CHAPTER 6

Design of Beam

CONTENTS

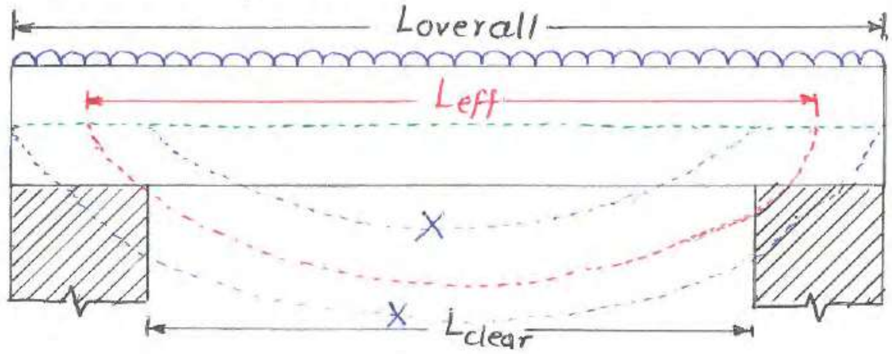
6. Design of Beam

6.1 Introduction:

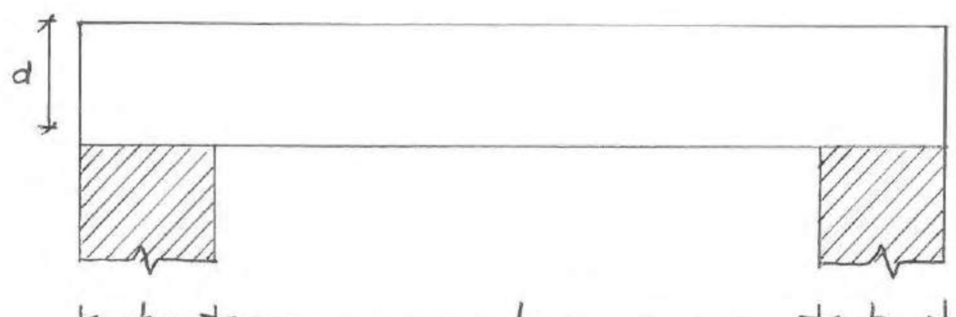
In previous chapters, analysis and design of section were covered. In this chapter, design of complete beam is going to be discussed.

6.2 Effective Span:

Portion of beam that effectively participates in bending is called effective span.



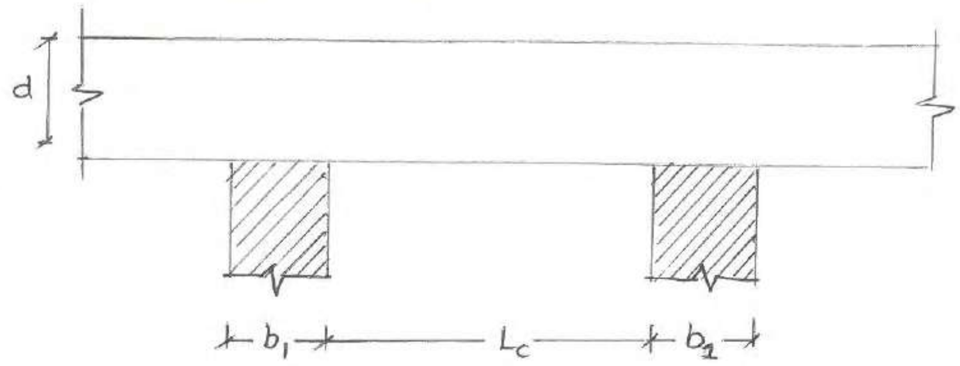
6.2.1 Simply Supported Beam/Slab:



$$L_{eff} = \text{Minimum} \begin{cases} \text{i)} \ L_c + d \\ \text{ii)} \ \dfrac{b_1}{2} + L_c + \dfrac{b_2}{2} \end{cases}$$

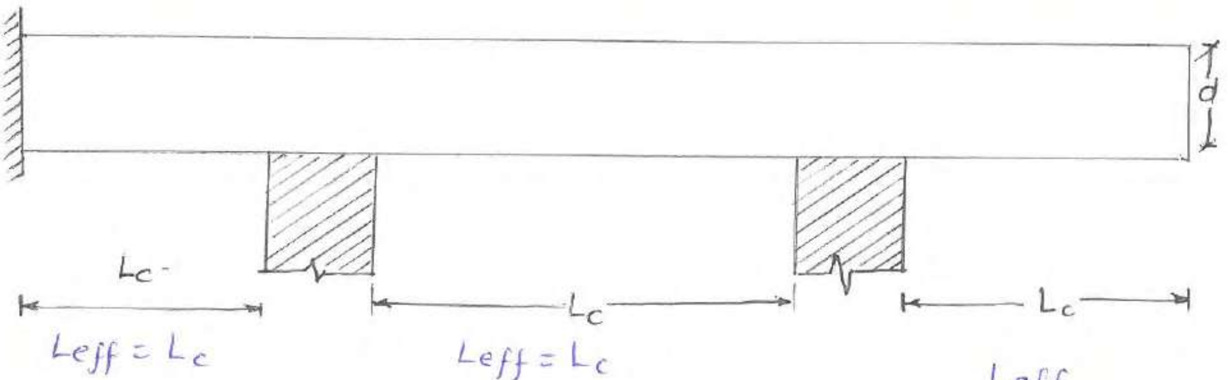
6.2.2 Continuous Beam:

Case I: If support width is less than $\frac{L_c}{12}$ then L_{eff} is same as 6.2.1



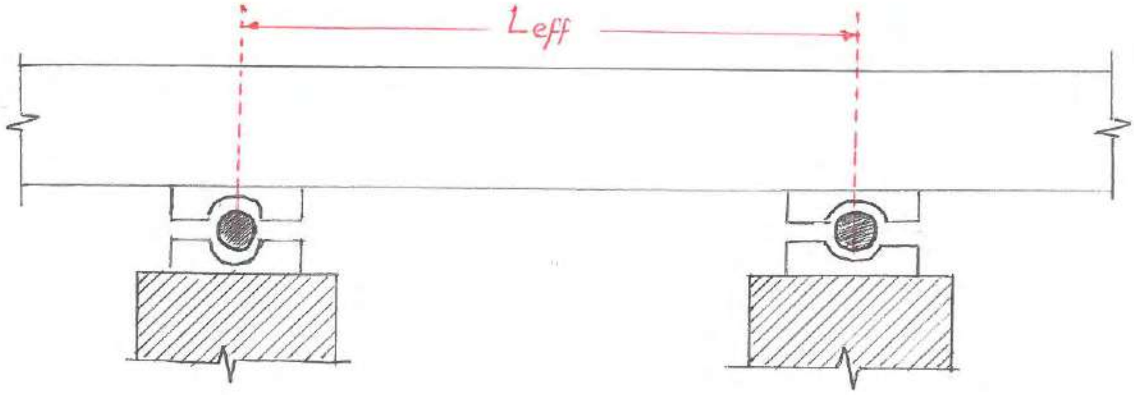
b_1 & $b_2 < \frac{L_c}{12}$ $\quad L_{eff} = \text{Minimum} \begin{cases} \text{i) } L_c + d \\ \text{ii) } \frac{b_1}{2} + L_c + \frac{b_2}{2} \end{cases}$

Case II: If support width is more than $\frac{L_c}{12}$ or 600 mm whichever is smaller then L_{eff} is calculated as follows.

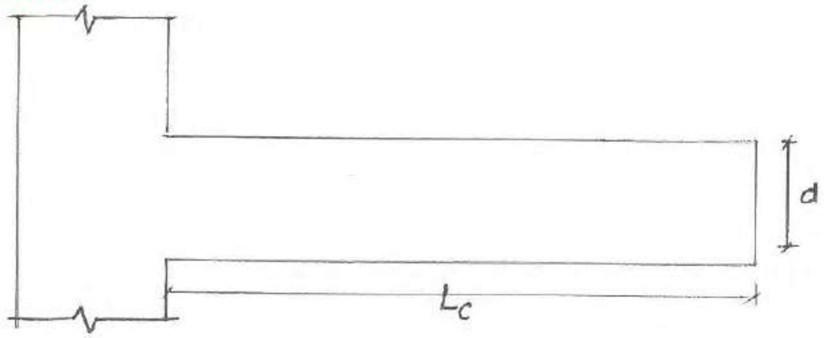


$= \text{Minimum} \begin{cases} \text{i) } L_c + \frac{d}{2} \\ \text{ii) } L_c + \frac{b}{2} \end{cases}$

Case III: If beam is continuous over Roller - rocker bearing then L_{eff} is c/c distance between bearings.



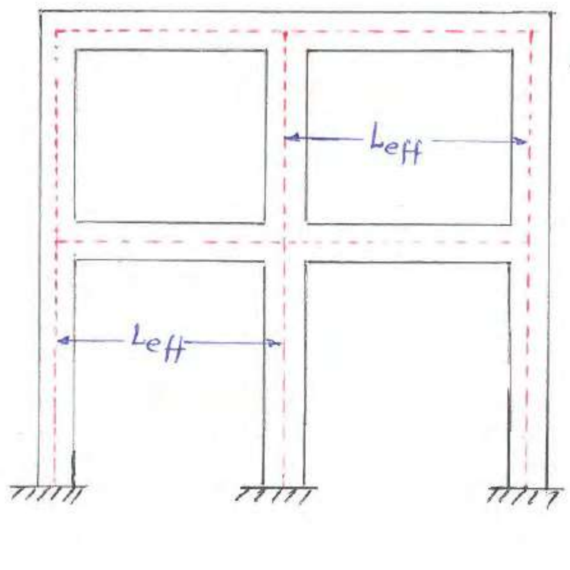
6.2.3 Cantilever:



$$L_{eff} = L_c + \frac{d}{2}$$

6.2.4 Rigid Frames:

L_{eff} is c/c distance between columns.



6.3 Longitudinal Reinforcement:

6.3.1 Longitudinal Tension Reinforcement:

$$\frac{A_{st,min}}{bd} > \frac{0.85}{f_y} \quad \cdots\cdots \text{(To prevent sudden failure and for ductility)}$$

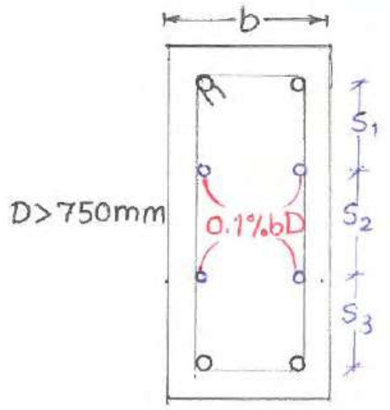
$$A_{st,max} = 0.04bD \quad \cdots\cdots \text{(Problem in compaction)}$$

6.3.2 Longitudinal Compression Reinforcement:

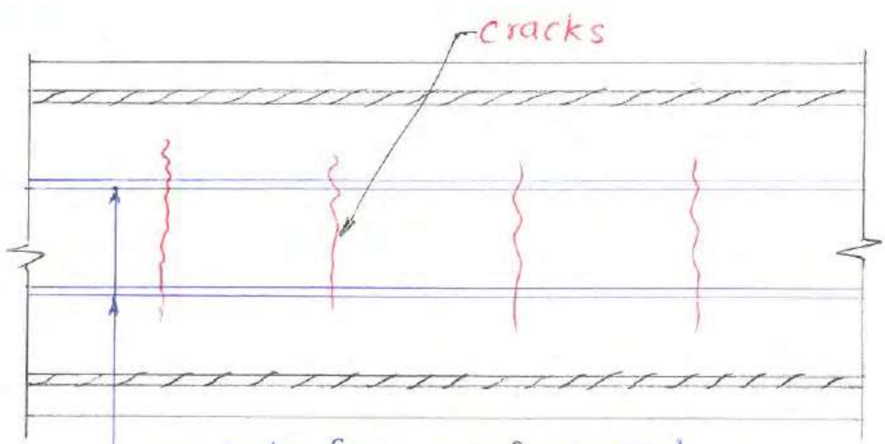
$A_{sc,min}$ = No value, but atleast 2-bars must be provided in compression zone for ductility and to hold stirrups.

$$A_{sc,max} = 0.04\,bD \quad \cdots\cdots \text{(Problem in compaction)}$$

6.3.3 Side Face Reinforcement:

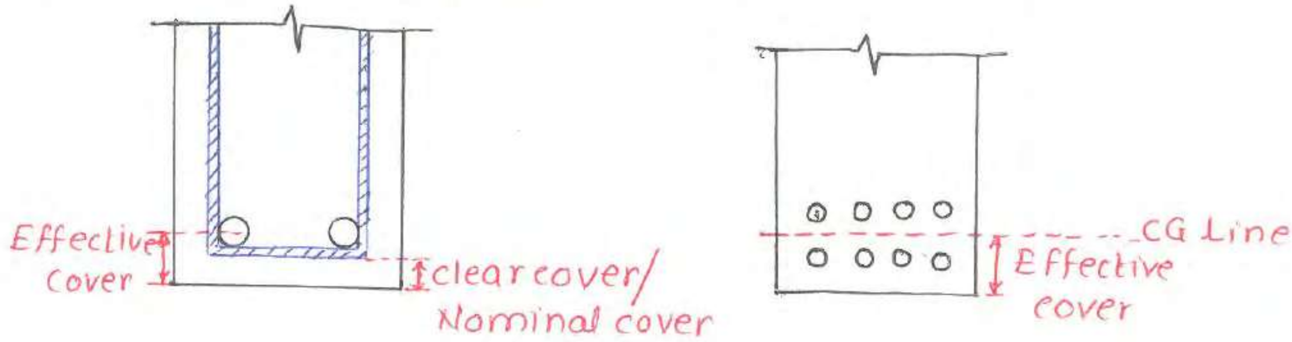


$S_i \not> 300$



For $D > 750$mm, 0.1% of gross web area is provided as side face reinforcement, equally distributed on both faces with spacing not more than 300mm. It is provided to take care of shrinkage of concrete along longitudinal direction (vertical cracks) and lateral buckling.

6.4 Minimum Nominal Cover:



- Minimum nominal cover is governed by following criteria.
  1. Type of Structural Member (Beam, slab, footing, etc.)
  2. Exposure condition (Mild, moderate, etc)
  3. Fire resistance in terms of hours (30 min to 4 hrs)

	Slab	Beam	Column	Footing
IS 456	20	20	40	50
SP 34	20	25	40	50

or dia. of bar whichever is greater.

6.5 Maximum Permissible Crack Width in RCC Structure:

Exposure Condition	Maximum Permissible crack width
• Mild (Cracks are not - harmful)	0.3mm
• Moderate & Severe (Cracks are harmful)	0.2mm
• Very Severe & Extreme	0.1mm

6.6 Horizontal Spacing of Reinforcement:

6.6.1 Minimum Horizontal Clear Spacing:

Maximum of following:

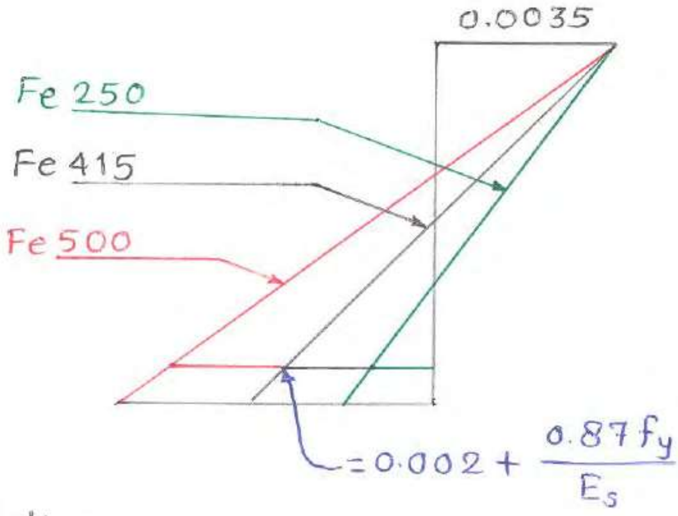
1) Dia. of bar if bars are of equal dia.
2) Dia. of larger dia. bar if bars are of unequal dia.
3) Nominal size of coarse aggregate + 5mm.

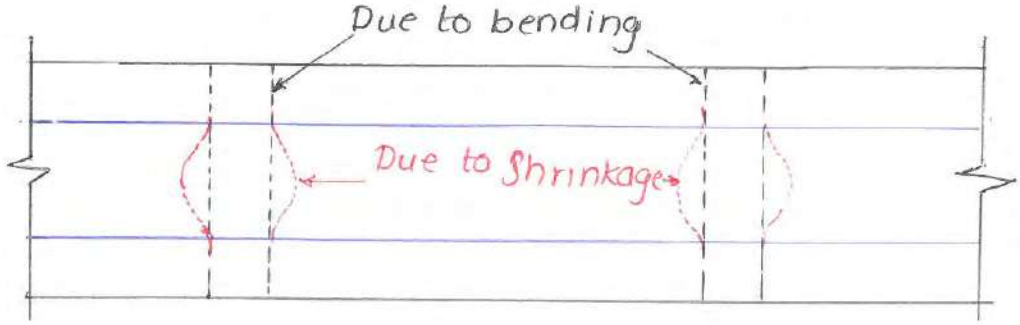
6.6.2. Maximum Horizontal c/c spacing:

Fe 250 → 300mm

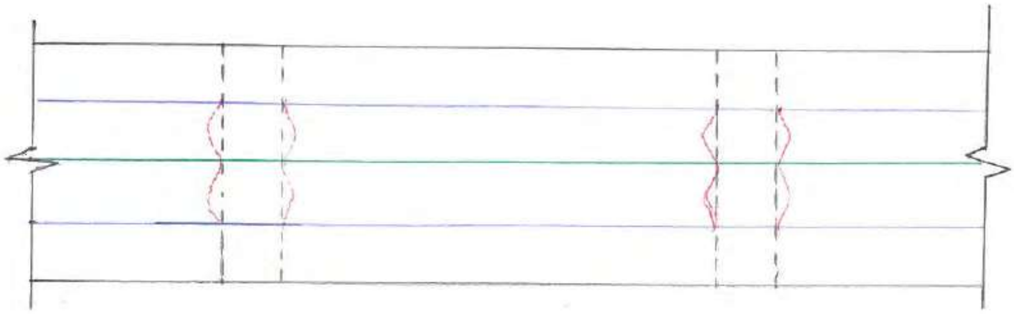
Fe 415 → 180mm

Fe 500 → 150mm.





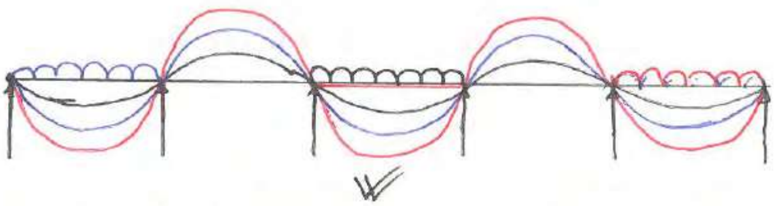
Plan.



Crack width of RCC structure depends on
Strain of Steel and
Shrinkage of concrete
Higher strain is required in high grade steel to get desired stress. so this produces wider cracks than low grade steel. Now, cummulative effect of shrinkage and strain of steel on crack width can be reduced by providing reduced spacing of reinforcement for higher grade.

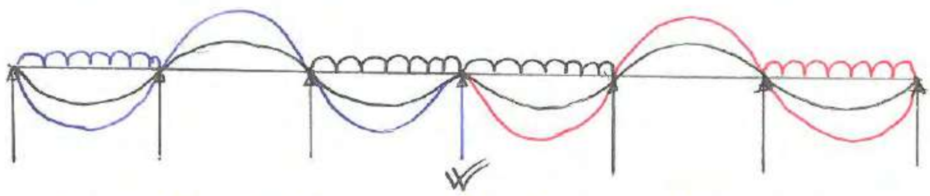
6.7 Arrangement of Loading for Maximum B.M.

Case I: For maximum sagging span moment



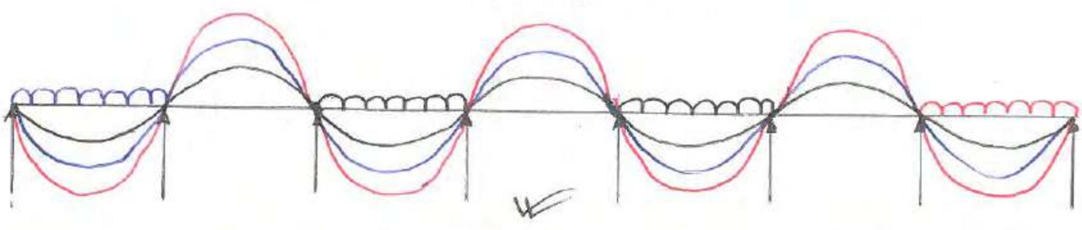
Loading should be applied on span under consideration + Alternate spans.

Case II: Maximum Hogging Support Moment:



Loading should be applied on adjacent spans + Alternate spans

Case III: For Maximum Hogging Span Moment:



Loading should be applied on adjacent spans + Alternate spans.

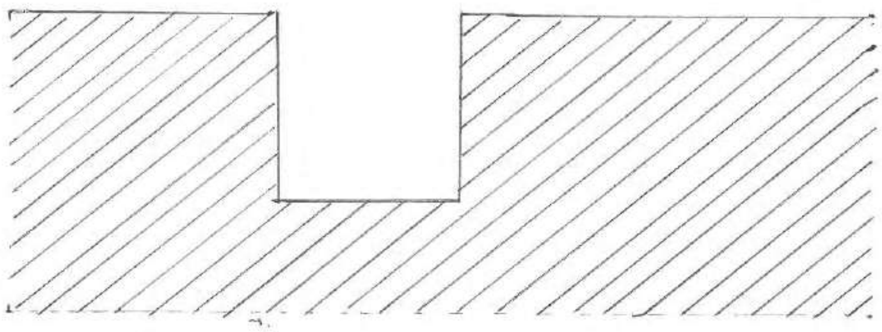
Case IV: For Maximum Sagging Support Moment:

Lading should be applied on next to adjacent spans + Alternate spans.

6.8 Lateral Stability of Beam:

1. Laterally Supported at Ends:

$$L_{unsupported} \ngtr \text{Minimum} \begin{cases} \text{i)}\ 60\,b \\ \text{ii)}\ \dfrac{250\,b^2}{d} \end{cases}$$



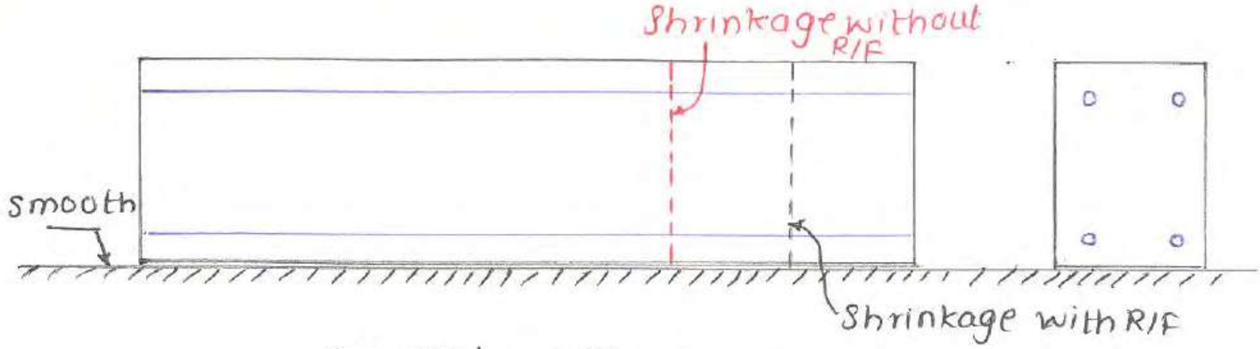
2. Cantilever:

$$L_{unsupported} \ngtr \text{Minimum} \begin{cases} \text{i)}\ 25\,b \\ \text{ii)}\ \dfrac{100\,b^2}{d} \end{cases}$$

6.9 Effect of Shrinkage:

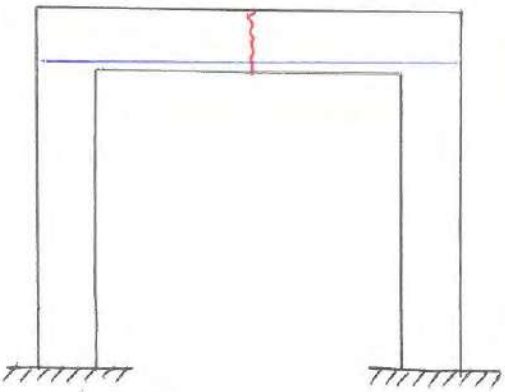
6.9.1 Stress due to Shrinkage:

Case I: Member is free to Shrink:



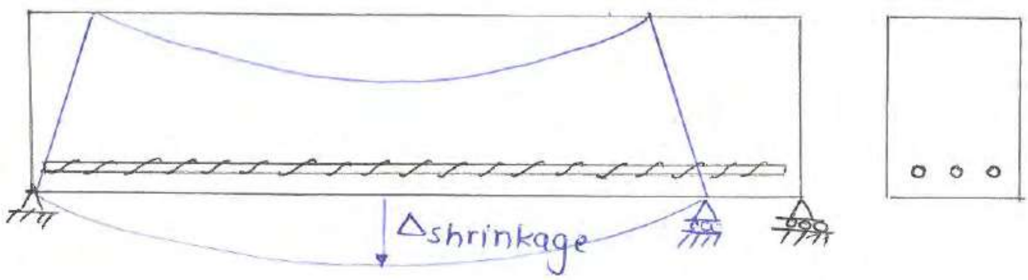
Concrent → Tension
R/F → Compression.

Case II: Member is not free to shrink:



Steel → Tension
Concrete → Tension.

6.9.2 Deflection due to Shrinkage:



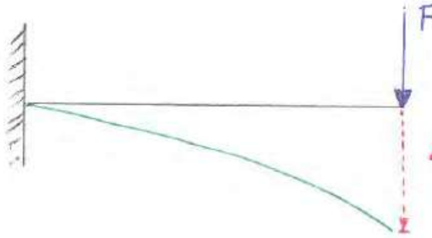
Due to presence of bottom steel, bottom fibres shrinks less than top fibres. This differential shrinkage produces downward deflection. Deflection due to shrinkage can be eliminated by providing equal % of top & bottom reinforcement.

6.10 Total Deflection of Beam:

$$\Delta_{Total} = \underbrace{\Delta_{Loading} + \Delta_{Temp.}}_{\text{Short term}} + \underbrace{\Delta_{Shrinkage} + \Delta_{Creep.}}_{\text{Long term}}$$

Short term modulus of Elasticity is used

Long term Modulus of Elasticity is used.



$$\Delta = \frac{PL^3}{3EI}$$

6.11 Deflection Criteria:

6.11.1 Deflection Limits:

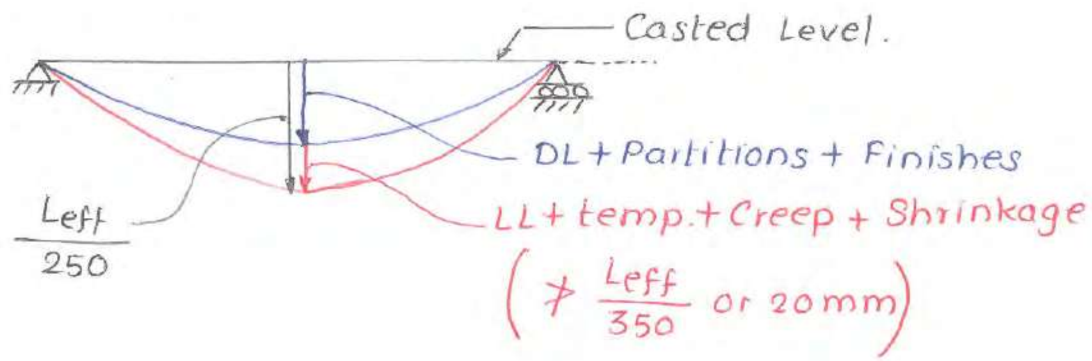
1) The final deflection due to all loads including the effect of temperature, creep and shrinkage and measured from the cast level of the supports of floors, roofs and other horizontal members should not normally exceed

$$\frac{L_{eff}}{250} \quad \text{(Dead Load + Live Load + Creep + Temp. + Shrinkage)}$$

This limitation is based on crack limitation with which code is very much concerned and to avoid pscytological fear of occupants or affect the appearanceof structure

2) The deflection including the effect of temperature, creep and shrinkage occuring after erection of partitions and the application of finishes, should not normally exceed

$$\frac{L_{eff}}{350} \text{ or 20mm whichever is less.}$$



6.11.2 Deflection Control:

Exact calculation of deflection and keeping it within permissible limits need lot of calculation. IS456 provides a simplified approach to keep deflection of beam within permissible limits.

If $\frac{L_{eff}}{d}$ ratio satisfies following conditions then beam is safe in deflection.

Support condition	Values.
Cantilever	7
Simply Supported	20
Continuous	26

$$\frac{L_{eff}}{d} < k_1 \, k_2 \, k_3 \, k_4 \text{ (value)}$$

where, k_i = Modification factor.

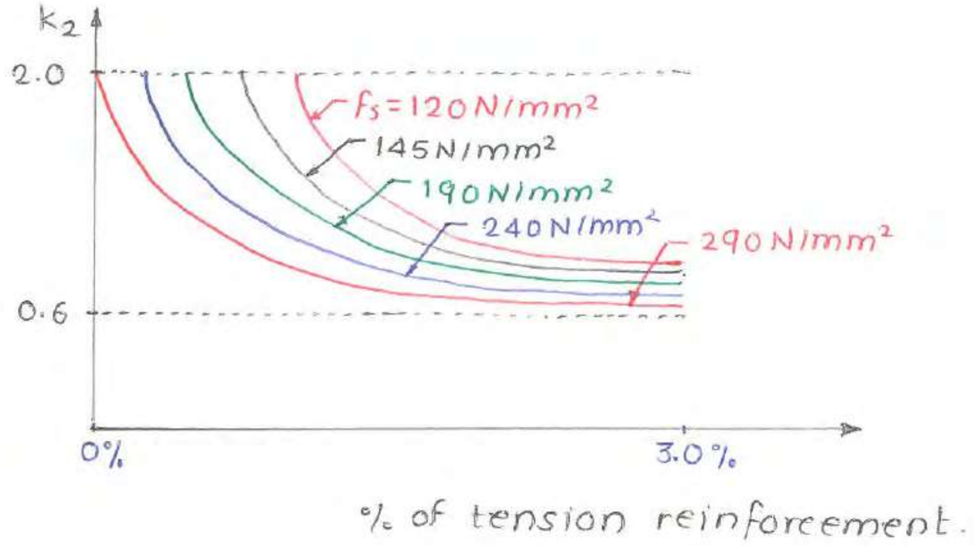
$k_1 \Rightarrow$ Depends on Span

$= 1$ (upto 10m)

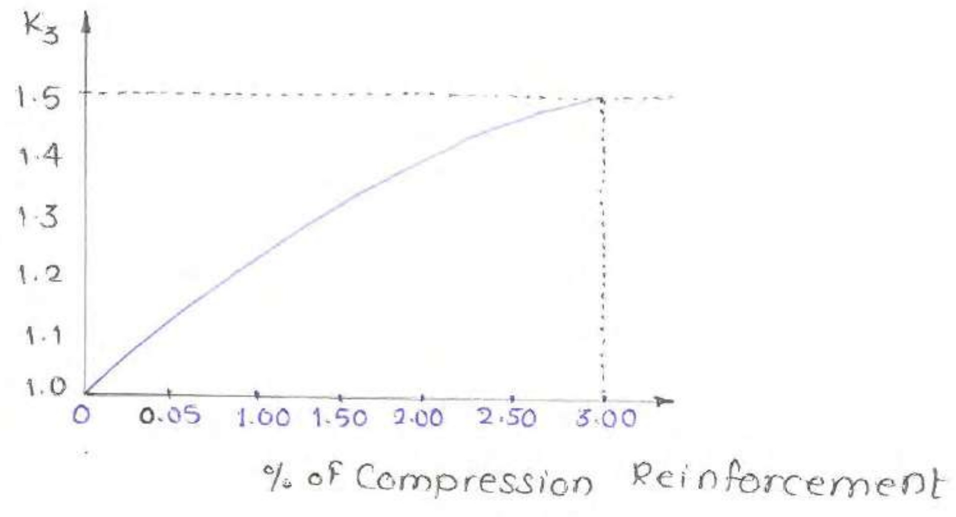
$= \frac{10}{\text{Span(m)}}$ (beyond 10m, NA for cantilever)

$k_2 \Rightarrow$ Depends on % of Tension Reinforcement.

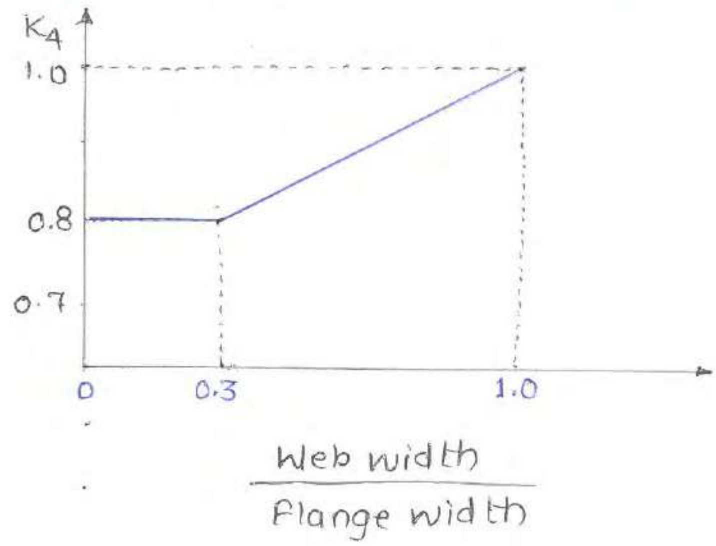
f_s = Stress level of steel $= 0.58 f_y \frac{A_{st,req}}{A_{st,provided}}$



$K_3 \Rightarrow$ Depends on % of Compression Reinforcement:



$K_4 \Rightarrow$ Depends on Ratio of Web width to Flange Width:



*Note:-

$$\frac{L_{eff}}{d} < K_1 K_2 K_3 K_4 \text{ (value)}$$

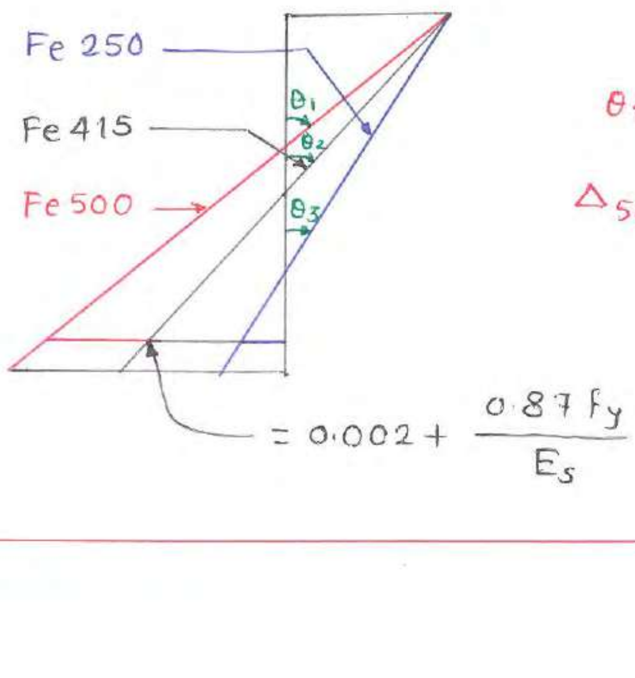
$$d > \frac{L_{eff}}{K_1 K_2 K_3 K_4 \text{ (value)}}$$

- Since K_4 is higher for rectangular section so depth requirement to satisfy deflection criteria is less for rectangular section than flanged section. It means flange section produces more deflection.

-

- As % of compression reinforcement increases, K_3 increases. It means depth requirement to satisfy deflection criteria decreases. In other words higher % of compression reinforcement produces less deflection.

- Higher grade of steel produces more deflection.



$\theta_1 > \theta_2 > \theta_3$

$\Delta_{500} > \Delta_{415} > \Delta_{250}$

6.12 Design of Simply Supported Singly Reinforced Beam of Rectangular Section:

Step 1: Assume suitable value of $\frac{b}{d}$ ratio.

$$(\text{Lateral Buckling})\ 0.3 < \frac{b}{d} < 0.7\ (\text{uneconomical})$$

$$\frac{b}{d} = 0.5 \quad (\text{for exam})$$

Step 2: Assume suitable value of d based on following criteria

- Thumb rule:

$$\frac{L_{eff}}{15} < d < \frac{L_{eff}}{10}$$

- Deflection criteria:

$$\frac{L_{eff}}{d} < k_1 k_2 k_3 k_4 (\text{value})$$

Where, $k_1 \rightarrow$ depends on span

$k_2 = 1$ (because % of tension reinforcement is not known)

$k_3 = 1$ (Singly reinforced)

$k_4 = 1$ (Rectangular)

Select suitable value of d based on above two criteria and calculate b and D accordingly.

* Note:

Since L_{eff} is not known so L_{clear} or c/c distance between supports can be used at the place of L_{eff} in above calculation for priliminary design.

Step 3: Calculate effective span.

Step 4: Calculate DL and design B.M.

Step 5: Calculate 'd' required for balanced section

$BM_u = M_{u,lim}$

$BM_u = Q\,bd^2$

$\Rightarrow d = ??$

'd' calculated here should be less than assumed in step 2. otherwise select 'd' suitably higher than calculated here and repeat Step 3, Step 4 and Step 5.

Step 6: Since section size provided in Step 2 is larger than required in step 5 so provided section is under reinforced.

$$A_{st} = \frac{0.5 f_{ck} bd}{f_y}\left[1 - \sqrt{1 - \frac{4.6\,BM_u}{f_{ck}\,bd^2}}\right]$$

Step 7: Ast calculated above should be within permissible limits

Step 8: Provide side face reinforcement if required.

Step 9: Apply check for deflection.

Ex. Design a simply supported singly reinforced beam of rectangular section spanning over clear span 11m and resting on 400mm thick wall on one side and 500mm on another side. It is subjected to superimposed LL of 30kN/m. M30, Fe500, Severe exposure.

⇒

Step 1: $\frac{b}{d} = 0.5$ (Assume)

Step 2: Effective depth

- Thumb rule.

$$\frac{L_{eff}}{15} < d < \frac{L_{eff}}{10}$$

$$\frac{11.45 \times 10^3}{15} < d < \frac{11.45 \times 10^3}{10}$$

$$\Rightarrow 763.33 < d < 1145 \text{mm}.$$

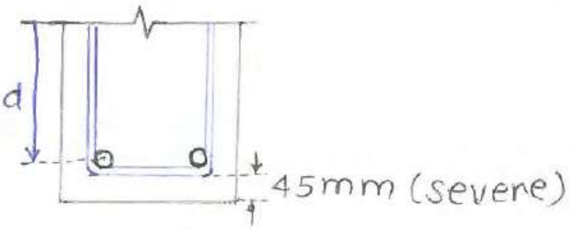
- Deflection Criteria:

$$\frac{L_{eff}}{d} < K_1 K_2 K_3 K_4 \text{ (value)}$$

$$\frac{11.45 \times 10^3}{d} < \left(\frac{10}{11.45}\right) (1)(1)(1)(20)$$

$$\Rightarrow d > 655.51 \text{mm}.$$

⇒ **Providing $d = 800$mm**
$b = 0.5d = 400$mm



D = d + allowance for reinforcement + Clear cover

= 800 + 30 (assumed) + 45 (Severe)

⇒ **D = 875mm**

Step3: Effective span

$$L_{eff} = \text{Minimum} \begin{cases} \text{i)}\ L_c + d = 11 + 0.8 = 11.8\,m \\ \text{ii)}\ \frac{b_1}{2} + L_c + \frac{b_2}{2} = \frac{0.4}{2} + 11 + \frac{0.5}{2} = 11.45\,m \end{cases}$$

$\Rightarrow$ $\boxed{L_{eff} = 11.45\,m}$

Step4: Loading - DL, LL and Factored B.M.

$DL = 0.4 \times 0.875 \times 1 \times 25 = 8.75\,kN/m$

$LL = 30\,kN/m$

Total factored load $= W_u = 1.5 \times (8.75 + 30) = 58.125\,kN/m$

$$BM_{u(mid\text{-}span)} = \frac{W_u L^2}{8}$$

$$= \frac{58.125 \times (11.45)^2}{8}$$

$\boxed{BM_u = 952.54\,kN\cdot m}$

Step5: 'd' required for balanced section

$BM_u = M_{u,lim}$

$BM_u = 0.133\, f_{ck}\, b d^2$

$952.54 \times 10^6 = 0.133 \times 30 \times 400 \times d^2$

$\Rightarrow$ $\boxed{d = 772.54\,mm < 800\,mm \Rightarrow OK.}$

Step 6: A_{st} Required.

$$A_{st} = \frac{0.5 f_{ck} b d}{f_y}\left[1 - \sqrt{1 - \frac{4.6\, BM_u}{f_{ck} b d^2}}\right]$$

$$= \frac{0.5 \times 30 \times 400 \times 800}{500}\left[1 - \sqrt{1 - \frac{4.6 \times 952.54 \times 10^6}{30 \times 400 \times 800^2}}\right]$$

$A_{st} = 3308.75\,mm^2$

Step 7: Permissible Limits for reinforcement.

- $\frac{A_{st,min}}{bd} > \frac{0.85}{f_y} \Rightarrow \frac{A_{st,min}}{400 \times 800} > \frac{0.85}{500}$

$A_{st,min} > 544\ mm^2$

• $A_{st} \leq$ Minimum
$$\begin{cases} \text{i)}\ A_{st,lim} = 0.414\left(\frac{f_{ck}}{f_y}\right)\cdot x_{u,lim}\cdot b \\ \quad = 0.414 \times \frac{30}{500} \times 0.46 \times 800 \times 400 \\ A_{st,lim} = 3656.45\ mm^2 \\ \text{ii)}\ 0.04bD = 0.04 \times 400 \times 800 = 14000\ mm^2. \end{cases}$$

$A_{st} \leq 3656.45\ mm^2$.

Providing 4-32 ϕ + 1-16 ϕ

Step 8: Side Face reinforcement: $A_{st} = 0.1\%\ bD$

$A_{st} = 0.001 \times 400 \times 875$

$A_{st} = 350\ mm^2$

Atleast 2-bars on each side face are required to satisfy maximum spacing criteria (300mm)

Providing 2-12 ϕ on each side face.

Step 9: Deflection Check:

$K_1 = \frac{10}{11.45} \Rightarrow K_1 = 0.87$

$K_2 \Rightarrow$ Depends on % of Tension reinforcement

$$P_t = \frac{A_{st}}{bD} \times 100$$

$$= \frac{4 \times \frac{\pi}{4} \times 32^2 + 1 \times \frac{\pi}{4} \times 16^2}{400 \times 875} \times 100$$

$P_t = 1.06\%$

$$f_s = 0.58 f_y \frac{A_{st,req}}{A_{st,provided}}$$

$$= 0.58 \times 500 \times \frac{3308.75}{4 \times \frac{\pi}{4} \times 32^2 + 1 \times \frac{\pi}{4} \times 16^2}$$

$f_s = 280.72\ N/mm^2$

$K_2 = 0.85$ .... (from graph 4 of IS456 $P_t = 1.06\%$ & $f_s = 280.72\ N/mm^2$)

$K_3 = 1$ .... (Singly reinforced)

$K_4 = 1$ ..... (Rectangular)

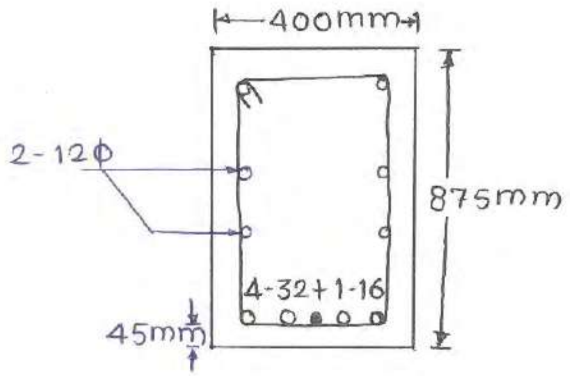
Now,

$$\frac{L_{eff}}{d} < K_1 K_2 K_3 K_4 \text{ (value)}$$

$$\frac{11.45 \times 10^3}{800} < 0.87 \times 0.85 \times 1 \times 1 \times (20)$$

$$14.31 < 14.79 \quad .... \quad OK.$$

Detailing:



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