Rational Numbers

FUNDAMENTALS

Rational Number:-

- A number which can be expressed as $\frac{x}{y}$, where x and y are Integers and $y \neq 0$ is called a rational number.

e.g., $\frac{1}{2}, \frac{2}{2}, \frac{-1}{2}, 0, \frac{3}{-2}$ etc.

- Set of rational number is denoted by Z.
- A Rational number may be positive, zero or negative
- If $\frac{x}{y}$ is a rational number and $\frac{x}{y} > 0$, then $\frac{x}{y}$ is called a positive Rational Number.

e.g., $\frac{1}{2}, \frac{2}{5}, \frac{-3}{-2}, -\left(-\frac{1}{2}\right)$ **etc.**

Negative Rational Numbers:-

- If $\frac{x}{y}$ is a rational number and $\frac{x}{y} < 0$, then $\frac{x}{y}$ is called a Negative Rational Number.

e.g., $\frac{-1}{2} . \frac{3}{-2}, \frac{-7}{11}$...... etc.

Standard form of Rational Number:-

- A Rational number $\frac{x}{y}$ is said to be m standard form, if x and y are integers having no common divisor other than one, where $y \neq 0$.

e.g., $\frac{-1}{2}, \frac{5}{6}, \frac{8}{11}$ ......etc.

Note:- There are infinite rational numbers between any two rational numbers.

Property of Rational Number

- Let x and y are two rational number and y > x, then the rational number between x and y is $\frac{1}{2}(x+y)$.

e.g., find 2 rational number between $\frac{1}{3}$ and $\frac{1}{2}$

Solution:- Let $x = \frac{1}{3}$ and $y = \frac{1}{3}$ and y > x.

Then, Rational no. between $\frac{1}{3}$ and $\frac{1}{2}$ is

$$\frac{1}{2}\left(\frac{1}{3}+\frac{1}{2}\right)=\frac{1}{2}\left(\frac{2+3}{6}\right)=\frac{5}{12}$$

Again Let $x=\frac{5}{12}$ and $y=\frac{1}{2}$ and y > x. then

Rational no. between $\frac{5}{12}$ and $\frac{1}{2}$ is

$$\frac{1}{2}\left(\frac{5}{12}+\frac{1}{2}\right)=\frac{1}{2}\left(\frac{5+6}{12}\right)=\frac{1}{2}\times\frac{11}{12}=\frac{11}{24}$$

Hence the Rational Numbers between $\frac{1}{3}$ and $\frac{1}{2}$ are $\frac{5}{12}$ and $\frac{11}{24}$.

- Let x and y are two rational number and y > x. Consider to find n rational numbers between x and y. Let d = $\frac{y-x}{n+1}$

Then 'n' rational number lying between x and y are $(x+d),(x+2d),(x+3d),\_\_\_(x+nd)$.

Example:- Find 9 rational number between 2 and 3.

Solution:- Let x = 2 and y = 3 then y > x

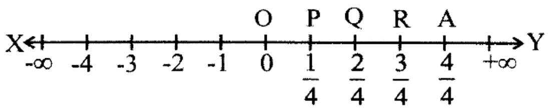
Now $\mathbf{d}=\frac{y-x}{n+1}=\frac{3-2}{9+1}=\frac{1}{10}$

Then, rational number are, 2 + 0.1, 2 + 0.2, 2 + 0.3, 2 + 0.4, 2 + 0.5, 2 + 0.6, 2 + 0.7, 2 + 0.8, 2 + 0.9 = 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8 and 2.9.

Representation of Rational Number on the Number line

- To represent - on the number line first we draw a number line XY.

Let O represent 0 (zero) and A represent 1. So divide OA into 4 equal parts, each point in the middle representing P, Q and R. Point R represent $\frac{3}{4}$.



Operations on Rational Numbers

- Addition of Rational Numbers:

Example: Find the sum of the rational numbers $\frac{-4}{9},\frac{15}{12}$ and $\frac{-7}{18}$.

Solution: $\frac{-4}{9}+\frac{15}{12}+\frac{-7}{18}=\frac{-16+45-14}{36}=\frac{15}{36}=\frac{5}{12}$

Properties of Addition of Rational Number

- Closure Property:- If a and b are two rational numbers, then a + b is always a rational number.

  E.g., Let $a = 3$, $b = -2,$ then

  $a + b = 3 + (-2) - 1$

- Commutative Property:- If a and b are two rational number then a + b = b + a.

  E.g., Let $a = \frac{1}{2}$ and $b = \frac{1}{3}$ then

  To check whether, a + b = b + a

$\Rightarrow \frac{1}{2} + = \frac{1}{3} + \frac{1}{2}$

$\Rightarrow \frac{5}{6} = \frac{5}{6}$

- Associative Property:-If a, b and c are three Rational number then,

  $a + (b + c) = (a + b) + c.$

E.g., $a = 1,$ $b = -2$ and $c = 3$ then,

$1 + (-2 + 3) = (1 - 2) + 3$

$1 + 1 = -1 + 3$

2 = 2

Existence of additive identity (property of zero):-

- Zero is the additive identity for any Rational Number because when zero is added to any Rational Number, then sum is the same given Number, (a + 0 = a).

E.g., $2 + 0 = 2, -2 + 0 = -2,$ $3 + 0 = 3,$ $\frac{-1}{2} + 0 = \frac{-1}{2}$

Existence of additive inverse;-

- Negative of rational number.

For $\frac{a}{b}$, it is $-\frac{a}{b}$

e.g., For $\frac{1}{2}$, it is $-\frac{1}{2}$

($-\frac{1}{2}$ is a additive inverse of $\frac{1}{2}$)

$-\frac{3}{2} \Rightarrow \frac{3}{2}$ ($\frac{3}{2}$ is a additive inverse of $-\frac{3}{2}$)

Note:- Additive inverse of the rational number '0' is 0 itself.

Subtraction of Rational Number:-

- Subtraction is inverse process of addition

If $\frac{p}{q}$ and $\frac{r}{s}$ be two rational number it follows

$\frac{r}{s}-\frac{p}{q}=\frac{r}{s}+\left(-\frac{p}{q}\right)$

e.g., subtract $\frac{-2}{7}$ from $\frac{3}{4}$.

Solution:- $\frac{3}{4}-\left(-\frac{2}{7}\right)=\frac{3}{4}+\frac{2}{7}=\frac{21+8}{28}=\frac{29}{28}$

Multiplication of Rational Number;

- The product of two rational numbers $=\frac{\text{The Product of the numerators}}{\text{Product of the denominators}}$

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then $\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d}$

Example:- Multiply $\frac{-17}{30}$ by $\frac{15}{-34}$

Solution:- $\frac{-17}{30}\times\frac{15}{34}=\frac{-17\times15}{30\times-34}=\frac{1}{4}$

Properties of multiplication of Rational Numbers:

- **Closure Property:-** If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then $\left(\frac{a}{b}\times\frac{c}{d}\right)$ is also a Rational Number.

e.g., $\frac{2}{3}\times\frac{3}{4}=\frac{2\times3}{3\times4}=\frac{1}{2}$

- **Commutative Property:**- If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then $\frac{a}{b}\times\frac{c}{d}=\frac{c}{d}\times\frac{a}{b}$

e.g., $\frac{2}{3}\times\frac{3}{4}=\frac{3}{4}\times\frac{2}{3}\Rightarrow\frac{6}{12}=\frac{6}{12}$

$\frac{1}{2}=\frac{1}{2}$

- **Associative Property:-** If $\frac{a}{b},\frac{c}{d}$ and $\frac{e}{f}$ are three rational numbers, then $\frac{a}{b}\times\left(\frac{c}{d}\times\frac{e}{f}\right)=\left(\frac{a}{b}\times\frac{c}{d}\right)\times\frac{e}{f}$

e.g., $\frac{1}{2}\times\left(\frac{2}{3}\times\frac{3}{4}\right)=\left(\frac{1}{2}\times\frac{2}{3}\right)\times\frac{3}{4}$

$\frac{6}{24} = \frac{6}{24}$

- **Existence of Multiplicative Identity:-** One is the multiplicative identity for any rational number because when 1 is multiplied to any Rational Number, Product is Given Rational Number itself.

e.g., $\left(\frac{p}{q} \times 1\right) = \frac{p}{q}$, $\left(\frac{3}{4} \times 1\right) = \frac{3}{4}$, $\left(\frac{-5}{2} \times 1\right) = \frac{-5}{2}$

- **Existence of Multiplicative inverse:-** for any non-zero rational number $\frac{a}{b}$, there exist a unique rational $\frac{b}{a}$ such that $\left(\frac{a}{b} \times \frac{b}{a}\right) = 1$.

Hence, we say that $\frac{a}{b}, \frac{b}{a}$ are multiplicative inverse of each other

e.g., (i) $\frac{2}{3} \times \frac{3}{2} = 1$

(ii) $\left(\frac{-3}{4} \times \frac{-4}{3}\right) = \frac{12}{12} = 1$

- **Distribution of Multiplication over Addition:-** for any three rational numbers $\frac{a}{b}, \frac{b}{a}$ and $\frac{e}{f}$

$\frac{a}{b} \times \left(\frac{c}{d} \times \frac{e}{f}\right) = \left(\frac{a}{b} \times \frac{c}{d}\right) + \left(\frac{a}{b} \times \frac{e}{f}\right)$. This property is called distributive property for multiplication over addition.

e.g., $\frac{1}{2} \times \left(\frac{2}{3} + \frac{3}{4}\right) = \frac{1}{2}\left(\frac{8+9}{12}\right) = \frac{1}{2} \times \frac{17}{12} = \frac{17}{24}$

- **Division of Rational Number:-** If $\frac{a}{b}$ is divided by $\frac{c}{d}$, then $\frac{a}{b}$ is the dividend, $\frac{c}{d}$ is the divisor and $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$ is the quotient.

Example:- $\frac{14}{57} \div \frac{42}{19} = \frac{14}{57} \times \frac{19}{42} = \frac{14 \times 19}{57 \times 42} = \frac{1}{9}$

- **Decimal representation of Rational Numbers:-** A rational number can be expressed as a terminating or non-terminating, recurring decimal.

For example:-

1. $\frac{1}{2}=0.5, \frac{1}{4}=0.25, \frac{1}{5}=0.2$ etc. are rational numbers which are terminating decimals.

2. $\frac{4}{3}=1.333.....=1.\bar{3}, \frac{1}{6}=0.1666.....=0.1\bar{6}, \ \frac{1}{7}=0.142857142857....=0.\overline{142857}, etc$

are non-terminating repeating decimals.

- If a rational number ($\neq$ integer) can be expressed in the form $\frac{p}{2^n \times 5^m}$, where $\mathbf{P} \in \mathbf{Z}, n \in W$ and $m \in W$, the rational number will be terminating decimal otherwise, rational number will be non-terminating recurring decimal.

For Example:

1. $\frac{3}{10}=\frac{3}{2^1 \times 5^1}$, So, $\frac{3}{10}$ is a terminating decimal.

2. $\frac{7}{250}=\frac{7}{2^1 \times 5^3}$, So, $\frac{7}{250}$ is a terminating decimal.

3. $\frac{8}{75}=\frac{8}{5^2 \times 3}$ is a non-terminating, recurring decimal.

- Non-terminating recurring decimal is also called periodic decimal.

Method of expressing recurring decimals as rational number:

- The recurring part of the non-terminating recurring decimal is called period and the number of digits in the recurring part is called periodicity.

Example:

1. $\frac{1}{3}=0.\bar{3}$, period = 3, Periodicity = 1

2. $\frac{7}{15}=0.4\bar{6}$, Period = 6, Periodicity = 1

3. $\frac{5}{13}=0.\overline{384615}$, Period=384615, Periodicity = 6

We can express non-terminating recurring decimals in the form of rational numbers.

Example-1:- Let us write $0.2\overline{45}$ in the form of rational number.

Solution:- Let x = $0.2\overline{45}$ (i)

Then $10x=2.4545$ .... (ii)

Also, $1000x=245.4545$ .... (iii)

On subtracting (ii) from (iii), we get: $990x=245 \Leftrightarrow x=\frac{245}{990}=\frac{49}{198}$.

Hence, $0.2\overline{45} = \frac{40}{198}$.

Example-2:- Let us find the rational form of $0.\overline{428571}$.

Solution:- The periodicity of the recurring decimal is 6. So multiply the decimal fraction by 10^6, $0.\overline{428571}$ = x (say)

$$10^6 = 1000000x = 428571.\overline{428571}$$

$$x = 0.\overline{428571}$$

$$99999x = 428571$$

$$\therefore x = \frac{428571}{999999} = \frac{3}{7}$$

Example-3:- Express $15.0\overline{2}$ as a rational Number

Solution:- Here, the whole number obtained by writing digits in there order =1502. The whole number made by the non-recurring digits in order = 150,

The number of digits after the decimal point = 2 (two)

The number of digits after the decimals point do not recur = one

$$\therefore 15.0\overline{2} = \frac{1502 - 150}{10^2 - 10^1} = \frac{1352}{90} = \frac{676}{45}$$