CAT 2020 Question Paper Slot 3

Quant

51. Two alcohol solutions, A and B, are mixed in the proportion 1:3 by volume. The volume of the mixture is then doubled by adding solution A such that the resulting mixture has 72% alcohol. If solution A has 60% alcohol, then the percentage of alcohol in solution B is

A 90%

B 94%

C 92%

D 89%

52. A batsman played n + 2 innings and got out on all occasions. His average score in these n + 2 innings was 29 runs and he scored 38 and 15 runs in the last two innings. The batsman scored less than 38 runs in each of the first n innings. In these n innings, his average score was 30 runs and lowest score was x runs. The smallest possible value of x is

A 4

B 3

C 2

D 1

53. Let m and n be positive integers, If $x^2 + mx + 2n = 0$ and $x^2 + 2nx + m = 0$ have real roots, then the smallest possible value of $m + n$ is

A 7

B 6

C 8

D 5

54. A contractor agreed to construct a 6 km road in 200 days. He employed 140 persons for the work. After 60 days, he realized that only 1.5 km road has been completed. How many additional people would he need to employ in order to finish the work exactly on time?

55. If $x_1 = -1$ and $x_m = x_{m+1} + (m + 1)$ for every positive integer m, then X_{100} equals

A -5050

B -5151

C -5051

D -5150

56. If $\log_a 30 = A, \log_a(\frac{5}{3}) = -B$ and $\log_2 a = \frac{1}{3}$, then $\log_3 a$ equals

A $\frac{2}{A+B-3}$

B $\frac{2}{A+B} - 3$

C $\frac{A+B}{2} - 3$

D $\frac{A+B-3}{2}$

57. Dick is thrice as old as Tom and Harry is twice as old as Dick. If Dick's age is 1 year less than the average age of all three, then Harry's age, in years, is

58. Vimla starts for office every day at 9 am and reaches exactly on time if she drives at her usual speed of 40 km/hr. She is late by 6 minutes if she drives at 35 km/hr. One day, she covers two-thirds of her distance to office in one-thirds of her usual total time to reach office, and then stops for 8 minutes. The speed, in km/hr, at which she should drive the remaining distance to reach office exactly on time is

A 29

B 26

C 28

D 27

59. Let m and n be natural numbers such that n is even and $0.2 < \frac{m}{20}, \frac{n}{m}, \frac{n}{11} < 0.5$. Then $m - 2n$ equals

A 3

B 1

C 2

D 4

60. How many integers in the set {100, 101, 102, ..., 999} have at least one digit repeated?

61. In the final examination, Bishnu scored 52% and Asha scored 64%. The marks obtained by Bishnu is 23 less, and that by Asha is 34 more than the marks obtained by Ramesh. The marks obtained by Geeta, who scored 84%, is

A 357

B 417

C 439

D 399

62. If a,b,c are non-zero and $14^a = 36^b = 84^c$, then $6b(\frac{1}{c} - \frac{1}{a})$ is equal to

63. A person invested a certain amount of money at 10% annual interest, compounded half-yearly. After one and a half years, the interest and principal together became Rs.18522. The amount, in rupees, that the person had invested is

64. A man buys 35 kg of sugar and sets a marked price in order to make a 20% profit. He sells 5 kg at this price, and 15 kg at a 10% discount. Accidentally, 3 kg of sugar is wasted. He sells the remaining sugar by raising the marked price by p percent so as to make an overall profit of 15%. Then p is nearest to

A 22

B 35

C 25

D 31

65. The points (2,1) and (-3,-4) are opposite vertices of a parallelogram.If the other two vertices lie on the line $x + 9y + c = 0$, then c is

A 12

B 13

C 15

D 14

66. A and B are two railway stations 90 km apart. A train leaves A at 9:00 am, heading towards B at a speed of 40 km/hr. Another train leaves B at 10:30 am, heading towards A at a speed of 20 km/hr. The trains meet each other at

A 11 : 45 am

B 11 : 20 am

C 11 : 00 am

D 10 : 45 am

67. Let N, x and y be positive integers such that $N = x + y, 2 < x < 10$ and $14 < y < 23$. If $N > 25$, then how many distinct values are possible for N?

68. Let k be a constant. The equations $kx + y = 3$ and $4x + ky = 4$ have a unique solution if and only if

A $| k | \neq 2$

B $| k | = 2$

C $k \neq 2$

D $k = 2$

69. How many of the integers 1, 2, … , 120, are divisible by none of 2, 5 and 7?

A 42

B 41

C 40

D 43

70. How many pairs(a, b) of positive integers are there such that $a \leq b$ and $ab = 4^{2017}$?

A 2018

B 2019

C 2017

D 2020

71. Anil, Sunil, and Ravi run along a circular path of length 3 km, starting from the same point at the same time, and going in the clockwise direction. If they run at speeds of 15 km/hr, 10 km/hr, and 8 km/hr, respectively, how much distance in km will Ravi have run when Anil and Sunil meet again for the first time at the starting point?

A 4.8

B 4.6

C 5.2

D 4.2

72. In a trapezium $ABCD$, AB is parallel to DC, BC is perpendicular to DC and $\angle BAD = 45^0$. If DC = 5cm, BC = 4 cm,the area of the trapezium in sq cm is

73. The area, in sq. units, enclosed by the lines $x = 2, y = \mid x - 2 \mid +4$, the X-axis and the Y-axis is equal to

A 10

B 6

C 8

D 12

74. If $f(x + y) = f(x)f(y)$ and $f(5) = 4$, then $f(10) - f(-10)$ is equal to

A 14.0625

B 0

C 15.9375

D 3

75. $\frac{2\times4\times8\times16}{(\log_2 4)^2(\log_4 8)^3(\log_8 16)^4}$ equals

76. The vertices of a triangle are (0,0), (4,0) and (3,9). The area of the circle passing through these three points is

A $\frac{14\pi}{3}$

B $\frac{123\pi}{7}$

C $\frac{12\pi}{5}$

D $\frac{205\pi}{9}$

Answers

Quant

51.**C**	52.**C**	53.**B**	54.**40**	55.**A**	56.**A**	57.**18**	58.**C**
59.**B**	60.**252**	61.**D**	62.**3**	63.**16000**	64.**C**	65.**D**	66.**C**
67.**6**	68.**A**	69.**B**	70.**A**	71.**A**	72.**28**	73.**A**	74.**C**
75.**24**	76.**D**						

Explanations

Quant

51. **C**

Initially let's consider A and B as one component

The volume of the mixture is doubled by adding A(60% alcohol) i.e they are mixed in 1:1 ratio and the resultant mixture has 72% alcohol.

Let the percentage of alcohol in component 1 be 'x'.

Using allegations , $\frac{(72-60)}{x-72} = \frac{1}{1}$ => x= 84

Percentage of alcohol in A = 60% => Let's percentage of alcohol in B = x%

The resultant mixture has 84% alcohol. ratio = 1:3

Using allegations , $\frac{(x-84)}{84-60} = \frac{1}{3}$

=> x= 92%

52. **C**

Given, $\frac{\text{sum of scores in n matches}+38+15}{n+2} = 29$

Given, $\frac{\text{sum of scores in n matches}}{n} = 30$

=> 30n + 53 = 29(n+2) => n=5

Sum of the scores in 5 matches = 29*7 - 38-15 = 150

Since the batsmen scored less than 38, in each of the first 5 innings. The value of x will be minimum when remaining four values are highest

=> 37+37+37+37 + x = 150

=> x = 2

53. **B**

To have real roots the discriminant should be greater than or equal to 0.

So, $m^2 - 8n \geq 0 \ \& \ 4n^2 - 4m \geq 0$

=> $m^2 \geq 8n \ \& \ n^2 \geq m$

Since m,n are positive integers the value of m+n will be minimum when m=4 and n=2.

∴ m+n=6.

54. **40**

Let the desired efficiency of each worker '6x' per day.

140*6x*200= 6 km ...(i)

In 60 days 60/200*6=1.8 km of work is to be done but actually 1.5km is only done.

Actual efficiency 'y'= 1.5/1.8 *6x =5x.

Now, left over work = 4.5km which is to be done in 140 days with 'n' workers whose efficiency is 'y'.

=> n*5x*140=4.5 ...(ii)

(i)/(ii) gives,

$\frac{(140 \cdot 6x \cdot 200)}{(n \cdot 5x \cdot 140)} = \frac{6}{4.5}$

=> n=180.

∴ Extra 180-140 =40 workers are needed.

55. **A**

$x_1 = -1$

$x_1 = x_2 + 2$ => $x_2 = x_1 - 2$ = -3

Similarly,

$x_3 = x_1 - 5 = -6$

$x_4 = -10$

.

.

The series is -1, -3, -6, -10, -15.....

When the differences are in AP, then the nth term is $-\frac{n(n+1)}{2}$

$x_{100} = -\frac{100(100+1)}{2} = -5050$

56. **A**

$\log_a 30 = A \; or \; \log_a 5 + \log_a 2 + \log_a 3 = A$.....(1)

$\log_a \left(\frac{5}{3}\right) = -B \; or \; \log_a 3 - \log_a 5 = B$.....(2)

and finally $\log_a 2 = 3$

Substituting this in (1) we get $\log_a 5 + \log_a 3 = A - 3$

Now we have two equations in two variables (1) and (2) . On solving we get

$\log_a 3 = \frac{(A+B-3)}{2} or \; \log_3 a = \frac{2}{A+B-3}$

57. **18**

Let tom's age = x

=> Dick=3x

=>harry = 6x

Given,

3x+1 = (x+3x+6x)/3

=> x= 3

Hence, Harry's age = 18 years

58. **C**

Let distance = d

Given, $\frac{d}{35} - \frac{d}{40} = \frac{6}{60}$

=> d = 28km

The actual time taken to travel 28km = 28/40 = 7/10 hours = 42 min.

Given time taken to travel 58/3 km = 1/3 *42 = 14 min.

Then a break of 8 min.

To reach on time, he should cover remaining 28/3 km in 20 min => Speed = $\frac{\left(\frac{28}{3}\right)}{\frac{20}{60}} = 28$ km/hr

59. **B**

$0.2 < \frac{n}{11} < 0.5$

=> 2.2<n<5.5

Since n is an even natural number, the value of n = 4

$0.2 < \frac{m}{20} < 0.5$ => 4< m<10. Possible values of m = 5,6,7,8,9

Since $0.2 < \frac{n}{m} < 0.5$, the only possible value of m is 9

Hence m-2n = 9-8 = 1

60. **252**

Total number of numbers from 100 to 999 = 900

The number of three digits numbers with unique digits:

_ _ _

The hundredth's place can be filled in 9 ways (Number 0 cannot be selected)

Ten's place can be filled in 9 ways

One's place can be filled in 8 ways

Total number of numbers = 9*9*8 = 648

Number of integers in the set {100, 101, 102, ..., 999} have at least one digit repeated = 900 - 648 = 252

61. **D**

Let the total marks be 100x

Marks obtained by Bishnu = 52x

Marks obtained by Asha = 64x

Marks obtained by Ramesh = 52x+23

Marks obtained by Ramesh = 64x-34

=> 52x+23 = 64x-34

=> x = $\frac{19}{4}$

Marks obtained by Geeta =84x = 84*19/4 = 399

62. **3**

Let $14^a = 36^b = 84^c$ = k

=> a = $\log_{14} k$, b = $\log_{36} k$, c=$\log_{84} k$

$6b(\frac{1}{c} - \frac{1}{a})$ = $6 \cdot \frac{1}{2} \log_6 k \left(\log_k 84 - \log_k 14\right)$ = 3

63. **16000**

Given,

Rate of interest = 10%

Since it is compounded half-yearly, R=5%

n=3

We know, A = $P\left(1 + \frac{R}{100}\right)^n$

18522 = $P\left(1 + 0.05\right)^3$

=> P = 16000

64. **C**

Let the cost price of 1kg of sugar = Rs 100

The total cost price of 35 kg = Rs3500

Marked up price per kg = Rs 120

GIven, the final profit is 15% => Final SP of 35 kg = 3500 *1.15 = Rs 4025

First 5 kg's are sold at 20% marked up price => $SP_1 = 5 \cdot 100 \cdot 1.2$ = Rs 600

Next 15 kgs are sold after giving 10% discount => $SP_2 = 15 \cdot 100 \cdot 1.2 \cdot 0.9 = 1620$

3kgs of sugar got wasted

=> 23 kg of sugar was sold at Rs (600 +1620) = Rs 2220

Remaining 12kg should be sold at Rs 4025 - 2220 = Rs1805

=> SP of 1kg = 1805/12 $\simeq 150$

Hence, the seller should further mark up by $\frac{(150-120)}{120} \cdot 100 = 25\%$

65. **D**

The midpoints of two diagonals of a parallelogram are the same

Hence the midpoint of (2,1) and (-3,-4) lie on $x + 9y + c = 0$

midpoint of (2,1) and (-3,-4) = ($\frac{2-3}{2}$, $\frac{1-4}{2}$) = (-1/2 , -3/2)

Keeping this cordinates in the above line equation, we get c = 14

66. **C**

The distance travelled by A between 9:00 Am and 10:30 Am is 3/2*40 =60 km.

Now they are separated by 30 km

Let the time taken to meet =t

Distance travelled by A in time t + Distance travelled by B in time t = 30

40t + 20t =30 => t=1/2 hour

Hence they meet at 11:00 AM

67. **6**

Possible values of x = 3,4,5,6,7,8,9

When x = 3, there is no possible value of y

When x = 4, the possible values of y = 22

When x = 5, the possible values of y=21,22

When x = 6, the possible values of y = 20.21,22

When x = 7, the possible values of y = 19,20,21,22

When x = 8, the possible values of y=18,19,20,21,22

When x = 9, the possible values of y=17,18,19,20,21,22

The unique values of N = 26,27,28,29,30,31

68. **A**

Two linear equations ax+by= c and dx+ ey = f have a unique solution if $\frac{a}{d} \neq \frac{b}{e}$

Therefore, $\frac{k}{4} \neq \frac{1}{k}$ => $k^2 \neq 4$

=> k $\neq$ |2|

69. **B**

The number of multiples of 2 between 1 and 120 = 60

The number of multiples of 5 between 1 and 120 which are not multiples of 2 = 12

The number of multiples of 7 between 1 and 120 which are not multiples of 2 and 5 = 7

Hence, number of the integers 1, 2, ... , 120, are divisible by none of 2, 5 and 7 = 120 - 60 - 12 - 7 = 41

70. **A**

$ab = 4^{2017} = 2^{4034}$

The total number of factors = 4035.

out of these 4035 factors, we can choose two numbers a,b such that a<b in [4035/2] = 2017.

And since the given number is a perfect square we have one set of two equal factors.

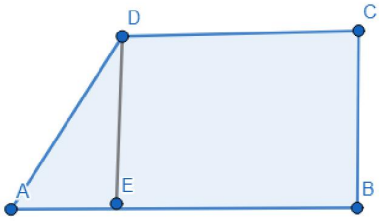
∴ many pairs(a, b) of positive integers are there such that $a \leq b$ and $ab = 4^{2017}$ = 2018.

71. **A**

Anil and Sunil will meet at a first point after LCM ($\frac{3}{15}, \frac{3}{10}$) = 3/5 hr

In the mean time, distance travelled by ravi = 8 * 3/5 = 4.8 km

72. **28**



Given, BC = DE = 4

CD = BE = 5

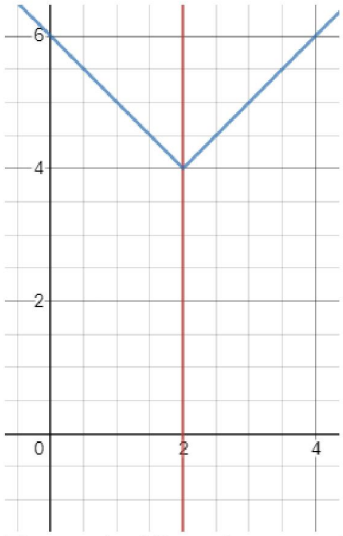
In triangle ADE, EAD=45^{0}$$

$\tan 45 = \frac{DE}{AE}$ => AE = 4

Area of trapezium = Area of rectangle BCDE + Area of triangle AED

= 20 + 8 = 28

73. **A**



The required figure is a trapezium with vertices A(0,0), B(2,0), C(2,4) and D(0,6)

AB = 2 BC = 4 and AD = 6

Area of trapezium = $\frac{1}{2}(sum\ of\ the\ opposite\ sides) \cdot height$ = $\frac{1}{2}(4+6) \cdot 2 = 10$

74. **C**

The given function is equivalent to f(x) = a^x

Given, f(5) = 4

=> $a^5 = 4 => \ a = 4^{\frac{1}{5}}$

=> f(x) = $4^{\frac{x}{5}}$

f(10) - f(-10) = 16 - 1/16 = 15.9375

75. **24**

$$\frac{(2\cdot4\cdot8\cdot16)}{(\log_2 2^2)^2\cdot\left(\log_{2^2} 2^3\right)^3\cdot\left(\log \quad 2\right.}$$

$$= \frac{2^{10}}{4\cdot\left(\frac{3}{2}\right)^3\cdot\left(\frac{4}{3}\right)^4} = 24$$

$$_{2^3 \quad 4}\Big)^4 .$$