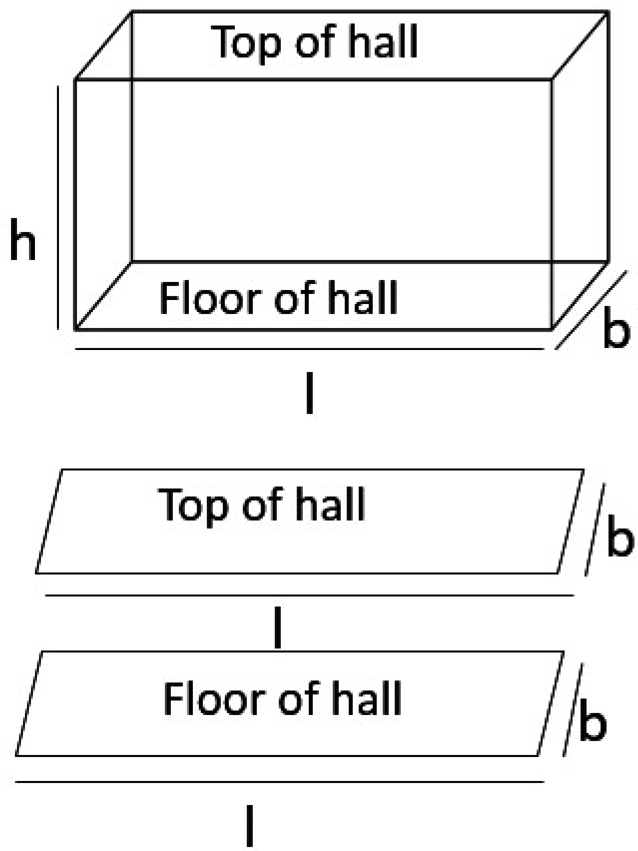
[Hint: Area of the four walls = Lateral surface area]

Solution:

Given: Perimeter of the floor of hall = 250 m, cost of painting the four walls = Rs. 15, 000

To Find: Height of the hall

Let length, breadth, and height of the rectangular hall be l m, b m, and h m respectively.



Area of four walls = 2lh + 2bh = 2(l + b) h

Perimeter of the floor of hall = 2(l + b)

Perimeter of the floor of hall = 250 m
Therefore, 2(l + b) = 250 m

Putting this value in the formula of Area of four walls we get,

Area of four walls = 2(l + b) h

Area of four walls = 250h m^2

Cost of painting per m^2 area = Rs 10

Cost of painting 250h m^2 area = Rs (250h × 10)

= Rs 2500h

However, it is given that the cost of painting the walls is Rs 15000

15000 = 2500h

h = $\frac{15000}{2500}$

h = $\frac{150}{25}$

h = 6

Therefore, the height of the hall is 6 m

Question :4 The paint in a certain container is sufficient to paint an area equal to 9.375 m^2 How many bricks of dimensions 22.5 cm × 10 cm × 7.5 cm can be painted out of this container?

Solution:

The dimensions of the brick are:

l = 22.5 cm

b = 10 cm

h = 7.5 cm

Total surface area of one brick = 2(lb + bh + lh)

= [2(22.5 ×10 + 10 × 7.5 + 22.5 × 7.5)] cm^2

= 2(225 + 75 + 168.75) cm^2

= (2 × 468.75) cm^2

= 937.5 cm^2

Area that can be painted by the paint of the container = 9.375 m^2

As 1 m2 = 10000 cm^2

= 93750 cm^2

No. of bricks = $\frac{Area\ that\ can\ be\ painted}{Surface\ are\ of\ \ brick}$

= $\frac{93750}{937.5}$

= 100

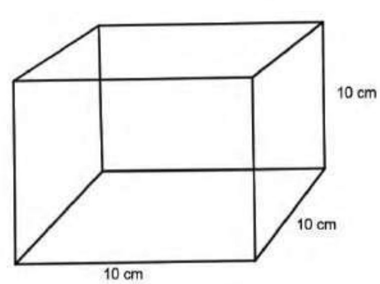
Therefore, 100 bricks can be painted out by the paint of the container.

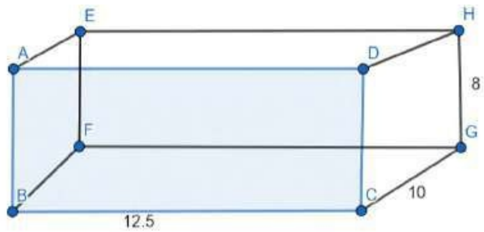
Question :5 A cubical box has each edge 10 cm and another cuboidal box is 12.5 cm long, 10 cm wide and 8 cm high.

(i) Which box has the greater lateral surface area and by how much?

(ii) Which box has the smaller total surface area and by how much?

Solution:





(i) Edge of cube = 10 cm

Length (l) of box = 12.5 cm

Breadth (b) of box = 10 cm

Height (h) of box = 8 cm

Lateral surface area (Cubical box) = $4(\text{edge})^2$

$= 4(10 \text{ cm})^2$

$= 400 \text{ cm}^2$

Lateral surface area (Cuboidal box) = $2[lh + bh]$

$= [2(12.5 \times 8 + 10 \times 8)] \text{ cm}^2$

$= (2 \times 180) \text{ cm}^2$

$= 360 \text{ cm}^2$

Clearly, the lateral surface area of the cubical box is greater than the lateral surface area of the cuboidal box

Lateral surface area of cubical box − Lateral surface area of cuboidal box $= 400 \text{ cm}^2 - 360 \text{ cm}^2 = 40 \text{ cm}^2$

Therefore, the lateral surface area of the cubical box is greater than the lateral surface area of the cuboidal box by 40 cm^2

(ii) Total surface area of cubical box = $6(\text{edge})^2$

$= 6(10 \text{ cm})^2$

$= 600 \text{ cm}^2$

Total surface area of cuboidal box = 2[lh + bh + lb]

$= [2(12.5 \times 8 + 10 \times 8 + 12.5 \times 10] \text{ cm}^2$

$= 610 \text{ cm}^2$

Clearly, the total surface area of the cubical box is smaller than that of the cuboidal box

Total surface area of cuboidal box − Total surface area of cubical box $=610 \text{ cm}^2 – 600 \text{ cm}^2$

$= 10 \text{ cm}^2$

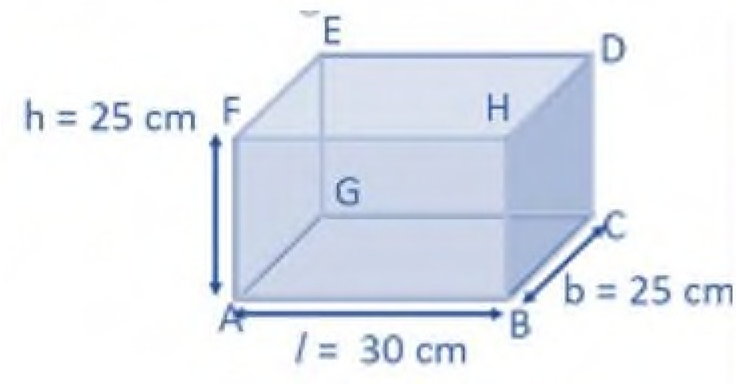
Therefore, the total surface area of the cubical box is smaller than that of the cuboidal box by 10 cm^2

Question :6 A small indoor greenhouse (herbarium) is made entirely of glass panes (including base) held together with tape. It is 30 cm long, 25 cm wide and 25 cm high.

(i) What is the area of the glass?

(ii) How much of tape is needed for all the 12 edges?

Solution:



(i) Length (l) of greenhouse = 30 cm

Breadth (b) of greenhouse = 25 cm

Height (h) of greenhouse = 25 cm

Total surface area of green house

= 2[lb + lh + bh]

$= [2(30 \times 25 + 30 \times 25 + 25 \times 25)]\ cm^2$

$= [2(750 + 750 + 625)]\ cm^2$

$= (2 \times 2125)\ cm^2$

$= 4250\ cm^2$

Therefore, the area of glass is $4250\ cm^2$

(ii) It can be observed that tape is required alongside AB, BC, CD, DA, EF, FG, GH, HE, AH, BE, DG, and CF

Total length of tape = 4(l + b + h)

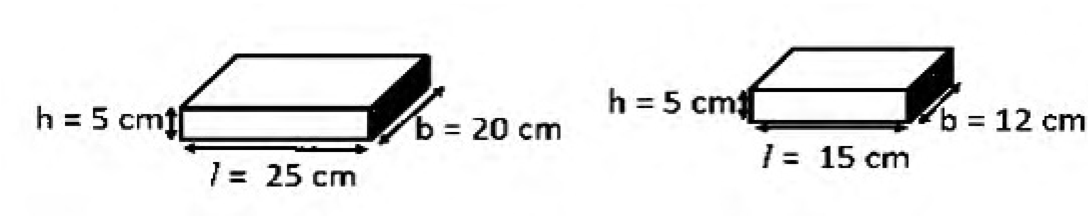
= [4(30 + 25 + 25)] cm

= 320 cm

Therefore, 320 cm tape is required for all the 12 edges

Question :7 Shanti Sweets Stall was placing an order for making cardboard boxes for packing their sweets. Two sizes of boxes were required. The bigger of dimensions25 cm × 20 cm × 5 cm and the smaller of dimensions 15 cm × 12 cm × 5 cm for all the overlaps, 5% of the total surface area is required extra. If the cost of the cardboard is Rs 4 for 1000 cm^2, find the cost of cardboard required for supplying 250 boxes of each kind

Solution:



We know total surface area of cuboid = 2(lb + lh + bh)

For bigger box

Length of bigger box = 25 cm

Breadth of bigger box = 20 cm

Height of bigger box = 5 cm

Total surface area of bigger box = 2(lb + lh + bh)

$= [2(25 \times 20 + 25 \times 5 + 20 \times 5)]\ cm^2$

$= [2(500 + 125 + 100)]\ cm^2$

$= 1450\ cm^2$

Extra area required for overlapping = 5% of the total surface area of bigger box

Extra area required for overlapping =

$= \frac{1450 \times 5}{100} cm^2$

$= 72.5 cm^2$

While considering all overlaps, total surface area of 1 bigger box = (1450 + 72.5) cm^2 =1522.5 cm^2

Area of cardboard sheet required for 250 such bigger boxes = (1522.5 × 250) cm^2

= 380625 cm^2

Similarly, total surface area of smaller box

= [2(15 ×12 + 15 × 5 + 12 × 5] cm^2

= [2(180 + 75 + 60)] cm^2

= (2 × 315) cm^2

= 630 cm^2

Therefore, extra area required for overlapping = 5% of the total surface area of smaller box

Therefore, extra area required for overlapping = $\frac{630 \times 5}{100}$ cm^2

= 31.5 cm^2

Total surface area of 1 smaller box while considering all overlaps = (630 + 31.5) cm^2

= 661.5 cm^2

Area of cardboard sheet required for 250 smaller boxes = (250 × 661.5) cm^2

= 165375 cm^2

Total cardboard sheet required

= (380625 + 165375) cm^2

= 546000 cm^2

Cost of 1000 cm^2 cardboard sheet = Rs 4

Cost of 546000 cm^2 cardboard sheet

$= \frac{546000 \times 4}{1000}$

= Rs 2184

Therefore, the cost of cardboard sheet required for 250 such boxes of each kind will be Rs 2184

Question :8 Parveen wanted to make a temporary shelter for her car, by making a box-like structure with tarpaulin that covers all the four sides and the top of the car (with the front face as a flap which can be rolled up). Assuming that the stitching margins are very small, and therefore negligible, how much tarpaulin would be required to make the shelter of height 2.5 m, with base dimensions 4 m × 3 m?

Solution:

Length (l) of shelter = 4 m

Breadth (b) of shelter = 3 m

Height (h) of shelter = 2.5 m

Tarpaulin will be required for the top and four wall sides of the shelter

Now the area of tarpaulin = Area of four sides + Area of the top

Area of Tarpaulin required = 2(lh + bh) + l b

= $[2(4 \times 2.5 + 3 \times 2.5) + 4 \times 3]\ m^2$

= $[2(10 + 7.5) + 12]\ m^2$

= $47\ m^2$

Therefore, 47 m^2 Tarpaulin will be required.

Exercise 13.2

Question :1 The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the base of the cylinder.

Solution: Given: Height (h) of cylinder = 14 cm, Curved surface area of cylinder = 88 cm^2

To Find: Diameter of the cylinder

Let the diameter of the cylinder be "d".

Formula used: Curved Surface Area of Cylinder = 2πrh

where, r = radius of the base of the cylinder and h = height of the cylinder

$2\pi rh = 88\ cm^2$ (r is the radius of the base of the cylinder)

[Now, we know that 2 times radius = diameter, 2r = d]

$\pi dh = 88\ cm^2$

$d = \frac{88 \times 7}{22 \times 14}$

$\frac{22}{7} \times d \times 14\ cm = 88\ cm^2$

$d = \frac{88 \times 7}{22 \times 14}$

d = 2cm

Therefore, the diameter of the base of the cylinder is 2 cm

Question :2 It is required to make a closed cylindrical tank of height 1 m and base diameter 140 cm from a metal sheet. How many square meters of the sheet are required for the same?

Solution: Height (h) of cylindrical tank = 1 m

Base radius (r) of cylindrical tank = ( ) cm

= 70 cm

= 0.7 m

Area of sheet required = Total surface area of tank

Area of sheet required = $2\pi r\,(r + h)$

Area of sheet required = $2 \times \frac{22}{7} \times 0.7(0.7 + 1)$

Area of sheet required = $2 \times 22 \ \times 0.1 \times 1.7$

Area of Sheet Required = 44 x 0.17

Area of sheet required = 7.48 m^2

Question :3 A metal pipe is 77 cm long. The inner diameter of a cross section is 4 cm, the outer diameter being 4.4 cm. Find its

(i) Inner curved surface area,

(ii) Outer curved surface area,

(iii) Total surface area

Solution:

Radius = $\frac{Diameter}{2}$

Inner Radius (r_1) = $\frac{4}{2}$ = 2 cm

Inner Radius (r_1) = $\frac{4.4}{2}$ = 2.2 cm

Height = Length = 77 cm

Curved Surface Area of Cylinder = 2 π r h

(i) C S A of inner surface of pipe = $2 \pi r_1 h$

Inner surface Area of pipe = $2 \times \frac{22}{7} \times 2 \times 77$

Inner surface Area of pipe = $968 cm^2$

(ii) CSA of outer surface of pipe = $2 \pi r^2 h$

$= 2 \times \frac{22}{7} \times 2.2 \times 77$

$= 2 \times 22 \times 2.2 \times 11$

$= 1064.8 cm^2$

(iii) TSA = Inner CSA + Outer CSA + Area of both circular ends of pipe

$= 2 \pi r_1 h + 2 \pi r_2 h + 2 \pi (r_2^2 - r_1^2)$

$= [968 + 1064.8 + 2\pi \{(2.2)^2 - (2)^2\}]$ cm^2

$= (2032.8 + 2 \text{ x } \frac{22}{7} \text{ x } 0.84)$ cm^2

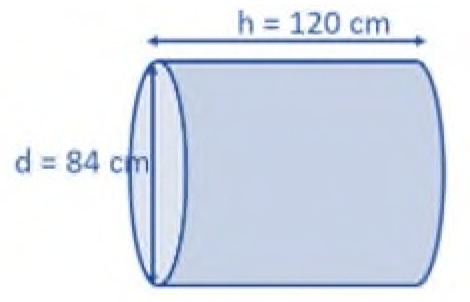
$= (2032.8 + 5.28)$ cm^2

$= 2038.08$ cm^2

Therefore, the total surface area of the cylindrical pipe is 2038.08 cm^2

Question :4 The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions to move once over to level a playground. Find the area of the playground in m^2.

Solution:



It can be observed that a roller is cylindrical

Height (h) of cylindrical roller = Length of roller

= 120 cm

Radius (r) of the circular end of roller = $\frac{84}{2}$ = 42 cm

Curved Surface Area (CSA) of roller = 2πrh

= 2 * $\frac{22}{7}$ * 42 * 120 cm^2

= 31680 cm^2

Area of field = 500 × CSA of roller

= (500 × 31680) cm^2

= 15840000 cm^2

= 1584 m^2 (∵ 1 m^2 = 10000 cm^2)

Question :5 A cylindrical pillar is 50 cm in diameter and 3.5 m in height. Find the cost of painting the curved surface of the pillar at the rate of Rs 12.50 per m^2

Solution:

Given:
Height (h) cylindrical pillar = 3.5 m

Radius (r) of the circular end of pillar = $\frac{50}{2}$ = 25 cm = 0.25 m

CSA of pillar = 2πrh

= 2 x $\frac{22}{7}$ x 0.25 x 3.5 m^2

= (44 x 0.125) m^2

= 5.5 m^2

Cost of painting 1 m^2 area = Rs 12.50

Cost of painting 5.5 m^2 area = Rs (5.5 × 12.50)

= Rs 68.75

Therefore, the cost of painting the CSA of the pillar is Rs 68.75

Question :6 Curved surface area of a right circular cylinder is 4.4 m^2. If the radius of the base of the cylinder is 0.7 m, find its height

Solution: To Find: Height of the cylinder

Given: Curved surface area of cylinder = 4.4 m^2 and radius of cylinder = 0.7 m

Concept Used:

Curved Surface Area of cylinder = 2πrh

where, r = radius of base of cylinder and h = height of cylinder

Explanation:

Let the height of the circular cylinder be h

2πrh = 4.4 m^2

(2 × 22/7 × 0.7 × h) m2 = 4.4 m^2

$4.4\ h = 4.4\ m^2$

$h = 1$ m

Therefore, the height of the cylinder is 1 m.

Question :7 The inner diameter of a circular well is 3.5 m. It is 10 m deep.

Find
(i) Its inner curved surface area,

(ii) The cost of plastering this curved surface at the rate of Rs 40 per m^2

Solution: Inner diameter of the circular well = 3.5 m

Inner radius (r) of circular well = $\frac{\text{Diameter}}{2}$ m

= 1.75 m

Depth (h) of circular well = 10 m

(i) Inner curved surface area = $2\pi rh$

$= 2 \times \frac{22}{7} \times 1.75 \times 10\ m^2$

$= (44 \times 0.25 \times 10)\ m^2$

$= 110\ m^2$

Therefore, the inner curved surface area of the circular well is 110 m^2.

(ii) Cost of plastering 1 m^2 area = Rs 40

Cost of plastering 110 m^2 area = Rs (110 × 40)

= Rs 4400

Therefore, the cost of plastering the CSA of this well is Rs 4400.

Question :8 In a hot water heating system, there is a cylindrical pipe of length 28 m and diameter 5 cm. Find the total radiating surface in the system.

Solution:

Height (h) of cylindrical pipe = Length of cylindrical pipe = 28 m

Radius (r) of circular end of pipe = $\frac{5}{2}$

= 2.5 cm

As 1 cm = $\frac{1}{100}$ m

= 0.025 m

CSA of cylindrical pipe = $2\pi rh$

= $2 \times \frac{22}{7} \times 0.025 \times 28$ m^2

= 4.4 m^2

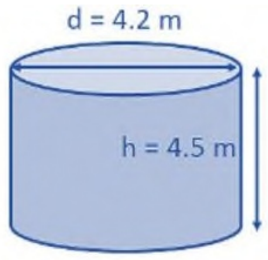
The area of the radiating surface of the system is 4.4 m^2

Question 9: Find

(i) The lateral or curved surface area of a closed cylindrical petrol storage tank that is 4.2 m in diameter and 4.5 m high.

(ii) How much steel was actually used, if $\frac{1}{12}$ of the steel actually used was wasted in making the tank

Solution:



Height (h) of cylindrical tank = 4.5 m

Radius (r) = ($\frac{4.2}{\ }$) = 2.1 m

(i) Lateral or curved surface area of tank = 2πrh

$= 2 \times \frac{22}{7} \times 2.1 \times 4.5$

$= (44 \times 0.3 \times 4.5)\ m^2$

$= 59.4\ m^2$

Therefore, the CSA of the tank is 59.4 m^2.

(ii) Total surface area of tank = 2πr (r + h)

$= 2 \times \frac{22}{7} \times 2.1 \times (2.1 + 4.5)\ m^2$

$= (44 \times 0.3 \times 6.6)\ m^2$

$= 87.12\ m^2$

Let x m^2 steel sheet be actually used in making the tank $\frac{1}{12}$ of steel was wasted.

TSA = amount of steel - the amount of steel used

$87.12\ m^2 = x - \frac{1}{12}x$

$x\ (1 - \frac{1}{12}) = 87.12\ m^2$

$x = (\frac{12}{11} \times 87.12)\ m^2$

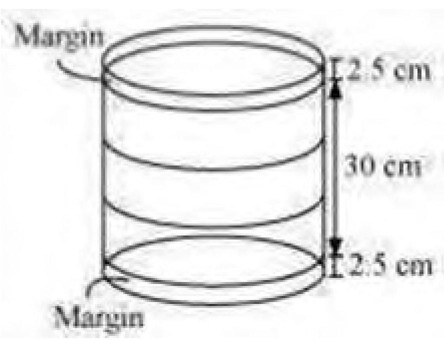
$x = 95.04\ m^2$

Therefore, 95.04 m^2 steel was used in making such a tank.

Question :10 In Fig. 13.12, you see the frame of a lampshade. It is to be covered with a decorative cloth. The frame has a base diameter of 20 cm and height of 30 cm. A margin of 2.5 cm is to be given for folding it over the top and bottom of the frame. Find how much cloth is required for covering the lampshade.

Solution:

Height (h) of the frame of lampshade = (2.5 + 30 + 2.5) cm = 35 cm



Radius (r) of the circular end of the frame of lampshade = $(\frac{20}{2})$ cm = 10 cm

Cloth required for covering the lampshade = Curved Surface Area of the Cylinder = $2\pi rh$

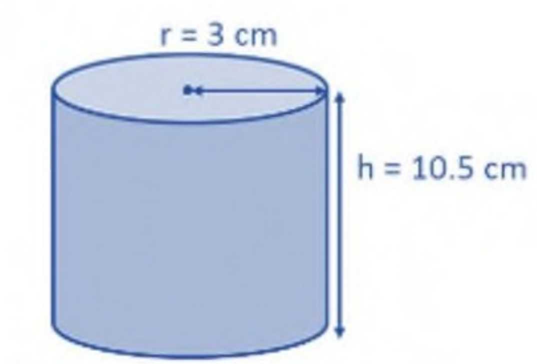
$= 2 \text{ x } \frac{22}{7} \text{ x } 10 \text{ x } 35 \text{ cm}^2$

$= 2200 \text{ cm}^2$

Hence, 2200 cm^2 cloth will be required for covering the lampshade.

Question :11 The students of a Vidyalaya were asked to participate in a competition for making and decorating penholders in the shape of a cylinder with a base, using cardboard. Each penholder was to be of radius 3 cm and height 10.5 cm. The Vidyalaya was to supply the competitors with cardboard. If there were 35 competitors, how much cardboard was required to be bought for the competition?

Solution:



Radius (r) of the circular end of cylindrical pen holder = 3 cm

Height (h) of pen holder = 10.5 cm

for pen holder the curved Surface Area and Area of Base will be needed
Surface area of 1 pen holder = CSA of pen holder + Area of base of pen holder

$= 2\pi rh + \pi r^2$

$= [2 \times \frac{22}{7} \times 3 \times 10.5 + \frac{22}{7} \times (3)^2] \text{ cm}^2$

$= (132 \times 1.5 + \frac{198}{7})\ cm^2$

$= \ (198 + \frac{198}{7})\ cm^2$

$= \frac{(198 \times 7)+198}{7}$

$= \frac{1386+198}{7}$

$= \frac{1584}{7}\ cm^2$

Area of cardboard sheet used by 1 competitor = $\frac{1584}{7}\ cm^2$

Area of cardboard sheet used by 35 competitors

$= \frac{1584}{7} \times 35$

$= 1584 \times 5$

$= 7920\ cm^2$

Exercise 13.3

Question :1 Diameter of the base of a cone is 10.5 cm and its slant height is 10 cm. Find its curved surface area.

Solution:

Radius (r) of the base of cone = $\frac{10.5}{2}$

= 5.25 cm

Slant height (l) of cone = 10 cm

CSA of cone = πrl

= $\frac{22}{7} \times 5.25 \times 10$

= $22 \times 0.75 \times 10$

= 165 cm^2

Therefore, the curved surface area of the cone is 165 cm^2

Question :2 Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m

Solution: Radius (r) = $\frac{24}{2}$ = 12 m

Slant height (l) = 21 m

TSA of cone = πr(r + l)

= $\frac{22}{7} \times 12\ (12 + 21)$

= $\frac{22}{7} \times 12 \times 33$

= 3.14×396

$= 1243.44\ m^2$

Question :3 The curved surface area of a cone is 308 cm^2 and its slant height is 14 cm. Find (i) radius of the base and (ii) total surface area of the cone.

Solution:

(i) Slant height (l) of cone = 14 cm

Let the radius of the circular end of the cone be "r".

CSA of cone = πrl

$308\ cm^2 = \quad \times r \times 14$

$r = \frac{308}{44}$

= 7 cm

Therefore, the radius of the circular end of the cone is 7 cm.

(ii) The total surface area of cone = CSA of cone + Area of base

$= \pi rl + \pi r^2$

$= [308 + \frac{22}{7} \times (7)^2]\ cm^2$

$= (308 + 154)\ cm^2$

$= 462\ cm^2$

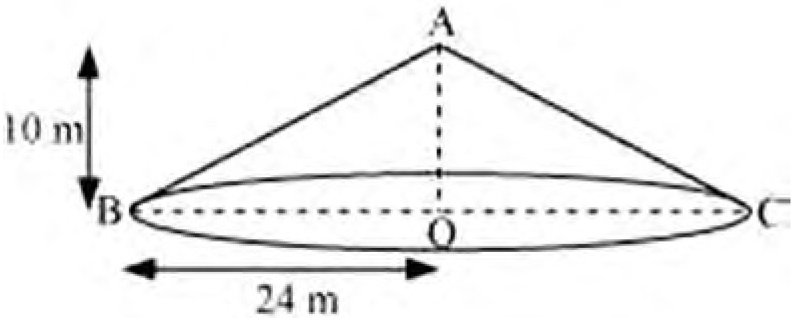
Therefore, the total surface area of the cone is 462 cm^2

Question :4 A conical tent is 10 m high and the radius of its base is 24 m. Find

(i) Slant height of the tent.

(ii) Cost of the canvas required to make the tent, if the cost of 1 m^2 canvas is Rs 70

Solution: (i) Let ABC be a conical tent.
Height (h) of conical tent = 10 m
Radius (r) of conical tent = 24 m



Let the slant height of the tent be l.
In ΔABO,
$AB^2 = AO^2 + BO^2$
$l^2 = h^2 + r^2$
$= (10\text{ m})^2 + (24\text{ m})^2$
$= 676\text{ m}^2$
l = 26 m

(ii) CSA of tent = $\pi r l$
$= \frac{22}{7} * 24 * 26$
$= \frac{13728}{7}\text{ m}^2$
Cost of 1 m^2 canvas = Rs 70
So, cost of $\frac{13728}{7}\text{ m}^2$ canvas = $(\frac{13728}{7}\text{ m}^2 * 70)$
= Rs 137280

Question :5 What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6 m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm (Use $\pi = 3.14$)

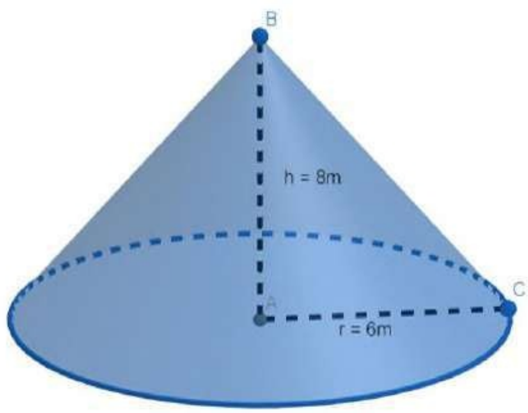
Solution:

To Find: Length of Tarpaulin

Concept Used:

Curved Surface Area of Cone = πrl

Diagram:



Height (h) = 8m

Radius (r) = 6m

Now, we know that,

According to Pythagoras theorem, $l2 = r2 + h2$

$\Rightarrow l2 = 62 + 82$

$\Rightarrow l2 = (62 + 82)$

$\Rightarrow l2 = 36 + 64$

$\Rightarrow l2 = 100$

$\Rightarrow l = \sqrt{100} = 10$ m

Therefore, slant height of the conical tent = 10 m.

CSA of conical tent = πrl

$= \pi \times 6\text{m} \times 10\text{m}$

$= 3.14 \times 6m \times 10m$

$= 188.4\ m^2$

Now, Let the length of tarpaulin sheet required be "x" m

As 20 cm will be wasted, therefore, the effective length will be = (x − 20 cm) = (x − 0.2 m)

Breadth of tarpaulin = 3 m

Area of sheet = CSA of tent

$\Rightarrow [(x - 0.2\ m) \times 3]\ m = 188.4\ m^2$

$\Rightarrow\ \ x - 0.2\ m = \frac{188.4}{3}$

$\Rightarrow x - 0.2\ m = 62.8\ m$

$\Rightarrow x = 63\ m$

Therefore, the length of the required tarpaulin sheet will be 63 m.

Question :6 The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white-washing its curved surface at the rate of Rs 210 per 100 m^2

Solution:

Slant height (l) of conical tomb = 25 m

Base radius (r) of tomb = $\frac{14}{2}$

= 7 m

CSA of conical tomb = πrl

$= \frac{22}{7} * 7 * 25$

$= 550\ m^2$

Cost of white-washing 100 m^2 area = Rs 210

Cost of white-washing 550 m^2 area = Rs ($\frac{210*550}{100}$)

= Rs. 1155

Therefore, it will cost Rs 1155 while white-washing such a conical tomb.

Question :7 A joker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps.

Solution: Radius (r) = 7cm

Height (h) = 24 cm

Slant height (l) = $\sqrt{7*7+24*24}$

= $\sqrt{625}$

= 25m

CSA (1 conical cap) = πrl

= $\left(\frac{22}{7}\times 7\times 25\right)$

= $550cm^2$

CSA of 10 conical caps = (10 × 550)

= $5500cm^2$

Question :8 A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height1 m. If the outer side of each of the cones is to be painted and the cost of painting is Rs 12 per m^2, what will be the cost of painting all these cones? (Use π = 3.14 and take $\sqrt{1.04}$ =1.02)

Solution:

Radius (r) = $\frac{40}{2}$ = 20 cm

= 0.2 m

Height (h) = 1 m

Slant height (l) = $\sqrt{1*1+0.2*0.2}$

$= \sqrt{1.04}$

= 1.02 m

Curved Surface Area = $\pi r l$

= (3.14 × 0.2×1.02) m^2

= 0.64056 m^2

CSA (50 cones) = 0.64056 * 50

= 32.028 m^2

Cost of painting 1 m^2 area = Rs 12

Cost of painting 32.028 m^2 area = Rs (32.028 × 12)
= Rs 384.336

= Rs 384.34 (approximately)

Therefore, it will cost Rs 384.34 in painting 50 such hollow cones.

Exercise 13.4

Question :1 Find the surface area of a sphere of radius:

(i) 10.5 cm

(ii) 5.6 cm

(iii) 14 cm

Answer: (i) Radius (r) of sphere = 10.5 cm

Surface area of sphere = $4\pi r^2$

$= 4 \times \frac{22}{7} \times 10.5 \times 10.5$

$= 88 \times 1.5 \times 10.5$

$= 1386 \text{ cm}^2$

(ii) Radius(r) of sphere = 5.6 cm

Surface area of sphere = $4\pi r^2$

$= 4 \times \frac{22}{7} \times 5.6 \times 5.6$

$= 88 \times 0.8 \times 5.6$

$= 394.24 \text{ cm}^2$

(iii) Radius (r) of sphere = 14 cm

Surface area of sphere = $4\pi r^2$

$= 4 \times \frac{22}{7} \times 14 \times 14$

$= 4 \times 44 \times 14$

$= 2464 \text{ cm}^2$

Question :2 Find the surface area of a sphere of diameter:

(i) 14 cm

(ii) 21 cm

(iii) 3.5 m

Answer: (i) Radius (r) of sphere = $\frac{Diameter}{2}$

$= \frac{14}{2}$

= 7 cm

Surface area of sphere = $4\pi r^2$

$= 4 \times \frac{22}{7} \times 7 \times 7$

$= 88 \times 7$

= 616 cm^2

(ii) Radius (r) of sphere = $\frac{Diameter}{2}$

$= \frac{21}{2}$

= 10.5 cm

Surface area of sphere = $4\pi r^2$

$= 4 \times \frac{22}{7} \times 10.5 \times 10.5$

= 1386 cm^2

(iii) Radius (r) of sphere = $\frac{Diameter}{2}$

$= \frac{3.5}{2}$

= 1.75 m

Surface area of sphere = $4\pi r^2$

$= 4 \times \frac{22}{7} \times 1.75 \times 1.75$

= 38.5 cm^2

Question :3 Find the total surface area of a hemisphere of radius 10 cm. (Use $\pi = 3.14$)

Answer: Radius (r) of hemisphere = 10 cm

Total surface area of hemisphere = CSA of hemisphere + Area of circular end of hemisphere

$= 2\pi r^2 + \pi r^2$

$= 3\pi r^2$

= [3 * 3.14 * 10 * 10]

$= 942\ cm^2$

Question :4 The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases

Answer:

Radius (r_1) of spherical balloon = 7 cm

Radius (r_2) of spherical balloon, when air is pumped into it = 14 cm

Ratio = $\frac{Initial\ surface\ area}{Surface\ area\ after\ pumping\ ballon}$

$= \frac{4\pi r_1 \times r_1}{4\pi r_2 \times r_2}$

$= \left(\frac{r_1}{r_2}\right)^2$

$= \left(\frac{7}{14}\right)^2 = \frac{1}{4}$

Therefore, the ratio between the surface areas is 1:4

Question :5 A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of tin-plating it on the inside at the rate of Rs 16 per 100 cm^2

Ans.: Inner radius (r) = $\frac{10.5}{2}$ = 5.25 cm

Surface area of hemispherical bowl = $2\pi r^2$

$= 2 * \frac{22}{7} * 5.25 * 5.25$

= 173.25 cm^2

Cost of tin-plating 100 cm^2 area = Rs 16

Cost of tin-plating 173.25 cm^2 area = $\left(\frac{16*173.25}{100}\right)$

= Rs 27.72

Question :6 Find the radius of a sphere whose surface area is 154 cm^2.

Ans.:

Let the radius of the sphere be r

Surface area of sphere = 154 cm^2

$4 \pi r^2 = 154$

$r^2 = \frac{154}{4\pi}$

$\Rightarrow r^2 = \frac{154 \times 7}{4 \times 22}$

$\Rightarrow r^2 = \frac{\overset{7}{\cancel{154}} \times 7}{4 \times \underset{1}{\cancel{22}}}$

$\Rightarrow r^2 = \frac{49}{1}$

$\Rightarrow r^2 = 12.25$

	3.5
3	12.25
	9
65	3.25
	3.25
	0

$\Rightarrow r = 3.5$

Therefore, the radius of the sphere whose surface area is 154 cm^2 is 3.5 cm

Question :7 The diameter of the moon is approximately one fourth of the diameter of the earth. Find the ratio of their surface areas.

Ans.: Let the diameter of earth be d. Therefore, the diameter of moon will be $\frac{d}{4}$

Radius (Earth) = $\frac{d}{2}$

Radius (Moon) = $\frac{1}{2} \times \frac{d}{4} = \frac{d}{8}$

Surface Area of moon = $4\pi\left(\frac{d}{8}\right)^2$

Surface Area of Earth = $4\pi\left(\frac{d}{2}\right)^2$

Ratio = $\frac{\text{Surface Area of moon}}{\text{Surface Area of Earth}}$

$= \frac{4\pi \frac{d}{64}}{4\pi \frac{d}{4}}$

$= \frac{4}{64}$

$= \frac{1}{16}$

Question :8 A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl.

Ans.: The inner radius of hemispherical bowl = 5 cm

The thickness of the bowl = 0.25 cm

Outer radius (r) of hemispherical bowl = (5 + 0.25) cm

= 5.25 cm

Outer CSA of hemispherical bowl = $2\pi r^2$

= 2 × × $(5.25)^2$

= 173.25 cm^2

Question :9 A right circular cylinder just encloses a sphere of radius r (see Fig. 13.22). Find

(i) Surface area of the sphere,

(ii) Curved surface area of the cylinder

(iii) Ratio of the areas obtained in (i) and (ii)

Fig. 13.22

Ans.: (i) Surface area of sphere = $4\pi r^2$

(ii) Height of cylinder = r + r = 2r

[As sphere touches both upper and lower surface of cylinder]

Radius of cylinder = r

CSA of cylinder = $2\pi rh$

$= 2\pi r\,(2r)$

$= 4\pi r^2$

(iii) Ratio = $\frac{4\pi r \times r}{4\pi r \times r}$

$= \frac{1}{1} = 1$

Ratio = 1:1

Exercise 13.5

Question :1 A matchbox measures 4 cm × 2.5 cm × 1.5 cm. What will be the volume of a packet containing 12 such boxes?

Answer:

Matchbox is a cuboid having its length (l), breadth (b), height (h) as 4 cm, 2.5 cm and 1.5 cm respectively

Volume of 1 match box = l × b × h

= $(4 \times 2.5 \times 1.5)\ cm^3$

= $15\ cm^3$

Volume of 12 such matchboxes = $(15 \times 12)\ cm^3$

= $180\ cm^3$

Therefore, the volume of 12 match boxes is $180\ cm^3$

Question :2 A cuboidal water tank is 6 m long, 5 m wide and 4.5 m deep. How many litres of water can it hold? (1 m3 = 1000 l)

Ans.:

The given cuboidal water tank has its length (l) as 6 m, breadth (b) as 5 m and height (h) as 4.5 m respectively

Volume of tank = l × b × h

= $(6 \times 5 \times 4.5)\ m^3$

= $135\ m^3$

Amount of water that 1 m3 volume can hold = 1000 litres

Amount of water that 135 m3 volume can hold = (135 × 1000) litres

= 135000 litres

Question :3 A cuboidal vessel is 10 m long and 8 m wide. How high must it be made to hold 380cubic metres of a liquid?

Ans.:

Let the height of the cuboidal vessel be h

Length (l) of vessel = 10 m

Width (b) of vessel = 8 m

The volume of vessel = 380 m^3

l × b × h = 380

[(10) (8) h] m^2 = 380 m^3

$= \frac{380}{80}$

$= \frac{38}{8}$

= 4.75 m

Therefore, the height of the vessel should be 4.75 m.

Question :4 Find the cost of digging a cuboidal pit 8 m long, 6 m broad and 3 m deep at the rate of Rs 30 per m^3

Ans.: The given cuboidal pit has its length (l) as 8 m, width (b) as 6 m, and depth (h) as 3 m respectively

Volume of pit = l × b × h

= (8 × 6 × 3) m^3

= 144 m^3

Cost of digging per m3 volume = Rs 30

Cost of digging 144 m3 volume = Rs (144 × 30)

= Rs 4320

Question :5 The capacity of a cuboidal tank is 50000 litres of water. Find the breadth of the tank, if its length and depth are respectively 2.5 m and 10 m

Ans.:

Let the breadth of the tank be b m

Length (l) and depth (h) of tank is 2.5 m and 10 m respectively

Volume of tank = l × b × h

= (2.5 × b × 10) m^3

= 25 b m^3

Capacity of tank = 25 b m^3 = 25000 b litres

[As 1 m^3 = 1000 litres]

25000 b = 50000

b = $\frac{50000}{25000}$

b = 2

Therefore, the breadth of the tank is 2 m

Question :6 A village, having a population of 4000, requires 150 litres of water per head per day. It has a tank measuring 20 m × 15 m × 6 m. For how many days will the water of this tank last?

Answer:

The given tank is cuboidal in shape having its length (l) as 20 m, breadth (b) as 15 m, and height (h) as 6 m respectively

Capacity of tank = l × b× h

= (20 × 15 × 6) m^3

= 1800 m^3

= 1800000 litres

Water consumed by the people of the village in 1 day = (4000 × 150) litres

= 600000 litres

Let water in this tank last for n days

Water consumed by all people of village in n days = Capacity of tank

n × 600000 = 1800000

n = 3

Therefore, the water of this tank will last for 3 days.

Question :7 A go down measures 40 m × 25 m × 15 m. Find the maximum number of wooden crates each measuring 1.5 m × 1.25 m × 0.5 m that can be stored in the go down.

Answer:

The dimensions of go down are:

length (l_1) as 40 m, breadth (b_1) as 25 m, height (h_1) as 15 m,

while the wooden crate has the dimensions:

length (l_2) as 1.5 m, breadth (b_2) as 1.25 m, and height (h_2) as 0.5 m respectively

Therefore,

Volume of go down = $l_1 \times b_1 \times h_1$= (40m × 25m × 15m)

= 15000 m^3

Volume of 1 wooden crate = $l_2 \times b_2 \times h_2$

= (1.5 m × 1.25m × 0.5m)

= 0.9375 m^3

Let n wooden crates can be stored in the go down

Therefore, volume of n wooden crates = Volume of go down

0.9375 × n = 15000

$n = \frac{15000}{0.9375} = 16000$

∴ The maximum number of wooden crates will be 16000.

Question :8 A solid cube of side 12 cm is cut into eight cubes of equal volume. What will be the side of the new cube? Also, find the ratio between their surface areas

Answer: Side (a) of cube = 12 cm

Volume of cube = $(a)^3 = (12\ cm)^3 = 1728\ cm^3$

Let the side of the smaller cube be a1

Volume = $(\frac{1728}{8})$

= 216 cm^3

$(a_1)^3 = 216\ cm^3$

a_1 = 6cm

Ratio = $\frac{Surface\ area\ of\ bigger\ cube}{smaller\ area\ of\ small\ cube}$

Ratio = $\frac{6 \times 12 \times 12}{6 \times 6 \times 6}$

Ratio = $\frac{4}{1}$

Ratio = 4 :1

Question :9 A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water will fall into the sea in a minute?

Ans.:

Rate of water flow = 2 km per hour

$= \left(\frac{2000}{60}\right)$m/ min

$= \left(\frac{100}{3}\right)$m/ min

Depth (h) of river = 3 m

Width (b) of river = 40 m

The volume of water flowed in 1 min = ($\frac{100}{3} \times 40 \times 3$)

= 4000 m^3

Therefore, in 1 minute, 4000 m^3 water will fall in the sea.

Exercise 13.6

Question :1 The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. How many litres of water can it hold? (1000 cm^3 = 1L)

Ans.:

Let the radius of the cylindrical vessel be r

Height (h) of vessel = 25 cm

Circumference of vessel = 132 cm

2∏r = 132 cm

r = $\frac{132 \times 7}{2 \times 22}$

= 21 cm

Volume of cylindrical vessel = $\pi r^2 h$

= $\frac{22}{7} \times 21 \times 21 \times 25$

= 34650 cm^3

= $\left(\frac{34650}{1000}\right)$ litres

= 34.65 litres

Question :2 The inner diameter of a cylindrical wooden pipe is 24 cm and its outer diameter is 28 cm. The length of the pipe is 35 cm. Find the mass of the pipe, if 1 cm^3 of wood has a mass of 0.6 g.

Ans.: Inner radius (r_1) = 24/2 = 12 cm

Outer radius (r_2) = 28/2 = 14 cm

Height (h) = Length = 35 cm

Volume = π $(r_2^2 - r_2^1)$ h

= 22/7 × (14^2 – 12^2) × 35

= 110 × 52

= 5720 cm^3

Mass of 1 cm^3 wood = 0.6 g

Mass of 5720 cm^3 wood = (5720 × 0.6) g

= 3432 g

= 3.432 kg

Question :3 A soft drink is available in two packs – (i) a tin can with a rectangular base of length 5 cm and width 4 cm, having a height of 15 cm and (ii) a plastic cylinder with a circular base of diameter 7 cm and height 10 cm. Which container has greater capacity and by how much?

Ans.: The tin can will be cuboidal in shape while the plastic cylinder will be cylindrical in shape

Length (l) of tin can = 5 cm

Breadth (b) of tin can = 4 cm

Height (h) of tin can = 15 cm

Capacity of tin can = l × b × h

= (5 × 4 × 15) cm^3

= 300 cm^3

Radius (r) of circular end of plastic cylinder = 3.5 cm

Height (H) of plastic cylinder = 10 cm

The capacity of plastic cylinder = $\pi r^2 H$

= (22/7) × 3.5 × 3.5 × 10

= 11 × 35

= 385 cm^3

Clearly, plastic cylinder has the greater capacity

Difference in capacity = (385 − 300) = 85 cm^3

Question :4 If the lateral surface of a cylinder is 94.2 cm^2 and its height is 5 cm, then find (i) radius of its base (ii) its volume. (Use π = 3.14)

Ans.: (i) Height (h) of cylinder = 5 cm

Let radius of cylinder be r

CSA of cylinder = 94.2 cm^2

$2\pi rh = 94.2$

(2 × 3.14 × r × 5) = 94.2

$r = \frac{94.2}{2 \times 3.14 \times 2}$

$= \frac{94.2}{31.4}$

r = 3 cm

(ii) Volume of cylinder = $\pi r^2 h$

= (3.14 × $(3)^2$ × 5)

= 3.14 x 9 x 5= 3.14 x 45

= 141.3 cm^3

Question :5 It costs Rs 2200 to paint the inner curved surface of a cylindrical vessel 10 m deep. If the cost of painting is at the rate of Rs 20 per m^2, find (i) inner curved surface area of the vessel,

(ii) Radius of the base,

(iii) Capacity of the vessel

Ans.:

Rs 20 is the cost of painting 1 m^2 area Rs 2200 is the cost of painting = ($\frac{1}{20}$× 2200) m^2

= 110 m^2

Therefore, the inner surface area of the vessel is 110 m^2

(ii) Let the radius of the base of the vessel be r

Height (h) = 10 m

Surface Area = 2πrh = 110 m^2

= $2 \times \frac{22}{7} \times r \times 10 = 110$

= $r = \frac{7}{4}$ m

r = 1.75 m

(iii) Volume = $\pi r^2 h$

= $\frac{22}{7} \times 1.75 \times 1.75 \times 10$

= 96.25 m^3

The capacity of the vessel is 96.25 m^3 or 96250 litres

Question :6 The capacity of a closed cylindrical vessel of height 1 m is 15.4 litres. How many square metres of metal sheet would be needed to make it?

Ans.: Let the radius of the circular end be "r".

The volume of cylindrical vessel = 15.4 litres

1 litre = 1/1000 m^3

= 0.0154 m^3

$r^2h = 0.0154\ m^3$

$(\frac{22}{7} \times r \times r \times 1)\ m = 0.0154\ m^3$

r = 0.07 m

TSA of vessel = $2\pi r\ (r + h)$

$= 2 \times \frac{22}{7} \times 0.07\ (0.07 + 1)$

$= 0.44 \times 1.07$

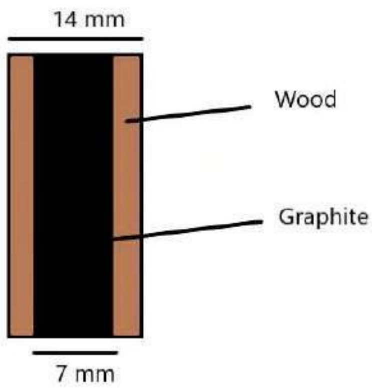
= 0.4708 m^2

Height (h) of cylindrical vessel = 1 m

Therefore, the metal sheet would be required to make the cylindrical vessel is 0.4708 m^2

Question :7 A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled in the interior. The diameter of the pencil is 7 mm and the diameter of the graphite is 1 mm. If the length of the pencil is 14 cm, find the volume of the wood and that of the graphite

Ans.:



Concept Used: Volume of the wood = Volume of a pencil – volume graphite

Volume of Cylinder = $\pi r^2 h$

Where r = radius of the cylinder and h = height of the cylinder

Given: Diameter of pencil = 7 mm

Diameter of graphite = 1 mm

Length of the pencil = 14 cm

Assumption: Let r^1 be the radius of pencil, r^2 be the radius of graphite and h be the height of pencil.

Explanation:

Radius

Radius (pencil), r_1 mm = 0.35 cm

Radius (graphite), r_2 mm = 0.05 cm

Height (Pencil), h = 14 cm

Volume of wood in pencil = $\pi (r_1^2 - r_2^2) h$

$= 22/7 [(0.35)^2 - (0.05)^2 \times 14]$

$= 22/7 (0.1225 - 0.0025) \times 14$

$= 44 \times 0.12$

$= 5.28\ cm^3$

Volume of graphite = $\pi r_2{}^2 h$

$= 22/7 \times (0.05)^2 \times 14$

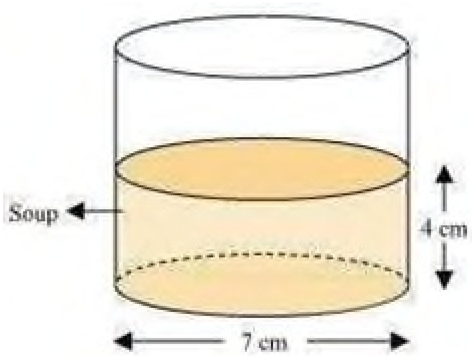
$= 44 \times 0.0025$

$= 0.11\ cm^3$

Hence, The volume of wood in pencil is $5.28\ cm^3$ and volume of graphite in pencil is $0.11\ cm^3$.

Question :8 A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7 cm. If the bowl is filled with soup to a height of 4 cm, how much soup the hospital has to prepare daily to serve 250 patients?

Ans.: Radius (r) of cylindrical bowl = $\frac{7}{2}$ = 3.5cm



Height (h) of bowl, up to which bowl is filled with soup = 4 cm

Volume of soup in 1 bowl = $\pi r^2 h$

$= \frac{22}{7} * 3.5 * 3.5 * 4$

$= (11 \times 3.5 \times 4)$

$= 154\ cm^3$

Volume of soup given to 250 patients $= (250 \times 154)\ cm^3$

$= 38500\ cm^3$

= 38.5 litres.

<u>Exercise 13.7</u>

Question :1 Find the volume of the right circular cone with

(i) Radius 6 cm, height 7 cm

(ii) Radius 3.5 cm, height 12 cm

Ans.: (i) Radius (r) = 6 cm

Height (h) = 7 cm

Volume = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 7$

= 12 × 22

= 264 cm^3

(ii) Radius (r) = 3.5 cm

Height (h) = 12 cm

Volume = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 12$

= 154 cm3

Question :2 Find the capacity in litres of a conical vessel with

(i) Radius 7 cm, slant height 25 cm

(ii) Height 12 cm, slant height 13 cm

v (i) Radius (r) = 7 cm

Slant height (l) = 25 cm

As we know $l^2 = h^2 + b^2$

So,

h = $\sqrt{625 - 49}$

h = $\sqrt{576}$

= 24 cm

Volume = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24$

= 154 × 8

= 1232 cm^3

= Capacity of conical vessel = $\left(\frac{1232}{1000}\right)$ litres

= 1.232 litres

(ii) Height (h) = 12 cm

Slant height (l) = 13 cm

r = $\sqrt{13^2 - 12^2}$

r = $\sqrt{16 - 144}$

r = $\sqrt{125}$

= 5 cm

Volume = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 12$

$= \frac{2200}{7} cm^3$

Capacity of conical vessel = $\left(\frac{2200}{7000}\right)$ litres

$= \frac{11}{35}$litres

Question :3 The height of a cone is 15 cm. If its volume is 1570 cm^3, find the radius of the base. (Use π = 3.14)

Ans.: Height (h) of cone = 15 cm

Let the radius be r

Volume of cone = 1570 cm^3

$\frac{1}{3}\pi r^2 h = 1570$

$\frac{1}{3} \times 3.14 \times r \times r \times 15 = 1570$

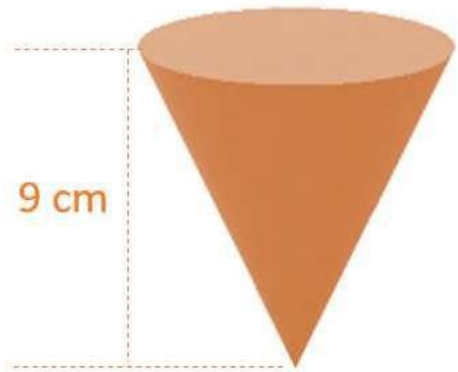
$$r^2 = \frac{\cancel{1570}^{5} \times}{\cancel{314}_{1}}$$

$$= \frac{15 \times 100}{15}$$

$r^2 = 100$

r = 10cm.

Question :4 If the volume of a right circular cone of height 9 cm is 48 π cm^3, find the diameter of its base



Answer:
Height (h) = 9 cm
Let the radius be r
Volume = $48\pi cm^3$
We know volume of cone = $\frac{1}{3}\pi r^2 h$
So,
$\frac{1}{3}\pi r^2 h = 48\pi$

$r^2 = 16$ cm

r = ±4 cm

As radius cannot be negative,
r = 4
Diameter = 2r
= 2 × 4

= 8 cm

Question :5 A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kiloliters?

Ans.:

Radius = 1.75 m

Height (h) = 12 m

Volume = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3}\times \pi \times 1.75 \times 1.75 \times 12$

= 38.5 m^3

Since, 1 m^3 = 1 kiloliter

Capacity of the pit = (38.5 × 1)

= 38.5 kiloliters

Question :6 The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28 cm, find

(i) Height of the cone

(ii) Slant height of the cone

(iii) Curved surface area of the cone

Ans.:

(i) Radius = 14 cm

Let the height be h

Volume = 9856 cm^3

$\frac{1}{3}\pi r^2 h = 9856$

$\frac{1}{3}\times \frac{22}{7} \times 14 \times 14 \times h = 9856$

h = 48 cm

(ii) Slant height (l) = $\sqrt{r * r + h * h}$

= $\sqrt{14 * 14 + 48 * 48}$

= $\sqrt{196 + 2304}$

= 50 cm

(iii) CSA = πrl

= $\frac{22}{7}$× 14 × 50

= 2200 cm^2

Question :7 A right triangle ABC with sides 5 cm, 12 cm, and 13 cm is revolved about the side 12 cm.

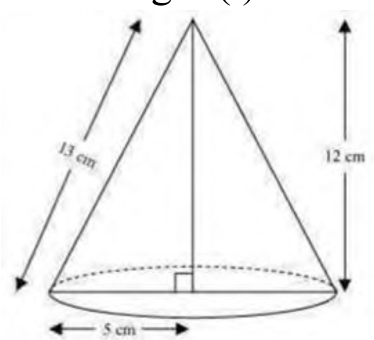
Find the volume of the solid so obtained.

Ans.: When right-angled ΔABC is revolved about its side 12 cm then a cone is formed having

Height (h) = 12 cm

Radius (r) = 5 cm

Slant height (l) = 13 cm



Volume = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3} \times \pi \times 5 \times 5 \times 12$

= 25 × 4 Π cm^3

= 100 Π cm^3

Now Π = 3.14So 100 π = 3.14 x 100 = 314 cm^3

Hence Volume of the figure = 314 cm^3

Question :8 If the triangle ABC in the Question 7 above is revolved about the side 5 cm, then find the volume of the solid so obtained. Find also the ratio of the volumes of the two solids obtained in Questions 7 and 8.

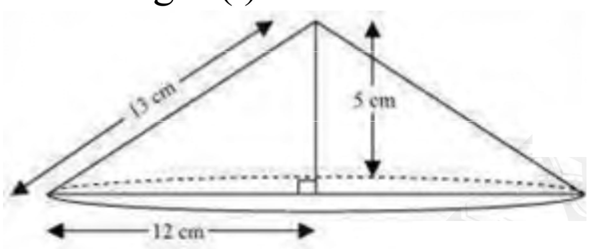
Ans.:

Case 1: When right-angled ΔABC is revolved about its side 5 cm a cone will be formed

Radius (r) = 12 cm

Height (h) = 5 cm

Slant height (l) = 13 cm

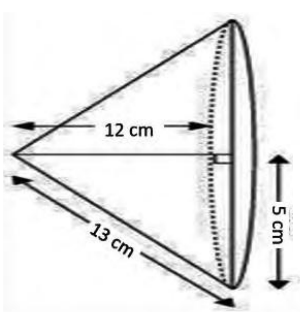


Volume = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3} \times \pi \times 12 \times 12 \times 5$

= $240\,\pi$ cm^3

Case 2: when a right triangle ABC is revolved about the side 12cm, a cone is formed as shown in the above figure, where



radius r = 5 cm

height h = 12 cm

and slant height l = 13cm

Now, volume of the cone = $\pi r^2 h/3$

$= \frac{1}{3}\pi 5^2 \times 12$

$= \frac{1}{3}\pi \times 5 \times 5 \times 12$

$= \pi \times 25 \times 4$

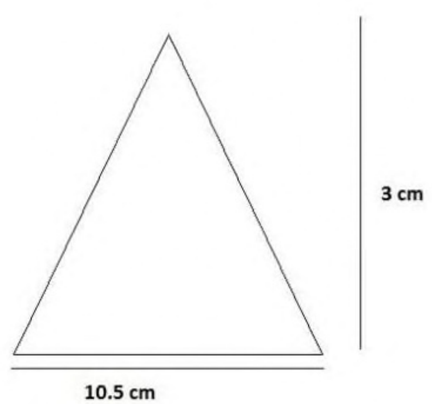
$= 100\pi$

Ratio $= \frac{100\pi}{240\pi}$

$= \frac{5}{12} = 5:12$

Question :9 A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required.

Ans.:



Radius $= \frac{Diameter}{2}$

Radius of Heap $= \frac{10.5}{2}$

Radius (r) = 5.25 m

Height (h) = 3 m

Volume of Heap = Volume of Cone

Volume of Heap = $\frac{1}{3}\pi r^2$h

Volume of Heap= $\frac{1}{3} \times \frac{22}{7} \times 5.25 \ \times 5.25 \times 3$

Volume of Heap = $22 \times 0.75 \times 0.75$

Volume of Heap = 86.625m^2

Volume of Heap = 86.625 m^3

Now for the Area of Canvas needed we need to calculate the curved surface Area of Cone.

Curved Surface Area of Cone = π r l

where r = radius and l = slant height

Slant Height is

given by l = $\sqrt{r^2 + h^2}$

l = $\sqrt{5.25^2 + 3^2}$ = $\sqrt{27.5625 + 9}$ = $\sqrt{36.5625}$ = 6.046

Area of canvas = $\frac{22}{7} \times 5.25 \times 6.046$

= 99.756 m^2

Exercise 13.8

Question :1 Find the volume of a sphere whose radius is

(i) 7 cm (ii) 0.63 m

Ans.: (i) Radius (r) = 7 cm

We know that volume of sphere is given by formula,

V = $\frac{4}{3}\pi r^3$

where, r is the radius of the sphere.

= $\frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$

= $\frac{4312}{3}$

= $1437\frac{1}{3}$ cm^3

(ii) Radius (r) = 0.63 m

We know that volume of sphere is given by formula,

V = $\frac{4}{3}\pi r^3$

where, r is the radius of the sphere.

= $\frac{4}{3} \times \frac{22}{7} \times (0.63)$ m^3

= 1.0478 m^3

Question :2 Find the amount of water displaced by a solid spherical ball of diameter

(i) 28 cm

(ii) 0.21 m

Ans.: Note: when a ball is dropped in any container, the volume of water displaced is same as volume of sphere as now sphere has taken the place of water. (i)

d = 28 cm As,

r = $\frac{d}{2} = \frac{28}{2}$

Radius (r) = 14 cm

Volume of water displaced = Volume of Sphere = $\frac{4}{3}\pi r^3$

= $\frac{4}{3} \times \frac{22}{7} \times (14)^3$

= $\frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14$

= $\frac{4}{3} \times 22 \times 2 \times 14 \times 14$

= $\frac{34469}{3}$

= 11498$\frac{2}{3} cm^3$

(ii) d = 0.21 m

As, r = $\frac{d}{2}$

= $\frac{0.21}{2}$

Radius (r) = 0.105 m

Volume of water displaced =Volume of Sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (0.105)^3$

$= \frac{4\times22\times105\times105\times105}{3\times7\times1000\times1000\times1000}$

$= \frac{4\times22\times\overset{35}{\cancel{105}}\times\overset{15}{\cancel{105}}\times105}{\underset{1}{\cancel{3}}\times\underset{1}{\cancel{7}}\times1000\times1000\times1000}$

$= \frac{4\times22\times35\times15\times105}{1000\times1000\times1000}$

$= \frac{4851\cancel{000}}{1000000\cancel{000}}$

$= 0.004851 m^3$

Question :3 The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm^3?

Ans.:

Given: Density of ball = 8.9 gm per cm^3

Diameter of the ball = 4.2 cm

To find: Mass of the ball

Formula to be used:

Density = $\frac{Mass}{Volume}$

Radius (r) = $\frac{\text{Diameter}}{2}$ = 2.1 cm

Volume of sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} x \frac{22}{7} x (2.1)^3$

= 38.808 cm^3

Density = $\frac{Mass}{Volume}$

Mass = Density × Volume

= 8.9 × 38.808 = 345.3912 gm

Question :4 The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Ans.:

Given: The diameter of the moon is approximately one-fourth of the diameter of the earth.

Let,

Diameter (Earth) = d, then Radius (Earth) = $\frac{d}{2}$

Diameter (Moon) = $\frac{d}{4}$, then Radius (Moon) = $\frac{d}{8}$

Volume of sphere = $\frac{4}{3}\pi r^3$

Volume (Moon) = $\frac{4\pi r^3}{3}$

$= \left(\frac{4}{3}\right) \times \pi \times (d/8)^3$

$= \left(\frac{1}{512}\right) \times \left(\frac{4}{3}\right) \pi d^3$

Volume (Earth) = $\frac{4\pi r^3}{3}$

$= \left(\frac{4}{3}\right) \times \pi \times \left(\frac{d}{2}\right)^3$

$= \left(\frac{1}{8}\right) \times \left(\frac{4}{3}\right) \pi d^3$

Now,

$$\frac{Voume\ of\ moon}{Volume\ of\ Earth} = \frac{\frac{1}{512}\times\frac{4}{3}\pi d^3}{\frac{1}{8}\times\frac{4}{3}\pi d^3} = \frac{1}{64}$$

Volume of moon $= \left(\frac{1}{64}\right)$ Volume of earth

Note: You can also solve this by taking diameter of earth as x and then diameter of moon will be $\frac{x}{4}$.

Question :5 How many litres of milk can a hemispherical bowl of diameter 10.5 cm hold?

Ans.: Radius � = 5.25 cm

Volume = $\frac{2}{3}\pi r^3$

= $\frac{2}{3}\times\frac{22}{7}\times(5.25)^3$

= $303.1875 cm^3$

= 303.1875 cm^3

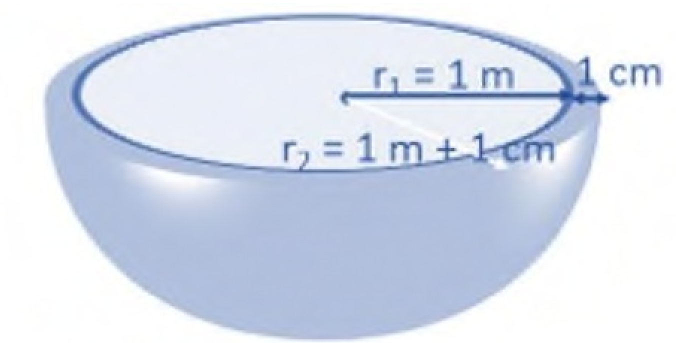
Capacity (Hemispherical bowl) =$\frac{303.1875}{10000}$ litres

= 0.3031875

= 0.303 litres (Approx)

Question :6 A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.

Ans.:



Inner radius (r_1) = 1 m

Thickness of iron sheet = 1 cm we know 1 cm =$\frac{1}{100}$ m = 0.01 m

External radius (r_2) = (1 + 0.01) cm

[External radius is inner radius + thickness of sheet]

= 1.01 cm

Volume = external volume – inner volume

= $\frac{2}{3}\pi r_2^3$ - $\frac{2}{3}\pi r_1^3$

$= \frac{2}{3}\pi\,(r_2^3 - r_1^3)$

$= \frac{2}{3} \times \frac{22}{7} \times [(1.01)^3 - (1)^3]$

$= \frac{44}{21} \times 0.030301$

$= 2.095 \times 0.030301$

$= 0.06348\ m^3$ (Approx)

Question :7 Find the volume of a sphere whose surface area is 154 cm^2.

Answer:

Let the radius be "r".

Given, Surface area = 154 cm^2

We know that Surface Area of a sphere is given by, $S = 4\pi r^2$

Therefore,

$4\pi r^2 = 154$

$4 \times \frac{22}{7} \times r \times r = 154$

$r^2 = \frac{154 \times 7}{4 \times 22} = \frac{7 \times 7}{2 \times 2}$

r = 3.5 cm

Now,

Volume = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (3.5)^3$

$= 179\frac{2}{3} cm^3$

$= 179.67\ cm^3$

Question :8 A dome of a building is in the form of a hemisphere. From inside, it was white-washed at the cost of Rs 498.96. If the cost of white-washing is Rs 2.00 per square meter, find the

(i) Inside surface area of the dome,

(ii) Volume of the air inside the dome

Ans.: (i) Cost of white-washing the dome from inside = Rs 498.96
Cost of white-washing 1 m^2 area = Rs 2

Therefore, CSA of the inner side of dome = $\frac{498.96}{2}$
= 249.48 m^2

(ii) Let the radius be r
CSA = 249.48 m^2
$2\pi r^2 = 249.48$
$2 \times \frac{22}{7} \times r^2 = 249.48$
r = 6.3 m

Volume of air inside the dome = Volume of hemispherical dome
$= \frac{2}{3}\pi r^3$
$= \frac{2}{3} \times \frac{22}{7} \times (6.3)^3$
= 523.908 m^3

Question :9 Twenty-seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S′. Find the
(i) Radius r ′ of the new sphere,
(ii) Ratio of S and S′
Answer:
(i) Volume of new sphere having radius r' = $\frac{4}{3}\pi r'^3$

(ii) Volume of 27 spheres having radius r = $27 \times \frac{4}{3}\pi r^3$

According to the question,

$\frac{4}{3}\pi r'^3 = 27 \times \frac{4}{3}\pi r^3$

$= r'3 = 27r^3$

$= \quad r'3 = (3r)^3$

$r' = 3r$

(ii) Ratio of surface area = $\frac{4\pi r^2}{4\pi r'^2}$

$= \frac{r^2}{(3r)^2} = \frac{r^2}{9r^2}$

$= \frac{1}{9} = 1:9$

Question :10 A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in mm3) is needed to fill this capsule?

Ans.: Radius (r) = 1.75 mm

Volume = πr^3

$= \quad (1.75)^3$

25

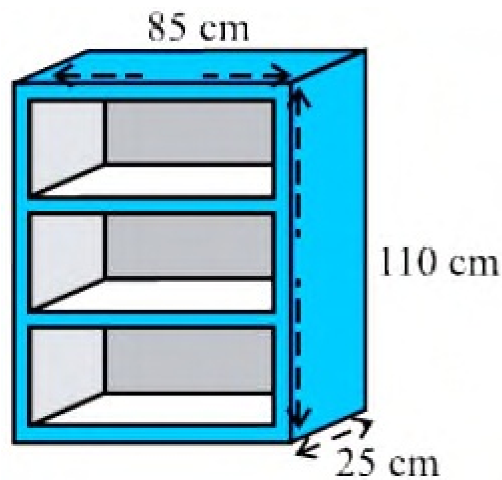
$= \frac{4}{3} \times \frac{22}{7} \times \frac{175}{100} \times \frac{175}{100} \times \frac{175}{1000}$

$= 22.458 mm^3$ Or

= 22.46 (Approx)

Exercise 13.9

Question :1 A wooden bookshelf has external dimensions as follows: Height = 110 cm, Depth = 25 cm, Breadth = 85 cm. The thickness of the plank is 5 cm everywhere. The external faces are to be polished and the inner faces are to be painted. If the rate of polishing is 20 paise per cm2 and the rate of painting is 10 paise per cm2, find the total expenses required for polishing and painting the surface of the bookshelf.



Ans.: External height (l) of book self = 85 cm

External breadth (b) of book self = 25 cm

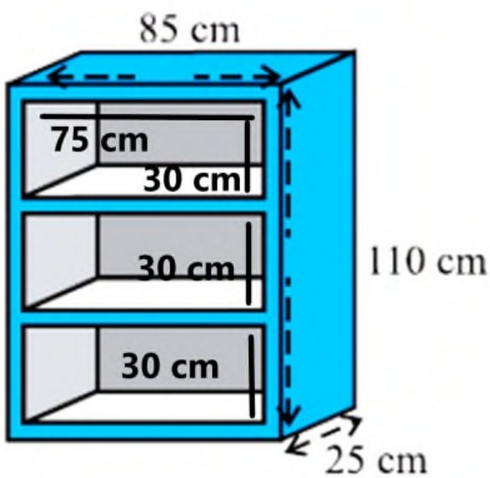
External height (h) of book self = 110 cm

The external surface area of the shelf while leaving out the front face of the shelf = lh + 2 (lb + bh)

= [85 × 110 + 2 (85 × 25 + 25 × 110)]

= (9350 + 9750)

= 19100 cm^2



we know each stripe on the front surface is also to be polished. which is 5 cm stretch.

Area of front face = [85 × 110 − 75 × 100 + 2 (75 × 5)]

= 1850 + 750

= 2600 cm^2

Area to be polished = (19100 + 2600) = 21700 cm^2

Cost of polishing 1 cm^2 area = Rs 0.20

Cost of polishing 21700 cm^2 area Rs (21700 × 0.20) = Rs 4340

It can be observed that length (l), breadth (b), and height (h) of each row of the bookshelf is 75 cm, 20 cm, and 30 cm respectively

Area to be painted in 1 row = 2 (l + h) b + lh

= [2 (75 + 30) × 20 + 75 × 30]

(4200 + 2250)

= 6450 cm^2

Area to be painted in 3 rows = (3 × 6450)

= 19350 cm^2

Cost of painting 1 cm^2 area = Rs 0.10

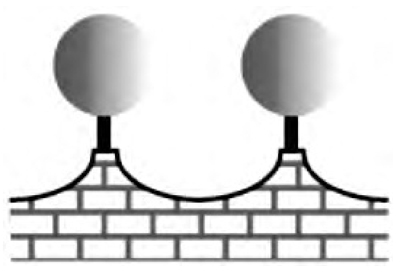
Cost of painting 19350 cm2 area = Rs (19350 × 0.1)

= Rs 1935

Total expense required for polishing and painting = Rs (4340 + 1935)

= Rs, 6275

Question :2 The front compound wall of a house is decorated by wooden spheres of diameter 21cm, placed on small supports. Eight such spheres are used for this purpose, and are to be painted silver. Each support is a cylinder of radius 1.5 cm and height7 cm and is to be painted black. Find the cost of paint required if silver paint costs 25 paise per cm^2 and black paint costs 5 paise per cm^2.



Ans.:

Radius (r) of wooden sphere 22 $\left(1\,\frac{21}{2}\right)$ = 10.5 cm

Surface area = 4 πr^2

$= 4 \times \frac{22}{7} \times 10.5 \times 10.5$

= 1386 cm^2

Radius of circular end (r1) = 1.5 cm

Height (h) = 7 cm

CSA = 2 πrh

$= 2 \times \frac{22}{7} \times 1.5 \times 7$

$= 66\ cm^2$

Area of the circular end of cylindrical support $= \pi r^2$

$= \frac{22}{7} \times 1.5 \times 1.5$

$= 7.07\ cm^2$

Area to be painted silver $= [8 \times (1386 - 7.07)]$

$= (8 \times 1378.93)$

$= 11031.44\ cm^2$

Cost of painting silver color = Rs (11031.44×0.25) = Rs 2757.86

Area to be painted black $= (8 \times 66)$

$= 528\ cm^2$

Cost for painting with black color = Rs (528×0.05) = Rs 26.40

Total cost in painting = Rs $(2757.86 + 26.40)$

= Rs 2784.26

Question :3 The diameter of a sphere is decreased by 25%. By what per cent does its curved surface area decrease?

Ans.:

Let the diameter be d

Radius $(r_1) = \frac{d}{2}$

New radius $(r_2) = \frac{d}{2}(1 - \frac{25}{100})$

$= \frac{3}{8}$ d

CSA (S_1) = 4r1^2

$= 4\pi (\frac{d}{2})^2$

$= \pi d^2$

CSA (S_2) = 4πr2^2

$= 4\pi (\frac{3d}{8})^2$

$= \frac{9}{6} \pi d^2$

Decrease in surface area = $S_1 - S_2$

$= \pi d^2 - d^2$

$= \frac{7}{16} \pi d^2$

Now,

Percentage decrease = $\frac{S_{1-} S_2}{S_1} * 100$

$= \frac{7\pi d * d}{16\pi d * d} * 100$

$= \frac{700}{16}$

= 43.75 %