MANIPAL

Engineering Entrance Exam

Solved Paper 2020

Physics

1. A body of mass m rises to a height $h = \frac{R}{5}$ from the earth's surface, where R is the earth's radius. If g is acceleration due to gravity at the earth's surface, the increase in potential energy will be

(a) mgh (b) $\frac{4}{5}mgh$ (c) $\frac{5}{6}mgh$ (d) $\frac{6}{7}mgh$

2. The energy required to break the covalent bond in a semiconductor is

(a) always 1 eV
(b) equal to the forbidden energy gap of semiconductor
(c) equal to fermi energy
(d) much less than fermi energy

3. When a glass prism of refracting angle 60° is immersed in a liquid its angle of minimum deviation is 30°. The critical angle of glass with respect to the liquid medium is

(a) 45° (b) 42° (c) 50° (d) 52°

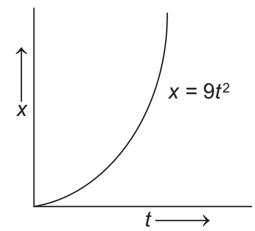
4. Faraday constant

(a) depends on the amount of the electrolyte
(b) depends on the current in the electrolyte
(c) is a universal constant
(d) depends on the amount of charge passed through the electrolyte

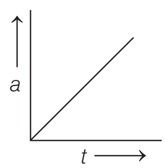
5. A comb is run through wet hair on a rainy day, then

(a) it will attract large number of small bits of paper
(b) it will not go through the hair
(c) it will not attract small bits of paper
(d) None of the above

6. The displacement-time graph of a particle moving along a straight line is shown in the figure.



The acceleration-time graph of this particle is

(a)


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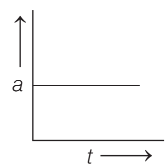

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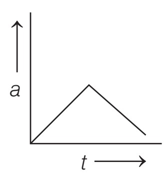

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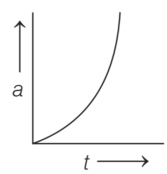


7. The maximum kinetic energy of photoelectrons coming out of a metal surface is 10 eV. The minimum voltage required to stop, the emission of electrons from this metal surface is

(a) 10 V (b) 5 V
(c) –5 V (d) –10 V

8. Thermal radiation exist in which part of electromagnetic spectrum?

(a) Ultraviolet (b) Infrared
(c) Visible (d) Violet

9. A rod of length L is composed of a uniform length $\frac{L}{2}$ of wood whose mass in m_w and a uniform length $\frac{L}{2}$ of brass whose mass is m_b. The moment of inertia I of the rod about an axis perpendicular to the rod and through its centre is equal to

(a) $(m_w + m_b)\frac{L^2}{6}$ (b) $(m_w + m_b)\frac{L^2}{2}$
(c) $(m_w + m_b)\frac{L^2}{12}$ (d) $(m_w + m_b)\frac{L^2}{3}$

10. A particle, doing simple harmonic motion, at a distance 3 cm from mean position has acceleration 12 cm/s^2. What is its time period?

(a) 0.5 s (b) 1s (c) 2s (d) 3.14 s

11. A step up transformer has turn ratio 10 : 1. A cell of emf 2 V is fed to the primary, then the secondary voltage developed is

(a) 20 V (b) 10 V (c) 2 V (d) zero

12. A man at a distance 11 km from two pillars wants to see two pillars separately. What will be the approximate distance between the pillars?

(a) 3 m (b) 1 m (c) 0.25 m (d) 0.5 m

13. Dimensions of Stefan's constant is

(a) $[MLT^{-3}\theta^{-4}]$ (b) $[MT^{-3}\theta^{-4}]$
(c) $[M^2T^{-3}\theta^{-4}]$ (d) $[M^2T^{-2}\theta^{-4}]$

14. Time period of oscillation of mass m suspended from a spring is T. What is the time period when the spring is cut in half and the same mass is suspended from one of the halves?

(a) $T/2$ (b) $T/\sqrt{2}$
(c) $\sqrt{2}\,T$ (d) $2\,T$

15. An electric current of 2 A passes through a wire of resistance 25 Ω. How much heat will be generated in 1 min?

(a) 6×10^3 J (b) 3.6×10^3 J
(c) 0.6×10^3 J (d) 0.36×10^3 J

16. A mass m hanging from a spring is doing simple harmonic motion with frequency f. If the mass is increased by 4 times, then frequency will be

(a) $2f$ (b) $f/2$ (c) $4f$ (d) $f/4$

17. Internal energy of a gas remains unchanged in

I. an isothermal process
II. an adiabatic process
III. a reversible process
IV. a cyclic process

Which of these are true?

(a) I and IV (b) I, III and IV
(c) III and IV (d) II and III

18. An electric charge in uniform motion produces

(a) only electric field
(b) only magnetic field
(c) Both electric and magnetic field
(d) Neither electric nor magnetic field

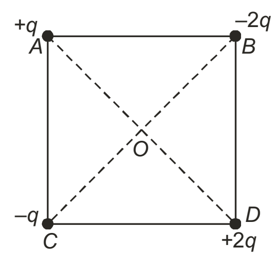
19. If λ is the incident wavelength and λ_0 is the threshold wavelength for a metal surface, photoelectric effect takes place only, if

(a) $\lambda \le \lambda_0$ (b) $\lambda \ge \lambda_0$
(c) $\lambda \ge 2\lambda_0$ (d) None of these

20. The mass number of an atom is 15 and its atomic number is 7. Now, this atom absorbs an α-particle and emits a proton. What will be the mass number of changed atom?

(a) 16 (b) 18 (c) 17 (d) 15

21. What is the direction of the electric field at the centre O of the square in the figure shown below? Given that, $q = 10$ nC and the side of the square is 5 cm.



(a) at 45° to OA upward
(b) at 135° to OA towards BD
(c) no direction, because $E = 0$
(d) None of the above

22. Which equation is valid for adiabatic process?

(a) $TV^{\gamma-1}$ = constant (b) $pV^{\gamma-1}$ = constant

(c) $T^{\gamma}V^{\gamma-1}$ = constant (d) $\frac{p^{\gamma-1}}{T^{\gamma-1}}$ = constant

23. Which of the following relation correct? (v_{rms}-root mean square velocity, $\bar{v}$-mean velocity and v_{mp}-most probable velocity)

(a) $v_{\text{rms}} > \bar{v} < v_{\text{mp}}$ (b) $v_{\text{rms}} < \bar{v} > v_{\text{mp}}$

(c) $v_{\text{rms}} > \bar{v} > v_{\text{mp}}$ (d) None of these

24. The effect of reverse bias in a junction diode on its potential barrier is

(a) increases (b) decreases

(c) remains same (d) None of these

25. During an experiment, an ideal gas is found to obey an additional law $Vp^2 =$ constant. The gas is initially at temperature T and volume V. The temperature of the gas will be following, when it expands to a volume 2 V?

(a) $\sqrt{2}\,T$ (b) $\sqrt{4}\,T$ (c) $\sqrt{6}\,T$ (d) $\sqrt{5}\,T$

26. In which of the following process, convection does not take place primarilly?

(a) Sea and land breeze

(b) Boiling of water

(c) Warming of glass bulb due to filament

(d) Heating air around furnace

27. The horizontal component of earth's magnetic field at a place is 0.4×10^4 T. If the angle of dip is 45° the value of total intensity of earth's magnetic field is

(a) 0.5×10^{-4} T (b) 0.4×10^{-4} T

(c) 0.5×10^{-6} T (d) 0.4×10^{-6} T

28. If an observer moves towards stationary source, then the apparent frequency is given by

(a) $f' = f\left(\frac{v + v_0}{v}\right)$ (b) $f' = f\left(\frac{v - v_0}{v}\right)$

(c) $f' = f\left(\frac{v}{v + v_0}\right)$ (d) $f' = f\left(\frac{v}{v - v_0}\right)$

29. Two coils have mutual inductance of 1.5 H. If current in primary coil is suddenly raised to 5 A in one milli second, the induced emf in the secondary coil is

(a) 75 V (b) 750 V

(c) 7500 V (d) 75000 V

30. The formula for magnetic field of a certain component is given by $B = \frac{\mu_0 Ni}{2\pi r}$,

where, N = total number of turns

i = current through it

r = radius.

The component is

(a) ring (b) solenoid

(c) toroid (d) None of these

31. A convex lens of focal length $0.12\ \text{m}$ produces an image, which is three times as long as the object. The distance between the object and the lens for a real image

(a) 0.16 m (b) – 0.16 m

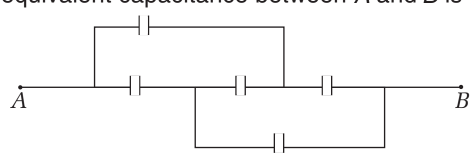
(c) 1.6 m (d) – 1.6 m

32. An object is moving in a circle at constant speed v. The magnitude of the rate of change of momentum of the object is

(a) zero (b) proportional to v^2

(c) proportional to v^3 (d) proportional to v

33. Five equal capacitors each with capacitance C are connected as shown in figure. Then, the equivalent capacitance between A and B is



(a) $5C$ (b) $\frac{C}{5}$ (c) $3C$ (d) C

34. The time period of a freely suspended magnet is 4 s. If it is broken in length into two equal parts and one part is suspended in the same way, then the time period will be

(a) 4 s (b) 2 s (c) 0.5 s (d) 0.25 s

35. A spring of force constant k is cut into two pieces such that one piece is double the length of other. Then, the long piece will have a force constant of

(a) $\frac{2}{3}k$ (b) $\frac{3}{2}k$ (c) $3k$ (d) $6k$

36. Half-life period of a radiactive substance is 10 min, then amount of substance decayed in 40 min will be

(a) 25% (b) 50%

(c) 75% (d) None of these

37. Two identical containers A and B with frictionless pistons contain the same ideal gas at the same temperature and the same volume V. The mass of the gas in A is m_A and that in B is m_B. The gas in each container is now allowed to expand isothermally to the same final volume $2V$. The changes in the pressure in A and B are to be found Δp and $1.5\ \Delta p$ respectively, then relation for masses will be

(a) $4m_A = 9m_B$ (b) $2m_A = 3m_B$
(c) $3m_A = 2m_B$ (d) $9m_a = 4m_B$

38. If one mole of a monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ is mixed with one mole diatomic gas $\left(\gamma = \frac{7}{5}\right)$, the value of γ for the mixture is

(a) 1.40 (b) 1.50 (c) 1.53 (d) 3.07

39. A solid sphere of mass 2 kg rolls on a smooth horizontal surface at 10 m/s and then rolls up a smooth 30° incline. The maximum height reached by the sphere is ($g = 9.8\ \text{m/s}^2$)

(a) 10 m (b) 4.9 m (c) 14.2 m (d) 7.1 m

40. How much deep inside the earth (radius R) should a man go, so that his weight becomes one-fourth of that on the earth's surface?

(a) $\frac{R}{2}$ (b) $\frac{3R}{4}$ (c) $\frac{R}{4}$ (d) $\frac{R}{3}$

41. Four projectiles are thrown with the same initial speed making angles 27°, 36°, 43°, 51°, with the horizontal. The range of projectiles will be largest for the projectile fired at angle

(a) 27° (b) 36° (c) 43° (d) 51°

42. A wire of resistance 5 Ω is drawn out so that its length is increased by twice its original length, its new resistance is

(a) 45 Ω (b) 54 Ω (c) 20 Ω (d) 5 Ω

43. The magnifying power of a telescope is m'. If the focal length of the eye piece is doubled, then its magnifying power will become

(a) $\sqrt{2}\ m$ (b) $3\ m$ (c) $2\ m$ (d) $\frac{m}{2}$

44. On a heater coil it is written that, 250 V, 500 W. What is the resistance of this coil?

(a) 62.5 Ω (b) 100 Ω (c) 200 Ω (d) 125 Ω

45. When a dielectric slab is introduced between the plates of a capacitor connected to a battery, then

(a) charge on capacitor increases
(b) potential difference across the capacitor increases
(c) energy stored increases
(d) capacity remains the same

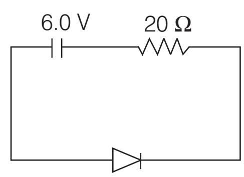
46. A concave lens is kept in contact with a convex lens of focal length 20 cm. The combination works as convex lens of focal length 50 cm. The power of concave lens is

(a) $P = -\ 3.0\ \text{D}$ (b) $P = +\ 3.0\ \text{D}$
(c) $P = -\ 0.3\ \text{D}$ (d) $P = +\ 0.3\ \text{D}$

47. The critical angle for glass water interface $\left(\text{if }_a\mu_g = \frac{3}{2},\ _a\mu_w = \frac{4}{3}\right)$

(a) $\sin^{-1}\left(\frac{8}{9}\right)$ (b) $\sin^{-1}\left(\frac{9}{8}\right)$
(c) $\sin^{-1}\left(\frac{3}{2}\right)$ (d) $\sin^{-1}\left(\frac{4}{3}\right)$

48. What is the current through the circuit and the pontential difference across the diode shown in figure. The drift current for the diode is 30μA.



(a) 30μ A, 5.99 V (b) 30μ A, 5 V
(c) 20μA, 6 V (d) 20μA, 5.99 V

49. A man who wears glasses of power 3 D must holds a newspaper at least 25 cm away to see the print clearly. How far away would the newspaper have to be if he took off the glasses and still wanted clear vision?

(a) 10 cm (b) 25 cm
(c) 1 m (d) – 1 m

50. The force per unit length between two parallel current carrying straight conductors separated by $2d$ is given by the formula

(a) $\frac{\mu_0}{4\pi}\frac{i_1 i_2}{d}$ (b) $\frac{\mu_0}{4\pi}\frac{i_1 i_2}{2d}$
(c) $\frac{\mu_0}{\pi}\frac{i_1 i_2}{2d}$ (d) None of these

Chemistry

1. Which of the following is not basic amino acid?

(a) Leucine (b) Lysine
(c) Arginine (d) Histidine

2. In qualitative analysis, NH_4Cl is added before NH_4OH

(a) to increase $[OH^-]$ concentration
(b) for making HCl
(c) to decrease $[OH^-]$ concentration
(d) statement is wrong

3. Certain electric current for half an hour can collect 11.2 L of hydrogen at NTP. Same current when passed through an electrolytic solution for one hour, can deposite how much silver?

(a) 216 g (b) 108 g (c) 47 g (d) 60 g

4. Which of the following complexes is an outer orbital complex?

(a) $[Co(NH_3)_6]^{3+}$ (b) $[Fe(CN)_6]^{4-}$
(c) $[Ni(NH_3)_6]^{2+}$ (d) $[Mn(CN)_6]^{4-}$

5. Which is true for a cyclic process?

(a) $\Delta E = 0$ (b) $\Delta E = q - W$
(c) $q = W$ (d) All of these

6. In the Cannizzaro reaction given below

$$2C_6H_5CHO \xrightarrow{OH^-} C_6H_5CH_2OH + C_6H_5COO^-$$

The slowest step is

(a) The attack of OH^- at the carbonyl group
(b) The transfer of hydride ion to the carbonyl group
(c) The abstraction of proton from the carboxylic acid
(d) The deprotonation of benzyl alcohol

7. The rate of a reaction doubles when the initial concentration of the reactant is made four fold. If the initial concentration is made 400 fold, then the rate will become

(a) 400 times (b) 200 times
(c) 40 times (d) 20 times

8. Which compound present in diesel?

(a) Cetane
(b) $TiCl_4$
(c) Cyclo pentandienyl manganese carbonyl
(d) Iso octane

9. Which of the following is an organometallic compound?

(a) Lithium acetate
(b) Methyl lithium
(c) Lithium dimethyl amide
(d) Lithium methoxide

10. If in the reaction, $N_2O_4 \rightleftharpoons 2NO_2$; α is the degree of dissociation of N_2O_4, then total number of moles at equilibrium is

(a) $(1-\alpha)$ (b) $(1+\alpha)$ (c) $(1-\alpha)^2$ (d) $(1-\alpha)^2$

11. $CHCl_3 \xrightarrow[h\nu]{[O]} A \xrightarrow{NH_3} B \xrightarrow[PoCl_3]{\text{Malonic acid}} C$

The end product C in the following reaction is used as

(a) explosive (b) hypnotic
(c) tear gas (d) analgesic

12. Which statement is false for white phosphorus (P_4) ?

(a) It has six P—P single bonds
(b) It has four P—P single bonds
(c) It has four lone pairs of electrons
(d) It has PPP angle 60°

13. Natural rubber is a polymer of monomer isoprene. During polymerisaton

(a) 1, 4 addition takes place
(b) 1, 2 addition takes place
(c) 1, 3 addition takes place
(d) both double bonds are converted into single bond

14. Match List I (species) with List II (hybridisation) and select the correct code given below

	List I		List II
A.	XeF_4	(i)	dsp^2
B.	H_2O	(ii)	sp^3
C.	PCl_5	(iii)	sp^3d^2
D.	$[Pt(NH_3)_4]^{2+}$	(iv)	sp^3d

Codes

	A	B	C	D
(a)	(iii)	(ii)	(iv)	(i)
(b)	(i)	(iv)	(ii)	(iii)
(c)	(iii)	(iv)	(ii)	(i)
(d)	(i)	(ii)	(iv)	(iii)

15. A new carbon-carbon bond formation is possible in :

I. Cannizzaro reaction

II. Friedel-Craft reaction

III. Clemmensen reduction

IV. Reimer-Tiemann reaction

(a) I, II and III (b) II, III and IV

(c) I and III (d) II and IV

16. Which of the following compounds formed during Perkin's reaction?

(a) Resorcinol (b) Cinnamic acid

(c) Benzaldehyde (d) Benzoin

17. The ratio of de-Broglie wavelengths for electron accelerated through 200 V and 50 V is

(a) 1 : 2 (b) 2 : 1 (c) 3 : 10 (d) 10 : 3

18. Malachite decomposed to give $A + CO_2 + H_2O$ and compound *A* on reduction with carbon gives $CO + B$. Here, *A* and *B* are

(a) CuO, Cu (b) Cu_2O, CuO

(c) Cu_2O, Cu (d) $CuCO_3$, Cu

19. Match List I with List II and choose correct answer from the codes given below

	List I		List II
A.	$NaNO_3$	(i)	Baking soda
B.	$Na(NH_4)HPO_4$	(ii)	Chile salt petre
C.	$NaHCO_3$	(iii)	Microcosmic salt
D.	$Na_2CO_3 \cdot 10H_2O$	(iv)	Washing soda

	A	B	C	D
(a)	(i)	(ii)	(iii)	(iv)
(b)	(ii)	(iii)	(i)	(iv)
(c)	(iii)	(i)	(ii)	(iv)
(d)	(iv)	(i)	(ii)	(iii)

20. When MnO_2 is heated with PbO_2 and conc. HNO_3, pink colour is obtained due to formation of

(a) $KMnO_4$ (b) $HMnO_4$

(c) $Pb(MnO_4)_2$ (d) $PbMnO_4$

21. Which is mismatched for NaCl crystal ?

(a) $\frac{r^+}{r^-} = 0.414$ to 0.732

(b) Coordination number = 6:6

(c) Edge of unit cell = $(r^+ + r^-)$

(d) Crystal structure = fcc

22. Which of the following ions has the highest magnetic moment ?

(a) Zn^{2+} (b) Ti^{3+} (c) Sc^{3+} (d) Mn^{2+}

23. Correct order for solubility of alkaline earth metals in water is

(a) $MgF_2 > CaF_2 > SrF_2 > BaF_2$

(b) $MgF_2 < CaF_2 < SrF_2 < BaF_2$

(c) $MgF_2 > CaF_2 < SrF_2 < BaF_2$

(d) $BaF_2 > MgF_2 > SrF_2 > CaF_2$

24. An organic compound *A* contains 20% C, 46.66% N and 6.66% H. It gives NH_3 gas on heating with NaOH. *A* can be

(a) CH_3CONH_2 (b) $C_6H_5CONH_2$

(c) NH_2CONH_2 (d) $CH_3NHCONH_2$

25. In the equilibrium mixture, $KI + I_2 \rightleftharpoons KI_3$; the concentration of KI and I_2 is made two fold and three fold respectively. The concentration of KI_3 becomes

(a) two fold (b) three fold

(c) five fold (d) six fold

26. $A \xrightarrow{HOH/H^+}$ glucose + fructose

$B \xrightarrow{HOH/H^+}$ glucose + glucose

$C \xrightarrow{HOH/H^+}$ glucose + galactose

The disaccharides *A*, *B* and *C* respectively are

(a) lactose, sucrose and maltose

(b) sucrose, maltose and lactose

(c) sucrose, lactose and maltose

(d) maltose, sucrose and lactose

27. *n*/*p* ratio during positron decay

(a) increases (b) decreases

(c) remains constant (d) All of these

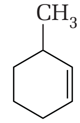
28. Number of carbon atoms in kerosene is

(a) C_{17-20} (b) C_{12-16} (c) C_{20-25} (d) C_{25-30}

29. Which of the following polymers can be used for lubrication and as an insulator?

(a) SBR (b) PVC (c) PTFE (d) PAN

30. The IUPAC name of



is

(a) 3-methyl cyclohexene

(b) 1-methyl cyclohex-2-ene

(c) 6-methyl cyclohexene
(d) 1-methyl cyclohex-5-ene

31. The correct representation of a complex ion is
(a) $[Co(H_2O)(NH_3)_4Cl]^{2+}$
(b) $[CoCl(H_2O)(NH_3)_4]^{2+}$
(c) $[Co(NH_3)_4Cl(H_2O)]^{2+}$
(d) $[Co(NH_3)_4(H_2O)Cl]^{2+}$

32. Which element has maximum first ionisation potential?
(a) Cs (b) F (c) Na (d) He

33. In which of the following $\Delta E = \Delta H$?
(a) $N_2O_4(g) \rightleftharpoons 2NO_2(g)$
(b) $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$
(c) $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$
(d) $H_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons H_2O(l)$

34. Calcination is used in metallurgy for removal of
(a) water and sulphide (b) water and CO_2
(c) CO_2 and H_2S (d) H_2O and H_2S

35. What is the value of x on the $[Ni(CN)_4]^x$ complex ion?
(a) +2 (b) – 2 (c) 0 (zero) (d) + 4

36. The activation energy of a reaction is zero. The rate constant of this reaction
(a) increases with an increase of temperature
(b) decreases with an increase of temperature
(c) decreases with decrease of temperature
(d) is independent of temperature

37. Which of the following does not exist?
I. $HO_3S—S—SO_3H$ II. HO—Te—OH
III. $HS—S_2—SH$ IV. HS—Po—OH
(a) Only II (b) Only III
(c) II and IV (d) I, III and IV

38. Which of the following notations shows the product incorrectly?
(a) ${}^{10}_{5}B(\alpha, n)\,{}^{13}_{7}N$ (b) ${}^{242}_{96}Cm(\alpha, 2n)\,{}^{243}_{97}Bk$
(c) ${}^{14}_{7}N(n, p)\,{}^{14}_{6}C$ (d) ${}^{28}_{14}Si(d, y)\,{}^{30}_{15}P$

39. Acidic dichromate ion reacts with hydrogen peroxide to give deep blue colour. This is due to the formation of
(a) $CrO(O)_2$ (b) CrO_5
(c) Both (a) and (b) (d) None of (a) and (b)

40. In blast furnace, the highest temperature is in
(a) reduction zone (b) slag zone
(c) fusion zone (d) combustion zone

41. The reagent which distinguishes formic acid and acetic acid is
(a) 2, 4-dinitrophenyl hydrazine
(b) $HgCl_2$
(c) C_2H_5ONa
(d) Hg_2Cl_2

42. Neon atom requires energy to remove one electron from its outermost orbit, so also to add one electron into the outermost orbit.
Which of the following is correct about fluorine?
I. Fluorine releases energy when an electron is removed.
II. Fluorine requires energy to add one electron.
(a) (I) and (II) are correct
(b) (I) is correct, (II) is false
(c) (I) is false, (II) is correct
(d) (I) and (II) are false

43. Liquid hydrocarbon is converted into mixture of gaseous hydrocarbons by
(a) cracking
(b) oxidation
(c) hydrolysis
(d) distillation under reduced pressure

44. Match List-I with List-II and select the correct answer from the given codes.

	List - I (Reaction)		List - II (Reagent/Catalyst)
A.	Cannizaro reaction	1.	$SnCl_2$ / HCl
B.	Stephen's reaction	2.	NaOH
C.	Clemmensen reduction	3.	Zn / Hg—conc.HCl
D.	Rosenmund's method	4.	$\frac{Pd / BaSO_4}{\text{Boiling xylene}}$

Codes

	A	B	C	D
(a)	1	2	3	4
(b)	2	1	3	4
(c)	4	3	2	1
(d)	1	4	2	3

45. The correct order of ionic radius of nitrogen family is

(a) $N^{3-} < P^{3-} < As^{3-} < Sb^{3-} < Bi^{3-}$

(b) $N^{3-} < P^{3-} < Sb^{3-}$

(c) $P^{3-} = As^{3-} > Bi^{3-}$

(d) $N^{3-} > Bi^{3-} > Sb^{3-}$

46. Which of the following statement is false?

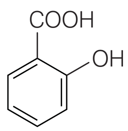
(a) Chlorophyll is responsible for the synthesis of carbohydrate in plants

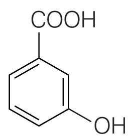
(b) In presence of oxygen, haemoglobin forms oxyhaemoglobin

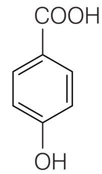
(c) Acetyl salicylic acid is known as aspirin

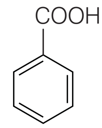
(d) Vitamin B_{12} contains Mg^{2+} ion

47. Among following, compound with the lowest pK_a value, is

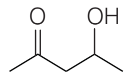
(a) 

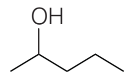
(b) 

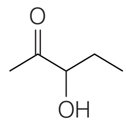
(c) 

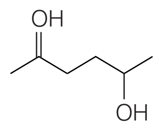
(d) 

48. Which one of the following will most readily be dehydrated in acidic solution?

(a) 

(b) 

(c) 

(d) 

49. Paracetamol is a following drug

(a) antipyretic (b) antiseptic

(c) antibiotic (d) anaesthetic

50. $K_2[HgI_4]$ detects the following ion

(a) Cl^- (b) NO_2^- (c) NO_3^- (d) NH_4^+

Mathematics

1. If A and B are two such events that $P(A \cup B) = P(A \cap B)$, then which of the following is true?

(a) $P(A) + P(B) = 0$

(b) $P(A) + P(B) = P(A)\,P(B/A)$

(c) $P(A) + P(B) = 2P(A)\,P(B/A)$

(d) None of the above

2. Equation of tangent to the circle $x^2 + y^2 - 2x - 2y + 1 = 0$ perpendicular to $y = x$ is given by

(a) $x + y \pm 1 = 0$ (b) $x + y = 2 \pm \sqrt{3}$

(c) $x - y \pm 3 = 0$ (d) $x - y \pm 1 = 0$

3. If point D divides the base BC of a ΔABC in the ratio $n : m$, then the value of $mBD^2 + nCD^2 + (m+n)AD^2$ is

(a) $mAC^2 + nAB^2$

(b) $(m+n)(AC^2 + AB^2)$

(c) $nAC^2 + mAB^2$

(d) None of the above

4. A man is standing on the horizontal plane. The angle of elevation of top of the pole is α. If he walks a distance double the height of the pole, then the elevation of the pole is 2α. The value of α is

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

5. If $f(x) = \begin{cases} \frac{\sin[x]}{[x]} & \text{for } [x] \neq 0 \\ 0 & \text{for } [x] = 0 \end{cases}$

where, $[x]$ denotes the greatest integer less than or equal to x, then $\lim_{x \to 0} f(x)$ is equal to

(a) 1 (b) -1

(c) 0 (d) Does not exist

6. The value of the angle between two straight lines $y = (2 - \sqrt{3})x + 5$ and $y = (2 + \sqrt{3})x - 7$ is

(a) 30° (b) 60°

(c) 45° (d) 90°

7. Equation of a plane passing through $(-1, 1, 1)$ and $(1, -1, 1)$ and perpendicular to $x + 2y + 2z = 5$ is

(a) $2x + 3y - 3z + 3 = 0$

(b) $x + y + 3z - 5 = 0$

(c) $2x + 2y - 3z + 3 = 0$

(d) $x + y + z - 3 = 0$

8. If $f'(x) > 0 \ \forall \ x \in R, f'(3) = 0$ and $g(x) = f(\tan^2 x - 2\tan x + 4), 0 < x < \frac{\pi}{2}$, then $g(x)$ is increasing in

(a) $\left(0, \frac{\pi}{4}\right)$ (b) $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ (c) $\left(0, \frac{\pi}{3}\right)$ (d) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

9. Probability of getting a total of 7 or 9 in a single throw of two dice is

(a) $\frac{5}{18}$ (b) $\frac{1}{6}$

(c) $\frac{1}{9}$ (d) None of these

10. What is compiler?

(a) Application software (b) System software

(c) Utility software (d) All of these

11. Let $f(x)$ be differentiable on the interval $(0, \infty)$ such that $f(1) = 1$ and

$$\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1 \text{ for each } x > 0.$$

Then, $f(x)$ is equal to

(a) $\frac{1}{3x} + \frac{2}{3}x^2$ (b) $-\frac{x}{3} + \frac{4x^2}{3}$

(c) $-\frac{1}{x}$ (d) $-\frac{1}{x} + \frac{2}{x^2}$

12. If $\int \sin\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}dx$

$$= A\sin^{-1} x + Bx\sqrt{1-x^2} + C,$$

then $A + B$ is equal to

(a) 10 (b) $\frac{1}{2}$

(c) 1 (d) $-\frac{1}{2}$

13. The coefficient of x^4 in $(1 + x + x^3 + x^4)^{10}$ is

(a) 210 (b) 100

(c) 310 (d) 110

14. The locus of centre of circles which cuts orthogonally the circle $x^2 + y^2 - 4x + 8 = 0$ and touches $x + 1 = 0$, is

(a) $y^2 + 6x + 7 = 0$ (b) $x^2 + y^2 + 2x + 3 = 0$

(c) $x^2 + 3y + 4 = 0$ (d) None of the above

15. Let $f(x) = \begin{vmatrix} \sin 3x & 1 & 2\left(\cos\frac{3x}{2} + \sin\frac{3x}{2}\right)^2 \\ \cos 3x & -1 & 2\left(\cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}\right) \\ \tan 3x & 4 & 1 + 2\tan 3x \end{vmatrix}$

Then, the value of $f'(x)$ at $x = (2n + 1)\pi, n \in I$ (the set of integers) is equal to

(a) $(-1)^n$ (b) $(-1)^{n+1}$

(c) 3 (d) 9

16. The condition for the line $lx + my + n = 0$ to be a normal to $\frac{x^2}{25} + \frac{y^2}{9} = 1$ is

(a) $\frac{l^2}{9} + \frac{m^2}{25} = \frac{n^2}{256}$ (b) $\frac{9}{m^2} + \frac{25}{l^2} = \frac{256}{n^2}$

(c) $\frac{l^2}{9} - \frac{m^2}{25} = \frac{n^2}{256}$ (d) None of these

17. The least value of a, for which the function $\frac{4}{\sin x} + \frac{1}{1 - \sin x} = a$ has atleast one solution in the interval $\left(0, \frac{\pi}{2}\right)$, is

(a) 9 (b) 4

(c) 5 (d) 1

18. If one line of regression coefficient is less than unity, then the other will be

(a) less than unity (b) equal to unity

(c) greater than unity (d) All of these

19. Three concurrent edges of a parallelopiped are given by

$\mathbf{a} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$,

$\mathbf{b} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

and $\mathbf{c} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$.

The volume of the parallelopiped is

(a) 14 cu units (b) 20 cu units

(c) 25 cu units (d) 60 cu units

20. Roots of equation $x^3 - 6x + 1 = 0$ lie in the interval

(a) (2, 3) (b) (3, 4) (c) (3, 5) (d) (4, 6)

21. If $\lim_{x \to 0} \frac{\sin(\sin x) - \sin x}{ax^3 + bx^5 + c} = \frac{-1}{12}$, then

(a) $a = 2, b \in R, c = 0$ (b) $a = -2, b \in R, c = 0$
(c) $a = 1, b \in R, c = 0$ (d) $a = -1, b \in R, c = 0$

22. According to Newton-Raphson method, the value of $\sqrt{12}$ upto three places of decimal will be

(a) 3.463 (b) 3.462
(c) 3.467 (d) None of these

23. If $\frac{(3-i)^2}{2+i} = A + iB$, where A and B are real numbers, then A and B are equal to

(a) $A = -4, B = 2$ (b) $A = 2, B = -4$
(c) $A = 2, B = 4$ (d) None of these

24. The radical centre of the system of circles,

$$x^2 + y^2 + 4x + 7 = 0,$$
$$2(x^2 + y^2) + 3x + 5y + 9 = 0$$
and $$x^2 + y^2 + y = 0$$ is

(a) $(-2, -1)$ (b) $(1, -2)$
(c) $(-1, -2)$ (d) None of these

25. The curve, for which the area of the triangle formed by X-axis, the tangent line at any point P and line OP is equal to a^2, is given by

(a) $y = x - Cx^2$ (b) $x = Cy \pm \frac{a^2}{y}$
(c) $y = Cx \pm \frac{a^2}{x}$ (d) None of these

26. Solution of the equation
$\cos^2 x \frac{dy}{dx} - (\tan 2x)y = \cos^4 x, |x| < \frac{\pi}{4}$, where $y\left(\frac{\pi}{6}\right) = \frac{3\sqrt{3}}{8}$, is given by

(a) $y \frac{\tan 2x}{1 - \tan^2 x} = 0$ (b) $y(1 - \tan^2 x) = C$
(c) $y = \sin 2x + C$ (d) $y = \frac{1}{2} \cdot \frac{\sin 2x}{1 - \tan^2 x}$

27. If $1, \omega$ and ω^2 are the cube roots of unity, then the value of $(1 - \omega + \omega^2)(1 + \omega - \omega^2)$ is equal to

(a) 4 (b) 0
(c) 2 (d) 3

28. The equation of the curve through the point (1, 0), whose slope is $\frac{y-1}{x^2 + x}$, is

(a) $2x(y-1) + x + 1 = 0$
(b) $(x+1)(y-1) + 2x = 0$
(c) $x(y-1)(x+1) + 2 = 0$
(d) $x(y+1) + y(x+1) = 0$

29. The number of points, where $f(x) = [\sin x + \cos x]$ (where $[\cdot]$ denotes the greatest integer function) and $x \in (0, 2\pi)$ is not continuous, is

(a) 3 (b) 4 (c) 5 (d) 6

30. The value of $\cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right)$ is equal to

(a) $\frac{x}{3}$ (b) $\frac{x}{4}$ (c) 1 (d) $\frac{x}{2}$

31. If $A(-1, 3, 2)$, $B(2, 3, 5)$ and $C(3, 5, -2)$ are vertices of a ΔABC, then angles of ΔABC are

(a) $\angle A = 90°, \angle B = 30°, \angle C = 60°$
(b) $\angle A = \angle B = \angle C = 60°$
(c) $\angle A = \angle B = 45°, \angle C = 90°$
(d) None of the above

32. $\lim_{x \to 0} \frac{\int_0^{x^2} \sin\sqrt{t}\, dt}{x^3}$ is equal to

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) 0 (d) ∞

33. If $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are three non-coplanar vectors, then $[\mathbf{a} \times \mathbf{b}\ \mathbf{b} \times \mathbf{c}\ \mathbf{c} \times \mathbf{a}]$ is equal to

(a) $[\text{a b c}]^3$ (b) $[\text{a b c}]^2$
(c) 0 (d) None of these

34. If geometric mean and harmonic mean of two numbers a and b are 16 and 64/5 respectively, then the value of $a : b$ is

(a) 4 : 1 (b) 3 : 2 (c) 2 : 3 (d) 1 : 4

35. If the sum of four numbers in GP is 60 and the arithmetic mean of the first and last numbers is 18, then the numbers are

(a) 3, 9, 27, 81 (b) 4, 8, 16, 32
(c) 2, 6, 18, 54 (d) None of these

36. The sum of the real solutions of equation $2|x|^2 + 51 = |1 + 20x|$ is

(a) 5 (b) 24
(c) 0 (d) None of these

37. The quadratic equation whose roots are $\frac{1}{3+\sqrt{2}}$ and $\frac{1}{3-\sqrt{2}}$, will be

(a) $7x^2 - 6x + 1 = 0$ (b) $6x^2 - 7x + 1 = 0$
(c) $x^2 - 6x + 7 = 0$ (d) $x^2 - 7x + 6 = 0$

38. Who said, "Number of transistors per square inch on integrated circuits double every year, since their invention will continue to do so in the foreseable future"?

(a) Alan Turing (b) Jon Von Neumann
(c) Herbert Simon (d) Gorden Moore

39. The number of vectors of unit length perpendicular to vectors $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\mathbf{b} = \hat{\mathbf{j}} + \hat{\mathbf{k}}$, is

(a) infinite (b) one (c) two (d) three

40. By trapezoidal rule, the approximate value of the integral $\int_0^6 \frac{dx}{1+x^2}$ is

(a) 1.3128 (b) 1.4108
(c) 1.4218 (d) None of these

41. The radius of a cylinder is increasing at the rate of 2 m/s and its height is decreasing at the rate of 3 m/s. When the radius is 3 m and height is 5 m, then the volume of the cylinder would change at the rate of

(a) $87\pi\ m^3/s$ (b) $33\pi\ m^3/s$
(c) $27\pi\ m^3/s$ (d) $15\pi m^3/s$

42. The point on the straight line $y = 2x + 11$ which is nearest to the circle $16(x^2 + y^2) + 32x - 8y - 50 = 0$, is

(a) $\left(\frac{9}{2}, 2\right)$ (b) $\left(\frac{9}{2}, -2\right)$ (c) $\left(-\frac{9}{2}, 2\right)$ (d) $\left(-\frac{9}{2}, -2\right)$

43. The locus of the extrimities of the latusrectum of the family of ellipses $b^2x^2 + y^2 = a^2b^2$ having a given major axis, is

(a) $x^2 \pm ay = a^2$ (b) $y^2 \pm bx = a^2$
(c) $x^2 \pm by = a^2$ (d) $y^2 \pm ax = b^2$

44. The solution of differential equation $(y \log x - 1)ydx = xdy$ is

(a) $y(\log e^x + Cx) = 1$ (b) $\left(\log\frac{x}{e} + Cx\right)x = y$
(c) $(\log Cx^2 + ex^2)y = x$ (d) None of these

45. If m things are distributed among a men and b women. Then, the chance that the number of things received by men is odd, is

(a) $\frac{(b-a)^m - (b+a)^m}{2(b+a)^m}$ (b) $\frac{(b+a)^m - (b-a)^m}{2(b+a)^m}$
(c) $\frac{(b+a)^m - (b-a)^m}{(b+a)^m}$ (d) None of these

46. The solution of the differential equation

$$\sqrt{a+x}\,\frac{dy}{dx} + xy = 0 \text{ is}$$

(a) $y = Ce^{\frac{2}{3}(2a-x)\sqrt{x+a}}$ (b) $y = Ce^{\frac{2}{3}(a-x)\sqrt{x+a}}$
(c) $y = Ce^{\frac{2}{3}(2a+x)\sqrt{x+a}}$ (d) $y = Ce^{-\frac{2}{3}(2a-x)\sqrt{x+a}}$

47. The values of a, if $f(x) = 2e^x - ae^{-x} + (2a+1)x - 3$ increases x, are in

(a) $[0, \infty)$ (b) $(-\infty, 0]$
(c) $(-\infty, \infty)$ (d) $(1, \infty)$

48. By Newton-Raphson method, the positive root of the equation $x^4 - x - 10 = 0$ is

(a) 1.871 (b) 1.868
(c) 1.856 (d) None of these

49. In ΔABC, if $3a = b + c$, then value of $\cot\frac{B}{2}\cot\frac{C}{2}$ will be

(a) 1 (b) 2 (c) $\sqrt{3}$ (d) $\sqrt{2}$

50. If $\sin^{-1}x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then value of x will be

(a) 0 (b) $\frac{1}{\sqrt{5}}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{\sqrt{3}}{2}$

51. The value of $\int_0^{\sqrt{\ln(\pi/2)}} \cos(e^{x^2})2x\, e^{x^2}dx$ is

(a) 1 (b) 1 + sin 1
(c) 1 – sin 1 (d) (sin 1) –1

52. API stands for

(a) Access Programming Interface
(b) Android Programming Interface
(c) Application Programming Interface
(d) None of the above

53. If $Z = i\log(2 - \sqrt{3})$, then the value of $\cos Z$ will be

(a) i (b) $2i$ (c) 1 (d) 2

54. $\lim_{x \to 2} \dfrac{2-\sqrt{2+x}}{2^{1/3}-(4-x)^{1/3}}$ is equal to

(a) $2\cdot 3^{-1/2}$ (b) $3\cdot 2^{-4/3}$
(c) $-3\cdot 2^{-4/3}$ (d) None of these

55. The three lines of a triangle are given by $(x^2-y^2)(2x+3y-6)=0$. If the point $(-2, \lambda)$ lies inside and $(\mu, 1)$ lies outside the triangle, then

(a) $\lambda \in \left(1, \dfrac{10}{3}\right); \mu \in (-3, 5)$
(b) $\lambda \in \left(2, \dfrac{10}{3}\right); \mu \in (-1, 1)$
(c) $\lambda \in \left(-1, \dfrac{9}{2}\right); \mu \in \left(-2, \dfrac{10}{3}\right)$
(d) None of the above

56. System software : Utility software : :

(a) Operating system: Anti-virus
(b) Anti-virus : Operating system
(c) Anti-virus : MS office
(d) MS office : Anti-virus

57. The number of common tangents to two circles $x^2+y^2=4$ and $x^2+y^2-8x+12=0$ is

(a) 1 (b) 2 (c) 3 (d) 4

58. If $2a+3b+6c=0$, then the equation $ax^2+bx+c=0$ has atleast one real root in

(a) $(0, 1)$ (b) $\left(0, \dfrac{1}{2}\right)$
(c) $\left(\dfrac{1}{4}, \dfrac{1}{2}\right)$ (d) None of these

59. If for all $x, y \in N$, there exists a function $f(x)$ satisfying $f(x+y)=f(x)\cdot f(y)$ such that $f(1)=3$ and $\sum_{x=1}^{n} f(x)=120$, then value of n will be

(a) 4 (b) 5
(c) 6 (d) None of these

60. If $f(x)=\begin{cases}\sin\left(\dfrac{\pi x}{2}\right), & \text{if } x<1\\ 3-2x, & \text{if } x\ge 1\end{cases}$, then $f(x)$ has

(a) local minimum at $x=1$
(b) local maximum at $x=1$
(c) Both local maximum and local minimum at $x=1$
(d) None of the above

61. The general solution of the differential equation $\dfrac{dy}{dx}=y\tan x-y^2\sec x$ is

(a) $\tan x=(C+\sec x)y$ (b) $\sec y=(C+\tan y)x$
(c) $\sec x=(C+\tan x)y$ (d) $\tan y=(C+\sec x)x$

62. The values of λ such that $(x, y, z)\ne(0, 0, 0)$ and $(\hat{\mathbf{i}}+\hat{\mathbf{j}}+3\hat{\mathbf{k}})x+(3\hat{\mathbf{i}}-3\hat{\mathbf{j}}+\hat{\mathbf{k}})y+(-4\hat{\mathbf{i}}+5\hat{\mathbf{j}})z$ $=\lambda(\hat{\mathbf{i}}x+\hat{\mathbf{j}}y+\hat{\mathbf{k}}z)$ are

(a) 0, 1 (b) –1, 1 (c) –1, 0 (d) –2, 0

63. In ΔABC, if $\cot A$, $\cot B$ and $\cot C$ are in AP, then a^2, b^2 and c^2 are in

(a) HP (b) AP
(c) GP (d) None of these

64. By Simpson's $\dfrac{1}{3}$rd rule, the approximate value of the integral $\int_1^2 e^{-x/2}dx$ using four intervals, is

(a) 0.377 (b) 0.487 (c) 0.477 (d) 0.387

65. A die is rolled three times. The probability of getting a larger number than the previous number is

(a) $\dfrac{5}{216}$ (b) $\dfrac{5}{54}$ (c) $\dfrac{1}{6}$ (d) $\dfrac{5}{36}$

66. If $f(x)=\log_e(6-|x^2+x-6|)$, then domain of $f(x)$ has how many integral values of x?

(a) 5 (b) 4
(c) Infinite (d) None of these

67. The graph of the equation $y^2+z^2=0$ in three dimensional space is

(a) YZ-plane (b) Z-axis (c) Y-axis (d) X-axis

68. If $Z_1=1+i$, $Z_2=-2+3i$ and $Z_3=\dfrac{ai}{3}$ are collinear, where $i^2=-1$, then the value of a will be

(a) –1 (b) 3 (c) 4 (d) 5

69. If a, b and c are in HP, then for any $n \in N$, which one of the following is true?

(a) $a^n+c^n<2b^n$ (b) $a^n+c^n>2b^n$
(c) $a^n+c^n=2b^n$ (d) None of these

70. The value of $\tan\left[2\tan^{-1}\left(\dfrac{1}{5}\right)-\dfrac{\pi}{4}\right]$ is

(a) $\dfrac{17}{7}$ (b) $\dfrac{-17}{7}$ (c) $\dfrac{7}{17}$ (d) $\dfrac{-7}{17}$

English and General Aptitude

Directions (Q. Nos. 1-5) *Read the given passage carefully and answer the questions that follow by selecting the most appropriate option.*

No one knows when or by whom rockets were invented. In all probability the rocket was not suddenly invented but evolved gradually over a long period of time, perhaps in different parts of the world at the same time. Some historians of rocketry, notably Willy Ley, trace the development of rockets to the 13th century China, a land noted in ancient times for its fire work display. In the year 1232 AD when the Mongols laid siege to the city of Kai-Feng Fu, the capital of Honan province, the Chinese defenders used weapons that were described as "arrows of flying fire". There is no explicit statement that these arrows were rockets, but some students have concluded that they were because the record does not mentioned bows or other means of shooting the arrows. In the same battle, we read, the defenders dropped from the walls of the city a kind of bomb described as *"heaven-shaking thunder*". From these meagre references some students have concluded that the Chinese, by the year 1232 had discovered gunpowder and had learned to use it to make explosive bombs as well as propulsive charges for rockets.

1. The passage gives primarily a history of
(a) the bravery of the Chinese
(b) the invention of rockets
(c) the attack on China by the Mongols
(d) the battle against the Chinese wall

2. According to this passage, rockets were invented by
(a) Willy Ley
(b) unknown people
(c) the Mongols
(d) the ruler of Honan province

3. According to this passage, rockets were
(a) a gift of God to the Chinese
(b) invented in the twentieth century
(c) invented in 1232 AD
(d) developed over many centuries

4. The phrase 'arrows of flying fire'
(a) means some ancient phenomenon in the skies
(b) refers to lightning and thunder
(c) is another name for rockets
(d) is assumed to refer to rockets

5. The bombs have been referred to as "heaven shaking thunder" because they
(a) contain gunpowder
(b) make thunderous noise
(c) are propelled by rockets
(d) seem to fall from heaven

Directions (Q. Nos. 6-8) *Complete the following sentence(s) with an appropriate word.*

6. The lawyer has plenty of ....
(a) criminals (b) buyers
(c) customers (d) clients

7. "I have brought the book. It's ..... !" Ravi said assertively to all the boys present.
(a) mine (b) my (c) me (d) myself

8. "The project is good, but there is .... missing to make it an excellent work," the engineer commented.
(a) everything (b) anything
(c) something (d) nothing

Directions (Q. Nos. 9-12) *In the following questions, select the antonym of the given words.*

9. Stingy
(a) Clean (b) Tight
(c) Generous (d) Cheap

10. Barren
(a) Fertile (b) Rich
(c) Prosperous (d) Positive

11. Virtue
(a) Vice (b) Failure (c) Fault (d) Offence

12. Nervous
(a) Flawless (b) Immature
(c) Smooth (d) Composed

Directions (Q. Nos. 13-15) *In the following questions, choose the alternative which best expresses the meaning of the word given in capital letters.*

13. Savour
(a) Taste (b) Protector
(c) Sour (d) Flavour

14. Rivalled

(a) Hatred (b) Revised
(c) Competed (d) Contradicted

15. Trimming

(a) Skimming (b) Arranging
(c) Planning (d) Cutting

Directions (Q. Nos. 16-20) *In each of the following questions, there is a certain relationship between two given words on side of* (: :) *and one word is given on another side* (: :) *while another word is to be found from the given alternatives, having the same relation with this word as the words of the given pair bear. Choose the correct alternative.*

16. Donkey : Brays :: Wolf : ?

(a) Bellows (b) Howls
(c) Whimpers (d) Roars

17. Astronauts : Space :: Argonauts : ?

(a) Fire (b) Ship
(c) Treasure (d) Sea

18. Orthopaedic : Bones :: Dermatologist : ?

(a) Feet (b) Skin
(c) Heart (d) Lungs

19. Basilica : Church :: Dormer : ?

(a) Window (b) Chapel
(c) Movie (a) Servant

20. Although : Nevertheless :: Though : ?

(a) Therefore (b) Yet
(c) However (d) Simultaneously

Directions (Q. Nos. 21-23) *In each of the following questions, a group of four words are given. Choose the word which is odd.*

21. Find the odd word.

(a) Sun (b) Mercury (c) Mars (d) Venus

22. Find the odd word.

(a) Stneng (b) Hesitant
(c) Daring (b) Brave

23. Find the odd word.

(a) Marigold (b) Tulip
(c) Lotus (d) Rose

24. In the English alphabet, find the position of *P* from right.

(a) 12 (b) 13 (c) 10 (d) 11

25. Which of the following letters is 14th to the right of 6th letter from the left in the English alphabet?

(a) R (b) P (c) W (d) T

26. If English alphabet is written in backward order, then what will be the 13th letter to the left of the 3rd letter from right?

(a) P (b) N (c) R (d) Q

Directions (Q. Nos. 27-28) *In the following series, replace the question mark (?) with the suitable option.*

27. 23, 28, 34, 41, 49, ?

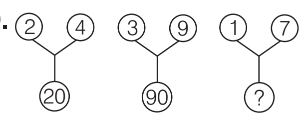
(a) 57 (b) 59 (c) 56 (d) 58

28. 11, 13, 17, 19, 23, 25, ?

(a) 29 (b) 27 (c) 31 (d) 37

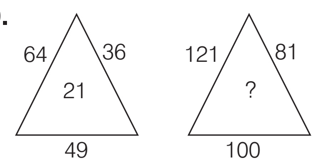
Directions (Q. Nos. 29-30) *In each of the following questions, a set of figures carrying certain characters is given. Assuming that the characters in each set follow a similar pattern, find the missing character in each case.*

29.



(a) 160 (b) 100 (c) 50 (d) 75

30.



(a) 40 (b) 30 (c) 20 (d) 10

Answers

Physics

1. (c)	**2.** (b)	**3.** (a)	**4.** (c)	**5.** (c)	**6.** (b)	**7.** (d)	**8.** (b)	**9.** (c)	**10.** (d)
11. (d)	**12.** (a)	**13.** (b)	**14.** (b)	**15.** (a)	**16.** (b)	**17.** (a)	**18.** (c)	**19.** (a)	**20.** (b)
21. (a)	**22.** (a)	**23.** (c)	**24.** (a)	**25.** (a)	**26.** (c)	**27.** (a)	**28.** (a)	**29.** (c)	**30.** (c)
31. (a)	**32.** (b)	**33.** (d)	**34.** (b)	**35.** (b)	**36.** (d)	**37.** (c)	**38.** (b)	**39.** (d)	**40.** (b)
41. (c)	**42.** (c)	**43.** (d)	**44.** (d)	**45.** (a)	**46.** (a)	**47.** (a)	**48.** (a)	**49.** (d)	**50.** (a)

Chemistry

1. (a)	**2.** (c)	**3.** (a)	**4.** (c)	**5.** (a)	**6.** (b)	**7.** (d)	**8.** (a)	**9.** (b)	**10.** (b)
11. (b)	**12.** (a)	**13.** (a)	**14.** (a)	**15.** (d)	**16.** (b)	**17.** (a)	**18.** (c)	**19.** (b)	**20.** (b)
21. (c)	**22.** (d)	**23.** (a)	**24.** (c)	**25.** (a)	**26.** (b)	**27.** (a)	**28.** (b)	**29.** (c)	**30.** (a)
31. (d)	**32.** (d)	**33.** (c)	**34.** (b)	**35.** (b)	**36.** (d)	**37.** (c)	**38.** (b)	**39.** (b)	**40.** (d)
41. (b)	**42.** (d)	**43.** (b)	**44.** (b)	**45.** (a)	**46.** (d)	**47.** (a)	**48.** (a)	**49.** (a)	**50.** (d)

Mathematics

1. (c)	**2.** (*)	**3.** (c)	**4.** (a)	**5.** (d)	**6.** (b)	**7.** (c)	**8.** (d)	**9.** (a)	**10.** (b)
11. (a)	**12.** (c)	**13.** (c)	**14.** (a)	**15.** (c)	**16.** (b)	**17.** (a)	**18.** (c)	**19.** (a)	**20.** (a)
21. (b)	**22.** (d)	**23.** (b)	**24.** (a)	**25.** (b)	**26.** (d)	**27.** (a)	**28.** (*)	**29.** (c)	**30.** (d)
31. (d)	**32.** (a)	**33.** (b)	**34.** (a)	**35.** (b)	**36.** (d)	**37.** (a)	**38.** (d)	**39.** (c)	**40.** (b)
41. (b)	**42.** (c)	**43.** (a)	**44.** (d)	**45.** (b)	**46.** (a)	**47.** (a)	**48.** (a)	**49.** (b)	**50.** (b)
51. (c)	**52.** (c)	**53.** (d)	**54.** (c)	**55.** (d)	**56.** (a)	**57.** (c)	**58.** (a)	**59.** (a)	**60.** (b)
61. (c)	**62.** (c)	**63.** (b)	**64.** (c)	**65.** (b)	**66.** (d)	**67.** (a)	**68.** (d)	**69.** (b)	**70.** (d)

English and General Aptitude

1. (b)	**2.** (b)	**3.** (d)	**4.** (d)	**5.** (b)	**6.** (d)	**7.** (a)	**8.** (c)	**9.** (c)	**10.** (a)
11. (a)	**12.** (a)	**13.** (a)	**14.** (c)	**15.** (d)	**16.** (b)	**17.** (d)	**18.** (b)	**19.** (a)	**20.** (c)
21. (a)	**22.** (b)	**23.** (b)	**24.** (d)	**25.** (d)	**26.** (a)	**27.** (d)	**28.** (a)	**29.** (c)	**30.** (b)

Note (*) None of the option is correct.

SOLUTIONS

Physics

1. *(c)* Increase in potential energy

= Final potential energy – Initial potential energy

$$= -\frac{GMm}{\left(R+\frac{R}{5}\right)} - \left(-\frac{GMm}{R}\right)$$

$$= -\frac{5}{6}\frac{GMm}{R} + \frac{GMm}{R}$$

$$= \frac{1}{6}\frac{GMm}{R} = \frac{1}{6}\frac{GMmR}{R^2}$$

$$= \frac{1}{6}mgR \qquad \left[\because g = \frac{GM}{R^2}\right]$$

$$= \frac{5}{6}mgh \qquad \left[\because h = \frac{R}{5}\right]$$

2. *(b)* To break the covalent bond in a semiconductor, energy equal to forbidden energy gap is required.

3. *(a)* Given, $A = 60^\circ$ and $\delta_m = 30^\circ$

$$\text{So, } \mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}} = \frac{\sin\left(\frac{60^\circ + 30^\circ}{2}\right)}{\sin\frac{60^\circ}{2}}$$

$$= \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{1}{\sqrt{2}} \times 2 = \frac{2\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}$$

$$= \frac{2\sqrt{2}}{2} = \sqrt{2}$$

We have, $\mu = \dfrac{1}{\sin C}$

or $\sqrt{2} = \dfrac{1}{\sin C}$

or $C = \sin^{-1}\left(\dfrac{1}{\sqrt{2}}\right) = 45^\circ$

4. *(c)* Faraday constant is a universal constant.

5. *(c)* If a comb is run through wet hair on rainy day, then it will not attract small bits of paper.

6. *(b)* Given, $x = 9t^2$

$$\therefore \quad v = \frac{dx}{dt} = \frac{d}{dt}(9t^2) = \frac{9d}{dt}(t^2) = 18\,t$$

also, $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = \dfrac{d}{dt}(18t) = 18$

So, the graph between acceleration and time is a line parallel to time axis as acceleration is constant.

7. *(d)* The minimum negative potential V_0 is called stopping potential.

Here, $K_{max} = 10$ eV

So, the stopping potential, $eV_0 = K_{max}$

$\Rightarrow \quad eV_0 = 10$ eV

or $\quad V_0 = -10$ V

8. *(b)* Thermal radiations exists in infrared part of the electromagnetic spectrum.

9. *(c)* For a thin uniform rod, moment of inertia about an axis through its centre perpendicular to length of rod, $I = \dfrac{1}{12}ML^2$

Here, $M = (m_w + m_b)$

$\therefore \quad I = \dfrac{1}{12}(m_w + m_b)\,L^2$

10. *(d)* The time period of a particle executing SHM is given as

$$T = 2\pi\sqrt{\frac{x}{a}} = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$

$$= 2\pi\sqrt{\frac{3\times10^{-2}}{12\times10^{-2}}} = 2\pi\times\frac{1}{2}$$

$$= \pi = 3.14 \text{ s}$$

11. *(d)* A transformer is essentially an AC device, it does not work on DC. So, voltage developed across secondary is zero.

12. *(a)* Resolving power of eye $= \left(\dfrac{1}{60}\right)^\circ = \dfrac{1}{60}\times\dfrac{\pi}{180}$

Let the minimum distance between the poles be *d*,

then, $\dfrac{d}{11000} = \dfrac{1}{60}\times\dfrac{\pi}{180}$

or $\quad d = 11000\times\dfrac{1}{60}\times\dfrac{\pi}{180} = 3$ m

13. *(b)* Stefan's constant, $\sigma = \frac{E}{T^4}$ (where, E is energy/s area)

$$= \frac{[ML^2 \ T^{-2}]}{[T]\ [L^2]\ [\theta^4]} = [MT^{-3}\theta^{-4}]$$

14. *(b)* Time period of oscillation of mass m suspended from a spring

$$T = 2\pi\sqrt{\frac{m}{k}}$$

If the spring is cut into two halves, then the new time period.

$$T' = 2\pi\sqrt{\frac{m}{2k}} = 2\frac{\pi}{\sqrt{2}}\sqrt{\frac{m}{k}} = \frac{T}{\sqrt{2}}$$

15. *(a)* Given, $I = 2.0$ A

$$R = 25\ \Omega$$
$$t = 1\text{ min } = 60\text{ s}$$

$\therefore$ Heat produced, $H = I^2\ Rt$

$$= (2)^2 \times 25 \times 60$$
$$= 4 \times 25 \times 60\text{ J}$$
$$= 6 \times 10^3\text{ J}$$

16. *(b)* Frequency for a mass m executing SHM

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad \text{...(i)}$$

When the mass is increased by 4 times, then the new frequency

$$f' = \frac{1}{2\pi}\sqrt{\frac{k}{4m}} = \frac{1}{2}\cdot\frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{f}{2}$$

17. *(a)* For isothermal process, $dU = 0$,

i.e., internal energy remains unchanged.

For a cyclic process, $dU = 0$,

i.e., internal energy remains unchanged.

18. *(c)* An electric charge in uniform motion produces both electric field and magnetic field.

19. *(a)* For photoelectric effect to take place, the incident wavelength (λ) should be equal to or smaller than the threshold wavelength (λ_0) *i.e.,*

$$\lambda \le \lambda_0$$

20. *(b)* $_7X^{15} + {}_2He^4 \longrightarrow {}_9Y^{19}$

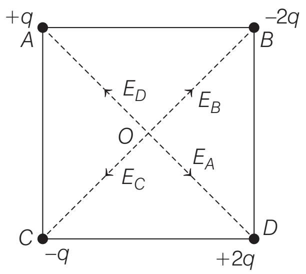
$$_9Y^{19} \longrightarrow {}_8Z^{18} + {}_1H^1$$

According to the conservation of mass number

$$19 = 18 + 1$$

So, the mass number of changed atom = 18

21. *(a)* $AD = BC = \sqrt{(5)^2 + (5)^2} = \sqrt{25+25} = \sqrt{2} \times 5\text{ cm}$



$\Rightarrow AO = BO = CO = OD = \frac{5\sqrt{2}}{2}$ cm

The electric field,

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

So, $\quad E_A = \frac{9 \times 10^9 \times 10 \times 10^{-9} \times 4}{25 \times 2}$

$= 7.2\ \ NC^{-1}$ along OD

$$E_B = \frac{9 \times 10^9 \times 2 \times 10 \times 10^{-9} \times 4}{25 \times 2}$$

$= 14.4\ NC^{-1}$ along OB

$$E_C = \frac{9 \times 10^9 \times 10 \times 10^{-9} \times 4}{25 \times 2}$$

$= 7.2$ along OC

$$E_D = \frac{9 \times 10^9 \times 2 \times 10 \times 10^{-9} \times 4}{25 \times 2}$$

$= 14.4$ along OA

Resultant of E_A and E_D, $E_1 = (14.4 - 7.2)$

$= 7.2\ NC^{-1}$ along OA.

Resultant of E_B and E_C, $E_2 = (14.4 - 7.2)$

$= 7.2\ NC^{-1}$ along OB.

Since, E_1 and E_2 are perpendicular to each other.

$\therefore E = \sqrt{E_1^2 + E_2^2}$ is along 45° to OA upward.

22. *(a)* For an adiabatic process, $TV^{\gamma-1}$ = constant.

23. *(c)* The correct relation is $v_{rms} > \bar{v} > v_{mp}$

As $\quad \bar{v} = 1.6\sqrt{\frac{RT}{M}}$, $v_{rms} = 1.73\sqrt{\frac{RT}{M}}$

and $\quad v_{mp} = 1.41\sqrt{\frac{RT}{M}}$

24. *(a)* Due to reverse biasing, in a junction diode the potential barrier increases.

25. *(a)* Given, $Vp^2 =$ constant

or $V\left[\frac{RT}{V}\right]^2 =$ constant or $\frac{T^2}{V} =$ constant

or $\frac{T'^2}{T^2} = \frac{V'}{V} \Rightarrow \frac{T'^2}{T^2} = \frac{2V}{V} \Rightarrow T' = \sqrt{2}T$

26. *(c)* Warming of glass bulb due to filament is an example of heat transfer due to radiation.

27. *(a)* Vertical component of earth's magnetic field is

$$B_V = B_H \tan\delta$$

where, B_H = horizontal component of earth's magnetic field

δ = angle of dip.

Total intensity of earth's magnetic field

$$B = \sqrt{B_H^2 + B_V^2}$$
$$= \sqrt{B_H^2 + B_H^2} \quad [\because \tan\delta = \tan 45° = 1]$$
$$= \sqrt{2}\, B_H$$
$$= 1.44 \times 0.4 \times 10^{-4}\ \text{T}$$
$$= 5.656 \times 10^{-5}\ \text{T}$$
$$= 0.5 \times 10^{-4}\ \text{T}$$

28. *(a)* Generalised formula for apparent frequency is

$$f_a = \left(\frac{v \pm v_o}{v \mp v_s}\right) f_0$$

According to the question source is stationary,

$$v_s = 0$$
$$f_a = f' = \left(\frac{v \pm v_o}{v}\right) f_0$$

Observer is moving towards the source

$$\Rightarrow \quad f_a = f'\left(\frac{v + v_o}{v}\right) f$$

where, v = speed of the wave

v_o = speed of the observer

f = original frequency of the source

29. *(c)* Magnetic flux linked with the secondary due to current in the primary.

$$\phi_2 = MI_1$$
$$\Rightarrow \quad \frac{d\phi_2}{dt} = M\frac{dI_1}{dt}$$

$\Rightarrow$ Induced emf in the secondary

$$\varepsilon_2 = \frac{d\phi_2}{dt} = M\frac{dI_1}{dt}$$

where, M = mutual inductance = 1.5 H

$$\frac{dI_1}{dt} = \frac{5}{1 \times 10^{-3}} = 5000\ \text{A/s}$$

$$\therefore \quad \varepsilon_2 = -1.5 \times 5000\ \text{V}$$
$$= 7.5 \times 10^3 V = 7500\ \text{V}$$

30. *(c)* Magnetic field at the centre of a toroid is given by

$$B = \frac{\mu_0 Ni}{2\pi r}$$

where, $n = \frac{N}{2\pi r}$ = number of turns per unit length of the toroid.

31. *(a)* Magnification produced by the lens

$$m = \frac{v}{u} = -3 \Rightarrow v = -3u$$

Applying lens of formula, we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{-3u} - \frac{1}{u} = \frac{1}{0.12}$$

or $$\frac{-1-3}{3u} = \frac{100}{12} = \frac{25}{3}$$

or $$-\frac{4}{3u} = \frac{25}{3}$$

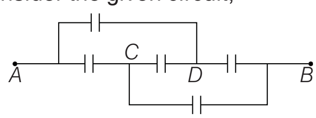
or $$u = -\frac{4}{25}\ \text{m} = 0.16\ \text{m}$$

32. *(b)* Rate of change of momentum of the object,

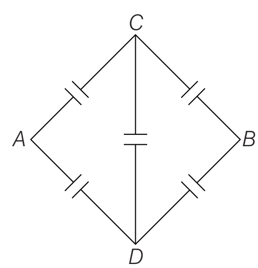
$$\frac{\Delta p}{\Delta t} = F_{\text{ext}} = \frac{mv^2}{r}$$

$$\frac{\Delta p}{\Delta t} \propto v^2 \quad \text{(where, } m \text{ and } r \text{ are constant)}$$

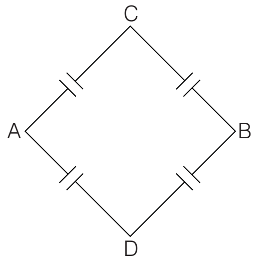
33. *(d)* Consider the given circuit,



The circuit can be redrawn as shown below.



The circuit is a balanced bridge. We can remove the capacitor between C and D.



Thus, equivalent capacitance between A and B is

$$C_{AB} = \frac{C}{2} + \frac{C}{2} = C$$

34. *(b)* Time period of a suspended magnet is given by

$$T = 2\pi \sqrt{\frac{I}{MB}}$$

where, I = moment of inertia of the magnet about the point of suspension.

M = magnetic moment

B = magnetic field

When the magnet is broken in length

$$T^{2'} = 2\pi\sqrt{\frac{I'}{M'B}}$$

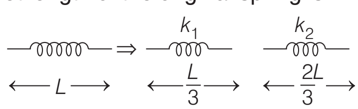
$$\Rightarrow \quad \frac{T}{T^{2'}} = \sqrt{\frac{I}{I'}} \times \sqrt{\frac{M'}{M}} = \sqrt{\frac{m\,L^2}{m'\,L'^2}} \times \sqrt{\frac{L'}{L}}$$

Here, m and m' are the masses of the magnet and L and L's are their lengths.

$$= \sqrt{\frac{m}{m'}} \times \sqrt{\frac{L}{L'}} = \sqrt{\frac{m}{m/2}} \times \sqrt{\frac{L}{L/2}} = \sqrt{2} \times \sqrt{2} = 2$$

$$\Rightarrow \quad T' = \frac{T}{2} = \frac{4}{2} = 2\text{ s}$$

35. *(b)* Let length of the original spring is L.



As we know that,

Spring constant, $k \propto \frac{1}{\text{length}}$

$$\Rightarrow \quad k \propto \frac{1}{L}$$

$$k_1 \propto \frac{3}{L}$$

$$k_2 \propto \frac{3}{2L}$$

$$\Rightarrow \quad \frac{k_2}{k} = \frac{3/2L}{1/L} = \frac{3}{2}$$

$$\Rightarrow \quad k_2 = \frac{3}{2}k$$

36. *(d)* Number of half-lives in 40 min

$$n = \frac{40}{T_{1/2}} = \frac{40}{10} = 4$$

Amount of the substance remaining after n half- lives

$$N = N_0\left(\frac{1}{2}\right)^n = N_0\left(\frac{1}{2}\right)^4 = \frac{N_0}{16}$$

where, N_0 is original amount of the substance.

Amount of the substance decayed in 40 min.

$$N' = N_0 - N = N_0 - \frac{N_0}{16} = \frac{15}{16}N_0$$

$$= \left(\frac{15}{16} \times 100\%\right) \text{ of } N_0$$

$$= 15 \times 6.25\ \% \text{ of } N_0$$

$$= 93.75\ \% \text{ of } N_0$$

37. *(c)* For isothermal expansion of an ideal gas

$$pV = \text{constant}$$

$$\Rightarrow \quad p\Delta V + V\Delta p = 0$$

For container A,

$$p_{iA}(V) + V(\Delta p) = 0$$

$$\Rightarrow \quad p_{iA} = -\frac{V\Delta P}{V} = -\Delta p \quad \ldots\text{(i)}$$

where, p_i is initial pressure.

For container B,

$$p_{iB}(V) + V(1.5\,\Delta p) = V$$

$$\Rightarrow \quad p_{iB} = -1.5\,\Delta p \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we have

$$p_{iA} = \frac{p_{iB}}{1.5} = \frac{2}{3}p_{iB}$$

$$\Rightarrow \quad m_A = \frac{2}{3}mB$$

$$\Rightarrow \quad 3m_A = 2mB \quad [\because p \propto m]$$

38. *(b)* For monoatomic gas,

$$\gamma_1 = \frac{C_{p_1}}{C_{V_1}} = \frac{5}{3} \quad \text{and} \quad C_{p_1} - C_{V_1} = R$$

this gives $C_{p_1} = \frac{5}{2}R$ and $C_{V_1} = \frac{3}{2}R$

For diatomic gas,

$$\gamma_2 = \frac{C_{p_2}}{C_{V_2}} = 7/5 \quad \text{and} \quad C_{p_2} - C_{V_2} = R$$

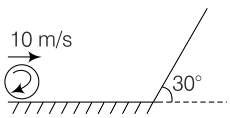
$$\Rightarrow \quad C_{p_2} = \frac{7}{2}R \quad \text{and} \quad C_{V_2} = \frac{5}{2}R$$

Now, we can write,

$$\gamma_{mix} = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 C_{V_1} + n_2 C_{V_2}}$$

$$= \frac{C_{p_1} + C_{p_2}}{C_{V_1} + C_{V_2}} = \frac{\frac{5}{2}R + \frac{7}{2}R}{\frac{3}{2}R + \frac{5}{2}R} = \frac{12}{8} = \frac{3}{2} = 1.5$$

39. *(d)* Applying conservation of mechanical energy, we can write



$$\frac{1}{2} I \omega^2 + \frac{1}{2} mv^2 = mgh$$

where, h is maximum height reached by the sphere, v is linear velocity and ω is angular velocity of the sphere.

$$\frac{1}{2} I \left(\frac{v}{r}\right)^2 + \frac{1}{2} mv^2 = mgh \qquad [\because v = \omega r]$$

$$\Rightarrow \quad \frac{1}{2} \times \frac{2}{5} mr^2 \times \frac{v^2}{r^2} + \frac{1}{2} mv^2 = mgh$$

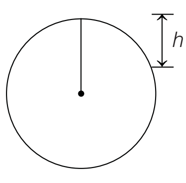
$$\frac{1}{5} mv^2 + \frac{1}{2} mv^2 = mgh$$

$$\frac{7}{10} mv^2 = mgh$$

$$h = \frac{7}{10} \times \frac{v^2}{g} = \frac{7}{10} \times \frac{10^2}{9.8} = \frac{70}{9.8}$$

$$= \frac{100}{14} = \frac{50}{7} = 7.1 \text{ m.}$$

40. *(b)* Suppose depth inside the earth is h.



As we know that, inside the earth

$$g_i \propto \left(\frac{R - h}{R^3}\right) \qquad \ldots\text{(i)}$$

On the surface of the earth

$$g_s \propto \frac{1}{R^2} \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{g_i}{g_s} = \frac{(R-h)R^2}{R^3} = \frac{R-h}{R}$$

$$\Rightarrow \quad \frac{1}{4} = \frac{R-h}{R}$$

$$\Rightarrow \quad R = 4R - 4h \Rightarrow 3R = 4h$$

$$\Rightarrow \quad h = \frac{3}{4}R.$$

41. *(c)* Range of a projectile is maximum for angle of projection of 45°.

The nearest value is 43°. Thus, among given values of angles of projection, range will be maximum for 43°.

42. *(c)* Resistance of a wire is given by

$$R = \frac{\rho L}{A}$$

where, $\rho =$ resistivity, $L =$ length of the wire

$A =$ area of cross- section of the wire

Equating initial and finial volumes of the wire, we can write

$$AL = A'L' = A' \times (2L) \quad \Rightarrow A' = \frac{A}{2}$$

$$\therefore \quad \frac{R'}{R} = \frac{\rho L'/A'}{\rho L/A} = \left(\frac{L'}{L}\right)\left(\frac{A}{A'}\right)$$

$$= \left(\frac{2L}{L}\right)\left(\frac{A}{A/2}\right) = 4$$

$$\Rightarrow \quad R' = 4R = 4(5) = 20\,\Omega.$$

43. *(d)* Magnifying power of a telescope is given by

$$m = \frac{f_o}{f_e}$$

If the focal length of the eye piece is doubled, the new magnifying power is given by

$$m' = \frac{f_o}{f_e'} = \frac{f_o}{2f_e}$$

$$\therefore \quad m' = \frac{m}{2}$$

44. *(d)* Given $V = 250$ V and $P = 500$ W

As we know that,

$$P = \frac{V^2}{R}$$

$\Rightarrow \quad R = V^2/P = \dfrac{250 \times 250}{500}$

$= \dfrac{625}{5} = 125\ \Omega.$

45. *(a)* As the battery remains connected. potential difference (V) of the capacitor remains constant. Suppose dielectric of dielectric constant K is introduced between the plates. Now, capcitance of the capacitor is

$$C' = KC_0$$

where, C_0 is original capacitance.

Initial energy stored in the capacitor

$$U_i = \frac{1}{2} C_0 V^2$$

Final energy stored in the capacitor

$$U_f = \frac{1}{2} C' V^2 = \frac{1}{2}(K C_0) V^2$$

Clearly, $\dfrac{U_f}{U_i} = K > 1 \Rightarrow U_f > U_i$

As, $Q = CV$

$\therefore$ V remains constant and C increases.

$\therefore$ Q increases .

46. *(a)* Power of the combination

$$P = P_1 + P_2$$

$$= P_{\text{concave}} + P_{\text{convex}}$$

$\Rightarrow \quad \dfrac{100}{50} = P_{\text{concave}} + \dfrac{100}{20}$

or $\quad 2 = P_{\text{concave}} + 5$

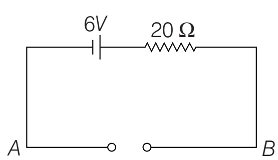
or $\quad P_{\text{concave}} = 2 - 5 = -3.0$ D

47. *(a)* Critical angle (θ_c) for glass water interface can be written as

$$\text{Sin}\ \theta_c = \frac{{}_a\mu_w}{{}_a\mu_g} = \frac{4/3}{3/2} = \frac{8}{9}$$

$\Rightarrow \quad \theta_c = \sin^{-1}(8/9)$

48. *(a)* In the given circuit, the diode is reverse biased. Only drift currant flows through the diode.



Consider the circuit in which by applying KVL, we can write

$$V_A - 6 + I \times 20 = V_B$$

$\Rightarrow \quad V_A - V_B = 6 - I \times 20 = 6 - 30 \times 10^{-6} \times 20$

$= 6 - 0.006 = -5.9994$ V

49. *(d)* Given, power P = 3D

$$f = \frac{1}{P} = \frac{1}{3}\ \text{m} = \frac{100}{3}\ \text{cm}.$$

$$u = -25\ \text{cm}.$$

Applying lens formula, we can write

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$\Rightarrow \quad \dfrac{1}{v} + \dfrac{1}{25} = \dfrac{3}{100}$

$\Rightarrow \quad \dfrac{1}{v} = \dfrac{3}{100} - \dfrac{1}{25} = \dfrac{3-4}{100} = \dfrac{-1}{100}$

$$v = -100\ \text{cm}.$$

Now, again using lens formula we can write

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

As, focal length of a far sighted person deceases

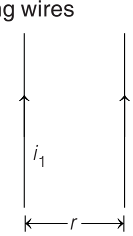
$\Rightarrow \quad \dfrac{1}{-100} - \dfrac{1}{u} > \dfrac{1}{1.7} = \dfrac{10}{17} \quad [\because f < 1.7\ \text{cm}]$

$\Rightarrow \quad \dfrac{1}{u} < \dfrac{-1}{100} - \dfrac{10}{17}$

$= \dfrac{-17-1000}{1700} = \dfrac{-1017}{1700}$

$\Rightarrow u > \dfrac{1700}{1000} \approx -1.7 = -1$ m (nearest option)

50. *(a)* Force per unit length between the two parallel current carrying wires

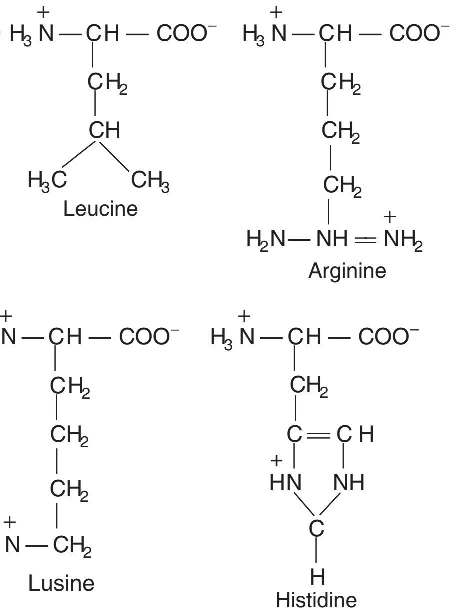


$$\frac{F}{L} = \frac{\mu_0}{2\pi r} i_1 i_2$$

$= \dfrac{\mu_0}{2\pi(2d)} i_1 i_2 = \dfrac{\mu_0\, i_1\, i_2}{4\pi d} \quad [\because r = 2d]$

Chemistry

1. *(a)*



Leucine is not a basic amino acid while others three i.e., arginine, lysine and histidine are basic amino acids as they have more number of amino groups than carboxyl groups.

2. *(c)* In qualitative analysis, NH_4Cl is added before NH_4OH to decrease $[OH^-]$ concentration. NH_4Cl suppresses the ionisation of NH_4OH due to common ion effect. Otherwise basic radicals of group V and group VI will be precipitated along with IIIrd group basic radicals.

3. *(a)* 11.2 L H_2 gas at NTP = 0.5 mole of H_2 gas =1 g of H_2 g

$\frac{w_1}{w_2} = \frac{E_1}{E_2} = \frac{1}{w_2} = \frac{1}{108}$ or $w_2 = 108$g silver is deposited in half an hour.

Therefore, $108 \times 2 = 216$ g silver will be deposited in one hour.

4. *(c)* A. $[Co(NH_3)_6]^{+3}$ inner orbital (d^2sp^3) complex

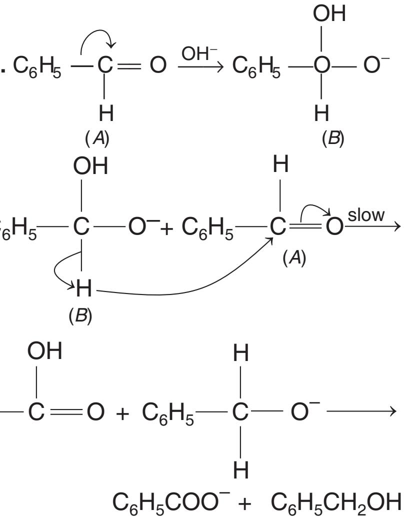
B. $[Fe(CN)_6]^{4-}$ inner orbital (d^2sp^3) complex

C. $[Ni(NH_3)_6]^{2+}$ outer orbital (sp^3d^2) complex

D. $[Mn(CN)_6]^{2-}$ inner orbital (d^2sp^3) complex

5. *(a)* For a cyclic process, $\Delta E = 0$. In cyclic process, system returns to its original state after a number of reactions.

6. *(b)* **Step I.** / **Step II.**



H^- is obtained by fission of C — H bond of (*B*). It is transferred to (II). This is the slowest step.

7. *(d)* $R = k[A]^x$ ...(i)

$$2R = k[4A]^x \quad \text{...(ii)}$$

Divide Eq. (ii) by (i)

$$2 = [4]^x$$

$$R = k[A]^{1/2}$$

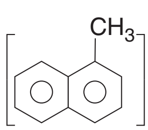
$$x = \frac{1}{2} = 0.5$$

$$R = k(400\,A)^{1/2}$$

$$\text{Rate} = 20k[A]^{1/2}$$

Rate becomes 20 times.

8. *(a)* Diesel is fraction of petroleum having $C_{15} - C_{18}$ composition and boiling point in the range of 573 – 673 K. Generally cetane $[CH_3 — (CH_2)_{14} — CH_3]$ and α-methyl naphthalene



are its main constituents.

9. *(b)* Organometalllic compounds must contain, at least one metal-carbon bond. Methyl lithium, CH_3 Li is an organometallic compound.

10. *(b)*

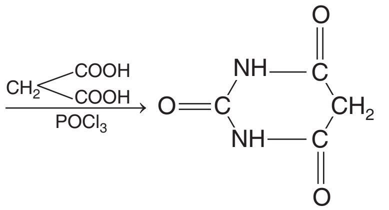
	N_2O_4	$\rightleftharpoons$	$2NO_2$
Initial	1		0
At. equil.	1		0

Total number of moles at equilibrium

$$= (1-\alpha) + 2\alpha = 1 + \alpha$$

11. *(b)*

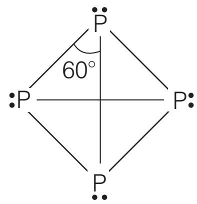
$$CHCl_3 \xrightarrow[h\nu]{[O]} COCl_2 \xrightarrow{NH_3} NH_2CONH_2$$



Barbituric acid

Barbituric acid and its derivatives are used in medicines as hypnotics and sedatives.

12. *(a)*

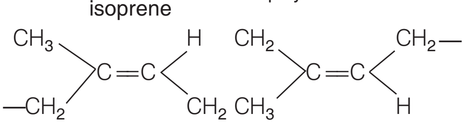


White phosphorus (P_4)
It has six P — P single bonds.

13. *(a)* Natural rubber is a linear polymers of isoprene and is also called as *cis*-1, 4-polyisoprene.

It is a linear 1, 4-polymer of isoprene (2-methyl-1,3-butadiene).

$$nCH_2=\underset{\text{isoprene}}{C(CH_3)-CH=CH_2} \xrightarrow[\text{polymerisation}]{\text{1, 4 addition}}$$



Natural rubber

14. *(a)*

A.	XeF_4	sp^3d^2
B.	H_2O	sp^3
C.	PCl_5	sp^3d
D.	$[Pt(NH_3)_4]^{2+}$	dsp^2

15. *(d)* A new carbon- carbon bond formation takes place in Friedel-Craft reaction and Reimer-Tiemann reaction.

16. *(b)* $C_6H_5CHO + (CH_3CO)_2O \xrightarrow[453\ K]{CH_3COONa}$

$$C_6H_5CH=CH\overset{O}{\overset{\|}{C}}-O-\overset{O}{\overset{\|}{C}}-CH_3 \xrightarrow[-CH_3COOH]{H_2O / H^+}$$

$$\underset{\text{Cinnamic acid}}{C_6H_5CH=CHCOOH}$$

17. *(a)* $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2m \times (KE)}}$

$$\lambda_1 = \frac{h}{\sqrt{2m \times 200}}$$

$$\lambda_2 = \frac{h}{\sqrt{2m \times 50}}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{1 \times \sqrt{50}}{\sqrt{200 \times 1}} = \frac{1}{2}$$

$\lambda_1 : \lambda_2 = 1 : 2$

18. *(c)* $CuCO_3 \cdot Cu(OH)_2 \longrightarrow \underset{(A)}{Cu_2O} + CO_2 + H_2O$

$$Cu_2O + C \longrightarrow \underset{(B)}{2Cu} + CO$$

19. *(b)*

	Formula	Name of the compound
A.	$NaNO_3$	Chile salt petre
B.	$Na(NH_4)HPO_4$	Microcosmic salt
C.	$NaHCO_3$	Baking soda
D.	$Na_2CO_3 \cdot 10H_2O$	Washing soda

20. *(b)* $2MnO_2 + 4HNO_3 \longrightarrow 2Mn(NO_3)_2 + 2H_2O + O_2$

$$2Mn(NO_3)_2 + 5PbO_2 + 6HNO_3 \longrightarrow \underset{\text{Permanganic acid (violet pink)}}{2HMnO_4} + 5Pb(NO_3)_2 + 2H_2O$$

21. *(c)* Edge length of (NaCl crystal) unit cell $= 2(r_{Na^+} + r_{Br^-})$

22. *(d)* Magnetic moment ∝ no. of unpaired electron.

$Zn^{2+} = 3d^{10}4s^0$ unpaired electron=0;

$\mu = 0$

$Ti^{3+} = 3d^{10}4s^0$ unpaired electron=1

$\mu = \sqrt{n(n+2)} = 1.73$

$Sc^{3+} = 3d^0 4s^0$ unpaired electron=0;

$\mu = 0$

$Mn^{2+} = 3d^5 4s^0$ unpaired electrons =5

$\mu = \sqrt{5(5+2)} = 5.91$

23. *(a)* Order of solubility of fluorides of alkaline earth metals in water is

Halides of alkaline earth metals (except Be) are ionic solids and are therefore are water soluble and their solubility in water decreases from Mg to Ba due to the decrease in the hydration energy.

However, fluorides of alkanline earth metals excepts BeF_2 are almost insoluble in water.

24. *(c)*

Element	% amount	Atomic mass	Mole ratio	Simplest ratio
C	20	12	$\frac{20}{12} = 1.66$	$\frac{1.66}{1.66} = 1$
N	46.66	14	$\frac{46.66}{14} = 3.33$	$\frac{3.33}{1.66} = 2$
H	6.66	1	$\frac{6.66}{1} = 6.66$	$\frac{6.66}{1.66} = 4$
O	26.68	16	$\frac{26.68}{16} = 1.66$	$\frac{1.66}{1.66} = 1$

$\therefore$ Empirical formula = CN_2H_4O

25. *(a)* $KI + I_2 \longrightarrow KI_3$ or $\underset{\text{from KI}}{I^- + I_2 \longrightarrow I_3^-}$

On increasing concentration of KI and I_2 two fold and three fold respectively, the concentration of KI_3 becomes two fold.

26. *(b)* $\underset{(C_{12}H_{22}O_{11})}{(+)\text{- Sucrose}} \xrightarrow{HOH/H^+} \underset{(C_6H_{12}O_6)}{D(+)\text{- glucose}} + \underset{(C_6H_{12}O_6)}{D(-)\text{- fructose}}$

$\underset{(C_{12}H_{22}O_{11})}{\text{Maltose}} \xrightarrow{HOH/H^+} \text{Glucose} + \text{Glucose}$

$\underset{(C_{12}H_{22}O_{11})}{\text{Lactose}} \xrightarrow{HOH/H^+} D(+)\text{- glucose}$

$+ D(+)\text{-galactose}$

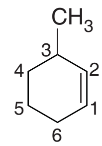
Hence, A = Sucrose, B = Maltose, C = Lactose

27. *(a)* A decay process is possible only if accompanied by the release of energy. There are three different decay processes that increase the *n/p* ratio. These are : (i) electron capture, (ii) position emission (β^+- emission) and (iii) alpha particle decay.

28. *(b)* Kerosene is a thin, clear liquid formed from hydro carbons obtained from the fractional distillation of petroleum between 150-275°C resulting in a mixture with a density of 0.78-0.81 g/cm^3 composed of carbon chains that typically contain between 6 and 16 carbon atoms per molecule.

29. *(c)* Poly tetrafluoro ethylene (PTFE) or teflon $-\!\!(CF_2 - CF_2)\!\!-_n$ is an addition homopolymer. It is used as lubricant, insulator and in making cooking wares.

30. *(a)* The IUPAC name of the given compound is 3-methyl cyclohexene.



31. *(d)* The correct representation of a complexionian is $[Co(NH_3)_4(H_2O)Cl]^{2+}$.

32. *(d)* Atoms and ions having inert gas configuration (such as helium), i.e. stable (half-filled) configuration have high ionisation potential, i.e. it is relatively easy to remove an electron from a partially filled valence shell, where Z_{eff} is lower but it is relatively difficult to remove an electron from an atom or ion that has a filled valence shell, where Z_{eff} is higher.

33. *(c)* $\Delta H = \Delta E + \Delta n_g RT$

where Δn_g = number of moles of gaseous product – number of moles of gaseous reactant

For $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$

$\Delta n_g = 2 - 2 = 0$

Thus, $\Delta H = \Delta E + 0 \times RT$ or $\Delta H = \Delta E$

34. *(b)* Calcination is used in metallurgy proceeds only with the expulsion of some small molecules like water, CO_2, SO_2 etc. without any chemical changes. e.g.

$2Al(OH)_3 \xrightarrow{1500°C} Al_2O_3 + 3H_2O\uparrow$

$FeCO_3 \xrightarrow{\Delta} FeO + CO_2\uparrow$

35. *(b)* $[Ni(CN)_4)]^x \equiv [Ni(CN)_4]^{2-}$

In this complex, the oxidation state of nickel is +2 and that of (CN) is – 1.

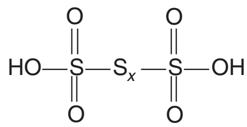
$\therefore \quad x = +2 + y(-1) = -2$

36. *(d)* From Arrhenius equation,

$$\log k = \log A - \frac{E_a}{2.303\,RT}$$

As the activation energy of a reaction is zero, the rate constant of this reaction becomes zero. It shows that the reaction is independent of temperature.

37. *(c)* I. $H_2S_3O_6$– trithionic acid is an example of polythionic acid.



III. H_2S_4 or HS—S—S—SH is an hydrosulfane.

The compounds (II) and (IV) do not exist.

38. *(b)* $^{13}_{7}N \xrightarrow{-_2He^4} {}^{10}_{5}B$

$^{242}_{96}Cm \xrightarrow{-_0n^1} {}^{244}_{94}Pu$

$^{14}_{7}N \xrightarrow{-_0n^1} {}^{14}_{6}C$

$^{30}_{15}P \xrightarrow[-g]{-_1H^1} {}^{28}_{14}Si$

39. *(b)* When cold H_2O_2 is added in cold mixture of $K_2Cr_2O_7$ and conc. H_2SO_4 (i.e. $K_2Cr_2O_7$ in acidic medium), a deep blue solution of blue perchromate is obtained as,

$$K_2Cr_2O_7 + H_2SO_4 \longrightarrow K_2SO_4 + H_2Cr_2O_7$$

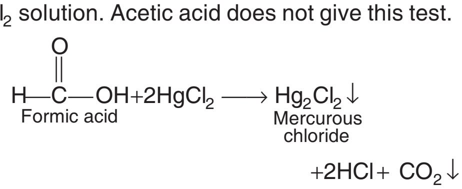
$$H_2Cr_2O_7 + 4\,H_2O_2 \longrightarrow \underset{\text{Blue perchromate}}{2CrO_5} + 5\,H_2O$$

40. *(d)* In blast furnace the highest temperature is in combustion zone. In the first stage, coke burns to produce CO_2 which rises temperature.

$$C + O_2 \longrightarrow CO_2 + \text{Heat}$$

This reaction is exothermic and hence temperature is 2170 K. It is known as combustion zone.

41. *(b)* $HgCl_2$ distinguishes formic acid and acetic acid. Formic acid produces a white precipitate with $HgCl_2$ solution. Acetic acid does not give this test.



42. *(d)* (i) Fluorine required energy when an electron is removed, i.e. ionisation energy.

(ii) Fluorine releases energy to add one electron, i.e. electron affinity ($-328\ \text{kJ mol}^{-1}$)

43. *(b)* Liquid hydrocarbon is converted into mixture of gaseous hydrocarbons by oxidation oxidation can be complete or partial. Complete oxidation is called combustion which is shown by the generalised reaction as

$$C_nH_{2n+2} + \left(\frac{3n+1}{2}\right)O_2 \xrightarrow{\text{Flame}} nCO_2 + (n+1)\,H_2O + \text{Heat}$$

44. *(b)* A → 2 B → 1 C → 3 D → 4

Cannizaro reaction – NaOH

Stephan's reaction – $SnCl_2$ / HCl

Clemmensen reduction – Zn/Hg –Conc. HCl

Rosenmund's method – $\dfrac{\text{Pd - BaSO}_4}{\text{Boiling xylene}}$

45. *(a)* Atomic raddii as well as ionic radii increases down the group. All the element of the nitrogen family contain same charge, i.e. – 3.

Thus, the correct order of ionic radius of nitrogen family is $N^{3-} < P^{3-} < As^{3-} < Sb^{3-} < Bi^{3-}$

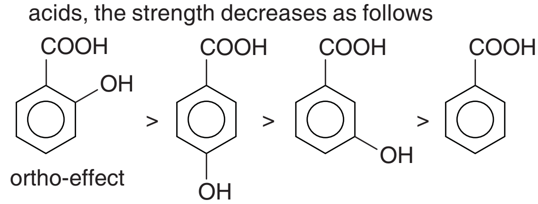
46. *(d)* Chlorophyll is responsible for the synthesis of glucose (carbohydrate) in plants.

In presence of oxygen, haemoglobin forms oxyhemoglobin.

Acetyl salicylic acid is known as aspirin

Vitamin B_{12} (Cyanocobalamin, $C_{63}H_{88}O_4N_{14}PCo$) contains Co^{2+} ion.

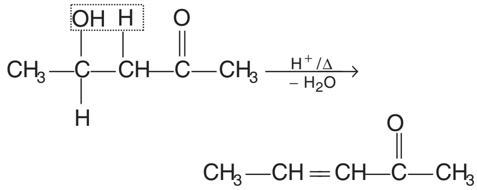
47. *(a)* Strength of acid is indicated by pK_a value. Higher the value of K_a or lower the value of pK_a, stronger is the acid. Among the given aromatic acids, the strength decreases as follows



ortho-effect
(lowest pK_a value)

***Ortho*-effect** The effect of a group is maximum at *ortho*-position due to nearness, is called *ortho* effect.

48. ***(a)***

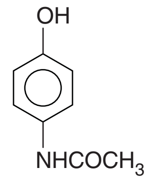


α,β-unsaturated ketones

Among the given compounds, the compound present in option (a) is readily dehydrated in acidic medium due to presence of acidic hydrogen with —OH group, In option (d), the strength of acidic hydrogen is less.

49. ***(a)*** The chemical substances which are used to bring down body temperature during high fever are called antipyretics,

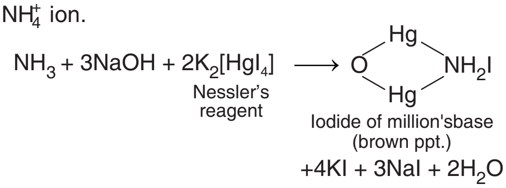
e.g. paracetamol, aspirin, phenacetin etc.



Paracetamol
(4-acetamidophenol)

50. ***(d)*** Analysis of NH_4^+ cation

Take small amount of salt in the test tube and add NaOH and heat. Add Nessler's reagent $[K_2H_gI_4]$ to the solution, brown ppt is obtained. It confirmed NH_4^+ ion.



Mathematics

1. ***(c)*** Given, $P(A \cup B) = P(A \cap B)$

$\Rightarrow \quad P(A) + P(B) - P(A \cap B) = P(A \cap B)$

$\Rightarrow \quad P(A) + P(B) = 2P(A \cap B)$

$\Rightarrow \quad P(A) + P(B) = 2 \times P(A)\, P(B/A)$

$$\left[\because P(B/A) = \frac{P(A \cap B)}{P(A)}\right]$$

2. ***(*)*** Let the equation of tangent be

$$y = mx + c \qquad \ldots(i)$$

Given equation of circle is

$$x^2 + y^2 - 2x - 2y + 1 = 0$$

Its centre is (1, 1)

and radius, $r = \sqrt{1^2 + 1^2 - 1} = \sqrt{1 + 1 - 1} = 1$

We know that, if line $y = mx + c$ be a tangent to the circle, then $c = \pm r\sqrt{1 + m^2}$

$\therefore \quad c = \pm 1\sqrt{1 + m^2} \qquad \ldots(ii)$

Since, the tangent line is perpendicular to $y = x$.

$\therefore \quad m \times 1 = -1 \qquad (\because m_1 m_2 = -1)$

$\Rightarrow \quad m = -1$

On putting $m = -1$ in Eq. (ii), we get

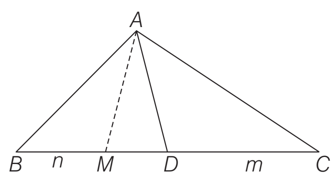
$$c = \pm 1\sqrt{1 + (-1)^2} = \pm\sqrt{1 + 1} = \pm\sqrt{2}$$

On putting the values of $m = -1$ and $c = \pm\sqrt{2}$ in Eq. (i), we get

$$y = -x \pm \sqrt{2}$$

3. ***(c)*** $mBD^2 + nCD^2 + (m + n)AD^2$

$= mBD^2 + nCD^2 + mAD^2 + nAD^2$



$= m(BD^2 + AD^2) + n(CD^2 + AD^2)$

$= m(AB^2 + 2DM \cdot DB) + n(AC^2 - 2DM \cdot DC)$

$= mAB^2 + m \cdot 2DM \cdot DB + nAC^2 - n \cdot 2DM \cdot DC$

$= mAB^2 + nAC^2 + 2DM(mDB - nDC)$

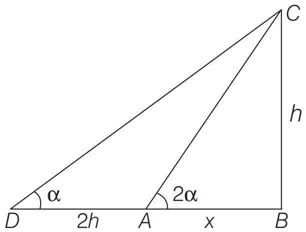
$= mAB^2 + nAC^2 + 2DM(mn - nm)$

$= mAB^2 + nAC^2$

4. ***(a)*** Let the height of the pole be $BC = h$ m.

In ΔABC,

$$\tan 2\alpha = \frac{h}{x} \qquad \ldots(i)$$



and in ΔDBC,

$$\tan\alpha = \frac{h}{2h + x}$$

$$\Rightarrow \quad \tan\alpha = \frac{\frac{h}{x}}{2\left(\frac{h}{x}\right) + 1}$$

$$\Rightarrow \quad \tan\alpha = \frac{\tan 2\alpha}{2\tan 2\alpha + 1}$$

$$\Rightarrow \quad \tan\alpha\,(2\tan 2\alpha + 1) = \tan 2\alpha$$

$$\Rightarrow \tan\alpha\left[2 \times \frac{2\tan\alpha}{1 - \tan^2\alpha} + 1\right] = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

$$\Rightarrow \quad [4\tan\alpha + 1 - \tan^2\alpha] = 2$$

$$\Rightarrow \quad \tan^2\alpha - 4\tan\alpha + 1 = 0$$

$$\Rightarrow \quad \tan\alpha = \frac{+4 \pm \sqrt{16 - 4}}{2 \times 1}$$

$$= \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2}$$

$$\Rightarrow \quad \tan\alpha = 2 \pm \sqrt{3}$$

Taking '–' sign, we get

$$\Rightarrow \quad \tan\alpha = 2 - \sqrt{3}$$

$$\Rightarrow \quad \tan\alpha = \tan 15^\circ$$

$$\Rightarrow \quad \alpha = 15^\circ \quad \text{or} \quad \frac{\pi}{12}$$

5. ***(d)*** Given, $f(x) = \begin{cases} \frac{\sin[x]}{[x]} & \text{for}[x] \neq 0 \\ 0 & \text{for}[x] = 0 \end{cases}$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{\sin(1 + [x])}{[x]}$$

$$= \frac{\sin[1 - 1]}{-1} = \frac{\sin 0}{-1} = 0$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{\sin(1 + [x])}{[x]}$$

$$= \frac{\sin(1 + 0)}{0} = \infty$$

Hence, limit does not exist.

6. ***(b)*** Given equation of straight lines are

$$y = (2 - \sqrt{3})x + 5$$

and $\quad y = (2 + \sqrt{3})x - 7$

On comparing with $y = mx + c$, we get

$$m_1 = 2 - \sqrt{3}$$

and $\quad m_2 = 2 + \sqrt{3}$

$$\therefore \quad \tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$= \frac{2 + \sqrt{3} - (2 - \sqrt{3})}{1 + (2 - \sqrt{3})(2 + \sqrt{3})}$$

$$= \frac{2\sqrt{3}}{1 + (4 - 3)} = \frac{2\sqrt{3}}{1 + 4 - 3}$$

$$\Rightarrow \quad \tan\theta = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

$$\Rightarrow \quad \theta = 60^\circ$$

7. ***(c)*** Equation of plane passing through $(-1, 1, 1)$ is

$$a(x + 1) + b(y - 1) + c(z - 1) = 0 \quad \ldots\text{(i)}$$

Also, it is passing through $(1, -1, 1)$.

$$\therefore \quad a(1 + 1) + b(-1 - 1) + c(1 - 1) = 0$$

$$\Rightarrow \quad 2a - 2b + 0c = 0 \quad \ldots\text{(ii)}$$

Also, required equation of plane (i) is perpendicular to $x + 2y + 2z = 5$.

$$\therefore \quad a \times 1 + b \times 2 + c \times 2 = 0$$

$$\Rightarrow \quad a + 2b + 2c = 0 \quad \ldots\text{(iii)}$$

Eqs. (ii) and (iii) are identical.

$$\therefore \quad \frac{a}{-4 - 0} = \frac{-b}{4 - 0} = \frac{c}{4 + 2}$$

$$\Rightarrow \quad \frac{a}{-4} = \frac{b}{-4} = \frac{c}{6}$$

$$\Rightarrow \quad \frac{a}{-2} = \frac{b}{-2} = \frac{c}{3} = \lambda \quad \text{(say)}$$

$$\Rightarrow \quad a = -2\lambda, b = -2\lambda, c = 3\lambda$$

On putting the values of a, b and c in Eq. (i), we get

$$-2\lambda(x + 1) - 2\lambda(y - 1) + 3\lambda(z - 1) = 0$$

$$\Rightarrow \quad \lambda[-2x - 2 - 2y + 2 + 3z - 3] = 0$$

$$\Rightarrow \quad -2x - 2y + 3z - 3 = 0$$

$$\Rightarrow \quad 2x + 2y - 3z + 3 = 0$$

8. ***(d)*** Given, $g(x) = f(\tan^2 x - 2\tan x + 4)$

On differentiating w.r.t. x, we get

$$g'(x) = f'(\tan^2 x - 2\tan x + 4) \times (2\tan x \sec^2 x - 2\sec^2 x)$$

$$= f'(\tan^2 x - 2\tan x + 4)\, 2\sec^2 x\,(\tan x - 1)$$

$\because \quad f'(x) > 0 \ \forall x \in \left(0, \frac{\pi}{2}\right)$

Also, $2\sec^2 x > 0 \ \forall \ x \in \left(0, \frac{\pi}{2}\right)\left[\because x \in \left(0, \frac{\pi}{2}\right), \text{given}\right]$

But $\tan x - 1 > 0 \ \forall \ x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$\therefore \quad g'(x) > 0 \ \forall \ x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

Hence, $g(x)$ is increasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

9. *(a)* The total number of sample points in a sample space of single throw of two dice, $n(S) = 36$

Let $E_1 =$ Event of getting a sum 7

$E_2 =$ Event of getting a sum 9

$\therefore \quad E_1 = \{(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)\}$

$\Rightarrow \quad n(E_1) = 6$

and $\quad E_2 = \{(3, 6), (6, 3), (4, 5), (5, 4)\}$

$\Rightarrow \quad n(E_2) = 4$

$\therefore \quad P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

and $\quad P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

Now, $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

$$= \frac{1}{6} + \frac{1}{9} = \frac{6+4}{36} = \frac{10}{36} = \frac{5}{18}$$

10. *(b)* Compiler is a system software which transforms source code written in a programming language into another computer language.

11. *(a)* Given, $\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$

Using L' Hospital's rule, we get

$$\lim_{t \to x} \frac{t^2 f'(x) - 2x f(t)}{-1} = 1$$

$\Rightarrow \quad x^2 f'(x) - 2x\, f(x) + 1 = 0$

$\Rightarrow \quad \frac{x^2 f'(x) - 2x f(x)}{(x^2)^2} + \frac{1}{x^4} = 0$

$\Rightarrow \quad \frac{d}{dx}\left(\frac{f(x)}{x^2}\right) = -\frac{1}{x^4}$

On integrating both sides, we get

$$\frac{f(x)}{x^2} = +\frac{1}{3x^3} + c \quad \Rightarrow \quad f(x) = \frac{1}{3x} + cx^2$$

Also, $\quad f(1) = 1 \quad \Rightarrow \quad 1 = \frac{1}{3 \times 1} + c(1)^2$

$\Rightarrow \quad 1 = \frac{1}{3} + c \quad \Rightarrow \quad \frac{2}{3} = c$

$\therefore \quad f(x) = \frac{1}{3x} + \frac{2}{3x^2}$

12. *(c)* We have,

$$\int \sin\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}dx = A\sin^{-1}x + Bx\sqrt{1-x^2} + C$$

Let $\quad I = \int \sin\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}dx$

Put $x = \cos 2\theta \quad \Rightarrow \quad dx = -2\sin 2\theta\, d\theta$

$\therefore \quad I = -\int \sin\left\{2\tan^{-1}\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}\right\}2\sin 2\theta\, d\theta$

$$= -\int \sin\left\{2\tan^{-1}\sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}}\right\}2\sin 2\theta\, d\theta$$

$$= -\int \sin\{2\tan^{-1}\sqrt{\tan^2\theta}\}2\sin 2\theta\, d\theta$$

$$= -\int \sin\{2\tan^{-1}\tan\theta\}2\sin 2\theta\, d\theta$$

$$= -\int [\sin(2\theta)]\ 2\sin 2\theta\, d\theta$$

$$= -\int 2\sin^2 2\theta\, d\theta = -\int (1 - \cos 4\theta)\, d\theta$$

$$= -\left[\theta - \frac{\sin 4\theta}{4}\right] + C$$

$$= \left[-\frac{1}{2}\cos^{-1}x + \frac{1}{4} \times 2\sin 2\theta \cos 2\theta\right] + C$$

$$= \left[-\frac{1}{2}\left(\frac{\pi}{2} - \sin^{-1}x\right) + \frac{1}{2}\sqrt{1-x^2} \times x\right] + C$$

$$= \frac{1}{2}\sin^{-1}x + \frac{x}{2}\sqrt{1-x^2} + \left(C - \frac{\pi}{4}\right)$$

But $\quad I = A\sin^{-1}x + Bx\sqrt{1-x^2} + C$

$\therefore \quad A = \frac{1}{2} \quad$ and $\quad B = \frac{1}{2}$

Hence, $\quad A + B = \frac{1}{2} + \frac{1}{2} = 1$

13. *(c)* Let $E = (1 + x + x^3 + x^4)^{10}$

$$= [1 + x + x^3(1+x)]^{10} = (1+x)^{10}(1+x^3)^{10}$$

$$= [{}^{10}C_0 + {}^{10}C_1 x^1 + {}^{10}C_2 x^2 + \ldots + {}^{10}C_9 x^9 + {}^{10}C_{10} x^{10}] \times [{}^{10}C_0 1 + {}^{10}C_1 1 \times (x^3) + {}^{10}C_2 (x^3)^2 + {}^{10}C_3 (x^3)^3 + \ldots]$$

$\therefore$ Coefficient of x^4 in $E = {}^{10}C_4 \times {}^{10}C_0 + {}^{10}C_1 \times {}^{10}C_1$

$$= \frac{10!}{6! \times 4!} \times 1 + 10 \times 10$$

$$= \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} + 100$$

$$= 210 + 100 = 310$$

14. *(a)* Let equation of circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Given equation of circle is

$$x^2 + y^2 - 4x + 8 = 0$$

The centres of above circles are $(-g, -f)$ and $(2, 0)$.

Condition of orthogonality is

$$2(g_1 g_2 + f_1 f_2) = c_1 + c_2$$

$$\therefore \quad 2(g \times (-2) + (f) \times 0) = c + 8$$

$$\Rightarrow \quad -4g = c + 8 \quad \ldots(\text{i})$$

Also, the assume circle touch the line $x + 1 = 0$.

$\therefore$ The perpendicular drawn from centre to the line is equal to radius.

$$\therefore \quad \frac{-g + 1}{\sqrt{1^2}} = \sqrt{g^2 + f^2 - c}$$

$$\Rightarrow \quad -g + 1 = \sqrt{g^2 + f^2 - c}$$

On squaring both sides, we get

$$g^2 + 1 - 2g = g^2 + f^2 - c$$

$$\Rightarrow \quad c = f^2 + 2g - 1$$

Putting the value of c in Eq. (i), we get

$$-4g = f^2 + 2g - 1 + 8$$

$$\Rightarrow f^2 + 2g + 4g + 7 = 0$$

$$\Rightarrow \quad f^2 + 6g + 7 = 0$$

$\therefore$ Locus of centre of circle is $y^2 + 6x + 7 = 0$.

15. *(c)* Given,

$$f(x) = \begin{vmatrix} \sin 3x & 1 & 2\left(\cos\frac{3x}{2} + \sin\frac{3x}{2}\right)^2 \\ \cos 3x & -1 & 2\left(\cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}\right) \\ \tan 3x & 4 & 1 + 2\tan 3x \end{vmatrix}$$

On differentiating w.r.t. x, we get

$$f'(x) = \begin{vmatrix} \frac{d}{dx}(\sin 3x) & 1 & 2\left(\cos\frac{3x}{2} + \sin\frac{3x}{2}\right)^2 \\ \frac{d}{dx}(\cos 3x) & -1 & 2\left(\cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}\right) \\ \frac{d}{dx}(\tan 3x) & 4 & 1 + 2\tan 3x \end{vmatrix}$$

$$+ \begin{vmatrix} \sin 3x & \frac{d}{dx}(1) & 2\left(\cos\frac{3x}{2} + \sin\frac{3x}{2}\right)^2 \\ \cos 3x & \frac{d}{dx}(-1) & 2\left(\cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}\right) \\ \tan 3x & \frac{d}{dx}(4) & 1 + 2\tan 3x \end{vmatrix}$$

$$+ \begin{vmatrix} \sin 3x & 1 & 2\frac{d}{dx}\left(\cos\frac{3x}{2} + \sin\frac{3x}{2}\right)^2 \\ \cos 3x & -1 & 2\frac{d}{dx}\left(\cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}\right) \\ \tan 3x & 4 & \frac{d}{dx}(1 + 2\tan 3x) \end{vmatrix}$$

$$= \begin{vmatrix} 3\cos 3x & 1 & 2\left(\cos\frac{3x}{2} + \sin\frac{3x}{2}\right)^2 \\ 3\sin 3x & -1 & 2\left(\cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}\right) \\ 3\sec^2 3x & 4 & 1 + 2\tan 3x \end{vmatrix}$$

$$+ \begin{vmatrix} \sin 3x & 0 & 2\left(\cos\frac{3x}{2} + \sin\frac{3x}{2}\right)^2 \\ \cos 3x & 0 & 2\left(\cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}\right) \\ \tan 3x & 0 & 1 + 2\tan 3x \end{vmatrix}$$

$$+ \begin{vmatrix} \sin 3x & 1 & 2 \times 2\left(\cos\frac{3x}{2} + \sin\frac{3x}{2}\right) \times \left(\frac{-3}{2}\sin\frac{3x}{2} + \frac{3}{2}\cos\frac{3x}{2}\right) \\ \cos 3x & -1 & 2\left(-2\cos\frac{3x}{2} \times \frac{3}{2}\sin\frac{3x}{2} - 2\sin\frac{3x}{2} \times \frac{3}{2}\cos\frac{3x}{2}\right) \\ \tan 3x & 4 & (0 + 2 \times 3\sec^2 3x) \end{vmatrix}$$

At $x = (2n+1)\pi$,

$$f'(x) = \begin{vmatrix} 3(-1) & 1 & 2(1) \\ 0 & -1 & 2(-1) \\ 3 & 4 & 1+0 \end{vmatrix} + 0$$

$$+ \begin{vmatrix} 0 & 1 & 4(0-1)\times\left[-\frac{3}{2}(-1)+\frac{3}{2}\times 0\right] \\ -1 & -1 & 0 \\ 0 & 4 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} -3 & 1 & 2 \\ 0 & -1 & -2 \\ 3 & 4 & 1 \end{vmatrix} + 0 + \begin{vmatrix} 0 & 1 & -6 \\ -1 & -1 & 0 \\ 0 & 4 & 0 \end{vmatrix}$$

$$= [-3(-1+8) - 1(0+6) + 2(0+3)] + [0 - 1(0-0) - 6(-4)]$$

$$= -21 + 24 = 3$$

16. *(b)* Given equation of line is $lx + my + n = 0$

and equation of ellipse is $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

$\therefore$ The equation of any normal to the ellipse is

$$5x\sec\theta - 3y\operatorname{cosec}\theta = 25 - 9$$

$\left(\because \text{the equation of any normal to the ellipse } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ is } ax\sec\theta - by\operatorname{cosec}\theta = a^2 - b^2\right)$

$$\Rightarrow \quad 5x\sec\theta - 3y\operatorname{cosec}\theta - 16 = 0 \quad \ldots\text{(i)}$$

As the Eq. (i) is the normal to the ellipse.

$$\therefore \quad \frac{5\sec\theta}{l} = \frac{-3\operatorname{cosec}\theta}{m} = \frac{-16}{n}$$

$$\Rightarrow \quad \cos\theta = \frac{5n}{-16l} \quad \text{and} \quad \sin\theta = \frac{3n}{16m}$$

$$\because \quad \cos^2\theta + \sin^2\theta = 1$$

$$\therefore \quad \left(\frac{5n}{16l}\right)^2 + \left(\frac{3n}{16m}\right)^2 = 1$$

$$\Rightarrow \quad \frac{25n^2}{256l^2} + \frac{9n^2}{256m^2} = 1$$

$$\Rightarrow \quad \frac{25}{l^2} + \frac{9}{m^2} = \frac{256}{n^2}$$

17. *(a)* Given, $\frac{4}{\sin x} + \frac{1}{1-\sin x} = a$

$$\Rightarrow \quad 4(1-\sin x) + \sin x = a\sin x(1-\sin x)$$

$$\Rightarrow \quad 4 - 4\sin x + \sin x = a\sin x - a\sin^2 x$$

$$\Rightarrow \quad a\sin^2 x - (3+a)\sin x + 4 = 0 \quad \ldots\text{(i)}$$

It is a quadratic equation in $\sin x$, so

$$D \geq 0$$

$$\Rightarrow \quad (3+a)^2 - 4\times 4a \geq 0$$

$$\Rightarrow \quad 9 + a^2 + 6a - 16a \geq 0$$

$$\Rightarrow \quad a^2 - 10a + 9 \geq 0$$

$$\Rightarrow \quad (a-1)(a-9) \geq 0$$

$$\Rightarrow \quad a \geq 9 \quad \text{or} \quad a \leq -9$$

Now, at $a = 9$, Eq. (i) becomes

$$9\sin^2 x - 12\sin x + 4 = 0$$

$$\Rightarrow \quad (3\sin x - 2)^2 = 0$$

$$\Rightarrow \quad \sin x = \frac{2}{3} < 0 \Rightarrow x \in \left(0, \frac{\pi}{2}\right)$$

Hence, least value of a is 9.

18. *(c)* If one line of regression coefficient is less than unity, then other will be greater than unity.

19. *(a)* Given edges of a parallelopiped are

$\mathbf{a} = 2\hat{i} - 3\hat{j} + \hat{k}, \mathbf{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\mathbf{c} = 2\hat{i} + \hat{j} - \hat{k}$

$\therefore$ Volume of parallelopiped $= [\mathbf{a}\,\mathbf{b}\,\mathbf{c}]$

$$= \left\|\begin{matrix} 2 & -3 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{matrix}\right\|$$

$$= |2(1-2) + 3(-1-4) + 1(1+2)|$$

$$= |-2 - 15 + 3| = 14 \text{ cu units}$$

20. *(a)* Let $f(x) = x^3 - 6x + 1$

Now, $f(2) = (2)^3 - 6\times 2 + 1$

$= 8 - 12 + 1 = -3$

and $f(3) = (3)^3 - 6\times 3 + 1$

$= 27 - 18 + 1 = 10$

Here, we see that $f(2)$ and $f(3)$ have opposite signs, so one of the roots lies in $(2, 3)$.

21. *(b)* Given, $\lim_{x\to 0} \frac{\sin(\sin x) - \sin x}{ax^3 + bx^5 + c} = -\frac{1}{12}$ ...(i)

$$\Rightarrow \quad \frac{\sin\sin 0 - \sin 0}{a(0)^3 + b(0)^5 + c} = -\frac{1}{12}$$

$$\Rightarrow \quad \frac{0}{c} = -\frac{1}{12} \Rightarrow c = 0$$

Applying L' Hospital's rule in Eq.(i),we get

$$\lim_{x\to 0} \frac{\cos(\sin x)\cos x - \cos x}{3ax^2 + 5bx^4 + 0} = -\frac{1}{12}$$

$$\Rightarrow \lim_{x\to 0} \frac{\cos x(\cos(\sin x) - 1)}{x^2(3a + 5bx)} = -\frac{1}{12}$$

$$\Rightarrow \lim_{x\to 0} \cos x \frac{2\sin^2\left(\frac{\sin x}{2}\right)}{x^2(3a + 5bx)} = -\frac{1}{12}$$

$$\Rightarrow \frac{\cos 0}{3a+5b\times 0}\times \lim_{x\to 0}\frac{2\sin^2\left(\frac{\sin x}{2}\right)}{4\left(\frac{\sin x}{2}\right)^2\times\left(\frac{x}{\sin x}\right)^2}=-\frac{1}{12}$$

$$\Rightarrow \frac{1}{3a+0}\times\frac{1}{2}\times\frac{1}{1}=-\frac{1}{12}\quad\left(\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right)$$

$$\Rightarrow \quad \frac{1}{6a}=-\frac{1}{12}\quad\Rightarrow\quad 6a=-12$$

$$\therefore \qquad a=-2$$

Hence, $a=-2, b\in R$ and $c=0$

22. *(d)* Let $x=\sqrt{12}$

$$\Rightarrow \qquad x^2=12$$

$$\Rightarrow \qquad x^2-12=0$$

Let $\qquad f(x)=x^2-12$

The first approximation in the Newton-Raphson method is given by

$$x_1=x_0-\frac{f(x_0)}{f'(x_0)}$$

$$=x_0-\frac{x_0^2-12}{2x_0}=\frac{{x_0}^2+12}{2x_0}$$

$$\because \qquad 3<\sqrt{12}<3.5$$

We can take $x_0=3.5$

$$\therefore \qquad x_1=\frac{(3.5)^2+12}{2\times 3.5}=\frac{12.25+12}{7}$$

$$=\frac{24.25}{7}=3.464$$

23. *(b)* Given equation is

$$\frac{(3-i)^2}{2+i}=A+iB$$

$$\Rightarrow \qquad \frac{9-1-6i}{2+i}=A+iB$$

$$\Rightarrow \qquad \frac{8-6i}{2+i}=A+iB$$

$$\Rightarrow \qquad \frac{2(4-3i)}{2+i}\times\frac{2-i}{2-i}=A+iB$$

$$\Rightarrow \qquad \frac{2[8-4i-6i-3]}{4+1}=A+iB$$

$$\Rightarrow \qquad \frac{2[5-10i]}{5}=A+iB$$

$$\Rightarrow \qquad 2-4i=A+iB$$

On equating the real and imginary parts from both sides, we get

$$A=2 \text{ and } B=-4$$

24. *(a)* Given system of circles is

$$x^2+y^2+4x+7=0 \qquad \ldots(\text{i})$$

$$2(x^2+y^2)+3x+5y+9=0$$

or $\qquad x^2+y^2+\frac{3}{2}x+\frac{5}{2}y+\frac{9}{2}=0 \qquad \ldots(\text{ii})$

and $\qquad x^2+y^2+y=0 \qquad \ldots(\text{iii})$

The radical centre can be obtained by solving the Eqs. (i), (ii) and (iii).

On subtracting Eq. (ii) from Eq. (i), we get

$$4x-\frac{3}{2}x-\frac{5}{2}y+7-\frac{9}{2}=0$$

$$\Rightarrow \qquad \frac{5}{2}x-\frac{5}{2}y+\frac{5}{2}=0$$

$$\Rightarrow \qquad x-y+1=0 \qquad \ldots(\text{iv})$$

On subtracting Eq. (iii) from Eq. (i), we get

$$4x-y+7=0 \qquad \ldots(\text{v})$$

On solving Eqs. (iv) and (v), we get

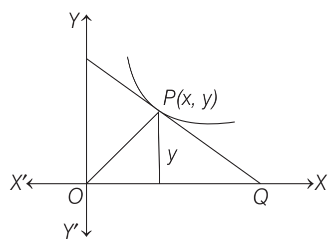
$$x=-2 \quad\text{and}\quad y=-1$$

Hence, radical centre is $(-2,-1)$.

25. *(b)* Tangent drawn at any point (x, y) is

$$Y-y=\frac{dy}{dx}(X-x)$$

When $Y=0, X=x-y\frac{dx}{dy}$



$$\because \qquad \text{Area of } \Delta OPQ=a^2 \qquad \text{(given)}$$

$$\therefore \qquad \left|\frac{1}{2}X\cdot y\right|=a^2$$

$$\Rightarrow \qquad \left|\left(x-y\frac{dx}{dy}\right)y\right|=2a^2$$

$$\Rightarrow \qquad xy-y^2\frac{dx}{dy}=\pm 2a^2$$

$$\Rightarrow \qquad \frac{dx}{dy}-\frac{x}{y}=\pm\frac{2a^2}{y^2}$$

Here, $P=-\frac{1}{y}$ and $Q=\pm\frac{2a^2}{y^2}$

$$\therefore \quad \text{IF} = e^{\int P\,dy} = e^{\int -\frac{1}{y}\,dy}$$

$$= e^{-\log y} = e^{\log \frac{1}{y}} = \frac{1}{y}$$

Hence, required solution is

$$x \times \frac{1}{y} = \int \pm \frac{2a^2}{y^2} \times \frac{1}{y}\,dy$$

$$\Rightarrow \quad \frac{x}{y} = \pm \frac{2a^2 y^{-2}}{-2} + C$$

$$\Rightarrow \quad x = Cy \pm \frac{a^2}{y}$$

which is the required curve.

26. ***(d)*** Given differential equation can be written as

$$\frac{dy}{dx} - \left(\frac{\tan 2x}{\cos^2 x}\right) y = \cos^2 x$$

Here, $P = -\dfrac{\tan 2x}{\cos^2 x} = \dfrac{-\sin 2x}{\cos 2x\left(\dfrac{\cos 2x + 1}{2}\right)}$

and $\quad Q = \cos^2 x$

$$\because \quad \text{IF} = e^{\int P\,dx} = e^{-\int \frac{2\sin 2x}{\cos 2x\,(\cos 2x + 1)}\,dx}$$

Put $\cos 2x = t \Rightarrow -2\sin 2x\,dx = dt$

$$\therefore \quad \text{IF} = e^{\int \frac{1}{t}\left(\frac{1}{t+1}\right)dt}$$

$$= e^{\int \left(\frac{1}{t} - \frac{1}{t+1}\right)dt}$$

$$= e^{[\log t - \log(t+1)]} = e^{\log \frac{t}{t+1}}$$

$$= e^{\log \frac{\cos 2x}{\cos 2x + 1}} = \frac{\cos 2x}{\cos 2x + 1}$$

Now, solution is

$$y \times \frac{\cos 2x}{\cos 2x + 1} = \int \frac{\cos 2x}{\cos 2x + 1} \times \cos^2 x\,dx + C$$

$$= \int \frac{\cos 2x}{2\cos^2 x} \times \cos^2 x\,dx + C$$

$$= \frac{1}{2}\int \cos 2x\,dx + C$$

$$= \frac{1}{2} \times \frac{\sin 2x}{2} + C$$

$$\Rightarrow \quad y\,\frac{\cos 2x}{\cos 2x + 1} = \frac{1}{4}\sin 2x + C \qquad \ldots\text{(i)}$$

But $\quad y\left(\dfrac{\pi}{6}\right) = \dfrac{3\sqrt{3}}{8}$

$$\therefore \frac{3\sqrt{3}}{8} \times \frac{\cos\left(2 \times \frac{\pi}{6}\right)}{\cos\left(2 \times \frac{\pi}{6}\right) + 1} = \frac{1}{4}\sin 2\left(\frac{\pi}{6}\right) + C$$

$$\Rightarrow \quad \frac{\frac{3\sqrt{3}}{8} \times \frac{1}{2}}{\frac{1}{2} + 1} = \frac{1}{4} \times \frac{\sqrt{3}}{2} + C$$

$$\Rightarrow \quad \frac{3\sqrt{3}}{2 \times 8 \times \frac{3}{2}} = \frac{\sqrt{3}}{8} + C$$

$$\Rightarrow \quad \frac{\sqrt{3}}{8} = \frac{\sqrt{3}}{8} + C \Rightarrow C = 0$$

From Eq. (i), we get

$$y\,\frac{\cos 2x}{\cos 2x + 1} = \frac{1}{4}\sin 2x + 0$$

$$\Rightarrow \quad y = \frac{1}{4}\frac{\sin 2x}{\frac{\cos 2x}{\cos 2x + 1}} = \frac{1}{4}\frac{\sin 2x}{\frac{\cos^2 x - \sin^2 x}{2\cos^2 x}}$$

$$= \frac{1}{2} \cdot \frac{\sin 2x}{1 - \tan^2 x}$$

27. ***(a)*** $(1 - \omega + \omega^2)(1 + \omega - \omega^2)$

$$= (1 + \omega^2 - \omega)(1 + \omega - \omega^2)$$

$$= (-\omega - \omega)(-\omega^2 - \omega^2) \quad (\because 1 + \omega + \omega^2 = 0)$$

$$= (-2\omega)(-2\omega^2)$$

$$= 4(\omega^3) = 4 \times 1 \qquad (\because \omega^3 = 1)$$

$$= 4$$

28. ***(*)*** Given, $\dfrac{dy}{dx} = \dfrac{y - 1}{x^2 + x}$

$$\Rightarrow \quad \frac{1}{y - 1}\,dy = \frac{dx}{x(x + 1)}$$

On integrating both sides, we get

$$\int \frac{1}{(y - 1)}\,dy = \int \left(\frac{1}{x} - \frac{1}{x + 1}\right)dx$$

$$\Rightarrow \quad \log|y - 1| = [\log|x| - \log|x + 1|] + C$$

$$\Rightarrow \quad \log|y - 1| = \log\left|\frac{x}{x + 1}\right| + C \qquad \ldots\text{(i)}$$

At point (1, 0), we get

$$\log|0 - 1| = \log\left(\frac{1}{2}\right) + C$$

$$\Rightarrow \quad C = -\log\frac{1}{2} + 0$$

From Eq. (i), we get

$$\log|y-1| = \log\left|\frac{x}{x+1}\right| - \log\frac{1}{2}$$

$$\Rightarrow \quad \log(y-1) = \log 2\left|\frac{x}{x+1}\right|$$

$$\Rightarrow \quad (y-1) = \frac{2x}{x+1}$$

$$\Rightarrow \quad (y-1)(x+1) = 2x$$

$$\Rightarrow \quad (y-1)(x+1) - 2x = 0$$

29. ***(c)*** Given, $f(x) = [\sin x + \cos x]$

$$= \left[\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right)\right]$$

$$= \left[\sqrt{2}\sin\left(x + \frac{\pi}{4}\right)\right]$$

We know that, greatest integer function is discontinuous on integer values.

Function $\sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$ will gives integer values at

$x = 90°, 135°, 180°, 270°, 315°,$

Hence, there are five points in the given interval, in which $f(x)$ is not continuous.

30. ***(d)*** Let $I = \cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right)$

$$= \cot^{-1}\left[\frac{\sqrt{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2} + \sqrt{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}}{\sqrt{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2} - \sqrt{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}}\right]$$

$$= \cot^{-1}\left(\frac{\sin\frac{x}{2} + \cos\frac{x}{2} + \cos\frac{x}{2} - \sin\frac{x}{2}}{\sin\frac{x}{2} + \cos\frac{x}{2} - \cos\frac{x}{2} + \sin\frac{x}{2}}\right)$$

$$= \cot^{-1}\left(\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right) = \cot^{-1}\left(\cot\frac{x}{2}\right) = \frac{x}{2}$$

31. ***(d)*** Given vertices of a ΔABC are $A(-1, 3, 2)$, $B(2, 3, 5)$ and $C(3, 5, -2)$.

Now DR's of $AB = (2+1, 3-3, 5-2) = (3, 0, 3)$

DR's of $BC = (3-2, 5-3, -2-5) = (1, 2, -7)$

and DR's of $CA = (-1-3, 3-5, 2+2)$

$= (-4, -2, 4)$

Now, the angle between AB and BC,

$$\cos B = \frac{|3\times1 + 0\times2 + 3\times(-7)|}{\sqrt{3^2+0^2+3^2}\sqrt{1^2+2^2+(-7)^2}}$$

$$= \frac{|3+0-21|}{\sqrt{9+0+9}\sqrt{1+4+49}}$$

$$= \frac{18}{3\sqrt{2}\times3\sqrt{6}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

angle between BC and CA,

$$\cos C = \frac{|1\times(-4) + 2(-2) + (-7)(4)|}{\sqrt{1^2+2^2+(-7)^2}\sqrt{(-4)^2+(-2)^2+(4)^2}}$$

$$= \frac{|-4-4-28|}{\sqrt{1+4+49}\sqrt{16+4+16}}$$

$$= \frac{36}{\sqrt{54}\sqrt{36}} = \frac{36}{3\sqrt{6}\times6}$$

$$= \frac{2}{\sqrt{2}\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}$$

and angle between AC and AB,

$$\cos A = \frac{|-4\times3 + (-2)\times0 + 4\times3|}{\sqrt{(-4)^2+(-2)^2+(4)^2}\sqrt{3^2+0^2+3^2}}$$

$$= |0|$$

$$\Rightarrow \quad A = 90°$$

32. ***(a)*** $\lim_{x\to0}\frac{\int_0^{x^2}\sin\sqrt{t}\,dt}{x^3}$ $\quad\left(\text{form}\frac{0}{0}\right)$

$$= \lim_{x\to0}\frac{\sin\sqrt{x^2}\times2x}{3x^2}$$

(using L' Hospital's rule)

$$= \lim_{x\to0}\frac{2}{3}\frac{\sin x}{x} = \frac{2}{3}\times1 = \frac{2}{3}$$

33. ***(b)*** $[a\times b \;\; b\times c \;\; c\times a]$

$= (a\times b)\cdot[(b\times c)\times(c\times a)]$

$= (a\times b)\cdot[((b\times c)\cdot a)\,c - ((b\times c)\cdot c)\,a]$

$= (a\times b)\cdot([b\;c\;a]\,c - [b\;c\;c]\,a)$

$= (a\times b\cdot c)[b\;c\;a] - [a\times b\cdot a]\;0$

$= [a\;b\;c]\,[a\;b\;c] - 0 = [a\;b\;c]^2$

34. ***(a)*** Geometric mean of a and $b = \sqrt{ab}$

$\Rightarrow \quad \sqrt{ab} = 16$ (given)

$\Rightarrow \quad ab = 256$ ...(i)

And harmonic mean of a and $b = \frac{2ab}{a+b}$

$$\therefore \quad \frac{2ab}{a+b} = \frac{64}{5} \quad \text{(given)}$$

$$\Rightarrow \quad \frac{2 \times 256}{a+b} = \frac{64}{5} \quad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad a + b = 40 \quad \ldots\text{(ii)}$$

Now, $(a-b) = \sqrt{(a+b)^2 - 4ab}$

$$= \sqrt{(40)^2 - 4 \times 256}$$

$$= \sqrt{1600 - 1024}$$

$$= \sqrt{576}$$

$$\Rightarrow \quad a - b = 24 \quad \ldots\text{(iii)}$$

On solving Eqs. (ii) and (iii), we get

$$a = 32 \quad \text{and} \quad b = 8$$

$$\therefore \quad a : b = 32 : 8 = 4 : 1$$

35. *(b)* Let four terms in a GP be $ar^3, ar, \frac{a}{r}$ and $\frac{a}{r^3}$.

According to the given condition,

$$ar^3 + ar + \frac{a}{r} + \frac{a}{r^3} = 60 \quad \ldots\text{(i)}$$

and

$$\frac{ar^3 + \frac{a}{r^3}}{2} = 18$$

$$\Rightarrow \quad ar^3 + \frac{a}{r^3} = 36 \quad \ldots\text{(ii)}$$

Now, from Eq. (i), we have

$$\left(ar + \frac{a}{r}\right) + ar^3 + \frac{a}{r^3} = 60$$

$$\Rightarrow \quad a\left(r + \frac{1}{r}\right) + 36 = 60 \quad \text{[from Eq. (ii)]}$$

$$\Rightarrow \quad a\left(r + \frac{1}{r}\right) = 24 \quad \ldots\text{(iii)}$$

On dividing Eq. (iii) by Eq. (ii), we get

$$\frac{a\left(r^3 + \frac{1}{r^3}\right)}{a\left(r + \frac{1}{r}\right)} = \frac{36}{24}$$

$$\Rightarrow \quad \frac{\left(r + \frac{1}{r}\right)\left(r^2 + \frac{1}{r^2} - 1\right)}{r + \frac{1}{r}} = \frac{3}{2}$$

$$\Rightarrow \quad 2\left(r^2 + \frac{1}{r^2} - 1\right) = 3$$

$$\Rightarrow \quad \frac{2(r^4 + 1 - r^2)}{r^2} = 3$$

$$\Rightarrow \quad 2r^4 + 2 - 2r^2 = 3r^2$$

$$\Rightarrow \quad 2r^4 - 5r^2 + 2 = 0$$

$$\Rightarrow \quad 2r^4 - 4r^2 - r^2 + 2 = 0$$

$$\Rightarrow \quad 2r^2(r^2 - 2) - 1(r^2 - 2) = 0$$

$$\Rightarrow \quad (r^2 - 2)(2r^2 - 1) = 0$$

$$\Rightarrow \quad r^2 = 2, 2r^2 = 1$$

$$\Rightarrow \quad r = \pm\sqrt{2}, r = \pm\frac{1}{\sqrt{2}}$$

On putting $r = \sqrt{2}$ in Eq. (iii), we get

$$a\left(\sqrt{2} + \frac{1}{\sqrt{2}}\right) = 24$$

$$\Rightarrow \quad a\left(\frac{2+1}{\sqrt{2}}\right) = 24$$

$$\Rightarrow \quad 3a = 24\sqrt{2}$$

$$\Rightarrow \quad a = 8\sqrt{2}$$

$\therefore$ Series becomes $8\sqrt{2}(\sqrt{2})^3, 8\sqrt{2}(\sqrt{2}), \frac{8\sqrt{2}}{\sqrt{2}}$ and $\frac{8\sqrt{2}}{(\sqrt{2})^3}$ *i.e.,* 32, 16, 8 and 4.

If we take $r = \frac{1}{\sqrt{2}}$, we get the series

4, 8, 16 and 32.

36. *(d)* Given, $2|x|^2 + 51 = |1 + 20x|$

Then, following two possible cases arise:

***Case* I** When $1 + 20x > 0$

$$\Rightarrow \quad x > -\frac{1}{20}$$

$$\therefore \quad 2x^2 + 51 = (1 + 20x)$$

$$\Rightarrow \quad 2x^2 - 20x + 50 = 0$$

$$\Rightarrow \quad x^2 - 10x + 25 = 0 \quad [\because 2 \neq 0]$$

$$\Rightarrow \quad (x-5)^2 = 0 \Rightarrow x = 5, 5$$

***Case* II** When $1 + 20x < 0$

$$\Rightarrow \quad x < -\frac{1}{20}$$

$$\therefore \quad 2x^2 + 51 = -(1 + 20x)$$

$$\Rightarrow \quad 2x^2 - 20x + 52 = 0$$

$$\Rightarrow \quad x^2 - 10x + 26 = 0$$

Here, $D < 0$

Thus, roots are imaginary.

Hence, sum of real roots $= 5 + 5 = 10$

37. *(a)* Given roots are $\frac{1}{3+\sqrt{2}}$ and $\frac{1}{3-\sqrt{2}}$

i.e. $\frac{1}{(3+\sqrt{2})} \times \frac{(3-\sqrt{2})}{(3-\sqrt{2})}$

and $\frac{1}{(3-\sqrt{2})} \times \frac{(3+\sqrt{2})}{(3+\sqrt{2})}$

$\Rightarrow \quad \frac{3-\sqrt{2}}{7}$ and $\frac{3+\sqrt{2}}{7}$

So, the required quadratic equation is

$$x^2 - (\text{Sum of the roots})\, x + \text{Product of the roots} = 0$$

$$\Rightarrow \quad x^2 - \left[\frac{3-\sqrt{2}}{7} + \frac{3+\sqrt{2}}{7}\right]x + \left(\frac{3-\sqrt{2}}{7}\right)\left(\frac{3+\sqrt{2}}{7}\right) = 0$$

$$\Rightarrow \quad x^2 - \left(\frac{6}{7}\right)x + \frac{9-2}{49} = 0$$

$$\Rightarrow \quad x^2 - \frac{6}{7}x + \frac{7}{49} = 0 \Rightarrow 7x^2 - 6x + 1 = 0$$

38. *(d)* Gorden Moore

39. *(c)* Vector which is perpendicular to vertors **a** and **b**, is $\mathbf{a} \times \mathbf{b}$ or $\mathbf{b} \times \mathbf{a}$.

Hence, two vectors are possible which are perpendicular to **a** and **b**.

40. *(b)* Given that,

$$I = \int_0^6 \frac{dx}{1+x^2} \qquad \ldots(i)$$

From Eq. (i), $f(x) = \frac{1}{1+x^2}$

Now, divide the interval [0, 6] into six parts each of width

$$h = \frac{6-0}{6} = 1.$$

The value of $f(x)$ are given below

x	0	1	2	3	4	5	6
***f*(*x*)**	1	0.5	0.2	0.1	0.0588	0.0385	0.027

The trapezoidal rule is

$$\int_{x_0}^{x_0+nh} y\,dx = \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + y_3 + \ldots + y_{n-1})]$$

$$\therefore \quad \int_0^6 \frac{1}{1+x^2}\,dx = \frac{1}{2}[(1+0.027) + 2\{0.5 + 0.2 + 0.1 + 0.0588 + 0.0385\}]$$

$$= \frac{1}{2}[1.027 + 2(0.8973)]$$

$$= \frac{1}{2}[1.027 + 1.7946]$$

$$= \frac{1}{2}[2.8216] = 1.4108$$

41. *(b)* Given,

$$\frac{dr}{dt} = 2 \text{ m/s}, \quad \frac{dh}{dt} = -3 \text{ m/s} \qquad \ldots(i)$$

$\because$ Volume of cylinder, $V = \pi r^2 h$

$$\therefore \quad \frac{dV}{dt} = \pi\left[2rh\frac{dr}{dt} + r^2\frac{dh}{dt}\right]$$

At $r = 3$ m and $h = 5$ m,

$$\frac{dV}{dt} = \pi\,[2 \times 3 \times 5 \times 2 - 9 \times 3] \quad [\text{using Eq. (i)}]$$

$$\Rightarrow \quad \frac{dV}{dt} = \pi\,[60 - 27] = 33\pi$$

$$\Rightarrow \quad \frac{dV}{dt} = 33\,\pi \text{ m}^3/\text{s}$$

42. *(c)* Let required point be (α, β) on the straight line $y = 2x + 11$, which is nearest to the circle

$$16(x^2 + y^2) + 32x - 8y - 50 = 0$$

$$\Rightarrow \quad x^2 + y^2 + 2x - \frac{1}{2}y - \frac{50}{16} = 0$$

$\therefore$ Centre of circle $= \left(-1, \frac{1}{4}\right)$

and radius, $r = \sqrt{1 + \frac{1}{16} + \frac{50}{16}} = \frac{\sqrt{67}}{4}$

Now, equation of straight line passing through centre $\left(-1, \frac{1}{4}\right)$ and (α, β) is

$$y - \frac{1}{4} = \left(\frac{\beta - \frac{1}{4}}{\alpha + 1}\right)(x+1) \qquad \ldots(i)$$

Now, gradient of this straight line $= \left(\frac{\beta - \frac{1}{4}}{\alpha + 1}\right)$

Since, straight line (i) is perpendicular to the line

$$y = 2x + 11$$

$$\therefore \quad \left(\frac{\beta - \frac{1}{4}}{\alpha + 1}\right) \times 2 = -1 \qquad [\because m_1 \cdot m_2 = -1]$$

$$\Rightarrow \quad 2\beta - \frac{1}{2} = -\alpha - 1$$

$$\Rightarrow \quad 2\beta + \alpha = -1 + \frac{1}{2} = \frac{-1}{2}$$

$$\Rightarrow \quad 4\beta + 2\alpha = -1 \qquad \text{...(ii)}$$

$\because$ Point (α, β) lies on straight line

$$y = 2x + 11$$

$$\therefore \quad \beta = 2\alpha + 11$$

$$\Rightarrow \quad \beta - 2\alpha = 11 \qquad \text{...(iii)}$$

On solving Eqs. (ii) and (iii), we get

$$5\beta = 10 \Rightarrow \beta = 2$$

and $2\alpha = 2 - 11 = -9 \Rightarrow \alpha = \frac{-9}{2}$

$\therefore$ Required point is $\left(\frac{-9}{2}, 2\right)$.

43. *(a)* Given equation is

$$b^2x^2 + y^2 = a^2b^2$$

$$\Rightarrow \quad \frac{x^2}{a^2} + \frac{y^2}{a^2b^2} = 1 \qquad \text{...(i)}$$

Above equation of an ellipse with semi-major axis (a) and semi-minor axis (ab).

Now, ecentricity,

$$e = 1 - \frac{a^2b^2}{a^2} \Rightarrow b^2 = 1 - e^2 \qquad \text{...(ii)}$$

Let (x, y) be extrimities of latusrectum, then

$$x = ae \text{ and } y = \pm \frac{a^2b^2}{a}$$

$$\Rightarrow \quad \frac{x}{a} = e \text{ and } \frac{y}{a} = \pm b^2$$

From Eq. (ii), we get

$$\pm \frac{y}{a} = 1 - \frac{x^2}{a^2} \Rightarrow \pm ay + x^2 = a^2$$

Hence, locus of latusrectum is $x^2 \pm ay = a^2$.

44. *(d)* Given differential equation is

$$(y \log x - 1)\, y\, dx = x dy$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{(y \log x - 1) y}{x}$$

$$= \frac{y^2 \log x}{x} - \frac{y}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{y}{x} = \frac{y^2 \log x}{x}$$

$$\Rightarrow \quad \frac{1}{y^2}\frac{dy}{dx} + \frac{y^{-1}}{x} = \frac{\log x}{x} \qquad \text{...(i)}$$

Put $\quad y^{-1} = v \Rightarrow -y^{-2}\frac{dy}{dx} = \frac{dv}{dx}$

$$\Rightarrow \quad y^{-2}\frac{dy}{dx} = -\frac{dv}{dx}$$

From Eq. (i), we have

$$-\frac{dv}{dx} + \frac{v}{x} = \frac{\log x}{x}$$

$$\Rightarrow \quad \frac{dv}{dx} - \frac{v}{x} = -\frac{\log x}{x} \qquad \text{...(ii)}$$

This is linear differential equation.

Here, $\text{IF} = e^{\int\left(-\frac{1}{x}\right)dx} = e^{-\log x} = \frac{1}{x}$

So, solution is $\quad v \cdot \text{IF} = \int \text{IF} \cdot Q\, dx + C$

$$v \cdot \frac{1}{x} = \int \frac{1}{x}\left(-\frac{\log x}{x}\right) dx + C$$

$$\Rightarrow \quad v \cdot \frac{1}{x} = -\int \frac{\log x}{x^2}\, dx + C$$

$$= -\left[\log x \left(\frac{-1}{x}\right) + \int \frac{1}{x} \cdot \frac{1}{x}\, dx\right] + C$$

$$\Rightarrow \quad \frac{1}{x \cdot y} = \frac{\log x}{x} + \frac{1}{x} + C \qquad \left[\because v = \frac{1}{y}\right]$$

$$\Rightarrow \quad 1 = y[\log x + 1 + Cx]$$

$$\Rightarrow \quad 1 = y[\log x + \log e + Cx] \qquad [\because 1 = \log e]$$

$$\Rightarrow \quad 1 = y[\log e \cdot x + Cx]$$

45. *(b)* A particular thing is received by a man with probability, $p = \frac{a}{a+b}$ and by a woman with probability, $q = \frac{b}{a+b}$.

Now, this experiment is repeated m times. Since, probability in each trial remains same for the men or women. Thus, we can apply binomial distribution.

Since, men should get odd number of things, so the random variable x would have $x = 1, 3, 5, \ldots$

$\therefore$ Required probability

$$= P(X = 1) + P(X = 3) + P(X = 5) + \ldots$$

$$= {}^mC_1 pq^{m-1} + {}^mC_3 p^3 q^{m-3} + {}^mC_5 p^5 q^{m-5} + \ldots$$

$$= \left[\frac{(q+p)^m - (q-p)^m}{2}\right]$$

$$= \left[\frac{1 - \left(\frac{b}{a+b} - \frac{a}{a+b}\right)^m}{2}\right] \quad [\because p + q = 1]$$

$$= \frac{(a+b)^m - (b-a)^m}{2(a+b)^m}$$

46. *(a)* Given differential equation is

$$\sqrt{a+x}\,\frac{dy}{dx} + xy = 0$$

$$\Rightarrow \quad \sqrt{a+x}\,\frac{dy}{dx} = -xy$$

$$\Rightarrow \quad \frac{dy}{y} = \frac{-x}{\sqrt{a+x}}\,dx$$

On integrating both sides, we get

$$\log y - \log C = \int \left[\frac{-x-a}{\sqrt{a+x}} + \frac{a}{\sqrt{a+x}}\right] dx$$

$$\Rightarrow \quad \log\left(\frac{y}{C}\right) = -\int \sqrt{x+a}\,dx + a\int \frac{1}{\sqrt{a+x}}\,dx$$

$$= -\frac{(x+a)^{3/2}}{3/2} + a\,\frac{(a+x)^{1/2}}{1/2}$$

$$\Rightarrow \quad \log\left(\frac{y}{C}\right) = -\frac{2}{3}(x+a)\sqrt{x+a} + 2a\sqrt{x+a}$$

$$= \frac{2}{3}(2a - x)\sqrt{x+a}$$

$$\Rightarrow \quad \frac{y}{C} = e^{\frac{2}{3}(2a-x)\sqrt{x+a}}$$

$$\Rightarrow \quad y = C \cdot e^{\frac{2}{3}(2a-x)\sqrt{x+a}}$$

which is the required solution.

47. *(a)* Given function is

$$f(x) = 2e^x - ae^{-x} + (2a+1)x - 3$$

For increasing function, $f'(x) > 0,\ \forall\, x$

$$\Rightarrow f'(x) = 2e^x + ae^{-x} + (2a+1) > 0$$

$$\Rightarrow \quad (2e^x + 1) + a(e^{-x} + 2) > 0$$

$$\Rightarrow \quad a(e^{-x} + 2) > -(1 + 2e^x)$$

$$\Rightarrow \quad a > \frac{-(1+2e^x)}{(e^{-x}+2)}$$

$$\Rightarrow \quad a > \frac{-(1+2e^x)}{(1+2e^x)} \cdot e^x$$

$$\Rightarrow \quad a > -e^x$$

$$\because \quad e^x \in (0, \infty) \Rightarrow -e^x \in (-\infty, 0)$$

and $\quad a > -e^x$

$$\therefore \quad a \in [0, \infty)$$

48. *(a)* Given, $f(x) = x^4 - x - 10$

We assume $x_0 = 2$ is the approximate root of $f(x)$.

Then, $\quad h = -\frac{f(x_0)}{f'(x_0)} = -\frac{f(2)}{f'(2)}$

$$\Rightarrow \quad h = -\left[\frac{(2)^4 - 2 - 10}{4(2)^3 - 1}\right]$$

$$\Rightarrow \quad h = -\left[\frac{16 - 12}{31}\right] = \frac{-4}{31}$$

$$\Rightarrow \quad h = -0.129$$

$\therefore$ Positive square root of $f(x)$ by Newton-Raphson method,

$$x_1 = x_0 + h = 2 + (-0.129)$$

$$= 2 - 0.129 = 1.871$$

49. *(b)* Given, $3a = b + c$

$$\because \quad \cot\left(\frac{B}{2}\right) = \sqrt{\frac{s(s-b)}{(s-a)(s-c)}}$$

and $\quad \cot\left(\frac{C}{2}\right) = \sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$

$$\therefore \cot\left(\frac{B}{2}\right) \cdot \cot\left(\frac{C}{2}\right) = \sqrt{\frac{s(s-b)}{(s-a)(s-c)}} \times \sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$$

$$= \frac{s}{(s-a)} = \frac{2s}{2s - 2a}$$

On putting $2s = a + b + c$, we get

$$\cot\left(\frac{B}{2}\right) \cdot \cot\left(\frac{C}{2}\right) = \frac{a+b+c}{a+b+c-2a}$$

$$\Rightarrow \cot\left(\frac{B}{2}\right) \cdot \cot\left(\frac{C}{2}\right) = \frac{a+3a}{a+3a-2a} = \frac{4a}{2a} = 2$$

50. *(b)* Given, $\sin^{-1} x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$

$$\Rightarrow \quad \sin^{-1} x = \frac{\pi}{2} - \cot^{-1}\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad \sin^{-1} x = \tan^{-1}\left(\frac{1}{2}\right)$$

$$\left[\because \tan^{-1}\theta + \cot^{-1}\theta = \frac{\pi}{2}\right]$$

$$\Rightarrow \quad x = \sin\left\{\tan^{-1}\left(\frac{1}{2}\right)\right\}$$

$$\Rightarrow \quad x = \sin\left\{\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)\right\}$$

$$\Rightarrow \quad x = \frac{1}{\sqrt{5}}$$

51. *(c)* Given, $I = \int_0^{\sqrt{\ln(\pi/2)}} \cos(e^{x^2}) \cdot 2xe^{x^2}\, dx$

Put $e^{x^2} = t \Rightarrow 2xe^{x^2}\, dx = dt$

Now, lower limit, $t = 1$

Upper limit, $t = e^{\ln \pi/2} = \frac{\pi}{2}$

$$\therefore \quad I = \int_1^{\pi/2} \cos(t)\, dt = [\sin t]_1^{\pi/2}$$

$$= \sin\left(\frac{\pi}{2}\right) - \sin 1$$

$$\Rightarrow \quad I = 1 - \sin 1$$

52. *(c)* Application Programming Interface.

53. *(d)* Given, $Z = i\log(2-\sqrt{3})$

$$\Rightarrow \cos Z = \cos[i\log(2-\sqrt{3})]$$

$$= \cos h\{\log(2-\sqrt{3})\}$$

$$[\because \cos ix = \cosh x, \text{ for } x \in R]$$

$$= \frac{e^{\log(2-\sqrt{3})} + e^{-\log(2-\sqrt{3})}}{2}$$

$$\left[\because \cos hx = \frac{e^x + e^{-x}}{2}\right]$$

$$= \frac{e^{\log(2-\sqrt{3})} + e^{\log\left(\frac{1}{2-\sqrt{3}}\right)}}{2}$$

$$= \frac{2-\sqrt{3} + \frac{1}{2-\sqrt{3}}}{2}$$

$$= \frac{(2-\sqrt{3})^2 + 1}{2(2-\sqrt{3})} = \frac{4+3-4\sqrt{3}+1}{2(2-\sqrt{3})}$$

$$= \frac{8-4\sqrt{3}}{2(2-\sqrt{3})} = \frac{4(2-\sqrt{3})}{2(2-\sqrt{3})} = 2$$

54. *(c)* Given, $\lim_{x \to 2} \frac{2-\sqrt{2+x}}{2^{1/3}-(4-x)^{1/3}}$ $\left[\frac{0}{0}\text{form}\right]$

$$= \lim_{x \to 2} \frac{0 - \frac{1}{2\sqrt{2+x}}}{0 - \frac{1}{3}\cdot\frac{(-1)}{(4-x)^{2/3}}}$$

$$= \lim_{x \to 2} \frac{-3(4-x)^{2/3}}{2\sqrt{2+x}}$$

$$= \frac{-3}{2} \times \frac{(4-2)^{2/3}}{\sqrt{2+2}} = \frac{-3}{2} \times \frac{2^{2/3}}{2}$$

$$= -3 \cdot 2^{\frac{2}{3}-2} = -3 \cdot 2^{-4/3}$$

55. *(d)* Three lines of triangle are given by

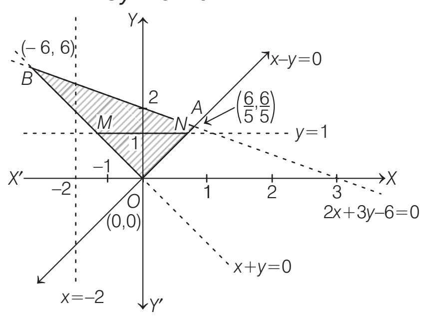
$$(x^2 - y^2)(2x + 3y - 6) = 0$$

$$\Rightarrow (x-y)(x+y)(2x+3y-6) = 0$$

$\therefore$ The three lines of triangle are

$$x - y = 0, x + y = 0$$

and $\quad 2x + 3y - 6 = 0$



From given lines of triangle, the required ΔOAB is formed.

$\because (-2, \lambda)$ lies inside the triangle.

$\therefore \quad 2(-2) + 3(\lambda) - 6 < 0$

and $\quad -2 + \lambda > 0$

$\Rightarrow \quad -4 + 3\lambda - 6 < 0$ and $\lambda > 2$

$\Rightarrow \quad 3\lambda < 10$ and $\lambda > 2$

$\Rightarrow \quad \lambda < \frac{10}{3}$ and $\lambda > 2$

$\therefore \quad \lambda \in \left(2, \frac{10}{3}\right)$ ...(i)

Now, $(\mu, 1)$ lies outside the triangle.

To find value of μ, we find the interval $[M, N]$ for values of x.

$$x + 1 \geq 0 \text{ and } x - 1 \leq 0$$

$\Rightarrow \quad x \geq -1$ and $x \leq 1$

$\therefore \quad x \in [-1, 1]$

$\because$ $(\mu, 1)$ lies outside the triangle.

$\therefore \quad \mu \in (-\infty, -1) \cup (1, \infty)$

or $\quad \mu \in R - [-1, 1]$

56. *(a)* System software : Utility software :: Operating system : Arti-virus

Utility software-Anti-virus

57. *(c)* Given equations of circles are

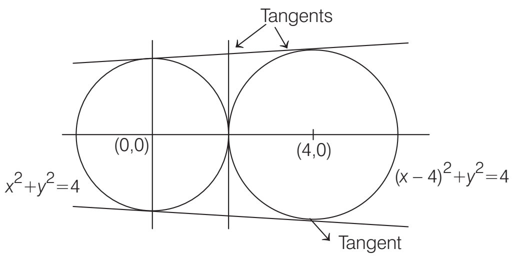
$$x^2 + y^2 = 4$$

and $\quad x^2 + y^2 - 8x + 12 = 0$

or $\quad x^2 + y^2 = (2)^2 \quad \ldots(i)$

$$(x-4)^2 + (y-0)^2 = (2)^2 \quad \ldots(ii)$$

The figure of both the circles are shown below :



From figure, we see that there is exactly three common tangents.

58. *(a)* Let $f(x) = \frac{1}{3}ax^3 + \frac{1}{2}bx^2 + cx$

Then, $f(x)$ is a polynomial.

So, it is continuous in R.

Now, $\quad f(0) = 0$

and $\quad f(1) = \frac{a}{3} + \frac{b}{2} + c = \frac{2a + 3b + 6c}{6}$

$\Rightarrow \quad f(1) = 0 \quad [\because 2a + 3b + 6c = 0$, given]

$\because f(x)$ is a polynomial, so it is differentiable in R, so in (0, 1).

Hence, by Rolle's theorem, there exists atleast one point $x \in (0, 1)$, there exists such that

$$f'(x) = 0$$

$\Rightarrow \quad ax^2 + bx + c = 0$

Hence, required interval is (0, 1).

59. *(a)* Given, for $x, y \in N$,

$$f(x + y) = f(x) \cdot f(y)$$

Then, function will be of the form

$$f(x) = a^x, \text{ where } a \in N \quad [\because a \neq 1]$$

$\because \quad f(1) = 3$

$\Rightarrow \quad f(1) = a^1 = 3$

$\Rightarrow \quad a = 3$

$\therefore$ Function is $f(x) = 3^x$.

Now, $\quad \sum_{x=1}^{n} f(x) = 120$

$\Rightarrow \quad \sum_{x=1}^{n} 3^x = 120$

$\Rightarrow 3 + 3^2 + 3^3 + \ldots + 3^n = 120$

$\Rightarrow \quad \frac{3(3^n - 1)}{3 - 1} = 120$

$\Rightarrow \quad 3^n = 1 + \frac{120 \times 2}{3}$

$\Rightarrow \quad 3^n = 81 = 3^4$

So, $\quad n = 4$

60. *(b)* Given function is

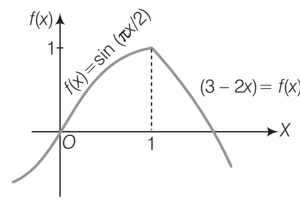
$$f(x) = \begin{cases} \sin\left(\frac{\pi x}{2}\right), & \text{if } x < 1 \\ 3 - 2x, & \text{if } x \leq 1 \end{cases}$$

$\because$ LHL and RHL of $f(x)$ is 1, when $x \to 1$

and $\quad f(1) = 3 - 2 = 1$

$\therefore \quad f(x)$ is continuous at $x = 1$.

Now, graph of function $f(x)$ is



From the above graph of function, we see that $f(x)$ has local maxima at $x = 1$.

61. *(c)* Given differential equation is

$$\frac{dy}{dx} = y\tan x - y^2 \sec x$$

$\Rightarrow \quad \frac{dy}{dx} - y\tan x = -y^2 \sec x$

$\Rightarrow \quad \frac{1}{y^2}\frac{dy}{dx} - \frac{\tan x}{y} = -\sec x \quad \ldots(i)$

Put $\quad \frac{-1}{y} = u \Rightarrow \frac{1}{y^2}\frac{dy}{dx} = \frac{du}{dx}$

From Eq. (i),

$$\frac{du}{dx} + \tan x \cdot u = -\sec x \quad \dots(ii)$$

This is linear differential equation of the form $\frac{du}{dx} + P \cdot u = Q$, where $P = \tan x$, $Q = -\sec x$

$$\therefore \quad \text{IF} = e^{\int \tan x\, dx} = e^{\log \sec x} = \sec x$$

Hence, general solution is

$$u \cdot \text{IF} = \int \text{IF} \cdot Q\, dx + C_1$$

$$\Rightarrow \quad u \cdot \sec x = \int (\sec x) \cdot (-\sec x)\, dx + C_1$$

$$\Rightarrow \quad u \cdot \sec x = -\int \sec^2 x\, dx + C_1$$

$$\Rightarrow \quad u \sec x = -\tan x + C_1$$

$$\Rightarrow \quad \frac{-\sec x}{y} = -\tan x + C_1 \qquad \left[\because u = \frac{-1}{y}\right]$$

$$\Rightarrow \quad \sec x = y(\tan x - C_1)$$

$$\Rightarrow \quad \sec x = y(\tan x + C) \quad [\text{where, } C = -C_1]$$

62. *(c)* Given,

$$(\hat{i} + \hat{j} + 3\hat{k})x + (3\hat{i} - 3\hat{j} + \hat{k})y + (-4\hat{i} + 5\hat{j})z = \lambda(\hat{i}x + \hat{j}y + \hat{k}z)$$

On equating the coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$ both sides, we have

$$x + 3y - 4z = \lambda x,$$
$$x - 3y + 5z = \lambda y$$
and
$$3x + y + 0 = \lambda z$$

Above three equations can be rewritten as

$$(1-\lambda)x + 3y - 4z = 0$$
$$x - (3+\lambda)y + 5z = 0$$
$$3x + y - \lambda z = 0$$

This is homogeneous system of equations in three variables x, y and z.

It is consistent and have non-zero solution.

i.e. $(x, y, z) \neq (0, 0, 0)$, if determinant of coefficient matrix is zero.

$$\Rightarrow \quad \begin{vmatrix} 1-\lambda & 3 & -4 \\ 1 & -(3+\lambda) & 5 \\ 3 & 1 & -\lambda \end{vmatrix} = 0$$

On expanding along first row, we have

$$(1-\lambda)[\lambda(3+\lambda) - 5] - 3(-\lambda - 15) - 4(1 + 9 + 3\lambda) = 0$$

$$\Rightarrow (1-\lambda)(\lambda^2 + 3\lambda - 5) + 3\lambda + 45 - 40 - 12\lambda = 0$$

$$\Rightarrow \lambda^2 + 3\lambda - 5 - \lambda^3 - 3\lambda^2 + 5\lambda - 9\lambda + 5 = 0$$

$$\Rightarrow \quad -\lambda^3 - 2\lambda^2 - \lambda = 0$$

$$\Rightarrow \quad \lambda(\lambda^2 + 2\lambda + 1) = 0$$

$$\Rightarrow \quad \lambda(\lambda + 1)^2 = 0$$

$$\Rightarrow \quad \lambda = 0, -1$$

63. *(b)* It is given, $\cot A$, $\cot B$ and $\cot C$ are in AP.

$$\Rightarrow \quad 2\cot B = \cot A + \cot C$$

$$\Rightarrow \quad 2\frac{\cos B}{\sin B} = \frac{\cos A}{\sin A} + \frac{\cos C}{\sin C}$$

$$\because \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$$

$$\Rightarrow \quad \frac{2\cos B}{kb} = \frac{\cos A}{ak} + \frac{\cos C}{ck}$$

$$\Rightarrow \quad \frac{2\cos B}{b} = \frac{\cos A}{a} + \frac{\cos C}{c}$$

$$\Rightarrow \quad \frac{2}{b}\left[\frac{a^2 + c^2 - b^2}{2ac}\right] = \frac{1}{a}\left[\frac{b^2 + c^2 - a^2}{2bc}\right] + \frac{1}{c}\left[\frac{a^2 + b^2 - c^2}{2ab}\right]$$

$$\Rightarrow \quad \frac{2(a^2 + c^2 - b^2)}{2abc} = \frac{1}{2abc}[(b^2 + c^2 - a^2) + (a^2 + b^2 - c^2)]$$

$$\Rightarrow \quad 2(a^2 + c^2 - b^2) = 2b^2$$

$$\Rightarrow \quad a^2 + c^2 = 2b^2$$

$\therefore$ a^2, b^2 and c^2 are in AP.

64. *(c)* Given integral is $\int_1^2 e^{-x/2}\, dx$

On dividing interval [1, 2] in four parts, we have

$$h = \frac{2-1}{4} = \frac{1}{4}$$

Now, value of $f(x) = e^{-x/2}$ is given below

x	1	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{7}{4}$	e
$f(x)$	0.6065	0.5352	0.4724	0.4168	0.3679

Now, Simpson's $\frac{1}{3}$ rule is

$$\int_{x_0}^{x_0 + nh} f(x)dx = \frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2})]$$

$$\therefore \int_1^2 e^{-x/2} dx = \frac{1}{12}[(0.6065 + 0.3679) + 4(0.5352 + 0.4168) + 2(0.4724)]$$

$$= \frac{1}{12}[0.9744 + 3.808 + 0.9448]$$

$$= \frac{1}{12} \times 5.7272 = 0.477$$

65. *(b)* When a die is rolled three times.

Then, cardinality of sample space $= 6^3$

Now, according to question, the favourable outcomes

$= \{(1, 2, 3), (1, 2, 4), (1, 2, 5), (1, 2, 6),$
$(1, 3, 4), (1, 3, 5), (1, 3, 6), (1, 4, 5),$
$(1, 4, 6), (1, 5, 6), (2, 3, 4), (2, 3, 5),$
$(2, 3, 6), (2, 4, 5), (2, 4, 6), (3, 4, 5),$
$(3, 4, 6), (4, 5, 6), (2, 5, 6), (3, 5, 6)\}$

$\therefore$ Required probability $= \dfrac{20}{6^3} = \dfrac{5}{54}$

66. *(d)* Given, $f(x) = \log_e(6 - |x^2 + x - 6|)$

The function $f(x)$ is defined, if

$$(6 - |x^2 + x - 6|) > 0$$

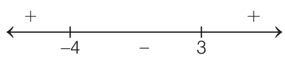
$\Rightarrow \quad |x^2 + x - 6| < 6$

$\Rightarrow \quad -6 < x^2 + x - 6 < 6$

If $\quad x^2 + x - 6 < 6$

$\Rightarrow \quad x^2 + x - 12 < 0$

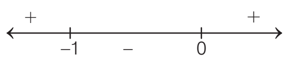
$\Rightarrow \quad (x + 4)(x - 3) < 0$



$\therefore \quad x \in (-4, 3) \quad$ ...(i)

Now, if $-6 < x^2 + x - 6$

$\Rightarrow \quad x^2 + x > 0 \Rightarrow x(x + 1) > 0$



$\therefore \quad x \in (0, \infty) \quad$ ...(ii)

From Eqs. (i) and (ii), we get

$$x \in (0, 3)$$

$\because f(x)$ has only two integral values.

$\therefore \quad x = 1, 2$

67. *(a)* The given equation is

$$y^2 + z^2 = 0$$

Above equation represents a point circle in the YZ-plane.

68. *(d)* Given complex numbers are

$$Z_1 = (1 + i) \equiv (1, 1) \equiv (a_1, b_1)$$

$$Z_2 = (-2 + 3i) \equiv (-2, 3) \equiv (a_2, b_2)$$

and $\quad Z_3 = \dfrac{ai}{3} \equiv \left(0, \dfrac{a}{3}\right) \equiv (a_3, b_3)$

These three points will be collinear, if

$$\frac{1}{2}[a_1(b_2 - b_3) + a_2(b_3 - b_1) + a_3(b_1 - b_2)] = 0$$

$\Rightarrow \quad 1\left(3 - \dfrac{a}{3}\right) - 2\left(\dfrac{a}{3} - 1\right) + 0(1 - 3) = 0$

$\Rightarrow \quad \dfrac{9 - a}{3} - 2\left(\dfrac{a - 3}{3}\right) + 0 = 0$

$\Rightarrow \quad 9 - a - 2a + 6 = 0$

$\Rightarrow \quad 3a = 15$

$\Rightarrow \quad a = 5$

69. *(b)* Given that, a, b and c are in HP.

$\therefore$ Harmonic mean of a and c is b.

and geometric mean of a and c is $\sqrt{ac}$.

$\because$ Geometric mean > Harmonic mean

$\Rightarrow \quad \sqrt{ac} > b \quad$ ...(i)

Now, for the positive numbers a^n and c^n, we have

$$\text{Geometric mean} = \sqrt{a^n \cdot c^n}$$

$$\text{and arithmetic mean} = \frac{a^n + c^n}{2}$$

$\because \quad$ AM > GM

$\therefore \quad \dfrac{a^n + c^n}{2} > \sqrt{a^n \cdot c^n}$

$\Rightarrow \quad a^n + c^n > 2\sqrt{a^n \cdot c^n}$

$\Rightarrow \quad a^n + c^n > 2(\sqrt{ac})^n > 2b^n \quad$ [using Eq. (i)]

$\Rightarrow \quad a^n + c^n > 2b^n$

70. *(d)* $\tan\left[2\tan^{-1}\left(\dfrac{1}{5}\right) - \dfrac{\pi}{4}\right] = \tan\left[\tan^{-1}\left(\dfrac{\frac{2}{5}}{1 - \frac{1}{25}}\right) - \dfrac{\pi}{4}\right]$

$$\left[\because 2\tan^{-1}\theta = \tan^{-1}\left(\frac{2\theta}{1 - \theta^2}\right)\right]$$

$$= \tan\left[\tan^{-1}\left(\frac{10}{24}\right) - \frac{\pi}{4}\right]$$

$$= \frac{\tan\left\{\tan^{-1}\left(\frac{10}{24}\right)\right\} - \tan\left(\frac{\pi}{4}\right)}{1 + \tan\left\{\tan^{-1}\left(\frac{10}{24}\right)\right\} \cdot \tan\left(\frac{\pi}{4}\right)}$$

$$= \frac{\frac{10}{24} - 1}{1 + \frac{10}{24} \cdot 1}$$

$$= \frac{10 - 24}{24 + 10}$$

$$= \frac{-14}{34} = \frac{-7}{17}$$

General Aptitude

16. *(b)* 'Brays' is the sound produced by Donkey, similarly howls is the sound produced by 'Wolf'.

17. *(d)* As 'astronauts' related to space, in the same manner Argonauts is related to 'sea'.

18. *(b)* As orthopaedic is the specialist of Bones, in the same manner dermatologist is the specialist of Skin.

19. *(a)* 'Basilica' is an early Christian church building, in the same manner, 'dormer' is a window that projects vertically from a sloping roof.

20. *(c)* As 'although' means nevertheless in the same manner, 'Though' means 'however'.

21. *(a)* Except Sun, all other are planets.

22. *(b)* Hesitant is different as all others are synonyms of each other.

23. *(b)* Except Tulip, all others are flowers whereas Tulip is a kind of plant.

24. *(d)* Position of P from left = 16

$\therefore$ Position of P from right = 27 − 16 = 11

25. *(d)* 14th to the right of 6th letter from the left

= (6 + 14)th letter from the left

= 20 th letter from the left

= *T*

26. *(a)* In backward order of alphabet,

3rd letter from right = *C*

13th letter to the left of 3rd letter from right = (13 + 3)th letter from the right

= 16 th letter from the right

= *P*

27. *(d)* The pattern is as follows

23 28 34 41 49 (58)

+5 +6 +7 +8 +9

$\therefore ? = 58$

28. *(a)* The pattern is as follows

11 → 17 (+6), 17 → 23 (+6), 23 → 29 (+6)

11 13 17 19 23 25 (29)

13 → 19 (+6), 19 → 25 (+6)

$\therefore ? = 29$

29. *(c)* In figure I, $2^2 + 4^2 = 4 + 16 = 20$

In figure II, $3^2 + 9^2 = 9 + 81 = 90$

Similarly in figure III, $1^2 + 7^2 = 1 + 49 = 50$

30. *(b)* In figure-I

$$\sqrt{64} + \sqrt{36} + \sqrt{49} = 8 + 6 + 7 = 21$$

Similarly, in figure II.

$$\sqrt{121} + \sqrt{81} + \sqrt{100} = 11 + 9 + 10 = 30$$