CHAPTER

3

Decimal Fractions

Fractions

If any unit be divided into any number of equal parts one or more of the these parts is called a fraction of the unit.

e.g., $\frac{4}{5}, \frac{3}{2}$ etc., are fractions and here the lower number is called as denominator and the upper number is called as numerator.

Decimal Fractions

Fractions that have powers of 10 m in the denominators are called as decimal fractions. *e.g.*, $\frac{1}{10}, \frac{1}{100}, \frac{1}{1000}$ etc.

e.g.,

$\frac{1}{10}$ is the tenth part of 1 and written as 0.1

$\frac{8}{10}$ is the 8 tenth part of 1 and written as 0.8

$\frac{7}{100}$ is the 7th hundredth part of 1 and written as 0.07

Important Rules and Formulae

Rule 1 While making addition or subtraction of decimal fractions the numbers are placed under each in such a way that the decimal point lie in column. Then, the numbers can be added or subtracted as usual.

Example 1. $51.3 + 7.078 + 1.38 + 0.9 = ?$

(a) 61.668 (b) 59.238
(c) 60.658 (d) None of these

Sol. (c)

$$\begin{array}{r} 51.3 \\ 7.078 \\ 1.38 \\ +0.9 \\ \hline 60.658 \end{array}$$

Example 2. $7.093 - 3.57 = ?$

(a) 3.523 (b) 3.513 (c) 3.143 (d) 3.532

Sol. (a) Here,

$$\begin{array}{r} 7.093 \\ -3.57 \\ \hline 3.523 \end{array}$$

Rule 2 While multiplying the two given numbers consider them without the decimal point. In the product the decimal point is marked off to obtain as many places of decimal as is the sum of the number of decimal places in the given numbers.

Example 3. The product of 3.5413×2.1 is

(a) 7.44673 (b) 7.43673
(c) 7.53673 (d) None of these

Sol. (b) For the product of 3.5413×2.1

Consider $35413 \times 21 = 743673$

Total decimal places $= 4 + 1 = 5$

$\therefore$ $3.5413 \times 2.1 = 7.43673$

Example 4. $5 \times 0.25 \times 6.301 \times 0.00394 = ?$

(a) 0.0310324220
(b) 0.0310324210
(c) 0.0310324250
(d) None of these

Sol. (c) $5 \times 0.25 \times 6.301 \times 0.00394$

Here, $5 \times 25 \times 6301 \times 394 = 310324250$

Total decimal places $(2 + 3 + 5) = 10$

$\therefore$ $5 \times 0.25 \times 6.301 \times 0.00394 = 0.0310324250$

Rule 3 While multiplying a decimal fraction by powers of 10 the product is obtained by shifting the decimal point to the right by as many places of decimal as is the power of 10.

Example 5. $9.4135 \times 1000 = ?$

(a) 0.0094135 (b) 9.4135
(c) 9413.5 (d) None of these

Sol. (c) $9.4135 \times 1000 = 9413.5$

Here, the decimal is shifted 3 places to the right as $1000 = 10^3$

Example 6. $0.031725 \times 100 = ?$

(a) 0.0031725 (b) 3.1725
(c) 0.031725 (d) None of these

Sol. (b) $0.031725 \times 100 = 3.1725$

Here, the decimal is shifted 2 places to the right as $100 = 10^2$

Example 7. $7.42139 \times 10 = ?$

(a) 74.2139 (b) 0.742139
(c) 7.42139 (d) None of these

Sol. (c) $7.42139 \times 10 = 74.2139$

Here, the decimal is shifted 1 place to the right.

Rule 4 While dividing a fractions number by powers of 10, the result is obtained by shifting the decimal point to the left by as many places of decimal as is the power of 10.

Example 8. $9.724 \div 100 = ?$

(a) 972.4 (b) 9.724
(c) 0.09724 (d) None of these

Sol. (c) $9.724 \div 100 = 0.09724$

Shift decimal two places to the left *i.e.*, $100 = 10^2$

Example 9. $\frac{3.242 \times 1.4}{100} = ?$

(a) 0.045388 (b) 4.5388
(c) 453.88 (d) None of these

Sol. (a) $\frac{3.242 \times 1.4}{100}$

Multiplying $3.242 \times 1.4 = 4.5388$

Now, divide 4.5388 by 100

So, $4.5388 \div 100 = 0.045388$

Shift the decimal two places to the left as $100 = 10^2$

Adding/Annexing zeros to the extreme right of a decimal fraction does not change its value.

Rule 5 While dividing the given decimal fraction without the decimal point by the given counting number.

Here, in the quotient, place the decimal point to have as many places of decimal as are these in the dividend.

Example 10. $0.01834 \div 13 = ?$

(a) 141.077 (b) 1.41077
(c) 0.00141077 (d) None of these

Sol. (c) Here, for $0.01834 \div 13$

So, $\frac{1834}{13} = 141.077$

So, $0.01834 \div 13 = 0.00141077$

Example 11. $0.01234 \div 2 = ?$

(a) 0.00617 (b) 617
(c) 0.617 (d) None of these

Sol. (a) Here, for $0.01234 \div 2$ So, $\frac{1234}{2} = 617$

So, $0.1234 \div 2 = 0.00617$

Here, place the decimal at the left of fifth digits.

Rule 6 For dividing a decimal fraction by a decimal fraction, multiply the dividend and the divisor by a suitable power of 10 to make the divisor a whole number and then proceed as in the previous rule.

Example 12. $0.0066 \times 0.22 = ?$

(a) 0.03 (b) 0.3
(c) 3 (d) None of these

Sol. (a) $0.0066 \div 0.22$

So, $\frac{0066 \times 1000}{22 \times 10000} = \frac{66}{22 \times 100} = \frac{3}{100} = 0.03$

Recurring Decimals

If in a decimal fraction, a set of figures is repeated or a figure is repeated, then the fraction is called recurring decimal.

e.g., $\frac{1}{7} = 0.1428571428571\ldots = 0.\overline{142857}$

$\frac{2}{3} = 0.66666\ldots = 0.\overline{6}$

$\frac{4}{3} = 1.33333\ldots = 1.\overline{3}$

Recurring decimal are of two types

1. **Pure Recurring Decimal** A decimal fraction in which all the figures after the decimal points are repeated, is called a pure recurring decimal.

   e.g., $0.786786786\ldots = 0.\overline{786}$

   $0.77777\ldots = 0.\overline{7}$

2. **Mixed Recurring Decimal** A decimal fraction in which some figures after the decimal point are not repeated while some are repeated, is called a mixed recurring decimal.

   e.g., $0.179999\ldots = 0.17\overline{9}$

   $0.053939\ldots = 0.0\overline{539}$

Rule 7 To compare the fraction using decimal, convert each one of the given fractions in decimal form and now arrange them in ascending or descending order as per requirement.

Example 13. Arrange $\frac{2}{3}, \frac{5}{6}, \frac{16}{25}, \frac{3}{7}$ in ascending order.

(a) $\frac{2}{3}, \frac{5}{6}, \frac{16}{25}$ and $\frac{3}{7}$ (b) $\frac{16}{25}, \frac{5}{6}, \frac{3}{7}$ and $\frac{2}{3}$
(c) $\frac{3}{7}, \frac{16}{25}, \frac{2}{3}$ and $\frac{5}{6}$ (d) None of these

Sol. (c) First, we convert each of the fraction in decimal form so here

$\frac{2}{3} = 0.\overline{666}, \frac{5}{6} = 0.8\overline{33}, \frac{16}{25}$

$= 0.64, \frac{3}{7} = 0.\overline{428571}$

Here, $0.\overline{428571} < 0.64 < 0.\overline{666} < 0.8\overline{333}$

Here, $\frac{3}{7} < \frac{16}{25} < \frac{2}{3} < \frac{5}{6}$ and so $\frac{3}{7}, \frac{16}{25}, \frac{2}{3}$ and $\frac{5}{6}$ are in ascending order.

To Convert a Recurring Decimals to Vulgar Fractions

Rule 8 In order to convert a pure recurring decimal fraction into vulgar fraction, we write the repeated figures only once in the numerator without decimal point and write as many nines in the denominator as the number of repeating figure.

e.g.
$$0.\overline{5} = \frac{5}{9}$$
$$0.\overline{45} = \frac{45}{99}$$

Rule 9 In the numerator take the difference between the number formed by all the digits after decimal point. (Here, repeated digits are taken only once) and that formed by digits which are not repeated. In the denominator we write a number formed by as many nines as are repeating digits followed by as many zeros as is the number of non-repeating digits. This is used to convert a mixed recurring decimal to vulgar fraction.

Example 14. The vulgar fraction of $0.12\overline{36}$ is

(a) $\frac{120}{823}$ (b) $\frac{102}{825}$

(c) $\frac{93}{825}$ (d) None of these

Sol. (b) $0.12\overline{36} = \frac{1236-12}{9900} = \frac{1224}{9900} = \frac{102}{825}$

Example 15. The vulgar fraction of $3.62\overline{4}$ is

(a) $\frac{1621}{450}$ (b) $\frac{1641}{455}$

(c) $\frac{1631}{450}$ (d) None of these

Sol. (c) $3.62\overline{4} = 3 + \frac{624-62}{900} = 3 + \frac{562}{900}$
$= 3 + \frac{281}{450} = \frac{1350+281}{450}$
$= \frac{1631}{450}$

HCF and LCM of Decimal Fractions

For finding the HCF or LCM of decimal fraction, first convert the given fraction with the same number of decimal places by annexing zeros if needed in the process. Now, we find the HCF or LCM of the numbers without decimal places. Finally, in the result mark as many decimal places as are there in each of the given numbers.

Example 16. The HCF and LCM of 1.600, 8 and 2.4 is

(a) 0.800 and 48 (b) 0.6000 and 38

(c) 0.900 and 42 (d) None of these

Sol. (a) We can write the given numbers as 1.600, 8.000, 2.400.
So, without decimal the number are 1600, 8000 and 2400.
Now, HCF of 1600, 8000 and 2400 is 800.
∴ HCF of 1.600, 8.000 and 2.400 is 0.800.
And here also LCM of 1600, 8000, 2400 is 48000.
∴ LCM of 1.600, 8.000 and 2.400 is 48.000.
So, LCM is 48.

Exercise

1. If $(15.9273 - x) = 11.0049$, then the value of x is
(a) 4.9224 (b) 0.4922 (c) 0.4294 (d) 6.932

2. If $(15.39 + 0.236 + 5.290 + 0.0002) = x$, then x is
(a) 0.20916 (b) 2.0916 (c) 209.16 (d) 20.9162

3. If $175 \times 1.24 = 217$, then the value of 1.75×124 is
(a) 217 (b) 0.0217
(c) 2.17 (d) None of these

4. If $111.744 + 28.8 = 3.88$, then the value of $1117.44 + 288$ is
(a) 3.88 (b) 0.388 (c) 388.0 (d) 38.8

5. $\frac{(0.5)^4 - (0.4)^4}{(0.5)^2 + (0.4)^2}$ is equal to
(a) 0.9 (b) 0.09
(c) 9 (d) 0.009

6. $\frac{0.004 \times 0.0008}{0.02}$ equals to
(a) 0.000016 (b) 0.00016
(c) 0.0016 (d) None of these

7. The value of $\frac{(0.0347)^3 + (0.9653)^3}{(0.0347)^2 - (0.0347)(0.9653) + (0.9653)^2}$
(a) 0 (b) 1
(c) 10 (d) 30

8. Which of the following sets of the fractions is in ascending order?
(a) $\frac{6}{8}, \frac{7}{9}, \frac{5}{6}, \frac{11}{13}$ (b) $\frac{5}{6}, \frac{6}{8}, \frac{7}{9}, \frac{11}{13}$
(c) $\frac{11}{13}, \frac{5}{6}, \frac{7}{9}, \frac{6}{8}$ (d) $\frac{11}{13}, \frac{7}{9}, \frac{6}{8}, \frac{5}{6}$

9. Which of the following fraction is smallest?
(a) $\frac{5}{6}$ (b) $\frac{2}{3}$
(c) $\frac{11}{13}$ (d) $\frac{7}{9}$

10. If $3.245 \times 10^k = 0.0003245$, then the value of k is
(a) 4 (b) −4 (c) 3 (d) 5

11. The value of $(0.\overline{6} + 0.\overline{8} + 0.\overline{7})$ is
(a) $2\frac{1}{8}$ (b) $2\frac{1}{9}$ (c) $2\frac{1}{3}$ (d) $1\frac{1}{9}$

12. The value of $(0.\overline{7} + 0.\overline{6} - 0.\overline{5})$ is
(a) $\frac{7}{9}$ (b) $\frac{8}{9}$ (c) $\frac{1}{3}$ (d) $\frac{4}{9}$

13. The value of $(6.\overline{88} - 2.\overline{58})$ is
(a) $4.\overline{30}$ (b) $4.\overline{29}$ (c) $3.\overline{22}$ (d) $4.\overline{38}$

14. If $2.5x = 0.5y$, then the value of $\frac{x+y}{x-y}$ is

(a) -1.3 (b) -1.5 (c) 1.5 (d) 1.3

15. What is the value of $27 \times 1.\overline{2} \times 5.526\overline{2} \times 0.\overline{6}$? **(CDS 2011 I)**

(a) $121.\overline{57}$ (b) $12.17\overline{5}$

(c) $121.7\overline{5}$ (d) None of these

16. 4.5 should be divided by what to get 20% of 90?

(a) 0.25 (b) 0.0025 (c) 2.5 (d) 0.025

17. 7.2 exceeds its one-tenth by

(a) 8.48 (b) 5.48 (c) 6.48 (d) 5.28

18. If $2.5252525\ldots = \frac{p}{q}$ (in the lowest form), then what is the value of $\frac{q}{p}$? **(CDS 2007 II)**

(a) 0.4 (b) 0.42525 (c) 0.0396 (d) 0.396

19. If $\sqrt{5} = 2.24$, then the value of $\frac{3\sqrt{5}}{2\sqrt{5} - 0.48}$ is

(a) 1.68 (b) 16.8 (c) 168 (d) 0.168

20. The square root of $\frac{0.324 \times 0.081 \times 4.624}{1.5625 \times 0.0289 \times 72.9 \times 64}$ is

(a) 2.4 (b) 0.024 (c) 0.24 (d) 24

21. $\frac{0.35 \times 0.0015}{0.25 \times 0.07}$ written as percentage is

(a) 0.03% (b) 3%

(c) 30% (d) None of these

22. What decimal of an hour is a second?

(a) 0.25 (b) 0.0256 (c) 0.00027 (d) 0.0125

23. $\left\{\frac{(0.1)^2 - (0.01)^2}{0.0001} + 1\right\}$ is equal to

(a) 1001 (b) 11 (c) 101 (d) 100

24. $\frac{3}{3 + \frac{0.3 - 3.03}{3 \times 0.91}}$ is equal to

(a) 0.75 (b) 1.5 (c) 15 (d) 0.15

25. $\frac{(0.87)^3 + (0.13)^3}{(0.87)^2 + (0.13)^2 - 0.87 \times 0.13} = ?$

(a) 0.13 (b) 0.87 (c) 1 (d) 0.74

26. What should be less by the multiplication of 0.527 and 2.013 to get 1?

(a) 0.939085 (b) 0.060851

(c) 1.91984 (d) 2.16085

27. 1 L of water weighs 1 kg. How many cubic millimetres of water will weigh 0.1 g?

(a) 100 (b) 1 (c) 10 (d) 0.1

28. The value of $4.1\overline{2}$ is

(a) $4\frac{11}{99}$ (b) $4\frac{11}{90}$ (c) $5\frac{2}{9}$ (d) $3\frac{2}{9}$

29. $0.34\overline{67} + 0.1\overline{33}$ is equal to

(a) 0.48 (b) $0.48\overline{01}$ (c) $0.\overline{48}$ (d) 0.48

30. The greatest fraction out of $\frac{2}{5}, \frac{5}{6}, \frac{11}{12}$ and $\frac{7}{8}$ is

(a) 7/8 (b) 5/6 (c) 11/12 (d) 2/5

31. Consider the following statements

I. $\frac{1}{22}$ cannot be written as a terminating decimal.

II. $\frac{2}{15}$ can be written as a terminating decimal.

III. $\frac{1}{16}$ can be written as a terminating decimal.

Which of the statements given above is/are correct? **(CDS 2008 I)**

(a) Only I (b) Only II (c) I and III (d) II and III

32. Consider the following decimal numbers

I. 1.16666666... II. 1.181181118...

III. 2.010010001... IV. 1.454545...

Which of the above numbers represent(s) rational number(s)? **(CDS 2007 I)**

(a) Only IV (b) II and III

(c) I and IV (d) None of these

33. If 1 is subtracted from the numerator of a fraction it becomes (1/3) and if 5 is added to the denominator the fraction becomes (1/4). Which fraction shall result, if 1 is subtracted from the numerator and 5 is added to the denominator? **(CDS 2008 I)**

(a) 5/12 (b) 7/23 (c) 1/8 (d) 2/3

34. Which one is the largest among the following? **(CDS 2009 II)**

(a) $0.7\underline{25}$ (b) $0.\underline{725}$ (c) 0.725 (d) 0.725

35. Which one of the following is a non-terminating and repeating decimal? **(CDS 2010 II)**

(a) $\frac{13}{8}$ (b) $\frac{3}{16}$ (c) $\frac{3}{11}$ (d) $\frac{137}{25}$

36. What is $3.\overline{76} - 1.\overline{4576}$ equal to? **(CDS 2011 II)**

(a) $2.3\overline{100191}$ (b) $2.3\overline{101091}$ (c) $2.31\overline{10091}$ (d) $2.311\overline{0901}$

Answers

1. (a)	2. (d)	3. (a)	4. (a)	5. (b)	6. (b)	7. (b)	8. (a)	9. (b)	10. (b)
11. (c)	12. (b)	13. (a)	14. (b)	15. (d)	16. (a)	17. (c)	18. (d)	19. (a)	20. (b)
21. (b)	22. (c)	23. (d)	24. (b)	25. (c)	26. (b)	27. (a)	28. (b)	29. (b)	30. (c)
31. (c)	32. (c)	33. (c)	34. (d)	35. (c)	36. (a)				

Hints and Solutions

1. $x = 15.9273 - 11.0049 = 4.9224$

2.
$$\begin{array}{r} 15.3900 \\ 0.2360 \\ 5.2900 \\ 0.0002 \\ \hline 20.9162 \end{array}$$
$\therefore \quad x = 20.9162$

3. $1.75 \times 124 = 175 \times 1.24 = 217$

4. Since, $\dfrac{111.744}{28.8} = 3.88 \Rightarrow \dfrac{111744}{28800} = 3.88$

Now, $\dfrac{1117.44}{288} = \dfrac{11744}{28800} = 3.88$

5. $\dfrac{((0.5)^2)^2 - ((0.4)^2)^2}{(0.5)^2 + (0.4)^2} = \dfrac{[(0.5)^2 - (0.4)^2][(0.5)^2 + (0.4)^2]}{[(0.5)^2 + (0.4)^2]}$

$= (0.5)^2 - (0.4)^2$

$= (0.5 - 0.4)(0.5 + 0.4)$

$= 0.1 \times 0.9 = 0.09$

6. $\dfrac{0.004 \times 0.0008}{0.02} = \dfrac{0.0000032}{0.02} = 0.00016$

7. Here, $\dfrac{a^3 + b^3}{a^2 - ab + b^2} = (a + b)$

$= 0.0347 + 0.9653 = 1.0$

8. Here, $\dfrac{6}{8} = 0.75, \dfrac{7}{9} = 0.\overline{7}, \dfrac{5}{6} = 0.8\overline{3}, \dfrac{11}{13} = 0.85$

So, $0.75 < 0.\overline{7} < 0.8\overline{3} < 0.85$

9. Here, $\dfrac{5}{6} = 0.8\overline{3}, \dfrac{2}{3} = 0.\overline{66}$

$\dfrac{11}{13} = 0.85, \dfrac{7}{9} = 0.\overline{77}$

So, $0.\overline{66}$ is the smallest, hence $\dfrac{2}{3}$ is true.

10. Here, $10^k = \dfrac{0.0003245}{3.245} = \dfrac{3.245 \times 10^{-4}}{3.245}$

$10^k = 10^{-4}$ So, $k = -4$

11. $0.\overline{6} + 0.\overline{8} + 0.\overline{7} = \dfrac{6}{9} + \dfrac{8}{9} + \dfrac{7}{9} = \dfrac{21}{9} = \dfrac{7}{3} = 2\dfrac{1}{3}$

12. $0.\overline{7} + 0.\overline{6} - 0.\overline{5} = \dfrac{7}{9} + \dfrac{6}{9} - \dfrac{5}{9} = \dfrac{13 - 5}{9} = \dfrac{8}{9}$

13. Here, $6.\overline{88} - 2.\overline{58}$

$= \left(6 + \dfrac{88}{99}\right) - \left(2 + \dfrac{58}{99}\right) = (6 - 2) + \left(\dfrac{88}{99} - \dfrac{58}{99}\right)$

$= 4 + \dfrac{88 - 58}{99}$

$= 4 + \dfrac{30}{99} = 4.\overline{30}$

14. $2.5x = 0.5y \Rightarrow \dfrac{x}{y} = \dfrac{0.5}{2.5} = 0.20$

Now, the expression is

$$\frac{x+y}{x-y} = \frac{x/y+1}{x/y-1} = \frac{0.20+1}{0.20-1} = \frac{1.20}{-0.80}$$

$= -1.5$

Shortcut method

$\dfrac{x}{y} = \dfrac{0.5}{2.5}$

$\dfrac{x+y}{x-y} = \dfrac{0.5+2.5}{0.5-2.5}$ (using dividendo and componendo)

$\dfrac{x+y}{x-y} = \dfrac{3}{-2}$

$= -1.5$

15. $27 \times 1.\overline{2} \times 5.526\overline{2} \times 0.\overline{6} = 27 \times 1\dfrac{2}{9} \times 5\dfrac{4736}{9000} \times \dfrac{6}{9}$

$= 27 \times \dfrac{11}{9} \times \dfrac{49736}{9000} \times \dfrac{6}{9}$

$= \dfrac{11 \times 49736 \times 2}{9000} = \dfrac{1094192}{9000} = 121.577$

16. 20% of $90 = \dfrac{20}{100} \times 90 = 18$

So, $\dfrac{45}{x} = 18 \Rightarrow x = \dfrac{45}{18}$

$x = 0.25$

17. $7.2 - \dfrac{7.2}{10} = 6.48$

18. $\dfrac{p}{q} = 2.5\overline{2}$ ...(i)

$\dfrac{100p}{q} = 252.\overline{52}$ ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get

$\dfrac{99p}{q} = 250 \Rightarrow \dfrac{q}{p} = \dfrac{99}{250} = 0.396$

19. $\dfrac{3\sqrt{5}}{2\sqrt{5} - 0.48} = \dfrac{3 \times 2.24}{2 \times 2.24 - 0.48}$

$= \dfrac{6.72}{4.48 - 0.48} = \dfrac{6.72}{4} = 1.68$

20. Given expression is

$= \dfrac{0.324 \times 0.081 \times 4.624}{15625 \times 0.0289 \times 72.9 \times 64}$

$= \dfrac{324 \times 81 \times 4624}{15625 \times 289 \times 729 \times 64} = \dfrac{9}{15625}$

and $\sqrt{\dfrac{9}{15625}} = \dfrac{3}{125} = 0.024$

21. $\frac{0.35\times0.0015}{0.25\times0.07}=\frac{35\times15}{25\times700}=\frac{3}{100}=\frac{3}{100}\times100=3\%$

22. Required decimal fraction $=\frac{1}{60\times60}$

$=\frac{1}{3600}=0.00027$

23. Here, $\left[\frac{(0.1)^2-(0.01)^2}{0.0001}+1\right]$

$=\frac{0.01-0.0001}{0.0001}+1=\left(\frac{0.0099}{0.0001}+1\right)=(99+1)=100$

24. Here, $\frac{3}{3+\frac{0.3-3.03}{3\times0.91}}=\frac{3}{3-\frac{273}{3\times91}}=\frac{3}{3-1}=\frac{3}{2}=1.5$

25. $\frac{a^3+b^3}{a^2+b^2-ab}=\frac{(a+b)(a^2+b^2-ab)}{(a^2+b^2-ab)}=(a+b)$

$=0.87+0.13$ (if $a=0.87$ and $b=0.13$)

26. $0.527\times2.013=1.060851$

Here, required answer $=1.060851-1=0.060851$

27. 1000 g is weight of 1000 cu cm.

1 g is the weight of 1 cu cm = 1000 cu mm.

$\frac{1}{10}$ g is the weight of $\left(\frac{1000}{10}\right)$ cu mm = 100 cu mm

28. $4.1\overline{2}=4+0.1\overline{2}=4+\frac{12-1}{90}=4+\frac{11}{90}=4\frac{11}{90}$

29. $0.34\overline{67}+0.1\overline{333}$

$=\frac{3467-34}{9900}+\frac{1333-13}{9900}$

$=\frac{3433+1320}{9900}=\frac{4753}{9900}$

$=\frac{4801-48}{9900}=0.4\overline{801}$

30. $\frac{2}{5}=0.4,\frac{5}{6}=0.8\overline{33},\frac{11}{12}=0.91\overline{6}$ and $\frac{7}{8}=0.875$

Clearly, the greatest fraction is $0.91\overline{6}$ *i.e.*, $\frac{11}{12}$.

31. We know that a number $\frac{p}{q}$ can be written as a terminating decimal when q is of the form $2^m\times5^n$ for some non-negative integers m and n.

In $\frac{1}{22}$ and $\frac{2}{15}$, 22 and 15 are not in the form of $2^m\times5^n$ but in $\frac{1}{16}$. 16 in the form of $2^4\times5^0$. Hence, $\frac{1}{16}$ can be written as a terminating decimal.

Hence, statements (I) and (III) are correct.

32. Since, 1.16666... and 1.454545... are recurring numbers and we know that recurring numbers represent rational numbers.

Hence, I and IV are rational numbers.

33. Let the numerator and denominator of a fraction are x and y, respectively.

By given condition, $\frac{x-1}{y}=\frac{1}{3}$

$\Rightarrow\quad 3x-3=y$

$\Rightarrow\quad 3x-y=3$ ...(i)

and $\frac{x}{y+5}=\frac{1}{4}$

$\Rightarrow\quad 4x-y=5$ ...(ii)

On solving Eqs. (i) and (ii), we get

$x=2$ and $y=3$

Hence, required fraction $=\frac{x-1}{y+5}=\frac{2-1}{3+5}=\frac{1}{8}$

34. (a) 0.725

(b) $0.72\overline{5}=0.7255\ldots$

(c) $0.7\overline{25}=0.7252525\ldots$

(d) $0.\overline{725}=0.725725725\ldots$

Hence, the largest number is $0.\overline{725}$.

35. $\because\frac{13}{8}=\frac{13}{8}\times\frac{125}{125}=\frac{1625}{1000}=1.625$

$\frac{3}{16}=\frac{3\times625}{16\times625}=\frac{1875}{10000}=0.1875$

$\frac{137}{25}=\frac{137\times4}{25\times4}=\frac{548}{100}=5.48$

From above, it is clear that all of these are terminating and decimals. Hence, $\frac{3}{11}$ is a non-terminating and repeating decimal because $\frac{3}{11}=0.272727\ldots$

$=0.\overline{27}$

36. $3.\overline{76}-1.4\overline{576}=3+0.\overline{76}-1-0.4\overline{576}$

$=3+\left(\frac{76-0}{99}\right)-1-\left(\frac{4576-4}{9990}\right)$

$=3+\frac{76}{99}-1-\frac{4572}{9990}$

$=2+\left(\frac{76}{99}-\frac{4572}{9990}\right)$

$=2+\frac{1}{9}\left(\frac{76}{11}-\frac{4572}{1110}\right)$

$=2+\frac{1}{9}\times\frac{(84360-50292)}{12210}$

$=2+\frac{1}{9}\times\frac{34068}{12210}=2+\frac{11356}{36630}$

$=2+0.\overline{3100191}$

$=2.\overline{3100191}$