☆ Root Locus :-

Purpose :-

- → To find the CL System Stability.
- → To find the range of K value for system Stability.
- → To find the K value to become System Marginal stable.
- → To find the natural freq. of oscillation (OR) Undamped oscillation when the system is Marginal Stable.
- → To find the K value to become the System undamped, underdamped, critical damped and overdamped system.
- → To find the relative stability. By using the relative stability concept we can find System time constant Setting time.

→ If the Root locus branches moves towards the left then the system is more Relative stable.

→ If the Root locus branches moves towards the Right then the system is less Relative Stable.

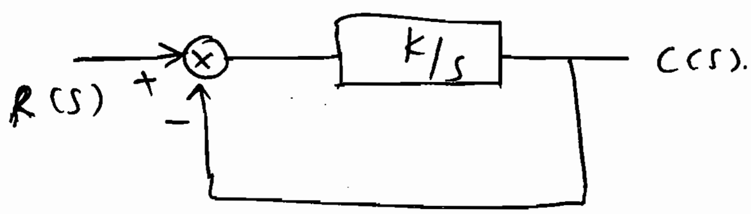
→ Best method to find the relative stability is Root Locus.

→ Best method to find absolute stability is RH - criteria.

* Defination of Root Locus:

→ 'Root' means roots of chara. eqn which is CL Poles. 'Locus' means path. Hence, Root Locus means CL poles path by varying K value from 0 to ∞.

[Q] Draw the Root locus to the given system.



Sol^n: Char. eq^n ⇒ $1 + GH(s) = 0$.

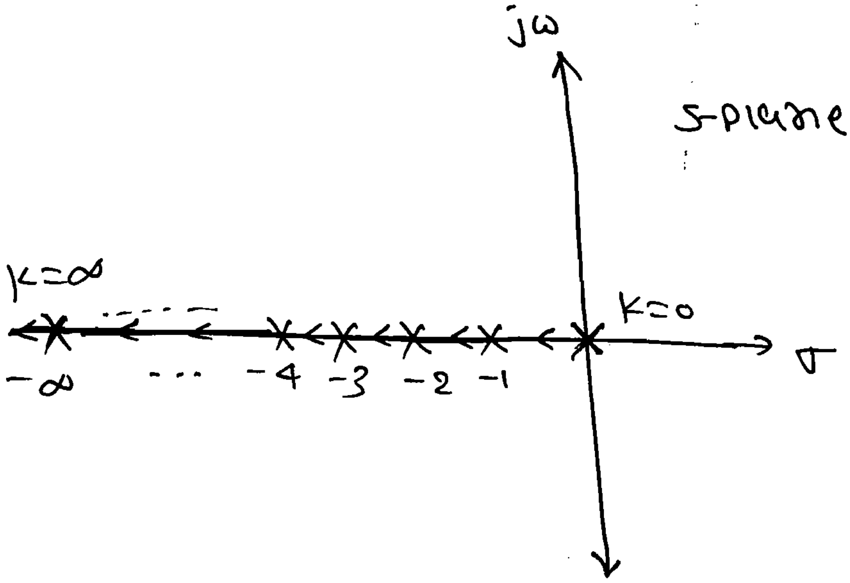
$$\Rightarrow 1 + \frac{K}{s} = 0$$

$$\boxed{s + k = 0.}$$

→ drawing a root locus means identifying the CL poles path.

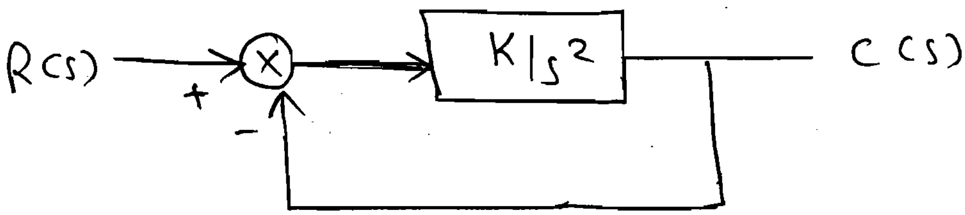
$$\underline{s = -k}$$

→ CL poles path is given by char. eq^n.



K value	pole location $s=-k$
0 →	0
1	−1
2	−2
3	−3
⋮	
∞	−∞.

Q Draw Root Locus.



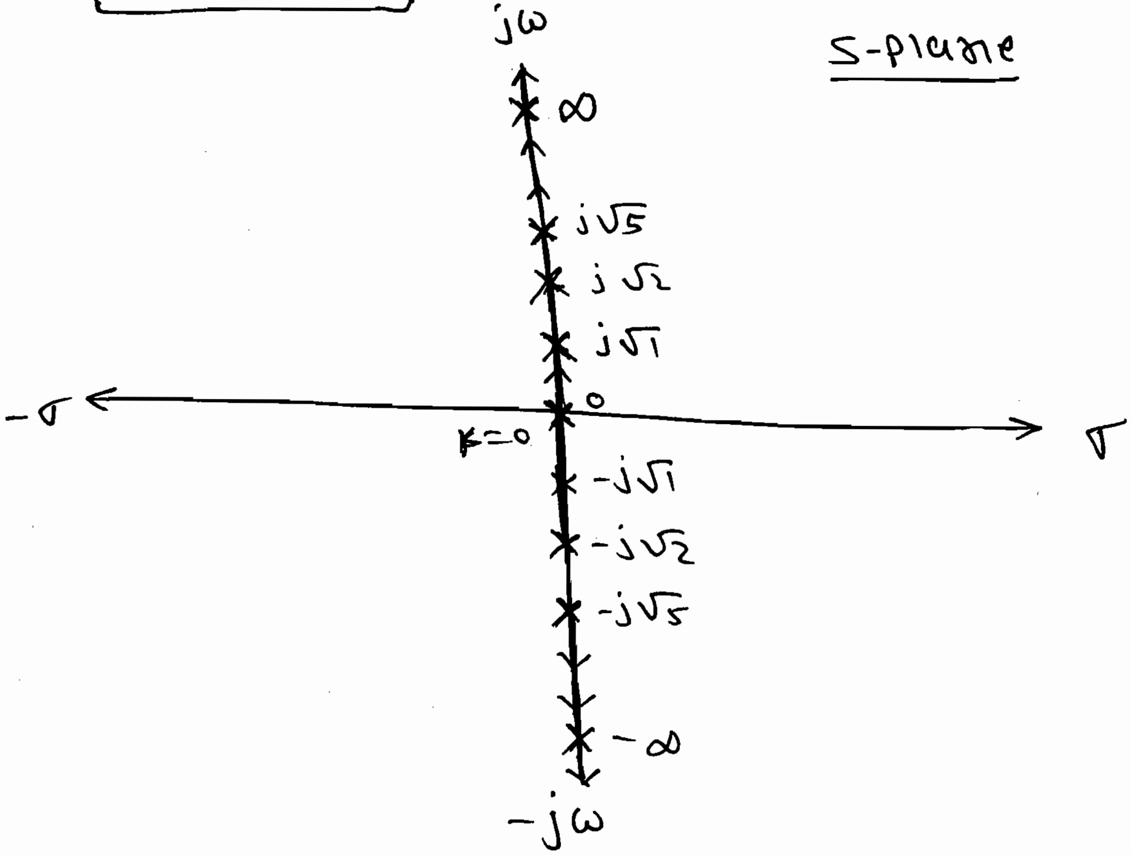
Sol^n: $\underrightarrow{CE}$ $1 + GH(s) = 0$

$$\therefore\ 1 + \frac{K}{s^2} = 0.$$

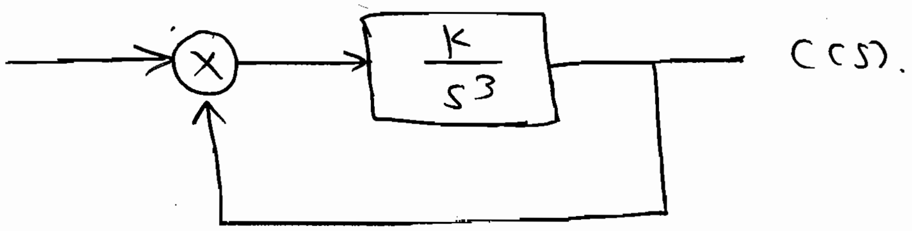
$\Rightarrow \quad s^2 + K = 0$

$$\boxed{s = \pm j\sqrt{K}}$$

$\Rightarrow$



$\boxed{a}$ Draw the Root Locus:



Soln:

$$\xrightarrow{CE} \quad 1 + \frac{K}{s^3} = 0$$

$$s^3 + K = 0.$$

$$s = ?$$

$\Rightarrow$ As order increases finding the roots for the char. eqn is very difficult. Hence we can not draw the Root Locus diagram by using char. eqn.

→ To draw a Root locus diagram we use the open loop transfer function, But the Stability analysis is for CL system.

*** Relationship between OL transfer f^n & CLTF Poles & Zeros:**

⇒ i] OLTF :

→ The CL Poles are given by char. eqn

$$1 + GH(s) = 0.$$

$$\Rightarrow \quad 1 + K \cdot \frac{N(s)}{D(s)} = 0.$$

$$\xrightarrow{CE} \quad D(s) + K \cdot N(s) = 0.$$

⇒ The CL Poles are nothing but the sum of OL poles, OL zeros with the f^n of system gain K.

$$\xrightarrow{OLTF} \quad G(s) \cdot H(s) = K \frac{N(s)}{D(s)} \quad \text{— (1).}$$

$$\xrightarrow{OL \text{ Poles}} \quad D(s) = 0.$$

$$\xrightarrow{OL \text{ zeros}} \quad N(s) = 0.$$

Case-I: $K = 0$, $K = \left| -\frac{D(s)}{N(s)} \right|$

$$\downarrow$$

$D(s) = 0$ ← CL Poles.

⇒ When $K=0$, CL Poles = OL Poles.

⇒ Case - II:

$$N(s)=0 \longrightarrow K=\pm\infty.$$

When $K=\infty$, CL Poles = OL zeros.	
When $K=-\infty$, CL Poles = OL zeros.	

⇒

When $K \Uparrow$ $\xrightarrow{0 \text{ to } \infty}$ $\underset{\text{OL Poles}}{\overset{K=0}{X}} \longrightarrow\longrightarrow\longrightarrow\longrightarrow\longrightarrow \underset{\text{OL zeros.}}{\overset{K=\infty}{O}}$

When $K \Uparrow$ $\xrightarrow{-\infty \text{ to } 0}$ $\underset{\text{OL Zeros}}{\overset{K=-\infty}{O}} \longrightarrow \underset{\text{OL Poles.}}{\overset{K=0}{X}}$

→ From above, we can conclude that when K increases from 0 to ∞ the direction of the root locus branch is from pole to zero because at OL poles K value is '0' and at OL zeros K value is '∞'.

⇒ If K increases from $-\infty$ to 0 then the direction of Root Locus Branch is from OL zero to OL poles.

because at OL zero, $K = -\infty$ and at OL Pole $K = 0$.

$\boxed{Q}$ Identify where the RL branch start and ends when K increased from 0 to ∞ for $G(s) \cdot H(s) = \frac{K(s+1)}{s(s+5)(s+10)}$.

Soln:

$$GH(s) = \frac{K(s+1)}{s(s+5)(s+10)}.$$

$\begin{bmatrix} \text{OL Poles} \\ K=0 \end{bmatrix} \Rightarrow s=0, \; s=-5, \; s=-10. \leftarrow$ start

$\begin{bmatrix} \text{OL Zero} \\ K=\infty \end{bmatrix} \Rightarrow s=-1, \; \underbrace{\infty, \infty}_{\text{angle of Asymptotes.}} \leftarrow$ end.

$\Rightarrow$ *** To draw a RL diagram, no. of Poles is must equal to no. of Zeros. If the Zeros are less we assume Zeros are at infinity. The direction of infinity is given by angle of asymptotes.

$\Rightarrow$ *** To draw a RL diagram, for one Pole we need one zero, because the RL branch start at poles and ends at Zeros.

* Angle & Magnitude Conditions:

→ The Construction Rules of Root Locus are formed by using the angle condition such that the Root Locus diagram gives the CL poles path and CL system stability.

→ The CL poles path given by Char. eqn,

(−ve FB)

CE →

$$1+G(s)\cdot H(s)=0.$$

$$GH(s)=-1+j0$$

A.C. →

$$\angle G(s)\cdot H(s)=\angle -1+j0$$

= odd multiples of $\pm 180^\circ$.

$$\boxed{\angle GH(s)=\pm(2q+1)\,180^\circ}$$

$$q=0,1,2,\ldots$$

(+ve FB)

CE →

$$1-G(s)\cdot H(s)=0$$

$$G(s)\cdot H(s)=1+j0$$

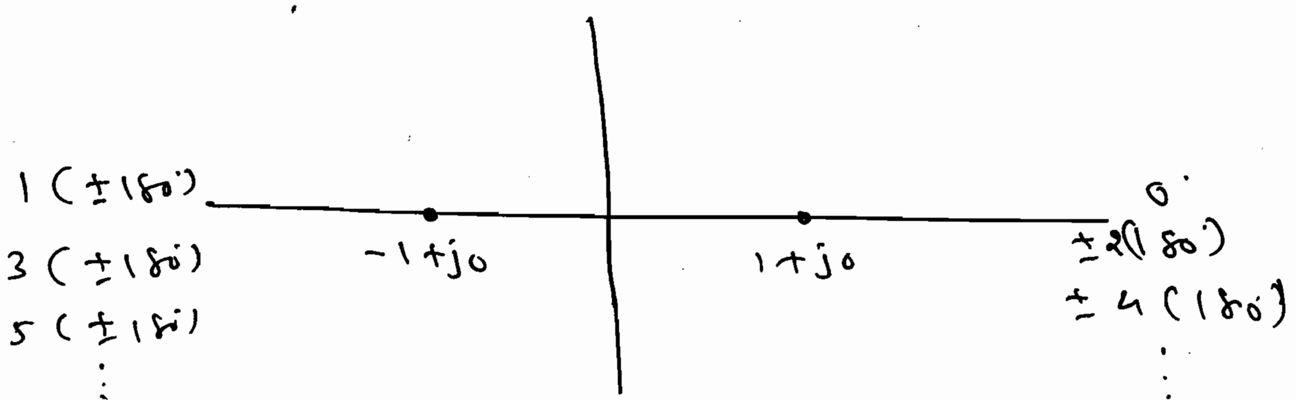
A.C. →

$$\angle G(s)\cdot H(s)=\angle 1+j0$$

= even multiples of $\pm 180^\circ$.

$$\boxed{\angle GH(s)=\pm 2q\,(180^\circ)}$$

$$q=0,1,2,\ldots$$



→ Roots which are formed by (-ve) FB Char. eqn are called Direct Root locus (DRL) (or) 180° Rules.

→ Roots which are formed by (+ve) FB Char. eqn are called Inverse Root locus (IRL) (or) Complementary Root locus (CRL) (or) 0° rules.

⇒

	DRL	IRL
R_3	→ odd	even (statement).
R_4	→ $2q+1$	$2q$ (formula).
Case (iii) R_6	→ Left most	Right Most (Statement)
R_8	→ 180°	0° (formula).

* Purpose of angle Condn:

⇒ To check the any point lies on Root Locus (or) not that means all the Points on the Root locus must satisfies the angle conditions.

Q Verify either the following points lies on the Root Locus ~~sco~~ or not to the following system,

$$G(s)\cdot H(s) = \frac{K}{s(s+5)(s+10)}$$ i) $s=-3$
ii) $s=-6$.

Sol^n: (i) $s=-3$.

$\xrightarrow{AC}$ $$\angle GH\Big|_{s=-3} = \frac{\angle K}{\angle -3 \angle +2 \angle 7}$$

$$= \frac{0^\circ}{\pm 180^\circ + 0^\circ + 0^\circ}$$

$$\angle GH\Big|_{s=-3} = \mp 180^\circ$$

Satisfies the odd multiple of 180°.

so, pole is on RL.

(ii) $s=-6$

$$\angle GH\Big|_{s=-6} = \frac{\angle K}{\angle -6 \angle -1 \angle 4}.$$

$$= \frac{0^\circ}{\pm 180^\circ \pm 180^\circ + 0^\circ}$$

$$= \mp 2(180^\circ)$$

→ Not satisfies the angle condition because even no. of 180°.

→ so, given root $s=-6$ do not lies on RL.

* Magnitude Condition:

$\Rightarrow$ $\xrightarrow{\text{M.C.}}$ $G(s) \cdot H(s) = -1 + j0.$

$$\boxed{|G(s) \cdot H(s)|_{\text{at any point which is on RL}} = 1.}$$

→ If given point is not on RL then M.C. is not valid.

→ so, the angle condⁿ must be satisfied to valid the Magnitude Condition (M.C.).

→ Magnitude Condition is valid when the given point is on RL. The given point on the RL is verified by angle condition that means to apply a magnitude condition the A.C. must be satisfied.

Purpose:

$\Rightarrow$ To find the system gain at any point which is on RL.

$\boxed{a}$ Find the system gain at a point $s = -5 + j5$ to the following system

i.e. $\dfrac{K}{s(s+10)}$.

Soln:

$$\xrightarrow{AC} \quad G(s)H(s)\Big|_{s=-5+j5} = \frac{\angle K}{\angle -5+j5 \ \angle 5+j5}$$

$$= \frac{0^\circ}{135^\circ + 45^\circ}$$

$$= -180^\circ \checkmark$$

Satisfied the AC.

So, given pole is on RL.

Now, $\xrightarrow{M.C.} \quad \left| GH(s) \right|_{s=-5+j5} = 1.$

$$\therefore \quad \left| \frac{K}{(-5+j5)(5+j5)} \right| = 1$$

$$\frac{K}{\sqrt{25+25} \cdot \sqrt{25+25}} = 1$$

$$\boxed{K = 50}$$

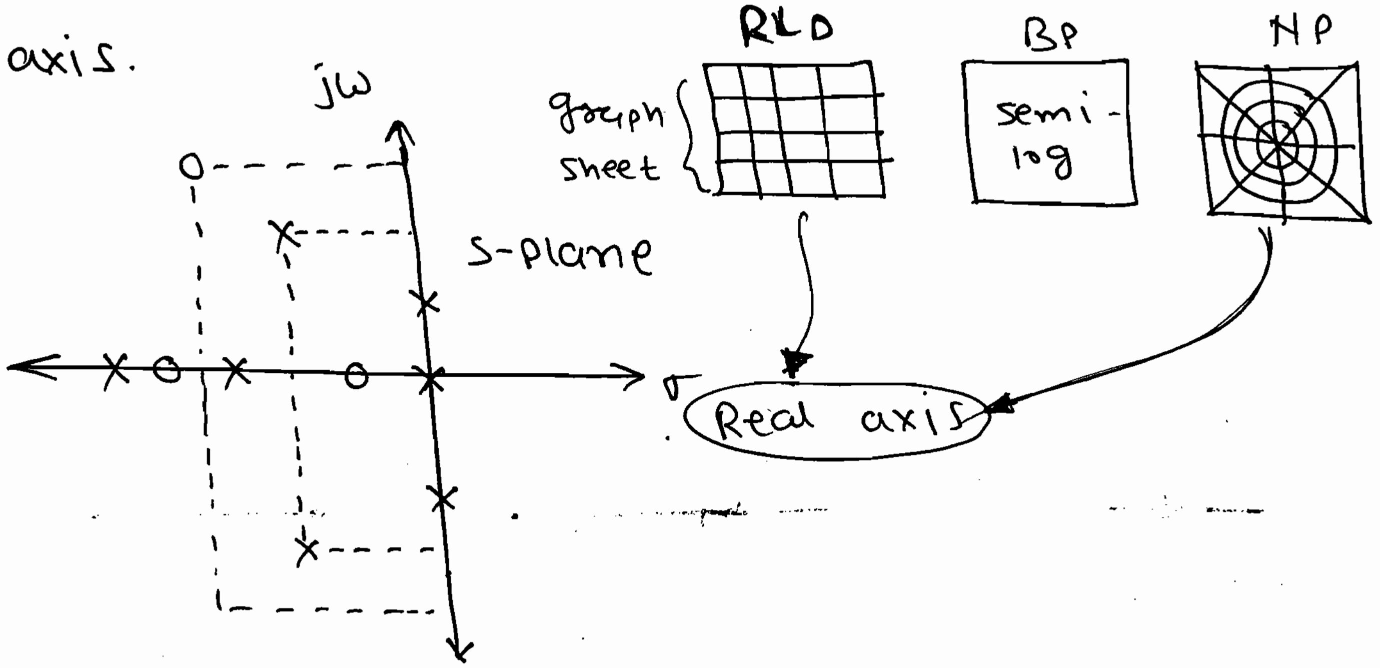
So, system gain at $s = -5 + j5$ is

$$\boxed{K = 50}$$

* Construction Rules for RL:-

Rule - 1 :- Symmetry:

⇒ The root locus diagram is Symm. about the real axis because the location of the poles and zeros in the s-plane Symm. about the real axis.



→ The Symm. not depends on poles and zeros location, it depends on the graph sheet and on which the plot is constructed the NP (Nyquist Plot) also Symm. about the real axis but not Bode plot because the bode plot drawn on non-linear graph sheet (semi-Log).

Rule - 2 :- No. of Loci (or) RL branches.

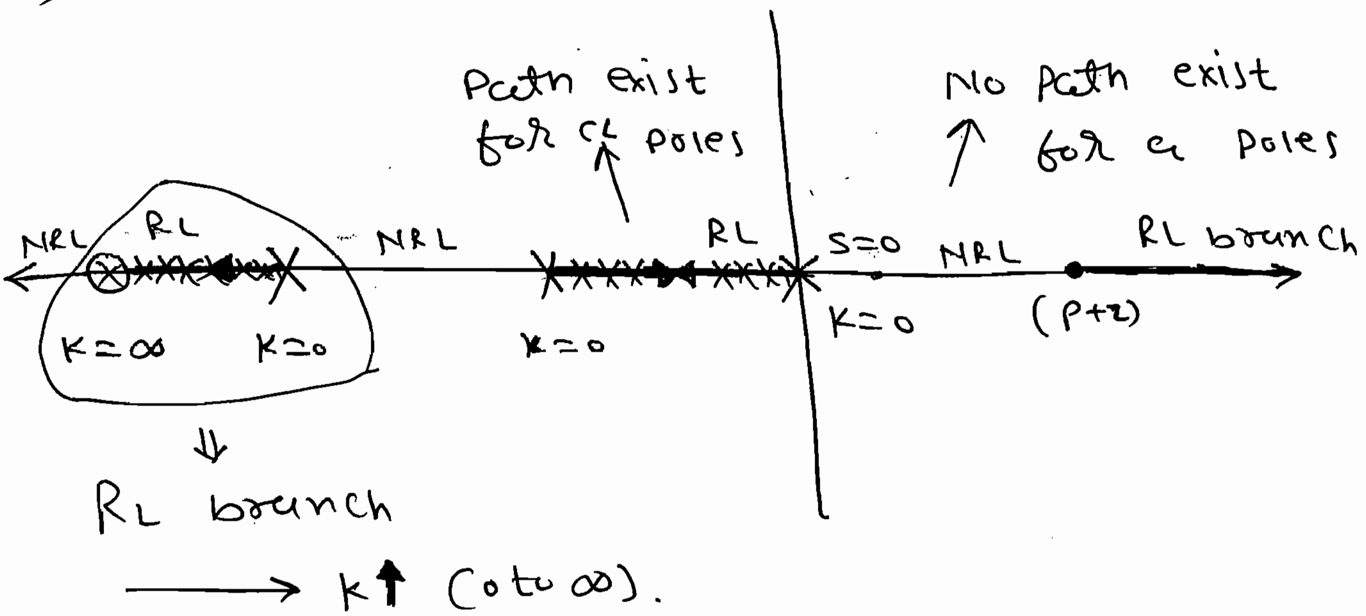
⇒ It depends on no. of poles and zeros.

Case-(i) : Poles > Zeros : No of Loci = No of pole.

Case-(ii) : Zeros > Poles : No of Loci = No. of Zero.
(Poles < zeros)

Rule - 3 :- Real axis Loci.

⇒



⇒ A Point exist on Real axis root locus branches, the sum of the the poles and zeros to the Right hand side of that point should be odd.

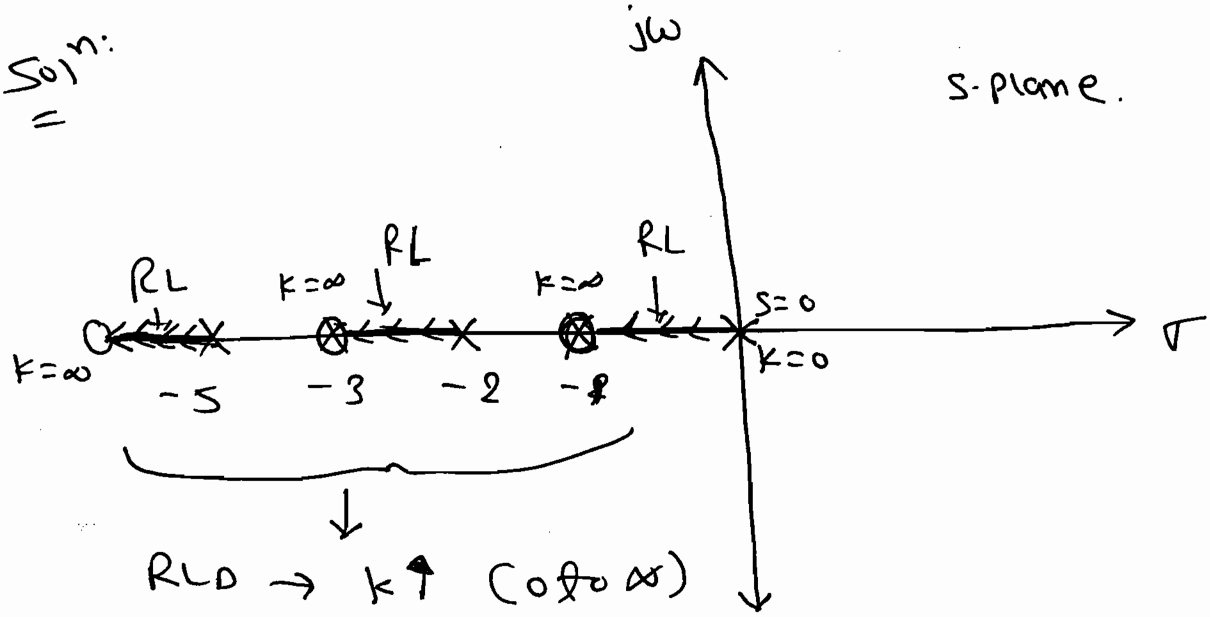
⇒ The poles moves only on the RL branches. Once the pole reach the zero then it become the complete root locus branch for that perticular pole where k increased from 0 to ∞.

⇒ At the position of poles and zeros never apply the angle and magnitude conditions because all the poles and zeros must lies on the RL branches. because that are starting and ending points of RL branches and the k value at pole is zero. and k value at zero ∞. So never apply magnitude and cond^n.

Q Identify the sections of Real axis which belongs to RL.

$$G(s) \cdot H(s) = \frac{k(s+1)(s+3)}{s(s+2)(s+5)}.$$

Sol^n:
=



[a] Identify the following points which are on RL branches to the following:

$$G(s).\ H(s) = \frac{k\ (s^2 + 2s+2)}{s^2\ (s+2)\ (s+4)(s+6)}.$$

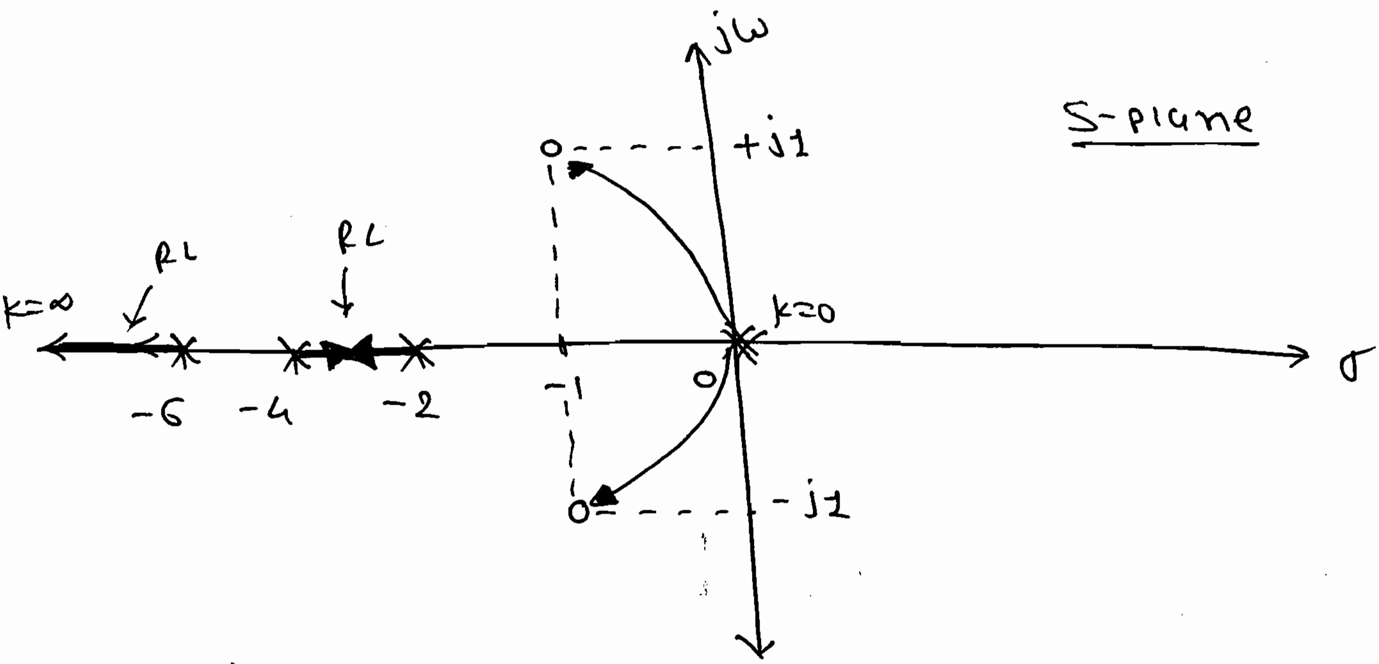
Pole:

s=0, s= -2, s= -4, s=-6, s=-1,

s=-3, s=-5, s=-6, s=-∞, s=-1+j1
= -1 -j1.

Soln:

$$G(s).H(s) = \frac{k\ (s+1+j)\ (s+1-j).}{s^2\ (s+2)\ (s+4)(s+6)}.$$



So,

Valid.

s=0

s= -2

s= -4

s= -6

s= -1+j

s= -1-j

s= -∞

Invalid

s=-5

s=-1

Rule - 4: Asumptotes:

⇒ Asumptotes are the RL branches which are approach to the infinity.

⇒ $$\boxed{\begin{array}{l} \text{No. of Asumptotes } N = P - Z. \\ \text{Angle of Asumptotes } \theta = \dfrac{(2q+1)180^\circ}{(P-Z)}, \\ q = 0, 1, 2, \ldots (P-Z-1). \end{array}}$$

Note: The Asumptotes gives the directiony of Zeros, when no. of Poles are greater than zeros.

Rule - 5 :- Centroid

⇒ The centroid is nothing but intersection point of asymptotes on the real axis.

⇒ $$\boxed{(\sigma)\ \text{Centroid} = \frac{\sum \text{Real part of Poles} - \sum \text{Real part of Zeros}}{(P-Z)}}$$

⇒ The centroid may be located anywhere on the real axis. It may (or) may not be on RL branch.

Q) Find the angle of Asymptotes and centroid $G(s)\cdot H(s) = \frac{K}{s(s+5)(s+10)}$.

Soln:

$P = 3, \quad Z = 0.$

$$\text{Centroid } (\sigma) = \frac{(-0-5-10) - (0)}{3-0}$$

$$= -15/3$$

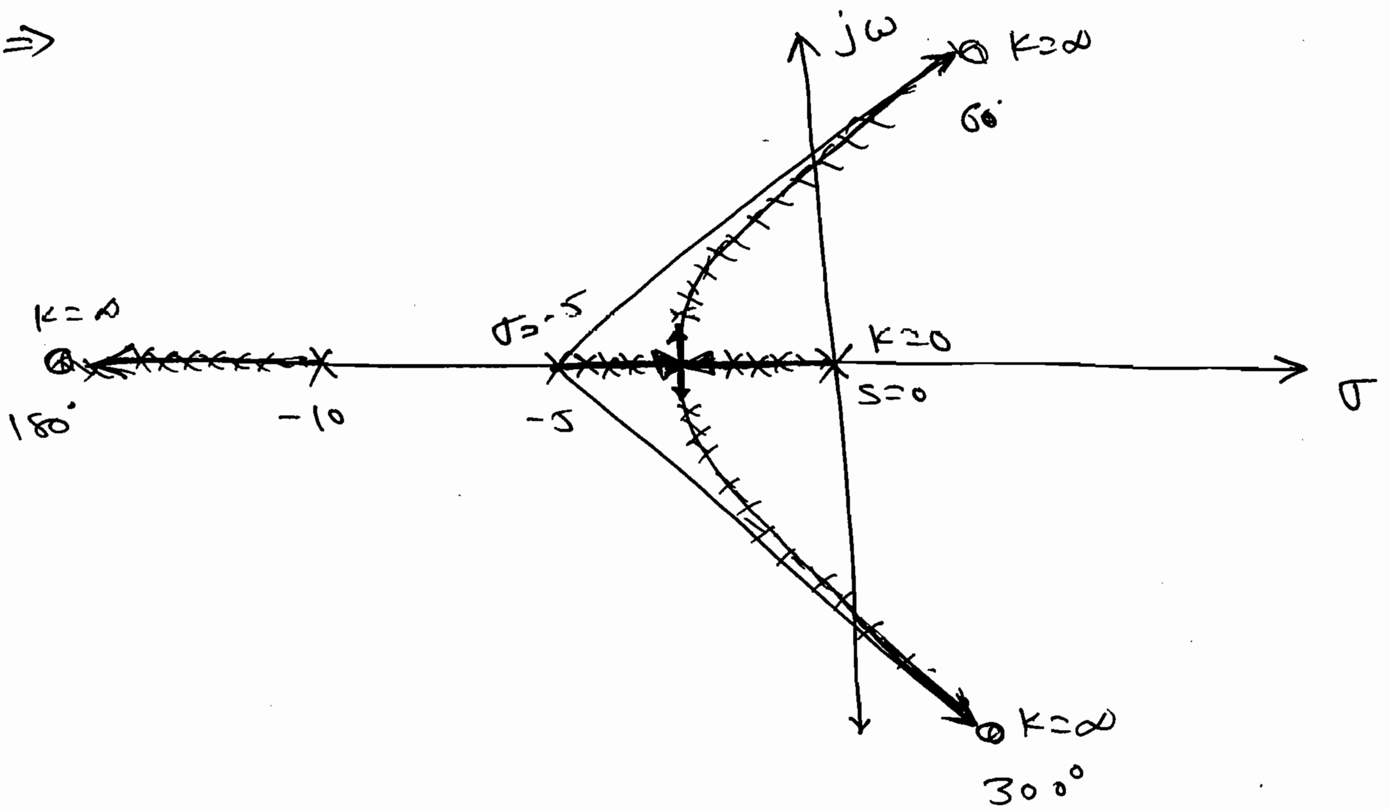
$$\boxed{\sigma = -5}$$

$$\therefore \quad \theta = \frac{(2q+1)\,180^\circ}{P-Z}$$

$$\therefore \quad \theta = \frac{(2q+1)\,\cancel{180^\circ}\,60^\circ}{\cancel{3}}$$

$$\theta = 60^\circ, 180^\circ, 300^\circ.$$

=>



=> At Collision $\frac{+180^\circ}{n} = \frac{\pm 180^\circ}{2} = \pm 90^\circ$.

=> The centroid is mainly required to draw the angle of asymptotes.

$\boxed{e}$ $G(s) \cdot H(s) = \dfrac{K(s+10)}{s^2(s+1)}$.

Soln:

angle $\theta = \dfrac{(2q+1)180°}{p-2}$

$= \dfrac{(2q+1)180°}{3-1}$

$= \dfrac{180°}{2}$ 90 $(2q+1)$

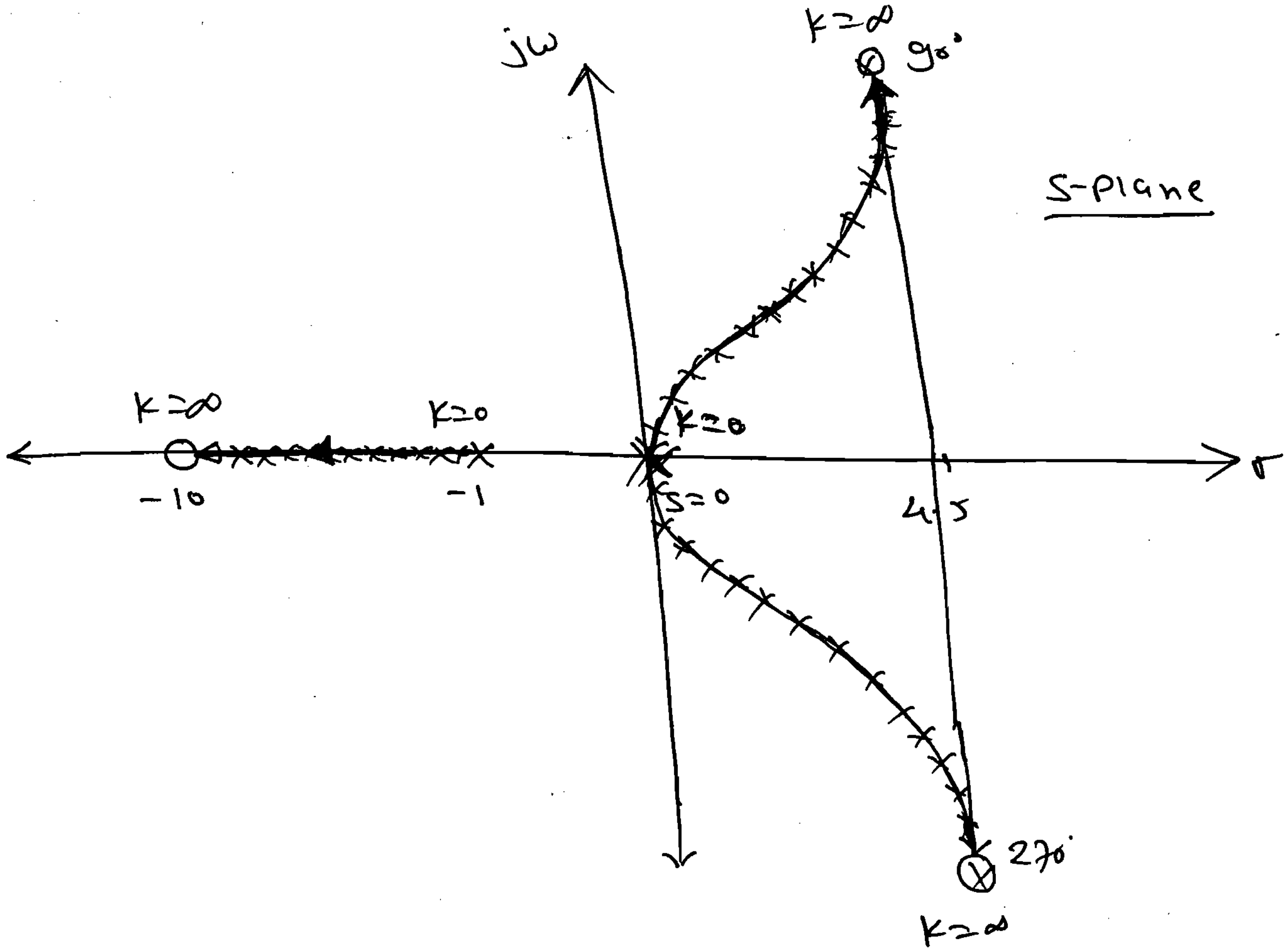
$\theta = 90°, 270°$

Centroid $= \dfrac{(-0-0-1)-(-10)}{3-1}$.

$= \dfrac{-1+10}{2}$

$\boxed{\sigma = 4.5}$

=)



Rule - 6 :- Break Point [Junction of 2 (or) more Poles].

⇒ The point at which two (or) more poles meet (or) two or more poles directly located at any point then it is called Break point.

⇒ Breakaway Point :-

→ The point at which the Root Locus branches leaves the real axis is called breakaway point.

⇒ Breakin Point :-

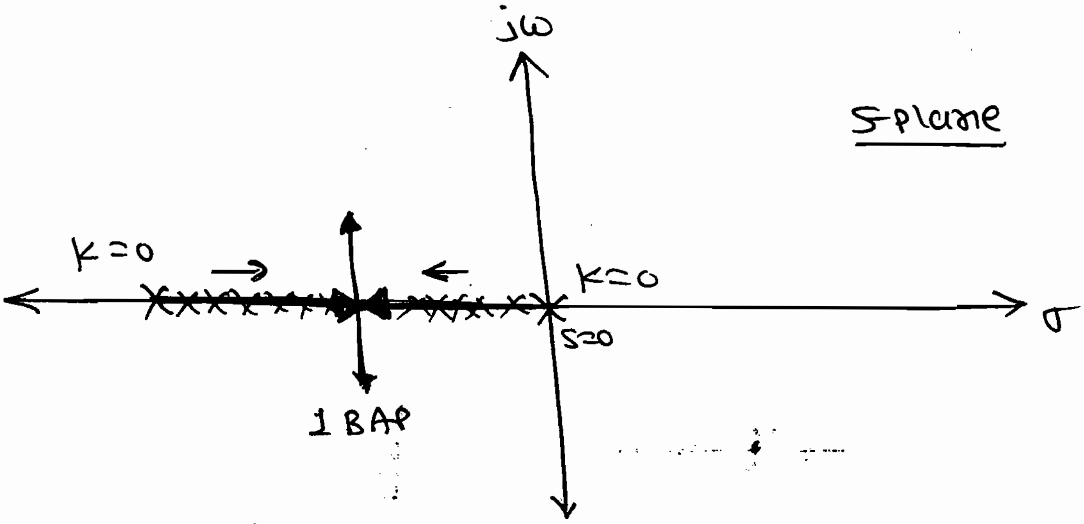
→ The point at which RL branches enters into the point on Real axis is called Breakin point.

* Finding the existence of the Break points:

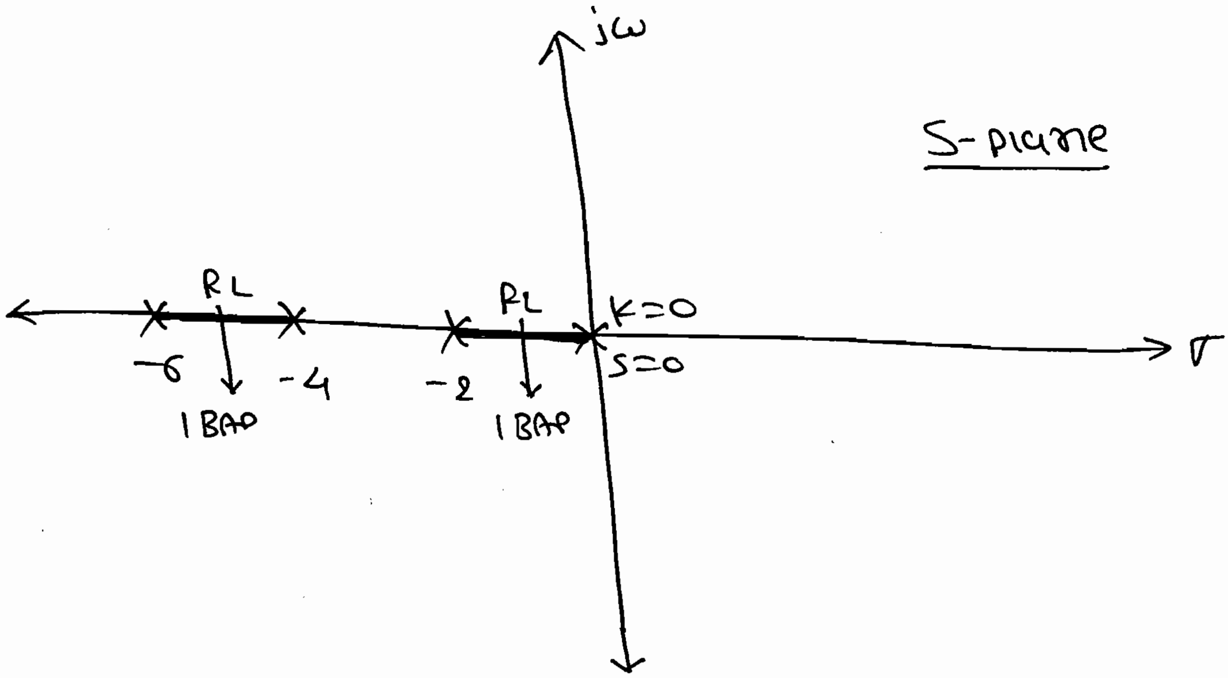
Case - I :

⇒ whenever there are two adjacently placed poles in between there exist a RL branch then there should be the minimum one break away point (BAP) in between adjacently placed poles.

$\Rightarrow$



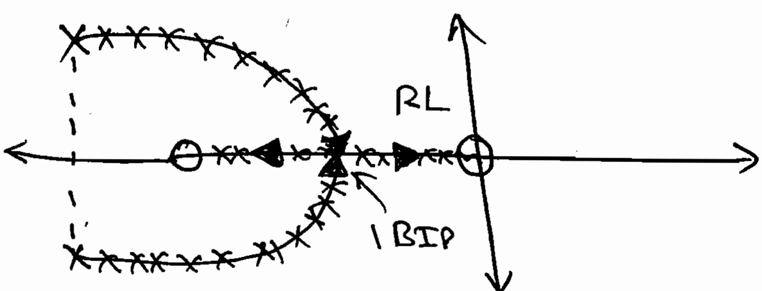
e.g. $G(s) = \dfrac{K}{s(s+2)(s+4)(s+6)}$.



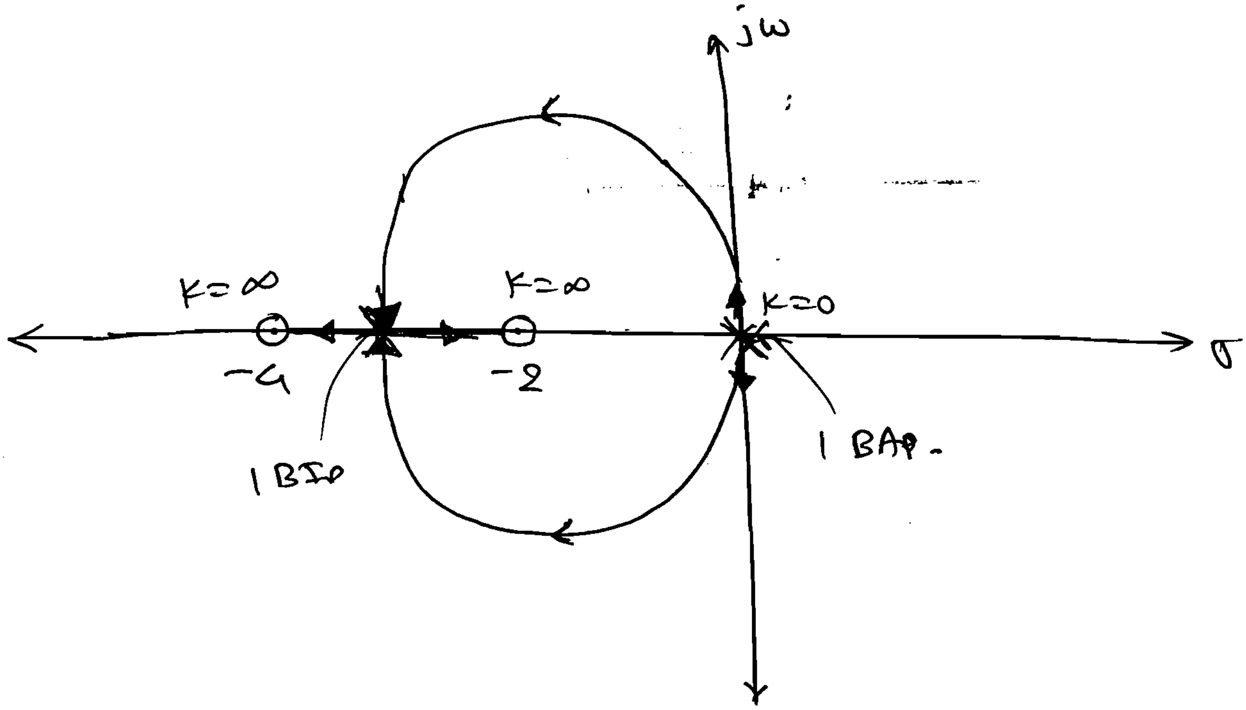
Case-II :-

$\Rightarrow$ Whenever there exist a two adjecently placed zero in betⁿ there exist a RL branch then there should be the minimum one break in point in betⁿ adjecently placed zero.

$\Rightarrow$



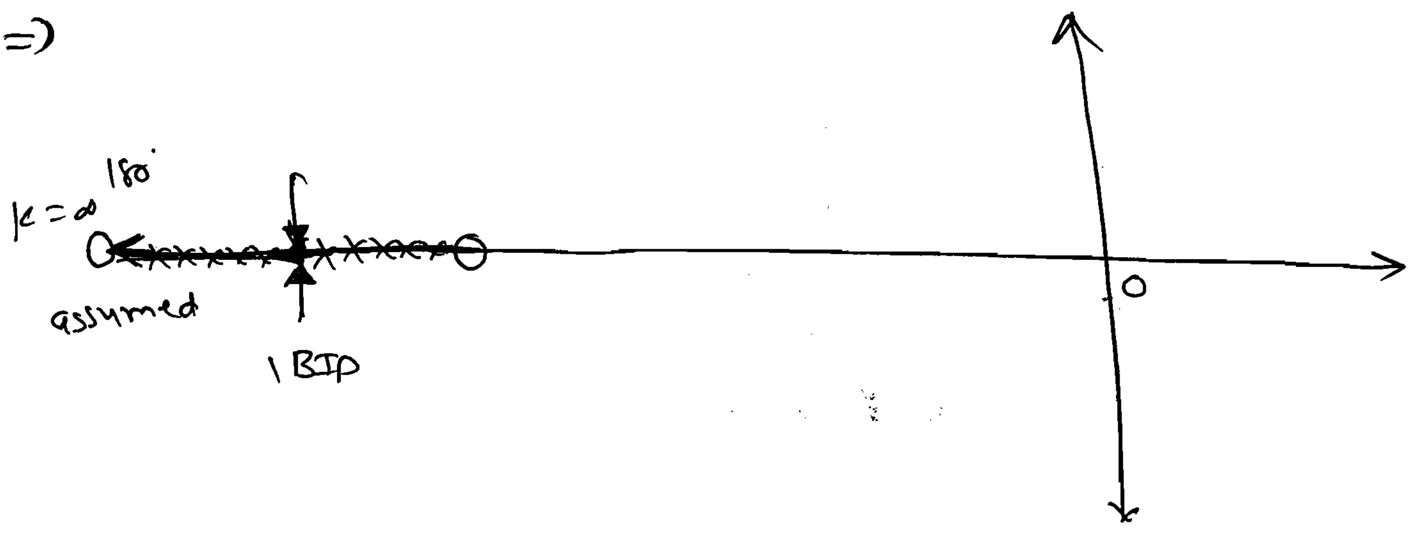
e.g. $G(s)H(s) = \frac{K(s+2)(s+4)}{s^2}$.



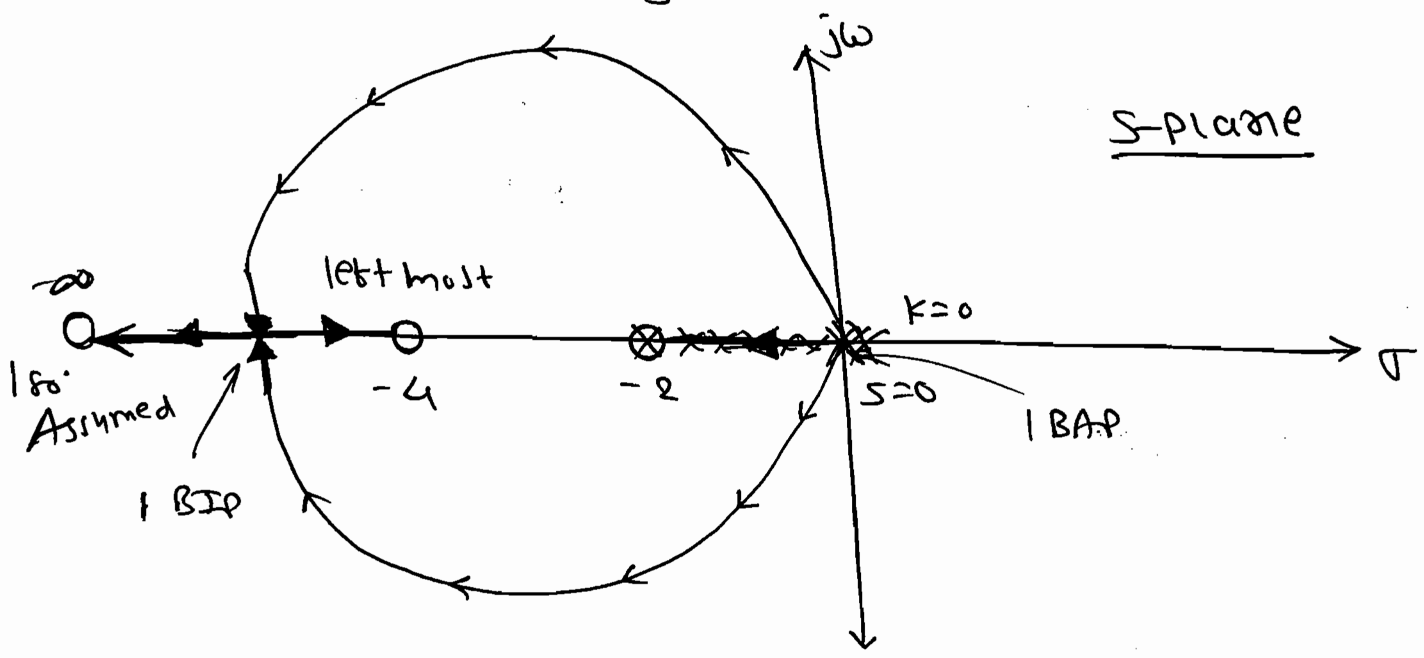
Case – III :- $\boxed{P>2}$

⇒ Whenever there exist left most side zero to the left most side of that zero there exist a RL branch then there should be the minimum one breaking point to the left most side of the zero when no. of poles are greater than no. of zeros.

⇒



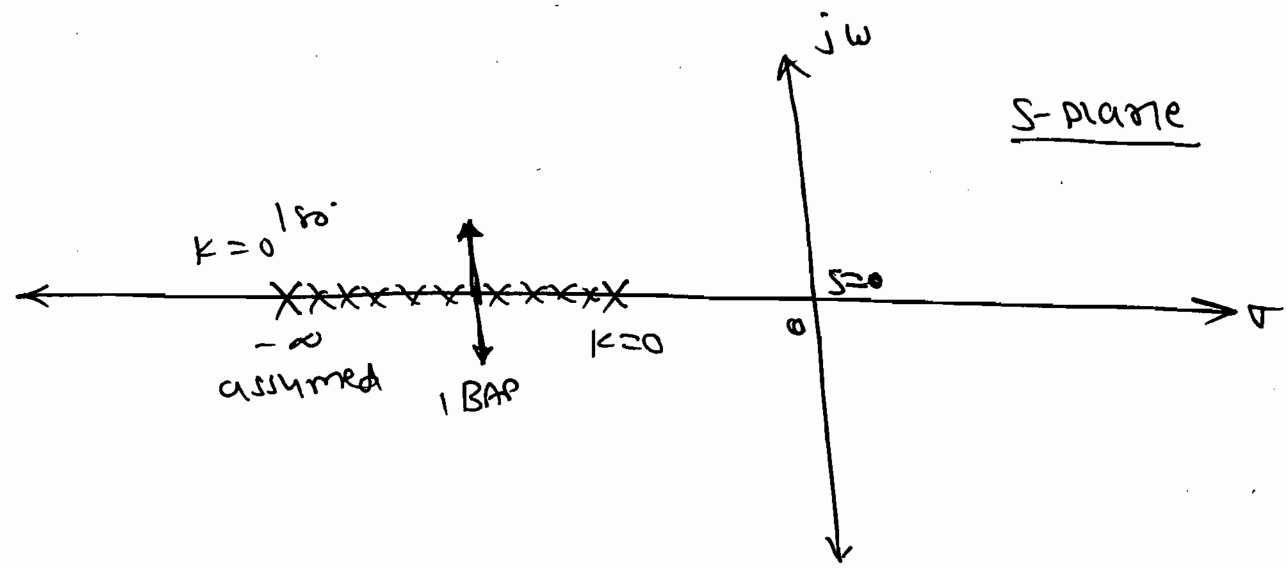
e.g. $G(s) \cdot H(s) = \dfrac{K(s+2)(s+4)}{s^3}$.



2 BPS

Case - IV: $P < Z$

=>



=> whenever there exist a left most side pole to the left most side of that pole there exist a RL branch then there should be a minimum one breakaway point to the left most side of the zero. when the no. of poles less than zeros only.

⇒ This case is practically not exist because the control systems are LPF that means the no. of poles must be greater than zeros only.

* Finding the location of Break points:

Step-1: Form the Char. equation.

Step-2: Rewrite the above eqn in the form of $K = F(s)$.

Step-3: Differentiate k with respect to 's' and make equal to '0'. The roots of $\frac{dK}{ds} = 0$ gives the valid and invalid Breakpoint.

→ The valid BP is the one which must be on RL branch (or), for valid B.P. K value in step-2 should be (+ve).

Q. Find the location of BP.

① $GH = \frac{K}{s(s+2)}$.

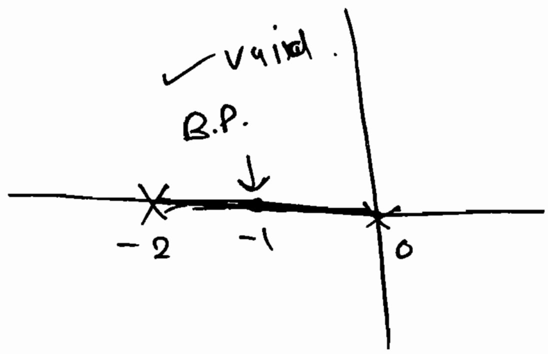
Soln:

$\xrightarrow{CE} 1 + GH = 0$

$1 + \frac{K}{s(s+2)} = 0$

$\Rightarrow K = -s^2 - 2s$

$\frac{dK}{ds} = -2s - 2 = 0 \Rightarrow \boxed{s = -1}$



Q ② $GH = \dfrac{K}{S(S+2)(S+4)}$.

Soln,

$$1 + GH = 0$$

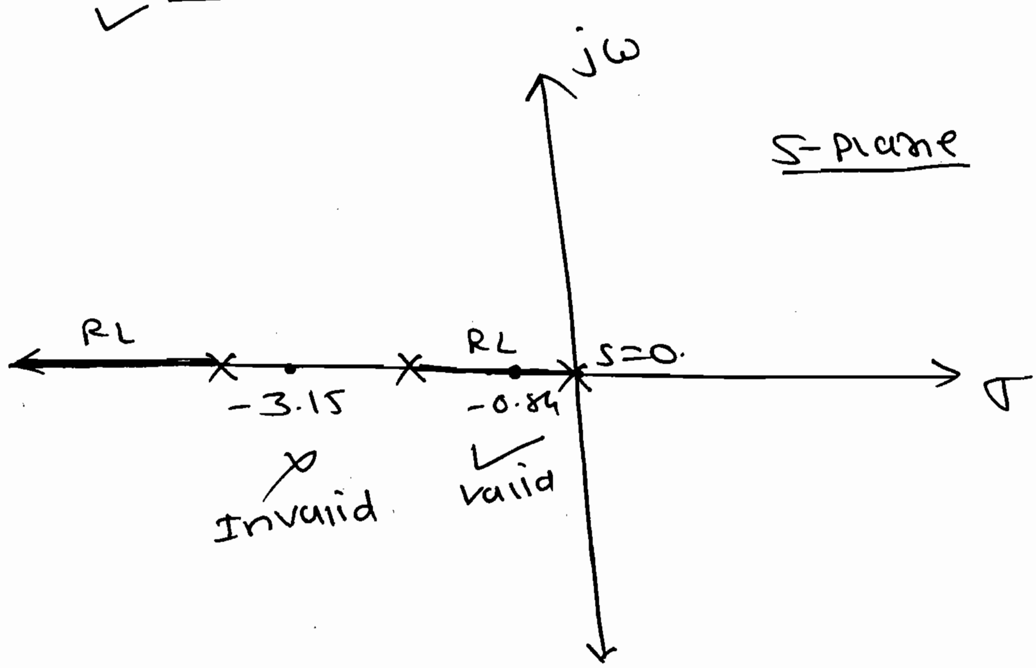
$$\therefore \quad 1 + \frac{K}{S(S+2)(S+4)} = 0.$$

$$K = -S^3 - 6S^2 - 8S.$$

$$\therefore \frac{dK}{dS} = -3S^2 - 12S - 8 = 0.$$

$$\boxed{S = -0.84} \checkmark, \quad \boxed{S = -3.15} \times$$

$\Rightarrow$



③ $GH = \dfrac{K(S+4)}{S(S+2)}$.

Soln

$$1 + GH = 0.$$

$$1 + \frac{K(S+4)}{S(S+2)} = 0.$$

$$\therefore \quad K = -\left[\frac{S^2+2S}{S+4}\right].$$

$$\therefore \frac{dK}{dS} = -\left[\frac{(S+4)(2S+2) - (S^2+2S)(1)}{(S+4)^2}\right] = 0.$$

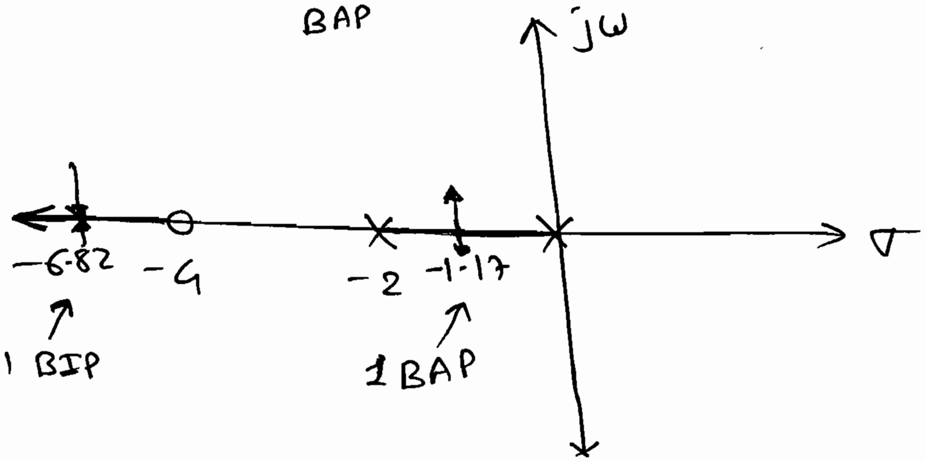
$\Rightarrow \quad -2s^2 - 8s - 2s - 8 + s^2 + 2s = 0.$

$\therefore \quad s^2 + 8s + 8 = 0.$

$s = -1.17, \; -6.82.$

↑ BAP   ↑ BIP.

$\Rightarrow$



Rule - 7 : Intersection Point with Imaginary axis:

$\Rightarrow$ The intersection point with Imaginary axis is obtained by RH criteria.

(i) Form the char. equation.

(ii) Write the Routh - tabular form.

(iii) Find the K marginal value.

(iv) Form the Auxiliary equation. The roots of auxiliary equation gives the valid or invalid intersection point with imaginary.

$\Rightarrow$ For valid intersection point with imaginary the K marginal be +ve.

Q. Find the intersection point with imaginary axis $G(s) \cdot H(s) = \frac{K}{s(s+2)(s+4)}$.

Soln:

$$CE \rightarrow 1 + GH(s) = 0$$

$$1 + \frac{K}{s(s^2+6s+8)} = 0.$$

$$\Rightarrow s^3 + 6s^2 + 8s + K = 0.$$

$$\begin{array}{c|cc} s^3 & 1 & 8 \\ s^2 & 6 & K \\ s^1 & \frac{48-K}{6} & \\ s^0 & K & \end{array}$$

for MS,

$$48 - K = 0$$

$$\Rightarrow \boxed{K_{mar} = 48.}$$

$$\therefore AE \rightarrow 6s^2 + K_{mar} = 0$$

$$6s^2 + 48 = 0$$

$$s^2 = -8$$

$$\boxed{s = \pm j\sqrt{8}.}$$

Rule - 8 :- Angle of Departure | Arrival :-

$\Rightarrow$ The angle of departure calculated at a complex conjugate poles and angle of Arrival calculated at a complex conjugate zero.

⇒ Angle of Departure :-

⇒ It gives that with what angle the pole depart or leaves from the initial position given by angle of departure.

→ $\phi_d = 180^\circ + \angle GH \Big|_{\text{at a (+ve) imag. Complex pole.}}$
→ $\phi_d = 180^\circ - \phi$ where, $\phi = \Sigma \phi_p - \Sigma \phi_z$.

⇒ Angle of Arrival :-

⇒ It gives that with what angle the poles arrives, terminates at the complex zero given by angle of arrival.

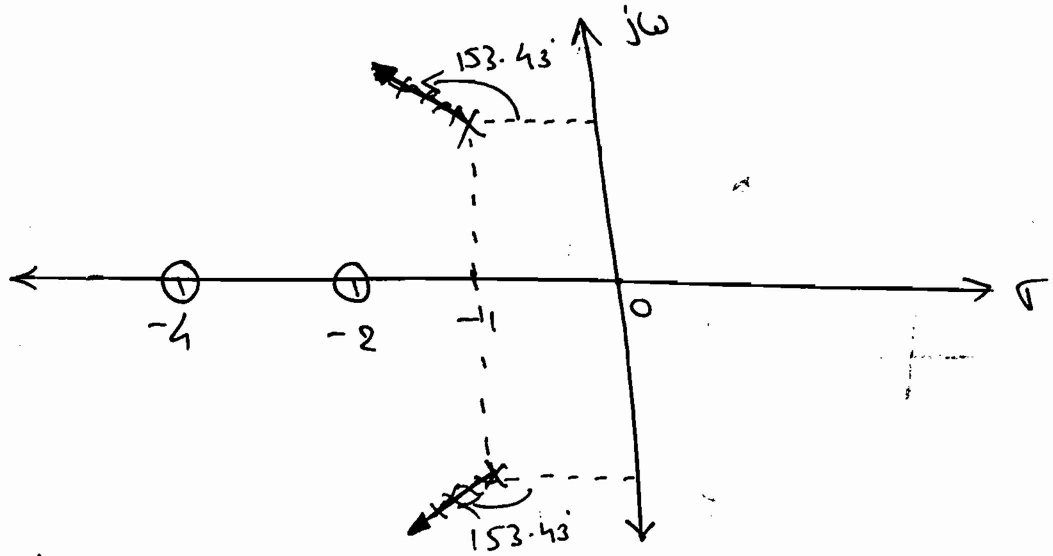
→ $\phi_a = 180 - \angle GH \Big|_{\text{at a (+ve) img. Complex zero.}}$
→ $\phi_a = 180 + \phi$ where, $\phi = \Sigma \phi_p - \Sigma \phi_z$.

$\boxed{Q}$ Calculate the angle of departure at a complex pole $G(s)\cdot H(s) = \dfrac{K(s+2)(s+4)}{s^2+2s+2}$.

Soln:

Poles: $s = -1 \pm j1$.

zeros: $s = -2, -4$.



$$\angle GH\Big|_{s=-1+j1} = \frac{\angle K \cdot \angle 1+j1 \cdot \angle 3+j1}{\angle 0^\circ \cdot \angle j2}$$

$$= \frac{0^\circ + 45^\circ + 18.43^\circ}{0^\circ + 90^\circ}$$

$$\angle GH = -26.27^\circ$$

$$\therefore \phi_d = 180^\circ + \angle GH = 180^\circ + (-26.27^\circ)$$

$$\boxed{\phi_d = 153.43^\circ}$$

$\boxed{Q}$ Calculate Angle angle $G(s)\cdot H(s) = \dfrac{K(s^2+2s+2)}{(s+2)(s+4)}$

Soln:

$$\angle GH\Big|_{s=-1+j} = \frac{\angle K \ \angle 0 \ \angle 2j}{\angle 1+1j \cdot \angle 3+1j}$$

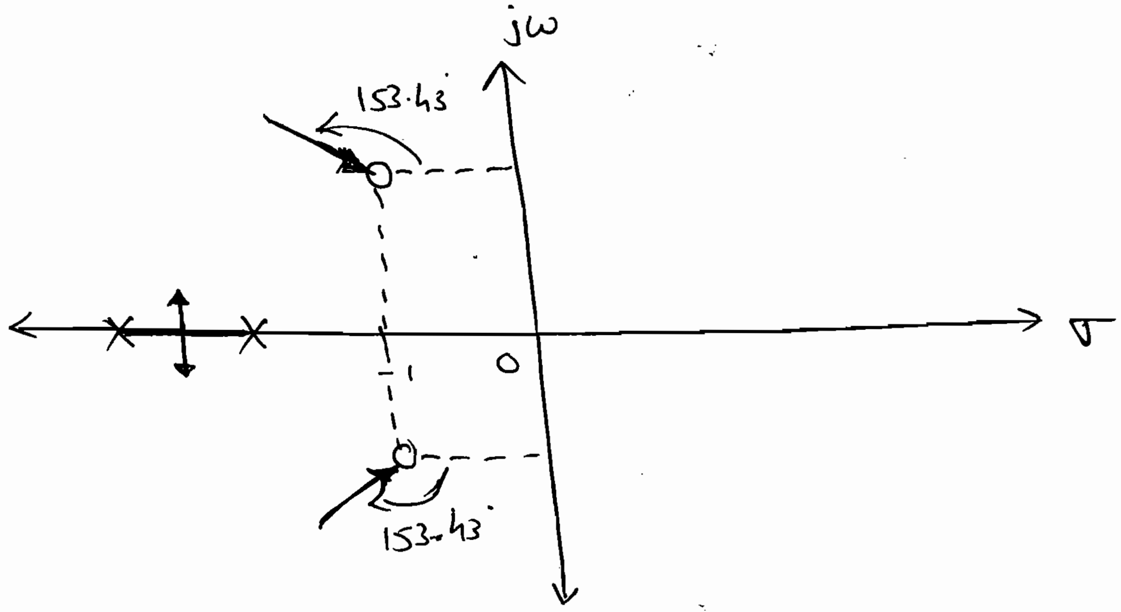
$$= \frac{0+0+90^\circ}{45^\circ + 18.43^\circ}.$$

$\angle GH = +26.27^\circ$

$\therefore \phi_a = 180^\circ - \angle GH$

$= 180^\circ - 26.27^\circ$

$$\boxed{\phi_a = 153.43^\circ}$$



Note:

→ Whenever all the zeros and poles are interchanged the angle of departure equal to angle of arrival. the break in point is equal to Breakaway point.

→ The shape of the RL diagram is also same except the direction.

* Procedure:

1. Identify the RL branches and B.P.
2. Find the centroid and angle of Asymptotes. (If required).
3. Find angle of departure and arrival.

(iv) Vary the K value from 0 to ∞ identify the path from pole to zero such that the Root locus diagram pole must reaches the zero.

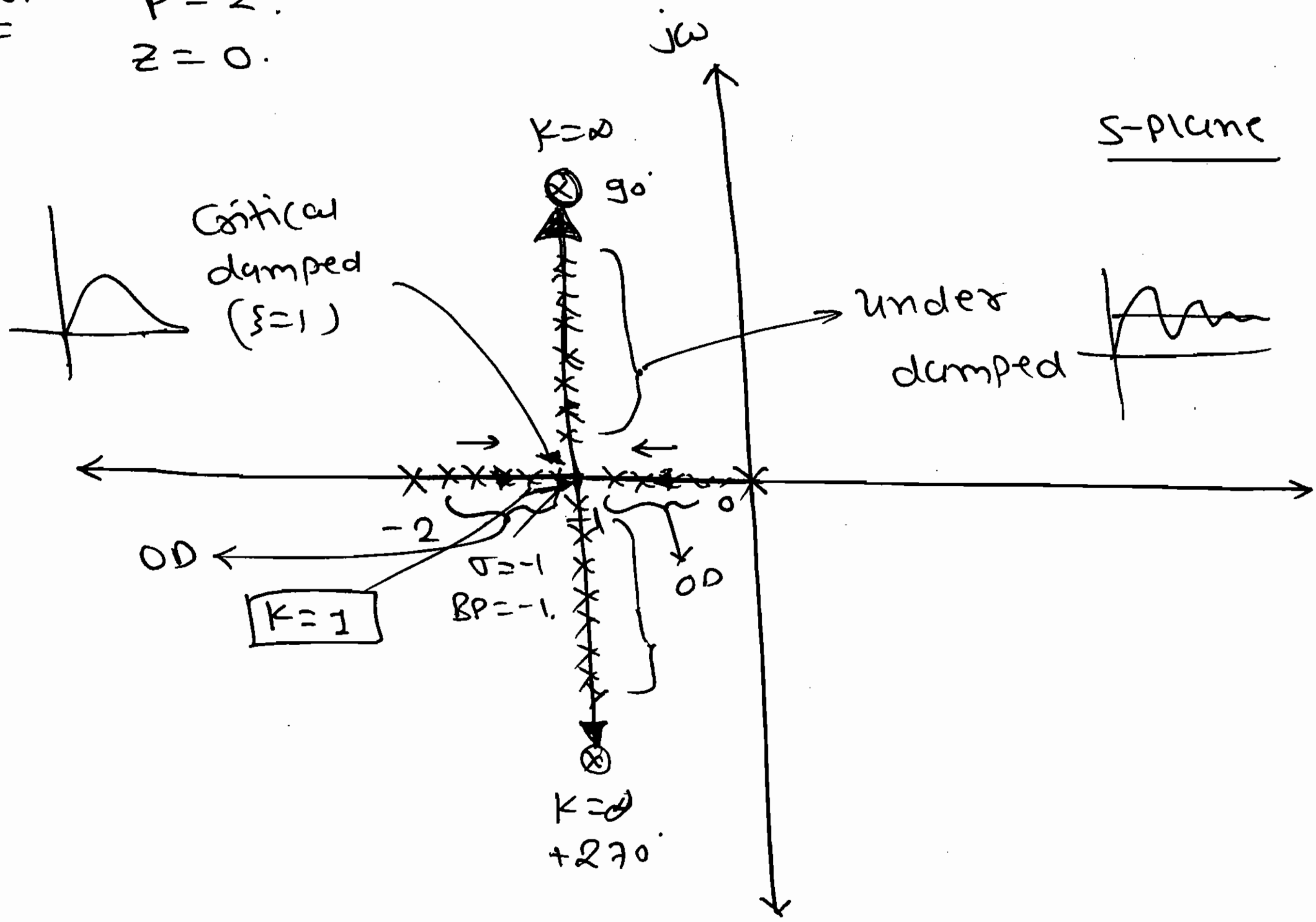
Q Draw the RL diagram to the following systems and find the CL system stability.

① $G(s) \cdot H(s) = \dfrac{K}{s(s+2)}$.

Soln:

$P = 2$.

$Z = 0$.



⇒ Centroid $\sigma = \dfrac{(0-2)-(0)}{2-0}$

$\boxed{\sigma = -1}$

$\Rightarrow \underrightarrow{A.A.}$ $\theta = \dfrac{(2q+1)}{P-2} \times 180^\circ$

$= \dfrac{(2q+1)}{2} \times 180^\circ$

$\theta = 90^\circ, 270^\circ$

$\Rightarrow \underrightarrow{B.P.}$ $s^2 + 2s = 0.$

$2s + 2 = 0.$

$\boxed{s = -1}$

$\Rightarrow$ The above system having over damped, critical damped and under damped nature but not undamped. To set the K values for different nature of the systems are required to find K value at the BP.

Method - I:

$\underrightarrow{M.C.}$ $\left| \dfrac{K}{s(s+2)} \right|_{s=-1} = 1.$

$\Rightarrow \left| \dfrac{K}{(-1)(1)} \right| = 1$

$\Rightarrow \boxed{K = 1}$

Method - II:

$K = \dfrac{\text{Product of lengths from the pt to poles}}{\text{Product of lengths from the pt to zeros.}}$

$$= \frac{(1)(1)}{\text{No zero}}$$

$$\boxed{K = 1}$$

$\Rightarrow$ $0 < K < 1 \longrightarrow$ OD.

$K = 1 \longrightarrow$ CD.

$K > 1 \longrightarrow$ underdamped.

$\Rightarrow$ The Root Locus diagram gives the CL poles path. But not the OL poles path.

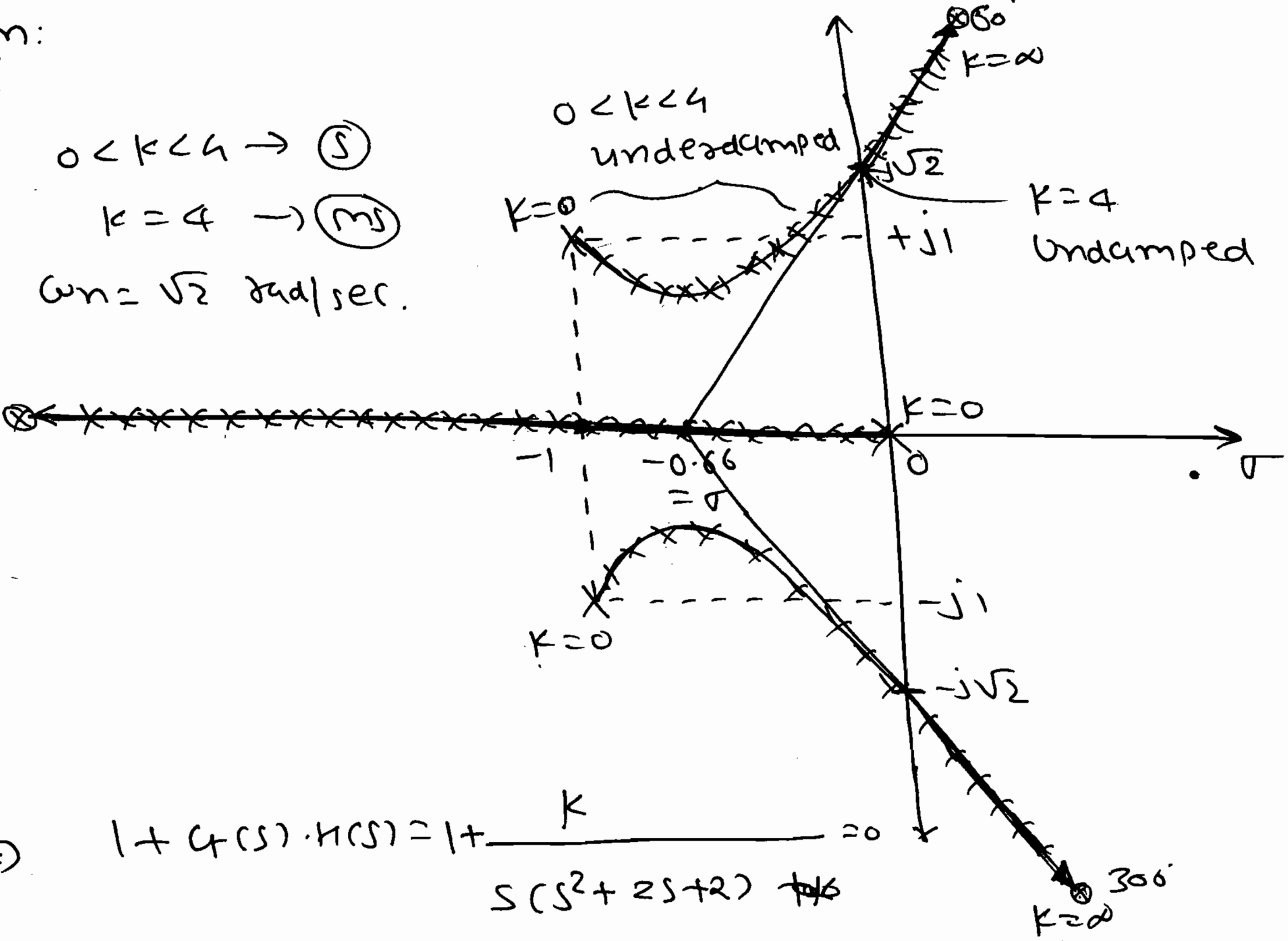
② $$G(s) \cdot H(s) = \frac{K}{s(s^2+2s+2)}.$$

Soln:

$0 < K < 4 \rightarrow$ S

$K = 4 \rightarrow$ MS

$\omega_n = \sqrt{2}$ rad/sec.



$$\Rightarrow 1 + G(s) \cdot H(s) = 1 + \frac{K}{s(s^2+2s+2)} = 0$$

$$= s^3 + 2s^2 + 2s + K = 0.$$

$\Rightarrow$ centroid $\sigma = + \dfrac{(-1-1) - 0}{P-Z}$

$= -2/3$

$\boxed{\sigma = -0.66}$

$\rightarrow$ BP $s^3 + 2s^2 + 2s = 0$

$3s^2 + 4s + 2 = 0.$

$s = -\frac{2}{3} + j2/3 \quad , -2/3 - j4/3 \times$ (Not valid)

$\rightarrow$ A.A. $\theta = \dfrac{(2q+1)180°}{P-Z}$

$= \dfrac{(2q+1)\cancel{180°}\,60°}{\cancel{3}}$

$\theta = 60°, 180°, 300°.$

$\rightarrow$ I.P $s^3 + 2s^2 + 2s + K = 0$

$\boxed{K = 4}$

AE $\rightarrow$ $2s^2 + \overset{K}{\cancel{2s}} = 0$

$\cancel{4s + 2 = 0}$

$2s^2 = -4$

$\boxed{s = \pm j\sqrt{2}.}$

$\therefore \quad j\omega_n = \pm j\sqrt{2}$

$\boxed{\omega_n = \sqrt{2} \text{ rad/sec}}$

$\Rightarrow$ Angle of Departure:

$\angle GH = \dfrac{\angle K}{\angle 1+j \quad \angle 2j} = \dfrac{0°}{135° + 90°} = -225°$

$$\therefore \quad \phi_d = 180^\circ + \angle GH$$

$$= 180^\circ - 225^\circ$$

$$\boxed{\phi_d = -45^\circ}$$

$\boxed{Q}$ $\quad GH = \dfrac{KS}{S^2+4}$.

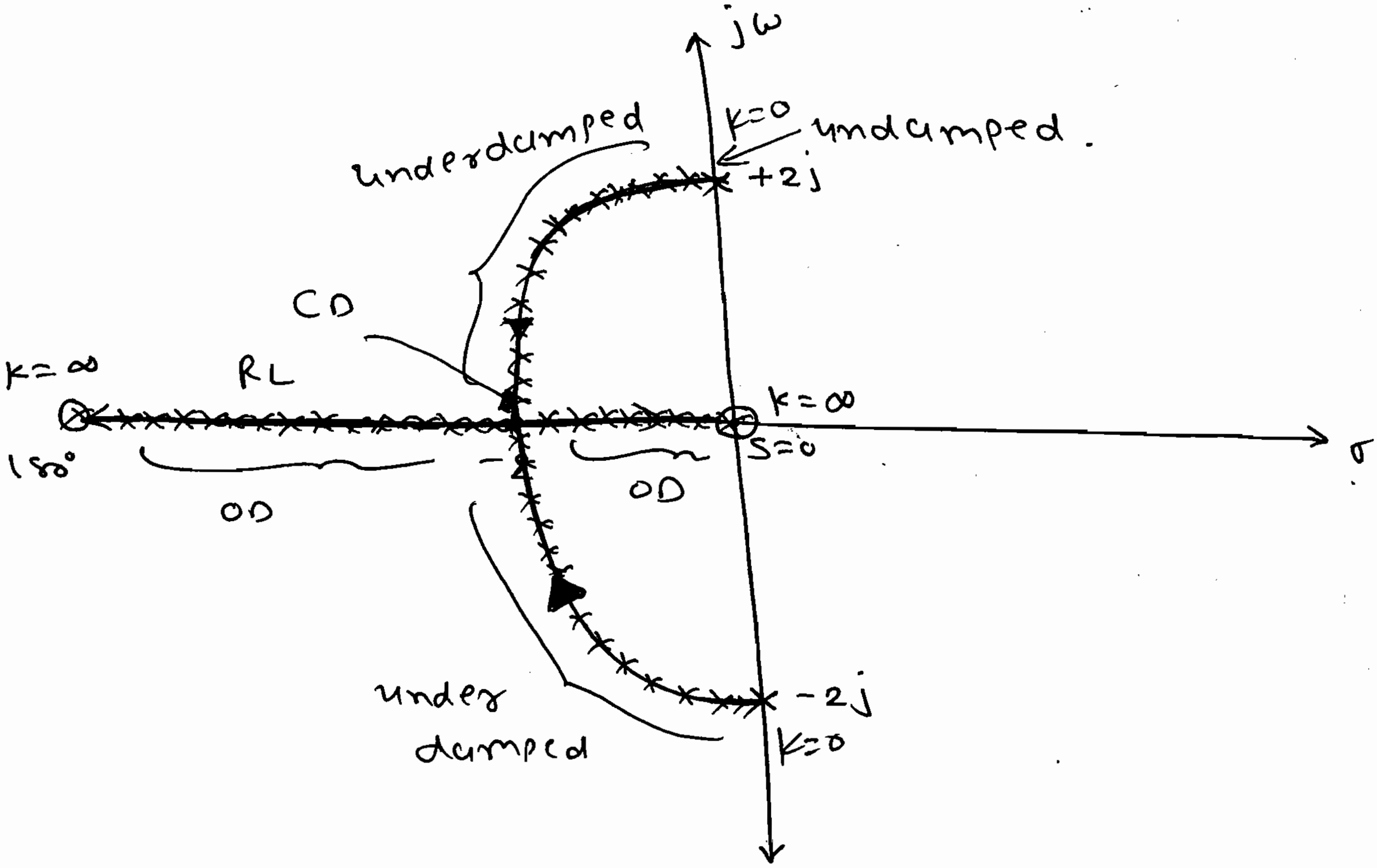
$\underline{\underline{\text{Soln}}}$:

$P = 2, \quad Z = 1.$

$P - Z = N = 1.$

$\xrightarrow{A.A.} \dfrac{(2q+1)180^\circ}{P-Z} = \dfrac{(2q+1)180^\circ}{1} = 180^\circ.$

<u>Note:</u> The centroid is required when the no. of asymptotes are more than 1.



$\xrightarrow{BP.}$ $\quad 1 + GH = 0.$

$$1 + \frac{KS}{S^2+4} = 0 \qquad \Rightarrow \quad \frac{dK}{dS} = -\left[\frac{S(2S) - S^2 - 4}{S^2}\right] = 0.$$

$$K = -\left[\frac{S^2+4}{S}\right] \qquad \Rightarrow \quad 2S^2 - S^2 = 4$$

$$S^2 = 4 \quad \Rightarrow \boxed{S = -2}, \quad \cancel{S = +2}$$

$$\angle GH\Big|_{s=2j} = \frac{\angle K \ \angle 2j}{\angle 0 \ \angle 4j} = 0^\circ$$

$$\phi_d = 180^\circ + \angle GH = 180^\circ$$

$\Rightarrow \xrightarrow{M.C.}$ $$\left| GH \right|_{s=-2} = 1.$$

$$\therefore \left| \frac{K(-2)}{4+4} \right| = 1.$$

$$\boxed{K=4}$$

$\Rightarrow$ $K=0 \rightarrow$ undamped.

$0 < K < 4 \rightarrow$ underdamped.

$K = 4 \rightarrow$ critical damped.

$\infty > K > 4 \longrightarrow$ OD.

$\boxed{a}$ $$GH(s) = \frac{K}{s^4-1}.$$

Soln.: Poles: $s^4 - 1 = 0$

$(s^2-1)(s^2+1) = 0$

$\Rightarrow s = \pm 1, \pm j.$

$\Rightarrow P - Z = 4 - 0 = 4 = N.$

$\xrightarrow{A.A.}$ $$\theta = \frac{(2q+1)}{4} \times \cancel{180} \ 45^\circ$$

$\theta = 45^\circ, 90^\circ, 135^\circ, 225, 315^\circ$

Centroid $$\sigma = \frac{(-1+1)-0}{4} = 0. \Rightarrow \boxed{\sigma = 0}$$

BP $\longrightarrow$ $4s^3 = 0$

$$\boxed{B.P = 0}$$

$\xrightarrow{A.C.}$ $\angle GH \Big|_{s=+j1} = \dfrac{\angle K}{\angle 1+j \ \angle -1+j \ \angle 0 \ \angle 2j}$

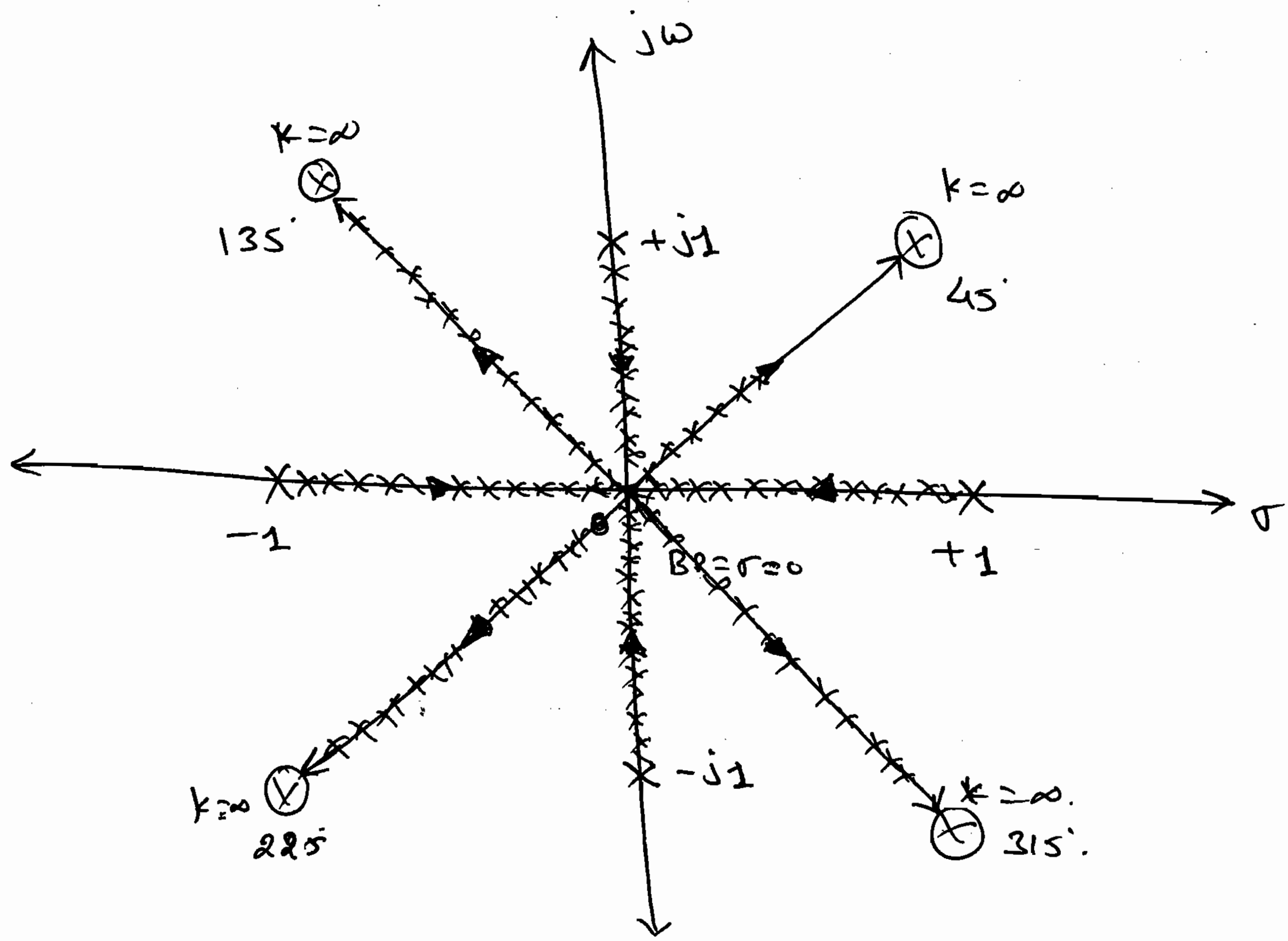
$= \dfrac{0^\circ}{45^\circ + 135^\circ + 90^\circ}$

$= -270^\circ$

$\therefore \ \phi_d = 180^\circ - 270^\circ = -90^\circ$

$\Rightarrow$ $\boxed{B_P = \sigma \Rightarrow \phi_d = \mp 90^\circ}$

$\Rightarrow$



$\Rightarrow$ $\xrightarrow{M.C.}$ $\left| \dfrac{K}{0-1} \right| = 1 \Rightarrow \boxed{K=1}$

$\Rightarrow$ Whenever BP = centroid and the RL diagram having complex conjugate poles then the angle of departure at complex conjugate pole is $\mp 90^\circ$.

⇒ Whenever BP = σ and all the poles symmetrical about BP then all the poles meet at the BP.

⇒ In the above root locus diagram four poles meet at the break point then K value at the BP is $K = 1$.

⇒ The CL TF at the BP is $\frac{C(s)}{R(s)} = \frac{1}{s^4}$.

$\boxed{a}$ $G_H = \frac{K(s+2)(s+4)}{(s^2+2s+2)}$.

Soln:

Pole: $s = -1 \pm j$

zero: $s = -2, -4$.

⇒ $N = P - 2 = 0 \Rightarrow \boxed{P=2}$

Note: whenever no. poles ≤ no. of zeros then centroid is not required and angle of asymptotes not exist.

BP. ⟶ $K = -\frac{(s^2+2s+2)}{(s^2+6s+8)}$

$$\frac{dK}{ds} = -\left[\frac{(s^2+6s+8)(2s+2) - (s^2+2s+2)(2s+6)}{(s^2+6s+8)^2}\right] = 0.$$

⇒ ~~$2s^3 + 12s^2 + 16s + 2s^2 + 12s + 16 - 2s^3 - 6s^2 - 4s^2$~~

~~$-12s - 12 = 0$.~~

~~$4s^2 + 16s + 4 = 0$~~

~~$8s^2 + 28s + 4 = 0$.~~

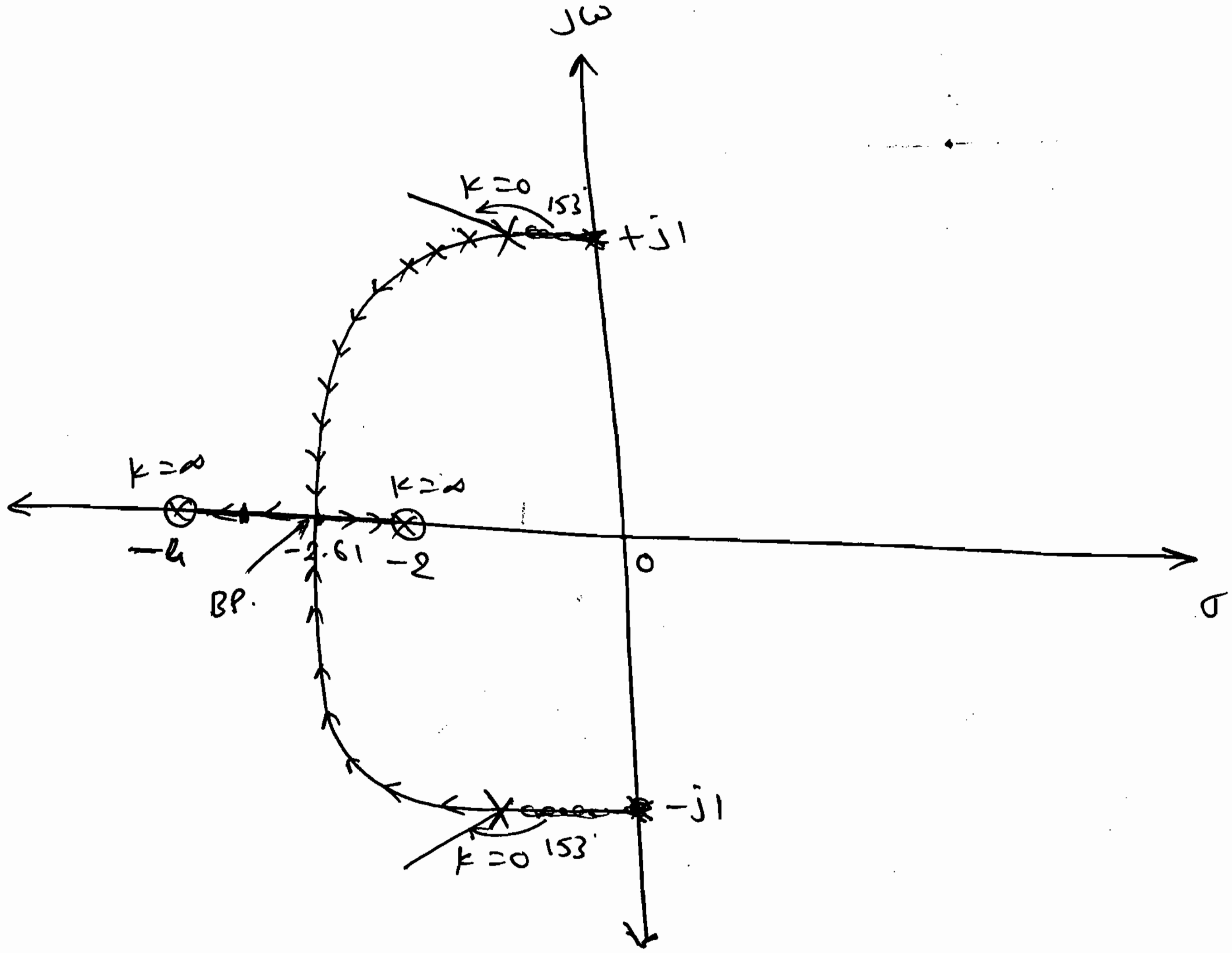
$\Rightarrow \quad \cancel{2}s^3 + 2s^2 + 12s^2 + 1\cancel{2}s + 16s + 16 - 2\cancel{s^3} - 6s^2$
$- 4s^2 - 1\cancel{2}s - 4s - 12 = 0.$

$4s^2 + 12s + 4 = 0$

$s = -0.38, \cancel{}$

$s = -2.61$ ✓

$\Rightarrow$



$\Rightarrow \quad \angle GH\Big|_{s=-1+j} = \dfrac{\angle K \angle 1+j \angle 3+j}{\angle 0^\circ \angle 2j}$

$= \dfrac{\angle 0^\circ + 45^\circ + 18.43^\circ}{90^\circ}$

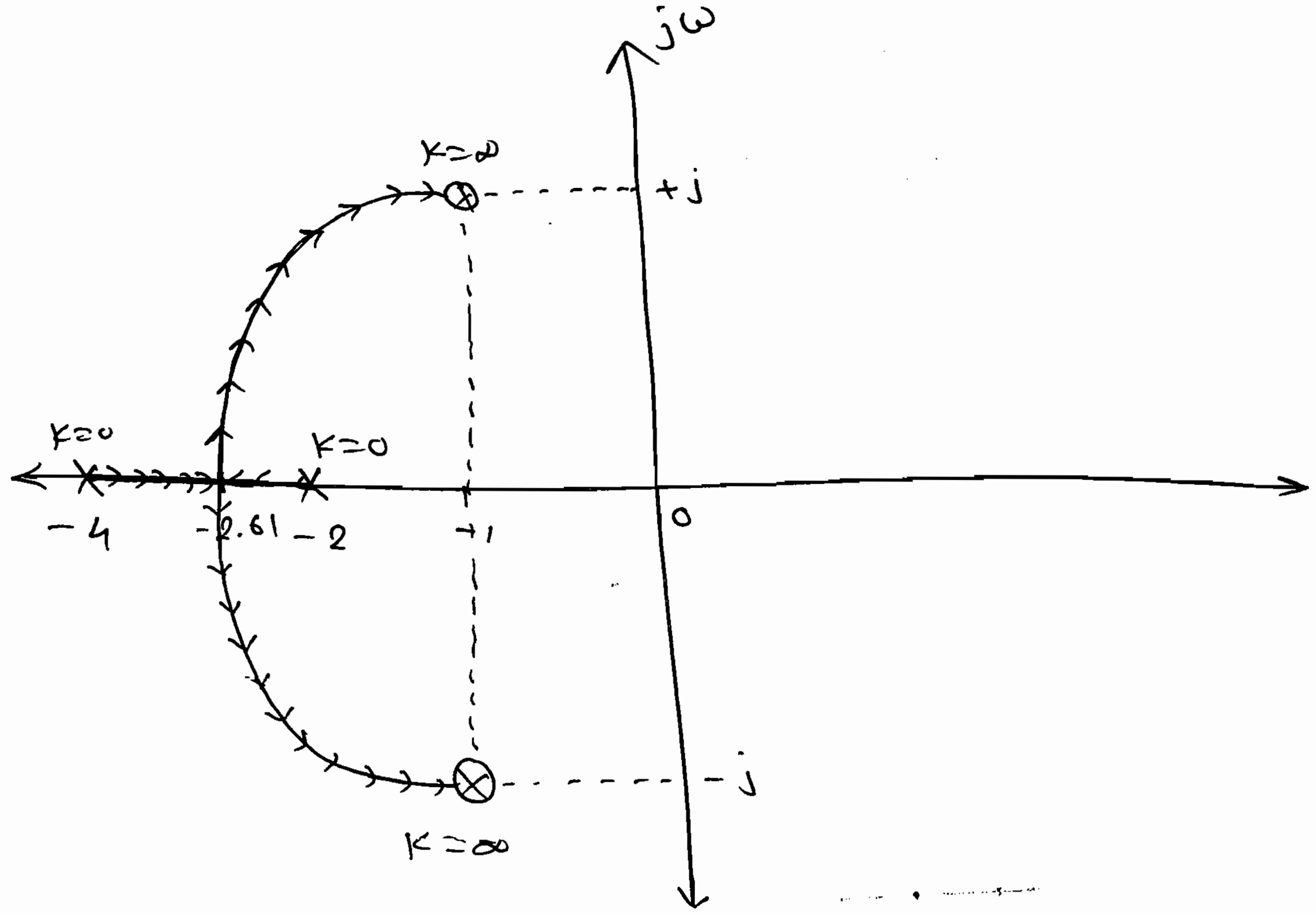
$= -26.27^\circ$

$\phi_d = 180^\circ + \angle GH$

$\phi_d = 180 - 26.27^\circ \quad \Rightarrow \quad \boxed{\phi_d = 153.67^\circ}$

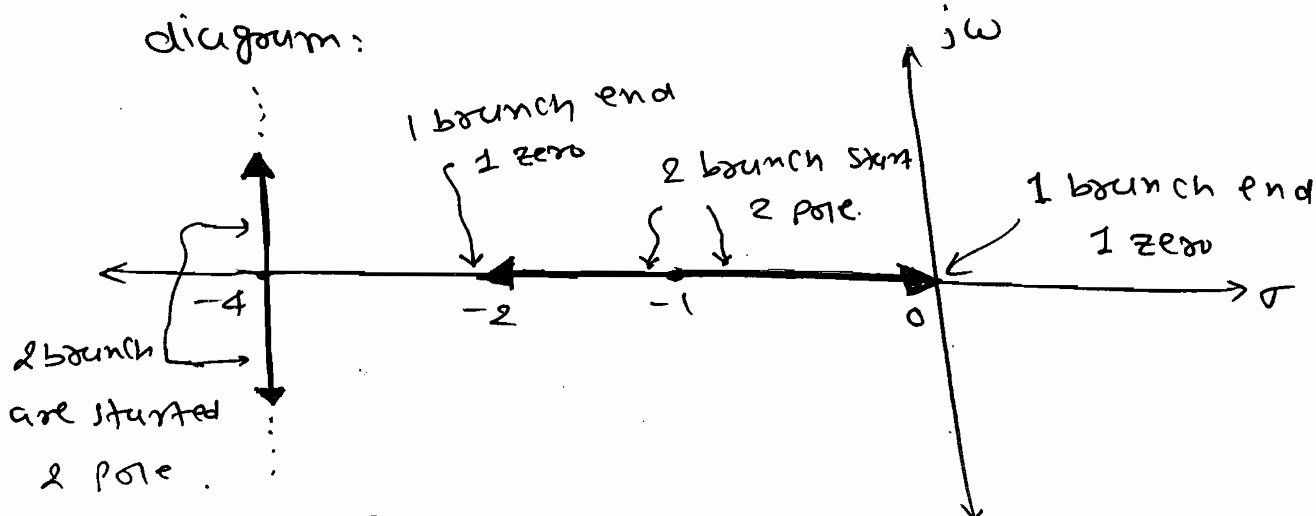
$\boxed{Q}$ $GH = \dfrac{K(s^2+2s+2)}{(s+2)(s+4)}$.

Soln:



Note: The correct Root locus diagram is the one that it should be symmetrical about the real axis and the direction of the root locus branch is from pole to zero when K is increase from 0 to ∞.

$\boxed{Q}$ Find the TF to the given root locus diagram:



$$\Rightarrow \quad \boxed{GH = \frac{Ks(s+2)}{(s+1)^2 (s+4)^2}}.$$

Q Draw the Root Locus of the following:

(1) $$GH = \frac{K}{s(s+1)^2 (s+2)}$$

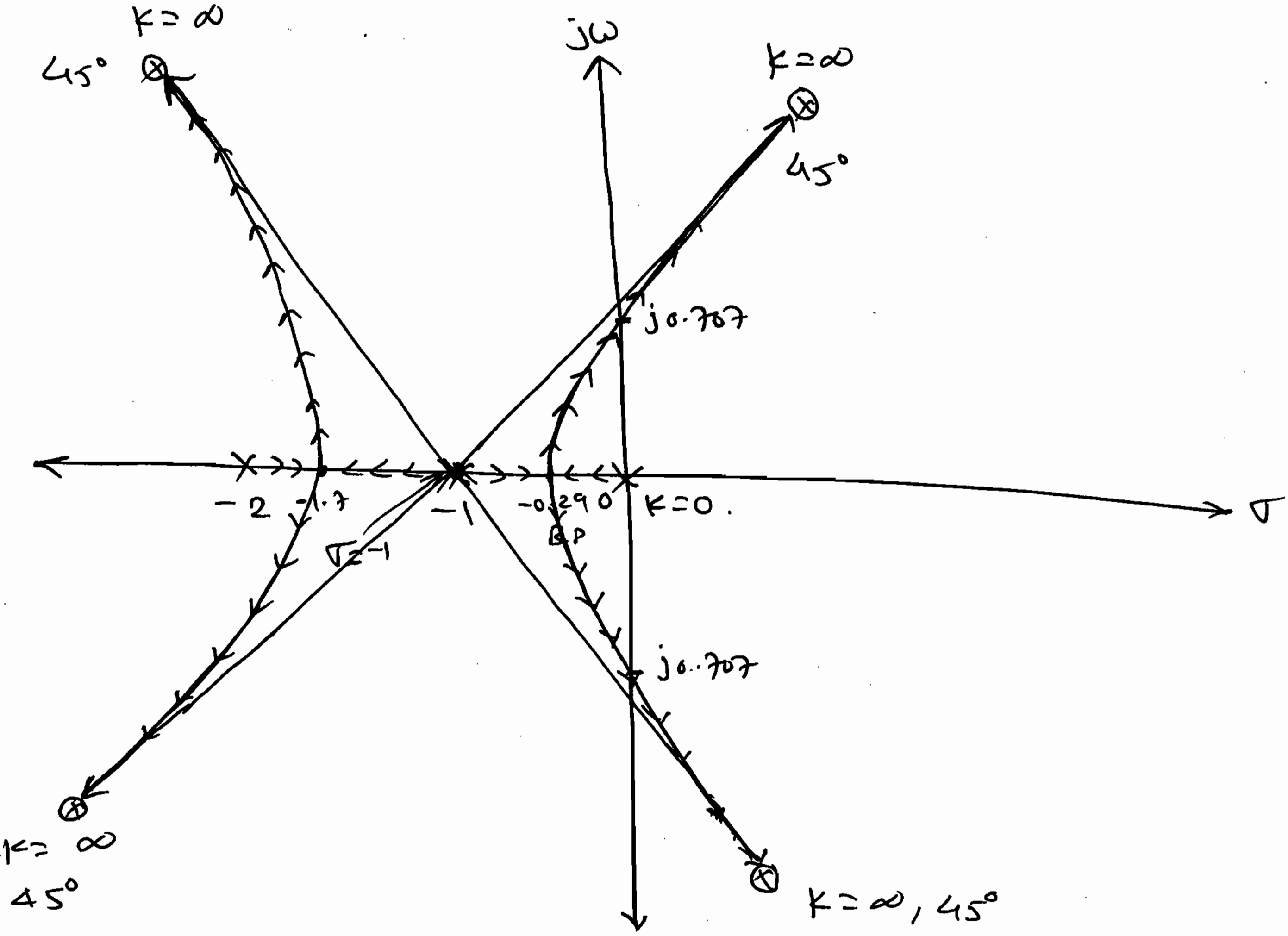
Sol^n: Poles: $P = 4$, $s=0$, $s=-1,-1$, $s=-2$

Zeros: $Z=0$

$\Rightarrow$ $P-Z = 4-0 = 4$, $N=4$,

$\xrightarrow{A.A.}$ $\theta = \frac{(2q+1)}{(P-Z)} \times 180^\circ = \frac{(2q+1)}{4} \times 180^\circ$

$\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$.



$\Rightarrow$ Centroid $\sigma = \frac{0-1-1-2-0}{4} = -1$.

$\underrightarrow{B.P}$ $K = -s(s+2)(s+1)^2.$

$\therefore K = -[(s^2+2s)(s^2+2s+1)].$

$\therefore K = -[s^4 + 2s^3 + s^2 + 2s^3 + 4s^2 + 2s].$

$$\frac{dK}{ds} = -[4s^3 + 12s^2 + \cancel{1}0s + 2] = 0.$$

$\therefore$ $\underrightarrow{B.P}$ $s = -0.292, -1.71, \cancel{-1}.$

$\therefore$

s^4	1	5	K
s^3	4	2	
s^2	$9/2$	K	
s^1	$\frac{9-4K}{9/2}$		
s^0	K		

$\Rightarrow$ $9 - 4K_{marg} = 0$

$K_{mar} = 9/4.$

$\therefore$ $\underrightarrow{AE}$ $9s^2 + K = 0$

$\frac{9s^2}{2} = -9/4 \cdot 2$

$$\boxed{s = \pm j1/\sqrt{2}}$$

2) $GH = \dfrac{K(s+1)^2}{s(s+2)}.$

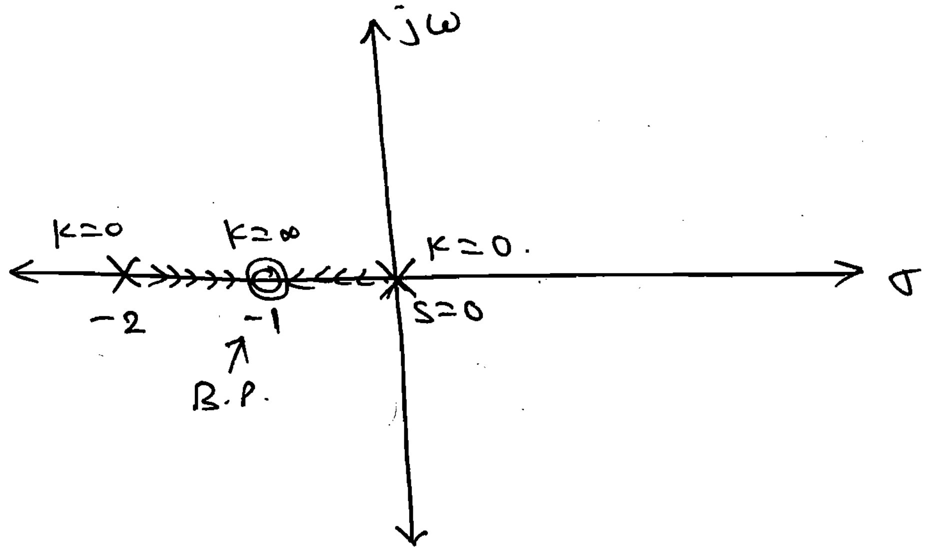
$\underline{Sol^n}$: Poles: $s = 0, -2 \Rightarrow P = 2$

zeros: $s = -1, -1, \Rightarrow Z = 2.$

$P - Z = N = 2 - 2 = 0 \Rightarrow$ No asymptotes.

No centroid.

$\Rightarrow$



$\Rightarrow$ $\xrightarrow{B.P}$ $K = -\dfrac{(s^2+2s)}{(s^2+2s+1)}$

$$\Rightarrow \frac{dK}{ds} = -\left[\frac{(s^2+2s+1)(2s+2) - (s^2+2s)(2s+2)}{0}\right] = 0.$$

$\Rightarrow$ $2s+2=0$

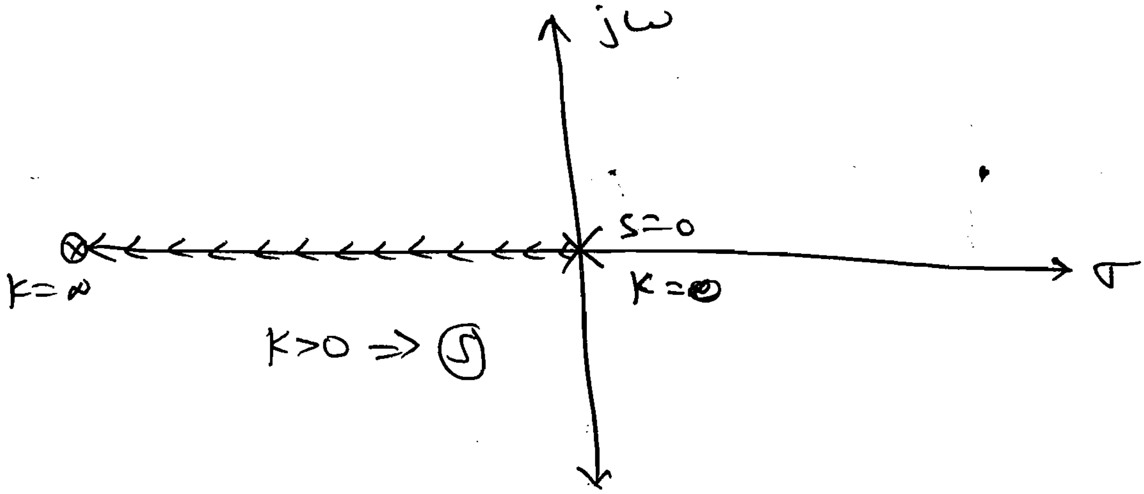
$\boxed{s=-1} \leftarrow$ B.P.

(3) $G_H = \dfrac{K}{s}$.

Note: → Whenever the transfer function consist only pole at origin then the Root Locus diagram is nothing but angle of asymptotes.

$\Rightarrow$ $P \Rightarrow 1 \Rightarrow s=0.$, $Z \Rightarrow 0.$

$P-Z = N = 1 \Rightarrow \theta = 180^\circ$

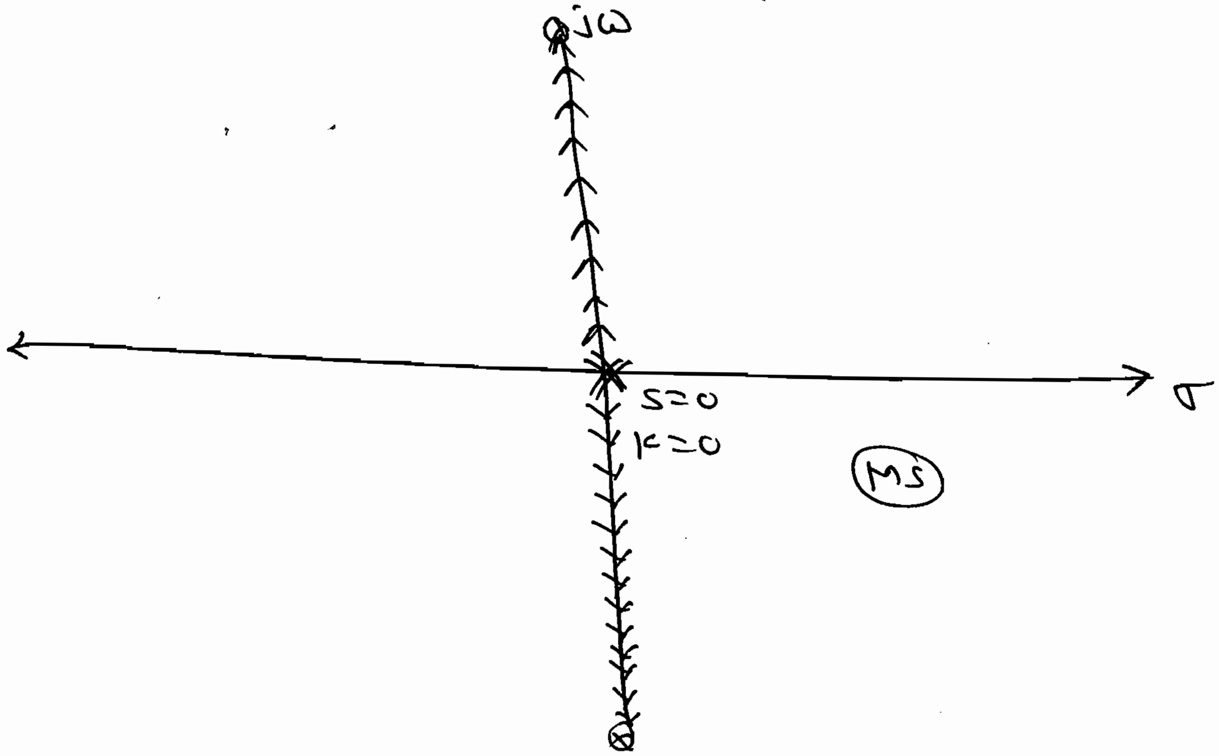


④ $GH = K/s^2$.

Soln: $1 + GH = 0 \Rightarrow s^2 + K = 0 \Rightarrow s = \pm j\sqrt{K}$.

$P = 2, \ Z = 0 \Rightarrow N = P - Z = 2$

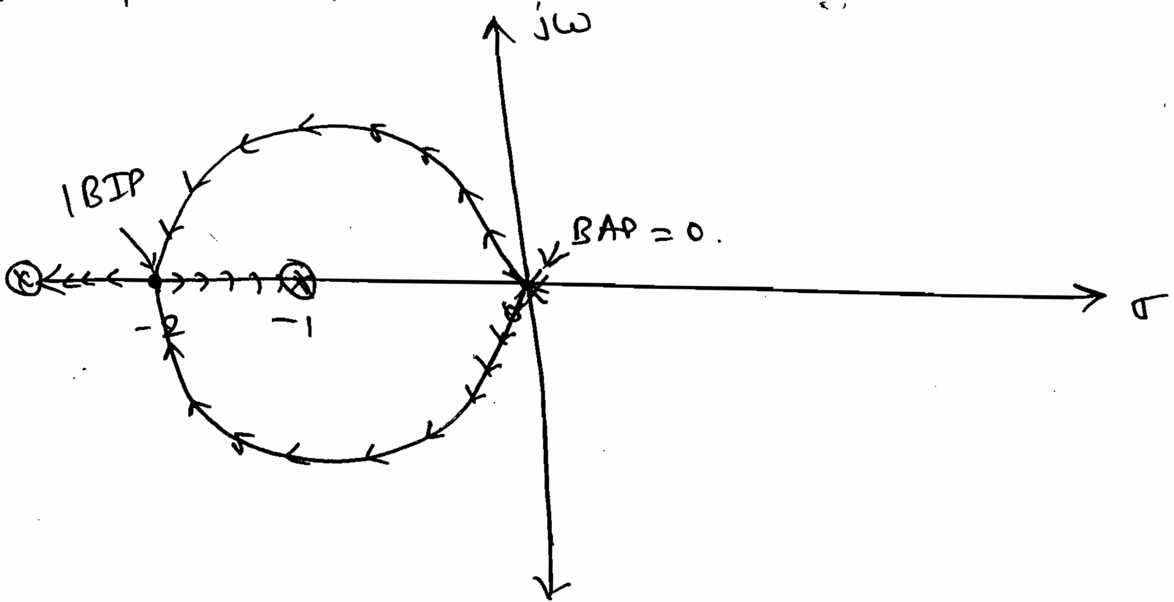
A.A $\rightarrow \theta = 90°, 270°$



Note: The above system is Marginal stable for all values of $K>0$. To make it stable we required to add a finite zero in the left side.

⑤ $\frac{K(s+1)}{s^2}$.

Soln: $P = 2, \ Z = 1, \ P - Z = N = 1 \Rightarrow \theta = 180°$.



$\rightarrow$ B.P. $\quad K = -\dfrac{s^2}{(s+1)}$.

$$\frac{dK}{ds} = -\left[\frac{(s+1)(2s) - s^2}{(s+1)^2}\right] = 0.$$

$$\Rightarrow \quad 2s^2 + 2s - s^2 = 0$$

$$s^2 + 2s = 0$$

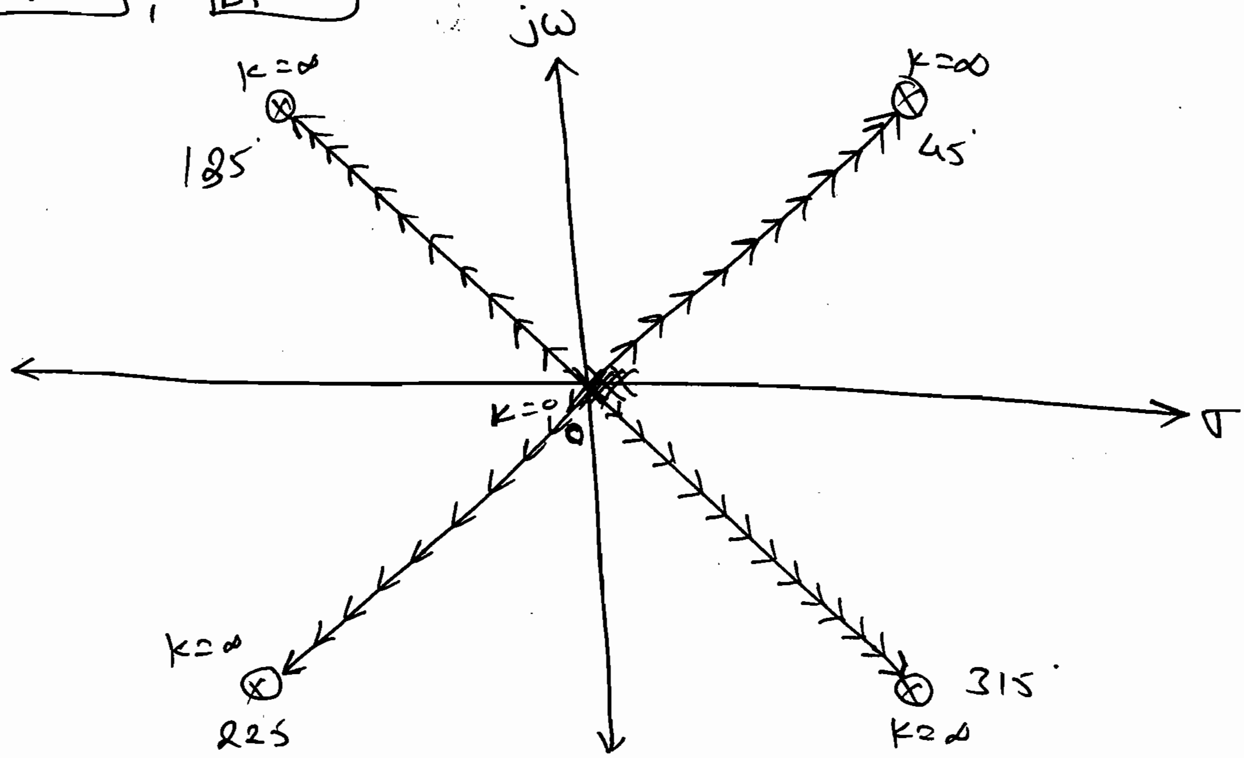
$$\Rightarrow \quad s = 0, -2$$

(6) $GH = \dfrac{K}{s^4}$.

Soln: $P = 4, \quad Z = 0 \quad \Rightarrow \quad P - Z = 4 = N \Rightarrow$

$\xrightarrow{A.A} \theta = 45°, 135°, 225°, 315°$.

$\boxed{\sigma = 0}$, $\boxed{BP = 0}$

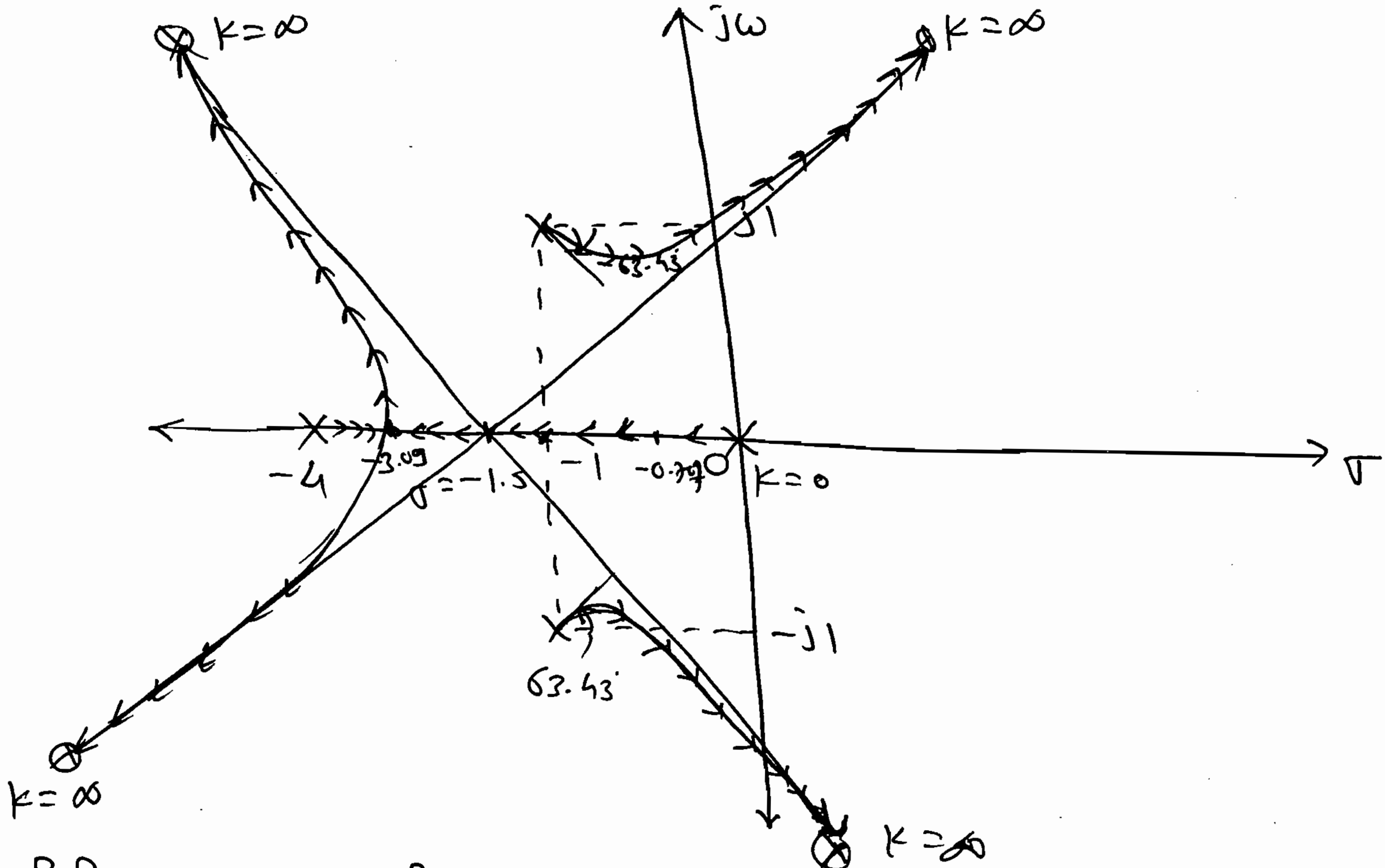


(7) $GH(s) = \dfrac{K}{s(s^2+2s+2)(s+4)}$.

Soln: $P = 4, \quad Z = 0 \quad \Rightarrow \quad P - Z = N = 4 \Rightarrow \theta = 45°, 135°, 225°, 315°$.

$$\sigma = \frac{-0 - 1 - 1 - 4}{4}.$$

$\boxed{\sigma = -1.5}$



B.P. $\rightarrow$ $K = -\left[(s^2+4s)\,(s^2+2s+2)\right].$

$\therefore K = -\left[s^4 + 2s^3 + 2s^2 + 4s^3 + 8s^2 + 8s\right].$

$\therefore K = -\left[s^4 + 6s^3 + 10s^2 + 8s\right].$

$$\frac{dK}{dK} = -\left[4s^3 + 18s^2 + 20s + 8\right] = 0.$$

B.P. $= -3.09,\ -0.703,\ -0.703.$

A.C. $\rightarrow$ $$\angle GH\Big|_{s=-1+j} = \frac{\angle K}{\angle -1+j\ \angle 2j\ \angle\ 3+j}$$

$$= \frac{0^\circ}{135^\circ + 90^\circ + 18.43}$$

$$= -243.43.$$

$\therefore\ \phi_d = 180 + \angle GH$

$$\boxed{\phi_d = -63.435^\circ}$$

* **Effect of addition of Poles and zeros:**

⇒ The addition of Poles and zeros only in the left of s-plane.

① **Addition of Poles:**

⇒ The root locus branches shifted towards the right of s-plane.

⇒ The relative stability of System decreases.

⇒ The range of K value for system stability decreases.

⇒ The system becomes more oscillatory.

⇒ The addition of Poles decreases the BW (higher cut of freq. decreases).

⇒ As BW decreases, the rise time increases ~~increases~~ and the system becomes has slow response.

⇒ The noise is eleminated.

⇒ The system gives the more accurate o/p.

⇒ As the root locus moving towards the right side time constant increases and setteling time also increases.

⇒ The damping ratio decreases.

⇒ As ξ decreases, % M_p increases.

② **Addition of Zeros:**

⇒ The root locus branches shifted towards the left side.

⇒ The system become more relative stable.

⇒ The range k value for system stability increases.

⇒ The system become less oscillatory.

⇒ If BW of the system increases (higher cut off freq. increases).

⇒ As BW increases, rise time decreases the system gives the quick response the noise signal enter into the system (less accurate).

⇒ As the root locus branches shifted toward left side, the time constant decreases and settling time also decreases.

⇒ The damping ratio ξ increases, % M_p decreases the system become more relative stable.

⇒ The addition of pole makes the system

more accurate.

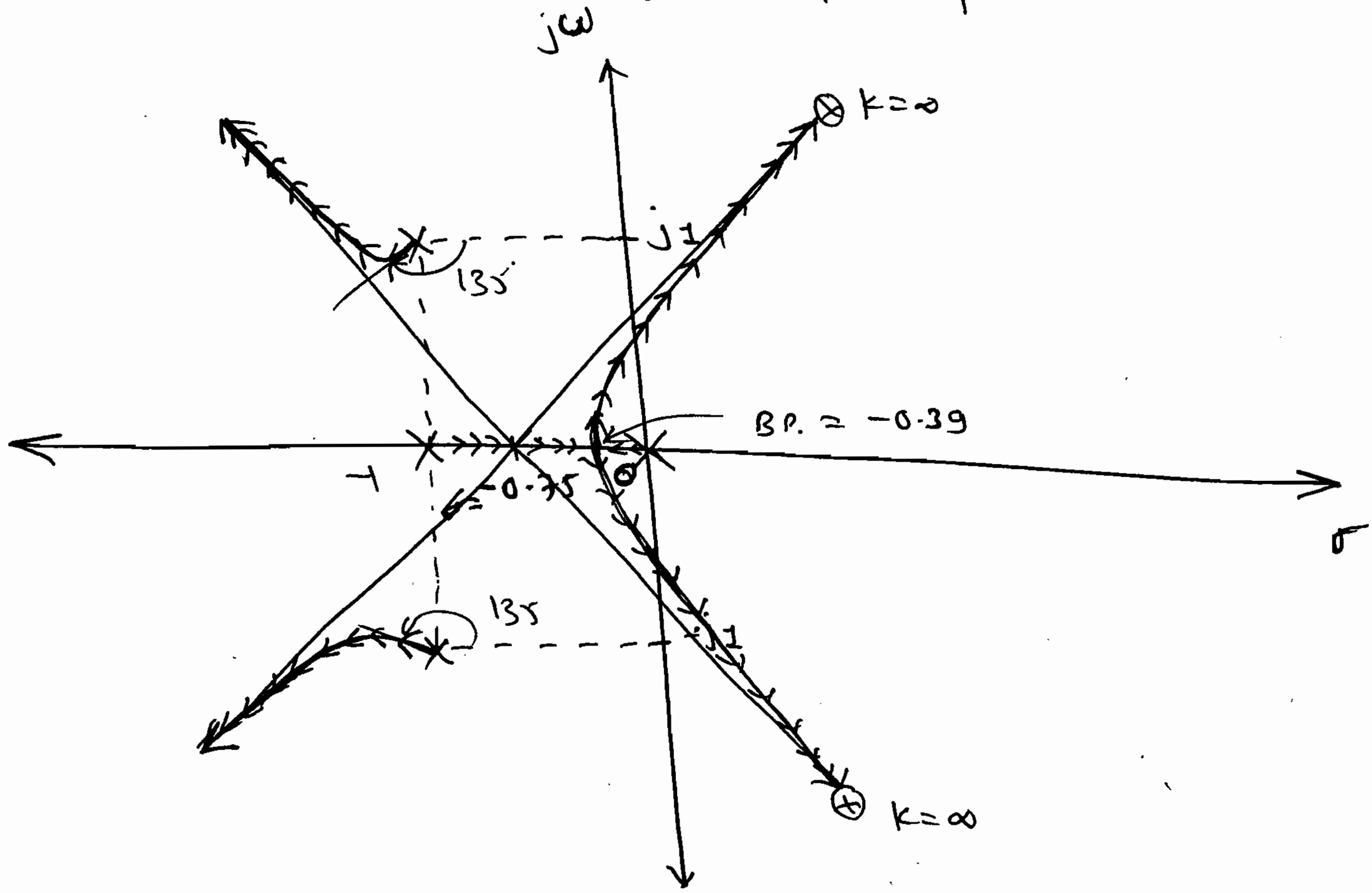
⇒ Addition of zero makes the system quick response.

$\boxed{8}$ $GH(s) = \dfrac{K}{s(s^2+2s+2)(s+1)}$.

Soln:

$P = 4, \quad z = 0, \quad P - z = 4 - 0 = 4 = N.$

$\theta = 45°, 135°, 225°, 315°.$



⇒ $\sigma = \dfrac{0-1-1-1}{4} = -3/4 = -0.75$

B.P. →

$K = -(s^2+s)(s^2+2s+2).$

$K = -[s^4 + 2s^3 + 2s^2 + s^3 + 2s^2 + 2s].$

$K = -[s^4 + 3s^3 + 4s^2 + 2s].$

⇒ $\dfrac{dK}{ds} = -[4s^3 + 9s^2 + 8s + 2] = 0.$

→ B.P. = $-0.39, -0.93, -0.93$

$\Rightarrow \xrightarrow{A.C.} \angle GH\Big|_{s=-1+j} = \dfrac{\angle K}{\angle 1+j \cdot \angle 2j \; \angle j}$

$\angle GH = \dfrac{0^\circ}{135^\circ + 90^\circ + 90^\circ}$

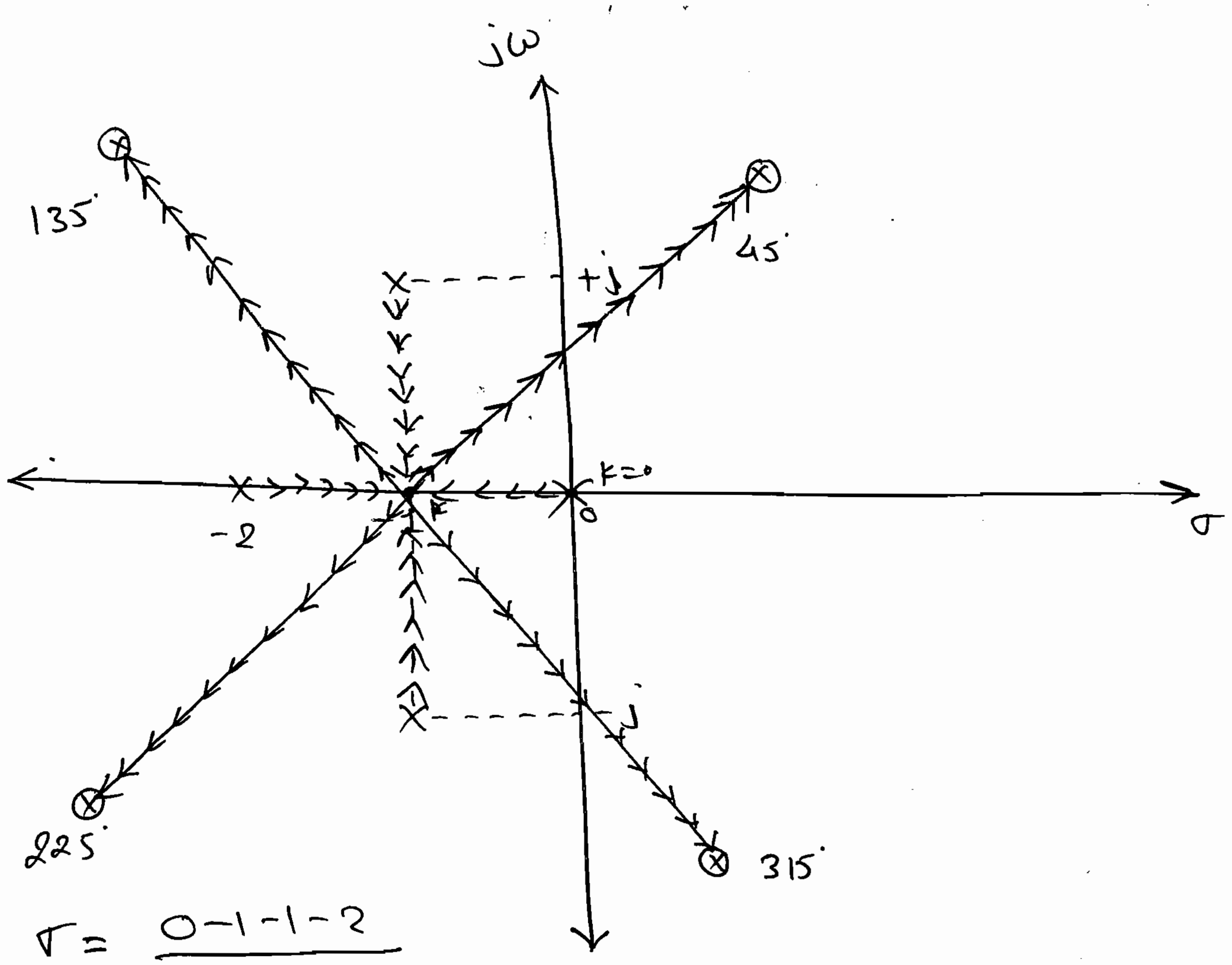
$= -315^\circ$

$\therefore \phi_d = 180 + \angle GH = 180 - 315$

$\boxed{\phi_d = -135^\circ}$

$\boxed{9}$ $GH(s) = \dfrac{K}{s(s^2+2s+2)(s+2)}$

Soln: $P=4,\ Z=0,\ P-Z = N = 4 \Rightarrow \theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ.$



$\Rightarrow \sigma = \dfrac{0-1-1-2}{4}$

$\boxed{\sigma = -1}$

Note: If BP = σ & all the poles symmetrical about BP then all the poles meet at the BP.

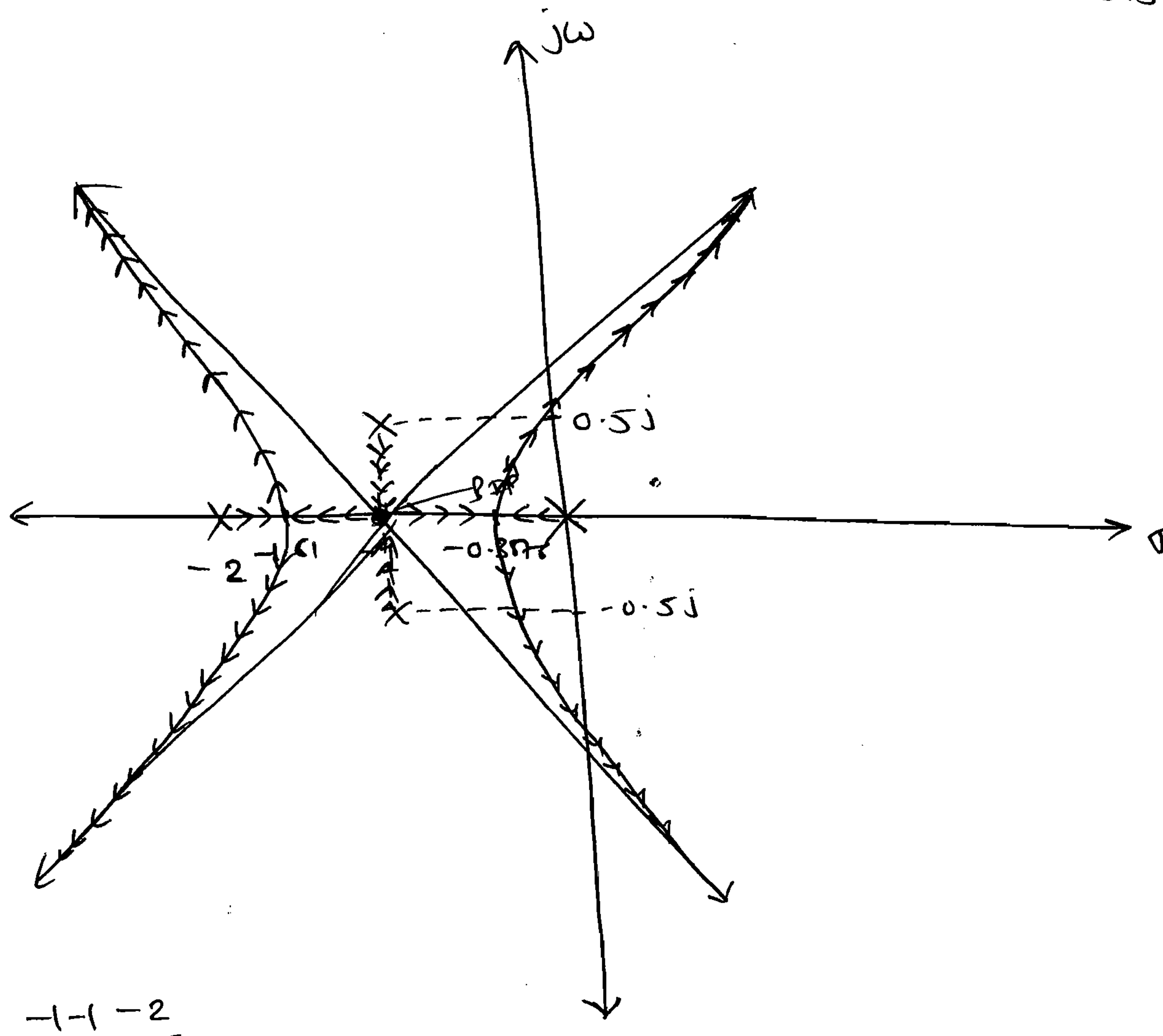
⇒ In the above diagram 4 poles meet at the B.P.

⇒ The K value at the B.P. is $|\times 1 \times 1 \times 1| = 1$.

The CLTF at the BP is $\frac{1}{(s+1)^4}$.

$\boxed{10}$ $GH = \frac{K}{s(s^2+2s+1.25)(s+2)}$.

Sol^n: $P = 4,\ z = 0,\ N = P - 2 = 4 \Rightarrow \theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ.$



$\sigma = \frac{-1-1-2}{4}$

$\boxed{\sigma = -1}$ $\boxed{\cancel{BP = -1}}$

$$K = -\left[(s^2+2s)(s^2+2s+1.25)\right].$$

$$\therefore K = -\left[s^4 + 2s^3 + 1.25s^2 + 2s^3 + 4s^2 + 2.5s\right].$$

$$K = -\left[s^4 + 4s^3 + 5.25s^2 + 2.5s\right]$$

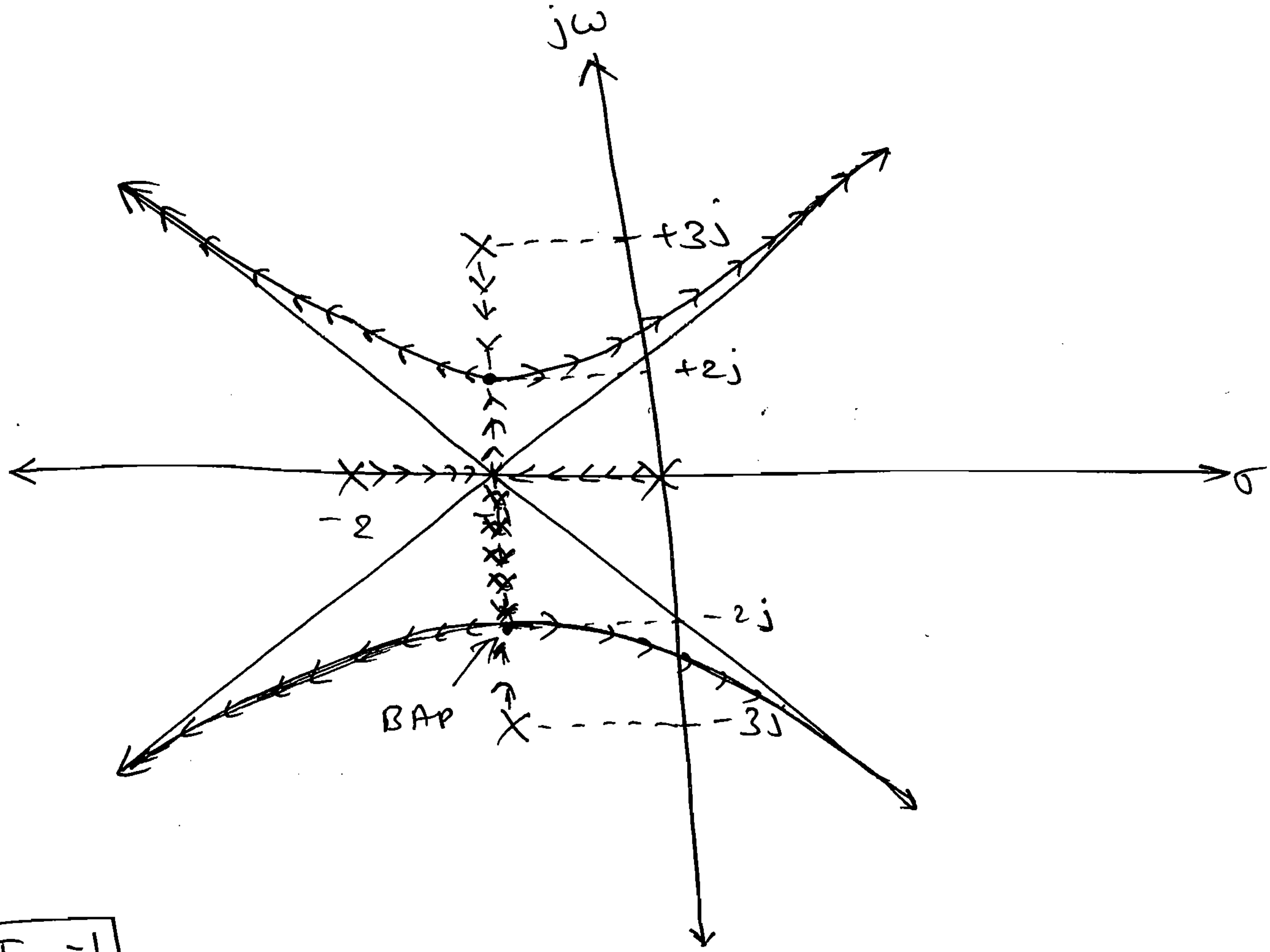
$$\therefore \frac{dK}{ds} = -\left[4s^3 + 12s^2 + 10.5s^2 + 2.5\right] = 0.$$

$$B.P. = -0.3876, \quad -1.61, \quad -1.$$

$$\underset{BAP}{\uparrow} \qquad \underset{BAP}{\uparrow} \qquad \underset{BIP}{\uparrow}$$

$\boxed{11}$ $$GH(s) = \frac{K}{s(s^2+2s+10)(s+2)}.$$

Soln:
$P = 4, \; Z = 0, \; P - Z = N = 4. \; , \theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ.$



$\therefore \boxed{\sigma = -1}$

$$K = -\left[(s^2+2s)(s^2+2s+10)\right].$$

$$\therefore K = -\left[s^4 + 2s^3 + 10s^2 + 2s^3 + 4s^2 + 20s \right].$$

$$\therefore \frac{dK}{ds} = -\left[4s^3 + 12s^2 + 28s + 20 \right] = 0.$$

$\xrightarrow{B.P.}$ $s = -1, \; -1 \pm 2i.$

Note: σ = Real part of Complex Pole.

- Symmetric → $\underbrace{\text{Symmetric}}_{1\ BP}$
- Complex Poles are very close to Real axis → $\underbrace{\text{Complex Poles are very close to Real axis}}$
  - 1 BIP & 2 BAP
  - ‖ 3 BP's
- Complex Poles are far away from Real axis → $\underbrace{\text{Complex Poles are far away from Real axis}}$
  - 3 BP's
    - 1 BIP
    - 1 Pair of Complex BAP's.

$\boxed{12}$ $$GH(s) = \frac{K}{s(s^2 + 6s + 10)}$$

Soln: Pole : 3
Zero : 0 $\Rightarrow P - Z = 3 \Rightarrow \theta = 60, 180, 300.$

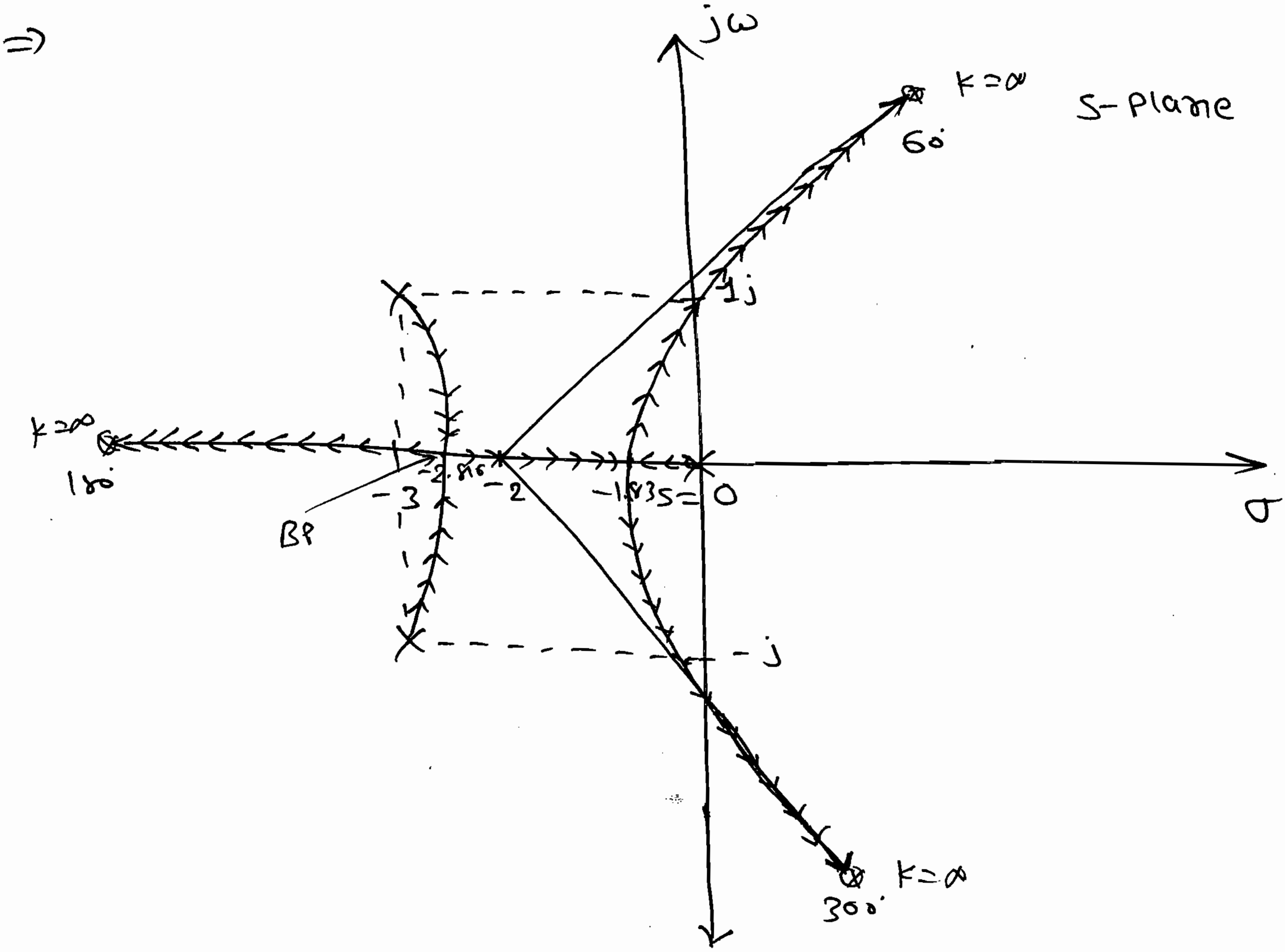
→ Poles: $s = -3 \pm j1$
$s = 0.$

→ Centroid : $$\sigma = \frac{-3 - 3 - 0 - 0}{3}$$

$$= -2$$

$$\boxed{\sigma = -2}$$

$\Rightarrow$



$\Rightarrow \xrightarrow{B.P.}$ $K = -[s^3 + 6s^2 + 10s]$.

$$\frac{dK}{ds} = -[3s^2 + 12s + 10] = 0.$$

$\Rightarrow$ B.P.: $s = -1.183, \; -2.816,$

$\boxed{12}$ $GH = \dfrac{K(s+1)}{s^2(s+K_1)}$.

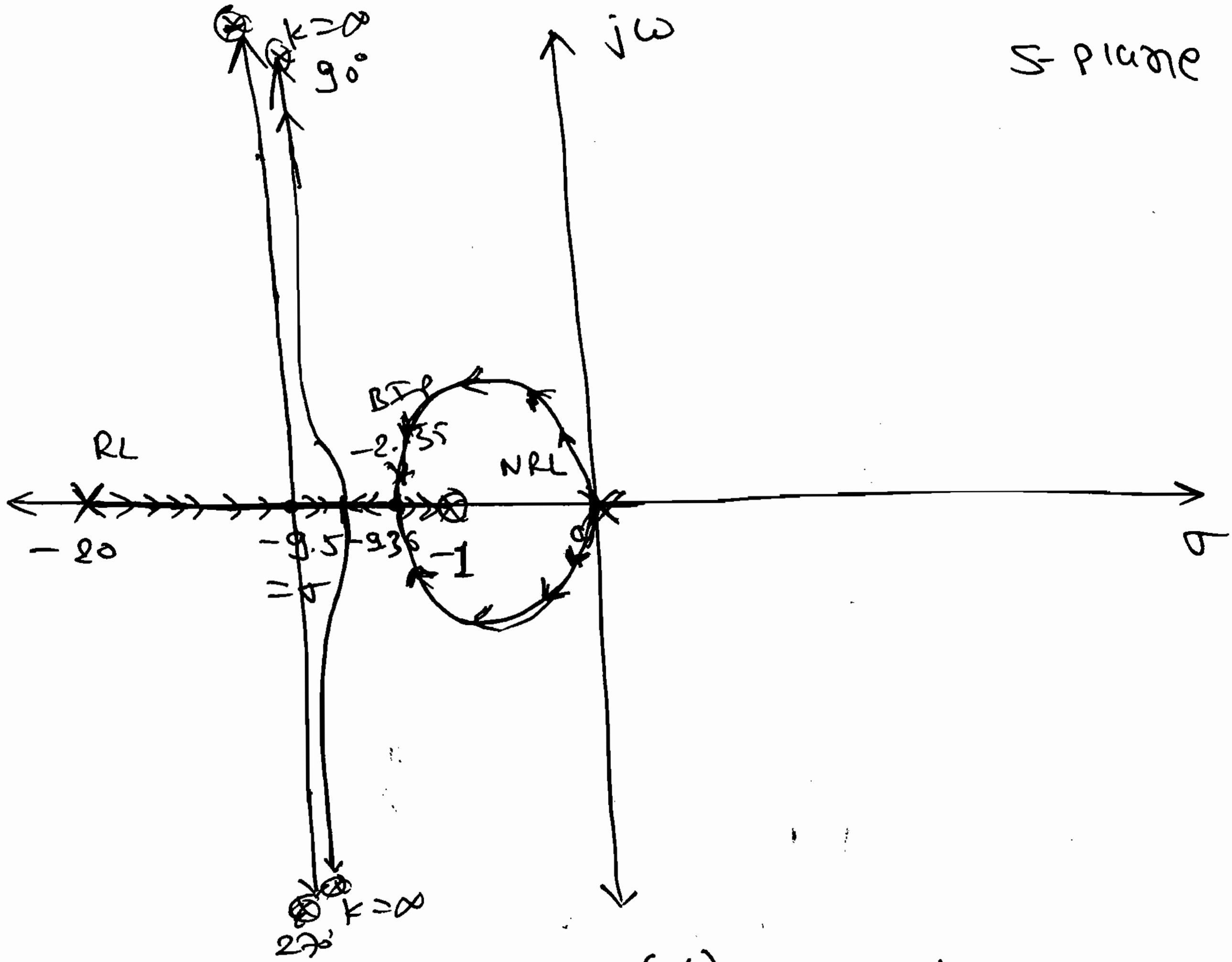
① $K_1 = 20$. ② $K_1 = 9$.

③ $K_1 = 2$. ④ $K_1 = 0.1$.

Soln: ① $K_1 = 20$

$\Rightarrow$ $GH = \dfrac{K(s+1)}{s^2(s+20)}$.

$\Rightarrow$ Poles: 3 $\Rightarrow$ $P - Z = N = 3 - 1 = 2$ $\Rightarrow \xrightarrow{A.A} \theta = 90°, 27°$
zeros: 1



$\Rightarrow$ Centroid $\sigma = \dfrac{-20-(-1)}{2} = \dfrac{-19}{2} = -9.5$

$\rightarrow$ B.P $\quad K = -\dfrac{s^2(s+20)}{(s+1)}$

$K = -\left(\dfrac{s^3+20s^2}{s+1}\right).$

$\Rightarrow \dfrac{dK}{ds} = -\left[\dfrac{(s+1)(3s^2+40s) - (s^3+20s^2)(1)}{(s+1)^2}\right] = 0$

$\Rightarrow 3s^3 + 40s^2 + 3s^2 + 40s - s^3 - 20s^2 = 0.$

$2s^3 + 23s^2 + 40s = 0$

$s = 0, -2.135, -9.36.$

So, $s=0 \Rightarrow$ BIP

$s=-9.36 \Rightarrow$ BAP.

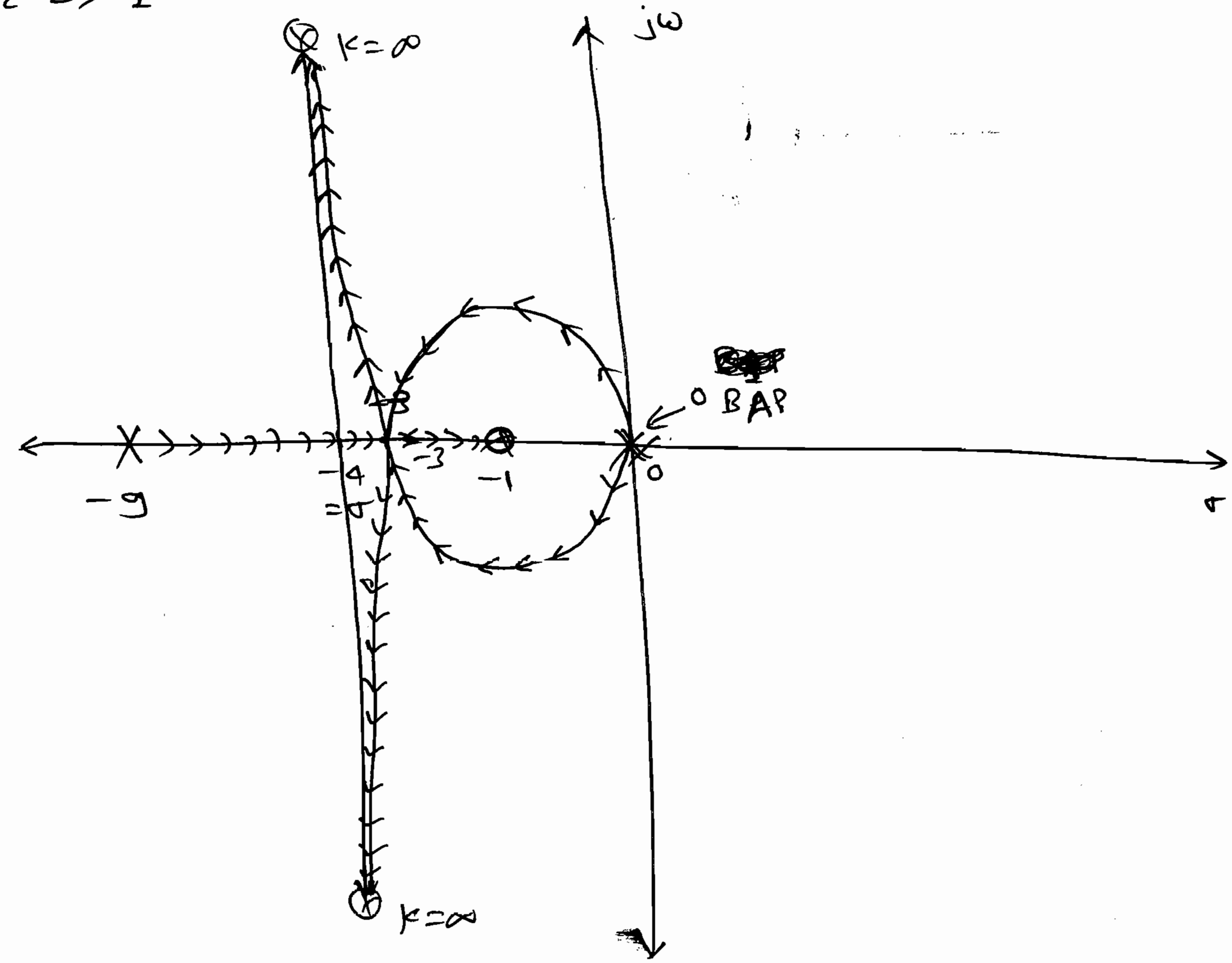
$s=-2.135 \Rightarrow$ BAP.

② $K_1 = 9$.

$$\Rightarrow \quad GH = \frac{K(s+1)}{s^2(s+9)}.$$

$P \Rightarrow 3$

$Z \Rightarrow 1$ $\Rightarrow$ $P - Z = N = 2$ $\Rightarrow$ $\xrightarrow{A.A}$ $\theta = 90°, 270°$.



$$\Rightarrow \quad \sigma = \frac{-9-0+1}{2} = -4.$$

$\xrightarrow{B.P.}$ $$K = -\frac{(s^3+9s^2)}{(s+1)}.$$

$$\Rightarrow \frac{dK}{ds} = -\left[\frac{(s+1)(3s^2+18s) - (s^3+9s^2)(1)}{(s+1)^2}\right] = 0$$

$$\Rightarrow \quad 3s^3 + 18s^2 + 3s^2 + 18s - s^3 + 9s^2 = 0.$$

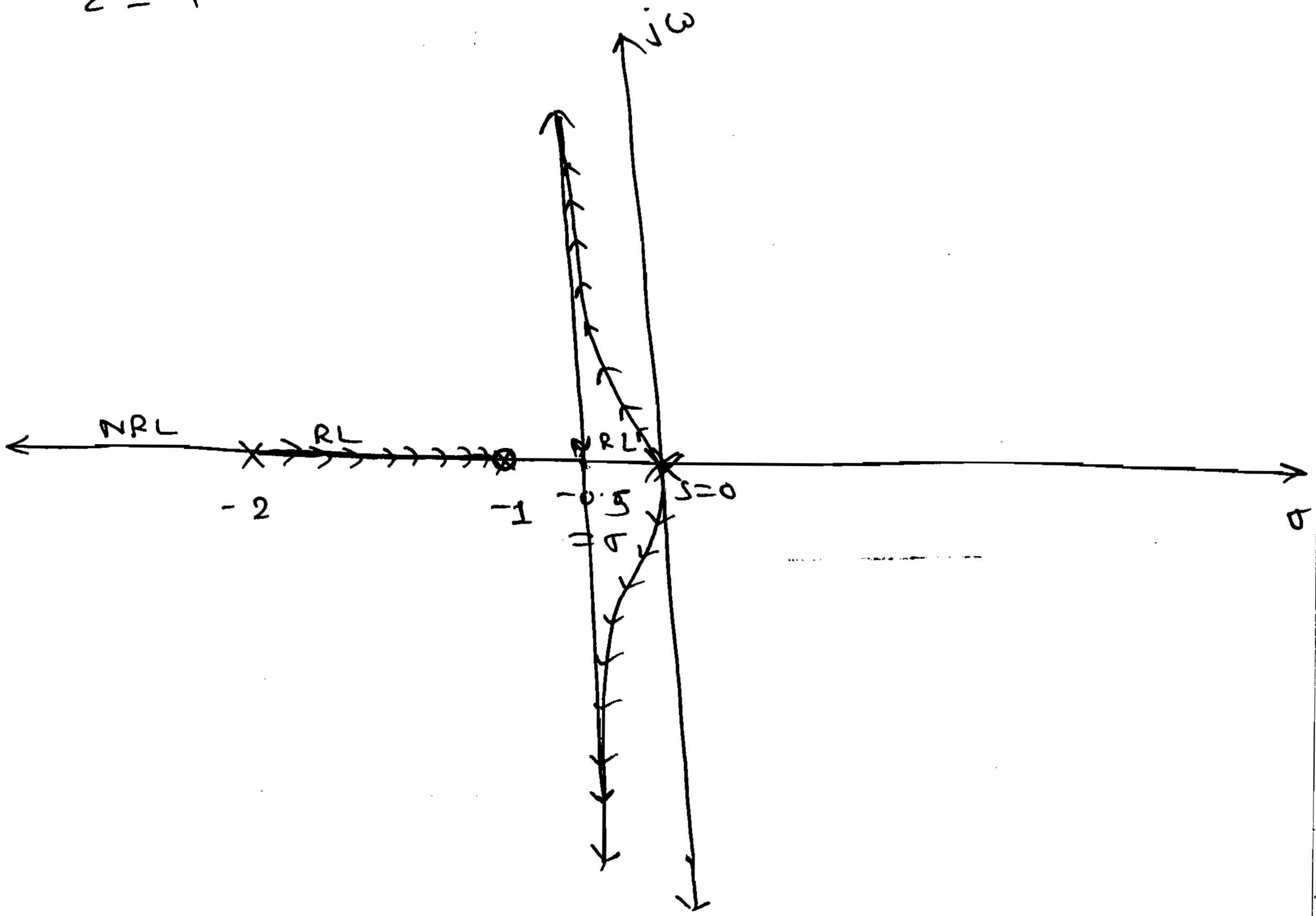
$$\Rightarrow \quad 2s^3 + 12s^2 + 18s = 0.$$

$\xrightarrow{B.P.}$ $s = 0,\ -3,\ -3$

③ $K_1 = 2$

$\Rightarrow GH = \dfrac{K(s+1)}{s^2(s+2)}.$

$\Rightarrow P = 3,\ Z = 1 \Rightarrow P - Z = N = 2 \Rightarrow \xrightarrow{A.A} \theta = 90^\circ, 270^\circ.$



$\Rightarrow \sigma = \dfrac{(-0-0-2)-(-1)}{2} = \dfrac{-2+1}{2} = -0.5.$

$\Rightarrow \xrightarrow{BP} K = -\dfrac{(s^3+2s^2)}{(s+1)}.$

$\Rightarrow \dfrac{dK}{ds} = -\left[\dfrac{(s+1)(3s^2+4s) - (s^3+2s^2)(1)}{(s+1)^2}\right] = 0.$

$\Rightarrow 3s^3 + 4s^2 + 3s^2 + 4s - s^3 - 2s^2 = 0.$

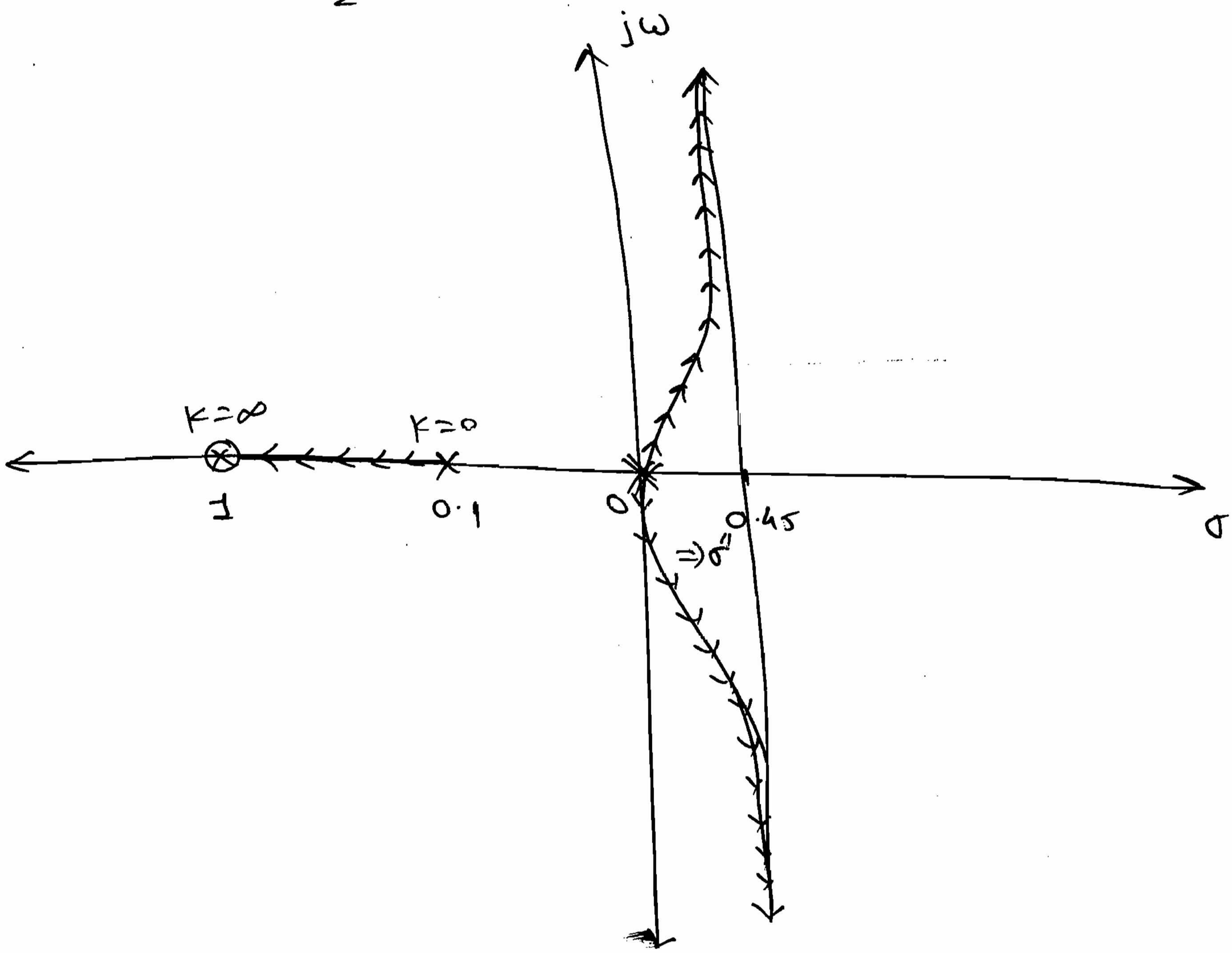
$2s^3 + 5s^2 + 4s = 0. \Rightarrow s = 0,\ \cancel{-0.005},\ -1.25 \pm 0.66i$

$\boxed{4}$ $K_1 = 0.1$

$$\Rightarrow \quad GH = \frac{K(s+1)}{s^2(s+0.1)}.$$

$$\Rightarrow \left.\begin{matrix} P = 3 \\ z = 1 \end{matrix}\right\} \Rightarrow P - z = N = 2 \Rightarrow \xrightarrow{A.A.} \theta = 90^\circ, 270^\circ.$$

$$\Rightarrow \sigma = \frac{(-0.1-0-0)-(-1)}{2} = \frac{-0.1+1}{2} = 0.45.$$



$\underrightarrow{B.P.}$ $$K = -\frac{(s^3+0.1s^2)}{(s+1)}.$$

$$\Rightarrow \frac{dK}{ds} = -\left[\frac{(s+1)(3s^2+0.2s)-(s^3+0.1s^2)}{(s+1)^2}\right] = 0.$$

$$\Rightarrow 3s^3 + 0.2s^2 + 3s^2 + 0.2s - s^3 - 0.1s^2 = 0.$$

$$2.9s^3 + 3.1s^2 + 0.2s = 0. \Rightarrow s = 0, -1, -0.068$$

Note: Whenever the Complex Poles are very Close to real axis the no. of break Points on the real axis increases.

$\boxed{Q}$ Draw the root Locus Diagram to the given Char. eqn by Considering

① K as a system gain.

② a as a system gain.

Soln: $CE \rightarrow s^2 + as + K = 0.$

Note: To draw a root locus diagram in the OLTF the system gain and its ~~is~~ Product term must be in numerator and remain all should be in denominator.

e.g $$G_H = \frac{K}{s^2 + s(K+2)+2}.$$

Not in standard form.

$CE \rightarrow$ $1 + G_H(s) = 0.$

$$1 + \frac{K}{s^2 + s(K+2)+2} = 0 \qquad G_H = \frac{K\,N(s)}{D(s)}.$$

$\Rightarrow$ $s^2 + Ks + 2s + 2 + K = 0.$

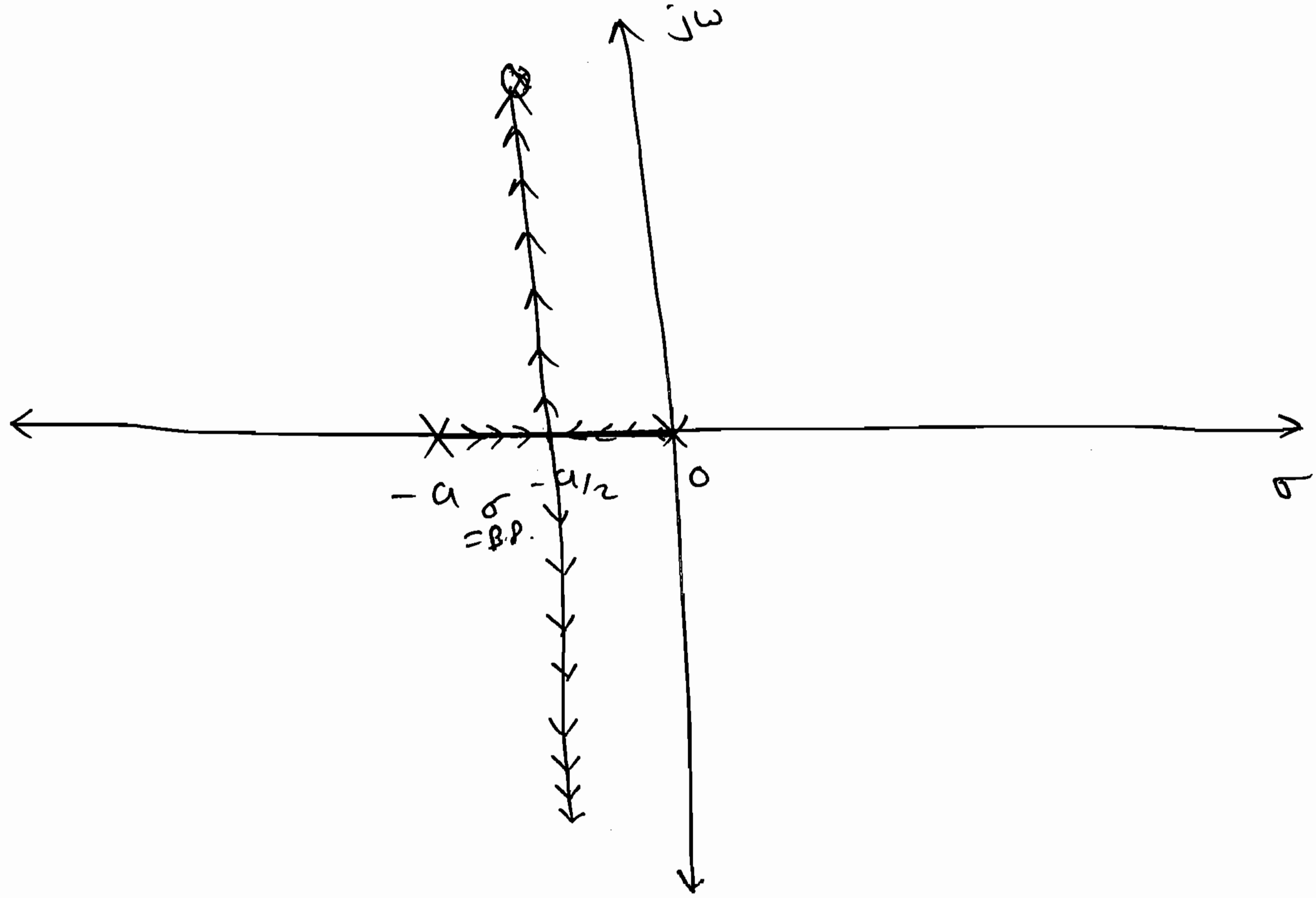
$\Rightarrow$ $s^2 + K(s+2) + 2s+2 = 0$ ~~⇒ GH=~~

$\Rightarrow$ $$G_H = \frac{K(s+2)}{s^2+2s+2}.$$

case - (i) given CE $\longrightarrow$ $s^2 + as + k = 0.$

$\Rightarrow$ $GH = \dfrac{k}{s^2+as}.$ "k" is system gain.

Poles: 2
zeros: 0 $\Rightarrow$ $N = P - Z = 2 - 0 = 2$ = $\pm 90°, 270°$



$\Rightarrow$ $\sigma = \dfrac{-0+a-0}{2} = -a/2$

B.P. $\longrightarrow$ $k = -(s^2+as)$

$\dfrac{dk}{ds} = -[2s+a] = 0$

$\xrightarrow{BP}$ $s = -a/2$

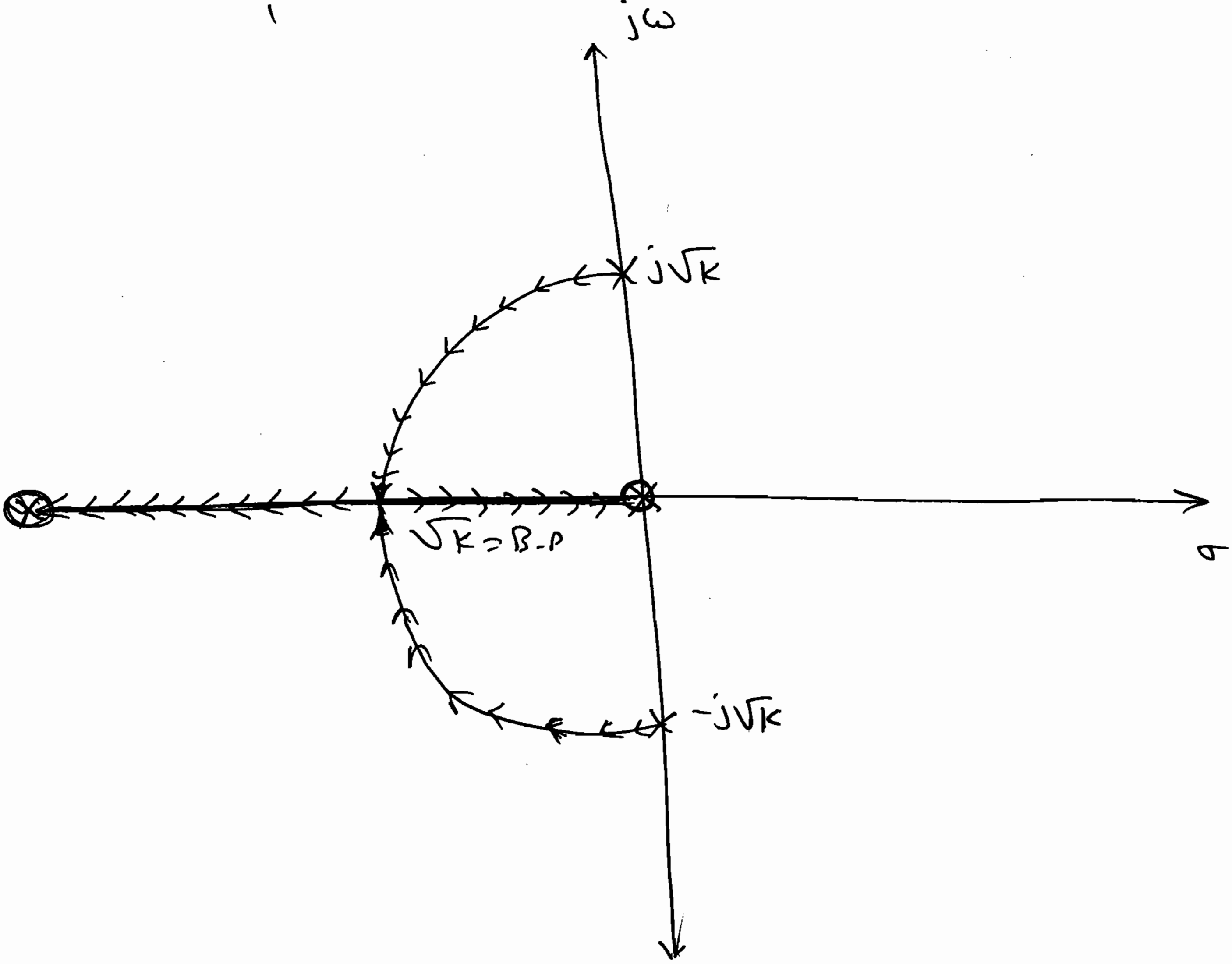
case - (ii) : "a" is system gain.

CE $\longrightarrow$ $s^2 + as + k = 0.$

$\Rightarrow$ $GH = \dfrac{as}{s^2+k}.$

$\left.\begin{matrix} P: 2 \\ Z = 1 \end{matrix}\right\} \Rightarrow \quad P - Z = N = 2 - 1 = 1 \quad \Rightarrow \quad \theta = 180.$

$\Rightarrow \sigma = \dfrac{0 - (-a)}{1} = a.$



$\xrightarrow{B.P.} \quad a = -\left[\dfrac{s^2 + K}{s}\right].$

$\Rightarrow \dfrac{da}{ds} = -\left[\dfrac{s(2s) - s^2 - K}{s^2}\right] = 0.$

$\Rightarrow \quad 2s^2 - s^2 - K = 0$

$s^2 = K$

$s = \pm \sqrt{K}$

$\xrightarrow{B.P} \quad s = -\sqrt{K}.$ ✓

$s = +\sqrt{K}$ ✗

$\boxed{Q}$ Draw the root-locus for $GH(s) = \frac{Ke^{-s}}{s(s+1)}$.

Sol^n:

Note: To draw a root locus diagram in the transfer f^n the s-term should not have the negative sign.

$$\Rightarrow GH(s) = \frac{Ke^{-s}}{s(s+1)} = \frac{K(1-s)}{s(s+1)} = \frac{-K(s-1)}{s(s+1)}.$$

By default -ve FB.

$\xrightarrow{CE}$ $1 + GH(s) = 0$

$$1 - \frac{K(s-1)}{s(s+1)} = 0.$$

IRL

$$\angle \frac{K(s-1)}{s(s+1)} = \angle(1+j0)$$

$= 0°$ (IRL)

+ve FB

$\longrightarrow$ CE $1 - GH(s) = 0$

$$1 + \frac{K(s-1)}{s(s+1)} = 0$$

DRL

$$\angle \frac{K(s-1)}{s(s+1)} = \angle -1+j0$$

$= 180°$ (DRL).

$\Rightarrow$ By default,

$K\uparrow$ (0 to ∞).

$\times \longrightarrow O$

$K=0$ $\quad$ $K=\infty$.

Now,

Poles : 2

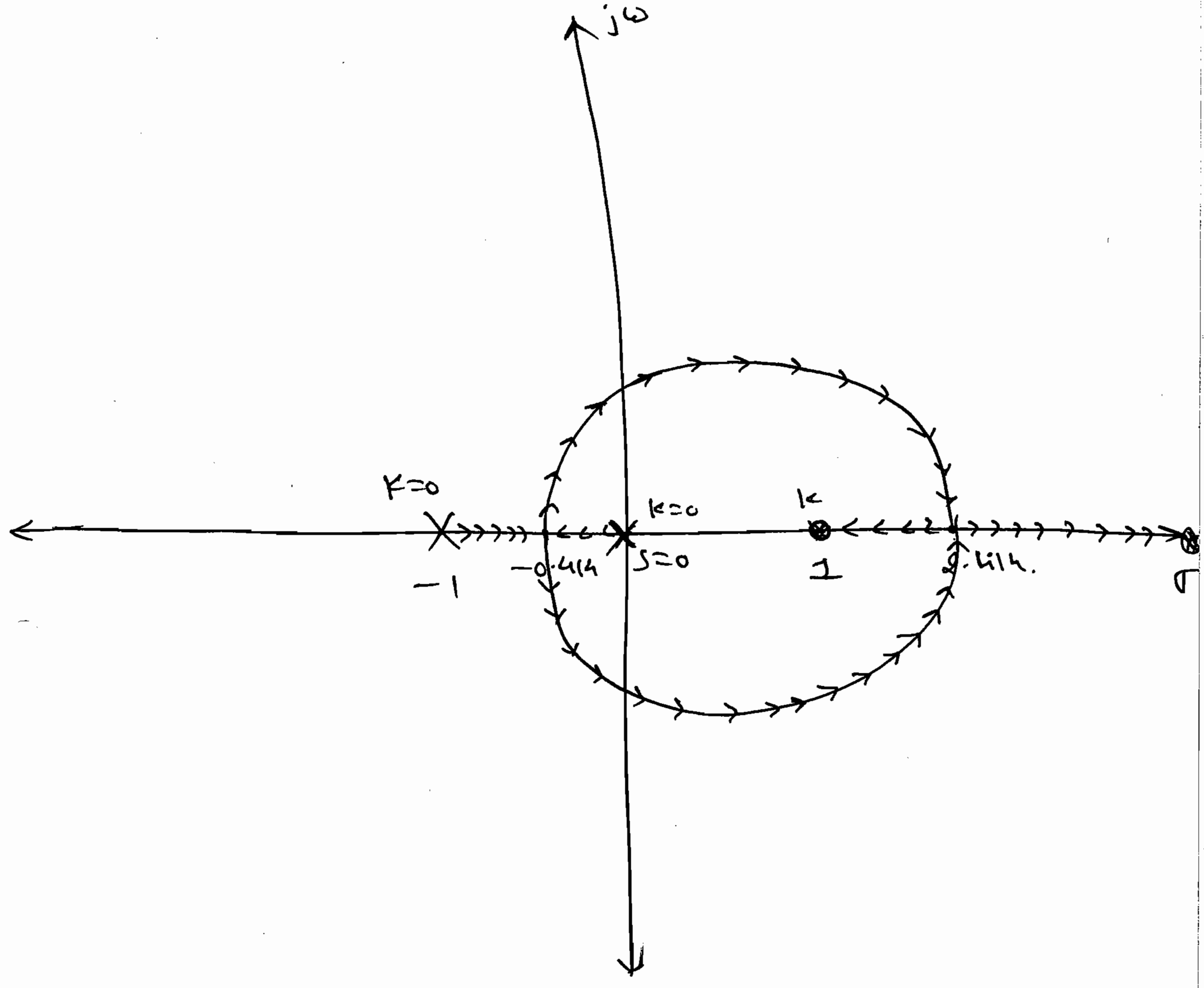
Zeros : 1

$\Rightarrow N = P - Z = 2 - 1 = 1$, $\Rightarrow \theta = \cancel{180}\ 0°$.

$\frac{2q}{(P-Z)} \times 180°$.

$$\sigma = \frac{0 - 1 - (1)}{1}.$$

$$\sigma = -2.$$



$$\Rightarrow \xrightarrow{BP} \; k = \frac{s(s+1)}{+(s-1)}.$$

$$\frac{dk}{ds} = \frac{(s-1)(2s+1) - s^2 - s}{(s-1)^2} = 0.$$

$$\therefore +2s^2 \cancel{+s} \mp 2s \mp 1 - s^2 - \cancel{s} = 0.$$

$$\cancel{2s^2 + s - 2 = 0}. \qquad s^2 - 2s - 1 = 0.$$

$$\cancel{s = -1, 2}. \qquad s = 2.414, \; -0.4142$$

BEP BAP.

* Verification Process to Select Ans.

$$\xrightarrow{CE} 1 + GH = 0$$

$$1 + \frac{K(1-s)}{s(s+1)} = 0.$$

$CE \rightarrow \quad s^2+s+k-ks=0.$

$k=0 \rightarrow \quad s^2+s=0 \Rightarrow s=0,-1.$

$k=1 \rightarrow \quad s^2+s+1-s=0 \Rightarrow s=\pm j1.$

$k=2 \rightarrow \quad s^2+s+2-2s=0.$

$s^2-s+2=0 \Rightarrow s=1\pm j1.3.$

Note: For a complete Root locus the range of K value is from $-\infty$ to $+\infty$.

Q) Draw the complete root locus the range of $K=-\infty$ to $+\infty$.

$$GH(s)=\frac{K}{s(s+2)}.$$

Soln:

IRL $\longrightarrow -\infty<K<0 \Rightarrow$ O→→→→X ($K=-\infty$, $K=0$)

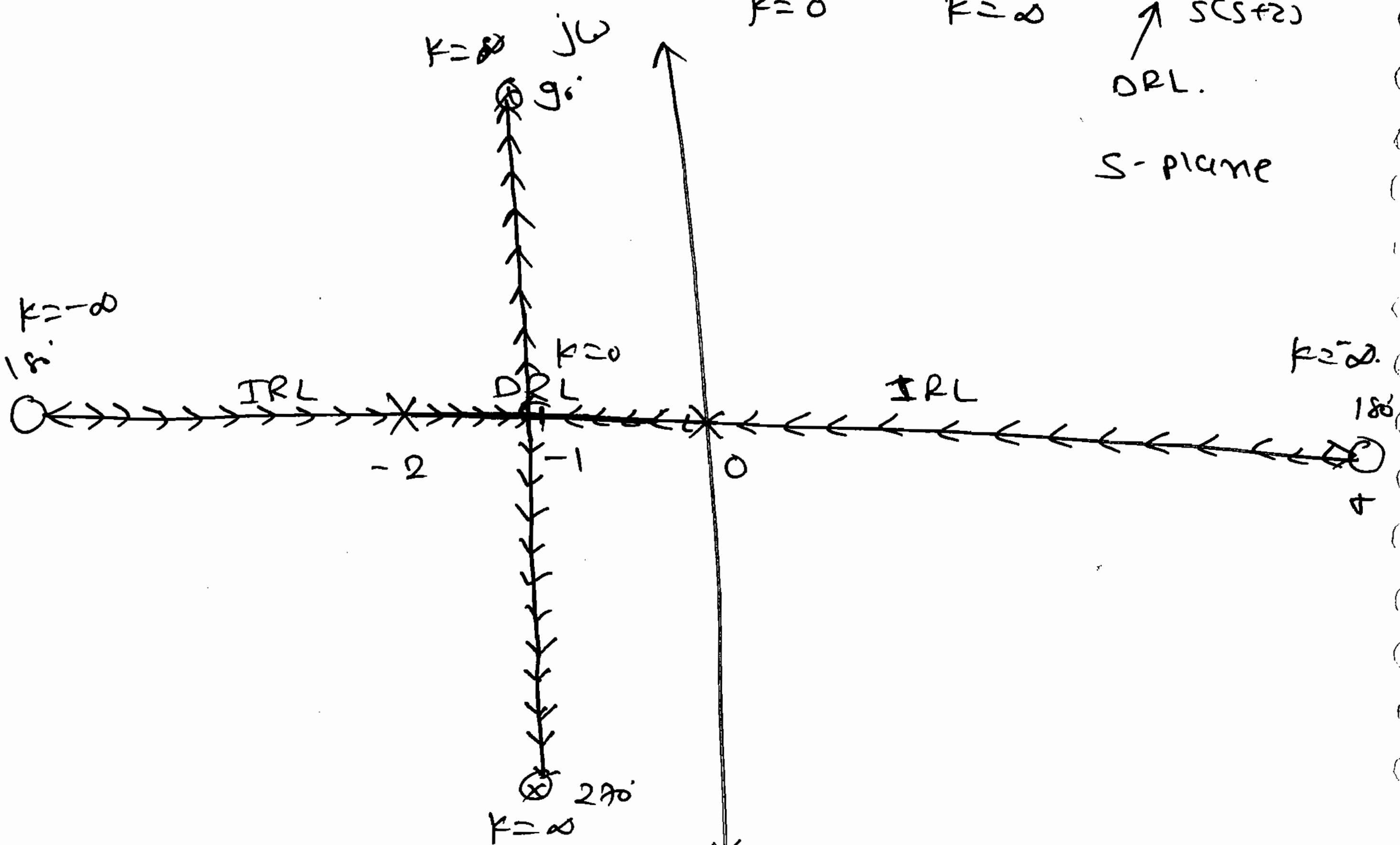
By default $CE \rightarrow 1+GH=0$

$1-\frac{K}{s(s+2)}=0$ — IRL

DRL $\longrightarrow 0<K<\infty \Rightarrow$ X→→→→→O ($K=0$, $K=\infty$)

$1+\frac{K}{s(s+2)}=0$ — DRL.

S-plane



$$\Rightarrow \quad \sigma = \frac{-2-0-0}{p-z}$$

$$= \frac{-2-0-0}{2-0}$$

$$\boxed{\sigma = -1}$$

$$\left.\begin{array}{l} p: 2 \\ z = 0 \end{array}\right\} \Rightarrow \quad N = p - z = 2.$$

$$\theta = \frac{(2q)180}{p-z}$$

$$= \frac{\cancel{2}q \times 180}{\cancel{2}}$$

$$\therefore \boxed{\theta = 0^\circ, 180^\circ}$$

$$BP \longrightarrow K = -s^2 - 2s$$

$$\frac{dK}{ds} = -2s - 2$$

$$\Rightarrow \boxed{s = -1}$$