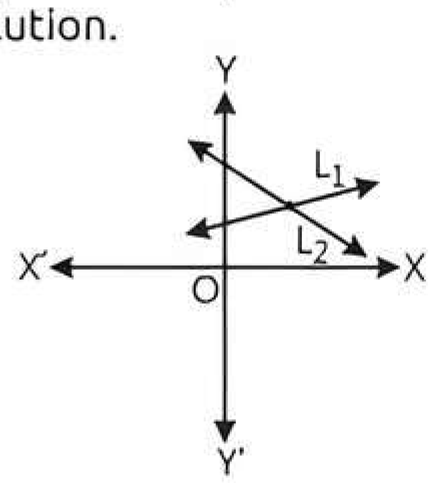
3 Pair of Linear Equations in Two Variables

Fastrack« Revision

- **Linear Equation in Two Variables:** An equation of the form $ax + by + c = 0$, where a, b and c are real numbers $(a \neq 0, b \neq 0)$, is called a linear equation in two variables x and y, *e.g.*, $7x - 3y - 2 = 0$, $4x + 9y = 14$, etc.
- **Solution of Linear Equation in Two Variables:** The values of the variables which satisfy the given linear equation is called solution of the linear equation. So, (x, y) is the solution of the linear equation $ax + by + c = 0$.
- **Simultaneous Linear Equations in Two Variables:** Two linear equations in two unknown variables x and y are said to form a system of simultaneous linear equations, if each of them is satisfied by the same pair of values of x and y.
The general form of a pair of linear equations is

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0$$

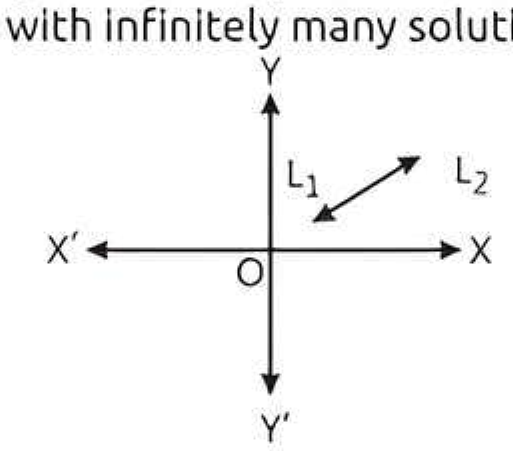
where a_1, a_2, b_1, b_2, c_1, c_2 are real numbers, such that $a_1^2 + b_1^2 \neq 0$, $a_2^2 + b_2^2 \neq 0$.
(*i.e.*, it means either a_1 or b_1 may be zero, but not both of them together should be zero. Similar condition exist for a_2 and b_2).
- **Consistent and Inconsistent Systems of Linear Equations**
  1. **Consistent System of Linear Equations:** A system of two linear equations in two unknown variables is said to be consistent, if it has at least one solution.
  Suppose two linear equations are $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.
  Case I: When two lines intersect each other at one point, then system of equations is called consistent with unique solution.



One (Unique) solution

Condition for Unique Solution: $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$

Case II: When two lines are coincident, *i.e.*, overlap each other, then system of equations is called consistent (dependent) with infinitely many solutions.

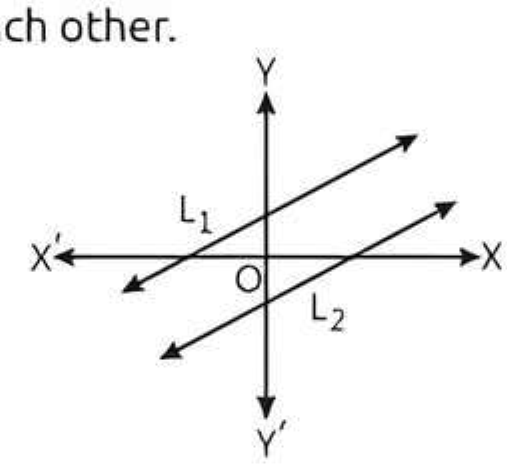


Infinitely many solutions

Condition for Infinitely Many Solutions:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

  2. **Inconsistent System of Linear Equations:** A system of two linear equations in two unknown variables is said to be inconsistent, if it has no solution at all, *i.e.*, lines are parallel to each other.



No solution

Condition for No Solution:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

- **Methods of Solving Linear Equations in Two Variables:** There are two methods to represent and solve these equations:
  1. **Graphical Method:** A pair of linear equations is represented by two lines in a graph, which are shown in above graphs.
  2. **Algebraic Method:** It includes two methods to find the solution of a pair of linear equations:
     (i) **Substitution Method:** If we have a pair of linear equations in two variables x and y, then we have to follow certain steps to solve them using substitution method:
     Step 1: Find the value of one variable, say y in terms of the other variable, *i.e.*, x from either equation whichever is convenient.
     Step 2: Substitute this value of y in the other equation and reduce it to an equation in one variable, *i.e.*, in terms of x, which can be solved.
     Step 3: Substitute the value of x (or y) obtained in step 2 in the first equation and obtain the value of other variable.
     Note: Sometimes while solving step 2, you can get statements with no variable. If this statement is true, you can conclude that the given pair of linear equations has infinitely many solutions. If the statement is false, then the pair of linear equations is inconsistent.
     (ii) **Elimination Method:** Study the steps to be followed in the elimination method:

Step 1: Multiply both the equations by some suitable non-zero constant to make the coefficient of one variable numerically equal.

Step 2: Add or subtract one equation from the other so that one variable gets eliminated.

Step 3: Solve the equation in one variable so obtained to get its value.

Step 4: Substitute the calculated value of variable in the given equations to find the value of the other variable.

Note:

- If in step 2, we obtain a true statement involving no variable, then the pair of linear equations has infinitely many solutions.
- If in step 2, we obtain a false statement involving no variable, then the pair of linear equations has no solution, *i.e.*, it is inconsistent.

Knowledge BOOSTER

1. *Sometimes, a pair of equations in two variables are not linear but can be reduced to linear form by making some suitable substitutions. Here, first we find the solution of new pair of linear equations and then find the solution for the given pair of equations.*
2. *Suppose speed of boat in still water be x km/h and speed of stream be y km/h. Then the speed of boat in downstream is $(x + y)$ km/h and speed of boat in upstream is $(x - y)$ km/h.*

Practice Exercise

Multiple Choice Questions

Q 1. Which of the following method(s) is/are used to find the solution of a pair of linear equations algebraically?

a. Substitution method

b. Elimination method

c. Either substitution or elimination method

d. Both substitution and elimination methods

Q 2. 3 chairs and 1 table cost ₹ 900; whereas 5 chairs and 3 tables cost ₹ 2100. If the cost of 1 chair is ₹ x and the cost of 1 table is ₹ y, then the situation can be represented algebraically as: *[CBSE 2023]*

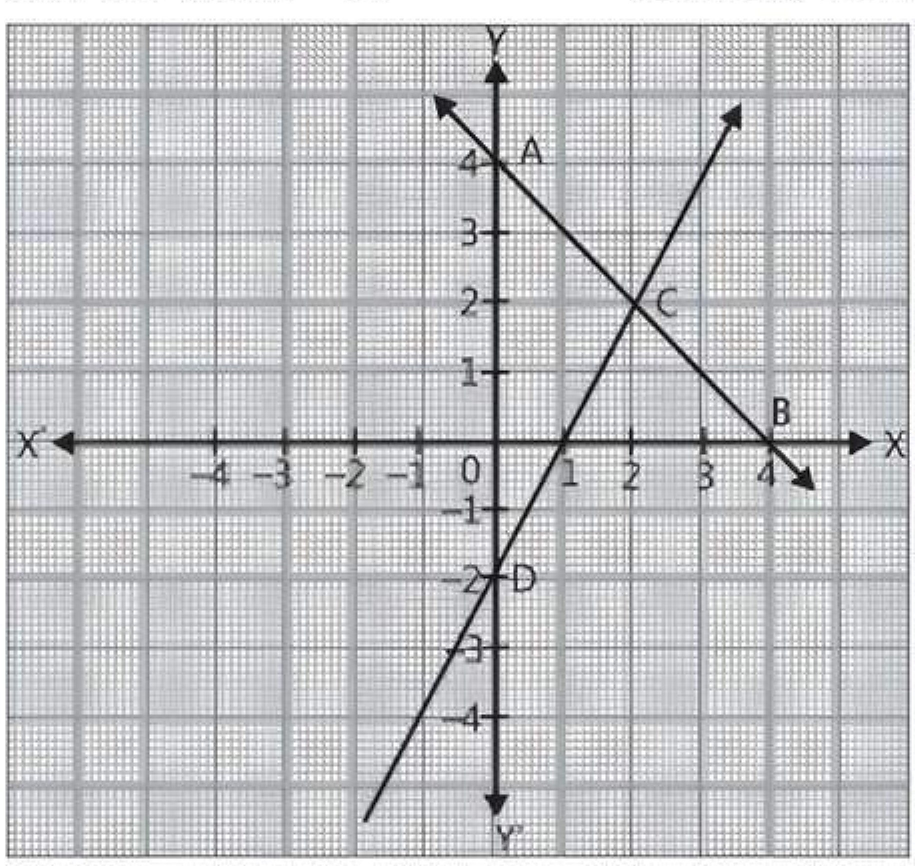
a. $3x + y = 900,\ 3x + 5y = 2100$

b. $x + 3y = 900,\ 3x + 5y = 2100$

c. $3x + y = 900,\ 5x + 3y = 2100$

d. $x + 3y = 900,\ 5x + 3y = 2100$

Q 3. Given below is the graph representing two linear equations by lines AB and CD respectively. What is the area of the triangle formed by these two lines and the line $x = 0$? *[CBSE SQP 2021 Term-I]*



a. 3 sq. units b. 4 sq. units

c. 6 sq. units d. 8 sq. units

Q 4. The point of intersection of the line represented by $3x - y = 3$ and Y-axis is given by: *[CBSE 2023]*

a. (0, −3) b. (0, 3) c. (2, 0) d. (−2, 0)

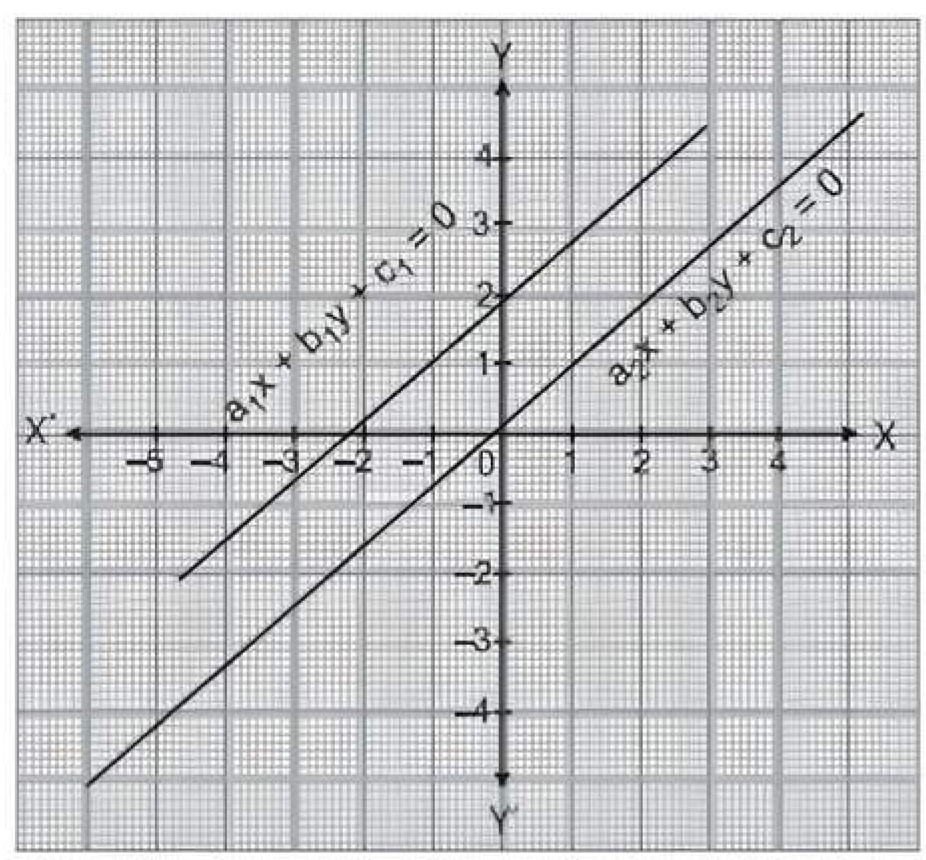
Q 5. The values of x and y satisfying the two equations $32x + 33y = 34$, $33x + 32y = 31$ respectively are: *[CBSE 2021 Term-I]*

a. −1, 2 b. − 1, 4 c. 1, −2 d. −1, −4

Q 6. If $217x + 131y = 913$, $131x + 217y = 827$, then $x + y$ is: *[CBSE SQP 2021 Term-I]*

a. 5 b. 6 c. 7 d. 8

Q 7. The given pair of linear equations is non-intersecting. Which of the following statements is true? *[CBSE SQP 2023-24]*



a. $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

b. $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

c. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} = \frac{c_1}{c_2}$

d. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Q 8. The pair of equations $x = a$ and $y = b$ graphically represents lines which are: *[CBSE 2023, CBSE SQP 2021 Term-I]*

a. parallel b. intersecting at (b, a)

c. coincident d. intersecting at (a, b)

Q 9. The pair of equations $2x - 3y + 4 = 0$ and $2x + y - 6 = 0$ has:

a. a unique solution

b. exactly two solutions

c. infinitely many solutions

d. no solution

Q 10. A pair of equations which has a unique solution $x = 2$ and $y = -3$, is:

a. $x + y = 1$ and $2x - 3y = -5$

b. $2x + 5y = -11$ and $4x + 10y = -22$

c. $2x - y = 1$ and $3x + 2y = 0$

d. $x - 4y - 14 = 0$ and $5x - y + 13 = 0$

Q 11. The number of solutions of $3^{x+y} = 243$ and $243^{x-y} = 3$ is: *[CBSE SQP 2021 Term-I]*

a. 0 b. 1 c. 2 d. infinite

Q 12. The pair of linear equations $2x = 5y + 6$ and $15y = 6x - 18$ represents two lines which are: *[CBSE 2023]*

a. intersecting

b. parallel

c. coincident

d. either intersecting or parallel

Q 13. If the pair of equations $3x - y + 8 = 0$ and $6x - ry + 16 = 0$ represent coincident lines, then the value of r is: *[CBSE 2023]*

a. $-\frac{1}{2}$ b. $\frac{1}{2}$

c. -2 d. 2

Q 14. Two lines are given to be parallel. The equation of one of the lines is $3x - 2y = 5$. The equation of the second line can be: *[CBSE 2021 Term-I]*

a. $9x + 8y = 7$ b. $-12x - 8y = 7$

c. $-12x + 8y = 7$ d. $12x + 8y = 7$

Q 15. The value of k for which the lines $5x + 7y = 3$ and $15x + 21y = k$ coincide is: *[CBSE SQP 2021 Term-I]*

a. 9 b. 5 c. 7 d. 18

Q 16. If the system of equations $2x + 3y = 6$ and $2ax + (a + b)y = 24$ has infinitely many solutions, then:

a. $a = 2b$ b. $b = 2a$

c. $a + 2b = 0$ d. $2a + b = 0$

Q 17. If the pair of equations $x + y = \sqrt{2}$ and $x \sin\theta + y \cos\theta = 1$ has infinitely many solutions, then θ is equal to:

a. $30°$ b. $45°$ c. $60°$ d. $90°$

Q 18. If $5x - 3y = 9$ and $(a - b)x - (a + b - 3)y = a - 4b$ represent coincident lines, then the values of a and b are:

a. $\frac{-11}{10}, \frac{9}{10}$ b. $\frac{-9}{10}, \frac{11}{10}$ c. $\frac{-33}{2}, 6$ d. $-6, \frac{33}{5}$

Q 19. Graphically, the pair of equations $6x - 3y + 10 = 0$ and $2x - y + 9 = 0$ represents two lines which are: *[NCERT EXEMPLAR]*

a. intersecting at exactly one point

b. intersecting at exactly two points

c. coincident

d. parallel

Q 20. The pair of equations $x + 3y + 5 = 0$ and $-3x - 9y + 2 = 0$ has:

a. a unique solution

b. exactly two solutions

c. infinitely many solutions

d. no solution

Q 21. Find the value of k for which the system of equations $x + 3y = 4$ and $3x + ky + 12 = 0$ are inconsistent.

a. $k = 12$ b. $k = -12$

c. $k = 9$ b. $k = -9$

Q 22. If $x = a$ and $y = b$ is the solution of the equations $x - y = 2$ and $x + y = 8$, then the value of ab is:

a. 15 b. 16 c. -15 d. 20

Q 23. The sum of the two digits of a two digit number is 9. The number obtained by interchanging the two digits exceeds the given number by 45. Find the number.

a. 72 b. 27 c. 54 d. 45

Q 24. The length of a room exceeds its breadth by 3 m. If the length is increased by 3 m and the breadth is decreased by 2 m, the area remains the same. Find the length and the breadth of the room.

a. 12 m, 9 m b. 17 m, 14 m

c. 15 m, 12 m d. 14 m, 11 m

Q 25. Aruna has only ₹ 1 and ₹ 2 coins with her. If the total number of coins that she has 50 and the amount of money with her is ₹ 75, then the number of ₹ 1 and ₹ 2 coins are, respectively: *[NCERT EXEMPLAR]*

a. 35 and 15 b. 35 and 20

c. 15 and 35 d. 25 and 25

Q 26. A fraction becomes 4 when 1 is added to both the numerator and denominator and it becomes 7 when 1 is subtracted from both the numerator and denominator. The numerator of the given fraction is:

a. 2 b. 3 c. 5 d. 15

Q 27. The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present ages (in year) of the son and the father are respectively:

a. 4 and 24 b. 5 and 30

c. 6 and 36 d. 3 and 24

Q 28. The sum of numerator and denominator of a proper fraction is 13 and their difference is 3. Find the fraction.

a. 5/8 b. 7/5

c. 3/5 d. 4/7

Q 29. A boat travels with a speed of 15 km/h in still water. In a river flowing at 5 km/h, the boat travels some distance downstream and then returns. The ratio of average speed to the speed in still water is:
a. 8 : 3 b. 3 : 8 c. 8 : 9 d. 9 : 8

Assertion & Reason Type Questions

Directions (Q. Nos. 30-34): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)
c. Assertion (A) is true but Reason (R) is false
d. Assertion (A) is false but Reason (R) is true

Q 30. Assertion (A): $x = 2, y = 1$ is a solution of pair of equations $3x - 2y = 4$ and $2x + y = 5$.
Reason (R): A pair of values (x, y) satisfying each one of the equations in a given system of two simultaneous linear equations in x and y is called a solution of the system of equations.

Q 31. Assertion (A): The system of equations $x + 2y - 5 = 0$ and $2x - 6y + 9 = 0$ has infinitely many solutions.
Reason (R): The system of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has infinitely many solutions when $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

Q 32. Assertion (A): Graphically, the pair of linear equations $2x - y - 5 = 0$ and $x - y - 3 = 0$ represent intersecting lines.
Reason (R): The linear equations $2x - y - 5 = 0$ and $x - y - 3 = 0$ meet the Y-axis at $(0, 3)$ and $(0, -5)$.

Q 33. Assertion (A): A two digit number, where ten's digit is greater than one's digit, is obtained by either multiplying sum of the digits by 8 and adding 1 or by multiplying the difference of digits by 13 and adding 2. The number is 41.
Reason (R): The linear equations used are $7x - 2y + 1 = 0$ and $12x + 23y + 2 = 0$.

Q 34. Assertion (A): If the system of equations $2x + 3y = 7$ and $2ax + (a + b)y = 28$ has infinitely many solutions, then $2a - b = 0$.
Reason (R): The system of equations $3x - 5y = 9$ and $6x - 10y = 8$ has a unique solution.

Fill in the Blanks Type Questions

Q 35. Graphically, the pair of equations $x = a$ and $y = b$ represents line are .... *[NCERT EXEMPLAR]*

Q 36. If a pair of linear equations is consistent, then the lines representing them are either .... or ....

Q 37. Dependent pair of linear equations is always ....

Q 38. The method in which we make the coefficient of one of the unknown variable same is .... method.

Q 39. The pair of equations $2x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ has .... solution. *[NCERT EXEMPLAR]*

Q 40. If $x = a$ and $y = b$ is the solution of the equations $x - y = 2$ and $x + y = 4$, then the values of a and b are respectively ....

Q 41. The value of c for which the pair of equations $cx + y = 3$ and $6x + 2y = 6$ will have infinitely many solutions, is ....

True/False Type Questions

Q 42. If two lines are parallel, then there is no solution.

Q 43. When two lines are coincident, then system of equations is called consistent with infinitely many solutions.

Q 44. The pair of linear equations $2px + 5y = 7$ and $6x - 5y = 11$ has a unique solution if $p \neq -3$.

Q 45. In substitution method, the value of one variable in terms of another variable and substitute it in other equation to solve the linear equations.

Q 46. The solution of the equations $3x - 5y = 4$ and $9x = 2y + 7$ is $x = -\frac{9}{13}$ and $y = \frac{5}{13}$.

Solutions

1. (d) Both substitution and elimination methods are used to solve the pair of linear equations algebraically.

2. (c) Given that, the cost of 1 chair is ₹ x and that of 1 table is ₹ y.
According to the question,
$$3x + y = 900$$
and
$$5x + 3y = 2100$$

3. (c) From the graph,
Coordinates of point $A \equiv (0, 4)$
Coordinates of point $D \equiv (0, -2)$
and coordinates of point $C \equiv (2, 2)$

$\therefore$ Area of $\Delta ACD = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times AD \times \left(\begin{array}{c}\text{perpendicular distance of} \\ \text{point C from } Y\text{-axis}\end{array}\right)$

$= \frac{1}{2} \times (4 + 2) \times 2 = 6$ sq. units

4. (a) Given equation of line is,

$$3x - y = 3 \quad ...(1)$$

and equation of Y-axis is,

$$x = 0 \quad ...(2)$$

Put the value of x in eq. (1), we get

$$3 \times 0 - y = 3 \Rightarrow y = -3$$

So, point of intersection is $(0, -3)$.

5. (a) Given equations are

$$32x + 33y = 34 \quad ...(1)$$

and $$33x + 32y = 31 \quad ...(2)$$

Adding eqs. (1) and (2), we get

$$65x + 65y = 65$$

$$\Rightarrow \quad x + y = 1 \quad \text{(dividing by 65)}$$

or $$y = 1 - x \quad ...(3)$$

Put the value of y in eq. (1), we get

$$32x + 33(1 - x) = 34$$

$$32x + 33 - 33x = 34$$

$$\Rightarrow \quad -x = 1 \Rightarrow x = -1$$

Put the value of x in eq. (3), we get

$$y = 1 - (-1) = 2$$

Hence, $x = -1, y = 2$

6. (a) Given pair of equations is,

$$217x + 131y = 913 \quad ...(1)$$

and $$131x + 217y = 827 \quad ...(2)$$

Adding both equations, we get

$$348x + 348y = 1740$$

$$\Rightarrow \quad x + y = \frac{1740}{348} = 5$$

7. (b) Given that the pair of linear equations is non-intersecting *i.e.* lines are parallel. So, the pair of linear equations is inconsistent, if it has no solution at all.

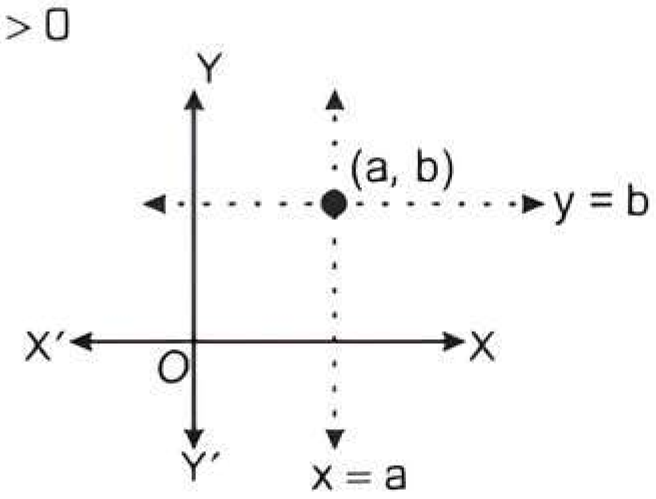
∴ Condition for no solution is:

$$\frac{a_1}{a_2} = \frac{b_1}{b_1} \neq \frac{c_1}{c_2}$$

8. (d) By graphically in every condition, if $a, b > 0$; $a, b < 0$; $a > 0, b < 0$; $a < 0, b > 0$ but $a = b \neq 0$.

The pair of equations $x = a$ and $y = b$ graphically represents lines which are intersecting at (a, b).

If $a, b > 0$



Similarly, in all cases, two lines intersect at (a, b).

9. (a) The given pair of equations is

$$2x - 3y + 4 = 0 \text{ and } 2x + y - 6 = 0$$

Subtracting second from first equation, we get

$$(2x - 3y + 4) - (2x + y - 6) = 0$$

$$\Rightarrow \quad -4y + 10 = 0 \Rightarrow y = \frac{10}{4}$$

$$\Rightarrow \quad y = \frac{5}{2}$$

Put $y = 5$ in equation $2x - 3y = 4$, we get

$$2x - 3\left(\frac{5}{2}\right) + 4 = 0 \Rightarrow 2x - \frac{15}{2} + 4 = 0$$

$$\Rightarrow \quad 2x - \frac{7}{2} = 0 \Rightarrow x = \frac{7}{4}$$

Hence, given pair of equations has a unique solution.

10. (b) Since, $x = 2, y = -3$ is a unique solution of any pair of equation, then these values must satisfy the pair of equations.

When $x = 2, y = -3$, then

(a) $x + y = 2 - 3 = -1 \neq 1$

Thus, it has no unique solution.

(b) $$2x + 5y = 2(2) + 5(-3)$$
$$= 4 - 15$$
$$= -11, \text{ which is true.}$$

and $$4x + 10y = 4(2) + 10(-3)$$
$$= 8 - 30$$
$$= -22, \text{ which is true.}$$

Thus, it has unique solution.

(c) $$2x - y = 2(2) - (-3)$$
$$= 4 + 3 = 7 \neq 1$$

Thus, it has no unique solution.

(d) $$x - 4y - 14 = 2 - 4(-3) - 14$$
$$= 2 + 12 - 14$$
$$= 0, \text{ which is true.}$$

and $$5x - y + 13 = 5(2) - (-3) + 13$$
$$= 10 + 3 + 13 = 26 \neq 0$$

Thus, it has no unique solution.

11. (b) Given that, $3^{x+y} = 243 = 3^5$

$$\Rightarrow \quad x + y = 5 \quad ...(1)$$

and $$243^{x-y} = 3$$

$$\Rightarrow \quad (3)^{5(x-y)} = 3^1$$

$$\Rightarrow \quad 5(x - y) = 1$$

$$\Rightarrow \quad x - y = \frac{1}{5} \quad ...(2)$$

Comparing eqs. (1) and (2) with $a_1x + b_1y + c = 0$ and $a_2x + b_2y + c_2 = 0$ respectively, we get

$$a_1 = 1, b_1 = 1, c_1 = -5$$

and $$a_2 = 1, b_2 = -1, c_2 = -1/5$$

Now, $$\frac{a_1}{a_2} = \frac{1}{1}, \frac{b_1}{b_2} = \frac{1}{-1} \text{ and } \frac{c_1}{c_2} = \frac{-5}{-1/5} = \frac{25}{1}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

∴ The pair of equations represent intersecting lines. So, it has a unique solution *i.e.* only one solution.

12. (c) Given pair of linear equations is,

$$2x = 5y + 6 \Rightarrow 2x - 5y - 6 = 0 \quad ...(1)$$

and $$15y = 6x - 18 \Rightarrow 6x - 15y - 18 = 0 \quad ...(2)$$

Comparing eqs. (1) and (2) with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ respectively, we get

$$a_1 = 2, \quad b_1 = -5, \quad c_1 = -6$$

and $$a_2 = 6, \quad b_2 = -15, \quad c_2 = -18$$

Now, $\frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-5}{-15} = \frac{1}{3}$ and $\frac{c_1}{c_2} = \frac{-6}{-18} = \frac{1}{3}$

$\because \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_1}$

$\therefore$ The pair of equations represent coincident lines.

13. (d) Given pair of linear equations is,

$$3x - y + 8 = 0 \quad ...(1)$$

and $$6x - ry + 16 = 0 \quad ...(2)$$

Comparing eqs. (1) and (2) with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$a_1 = 3, b_1 = -1, c_1 = 8$ and $a_2 = 6, b_2 = -r, c_2 = 16$

Since, the given pair of linear equations represent coincident lines *i.e.* it has infinitely many solutions.

$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \quad \frac{3}{6} = \frac{-1}{-r} = \frac{8}{16}$

Taking $\frac{3}{6} = \frac{-1}{-r}$, we get

$$r = \frac{6}{3} = 2$$

14. (c)

TR!CK

The condition for pair of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ parallel is, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

Given equation is $3x - 2y = 5$.

(a) Consider second equation is $9x + 8y = 7$.

Here, $\frac{a_1}{a_2} = \frac{3}{9} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-2}{8} = \frac{-1}{4}$

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, which is not satisfied the condition of parallel lines.

(b) Consider second equation is $-12x - 8y = 7$.

Here $\frac{a_1}{a_2} = \frac{3}{-12} = \frac{-1}{4}, \frac{b_1}{b_2} = \frac{-2}{-8} = \frac{1}{4}$

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, which is not satisfied the condition of parallel lines.

(c) Consider second equation is $-12x + 8y = 7$

Here, $\frac{a_1}{a_2} = \frac{3}{-12} = \frac{-1}{4}, \frac{b_1}{b_2} = \frac{-2}{8} = -\frac{1}{4}$

and $\frac{c_1}{c_2} = \frac{-5}{-7} = \frac{5}{7}$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, which is satisfied the condition of parallel lines.

15. (a) Given pair of lines is

$$5x + 7y = 3 \quad \Rightarrow \quad 5x + 7y - 3 = 0$$

and $15x + 21y = k \quad \Rightarrow \quad 15x + 21y - k = 0$

Compare these lines with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ respectively, we get

$$a_1 = 5, \quad b_1 = 7, \quad c_1 = -3$$

and $$a_2 = 15, \quad b_2 = 21, \quad c_2 = -k$$

Condition for coincide lines is:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad \Rightarrow \quad \frac{5}{15} = \frac{7}{21} = \frac{-3}{-k}$$

$$\Rightarrow \quad \frac{1}{3} = \frac{1}{3} = \frac{3}{k} \quad \Rightarrow \quad \frac{1}{3} = \frac{3}{k} \Rightarrow k = 9$$

16. (b) The given pair of linear equations is

$$2x + 3y - 6 = 0 \text{ and } 2ax + (a+b)y - 24 = 0$$

Here, $a_1 = 2, b_1 = 3, c_1 = -6, a_2 = 2a, b_2 = a + b, c_2 = -24$

The given pair of linear equations has infinitely many solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

$$\Rightarrow \quad \frac{2}{2a} = \frac{3}{a+b} = \frac{-6}{-24} \quad \Rightarrow \quad \frac{1}{a} = \frac{3}{a+b} = \frac{1}{4}$$

$$\Rightarrow \quad \frac{1}{a} = \frac{3}{a+b} \quad \Rightarrow a + b = 3a \Rightarrow b = 2a$$

17. (b)

TR!CK

The condition for pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has infinitely many solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

Here, $\frac{\sin\theta}{1} = \frac{\cos\theta}{1} = \frac{1}{\sqrt{2}}$

$$\Rightarrow \quad \sin\theta = \frac{1}{\sqrt{2}}, \cos\theta = \frac{1}{\sqrt{2}}$$

$$\therefore \quad \theta = 45^\circ$$

18. (c) The pair of equations is $5x - 3y - 9 = 0$

and $(a - b)x - (a + b - 3)y - (a - 4b) = 0$

Here, $a_1 = 5, b_1 = -3, c_1 = -9, a_2 = a - b$

$$b_2 = -(a + b - 3), c_2 = -(a - 4b)$$

For the equations to be coincident, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{5}{a-b}=\frac{-3}{-(a+b-3)}=\frac{-9}{-(a-4b)}$

$\Rightarrow \quad \frac{5}{a-b}=\frac{3}{a+b-3}$ and $\frac{3}{a+b-3}=\frac{9}{a-4b}$

$\Rightarrow \quad 5a+5b-15=3a-3b$

and $\quad 3a-12b=9a+9b-27$

$\Rightarrow \quad 2a+8b-15=0 \quad$ ...(1)

and $\quad 6a+21b-27=0$

or $\quad 2a+7b-9=0 \quad$ ...(2)

Subtracting eq. (2) from eq. (1), we get

$b-6=0 \Rightarrow b=6$

Substituting this value of b in eq. (1), we get

$2a+8\times6-15=0$

$\Rightarrow \quad 2a+48-15=0$

$\Rightarrow \quad 2a=-33 \Rightarrow a=\frac{-33}{2}$

19. (d) Given pair of equations is,

$6x-3y+10=0 \quad$ ...(1)

and $\quad 2x-y+9=0 \quad$ ...(2)

Comparing eqs. (1) and (2) with $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ respectively, we get

$a_1=6$, $b_1=-3$, $c_1=10$ and $a_2=2$, $b_2=-1$, $c_2=9$.

Now, $\frac{a_1}{a_2}=\frac{6}{2}=\frac{3}{1};\ \frac{b_1}{b_2}=\frac{-3}{-1}=\frac{3}{1};\ \frac{c_1}{c_2}=\frac{10}{9}$

$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

∴ The pair of equations represents two lines which are parallel.

20. (d) The given equations are $x+3y+5=0$ and $-3x-9y+2=0$.

Here, $a_1=1, b_1=3, c_1=5, a_2=-3, b_2=-9, c_2=2$

Now, $\frac{a_1}{a_2}=\frac{-1}{3}, \frac{b_1}{b_2}=\frac{3}{-9}=\frac{-1}{3}$ and $\frac{c_1}{c_2}=\frac{5}{2}$

$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

∴ The given pair of equations has no solution.

21. (c) The given equations are

$x+3y-4=0$ and $3x+ky+12=0$

Here, $a_1=1, b_1=3, c_1=-4, a_2=3, b_2=k, c_2=12$

For inconsistent equations,

$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2} \Rightarrow \frac{1}{3}=\frac{3}{k}\neq\frac{-4}{12}$

$\Rightarrow k=9$ and $k\neq-9$

22. (a) Since, $x=a$ and $y=b$ is the solution of given equations $x-y=2$ and $x+y=8$.

∴ $\quad a-b=2 \quad$ ...(1)

and $\quad a+b=8 \quad$ (put $x=a$ and $y=b$) ...(2)

Adding eqs. (1) and (2), we get

$2a=10 \Rightarrow a=5$

Put $a=5$ in eq. (1), we get

$5-b=2 \Rightarrow b=3$

∴ $\quad ab=5\times3=15$

23. (b) Let the digits at unit's and ten's place of the number be x and y respectively.

∴ The number is $10y+x$.

Number obtained by interchanging the digits

$=10x+y$

According to the question, $x+y=9 \quad$ ...(1)

and $\quad 10x+y-(10y+x)=45$

$\Rightarrow \quad 9x-9y=45$

$\Rightarrow \quad x-y=5 \quad$ ...(2)

On adding eqs. (1) and (2), we get

$2x=14 \Rightarrow x=7$

Substituting the value of x in eq. (1), we get

$y=9-7=2$

Hence, the number is $10(2)+7=27$.

24. (c) Let length of the room be x m and breadth be y m.

Then, $\quad x=y+3 \Rightarrow x-y=3 \quad$ ...(1)

and $\quad (x+3)(y-2)=xy$

$\Rightarrow \quad -2x+3y=6 \quad$ ...(2)

Multiplying eq. (1) by 2, we get

$2x-2y=6 \quad$ ...(3)

On adding eqs. (2) and (3), we get $y=12$

Substituting $y=12$ in eq. (1), we get

$x=12+3=15$

Hence, the length and breadth of the room are 15 m and 12 m respectively.

25. (d) Let the number of ₹ 1 coins $=x$

and $\quad$ number of ₹ 2 coins $=y$

According to the question, $x+y=50 \quad$ ...(1)

and $\quad (x\times1)+(y\times2)=75$

$\Rightarrow \quad x+2y=75 \quad$ ...(2)

Subtracting eq. (1) from eq. (2), we get

$y=75-50=25$

From eq. (1), $\quad x+25=50 \Rightarrow x=25$

Hence, number of ₹ 1 and ₹ 2 coins are respectively 25 and 25 respectively.

26. (d) Let the fraction be $\frac{x}{y}$.

Then according to the given conditions,

$\frac{x+1}{y+1}=4$ and $\frac{x-1}{y-1}=7$

$x+1=4y+4 \Rightarrow x-4y=3 \quad$ ...(1)

and $\quad x-1=7y-7 \Rightarrow x-7y=-6 \quad$ ...(2)

On solving eqs. (1) and (2), we get

$x=15, \ y=3$

Hence, numerator is 15.

27. (c) Let the present ages of father and son be x years and y years respectively.

According to the given conditions,

$$x = 6y \quad \text{...(1)}$$

and $\quad x + 4 = 4(y + 4) \Rightarrow x - 4y = 12 \quad \text{...(2)}$

Substitute $x = 6y$ in eq. (2), we get

$$6y - 4y = 12 \Rightarrow 2y = 12$$

$\Rightarrow \quad y = 6$

Put $y = 6$ in eq. (1), we get

$$x = 6 \times 6 = 36$$

Hence, present age of father is 36 years and that of son is 6 years.

28. (a) Let the denominator and numerator of the fraction be x and y respectively.

According to the question,

$$x + y = 13 \quad \text{...(1)}$$

and $\quad x - y = 3 \quad \text{...(2)}$

or $\quad x - y = -3 \quad \text{...(3)}$

On adding eqs. (1) and (2), we get

$$2x = 16$$

$\Rightarrow \quad x = 8$

From eq. (1), we get

$$y = 13 - 8 = 5$$

$\therefore$ The fraction is 5/8.

Again adding eqs. (1) and (3), we get

$$2x = 10 \quad \Rightarrow \quad x = 5$$

Put $x = 5$ in eq. (1), we get

$$5 + y = 13 \quad \Rightarrow \quad y = 8$$

$\therefore$ The fraction is $\frac{8}{5}$.

29. (c)

TiP

The term upstream refers to the direction towards the source of the river, i.e., against the direction of flow. Likewise, the term downstream refers the direction towards the flow of the river.

Let $\quad$ distance $= d$

Time taken by upstream $= \frac{d}{15 - 5} = \frac{d}{10}$

$$\left(\because \text{time} = \frac{\text{distance}}{\text{speed}}\right)$$

Time taken by downstream $= \frac{d}{15 + 5} = \frac{d}{20}$

Hence, average speed

$$= \frac{2d}{\frac{d}{10} + \frac{d}{20}} = \frac{2d \times 20}{3d} = \frac{40}{3} \text{ km/h}$$

$\therefore \quad$ Ratio $= \frac{40}{3} : 15 = 40 : 45 = 8 : 9$

30. (a) **Assertion (A):** The given system of equations is

$$3x - 2y = 4 \quad \text{...(1)}$$

and $\quad 2x + y = 5 \quad \text{...(2)}$

Putting $x = 2$ and $y = 1$ in eq. (1), we get

$$\text{LHS} = 3 \times 2 - 2 \times 1 = 4 = \text{RHS}$$

Putting $x = 2$ and $y = 1$ in eq. (2), we get

$$\text{LHS} = 2 \times 2 + 1 \times 1 = 5 = \text{RHS}$$

Thus, $x = 2$ and $y = 1$ satisfy both the equations of the given system.

Hence, $x = 2$, $y = 1$ is a solution of the given pair of equations.

So, Assertion (A) is true.

Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

31. (d) **Assertion (A):** The given system of equations is

$$x + 2y - 5 = 0 \quad \text{and} \quad 2x - 6y + 9 = 0$$

Here. $\quad \frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{-6} = \frac{-1}{3}, \frac{c_1}{c_2} = \frac{-5}{9}$

Since, $\quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\therefore$ The given system of equations has a unique solution.

So, Assertion (A) is false.

Reason (R): It is true to say that the system of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has infinitely many solutions, when $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

Hence, Assertion (A) is false but Reason (R) is true.

32. (c) **Assertion (A):** The given system of linear equations are

$$2x - y - 5 = 0 \quad \text{...(1)}$$

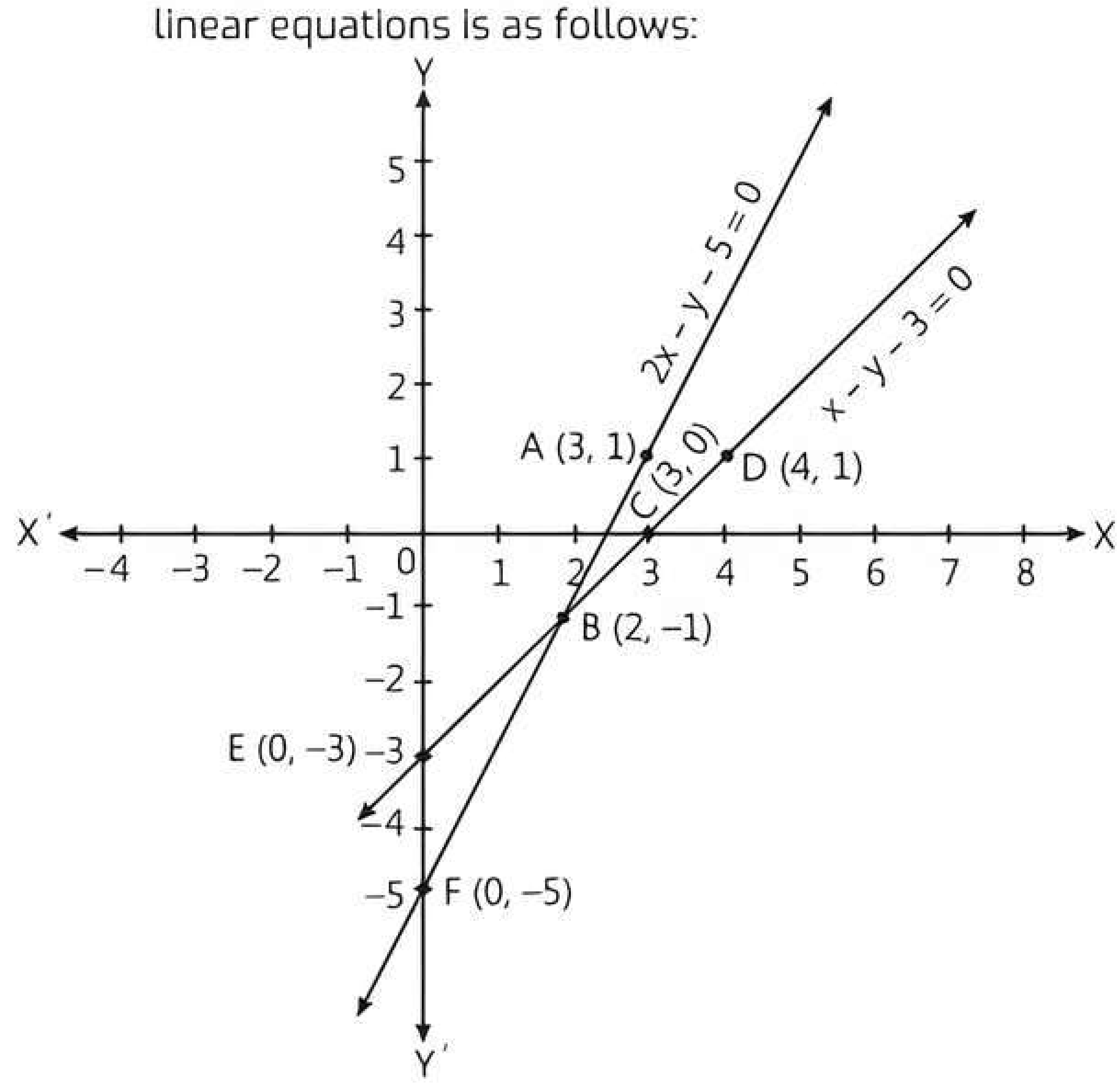
and $\quad x - y - 3 = 0 \quad \text{...(2)}$

Table for eqs. (1) and (2) are given below:

x	3	2
y	1	–1

x	3	4
y	0	1

The graphical representation of the given pair of linear equations is as follows:



In the graph, we observe that the two lines intersect at the point B(2, –1).

So, $x = 2$, $y = -1$ is the required solution of the given pair of linear equations.

So, Assertion (A) is true.

Reason (R): We observe from the graph that the lines (1) and (2) meet the Y-axis at the points $E(0, -3)$ and $F(0, -5)$ respectively.

So, Reason (R) is false.

Hence, Assertion (A) is true but Reason (R) is false.

33. (c) **Assertion (A):** Let the digit at unit's place be x and the digit at ten's place be y. Then number $= 10y + x$

According to the question, we have

$$10y + x = 8(x + y) + 1$$

$$\Rightarrow \quad 7x - 2y + 1 = 0 \quad \text{...(1)}$$

and $$10y + x = 13(y - x) + 2$$

$$\Rightarrow \quad 14x - 3y - 2 = 0 \quad \text{...(2)}$$

By using elimination method,

Multiplying eq. (1) by 2 and subtracting from eq. (2), we get

$$(14x - 3y - 2) - 2 \times (7x - 2y + 1) = 0$$

$$\Rightarrow \quad 14x - 3y - 2 - 14x + 4y - 2 = 0$$

$$\Rightarrow \quad y - 4 = 0$$

$$\Rightarrow \quad y = 4$$

Put the value of y in eq. (1), we get

$$7x - 2 \times 4 + 1 = 0$$

$$\Rightarrow \quad 7x = 7$$

$$\Rightarrow \quad x = 1$$

Hence, the number $= 10y + x = 10 \times 4 + 1 = 41$.

So, Assertion (A) is true.

Reason (R): It is false statement.

Hence, Assertion (A) is true but Reason (R) is false.

34. (c) **Assertion (A):** Given system of equations has infinitely many solutions, if

$$\frac{2}{2a} = \frac{3}{a+b} = \frac{-7}{-28}$$

i.e., $$\frac{1}{a} = \frac{3}{a+b} = \frac{1}{4}$$

$$\therefore \quad 3a = a + b \Rightarrow 2a - b = 0$$

So, Assertion (A) is true.

Reason (R): For unique solution, condition is

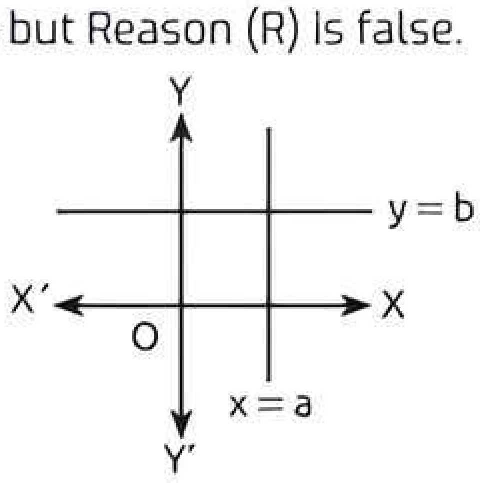
$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

But here $$\frac{3}{6} = \frac{-5}{-10} \Rightarrow \frac{1}{2} = \frac{1}{2}$$

So, Reason (R) is false.

Hence, Assertion (A) is true but Reason (R) is false.

35. Graphically, the pair of equations $x = a$ and $y = b$ represents lines are intersecting.



36. intersecting, coincident

37. consistent.

38. elimination

39. Given system of equations are

$$2x + 2y + 5 = 0 \quad \text{...(1)}$$

and $$-3x - 6y + 1 = 0 \quad \text{...(2)}$$

Multiplying eq. (1) by 3 and then adding it with eq. (2), we get

$$6x - 3x + 15 + 1 = 0$$

$$\Rightarrow \quad 3x + 16 = 0$$

$$\Rightarrow \quad x = -\frac{16}{3}$$

Put $x = -\frac{16}{3}$ in eq. (1), we get

$$2\left(\frac{-16}{3}\right) + 2y + 5 = 0$$

$$\Rightarrow \quad 2y = -5 + \frac{32}{3} \Rightarrow y = \frac{17}{6}$$

Hence, it has unique solution.

40. Since, a and b are the solutions of the given system of equations.

$$\therefore \quad a - b = 2 \quad \text{and} \quad a + b = 4$$

Adding both equations, we get

$$2a = 6 \Rightarrow a = 3$$

Now, $3 - b = 2 \Rightarrow b = 1$

Hence, $a = 3, b = 1$

41. Given pair of equations is

$$cx + y - 3 = 0 \quad \text{and} \quad 6x + 2y - 6 = 0$$

Here, $a_1 = c, \quad b_1 = 1, \quad c_1 = -3$

and $a_2 = 6, \quad b_2 = 2, \quad c_2 = -6$

Now, $$\frac{a_1}{a_2} = \frac{c}{6}, \frac{b_1}{b_2} = \frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{-3}{-6} = \frac{1}{2}$$

TR!CK

The condition of infinitely many solutions is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\therefore \quad \frac{c}{6} = \frac{1}{2} = \frac{1}{2} \Rightarrow \frac{c}{6} = \frac{1}{2}$$

$$\Rightarrow \quad c = 3$$

42. True

43. True

44. The condition for unique solution is

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\therefore \quad \frac{2p}{6} \neq \frac{5}{-5} \Rightarrow p \neq -\frac{6}{2}$$

$$\Rightarrow \quad p \neq -3$$

So, given statement is true.

45. True

46. Given equations are

$$3x - 5y = 4 \quad \text{...(1)}$$

and $$9x - 2y = 7 \quad \text{...(2)}$$

Multiplying eq. (1) by 3 and subtracting resultant equation from eq. (2), we get

$$(9x - 2y) - (9x - 15y) = 7 - 12$$

$\Rightarrow \quad 13y = -5 \Rightarrow y = -\frac{5}{13}$

Put $y = -\frac{5}{13}$ in eq. (1), we get

$$3x - 5\left(\frac{-5}{13}\right) = 4$$

$\Rightarrow \quad 39x + 25 = 4 \times 13$

$\Rightarrow \quad 39x = 52 - 25$

$\Rightarrow \quad x = \frac{27}{39} = \frac{9}{13}$

Hence, solution is $x = \frac{9}{13}$ and $y = -\frac{5}{13}$.

So, the given statement is false.

Case Study Based Questions

Case Study 1

A book store shopkeeper gives books on rent for reading. He has variety of books in his store related to fiction, stories and quizzes, etc. He takes a fixed charge for the first two days and an additional charge for subsequent day. Amrita paid ₹ 22 for a book and kept for 6 days; while Radhika paid ₹ 16 for keeping the book for 4 days.

Assume that the fixed charge be ₹ x and additional charge (per day) be ₹ y.

Based on the above information, solve the following questions:

Q 1. The situation of amount paid by Radhika, is algebraically represented by:

a. $x - 4y = 16$
b. $x + 4y = 16$
c. $x - 2y = 16$
d. $x + 2y = 16$

Q 2. The situation of amount paid by Amrita, is algebraically represented by:

a. $x - 2y = 11$ b. $x - 2y = 22$
c. $x + 4y = 22$ d. $x - 4y = 11$

Q 3. What are the fixed charges for a book?

a. ₹ 9 b. ₹ 10
c. ₹ 13 d. ₹ 15

Q 4. What are the additional charges for each subsequent day for a book?

a. ₹ 6 b. ₹ 5 c. ₹ 4 d. ₹ 3

Q 5. What is the total amount paid by both, if both of them have kept the book for 2 more days?

a. ₹ 35 b. ₹ 52 c. ₹ 50 d. ₹ 58

Solutions

1. Given, Radhika paid ₹ 16 for keeping the book for 4 days. Therefore, algebraically represented by her is
$$x + 2y = 16$$
So, option (d) is correct.

2. Given, Amrita paid ₹ 22 for keeping the book for 6 days. Therefore, algebraically represented by her is
$$x + 4y = 22$$
So, option (c) is correct.

3. We have,
$$x + 2y = 16 \quad ...(1)$$
and
$$x + 4y = 22 \quad ...(2)$$
Multiplying eq. (1) by 2 and subtracting eq. (2) from eq. (1), we get
$$2x - x = 32 - 22 \Rightarrow x = 10$$
Hence, fixed charges for a book is ₹ 10.
So, option (b) is correct.

4. We have, $x = 10$
Put $x = 10$ in the equation $x + 2y = 16$, we get
$$10 + 2y = 16$$
$\Rightarrow \quad 2y = 6 \Rightarrow y = 3$
Hence, additional charges for each subsequent day for a book is ₹3.
So, option (d) is correct.

5. If both of them kept 2 days more, then the total amount paid by both of them
$$= 22 + 16 + 2\,(2y) = 38 + 4 \times 3$$
$$= 38 + 12 = ₹\,50$$
So, option (c) is correct.

Case Study 2

It is common that Governments revise travel fares from time to based on various factors such as inflation (a general increase in prices and fall in the purchasing value of money) on different types of vehicles like auto rickshaws, taxis, radio cab, etc. The auto charges in a city comprise of a fixed charge together with the charge for the distance covered. Study the following situations:

Name of the city	Distance travelled (km)	Amount (₹)
City A	10	75
	15	110
City B	8	91
	14	145

Situation 1: In city A, for a journey of 10 km, the charge paid is ₹ 75 and for a journey of 15 km, the charge paid is ₹ 110.

Situation 2: In city B, for a journey of 8 km, the charge paid is ₹ 91 and for a journey of 14 km, the charge paid is ₹ 145.

Based on the above information, solve the following questions:

Refer to city A

Q 1. If the fixed charges of auto rickshaw is ₹ x and the running charges is ₹ y per km, the pair of linear equations representing the situation is:

a. $x + 10y = 110, x + 15y = 75$

b. $x + 10y = 75, x + 15y = 110$

c. $10x + y = 110, 15x + y = 75$

d. $10x + y = 75, 15x + y = 110$

Q 2. A person travels a distance of 50 km. The amount he has to pay is:

a. ₹ 155 b. ₹ 255 c. ₹ 355 d. ₹ 455

Refer to city B

Q 3. If the fixed charges of taxi is ₹ x and the running charges is ₹ y per km, the pair of linear equations representing the situation is:

a. $x + 8y = 91, x + 14y = 145$

b. $x + 8y = 145, x + 14y = 91$

c. $8x + y = 91, 14x + y = 145$

d. $8x + y = 145, 14x + y = 91$

Q 4. What will a person have to pay for travelling a distance of 30 km?

a. ₹ 185 b. ₹ 289

c. ₹ 275 d. ₹ 305

Q 5. Which one of the city fixed a minimum charges for auto rickshaw?

a. City A b. City B

c. Can't determined d. None of these

Solutions

1. Given that, the fixed charges of auto rickshaw is ₹ x and the running charges is ₹ y per km.

From City A:

Fixed charges + 10 × running charges = Total amount

$\therefore \quad x + 10y = 75 \quad ...(1)$

and fixed charges + 15 × running charges = Total amount

$\therefore \quad x + 15y = 110 \quad ...(2)$

Therefore, eqs. (1) and (2) represent the pair of linear equations.

So, option (b) is correct.

COMMON ERR(!)R

Some students frame this word problem into wrong equation, lead to get incorrect answer.

2. Subtracting eq. (1) from eq. (2), we get

$$\begin{array}{r} x + 15y = 110 \\ \underline{-x - 10y = -75} \\ 5y = 35 \end{array}$$

$$\Rightarrow \quad y = \frac{35}{5} = 7$$

Put the value of y in eq. (1), we get

$$x + 10 \times 7 = 75$$

$$\Rightarrow \quad x = 75 - 70 = 5$$

$\therefore$ The amount he has to pay for travelled a distance of 50 km

$$= ₹(x + 50y)$$

$$= ₹(5 + 50 \times 7) = ₹(5 + 350)$$

$$= ₹\,355$$

So, option (c) is correct.

3. Given that, the fixed charges of taxi is ₹ x and the running charges is ₹ y per km.

From City B:

Fixed charges + 8 × running charges = Total amount

$\therefore \quad x + 8y = 91 \quad ...(3)$

and fixed charges + 14 × running charges = Total amount

$\therefore \quad x + 14y = 145 \quad ...(4)$

Therefore, eqs. (3) and (4) represent the pair of linear equations.

So, option (a) is correct.

COMMON ERR(!)R

Some students frame this word problem into wrong equation, lead to get incorrect answer.

4. Subtracting eq. (3) from eq. (4), we get

$$\begin{array}{r} x + 14y = 145 \\ \underline{-x - 8y = -91} \\ 6y = 54 \end{array}$$

$$\Rightarrow \quad y = \frac{54}{6} = 9$$

Put the value of y in eq. (3), we get

$$x + 8 \times 9 = 91$$

$$\Rightarrow \quad x = 91 - 72 = 19$$

$\therefore$ The amount he has to pay for travelling a distance of 30 km $= ₹\,(x + 30y)$

$$= ₹\,(19 + 30 \times 9)$$

$$= ₹\,(19 + 270)$$

$$= ₹\,289$$

So, option (b) is correct.

5. City A fixed a minimum charges for auto rickshaw (*i.e.* ₹ 5).

So, option (a) is correct.

Case Study 3

Two schools 'P' and 'Q' decided to award prizes to their students for two games of Hockey ₹ x per student and Cricket ₹ y per student. School 'P' decided to award a total of ₹ 9500 for the two games to 5 and 4 students respectively; while school 'Q' decided to award ₹ 7370 for the two games to 4 and 3 students respectively.

Based on the above information, solve the following questions: **[CBSE 2023]**

Q 1. Represent the following information algebraically (in terms of x and y).

Q 2. What is the prize amount for hockey?

Or

Prize amount on which game is more and by how much?

Q 3. What will be the total prize amount if there are 2 students each from two games?

Solutions

1. Given that, two schools 'P' and 'Q' decided to award prizes to their students for two games of Hockey ₹x per student and Cricket ₹ y per student.

First Condition: School 'P' decided to award a total of ₹ 9500 for the two games to 5 and 4 students respectively.

i.e., $5x + 4y = 9500$ ...(1)

Second Condition: School 'Q' decided to award ₹7370 for the two games to 4 and 3 students respectively.

I.e., $4x + 3y = 7370$ ...(2)

2. Multiplying eq. (1) by 3 and eq. (2) by 4, we get

$15x + 12y = 28500$ ...(3)

$16x + 12y = 29480$ ...(4)

Subtracting eq. (3) from eq. (4), we get

$(16x + 12y) - (15x + 12y) = 29480 - 28500$

$\Rightarrow \quad x = 980$

$\therefore$ Prize amount for Hockey = ₹ 980 per student

Or

Multiplying eq. (1) by 4 and eq. (2) by 5, we get

$20x + 16y = 38000$ ...(1)

and $20x + 15y = 36850$ ...(1)

Subtracting eq. (4) from eq. (3), we get

$(20x + 16y) - (20x + 15y) = 38000 - 36850$

$\Rightarrow \quad y = 1150$

$\therefore$ Prize amount for Cricket = ₹1150 per student

Put the value of y in eq. (1), we get

$5x + 4 \times 1150 = 9500$

$\Rightarrow \quad 5x = 9500 - 4600 = 4900$

$\Rightarrow \quad x = \frac{4900}{5} = 980$

$\therefore$ Prize amount for Hockey = ₹ 780 per student

$\Rightarrow$ Prize amount per student of Cricket is greater by ₹ (1150 − 980) = ₹ 170.

3. Total prize amount if there are 2 students each from 2 games = $2(x + y)$

$= 2(980 + 1150)$

$= 2 \times 2130 =$ ₹ 4260

Case Study 4

A coaching institute of Mathematics conducts classes in two batches I and II and fees for rich and poor children are different. In batch I, there are 20 poor and 5 rich children, whereas in batch II, there are 5 poor and 25 rich children. The total monthly collection of fees from batch I is ₹ 9000 and from batch II is ₹ 26000. Assume that each poor child pays ₹x per month an each rich child pays ₹ y per month.

Based on the above information, solve the following questions: **[CBSE 2023]**

Q 1. Represent the information given above in terms of x and y.

Q 2. Find the monthly fee paid by a poor child.

Or

Find the difference in the monthly fee paid by a poor child and a rich child.

Q 3. If there are 10 poor and 20 rich children in batch II, what is the total monthly collection of fees from batch II?

Solutions

1. Given that, each poor child pays ₹ x per month and each rich child pays ₹ y per month.

First Condition: In batch I, there are 20 poor and 5 rich children. The total monthly collection of fees from batch I is ₹ 9000.

i.e., $20x + 5y = 9000$ ...(1)

Second Condition: In batch II, there are 5 poor and 25 rich children. The total monthly collection of fees from batch II is ₹26000.

i.e., $5x + 25y = 26000$ ...(2)

2. Multiplying eq. (1) by 5 and subtract it from eq. (2), we get

$(5x+25y)-5(20x+5y)=26000-5\times 9000$

$\Rightarrow \quad 5x+25y-100x-25y=26000-45000$

$\Rightarrow \quad -95x=-19000$

$\Rightarrow \quad x=\dfrac{-19000}{-95}=200$

Hence, monthly fee paid by a poor child is ₹200.

Or

Multiplying eq. (2) by 4 and subtract it from eq. (1), we get

$(20x+5y)-4(5x+25y)=9000-4\times 26000$

$\Rightarrow \quad 20x+5y-20x-100y=9000-104000$

$\Rightarrow \quad -95y=-95000$

$\Rightarrow \quad y=\dfrac{-95000}{-95}=1000$

∴ Monthly fee paid by a rich child is ₹ 1000.

Put the value of y in eq. (1), we get

$20x+5\times 1000=9000$

$\Rightarrow \quad 20x=9000-5000=4000$

$\Rightarrow \quad x=\dfrac{4000}{20}=200$

∴ Monthly fee paid by a poor child is ₹ 200.

∴ The difference in the monthly fee paid by a poor child and a rich child = ₹ (1000 − 200) = ₹ 800.

3. Given that, there are 10 poor and 20 rich children in batch II = $10x+20y$.

∴ Total monthly collection of fees from batch II

$=10\times 200+20\times 1000$

$=2000+20000$

$=₹22000$

Case Study 5

Gagan went to a fare. He ate several rural delicacies such as jalebis, chaat etc. He also wanted to play the ring game in which a ring is thrown on the items displayed on the table and the balloon shooting game.

The cost of three balloon shooting games exceeds the cost of four ring games by ₹ 4. Also, the total cost of three balloon shooting games and four ring games is ₹ 20.

Based on the above information, solve the following questions:

Q 1. Taking the cost of one ring game to be ₹ x and that of one balloon game as ₹ y, find the pair of linear equations describing the given statement.

Q 2. Find the cost of one ring game and one balloon game.

Solutions

1. Given, the cost of one ring game = ₹ x and cost of one balloon game = ₹ y.

According to the question,

$3y=4x+4$ or $4x-3y=-4$ ...(1)

and $4x+3y=20$ ...(2)

2. Solving the equations $4x-3y=-4$ and $4x+3y=20$ by the method of substitution.

From eq. (1), $4x=3y-4$ ...(3)

Substituting the value of $4x$ in eq. (2),

$(3y-4)+3y=20$

$\Rightarrow \quad 6y-4=20$

$\Rightarrow \quad 6y=24$

$\Rightarrow \quad y=4$

Substituting $y=4$ in eq. (3),

$4x=3\times 4-4=8 \Rightarrow x=2$

Therefore, cost of one ring game = ₹ 2 and cost of one balloon game = ₹ 4.

Very Short Answer Type Questions

Q 1. Find the solution of the equations $2x+y-6=0$ and $4x-2y-4=0$.

Q 2. How many solutions does the pair of equations $y=0$ and $y=-5$ have?

Q 3. If $47x+31y=18$ and $31x+47y=60$, then find the value of $x+y$.

Q 4. If $49x+51y=499$, $51x+49y=501$, then find the values of x and y. *[CBSE SQP 2022-23]*

Q 5. Find the conditions to be satisfied by coefficients for which the following pair of equations $ax+by+c=0$, $dx+ey+f=0$ represent coincident lines.

Q 6. Given the pair of equations $ax+(a-1)y=1$ and $(a+1)x-ay=1$. For which one of the following values of a, there is no common solution of the given pair of equations?

Q 7. Find the values of k for which the system of equations $x+ky=0, 2x-y=0$ has unique solution?

Q 8. If $ax+by=a^2-b^2$ and $bx+ay=0$, find the value of $(x+y)$.

Q 9. If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then find the value of k. *[NCERT EXEMPLAR; V. Imp.]*

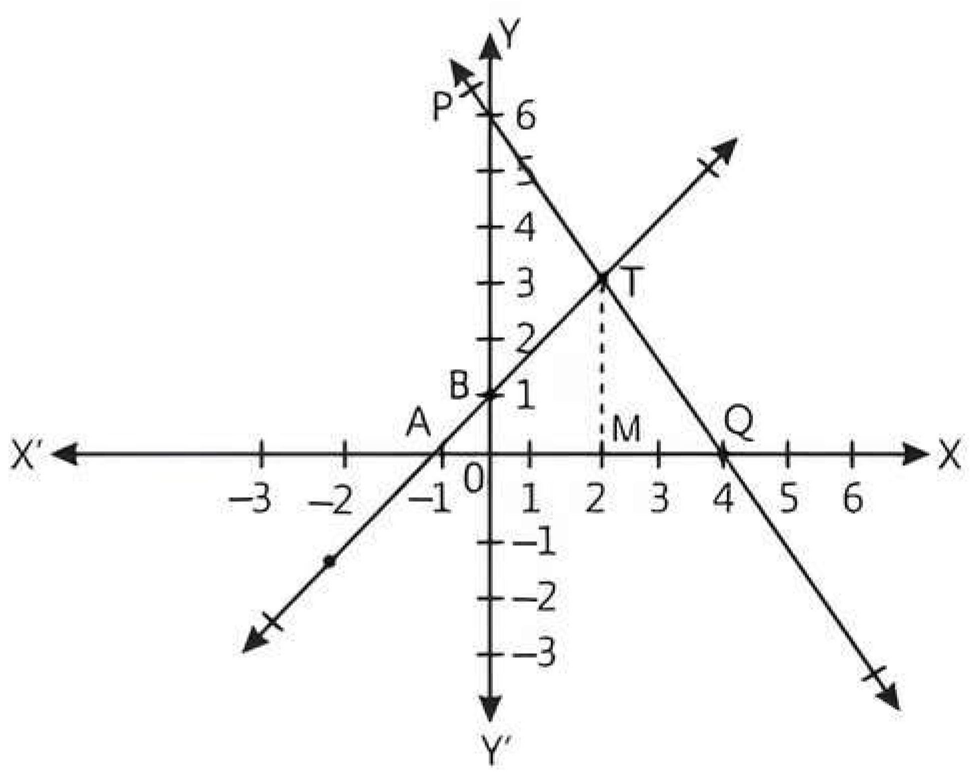
Q 10. Find the value of c for which the pair of equations $cx - y = 2$ and $6x - 2y = 3$ will have infinitely many solutions. *[NCERT EXEMPLAR]*

Q 11. Find the larger of two complementary angles exceeds the smaller by 16 degrees. Find them.

Q 12. Find the value of k for which the system of linear equations $x + 2y = 3$ and $5x + ky + 7 = 0$ is inconsistent. *[CBSE 2020]*

Short Answer Type-I Questions

Q 1. A student Ajay, was given a task to solve the pair of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$ graphically. To draw the graph, he find three solutions of each of the given equations. After plotting the points on graph paper, he draw the lines AB and PQ, representing the equations as shown in the given graph.



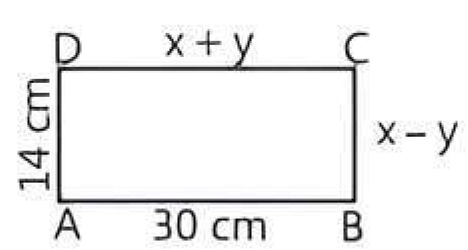
Based on the above graph, solve the following questions:

(i) Identify equation of lines AB and PQ. Also find the solution of the equation from the graph.

(ii) Determine the area of the triangle formed by these lines and the X-axis.

Q 2. If $217x + 131y = 913$ and $131x + 217y = 827$, then solve the equations for the values of x and y. *[CBSE 2023]*

Q 3. In figure, ABCD is a rectangle. Find the values of x and y. *[CBSE 2018]*



Q 4. Find whether the lines representing the following pair of linear equations intersect at a point or parallel or coincident:

$2x - 3y + 6 = 0$ and $4x - 5y + 2 = 0$ *[CBSE 2016]*

Q 5. Given the linear equation $3x + 4y = 9$. Write another linear equation in these two variables such that the geometrical representation of the pair so formed is:

(i) intersecting lines

(ii) coincident lines

(iii) parallel lines *[CBSE 2015]*

Q 6. Find the value(s) of k for which the pair of equations $kx + 2y = 3$ and $3x + 6y = 10$ has a unique solution. *[CBSE 2019]*

Q 7. For what value of k, does the system of linear equations $2x + 3y = 7$ and $(k - 1)x + (k + 2)y = 3k$ have an infinite number of solutions? *[CBSE 2019]*

Q 8. Find the value of k for which the following pair of linear equations have infinitely many solutions,

$2x + 3y = 7, (k + 1)x + (2k - 1)y = 4k + 1$. *[CBSE 2019]*

Q 9. Find the value of k for which the system of equations $3x + 5y = 0, kx + 10y = 0$ has no solution. *[CBSE 2017]*

Q 10. The sum of two numbers is 85. If the larger number is 5 more than four times of the smaller number, find the numbers.

Q 11. I am three times old as my son. Five years later, I shall be two and a half times as old as my son. How old am I and how old is my son?

Q 12. The total expenditure per month of a household consists of a fixed rent of the house and mess charges depending upon the number of people sharing the house. The total monthly expenditure is ₹ 3900 for 2 people and ₹ 7500 for 5 people. Find the rent of the house and the mess charges per head per month.

Q 13. If $x + 1$ is a factor of $2x^3 + ax^2 + 2bx + 1$, then find the values of a and b, given that $2a - 3b = 4$. *[NCERT EXEMPLAR]*

Short Answer Type-II Questions

Q 1. If $2x + y = 23$ and $4x - y = 19$, find the values of $(5y - 2x)$ and $\left(\frac{y}{x} - 2\right)$. *[CBSE 2019]*

Q 2. Solve:

$$\frac{x}{a} + \frac{y}{b} = a + b; \frac{x}{a^2} + \frac{y}{b^2} = 2, a, b \neq 0$$ *[CBSE 2017]*

Q 3. If the system of linear equations $2x + 3y = 7$ and $2ax + (a + b)y = 28$ have infinite number of solutions, then find the values of a and b. *[CBSE 2023]*

Q 4. Find the two numbers whose sum is 75 and difference is 15.

Q 5. A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When student A takes food for 25 days she has to pay ₹ 4500 as hostel charges whereas student B, who takes food for 30 days, pays ₹ 5200 as hostel charges. Find the fixed charges and the cost of food per day. [CBSE 2019]

Q 6. There are some students in two examination halls A and B. To make the number of students equal in each hall, 10 students are sent from A to B. But, if 20 students are sent from B to A, the number of students in A becomes double the number of students in B. Find the number of students in both the halls. [NCERT EXEMPLAR]

Q 7. A fraction becomes $\frac{1}{3}$ when 2 is subtracted from the numerator and it becomes $\frac{1}{2}$ when 1 is subtracted from the denominator. Find the fraction. [CBSE 2019]

Q 8. The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there? [CBSE SQP 2023-24]

Q 9. The monthly incomes of A and B are in the ratio of 8 : 7 and their expenditures are in the ratio of 19 : 16. If each saves ₹ 5000 per month, find the monthly income and expenditures of each.

Q 10. A father's age is three times the sum of the ages of his two children. After 5 years, his age will be two times the sum of their ages. Find the present age of father. [CBSE 2019]

Q 11. The sum of two numbers is 8 and the sum of their reciprocals is $\frac{8}{15}$. Find the numbers.

Q 12. A person can row a boat at the rate of 5km/h in still water. He takes thrice as much time in going 40 km upstream as in going 40 km downstream. Find the speed of the stream. [NCERT EXEMPLAR; CBSE 2016]

Q 13. A train covered a certain distance at a uniform speed. If the train would have been 6 km/h faster, it would have taken 4 hours less than the scheduled time. And, if the train were slower by 6 km/hr it would have taken 6 hours more than the scheduled time. Find the length of the journey.

[CBSE SQP 2022-23]

Q 14. Anuj had some chocolates and he divided them into two lots A and B. He sold the first lot at the rate of ₹ 2 for 3 chocolates and the second lot at the rate of ₹ 1 per chocolates, and got a total of ₹ 400. If he had sold the first lot at the rate of ₹ 1 per chocolate, and the second lot at the rate of ₹4 for 5 chocolates, his total collection would have been ₹ 460. Find the total number of chocolate he had. [CBSE SQP 2022-23]

Q 15. Solve: $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2;\ \frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$

[CBSE SQP 2023-24]

Long Answer Type Questions

Q 1. Solve the following pair of linear equations graphically: $6x-y+4=0$ and $2x-5y=8$.
Shade the region bounded by the lines and Y-axis. [CBSE 2016]

Q 2. Draw the graphs of the equations $x-y+1=0$ and $3x+2y-12=0$. Determine the coordinates of the vertices of the triangle formed by these lines and the Y-axis and shade the triangular region. Also calculate the area bounded by these lines and Y-axis. [NCERT EXERCISE; CBSE 2017]

Q 3. For which values of a and b will the following pair of linear equations have an infinite number of solutions?

$$2x+3y=7$$
$$(a-b)x+(a+b)y=3a+b-2$$

Q 4. A and B are friends and their ages differ by 2 yr. A's father D is twice as old as A and B is twice as old as his sister C. The age of D and C differ by 40 yr. Find the ages of A and B.

Q 5. A man sold a chair and a table together for ₹ 1520 thereby making a profit of 25% on the chair and 10% on table. By selling them together for ₹ 1535, he would have made a profit of 10% on the chair and 25% on the table. Find the cost price of each.

Q 6. A person invested some amount at the rate of 12% simple interest and some other amount at the rate of 10% simple interest. He received yearly interest of ₹ 130. But, if he had interchanged the amounts invested, he would have received ₹4 more as interest. How much amount did he invest at different rates?

Q 7. Places A and B are 160 km apart on a highway. One car starts from A and another from B at the same time. If they travel in the same direction, they meet in 8 hours. But if they travel towards each other, they meet in 2 hours. Find the speed of each car.

Solutions

Very Short Answer Type Questions

1. Given equations are

$$2x + y - 6 = 0 \quad ...(1)$$

and $$4x - 2y - 4 = 0 \quad ...(2)$$

Multiplying eq. (1) by 2 and adding eqs. (1) and (2), we get

$$4x - 12 + (4x - 4) = 0$$

$\Rightarrow$ $8x - 16 = 0 \Rightarrow x = 2$

Put $x = 2$ in eq. (1), we get

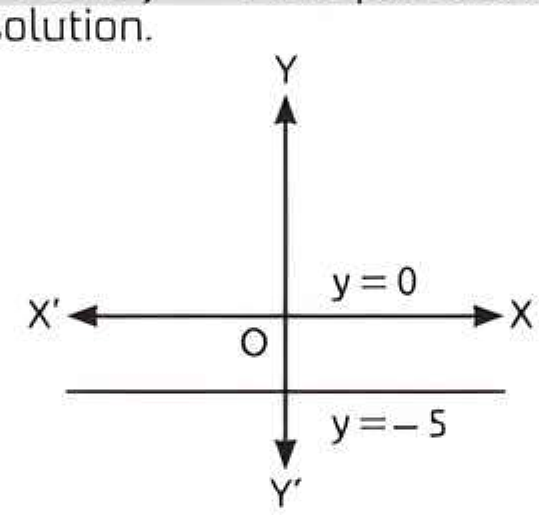
$$2(2) + y - 6 = 0$$

$\Rightarrow$ $4 + y - 6 = 0$

$\Rightarrow$ $y = 2$

Hence, $x = 2$ and $y = 2$

2. Since, $y = 0$ and $y = -5$ are parallel lines, hence they have no solution.



TR!CK

Parallel lines never intersect each other.

3. Given,

$$47x + 31y = 18 \quad ...(1)$$

and $$31x + 47y = 60 \quad ...(2)$$

Adding eqs. (1) and (2), we get

$$78x + 78y = 78$$

$\Rightarrow$ $x + y = 1$ (divide both sides by 78)

COMMON ERR(!)R

Students do error in simplifying these type of equations. They solve these equations by elimination method.

4. Given equations are

$$49x + 51y = 499 \quad ...(1)$$

and $$51x + 49y = 501 \quad ...(2)$$

Adding eqs. (1) and (2), we get

$$100x + 100y = 1000$$

$\Rightarrow$ $x + y = 10 \quad ...(3)$

Subtracting eq. (1) from eq. (2), we get

$$2x - 2y = 2$$

$\Rightarrow$ $x - y = 1 \quad ...(4)$

Adding eqs. (3) and (4), we get

$$2x = 11$$

$\Rightarrow$ $x = \frac{11}{2}$

Put $x = \frac{11}{2}$ in eq. (3), we get

$$\frac{11}{2} + y = 10$$

$\Rightarrow$ $y = 10 - \frac{11}{2} \Rightarrow y = \frac{9}{2}$

5. The given pair of equations is

$$ax + by + c = 0 \text{ and } dx + ey + f = 0$$

For coincident lines,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{a}{d} = \frac{b}{e} = \frac{c}{f}$$

$\Rightarrow$ $ae = bd$ and $bf = ce$

6. The condition for there is no common point, is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{a}{a+1} = \frac{a-1}{-a} \neq 1$$

$\Rightarrow$ $-a^2 = a^2 - 1 \Rightarrow 2a^2 = 1$

$\Rightarrow$ $a^2 = \frac{1}{2} \Rightarrow a = \pm\frac{1}{\sqrt{2}}$

7. Given, $x + ky = 0$, $2x - y = 0$

Here, $a_1 = 1, b_1 = k, c_1 = 0,$

and $a_2 = 2, b_2 = -1, c_2 = 0$

$\because$ System of equations has unique solution.

$\therefore$ $\frac{a_1}{b_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{1}{2} \neq \frac{k}{-1} \Rightarrow k \neq -\frac{1}{2}$

8. Given equations are

$$ax + by = a^2 - b^2 \quad ...(1)$$

and $$bx + ay = 0 \quad ...(2)$$

On adding both equations, we get

$$(ax + by) + (bx + ay) = a^2 - b^2$$

$\Rightarrow$ $(a + b)x + (a + b)y = a^2 - b^2$

$\Rightarrow$ $(a + b)(x + y) = (a + b)(a - b)$

Hence, $x + y = a - b$.

COMMON ERR(!)R

Students do error in simplifying these type of equations.

9. Since, the given lines are parallel, so it has no solution.

TR!CK

Condition for no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\therefore$ $\frac{3}{2} = \frac{2k}{5} \neq \frac{-2}{1}$

$\Rightarrow$ $4k = 15$

Hence, $k = \frac{15}{4}$

10. Given equations are,

$$cx - y - 2 = 0 \quad ...(1)$$

and $$6x - 2y - 3 = 0 \quad ...(2)$$

Here, $a_1 = c, b_1 = -1, c_1 = -2$

$a_2 = 6, b_2 = -2, c_2 = -3$

Condition for infinitely many solutions is

$$\frac{a_1}{c_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{c}{6} = \frac{-1}{-2} = \frac{-2}{-3}$$

$$\Rightarrow \quad \frac{c}{b} = \frac{1}{2} \text{ and } \frac{c}{6} = \frac{2}{3}$$

$$\Rightarrow \quad c = \frac{6}{2} = 3 \text{ and } c = \frac{6 \times 2}{3} = 4$$

Here, $c = 3$ and 4, which are not possible.

Since, these values are not satisfying the given condition, so no value of c will exist.

COMMON ERR(!)R

Sometimes students do not crosschecking the values of c with the given pair of equations. Finally get the wrong answer.

11. Let the two angles be x and y where $x > y$.

According to the question,

$$x + y = 90^\circ \quad ...(1)$$

and
$$x - y = 16^\circ \quad ...(2)$$

On adding eqs. (1) and (2), we get

$$2x = 106^\circ \Rightarrow x = 53^\circ$$

Substituting the value of x in eq. (1), we get

$$y = 90^\circ - 53^\circ = 37^\circ$$

12. Given, $x + 2y = 3$ and $5x + ky + 7 = 0$

Here, $a_1 = 1, b_1 = 2, c_1 = -3$

and $a_2 = 5, b_2 = k, c_2 = 7$

Condition of inconsistency is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{1}{5} = \frac{2}{k} \Rightarrow k = 10$$

Short Answer Type-I Questions

1. (i) For $x - y + 1 = 0$

Let $x = 0 \Rightarrow 0 - y + 1 = 0 \Rightarrow y = 1$

Let $y = 0 \Rightarrow x - 0 + 1 = 0 \Rightarrow x = -1$

x	0	−1
y	1	0

So, line AB passes through the points B (0, 1) and A (−1, 0).

So, line AB represents the equation

$$x - y + 1 = 0$$

For $3x + 2y - 12 = 0$

Let $x = 0 \Rightarrow 3 \times 0 + 2y - 12 = 0 \Rightarrow y = \frac{12}{2} = 6$

Let $y = 0 \Rightarrow 3x + 2 \times 0 - 12 = 0 \Rightarrow x = \frac{12}{3} = 4$

x	0	4
y	6	0

So, line PQ passes through the points P (0, 6) and Q (4, 0).

So, line PQ represents the equation

$$3x + 2y - 12 = 0$$

From the graph given in question, we observe that the two lines AB and PQ intersect at the point T (2, 3). So, $x = 2$ and $y = 3$ is the required solution for the pair of linear equations.

(ii) The coordinates of the vertices of the triangle formed by these lines and the X-axis are (−1, 0), (2, 3) and (4, 0).

$\therefore$ Area of Δ ATQ $= \frac{1}{2} \times$ Base $\times$ Height

$$= \frac{1}{2} \times AQ \times TM$$

$$= \frac{1}{2} \times (1+4) \times 3$$

$$= \frac{15}{2} = 7.5 \text{ sq. units}$$

2. Given equations are

$$217x + 131y = 913 \quad ...(1)$$

$$131x + 217y = 827 \quad ...(2)$$

Adding eqs. (1) and (2), we get

$$348x + 348y = 1740$$

$$\Rightarrow \quad x + y = 5 \quad ...(3)$$

Subtracting eq. (2) from eq. (1), we get

$$86x - 86y = 86$$

$$\Rightarrow \quad x - y = 1 \quad ...(4)$$

Adding eqs. (3) and (4), we get

$$(x + y) + (x - y) = 5 + 1 \Rightarrow 2x = 6$$

$$\Rightarrow \quad x = 3.$$

Put the value of x in eq. (3), we get

$$3 + y = 5 \Rightarrow y = 5 - 3 = 2$$

Hence, $x = 3$ and $y = 2$.

3. Since, ABCD is a rectangle.

TR!CK

Opposite sides of a rectangle are equal.

So, $AB = DC$

$$30 = x + y \quad ...(1)$$

and $AD = BC$

$$14 = x - y \quad ...(2)$$

On adding eqs. (1) and (2), we get

$$(x + y) + (x - y) = 30 + 14$$

$$\Rightarrow \quad 2x = 44$$

$$\Rightarrow \quad x = \frac{44}{2} \Rightarrow x = 22$$

On putting the value of 'x' in eq. (1), we get

$$30 = 22 + y$$

$$\Rightarrow \quad y = 30 - 22 = 8$$

Hence, $x = 22$ and $y = 8$.

4. Given equations are,

$$2x - 3y + 6 = 0 \quad ...(1)$$

and
$$4x - 5y + 2 = 0 \quad ...(2)$$

Here, $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{-5} = \frac{3}{5}$

Since, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, given pair of linear equations intersect at one point.

5. Given equation is $3x + 4y = 9$.

Another linear equation must be in such a way that

(i) For intersecting lines,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Hence, another equation is $4x + 2y = 8$.

(ii) For coincident lines,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So, we multiply given equation by any constant.

$6x + 8y = 18$ (multiply by 2)

Hence, another equation is $6x + 8y = 18$.

(iii) For parallel lines,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, another equation is $3x + 4y = 8$.

6. Given equations are,

$kx + 2y = 3$

and $3x + 6y = 10$

Here, $a_1 = k, b_1 = 2, c_1 = -3$

and $a_2 = 3, b_2 = 6, c_2 = -10$

For unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{k}{3} \neq \frac{2}{6} \Rightarrow k \neq 1$$

Hence, the given system of equations will have a unique solution for all real values of k other than 1.

7. Given system of linear equations are

$2x + 3y - 7 = 0$

and $(k-1)x + (k+2)y - 3k = 0$

Here $a_1 = 2, b_1 = 3, c_1 = -7$

and $a_2 = k-1, b_2 = k+2, c_2 = -3k$

Condition for infinitely many solutions is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\therefore \quad \frac{2}{k-1} = \frac{3}{k+2} = \frac{-7}{-3k}$$

Consider Ist and IInd terms,

$$\frac{2}{k-1} = \frac{3}{k+2} \Rightarrow 2k + 4 = 3k - 3$$

$\Rightarrow 3k - 2k = 4 + 3 \Rightarrow k = 7$

Hence, given system of equations gives the infinite solutions, when $k = 7$.

8. Given pair of linear equations are

$2x + 3y = 7, (k+1)x + (2k-1)y = 4k + 1$

Here, $a_1 = 2, b_1 = 3, c_1 = -7$

and $a_2 = (k+1), b_2 = (2k-1), c_2 = -(4k+1)$

Condition for infinitely many solutions is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\therefore \quad \frac{2}{k+1} = \frac{3}{2k-1} = \frac{-7}{-(4k+1)}$$

Consider Ist and IInd terms,

$$\frac{2}{k+1} = \frac{3}{2k-1}$$

$\Rightarrow 4k - 2 = 3k + 3 \Rightarrow 4k - 3k = 3 + 2 \Rightarrow k = 5$

Consider IInd and IIIrd terms,

$$\frac{3}{2k-1} = \frac{-7}{-(4k+1)} \Rightarrow 12k + 3 = 14k - 7$$

$\Rightarrow 14k - 12k = 3 + 7 \Rightarrow 2k = 10 \Rightarrow k = 5$

Hence, $k = 5$

9. Given equations are

$3x + 5y = 0$ ...(1)

and $kx + 10y = 0$ ...(2)

Here, $a_1 = 3, b_1 = 5, c_1 = 0$

and $a_2 = k, b_2 = 10, c_2 = 0$

For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \frac{3}{k} = \frac{5}{10} \Rightarrow k = \frac{30}{5} = 6$$

Hence, $k = 6$.

10. Let the two numbers be x and y such that $x > y$.

Given, sum of two numbers = 85

and larger number = 4 × smaller number + 5.

$\therefore \quad x + y = 85$ ...(1)

and $x = 4y + 5$ ...(2)

Put $x = 4y + 5$ in eq. (1), we get

$4y + 5 + y = 85$

$\Rightarrow 5y = 80 \Rightarrow y = 16$

Put $y = 16$ in eq. (1), we get

$x + 16 = 85$

$\Rightarrow x = 69$

Hence, numbers are 16 and 69.

11. Let my age be x years and my son's age be y years. Then,

$x = 3y$ ...(1)

Five years later, my age will be $(x + 5)$ years and my son's age will be $(y + 5)$ years.

$$\therefore \quad x + 5 = 2\frac{1}{2}(y+5)$$

$$\Rightarrow \quad x + 5 = \frac{5}{2}(y+5)$$

$\Rightarrow 2x + 10 = 5y + 25$

$\Rightarrow 2x - 5y = 15$

$\Rightarrow 2(3y) - 5y = 15$ [from eq. (1)]

$\Rightarrow 6y - 5y = 15$

$\Rightarrow y = 15$

Put $y = 15$ in eq. (1), we get

$x = 3 \times 15$

$\Rightarrow x = 45$

Hence, my present age and my son's present age are 45 years and 15 years respectively.

12. Let the monthly rent of the house be ₹ x and the mess charges per head per month be ₹ y.

According to the given condition, we have

$x + 2y = 3900$ ...(1)

and $x + 5y = 7500$ ...(2)

Subtracting eq. (1) from eq. (2), we get

$3y = 3600 \Rightarrow y = 1200$

Put $y = 1200$ in eq. (1), we get

$x + 2 \times 1200 = 3900 \Rightarrow x = 3900 - 2400$

$\Rightarrow \quad x = 1500$

Hence, monthly rent = ₹ 1500 and mess charges per head per month = ₹ 1200.

13. Let a factor of $f(x) = 2x^3 + ax^2 + 2bx + 1$ be $(x + 1)$.

$\therefore \quad f(-1) = 0$

$\Rightarrow \quad 2(-1)^3 + a(-1)^2 + 2b(-1) + 1 = 0$

$\Rightarrow \quad -2 + a - 2b + 1 = 0$

$\Rightarrow \quad a - 2b = 1 \quad ...(1)$

and $\quad 2a - 3b = 4$ (given) $\quad ...(2)$

Multiplying eq. (1) by 2 and then subtract it from eq. (2), we get

$(2a - 3b) - (2a - 4b) = 4 - 2$

$\Rightarrow \quad 2a - 3b - 2a + 4b = 2 \Rightarrow b = 2$

Put the value of b in eq. (1), we get

$a - 2(2) = 1 \Rightarrow a - 4 = 1$

$\Rightarrow \quad a = 5$

Therefore, $a = 5$ and $b = 2$.

Short Answer Type-II Questions

1. Given linear equations are

$2x + y = 23 \quad ... (1)$

and $\quad 4x - y = 19 \quad ... (2)$

Adding eqs. (1) and (2),

$$\begin{array}{r} 2x + y = 23 \\ \underline{4x - y = 19} \\ 6x = 42 \end{array}$$

$\therefore \quad x = \frac{42}{6} = 7$

Now, put the value of x in eq. (1), we get

$y = 23 - 2x = 23 - 2 \times 7 = 23 - 14 = 9$

So, $5y - 2x = 5 \times 9 - 2 \times 7 = 45 - 14 = 31$

and $\frac{y}{x} - 2 = \frac{9}{7} - 2 = \frac{9-14}{7} = \frac{-5}{7}$

2. Given equations are

$\frac{x}{a} + \frac{y}{b} = a + b \quad ...(1)$

and $\quad \frac{x}{a^2} + \frac{y}{b^2} = 2 \quad ...(2)$

On multiplying eq. (1) by $\frac{1}{b}$ and subtracting from eq. (2), we get

$$\left(\frac{x}{a^2} + \frac{y}{b^2}\right) - \left(\frac{x}{ab} + \frac{y}{b^2}\right) = 2 - \left(\frac{a+b}{b}\right)$$

$\Rightarrow \quad x\left\{\frac{1}{a^2} - \frac{1}{ab}\right\} = 2 - \frac{a}{b} - 1$

$\Rightarrow \quad \frac{x}{a}\left\{\frac{1}{a} - \frac{1}{b}\right\} = 1 - \frac{a}{b}$

$\Rightarrow \quad \frac{x}{a}\left(\frac{b-a}{ab}\right) = \frac{b-a}{b} \Rightarrow x = \frac{(b-a)}{b} \times \frac{ab}{(b-a)} \times a$

$\therefore \quad x = a^2$

On putting the value of 'x' in eq. (1), we get

$\frac{a^2}{a} + \frac{y}{b} = a + b \quad \Rightarrow \quad a + \frac{y}{b} = a + b$

$\Rightarrow \quad \frac{y}{b} = b \quad \Rightarrow \quad y = b^2$

Hence, $x = a^2$ and $y = b^2$.

3. Given system of linear equations is:

$2x + 3y = 7 \Rightarrow 2x + 3y - 7 = 0 \quad ...(1)$

$2ax + (a + b)y = 28 \Rightarrow 2ax + (a + b)\,y - 28 = 0 \quad ...(2)$

Compare it with $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ respectively, we get

$a_1 = 2, b_1 = 3, c_1 = -7$

and $\quad a_2 = 2a, b_2 = (a + b), c_2 = -28$

Since, the given system of linear equations have infinite number of solutions.

$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \quad \frac{2}{2a} = \frac{3}{a+b} = \frac{-7}{-28}$

Taking Ist and IInd terms, $\frac{2}{2a} = \frac{3}{a+b} \Rightarrow a + b = 3a$

$\Rightarrow \quad b = 2a \quad ...(1)$

Taking Ist and IIIrd terms, $\frac{2}{2a} = \frac{-7}{-28} \Rightarrow \frac{1}{a} = \frac{1}{4} \Rightarrow a = 4$

Put the value of 'a' in eq. (1), we get

$b = 2 \times 4 = 8$

Hence, $\quad a = 4$ and $b = 8$.

4. Let the two numbers be x and y.

According to the question,

$x + y = 75 \quad ...(1)$

and $\quad x - y = \pm 15 \quad ...(2)$

TiP

Take ± sign with resultant quantity of difference of two numbers.

Taking positive sign:

On adding eqs. (1) and (2), we get

and $\quad (x + y) + (x - y) = 75 + 15$

$\Rightarrow \quad 2x = 90$

$\Rightarrow \quad x = 45$

On putting the value of 'x' in eq. (1), we get

$45 + y = 75$

$\Rightarrow \quad y = 75 - 45 = 30$

$\therefore \quad x = 45$ and $y = 30$

Taking negative sign:

On adding eqs. (1) and (2), we get

$(x + y) + (x - y) = 75 - 15$

$\Rightarrow \quad 2x = 60$

$\Rightarrow \quad x = 30$

On putting the value of x in eq. (1), we get

$30 + y = 75$

$\Rightarrow \quad y = 75 - 30 = 45$

$\therefore \quad x = 30$ and $y = 45$

Hence, required numbers are (45, 30) or (30, 45).

5. Let x be the fixed charge of the food and y be the cost of food per day.

According to the given conditions,

$x + 25y = 4500 \quad ...(1)$

and $\quad x + 30y = 5200 \quad ...(2)$

On subtracting eq. (1) from eq. (2), we get

$$\begin{array}{r} x+30y=5200 \\ x+25y=4500 \\ - \quad - \quad - \\ \hline 5y=700 \end{array}$$

$\Rightarrow \quad y=140$

On substituting $y=140$ in eq. (1), we get

$x+25\times140=4500$

$\Rightarrow \quad x=4500-3,500$

$\Rightarrow \quad x=1000$

Hence, fixed charges is ₹ 1000 and cost of food per day is ₹ 140.

COMMON ERR!R

Some students frame this word problem into wrong equation, lead to get incorrect answer. So, adequate practice should be needed on solving such type of problem.

6. Let the number of students in halls A and B be x and y respectively.

According to first condition,

$x-10=y+10$

$\Rightarrow \quad x-y=10+10=20 \quad$ ...(1)

According to second condition,

$x+20=2(y-20)$

$\Rightarrow \quad x+20=2y-40$

$\Rightarrow \quad x-2y=-40-20=-60 \quad$ ...(2)

On subtracting eq. (2) from eq. (1), we get

$$\begin{array}{r} x-y=20 \\ x-2y=-60 \\ - \quad + \quad + \\ \hline y=80 \end{array}$$

On substituting the value of 'y' in eq. (1), we get

$x-80=20$

$\Rightarrow \quad x=20+80=100$

Hence, number of students in hall A and B are 100 and 80 respectively.

COMMON ERR!R

Some candidates frame this word problem into wrong equation, lead to get incorrect answer.

7. Let fraction be $\frac{x}{y}$, where $y\neq0$

According to the given condition,

When 2 is subtracted from numerator then fraction becomes $\frac{1}{3}$.

i.e. $\quad \frac{x-2}{y}=\frac{1}{3} \Rightarrow 3x-6=y \quad$ ...(1)

and when 1 is subtracted from denominator then fraction becomes $\frac{1}{2}$.

i.e. $\quad \frac{x}{y-1}=\frac{1}{2}$

$\Rightarrow \quad 2x=y-1 \quad$ ...(2)

From eq. (1), put the value of y in eq. (2), we get

$2x=(3x-6)-1$

$\Rightarrow \quad 2x=3x-7$

$\Rightarrow \quad 3x-2x=7$

$\Rightarrow \quad x=7$

Now, put $x=7$ in eq. (1), we get

$3\times7-6=y$

or $\quad y=21-6$

or $\quad y=15$

$\therefore$ Required fraction $=\frac{x}{y}=\frac{7}{15}$

COMMON ERR!R

Some students confused between numerator and denominator of a fraction. So, adequate practice is required.

8. Let the digit at ten's place be x and the digit at one's place be y.

So, the required number is $10x+y$.

The number obtained on reversing the digits is $10y+x$.

According to first condition,

$(10x+y)+(10y+x)=66$

$\Rightarrow \quad 11x+11y=66$

$\Rightarrow \quad 11(x+y)=66$

$\Rightarrow \quad x+y=\frac{66}{11}=6 \quad$ ...(1)

According to second condition,

$x-y=2 \quad$ ...(2)

or $\quad x-y=-2 \quad$ ...(3)

TiP

It is not given that which number is greater, so we can take here $x-y=-2$ also.

On adding eqs. (1) and (2), we get

$$\begin{array}{r} x+y=6 \\ x-y=2 \\ \hline 2x=8 \end{array}$$

$\Rightarrow \quad x=\frac{8}{2}=4$

On substituting the value of 'x' in eq. (1), we get

$4+y=6 \Rightarrow y=6-4=2$

So, the number is 42.

Again, on adding eqs. (1) and (3), we get

$$\begin{array}{r} x+y=6 \\ x-y=-2 \\ \hline 2x=4 \end{array}$$

$\Rightarrow \quad x=\frac{4}{2}=2 \quad \Rightarrow \quad x=2$

On substituting the value of 'x' in eq. (1), we get

$2+y=6 \Rightarrow y=6-2=4$

So, the number is 24.

Hence, the required number is 42 or 24.

COMMON ERR!R

Sometimes students take only + sign with resultant quantity of difference of two numbers.

9. Let the monthly incomes of A and B be $8x$ and $7x$ and their expenditures be $19y$ and $16y$ respectively.

TiP

Adequate practice should be needed on solving such type of problem.

According to given conditions,

$$8x - 19y = 5000 \quad ...(1)$$

and $$7x - 16y = 5000 \quad ...(2)$$

On multiplying eq. (1) by 7 and eq. (2) by 8 and then subtracting, we get

$$56x - 133y = 35000$$
$$56x - 128y = 40000$$
$$\underline{- \quad + \quad -}$$
$$-5y = -5000$$

$\Rightarrow$ $$y = \frac{5000}{5} = 1000$$

On substituting the value of 'y' in eq. (1), we get

$$8x - 19 \times 1000 = 5000$$

$\Rightarrow$ $$8x = 5000 + 19000$$

$\Rightarrow$ $$8x = 24000$$

$\Rightarrow$ $$x = \frac{24000}{8} \Rightarrow x = 3000$$

Hence, A's income $= ₹\ 8 \times 3000 = ₹\ 24000$

A's expenditures $= ₹\ 19 \times 1000 = ₹\ 19000$

and B's income $= ₹\ 7 \times 3000 = ₹\ 21000$

B's expenditures $= ₹\ 16 \times 1000 = ₹\ 16000$

10. Let father's age be x and sum of ages of his children be y years.

TiP

Adequate practice should be needed on solving such type of word problem.

After 5 years, father's age will be $(x + 5)$ years and sum of ages of his children will be $(y + 10)$ years.

According to the given conditions,

Father's age $= 3 \times$ Sum of ages of his children

i.e. $$x = 3y \quad ... (1)$$

and after 5 years,

Father's age $= 2 \times$ Sum of ages of his children

i.e. $$x + 5 = 2 \times (y + 10)$$

$\Rightarrow$ $$x + 5 = 2y + 20$$

$\Rightarrow$ $$x - 2y = 15 \quad ...(2)$$

From eq. (1), put the value of x in eq. (2), we get

$$3y - 2y = 15 \quad \Rightarrow \quad y = 15 \text{ years}$$

Now put $y = 15$ in eq. (1), we get

$$x = 3 \times 15 = 45 \text{ years}$$

Hence, present age of father is 45 years.

COMMON ERR(!)R

Some students frame this word problem into wrong equation, lead to get incorrect answer.

11. Let the required numbers be x and y.

According to first condition,

$$x + y = 8 \quad ...(1)$$

According to second condition,

$$\frac{1}{x} + \frac{1}{y} = \frac{8}{15}$$

$\Rightarrow$ $$\frac{y + x}{xy} = \frac{8}{15} \Rightarrow \frac{8}{xy} = \frac{8}{15} \quad (\because x + y = 8)$$

or $$xy = 15$$

Now, $$x - y = \sqrt{(x + y)^2 - 4xy} = \sqrt{8^2 - 4 \times 15}$$
$$= \sqrt{64 - 60} = \sqrt{4} = 2$$

$\therefore$ $$x - y = 2 \quad ...(2)$$

On adding eqs. (1) and (2), we get

$$x + y = 8$$
$$x - y = 2$$
$$2x = 10 \quad \Rightarrow \quad x = \frac{10}{2} = 5$$

On substituting the value of 'x' in eq. (1), we get

$$5 + y = 8 \quad \Rightarrow \quad y = 8 - 5 = 3$$

Hence, the required numbers are 5 and 3.

COMMON ERR(!)R

Students do error in simplifying these type of equations.

12. Let the speed of stream be x km/h.

$\therefore$ Speed of the boat rowing upstream

$$= (5 - x) \text{ km/h}$$

Speed of boat rowing downstream

$$= (5 + x) \text{ km/h}$$

TiP

Downstream means when we row with the flow of water so our speed increases, that's why we add both the speeds. Upstream means when we row against the flow of water so our speed decreases, that's why we subtract speed of current from speed of stream.

According to the question,

$$3 \times \frac{40}{5 + x} = \frac{40}{5 - x} \quad \left(\because \text{time} = \frac{\text{distance}}{\text{speed}}\right)$$

$\Rightarrow$ $$\frac{3}{5 + x} = \frac{1}{5 - x}$$

$\Rightarrow$ $$15 - 3x = 5 + x$$

$\Rightarrow$ $$4x = 10$$

$\Rightarrow$ $$x = \frac{10}{4} = 2.5$$

Hence, speed of the stream is 2.5 km/h.

13. Let the actual speed of the train be x km/h and let the actual time taken by y hours.

Distance covered $= xy$ km $\quad ...(1)$

If the speed is increased by 6 km/h, then time of journey is reduced by 4 hours i.e., when speed is $(x + 6)$ km/h, time or journey is $(y - 4)$ hours.

$\therefore$ Distance covered $= (x + 6)\ (y - 4)$

$\Rightarrow$ $$xy = (x + 6)(y - 4)$$

$\Rightarrow$ $$xy = xy - 4x + 6y - 24$$

$\Rightarrow$ $$-4x + 6y - 24 = 0$$

$\Rightarrow$ $$-2x + 3y - 12 = 0 \quad ...(2)$$

Similarly, $xy = (x - 6)\ (y + 6)$

$\Rightarrow$ $$xy = xy + 6x - 6y - 36$$

$\Rightarrow$ $$6x - 6y - 36 = 0$$

$\Rightarrow$ $$x - y - 6 = 0 \quad ...(3)$$

Multiplying eq. (3) by 2 and adding it with eq. (2), we get

$$2(x-y-6)-2x+3y-12=0$$

$\Rightarrow \quad y-24=0 \Rightarrow y=24$

Put $y=24$ in eq. (3), we get

$$x-24-6=0 \Rightarrow x=30$$

Putting the values of x and y in eq. (1), we obtain Distance $= (30\times 24)$km $= 720$ km.

Hence, the length of the journey is 720 km.

14. Let the number of chocolates in lot A be x and the number of chocolates in lob B be y.

Then total number of chocolates $= x+y$

Price of 1 chocolate = ₹ 2/3, so for x chocolate $=\frac{2}{3}x$

and price of y chocolates at the rate of ₹ 1 per chocolate $= y$.

$\therefore$ By the given condition, $\frac{2}{3}x+y=400$

$\Rightarrow \quad 2x+3y=1200 \quad$ ...(1)

Similarly, $\quad x+\frac{4}{5}y=460$

$\Rightarrow \quad 5x+4y=2300 \quad$ ...(2)

Multiplying eq. (1) by 4 and eq. (2) by 3 and then subtracting eq. (2) and from eq. (1),

$$4(2x+3y-1200)-3(5x+4y-2300)=0$$

$\Rightarrow \quad 8x-15x-4800+6900=0$

$\Rightarrow \quad -7x+2100=0$

$\Rightarrow \quad x=\frac{2100}{7}=300$

Put $x=300$ in eq. (1), we get

$2\times 300+3y=1200 \Rightarrow 3y=1200-600$

$\Rightarrow \quad y=\frac{600}{3}=200$

Thus, $x=300$ and $y=200$

$\therefore x+y=300+200=500$

So, Anuj had 500 chocolates.

15. Given equations are

$$\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2 \quad \text{...(1)}$$

and $\quad \frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1 \quad$ ... (2)

Let $u=\frac{1}{\sqrt{x}}$ and $v=\frac{1}{\sqrt{y}}$, then equations become

$2u+3v=2 \quad$ ...(3)

and $\quad 4u-9v=-1 \quad$ ...(4)

Multiply by 3 in eq. (3) and add with eq. (4), we get

$$(6u+9v)+(4u-9v)=6+(-1)$$

$\Rightarrow \quad 10\,u=5 \Rightarrow u=\frac{5}{10}=\frac{1}{2}$

Put the value of u in eq. (3), we get

$$2\times\frac{1}{2}+3v=2 \Rightarrow 3v=2-1=1 \Rightarrow v=\frac{1}{3}$$

$$u=\frac{1}{\sqrt{x}} \text{ and } v=\frac{1}{\sqrt{y}}$$

$$\frac{1}{2}=\frac{1}{\sqrt{x}} \text{ and } \frac{1}{3}=\frac{1}{\sqrt{y}}$$

$\Rightarrow \quad \sqrt{x}=2$ and $\sqrt{y}=3$

$\Rightarrow \quad x=4$ and $y=9$

Long Answer Type Questions

1.

TiP

An easy way of getting a solution of an equation is to take $x=0$ and get the corresponding value of y. Similarly, we can put $y=0$ and obtain the corresponding value of x.

For $6x-y+4=0$, when $x=0$, ...(1)

$6\times 0-y=-4$

$\Rightarrow \quad y=4$

and when $y=0$, $\quad 6x-0=-4$

$\Rightarrow \quad x=\frac{-4}{6}=-\frac{2}{3}$

x	0	–2/3
y	4	0

For $2x-5y=8$, when $x=0$, ...(2)

$2\times 0-5y=8$

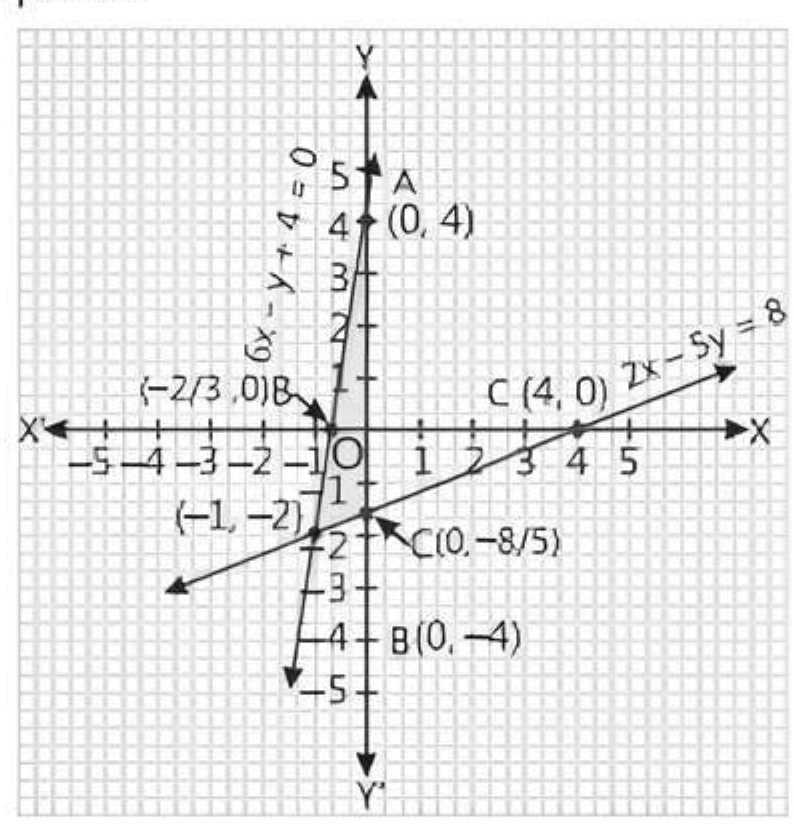
$\Rightarrow \quad y=\frac{8}{-5}=-\frac{8}{5}$

and when $y=0$, $2x-5\times 0=8$

$\Rightarrow \quad x=\frac{8}{2}=4$

x	0	4
y	–8/5	0

On a graph paper, draw X and Y-axes and plot the above points.



Both the lines intersect at (–1, –2).

Hence, (–1, –2) is the solution of given pair of linear equations.

2. For $x-y+1=0$, when $x=0$, $0-y+1=0 \Rightarrow y=1$

and $\quad$ when $y=0$, $x-0+1=0 \Rightarrow x=-1$

x	–1	0
y	0	1

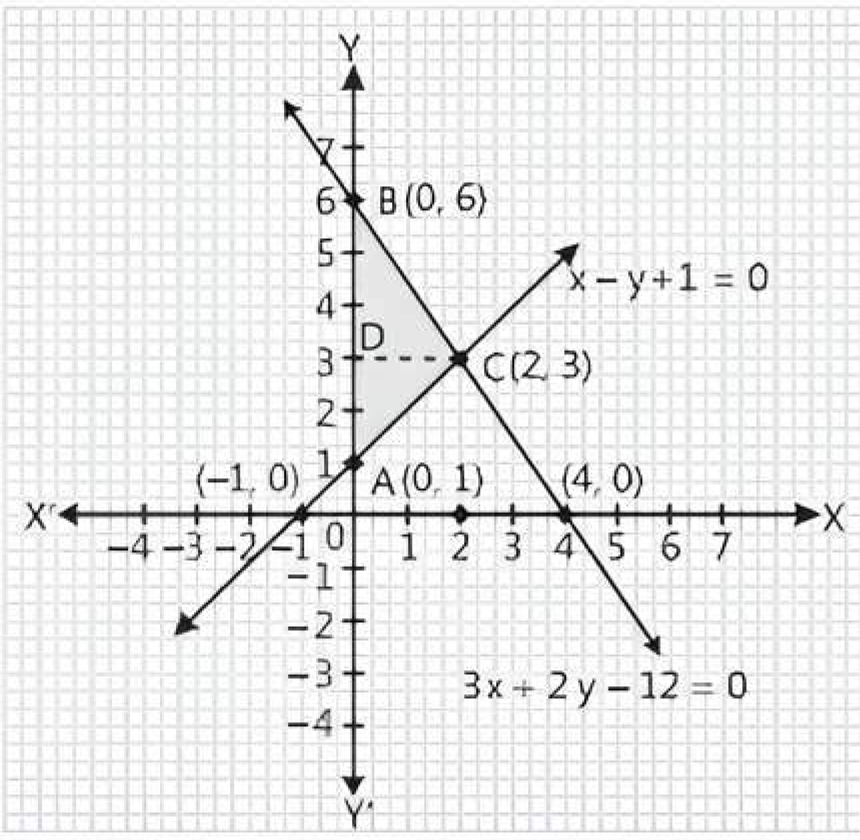
For $3x + 2y - 12 = 0$, when $x = 0$,

$$3 \times 0 + 2y = 12 \Rightarrow y = 6$$

and when $y = 0$, $3x + 2 \times 0 = 12 \Rightarrow x = 4$

x	0	4
y	6	0

The graphic representation is as follows:



From the figure, it can be observed that these lines are intersecting each other at point (2, 3) and Y-axis at (0, 1) and (0, 6). So, the vertices of the triangle are (2, 3), (0, 1) and (0, 6).

So, $x = 2$ and $y = 3$ is the solution of equations.

TR!CK

Area of triangle $= \frac{1}{2} \times$ *base* $\times$ *altitude*

$\therefore$ Area bounded by ΔABC

$$= \frac{1}{2} \times AB \times CD = \frac{1}{2} \times 5 \times 2$$

(here, $AB = 4 + 1 = 5$ and $CD = 2$)

$= 5$ sq. units

3. Given equations are

$$2x + 3y - 7 = 0 \quad ...(1)$$

and $(a - b)x + (a + b)y - (3a + b - 2) = 0 \quad ...(2)$

Here, $\frac{a_1}{a_2} = \frac{2}{a - b}, \frac{b_1}{b_2} = \frac{3}{a + b}$

and $\frac{c_1}{c_2} = \frac{-7}{-(3a + b - 2)} = \frac{7}{(3a + b - 2)}$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{2}{a - b} = \frac{3}{a + b} = \frac{7}{3a + b - 2}$$

Take Ist and IIIrd terms, $\frac{2}{a - b} = \frac{7}{3a + b - 2}$

$\Rightarrow 6a + 2b - 4 = 7a - 7b \Rightarrow a - 9b = -4 \quad ...(3)$

Again take Ist and IInd terms, $\frac{2}{a - b} = \frac{3}{a + b}$

$\Rightarrow 2a + 2b = 3a - 3b \Rightarrow a - 5b = 0 \quad ...(4)$

On subtracting eq. (3) from eq. (4), we get

$$4b = 4 \Rightarrow b = 1$$

On substituting the value of 'b' in eq. (4), we get

$$a - 5 \times 1 = 0$$

$$\Rightarrow a = 5$$

Hence, for $a = 5$ and $b = 1$, the given equations have infinitely many solutions.

4. Let the ages of A and B be x and y years respectively. Then,

$$x - y = \pm 2 \quad \text{(given)}$$

D's age $= 2x$ years and C's age $= \frac{y}{2}$ years

Clearly, D is older than C.

$$\therefore \quad 2x - \frac{y}{2} = 40 \Rightarrow 4x - y = 80$$

Thus, we have the following two systems of linear equations:

$$x - y = 2 \quad ....(1)$$

and $4x - y = 80 \quad ....(2)$

$$x - y = -2 \quad ....(3)$$

and $4x - y = 80 \quad ....(4)$

Subtracting eq. (1) from eq. (2), we get

$$3x = 78 \Rightarrow x = 26$$

Putting $x = 26$ in eq. (1), we get $y = 24$

Subtracting eq. (4) from eq. (3), we get

$$-3x = -82 \Rightarrow x = \frac{82}{3} = 27\frac{1}{3}$$

Putting $x = \frac{82}{3}$ in eq. (3), we get

$$y = \frac{82}{3} + 2 = \frac{88}{3} = 29\frac{1}{3}$$

Hence, A's age = 26 years and B's age = 24 years

or A's age $= 27\frac{1}{3}$ years and B's age $= 29\frac{1}{3}$ years.

5. Let the cost price of one chair be ₹ x and that of one table be ₹ y.

Profit on a chair = 25%.

$\therefore$ Selling price of one chair $= x + \frac{25}{100}x = \frac{125}{100}x$

Profit on a table = 10%

$\therefore$ Selling price of one table $= y + \frac{10y}{100} = \frac{110}{100}y$

According to the given condition, we have

$$\frac{125}{100}x + \frac{110}{100}y = 1520$$

$$\Rightarrow 125x + 110y = 152000$$

$$\Rightarrow 25x + 22y = 30400 \quad ....(1)$$

If profit on a chair is 10% and on a table is 25%, then total selling price is ₹ 1535.

$$\therefore \left(x + \frac{10}{100}x\right) + \left(y + \frac{25}{100}y\right) = 1535$$

$$\Rightarrow \frac{110}{100}x + \frac{125}{100}y = 1535$$

$$\Rightarrow 110x + 125y = 153500$$

$$\Rightarrow 22x + 25y = 30700 \quad ....(2)$$

Subtracting eq. (2) from eq. (1), we get

$$3x - 3y = -300$$

$$\Rightarrow \quad x - y = -100 \quad ...(3)$$

Adding eqs. (2) and (1),

$$47x + 47y = 61100 \Rightarrow x + y = 1300 \quad ...(4)$$

Solving eqs. (3) and (4),

$$x = 600 \text{ and } y = 700$$

Hence, the cost price of a chair is ₹ 600 and that of a table is ₹ 700.

6. Suppose the person invested ₹ x at the rate of 12% simple interest and ₹ y at the rate of 10% simple interest. Then,

$$\text{Yearly interest} = \frac{12x}{100} + \frac{10y}{100}$$

According to the given condition,

$$\frac{12x}{100} + \frac{10y}{100} = 130$$

$$\Rightarrow \quad 12x + 10y = 13000$$

$$\Rightarrow \quad 6x + 5y = 6500 \quad \text{(divide by 2)} \ ...(1)$$

If the invested amounts are interchanged, then yearly interest increases by ₹ 4.

$$\therefore \quad \frac{10x}{100} + \frac{12y}{100} = 134$$

$$\Rightarrow \quad 10x + 12y = 13400$$

$$\Rightarrow \quad 5x + 6y = 6700 \quad \text{(divide by 2)} \ ...(2)$$

Subtracting eq. (2) from eq. (1), we get

$$x - y = -200 \quad ...(3)$$

Adding eqs. (2) and (1), we get

$$11x + 11y = 13200$$

$$\Rightarrow \quad x + y = 1200 \quad ...(4)$$

Adding eqs. (3) and (4), we get

$$2x = 1000 \Rightarrow x = 500$$

Putting $x = 500$ in eq. (3), we get $y = 700$

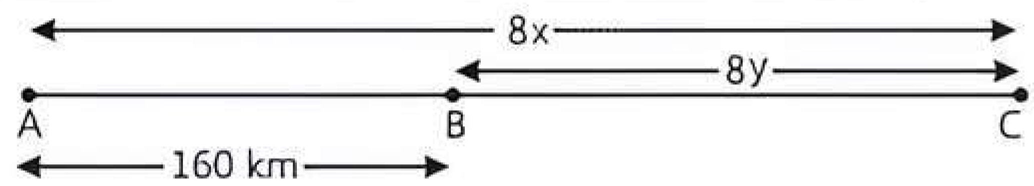
Thus, the person invested ₹ 500 at the rate of 12% per year and ₹ 700 at the rate of 10% per year.

7. Let the speed of car starting from place A be x km/h and that of another car starting from place B be y km/h.

TiP

Emphasis on solving such type of application based problem.

Case I: When two cars move in same direction.



Distance covered (AC) by first car in 8 h = $8x$

($\because$ distance = time × speed)

Distance covered (BC) by second car in 8 h = $8y$

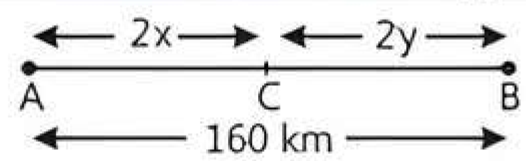
Now, $AC - BC = 160$ (given)

$$\therefore \quad 8x - 8y = 160$$

$$\Rightarrow \quad 8(x - y) = 160$$

$$\Rightarrow \quad x - y = \frac{160}{8} = 20 \quad ...(1)$$

Case II: When two cars move in opposite directions.



Distance covered (AC) by first car in 2 h = $2x$

Distance covered (BC) by second car in 2 h = $2y$

Now, $AC + CB = 160$

$$\therefore \quad 2x + 2y = 160$$

$$\Rightarrow \quad 2(x + y) = 160$$

$$\Rightarrow \quad x + y = \frac{160}{2} = 80 \quad ...(2)$$

On adding eqs. (1) and (2), we get

$$x - y = 20$$
$$x + y = 80$$
$$2x = 100$$

$$\Rightarrow \quad x = \frac{100}{2} = 50$$

On substituting the value of 'x' in eq. (1), we get

$$50 - y = 20$$

$$\Rightarrow \quad -y = 20 - 50 = -30$$

$$\Rightarrow \quad y = 30$$

Hence, the speed of car that starts from place A is 50 km/h and that of another car starting from place B is 30 km/h.

COMMON ERROR

Sometimes, students frame this word problem into wrong equation, lead to get incorrect answer.

Chapter Test

Multiple Choice Questions

Q 1. The pair of equations $3x - 4y - 11 = 0$ and $7x - 5y - 4 = 0$ has:

a. a unique solution

b. exactly two solutions

c. no solution

d. infinitely many solutions

Q 2. If the system of equations $3x + y = 1$ and $(2k - 1)x + (k - 1)y = 2k + 1$ is inconsistent, then k is equal to:

a. 0

b. 2

c. −1

d. 1

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q 3. Assertion (A): If the pair of equations $x + 2y + 7 = 0$ and $3x + ky + 21 = 0$ represents coincident lines, then value of k is 6.

Reason (R): The pair of linear equations are coincident lines, if they have no solution.

Q 4. Assertion (A): The graph of lines $4x + 3y = 12$ and $8x + 6y = 48$ is parallel.

Reason (R): The graph of linear equation $ax + b = 0$, where $a \neq 0$ is parallel to Y-axis.

Fill in the Blanks

Q 5. Graphically, the pair of equations intersect at .... point.

Q 6. If the lines given by $3x + 4ky = 2$ and $2x + 7y = 1$ are parallel, then the value of k is ....

True/False

Q 7. The pair of equations $y = 0$ and $y = -7$ has no solution.

Q 8. The system of equations $ax + 3y = 1, -12y + ax = 2$ has unique solution for all real values of a.

Case Study Based Question

Q 9. Mr Gaurav decided to go to an amusement park along with his family. The cost of an entry ticket was ₹ 25.00 for children and ₹ 50.00 for adults. On that particular day, attendance at the circus was 2000 and the total revenue accumulated was ₹ 70000.

Based on the above information, solve the following questions:

(i) If we consider the number of children and adults who bought ticket on that day as x and y respectively, form the pair of linear equations describing the above situation.

(ii) Find the number of adults who bought tickets on that particular day.

Or

Find the difference between the number of children and adults who bought tickets on that particular day.

(iii) Find the total revenue if the cost of an entry ticket is ₹45 for children and ₹ 65 for adults.

Very Short Answer Type Questions

Q 10. Find the solution of the following equations:

$$2x + 3y = 17 \quad \text{and} \quad 3x - 2y = 6$$

Q 11. Show that system of equations $6x + 5y = 11$ and $9x + \frac{15}{2}y = 21$ has no solution.

Short Answer Type-I Questions

Q 12. The cost of 4 pens and 4 pencils boxes is ₹100. Three times the cost of a pen is ₹ 15 more than the cost of a pencil box. Form the pair of linear equations for the above situation. Find the cost of a pen and a pencil box.

Q 13. Five years hence, father's age will be three times the age of his son. Five years ago, father was seven times as old as his son. Find their present ages.

Short Answer Type-II Questions

Q 14. For which value(s) of λ does the pair of linear equations $\lambda x + y = \lambda^2$ and $x + \lambda y = 1$ have:

(i) no solution.

(ii) infinitely many solutions.

(iii) a unique solution.

Q 15. The sum of two numbers is 25 and their difference is 1. Find the numbers.

Long Answer Type Question

Q 16. A railway half ticket cost half the full fare but the reservation charges are the same on a half ticket as on a full ticket. One reserved first class ticket from the stations A to B costs ₹ 2530. Also, one full reserved first class ticket and one reserved first class half ticket from stations A to B costs ₹ 3810. Find the full first class fare from stations A to B and also the reservation charges for a ticket.