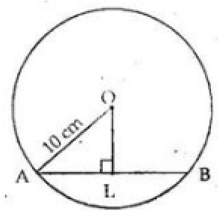
Circle

Exercise 11A

Question 1:

Let AB be a chord of the given circle with centre O and radius 10 cm. Then, OA = 10 cm and AB = 16 cm. From O, draw $OL \perp AB$. We know that the perpendicular from the centre of a circle to a chord bisects the chord.



$$\therefore \quad AL = \frac{1}{2} \times AB = \left(\frac{1}{2} \times 16\right) cm = 8 \text{ cm}.$$

From right angled ΔOLA, we have

$$OA^2 = OL^2 + AL^2$$

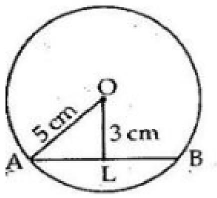
$$\Rightarrow \quad OL^2 = OA^2 - AL^2 = 10^2 - 8^2 = 100 - 64 = 36$$

$$\therefore \quad OL = \sqrt{36} = 6 \text{ cm}.$$

$\therefore$ The distance of the chord from the centre is 6 cm.

Question 2:

Let AB be the chord of the given circle with centre O and radius 5 cm.
From O, draw $OL \perp AB$
Then, OA = 5 cm and OL = 3 cm [given]
We know that the perpendicular from the centre of a circle to a chord bisects the chord.



Now, in right angled ΔOLA, we have

$$OA^2 = AL^2 + OL^2$$

$$\Rightarrow \quad AL^2 = OA^2 - OL^2$$

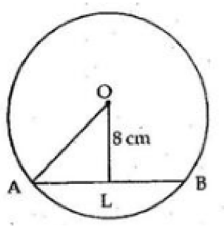
$$\Rightarrow \quad AL^2 = 5^2 - 3^2 = 25 - 9 = 16$$

$$\therefore \quad AL = \sqrt{16} = 4 \text{ cm}$$

So, $AB = 2\,AL = (2 \times 4) cm = 8 \text{ cm}$

$\therefore$ the length of the chord is 8 cm.

Let AB be the chord of the given circle with centre O. Draw $OL \perp AB$.



Then, OL is the distance from the centre to the chord.
So, we have AB = 30 cm and OL = 8 cm

We know that the perpendicular from the centre of a circle to a circle bisects the chord.

$$\therefore \quad AL = \frac{1}{2} \times AB$$
$$= \left(\frac{1}{2} \times 30\right) \text{cm} = 15 \text{ cm}$$

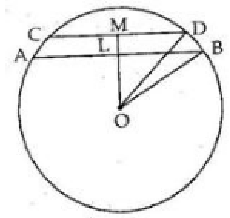
Now, in right angled ΔOLA we have,

$$OA^2 = OL^2 + AL^2$$
$$= 8^2 + 15^2$$
$$= 64 + 225 = 289$$
$$\therefore \quad OA = \sqrt{289} = 17 \text{ cm}$$

$\therefore$ the radius of the circle is 17 cm.

Question 4:

(i)Let AB and CD be two chords of a circle such that AB || CD which are on the same side of the circle. Also AB = 8 cm and CD = 6 cm. OB = OD = 5 cm. Join OL and LM.
Since the perpendicular from the centre of a circle to a chord bisects the chord.



We have $\quad LB = \frac{1}{2} \times AB$
$$= \left(\frac{1}{2} \times 8\right) \text{cm} = 4 \text{ cm}$$

and $\quad MD = \frac{1}{2} \times CD$
$$= \left(\frac{1}{2} \times 6\right) \text{cm} = 3 \text{ cm}$$

Now in right angled Δ BLO

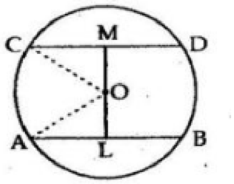
$$OB^2 = LB^2 + LO^2$$
$$\Rightarrow \quad LO^2 = OB^2 - LB^2$$
$$\Rightarrow \quad = 5^2 - 4^2$$
$$= 25 - 16 = 9$$
$$\therefore \quad LO = \sqrt{9} = 3 \text{ cm.}$$

Again in right angled ΔDMO

$$OD^2 = MD^2 + MO^2$$
$$\Rightarrow \quad MO^2 = OD^2 - MD^2$$
$$= 5^2 - 3^2$$
$$= 25 - 9 = 16$$
$$\Rightarrow \quad MO = \sqrt{16} = 4 \text{ cm}$$

$\therefore$ The distance between the chords $= (4-3)$ cm $= 1$ cm.

(ii)Let AB and CD be two chords of a circle such that AB || CD and they are on the opposite sides of the centre. AB = 8 cm and CD = 6 cm. Draw $OL \perp AB$ and $OM \perp CD$.



Join OA and OC

Then OA = OC = 5cm (radius)

Since the perpendicular from the centre of a circle to a chord bisects the chord, we have,

$$AL = \frac{1}{2}AB$$
$$= \left(\frac{1}{2} \times 8\right) \text{ cm} = 4 \text{ cm.}$$

Also $$CM = \frac{1}{2}CD$$
$$= \left(\frac{1}{2} \times 6\right) \text{cm} = 3 \text{ cm}$$

Now in right angled Δ OLA, we have

$$OA^2 = AL^2 + OL^2$$
$$\Rightarrow \quad OL^2 = OA^2 - AL^2$$
$$= 5^2 - 4^2$$
$$= 25 - 16 = 9 \text{ cm}$$
$$\therefore \quad OL = \sqrt{9} = 3 \text{ cm}$$

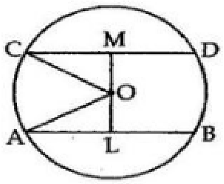
Again in right angled ΔOMC, we have

$$OC^2 = OM^2 + CM^2$$
$$\Rightarrow \quad OM^2 = OC^2 - CM^2$$
$$= 5^2 - 3^2$$
$$= 25 - 9 = 16$$
$$\Rightarrow \quad OM = \sqrt{16} = 4 \text{cm}$$

$\therefore$ the distance between the chords = (4 + 3)cm = 7 cm

Question 5:

Let AB and CD be two chords of a circle having centre O.
AB = 30 cm and CD = 16 cm.



Join AO and OC which are its radii. So AO = 17 cm and CO = 17 cm.
Draw $OM \perp CD$ and $OL \perp AB$.
Since the perpendicular from the centre of a circle to a chord bisects the chord, we have

$$AL = \frac{1}{2} \times AB$$
$$= \left(\frac{1}{2} \times 30\right) cm = 15\ cm$$
$$CM = \frac{1}{2} \times CD$$
$$= \left(\frac{1}{2} \times 16\right) cm = 8\ cm.$$

Now, in right angled Δ ALO, we have

$$AO^2 = OL^2 + AL^2$$
$$\Rightarrow \quad LO^2 = AO^2 - AL^2$$
$$= 17^2 - 15^2$$
$$= 289 - 225 = 64$$
$$\Rightarrow \quad LO = \sqrt{64} = 8\ cm$$

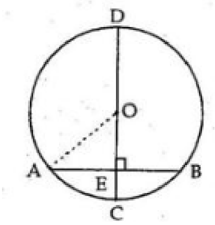
Again, in right angled Δ CMO, we have

$$CO^2 = CM^2 + OM^2$$
$$\Rightarrow \quad OM^2 = CO^2 - CM^2$$
$$= 17^2 - 8^2$$
$$= 289 - 64 = 225$$
$$\Rightarrow \quad OM = \sqrt{225} = 15\ cm$$

$\therefore$ Distance between the chords $= OM + OL = (8 + 15)cm$
$= 23$ cm.

Question 6:

CD is the diameter of a circle with centre O, and is perpendicular to chord AB. Join OA.



AB = 12 cm and CE = 3 cm [Given]

Let OA = OC = r cm

Then, OE = (r – 3) cm

Since the perpendicular from the centre of a circle to a chord bisects the chord, we have

$$AE = \frac{1}{2} \times AB$$

$$= \left(\frac{1}{2} \times 12\right) cm = 6\ cm$$

Now, in right angled ΔOEA,

$$OA^2 = OE^2 + AE^2$$

$$\Rightarrow \quad r^2 = (r-3)^2 + 6^2$$

$$\Rightarrow \quad = r^2 - 6r + 9 + 36$$

$$\Rightarrow \quad r^2 - r^2 + 6r = 45$$

$$\Rightarrow \quad 6r = 45$$

$$\Rightarrow \quad r = \frac{45}{6} = 7.5\ cm$$

$\therefore$ OA, the radius of the circle is 7.5 cm.

Question 7:

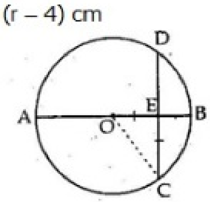
AB is the diameter of a circle with centre O which bisects the chord CD at point E.

CE = ED = 8cm and EB = 4cm. Join OC.

Let OC = OB = r cm.

Then,

OE = (r – 4) cm



Now, in right angled ΔOEC

$$OC^2 = OE^2 + EC^2$$

$$r^2 = (r-4)^2 + 8^2$$

$$\Rightarrow \quad r^2 = r^2 - 8r + 16 + 64$$

$$\Rightarrow \quad r^2 = r^2 - 8r + 80$$

$$\Rightarrow \quad r^2 - r^2 + 8r = 80$$

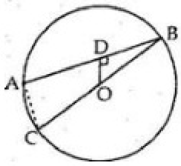
$$\Rightarrow \quad 8r = 80$$

$$\Rightarrow \quad r = \frac{80}{8} = 10\ cm$$

$\therefore$ the radius of the circle is 10 cm.

Question 8:

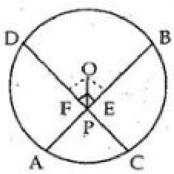
Given: $OD \perp AB$ of a circle with centre O. BC is a diameter.
To Prove: $AC \parallel OD$ and $AC = 2 \times OD$
Construction: Join AC.



Proof: We know that the perpendicular from the centre of the circle to a chord bisects the chord.
Here $OD \perp AB$
$\Rightarrow$ D is the mid – point of AB
$\Rightarrow \quad AD = BD$
Also, O is the mid – point of BC
$\therefore \quad OC = OB$
Now, in $\triangle ABC$, D is the midpoint of AB and O is the midpoint of BC.
Midpoint Theorem: The line segment joining the midpoints of any two sides of a triangle is parallel to the third side and equal to half of it.
$\therefore\ OD \parallel AC$ and $OD = \frac{1}{2}AC$
$\therefore \quad AC = 2 \times OD$

Question 9:

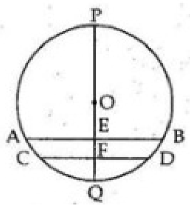
Sol.9. Given: O is the centre in which chords AB and CD intersects at P such that PO bisects $\angle BPD$.
To Prove: $AB = CD$
Construction: Draw $OE \perp AB$ and $OF \perp CD$



Proof: In $\triangle OEP$ and $\triangle OFP$
$\angle OEP = \angle OFP$ [Each equal to 90°]
$OP = OP$ [common]
$\angle OPE = \angle OPF$ [Since OP bisects $\angle BPD$]
Thus, by Angle-Side-Angle criterion of congruence, have,
$\therefore \quad \triangle OEP \cong \triangle OFP$ [By ASA]
The corresponding the parts of the congruent triangles are equal
$\Rightarrow \quad OE = OF$ [C.P.C.T.]
$\Rightarrow$ Chords AB and CD are equidistant from the centre O.
$\Rightarrow \quad AB = CD$ [$\because$ chords equidistant from the centre are equal]
$\therefore \quad AB = CD$

Question 10:

Given: AB and CD are two parallel chords of a circle with centre O. POQ is a diameter which is perpendicular to AB.
To Prove: $PF \perp CD$ and $CF = FD$

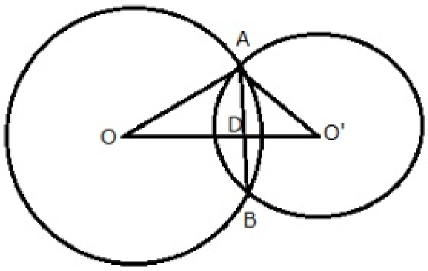


Proof: $AB \parallel CD$ and POQ is a diameter.
$\angle PEB = 90°$ [Given]
Then, $\angle PFD = \angle PEB$ [$AB \parallel CD$, Corresponding angles]
Thus, $PF \perp CD$
So, $OF \perp CD$
We know that, the perpendicular from the centre of a circle to chord, bisects the chord.
$\therefore \quad CF = FD.$

Question 11:

If possible let two different circles intersect at three distinct point A, B and C.
Then, these points are noncollinear. So a unique circle can be drawn to pass through these points. This is a contradiction.

Question 12:



$$OA = 10\text{cm} \quad \text{and } AB = 12 \text{ cm}$$

$$\therefore \quad AD = \frac{1}{2} \times AB$$

$$AD = \left(\frac{1}{2} \times 12\right) \text{cm} = 6 \text{ cm}$$

Now in right angled ΔADO,

$$OA^2 = AD^2 + OD^2$$

$$\Rightarrow \quad OD^2 = OA^2 - AD^2$$

$$= 10^2 - 6^2$$

$$= 100 - 36 = 64$$

$$\therefore \quad OD = \sqrt{64} = 8 \text{ cm}$$

Again, we have $O'A = 8$ cm.

In right angle $\Delta\ ADO'$

$$O'A^2 = AD^2 + O'D^2$$

$$\Rightarrow \quad O'D^2 = O'A^2 - AD^2$$

$$= 8^2 - 6^2$$

$$= 64 - 36 = 28$$

$$O'D = \sqrt{28} = 2\sqrt{7} \text{ cm}$$

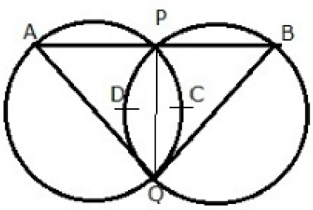
$$\therefore \quad OO' = (OD + O'D)$$

$$= (8 + 2\sqrt{7}) \text{ cm}$$

$\therefore$ the distance between their centres is $(8 + 2\sqrt{7})$ cm.

Question 13:

Given: Two equal cirles intersect at points P and Q. A straight line through P meets the circles in A and B.
To Prove : QA = QB
Construction: Join PQ



Proof : Two circles will be congruent if and only if they have equal radii.
If two chords of a circle are equal then their corresponding arcs are congruent.
Here PQ is the common chord to both the circles.
Thus, their corresponding arcs are equal.

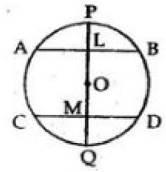
So, arc PCQ = arc PDQ

$\therefore \quad \angle QAP = \angle QBP$ [congruent arcs have the same degree mesure]

$\therefore \quad QA = QB$ [isosceles triangle, base angles are equal]

Question 14:

Given: AB and CD are the two chords of a circle with centre O.
Diameter POQ bisects them at L and M.
To Prove : AB ∥ CD.



Proof : AB and CD are two chords of a circle with centre O.
Diameter POQ bisects them at L and M.

Then, $OL \perp AB$

and, $OM \perp CD$

$\therefore$ $\angle ALM = \angle LMD$

$\therefore$ $AB \parallel CD$ [alternate angles are equal]

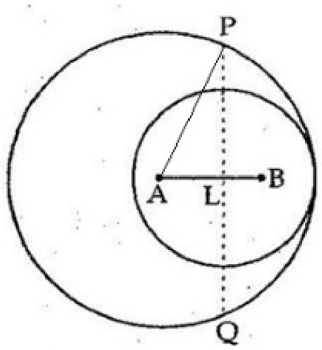
Question 15:

Two circles with centres A and B, having radii 5 cm and 3 cm touch each other internally.
The perpendicular bisector of AB meets the bigger circle in P and Q.
Join AP.
Let PQ intersect AB at L.

Then, $AB = (5-3)$ cm $= 2$ cm.

Since PQ is the perpendicular bisector of AB, we have

$$AL = \frac{1}{2} \times AB$$
$$= \left(\frac{1}{2} \times 2\right) \text{cm} = 1 \text{ cm}$$



Now, in right angle ΔPLA

$\therefore$ $AP^2 = AL^2 + PL^2$

$\Rightarrow$ $PL = \sqrt{AP^2 - AL^2}$ cm

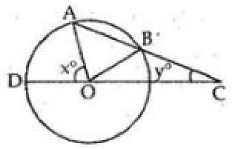
$= \sqrt{(25-1)}$ cm $= \sqrt{24}$ cm $= 2\sqrt{6}$ cm

$\therefore$ $PQ = (2 \times PL) = (2 \times 2\sqrt{6})$ cm $= 4\sqrt{6}$ cm

$\therefore$ the length of $PQ = 4\sqrt{6}$ cm

Question 16:

Given: AB is a chord of a circle with centre O. AB is produced to C such that BC = OB. Also, CO is joined to meet the circle in D. $\angle ACD = y^\circ$ and $\angle AOD = x^\circ$.



To Prove : $x = 3y$

Proof : $OB = BC$ [Given]

$\therefore \angle BOC = \angle BCO = y^\circ$ [isosceles triangle]

Ext. $\angle OBA = \angle BOC + \angle BCO = (2y)^\circ$

Again, $OA = OB$ [radii of same circle]

$\therefore \angle OAB = \angle OBA = (2y)^\circ$ [isosceles triangle]

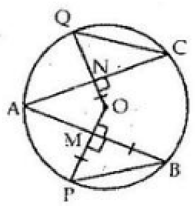
Ext. $\angle AOD = \angle OAC + \angle ACO$

$= \angle OAB + \angle BCO = 3y^\circ$

$\therefore x^\circ = 3y^\circ$ [$\because \angle AOD = x$ (given)]

Question 17:

Given: AB and AC are chords of the circle with centre O. $AB = AC, OP \perp AB$ and $OQ \perp AC$.



To Prove : $PB = QC$

Proof: $AB = AC$ [Given]

$\therefore \frac{1}{2}AB = \frac{1}{2}AC$ [Divide by 2]

The perpendicular from the centre of a circle to a chord bisects the chord.

$\Rightarrow$ $MB = NC$.....(1)

Equal chords of a circle are equidistant from the centre.

$\Rightarrow$ $OM = ON$

Also, $OP = OQ$ [Radii]

$\Rightarrow$ $OP - OM = OQ - ON$

$\Rightarrow$ $PM = QN$......(2)

Now consider the triangles, ΔMPB and ΔNQC:

$MB = NC$ [from (1)]

$\angle PMB = \angle QNC$ [right angle, given]

$PM = QN$ [from (2)]

Thus, by Side-Angle-Side criterion of congruence, we have

$\therefore \Delta MPB \cong \Delta NQC$ [S.A.S]

The corresponding parts of the congruent triangles are equal.

$\therefore PB = QC$ [by c.p.c.t]

Question 18:

Given: BC is a diameter of a circle with centre O. AB and CD are two chords such that AB ∥ CD.

To Prove: AB = CD

Construction: Draw OL ⊥ AB and OM ⊥ CD.

Proof: In △OLB and △OMC

$\angle OLB = \angle OMC$ [Perpendicular bisector, angle = 90°]

$\angle OBL = \angle OCD$ [AB ∥ CD, BC is a transversal, thus alternate interior angles are equal]

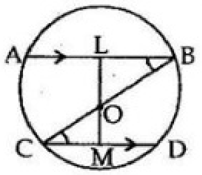
$OB = OC$ [Radii]

Thus by Angle-Angle-Side criterion of congruence, we have

$\therefore \quad \triangle OLB \cong \triangle OMC$ [By AAS]

The corresponding parts of the congruent triangle are equal.

$\therefore \quad OL = OM$ [C.P.C.T.]



But the chords equidistant from the centre are equal.

$\therefore \quad AB = CD$

Question 19:

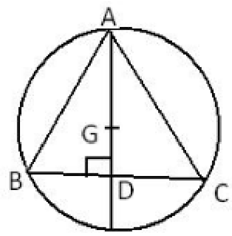
Let △ABC be an equilateral triangle of side 9 cm.

Let AD be one of its medians.

Then, $AD \perp BC$

and $BD = \frac{1}{2} \times BC$

$= \left(\frac{1}{2} \times 9\right) cm = 4.5$ cm.



$\therefore$ In right angled △ADB,

$$AB^2 = AD^2 + BD^2$$

$$\Rightarrow \quad AD^2 = AB^2 - BD^2$$

$$\Rightarrow \quad AD = \sqrt{AB^2 - BD^2}$$

$$= \sqrt{(9)^2 - \left(\frac{9}{2}\right)^2}\ cm = \frac{9\sqrt{3}}{2}\ cm$$

In an equilateral triangle, the centroid and circumcentre coincide and AG : GD = 2 : 1

$$\therefore \quad \text{radius } AG = \frac{2}{3} AD$$

$$= \left(\frac{2}{3} \times \frac{9\sqrt{3}}{2}\right) cm = 3\sqrt{3}\ cm$$

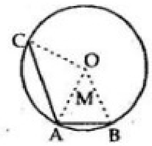
$\therefore$ The radius of the circle is $3\sqrt{3}$ cm.

Question 20:

Given : AB and AC are two equal chords of a circle with centre O

To Prove: $\angle OAB = \angle OAC$

Construction: Join OA, OB and OC.



Proof: In ΔOAB and ΔOAC,

$AB = AC$ [Given]

$OA = OA$ [common]

$OB = OC$ [Radii]

Thus by Side-Side-Side criterion of congruence, we have

$\therefore \quad \Delta OAB \cong OAC$ [by SSS]

The corresponding parts of the congruent triangles are equal.

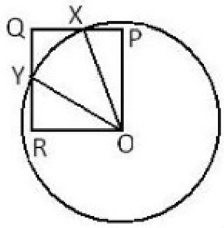
$\Rightarrow \quad \angle OAB = \angle OAC$ [by C.P.C.T.]

Therefore, O lies on the bisector of $\angle BAC$

Question 21:

Given: OPQR is a square. A circle with centre O cuts the square in X and Y.

To Prove: $QX = QY$



Construction: Join OX and OY.

Proof: In ΔOXP and ΔOYR

$\angle OPX = \angle ORY$ [Each equal to 90°]

$OX = OY$ [Radii]

$OP = OR$ [Sides of a square]

Thus by Right Angle-Hypotenuse-Side criterion of congruence, we have,

$\therefore \quad \Delta OXP \cong \Delta OYR$ [by RHS]

The corresponding parts of the congruent triangles are equal.

$\Rightarrow \quad PX = RY$ [by C.P.C.T.]

$\Rightarrow \quad PQ - PX = QR - RY$ [$\because PQ = QR$]

$\therefore \quad QX = QY.$

Exercise 11B

Question 1:

(i) Join BO.

In ΔBOC we have

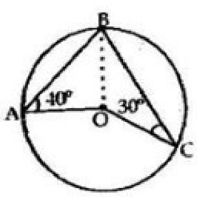
$OC = OB$ [Each equal to the radius]

$\Rightarrow \quad \angle OBC = \angle OCB$ [$\because$ base angles of an isosceles triangle are equal]

$\Rightarrow \quad \angle OBC = 30°$ [$\because \angle OCB = 30°$]

Thus, we have,

$\angle OBC = 30°$ ......(1)



Now, in ΔBOA, we have

$OB = OC$ [Each equal to the radius]

$\Rightarrow \angle OAB = \angle OBA$ $\left[\begin{array}{l}\because \text{base angles of an isosceles} \\ \text{triangle are equal}\end{array}\right]$

$\Rightarrow \angle OBA = 40^\circ$ $[\because \angle OAB = 40^\circ, \text{given}]$

Thus, we have,

$\angle OBA = 40^\circ$ .....(2)

$\therefore \angle ABC = \angle OBC + \angle OBA$

$\Rightarrow = 30^\circ + 40^\circ$ [from (1) and (2)]

$\Rightarrow \angle ABC = 70^\circ$

The angle subtended by an arc of a circle at the centre is double the angle subtended by the arc at any point on the circumference.

$\therefore \angle AOC = 2 \times \angle ABC$

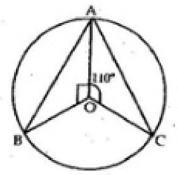
$= 2 \times 70^\circ = 140^\circ$

(ii) $\angle BOC = 360^\circ - (\angle AOB + \angle AOC)$

$= 360^\circ - (90^\circ + 110^\circ)$

$= 360^\circ - 200^\circ = 160^\circ$

We know that $\angle BOC = 2\angle BAC$



$\Rightarrow \angle BAC = \frac{160^\circ}{2} = 80^\circ$ $[\because \angle BOC = 160^\circ]$

$\therefore \angle BAC = 80^\circ$.

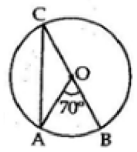
Question 2:

(i)

The angle subtended by an arc of a circle at the centre is double the angle subtended by the arc at any point on the circumference.

$\therefore \angle AOB = 2\angle OCA$

$\Rightarrow \angle OCA = \frac{70}{2} = 35^\circ$ $[\because \angle AOB = 70^\circ]$



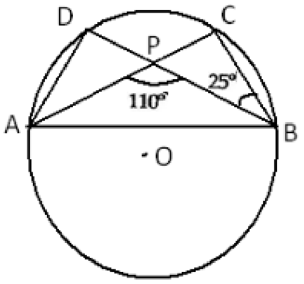
(ii) The radius of the circle is

$OA = OC$

$\Rightarrow \angle OAC = \angle OCA$ [base angles of an isosceles triangle are equal

$\Rightarrow \angle OAC = 35^\circ$ $[\text{as} \angle OCA = 35^\circ]$

Question 3:



It is clear that $\angle ACB = \angle PCB$
Consider the triangle ΔPCB.
Applying the angle sum property, we have,

$\angle PCB = 180° - (\angle BPC + \angle PBC)$

$= 180° - (180° - 110° + 25°)$ [$\angle APB$ and $\angle BPC$ are linear pair; $\angle PBC = 25°$, given]

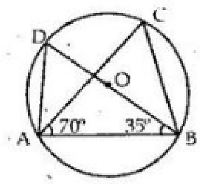
$= 180° - (70° + 25°)$

$\angle PCB = 180° - 95° = 85°$

Angles in the same segment of a cirlce are equal.

$\therefore \angle ADB = \angle ACB = 85°$

Question 4:



It is clear that, BD is the diameter of the circle.
Also we know that, the angle in a semicircle is a right angle.

$\therefore \angle BAD = 90°$

Now consider the triangle, ΔBAD

$\Rightarrow \angle ADB = 180° - (\angle BAD + \angle ABD)$ [Angle sum property]

$\Rightarrow \quad = 180° - (90° + 35°)$ [$\angle BAD = 90°$ and $\angle ABD = 35°$]

$\Rightarrow \quad = 180° - 125°$

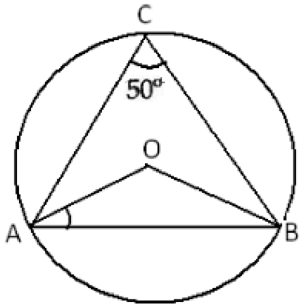
$\Rightarrow \angle ADB = 55°$

Angles in the same segment of a circle are equal.

$\therefore \angle ACB = \angle ADB = 55°$

$\therefore \angle ACB = 55°$

Question 5:



The angle subtended by an arc of a circle at the centre is double the angle subtended by the arc at any point on the circumference.

$\therefore \quad \angle AOB = 2\angle ACB$

$= 2 \times 50^\circ \quad$ [Given]

$\Rightarrow \quad \angle AOB = 100^\circ \quad ....(1)$

Consider the triangle ΔOAB

$OA = OB \quad$ [radius of the circle]

$\angle OAB = \angle OBA \quad$ [base angles of an isosceles triangle are equal]

Thus we have

$\angle OAB = \angle OBA \quad ....(2)$

By angle sum property, we have

Now $\angle AOB + \angle OAB + \angle OBA = 180^\circ$

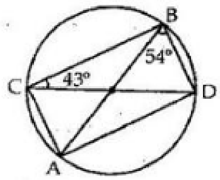
$\Rightarrow \quad 100^\circ + 2\angle OAB = 180^\circ \quad$ [from (1) and (2)]

$\Rightarrow \quad 2\angle OAB = 180^\circ - 100^\circ = 80^\circ$

$\Rightarrow \quad \angle OAB = \dfrac{80^\circ}{2} = 40^\circ$

$\therefore \quad \angle OAB = 40^\circ$

Question 6:



(i) Angles in the same segment of a circle are equal.

$\angle ABD$ and $\angle ACD$ are in the segment AD.

$\therefore \quad \angle ACD = \angle ABD$

$= 54^\circ \quad$ [Given]

(ii) Angles in the same segment of a circle are equal.

$\angle BAD$ and $\angle BCD$ are in the segment BD.

$\therefore \quad \angle BAD = \angle BCD$

$= 43^\circ \quad$ [Given]

(iii) Consider the ΔABD.

By Angle sum property we have

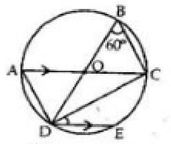
$\angle BAD + \angle ADB + \angle DBA = 180^\circ$

$\Rightarrow \quad 43^\circ + \angle ADB + 54^\circ = 180^\circ$

$\Rightarrow \quad \angle ADB = 180^\circ - 97^\circ = 83^\circ$

$\Rightarrow \quad \angle BDA = 83^\circ$

Question 7:



Angles in the same segment of a circle are equal.
$\angle CAD$ and $\angle CBD$ are in the segment CD.

$\therefore \quad \angle CAD = \angle CBD = 60^\circ \qquad$ [Given]

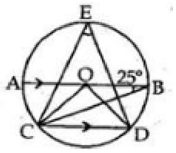
We know that an angle in a semi circle is a right angle.

$\therefore \quad \angle ADC = 90^\circ \qquad$ [angle in a semicircle]

$$\therefore \quad \angle ACD = 180^\circ - (\angle ADC + \angle CAD)$$
$$= 180^\circ - (90^\circ + 60^\circ)$$
$$= 180^\circ - 150^\circ = 30^\circ$$

$\therefore \quad \angle CDE = \angle ACD = 30^\circ \qquad$ [AC ∥ DE and CD is a transversal, thus alternate angles are equal]

Question 8:



Join CO and DO, $\angle BCD = \angle ABC = 25^\circ$ [alternate interior angles]

The angle subtended by an arc of a circle at the centre is double the angle subtended by the arc at any point on the circumference.

$$\therefore \angle BOD = 2\angle BCD$$
$$= 50^\circ \qquad [\angle BCD = 25^\circ]$$

Similarly,

$$\angle AOC = 2\angle ABC$$
$$= 50^\circ$$

AB is a straight line passing through the centre.

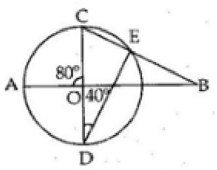
$$\therefore \quad \angle AOC + \angle COD + \angle BOD = 180^\circ$$
$$\Rightarrow \quad 50^\circ + \angle COD + 50^\circ = 180^\circ$$
$$\Rightarrow \quad \angle COD = 180^\circ - 100^\circ = 80^\circ$$
$$\therefore \quad \angle CED = \frac{1}{2}\angle COD$$
$$= \frac{80^\circ}{2} = 40^\circ$$
$$\therefore \quad \angle CED = 40^\circ$$

Question 9:

(i) $\angle CED = 90^\circ$

In $\triangle CED$, we have

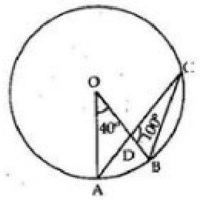
$$\angle CED + \angle EDC + \angle DCE = 180^\circ$$
$$\Rightarrow \quad 90^\circ + 40^\circ + \angle DCE = 180^\circ$$
$$\therefore \quad \angle DCE = 180^\circ - 130^\circ$$
$$\angle DCE = 50^\circ \qquad \ldots(1)$$



(ii) $\angle AOC$ and $\angle BOC$ are linear pair.

$$\therefore \quad \angle BOC = (180^\circ - 80^\circ) = 100^\circ \qquad \ldots..(2)$$
$$\therefore \angle ABC = 180^\circ - (\angle BOC + \angle DCE)$$
$$= 180^\circ - (100^\circ + 50^\circ) \qquad \text{[from (1) and (2)]}$$
$$= 180^\circ - 150^\circ = 30^\circ$$

Question 10:



The angle subtended by an arc of a circle at the centre is double the angle subtended by the arc at any point on the circumference.

$\therefore\ \angle AOB = 2\angle ACB$

$\Rightarrow\ \quad = 2\angle DCB \quad [\because \angle ACB = \angle DCB]$

$\Rightarrow \angle DCB = \frac{1}{2}\angle AOB$

$= \left(\frac{1}{2} \times 40\right) = 20°$

Consider the ΔDBC;

By angle sum property, we have

$\angle BDC + \angle DCB + \angle DBC = 180°$

$\Rightarrow\ 100° + 20° + \angle DBC = 180°$

$\Rightarrow\ \angle DBC = 180° - 120° = 60°$

$\Rightarrow\ \angle OBC = \angle DBC = 60°$

$\therefore\ \angle OBC = 60°$

Question 11:

Join OB.

$\therefore\ OA = OB \quad$ [Radius]

$\therefore\ \angle OBA = \angle OAB = 25°$ [base angles are equal in isosceles triangle

Now in ΔOAB, we have

$\Rightarrow\ \angle OAB + \angle OBA + \angle AOB = 180°$

$\Rightarrow\ 25° + 25° + \angle AOB = 180°$

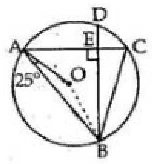
$\Rightarrow\ \angle AOB = 180° - 50° = 130°$

The angle subtended by an arc of a circle at the centre is double the angle subtended by the arc at any point on the circumference.

$\therefore\ \angle AOB = 2\angle ACB$

$\Rightarrow\ \angle ACB = \frac{1}{2}\angle AOB = \frac{1}{2} \times 130 = 65°$

$\Rightarrow\ \angle ECB = 65°$



Consider the right triangle ΔBEC.

We know that the sum of three angles in a triangle is 180°.

$\Rightarrow \angle EBC + \angle BEC + \angle ECB = 180°$

$\Rightarrow\ \angle EBC + 90° + 65° = 180°$

$\Rightarrow\ \angle EBC = 180° - 155° = 25°$

$\therefore\ \angle EBC = 25°$

Question 12:

$$OB = OC \qquad \text{[Radius]}$$

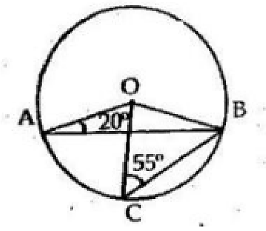
$\Rightarrow \quad \angle OBC = \angle OCB = 55°$ [base angles in an isosceles triangle are equal]

Consider the triangle ΔBOC.

By angle sum property, we have

$$\angle BOC = 180° - (\angle OCB + \angle OBC)$$
$$= 180° - (55° + 55°)$$
$$= 180° - 110° = 70°$$

$\therefore \quad \angle BOC = 70°$



Again, $\quad OA = OB$

$\Rightarrow \quad \angle OBA = \angle OAB = 20°$ [base angles in an isosceles triangle are equal]

Consider the triangle ΔAOB.

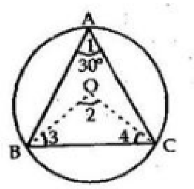
By angle sum property, we have

$$\Rightarrow \quad \angle AOB = 180° - (\angle OAB + \angle OBA)$$
$$= 180° - (20° + 20°)$$
$$= 180° - 40° = 140°$$

$$\therefore \quad \angle AOC = \angle AOB - \angle BOC$$
$$= 140° - 70° = 70°$$

$\therefore \quad \angle AOC = 70°$

Question 13:



Join OB and OC.

The angle subtended by an arc of a circle at the centre is double the angle subtended by the arc at any point on the circumference.

$$\therefore \quad \angle BOC = 2\angle BAC$$
$$= 2 \times 30° \qquad [\because \angle BAC = 30°]$$
$$= 60° \qquad ....(1)$$

Now consider the triangle ΔBOC.

$$OB = OC \qquad \text{[radii]}$$

$\Rightarrow \quad \angle OBC = \angle OCB \quad ....(2)$

[base angles in an isosceles triangle are equal]

Now, in ΔBOC, we have

$$\angle BOC + \angle OBC + \angle OCB = 180°$$

$\Rightarrow \quad 60° + \angle OCB + \angle OCB = 180° \quad$ [from (1) and (2)]

$$\Rightarrow \quad 2\angle OCB = 180° - 60°$$
$$\Rightarrow \quad = 120°$$
$$\Rightarrow \quad \angle OCB = \frac{120°}{2} = 60°$$

$\Rightarrow \quad \angle OBC = 60° \quad$ [from (2)]

Thus, we have, $\angle OBC = \angle OCB = \angle BOC = 60°$

So, ΔBOC is an equilateral triangle

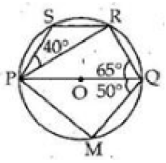
$\Rightarrow \quad OB = OC = BC$

$\therefore$ BC is the radius of the circumference.

Question 14:

Consider the triangle, ΔPRQ.
PQ is the diameter.
The angle in a semicircle is a right angle.
$\Rightarrow \angle PRQ = 90^\circ$
By the angle sum property in ΔPRQ, we have,

$$\angle QPR + \angle PRQ + \angle PQR = 180^\circ$$
$$\Rightarrow \angle QPR + 90^\circ + 65^\circ = 180^\circ$$
$$\Rightarrow \angle QPR = 180^\circ - 155^\circ = 25^\circ \quad ....(1)$$



Now consider the triangle ΔPQM.
Since PQ is the diameter, $\angle PMQ = 90^\circ$
Again applying the angle sum property in ΔPQM, we have

$$\angle QPM + \angle PMQ + \angle PQM = 180^\circ$$
$$\Rightarrow \angle QPM + 90^\circ + 50^\circ = 180^\circ$$
$$\Rightarrow \angle QPM = 180^\circ - 140^\circ = 40^\circ$$

Now in quadrilateral PQRS

$$\angle QPS + \angle SRQ = 180^\circ$$
$$\Rightarrow \angle QPR + \angle RPS + \angle PRQ + \angle PRS = 180^\circ \quad [\text{from } (1)]$$
$$\Rightarrow 25^\circ + 40^\circ + 90^\circ + \angle PRS = 180^\circ$$
$$\Rightarrow \angle PRS = 180^\circ - 155^\circ = 25^\circ$$
$$\therefore \angle PRS = 25^\circ$$

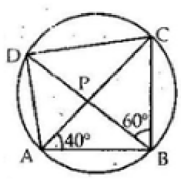
Exercise 11C

Question 1:

$\angle BDC = \angle BAC = 40^\circ$ [angles in the same segment]

In ΔBCD, we have

$$\angle BCD + \angle BDC + \angle DBC = 180^\circ$$
$$\therefore \angle BCD + 40^\circ + 60^\circ = 180^\circ$$
$$\Rightarrow \angle BCD = 180^\circ - 100^\circ = 80^\circ$$
$$\therefore \angle BCD = 80^\circ$$



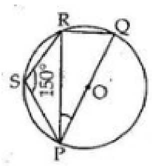
(ii) Also $\angle CAD = \angle CBD$ [angles in the same segment]

$\therefore \angle CAD = 60^\circ$ [$\because \angle CBD = 60^\circ$]

Question 2:

In cyclic quadrilateral PQRS

$$\angle PSR + \angle PQR = 180^\circ$$
$$\Rightarrow \quad 150^\circ + \angle PQR = 180^\circ$$
$$\Rightarrow \quad \angle PQR = 180^\circ - 150^\circ = 30^\circ \quad \ldots\ldots(i)$$
$$\text{Also,} \quad \angle PRQ = 90^\circ \quad \ldots\ldots(ii)$$
[angle ina semi circle]

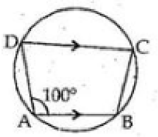


Now in ΔPRQ we have

$$\angle PQR + \angle PRQ + \angle RPQ = 180^\circ$$
$$\Rightarrow \quad 30^\circ + 90^\circ + \angle RPQ = 180^\circ \quad [\text{from (i)and(ii)}]$$
$$\Rightarrow \quad \angle RPQ = 180^\circ - 120^\circ = 60^\circ$$
$$\therefore \quad \angle RPQ = 60^\circ$$

Question 3:

In cyclic quadrilateral ABCD, AB $\parallel$ DC and $\angle BAD = 100^\circ$



$$\text{(i)} \quad \angle BCD + \angle BAD = 180^\circ$$
$$\Rightarrow \quad \angle BCD + 100^\circ = 180^\circ$$

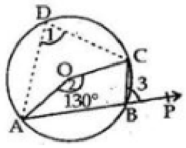
$$\Rightarrow \quad \angle BCD = 180^\circ - 100^\circ = 80^\circ$$
$$\text{(ii)} \quad \text{Also,} \quad \angle ADC = \angle BCD = 80^\circ$$
$$\therefore \quad \angle ADC = 80^\circ$$
$$\text{(iii)} \quad \angle ABC = \angle BAD = 100^\circ$$
$$\therefore \quad \angle ABC = 100^\circ$$

Question 4:

Take a point D on the major arc CA and join AD and DC

$$\therefore \quad \angle 2 = 2\angle 1$$

[Angle subtended by an arc is twice the angle subtended by it on the circumference in the alternate segment.]



$$\therefore \quad 130^\circ = 2\angle 1$$
$$\Rightarrow \quad \angle 1 = 65^\circ \quad \ldots\ldots\ldots\ldots(i)$$

$$\angle PBC = \angle 1$$
[$\because$ exterior angle of a cyclic quadrilateral interior opposite angle]
$$\therefore \quad \angle PBC = 65^\circ$$

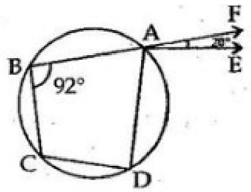
Question 5:

ABCD is a cyclic quadrilateral

$\therefore \quad \angle ABC + \angle ADC = 180°$

$\Rightarrow \quad 92° + \angle ADC = 180°$

$\Rightarrow \quad \angle ADC = 180° - 92° = 88°$



Also, $AE \parallel CD$

$\therefore \quad \angle EAD = \angle ADC = 88°$

$\therefore \quad \angle BCD = \angle DAF$

[$\because$ exterior angle of a cyclic quadrilateral = int.opp.angle]

$\therefore \quad \angle BCD = \angle EAD + \angle EAF$

$= 88° + 20° \quad [\because \angle FAE = 20°(\text{given})]$

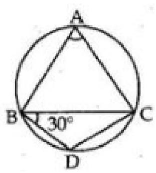
$= 108°$

$\therefore \quad \angle BCD = 108°$

Question 6:

$BD = DC$

$\therefore \quad \angle BCD = \angle CBD = 30°$



In ΔBCD, we have

$\angle BCD + \angle CBD + \angle CDB = 180°$

$\Rightarrow \quad 30° + 30° + \angle CDB = 180°$

$\Rightarrow \quad \angle CDB = 180° - 60°$

$= 120°$

The opposite angles of a cyclic quadrilateral are supplementary.

ABCD is a cyclic quadrilateral and thus,

$\angle CDB + \angle BAC = 180°$

$= 180° - 120° [\because \angle CDB = 120°]$

$= 60°$

$\therefore \quad \angle BAC = 60°$

Question 7:

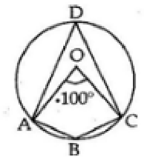
Angle subtended by an arc is twice the angle subtended by it on the circumference in the alternate segment.
Here arc ABC makes $\angle AOC = 100°$ at the centre of the circle and $\angle ADC$ on the circumference of the circle

$\therefore \angle AOC = 2\angle ADC$

$\Rightarrow \angle ADC = \frac{1}{2}(\angle AOC)$

$\Rightarrow \quad = \frac{1}{2} \times 100° \quad [\angle AOC = 100°]$

$\Rightarrow \angle ADC = 50°$



The opposite angles of a cyclic quadrilateral are supplementary,
ABCD is a cyclic quadrilateral and thus,

$\angle ADC + \angle ABC = 180°$

$= 180° - 50° \, [\because \angle ADC = 50°]$

$= 130°$

$\therefore \quad \angle ABC = 130°$

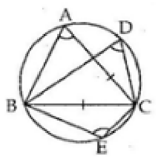
$\therefore \angle ADC = 50° \quad \text{and} \quad \angle ABC = 130°$

Question 8:

Δ ABC is an equilateral triangle.

$\therefore$ Each of its angle is equal to $60°$

$\Rightarrow \angle BAC = \angle ABC = \angle ACB = 60°$



(i) Angles in the same segment of a circle are equal.

$\therefore \angle BDC = \angle BAC$

$= 60° \quad [\because \angle BAC = 60°]$

$\Rightarrow \angle BDC = 60°$

(ii) The opposite angles of a cyclic quadrilateral are supplementary
ABCE is a cyclic quadrilateral and thus,

$\angle BAC + \angle BEC = 180°$

$\angle BEC = 180° - 60° \, [\because \angle BAC = 60°]$

$= 120°$

$\Rightarrow \quad \angle BEC = 120°$

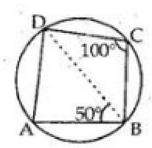
Question 9:

ABCD is a cyclic quadrilateral.

$\therefore \quad \angle A + \angle C = 180° \quad \begin{bmatrix} \text{opp. angle of a cyclic quadrilateral} \\ \text{are supplementary} \end{bmatrix}$

$\Rightarrow \quad \angle A + 100° = 180°$

$\Rightarrow \quad \angle A = 180° - 100° = 80°$



Now in ΔABD, we have

$\angle A + \angle ABD + \angle ADB = 180°$

$\Rightarrow \quad 80° + 50° + \angle ADB = 180°$

$\Rightarrow \quad \angle ADB = 180° - 130° = 50°$

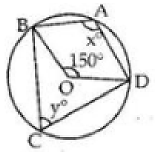
$\therefore \quad \angle ADB = 50°$

Question 10:

O is the centre of the circle and $\angle BOD = 150°$

$\therefore$ Reflex $\angle BOD = (360° - \angle BOD)$

$= (360° - 150°) = 210°$



Now, $x = \frac{1}{2}(\text{reflex} \angle BOD)$

$= \frac{1}{2} \times 210° = 105°$

$\therefore$ $x = 105°$

Again, $x + y = 180°$

$\Rightarrow$ $105° + y = 180°$

$\Rightarrow$ $y = 180° - 105° = 75°$

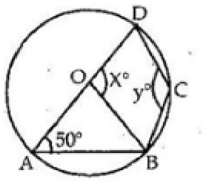
$\therefore$ $y = 75°$

Question 11:

O is the centre of the circle and $\angle DAB = 50°$

$OA = OB$ [Radii]

$\Rightarrow$ $\angle OBA = \angle OAB = 50°$



In ΔOAB we have

$\angle OAB + \angle OBA + \angle AOB = 180°$

$\Rightarrow 50° + 50° + \angle AOB = 180°$

$\Rightarrow$ $\angle AOB = 180° - 100° = 80°$

Since, AOD is a straight line,

$\therefore$ $x = 180° - \angle AOB.$

$= 180° - 80° = 100°$

$\therefore$ $x = 100°$

The opposite angles of a cyclic quadrilateral are supplementary.

ABCD is a cyclic quadrilateral and thus,

$\angle DAB + \angle BCD = 180°$

$\angle BCD = 180° - 50°$ [$\because \angle DAB = 50°$, given]

$= 130°$

$\Rightarrow$ $y = 130°$

Thus, $x = 100°$ and $y = 130°$

Question 12:

ABCD is a cyclic quadrilateral.

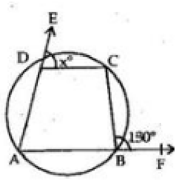
We know that in a cyclic quadrilateral exterior angle = interior opposite angle.

$\therefore$ $\angle CBF = \angle CDA = (180° - x)$

$\Rightarrow$ $130° = 180° - x$

$\Rightarrow$ $x = 180° - 130° = 50°$

$x = 50°$



Question 13:

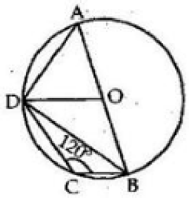
AB is a diameter of a circle with centre O and DO || CB,
$\angle BCD = 120°$

(i) Since ABCD is a cyclic quadrilateral

$\therefore \quad \angle BCD + \angle BAD = 180°$

$\Rightarrow \quad 120° + \angle BAD = 180°$

$\Rightarrow \quad \angle BAD = 180° - 120° = 60°$



(ii) $\angle BDA = 90°$ [angle in a semi circle]

In ΔABD we have

$\angle BDA + \angle BAD + \angle ABD = 180°$

$\Rightarrow \quad 90° + 60° + \angle ABD = 180°$

$\Rightarrow \quad \angle ABD = 180° - 150° = 30°$

(iii) OD = OA.

$\Rightarrow \quad \angle ODA = \angle OAD = \angle BAD = 60°$

$\therefore \quad \angle ODB = 90° - \angle ODA$

$= 90° - 60° = 30°$

Since DO ‖ CB, alternate angles are equal

$\Rightarrow \quad \angle CBD = \angle ODB$

$= 30°$

(iv) $\angle ADC = \angle ADB + \angle CDB$

$= 90° + 30° = 120°$

Also, in ΔAOD, we have

$\angle ODA + \angle OAD + \angle AOD = 180°$

$\Rightarrow \quad 60° + 60° + \angle AOD = 180°$

$\Rightarrow \quad \angle AOD = 180° - 120° = 60°$

Since all the angles of ΔAOD are of 60° each

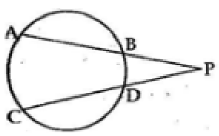
$\therefore \quad \Delta$ AOD is an equilateral triangle.

Question 14:

AB and CD are two chords of a circle which interect each other at P, outside the circle. AB = 6cm, BP = 2 cm and PD = 2.5 cm

Therefore, AP x BP = CP x DP

Or, 8 x 2 = (CD + 2.5) x 2.5 cm [as CP = CD + DP]



Let x = CD

Thus, $8 \times 2 = (x + 2.5) \times 2.5$

$\Rightarrow \quad 16 \text{ cm} = 2.5x + 6.25 \text{ cm}$

$\Rightarrow \quad 2.5x = (16 - 6.25) \text{cm}$

$\Rightarrow \quad 2.5x = 9.75 \text{cm}$

$\Rightarrow \quad x = \frac{9.75}{2.5} = 3.9 \text{cm}$

$\therefore \quad x = 3.9 \text{ cm}$

Therefore, CD = 3.9 cm

Question 15:

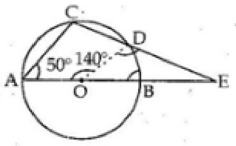
O is the centre of a circle having $\angle AOD = 140°$ and $\angle CAB = 50°$

(i) $\angle BOD = 180° - \angle AOD$

$= 180° - 140° = 40°$

$OB = OD$

$\therefore \angle OBD = \angle ODB$



In ΔOBD, we have

$\angle BOD + \angle OBD + \angle ODB = 180°$

$\Rightarrow \angle BOD + \angle OBD + \angle OBD = 180°$ $[\because \angle OBD = \angle ODB]$

$\Rightarrow 40° + 2\angle OBD = 180°$ $[\because \angle BOD = 40°]$

$\Rightarrow 2\angle OBD = 180° - 40° = 140°$

$\Rightarrow \angle OBD = \angle ODB = \frac{140}{2} = 70°$

Also, $\angle CAB + \angle BDC = 180°$ $[\because ABCD \text{ is cyclic}]$

$\Rightarrow \angle CAB + \angle ODB + \angle ODC = 180°$

$\Rightarrow 50° + 70° + \angle ODC = 180°$

$\Rightarrow \angle ODC = 180° - 120° = 60°$

$\therefore \angle ODC = 60°$

$\therefore \angle EDB = 180° - (\angle ODC + \angle ODB)$

$= 180° - (60° + 70°)$

$= 180° - 130° = 50°$

(ii) $\angle EBD = 180° - \angle OBD$

$= 180° - 70° = 110°$

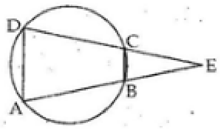
Question 16:

Consider the triangles, ΔEBC and ΔEDA

Side AB of the cyclic quadrilateral ABCD is produced to E

$\therefore \angle EBC = \angle CDA$

$\Rightarrow \angle EBC = \angle EDA$ .....(i)



Again, side DC of the cyclic quadrilateral ABCD isproduced toE.

$\therefore \angle ECB = \angle BAD$

$\Rightarrow \angle ECB = \angle EAD$ .....(ii)

and $\angle BEC = \angle DEA$ [each equal to $\angle E$]....(iii)

Thus from (i), (ii) and (iii), we have

$\therefore \Delta EBC \cong \Delta EDA$

Question 17:

ΔABC is an isosceles triangle in which AB = AC and a circle passing through B and C intersects AB and AC at D and E.

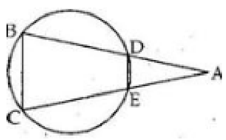
Since $AB = AC$

$\therefore \angle ACB = \angle ABC$

So, ext. $\angle ADE = \angle ACB = \angle ABC$

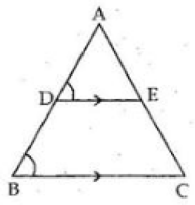
$\therefore \angle ADE = \angle ABC$

$\Rightarrow DE \parallel BC$.



Question 18:

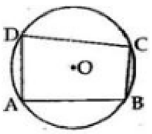
ΔABC is an isosceles trianglein which $AB = AC$. D and E are the mid points of AB and AC respectively.



$\therefore \quad DE \parallel BC$
$\Rightarrow \quad \angle ADE = \angle ABC \quad \ldots(i)$
Also, $\quad AB = AC \quad$ [Given]
$\Rightarrow \quad \angle ABC = \angle ACB \quad \ldots(ii)$
$\therefore \quad \angle ADE = \angle ACB \quad$ [From (i) and(ii)]
Now, $\angle ADE + \angle EDB = 180° \quad$ [$\because$ ADBis a straightline]
$\therefore \quad \angle ACB + \angle EDB = 180°$
$\Rightarrow$ The opposite angles are supplementary.
$\Rightarrow$ D, B, C and E are concyclic
i.e. D, B, C and E is a cyclic quadrilateral.

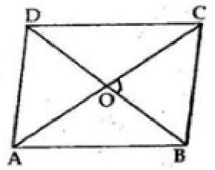
Question 19:

Let ABCD be a cyclic quadrilateral and let O be the centre of the circle passing through A, B, C, D.
Then each of AB, BC, CD and DA being a chord of the circle, its right bisector must pass through O.
$\therefore$ the right bisectors of AB, BC, CD andDA pass through and are concurrent.



Question 20:

ABCD is a rhombus.
Let the diagonals AC and BD of the rhombus ABCD intersect at O.
But, we know, that the diagonals of a rhombus bisect each other at right angles.
So, $\angle BOC = 90°$
$\therefore$ $\angle BOC$ lies in a circle.

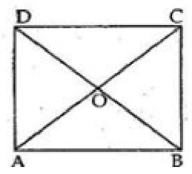


Thus the circle drawn with BC as diameter will pass through O

Similarly, all the circles described with AB, AD and CD as diameters will pass through O.

Question 21:

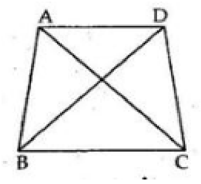
ABCD is a rectangle.
Let O be the point of intersection of the diagonals AC and BD of rectangle ABCD.



Since the diagonals of a rectangle are equal and bisect each other.
$\therefore OA = OB = OC = OD$
Thus, O is the centre of the circle through A, B, C, D.

Question 22:

Let A, B, C be the given points.
With B as centre and radius equal to AC draw an arc.
With C as centre and AB as radius draw another arc, which cuts the previous arc at D.



Then D is the required point BD and CD.
In ΔABC and ΔDCB
$AB = DC$
$AC = DB$
$BC = CB$ [common]
$\therefore$ $\Delta ABC \cong \Delta DCB$ [by SSS]
$\Rightarrow$ $\angle BAC = \angle CDB$ [C.P.C.T]
Thus, BC subtends equal angles, $\angle BAC$ and $\angle CDB$ on the same side of it.

$\therefore$ Points A, B, C, D are concyclic.

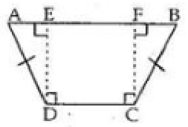
Question 23:

ABCD is a cyclic quadrilateral
$\angle B - \angle D = 60°$ ......(i)
and $\angle B + \angle D = 180°$ ....(ii)
Adding (i) and (ii) we get,
$2\angle B = 240°$
$\therefore$ $\angle B = \frac{240}{2} = 120°$
Substituting the value of $\angle B = 120°$ in (i) we get
$120° - \angle D = 60°$
$\Rightarrow$ $\angle D = 120° - 60° = 60°$
The smaller of the two angles i.e. $\angle D = 60°$

Question 24:

ABCD is a quadrilateral in which $AD = BC$ and $\angle ADC = \angle BCD$

Draw $DE \perp AB$ and $CF \perp AB$



Now, in ΔADE and ΔBCF, we have

$\angle AED = \angle BFC$ [each equal to 90°]

$\angle ADE = \angle ADC - 90° = \angle BCD - 90° = \angle BCF$

$AD = BC$ [given]

Thus, by Angle-Angle-Side criterionof congruence, we have

$\therefore \quad \Delta ADE \cong \Delta BCF$ [by AAS congruence]

The corresponding parts of the congruent triangles are equal.

$\therefore \quad \angle A = \angle B$

Now, $\angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow \quad 2\angle B + 2\angle D = 360°$

$\Rightarrow \quad 2(\angle B + \angle D) = 360°$

$\Rightarrow \quad \angle B + \angle D = \dfrac{360}{2} = 180°$

$\therefore$ ABCD is a cyclic quadrilateral.

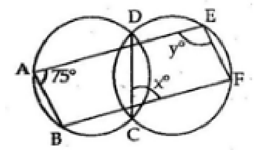
Question 25:

If one side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle.

$\Rightarrow \quad \angle BAD = \angle DCF = 75°$

$\therefore \quad \angle DCF = x = 75°$

$\therefore \quad x = 75°$



The opposite angles of the opposite angles of a cyclic quadrilateral is 180°

$\Rightarrow \quad \angle DCF + \angle DEF = 180°$

$\Rightarrow \quad 75° + \angle DEF = 180°$

$\Rightarrow \quad \angle DEF = 180° - 75° = 105°$

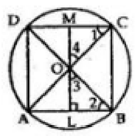
As $\quad \angle DEF = y° = 105°$

$\therefore \quad x = 75°$ and $y = 105°$

Question 26:

Given: Let ABCD be a cyclic quadrilateral whose diagonals AC and BD intersect at O at right angles.

Let $OL \perp AB$ such that LO produced meets CD at M.



To Prove: $CM = MD$

Proof: $\angle 1 = \angle 2$ [angles in the same segment]

$\angle 2 + \angle 3 = 90°$ [$\because \angle OLB = 90°$]

$\angle 3 + \angle 4 = 90°$ [$\because$ LOM is a straight line and $\angle BOC = 90°$]

$\therefore \quad \angle 2 + \angle 3 = \angle 3 + \angle 4$

$\Rightarrow \quad \angle 2 = \angle 4$

Thus, $\quad \angle 1 = \angle 2$

and $\quad \angle 2 = \angle 4$

$\Rightarrow \quad \angle 1 = \angle 4$

$\therefore \quad OM = CM$

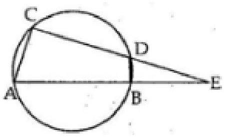
Similarly, $\quad OM = MD$

Hence, $\quad CM = MD$.

Question 27:

Chord AB of a circle is produced to E.
If one side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle.

$\therefore$ Ext. $\angle BDE = \angle BAC = \angle EAC$ ....(1)



Chord CD of a circle is produced to E

$\therefore$ Ext. $\angle DBE = \angle ACD = \angle ACE$.....(2)

Consider the triangles ΔEDB and ΔEAC.

$\angle BDE = \angle CAE$ [from(1)]

$\angle DBE = \angle ACE$ [from(2)]

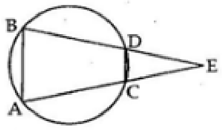
$\angle E = \angle E$ [common]

$\therefore$ $\Delta EDB \sim \Delta EAC$.

Question 28:

Given: AB and CD are two parallel chords of a circle BDE and ACE are straight lines which intersect at E.
If one side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle.

$\therefore$ Ext$\angle EDC = \angle A$ and, Ext$\angle DCE = \angle B$



Also, $AB \parallel CD$

$\Rightarrow$ $\angle EDC = \angle B$

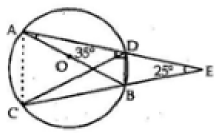
and $\angle DCE = \angle A$

$\therefore$ $\angle A = \angle B$

$\therefore$ Δ AEB is isosceles.

Question 29:

AB is a diameter of a circle with centre O. ADE and CBE are straight lines, meeting at E, such that $\angle BAD = 35°$ and $\angle BED = 25°$.
Join BD and AC.



(i) Now, $\angle BDA = 90° = \angle EDB$ [angle in a semi circle]

$\Rightarrow$ $\angle EBD = 180° - (\angle EDB + \angle BED)$
$= 180° - (90° + 25°)$
$= 180° - 115° = 65°$

$\therefore$ $\angle DBC = (180° - \angle EBD)$
$= 180° - 65° = 115°$

$\therefore$ $\angle DBC = 115°$

(ii) Again, $\angle DCB = \angle BAD$ [angle in the same segment]

Since, $\angle BAD = 35°$

$\therefore$ $\angle DCB = 35°$

(iii) $\angle BDC = 180° - (\angle DBC + \angle DCB)$
$= 180° - (\angle DBC + \angle BAD)$
$= 180° - (115° + 35°)$
$= 180° - 150° = 30°$

$\therefore$ $\angle BDC = 30°$