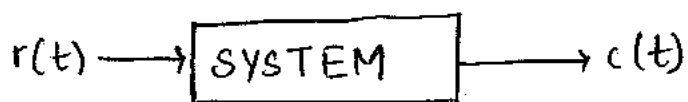


Ch-5 (R-H criteria) Stability Criteria



For stable system,

$$c(t)|_{t \rightarrow \infty} = \lim_{t \rightarrow \infty} c(t) = \text{finite} \rightarrow \text{stable}$$
$$= \infty \rightarrow \text{unstable}$$

→ For bounded i/p → output should be bounded
BIBO → stable

For an LTI system, system is said to be stable if it satisfies following conditions:-

① i/p is bounded, o/p must be bounded

② If i/p signal, system is zero the output must be zero irrespective of all initial conditions

- Stability are classified in 3 ways:-

① Conditional stable system

- Here system is stable for certain range of system component

for eg:- $0 < k < 100$ where k is parameter of system components

② Absolutely stable system

- Here the system is stable for all the values of system components marginal or limitedly stable system

- A linear time invariant system is said to be stable if for the bounded input, output maintains constant amplitude and frequency of oscillation [sustained oscillation]

- The non-repeated poles on ~~real~~^{imaginary} axis gives constant amplitude and f.o.o and system is marginal stable.

* Techniques used to check stability

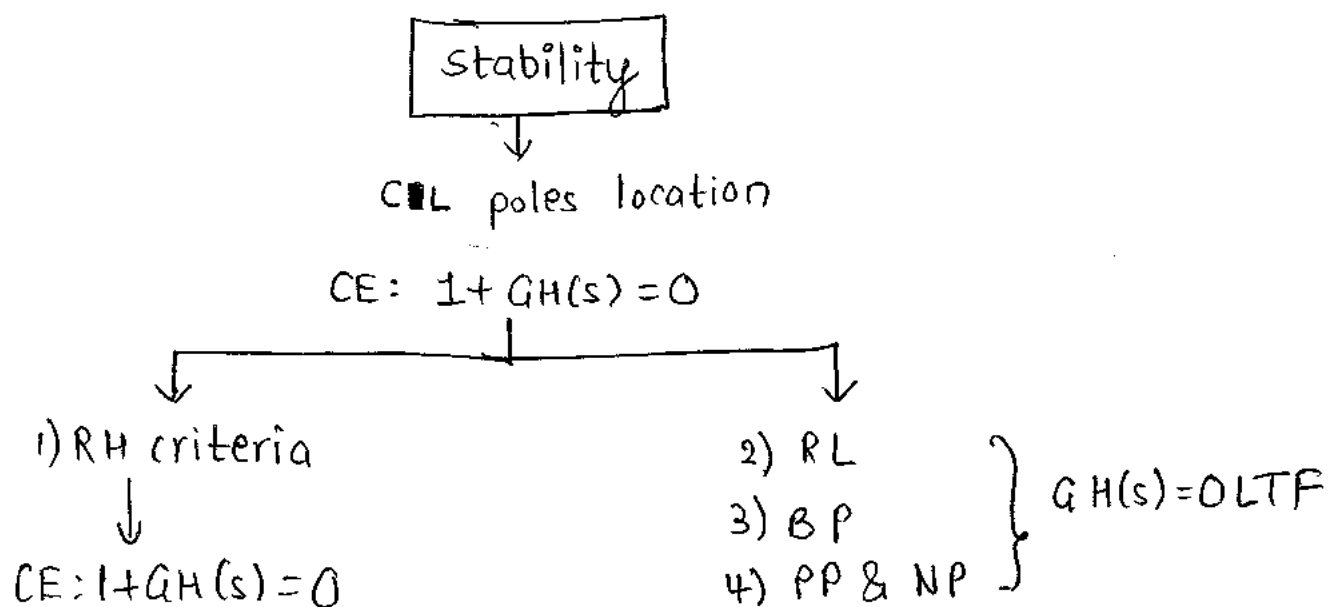
① Routh-Hurwitz criteria [RH criteria]

② Root locus

③ Bode plot

④ Polar plot and nyquist plot $\left. \begin{array}{l} f: 0 \text{ to } \infty \\ f: -\infty \text{ to } \infty \end{array} \right\} \rightarrow \text{frequency response}$

⑥ Nicholas chart



Purpose:-

- ① To find out close loop system stability
- ② To find no. of ^{cl}poles^{lies} on right, left and imaginary axis of s-plane.
- ③ The main purpose of RH criteria is to find no. of poles right hand side on s-plane.
- ④ For finding range of k value for close loop system stability.
- ⑤ To find k value
- ⑥ To become a system marginal stable or undamped system
- ⑦ To find relative stability. By using relative stability concept we can find system time constant and settling time.
- ★ ⑧ To find close loop system stability by using RH criteria required characteristics equation
i.e. $1 + GH(s) = 0$ and roots of C.E. gives close loop poles location.

From C.E.

The n^{th} order general form of C.E.

$$G_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3} + \dots + a_n s^0 = 0$$

S^n	a_0	a_2	a_4	$a_6 \dots \dots \dots$	a_n
S^{n-1}	a_1	a_3	a_5	$a_7 \dots \dots \dots$	a_{n-1}
S^{n-2}	$\frac{a_1 a_2 - a_0 a_3}{a_1}$		$\frac{a_1 a_4 - a_0 a_5}{a_1}$	$\frac{a_1 a_6 - a_0 a_7}{a_1}$	
S^{n-3}					
$\vdots$					
S^0	a_n				

* Conditions for system stability

- A system is said to be stable if all the co-efficients in first column of RH table have same sign and no co-efficient is zero.
- The no. of sign changes in 1st column $\hat{=}$ no. of close loops poles lies in right hand side of s-plane & system becomes unstable.

Q:- check the system stability for following using RH criteria of characteristics eqⁿ given:-

① $as + b = 0$

s^1	a	$+$	$-$
s^0	b	$+$	$-$

→ System is stable if $a > 0, b > 0$ OR $a < 0, b < 0$

② $as^2 + bs + c = 0$

s^2	a	c	$+$	$-$
s^1	b		$+$	$-$
s^0	c		$+$	c

→ System is stable if $a, b, c > 0$ OR $a, b, c < 0$

★
③★ $as^3 + bs^2 + cs + d = 0$
★

s^3	a	c	$+$	
s^2	b	d	$+$	
s^1	$bc - ad$		$+$	
s^0	d	$-$	$+$	

→ System is stable if $a, b, d > 0$

$$\frac{bc - ad}{b} > 0$$

$cb > ad \rightarrow s$

$cb < ad \rightarrow \text{US}$

$cb = ad \rightarrow \text{MS} \rightarrow \text{then there will be f.o.o.}$

If it is marginal stable system. then we have to also find frequency of oscillation.

$$\boxed{cb=ad} \rightarrow \textcircled{\text{MS}}$$

$$bs^2 + ds = 0$$

$s^2 \rightarrow$ column n see

$$s^2 = -\frac{d}{b}$$

$$s = \pm j \sqrt{\frac{d}{b}}$$

$$j\omega_n = \pm j \sqrt{\frac{d}{b}}$$

$$\boxed{\omega_n = \sqrt{\frac{d}{b}}} \text{ rad/sec.}$$

(outer product) ad Short-cut

$$as^3 + bs^2 + cs + d = 0$$

cb (Inner product)

$$cb > ad \rightarrow \textcircled{\text{S}}$$

$$cb < ad \rightarrow \textcircled{\text{US}}$$

$$cb = ad \rightarrow \textcircled{\text{MS}}$$

For stable
Inner product > Outer product
co-efficient co-efficient

For f.o.o.,

$$\rightarrow bs^2 + d = 0$$

$$\textcircled{4} s^3 + 8s^2 + 4s + 32 = 0$$

s^3	1	4
s^2	8	32
s^1	0	4
s^0	32	

$\textcircled{\text{MS}}$

$$8s^2 + 32 = 0$$

$$s^2 = -4$$

$$s = \pm j2$$

$$j\omega = \pm j2$$

$$\boxed{\omega = 2 \text{ rad/sec.}}$$

$$(7) s^2 + 10s + 10 = 0$$

$$\begin{array}{c|cc} s^2 & 1 & 10 \\ s^1 & 10 & \\ s^0 & 10 & \end{array}$$

(stable)

$$(5) s + 10 = 0$$

$$\begin{array}{c|cc} s^1 & 1 & + \\ s^0 & 10 & + \end{array}$$

(Stable)

$$(6) s^2 + 25 = 0$$

$$\begin{array}{c|cc} s^2 & 1 & 25 \\ s^1 & 0 & \\ s^0 & 25 & \end{array}$$

(ms)

$\omega = 5 \text{ rad/sec.}$

$$(8) s^3 + 25s^2 + 8s + 10 = 0$$

$$25 \cdot 8 > 10$$

$$200 > 10 \rightarrow \text{(stable)}$$

$$(9) s^4 + 2s^3 + 3s^2 + 2s + 1 = 0$$

$$\begin{array}{c|cccc} s^4 & 1 & 3 & 1 & \\ s^3 & 2 & 2 & & \\ s^2 & 2 & 1 & & \\ s^1 & 1 & & & \\ s^0 & 1 & & & \end{array}$$

(stable)

$$(10) s^4 + 2s^3 + 3s^2 + s + 2 = 0$$

s^4	1	3	2	+
s^3	2	1		+
s^2	$5/2$	2		+
s^1	$-3/5$			-
s^0	2			+

unstable

No. of sign changes = 2 = No. of poles on R.H.S.

$$(11) s^4 + 2s^3 + 2s^2 + 4s + 8 = 0$$

Difficulty

s^4	1	2	8	+
s^3	2	4		+
s^2	$\emptyset \rightarrow \epsilon$	8		+
s^1	$\frac{4\epsilon - 16}{\epsilon}$			-
s^0	8			+

Difficulty no. ①

- Whenever any one element is zero in 1st column then replace that zero by smallest +ve constant ϵ and continue Routh's stability.

- finally for $\epsilon \rightarrow 0$ check no. of sign changes

$$\lim_{\epsilon \rightarrow 0} \frac{4\epsilon - 16}{\epsilon} = \lim_{\epsilon \rightarrow 0} 4 - \frac{16}{\epsilon} = -\infty$$

No. of sign changes = 2 = no. of poles on RHS

unstable

$$(12) s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15 = 0$$

s^5	1	2	3	+
s^4	1	2	15	+
s^3	0(ϵ)	-12		+
s^2	$\frac{2\epsilon+12}{\epsilon}$	15		+
s^1	A			-
s^0	15			+

unstable

$$A = \frac{-12(\epsilon+6) - 15\epsilon}{\epsilon+6}$$

No. of sign changes = 2 = no. of poles lies on R.H.S. of s-plane

$$\lim_{\epsilon \rightarrow 0} \frac{2\epsilon+12}{\epsilon} = +\infty$$

$$\lim_{\epsilon \rightarrow 0} \frac{-12(\epsilon+6) - 15\epsilon}{\epsilon+6} = \frac{-12(6)}{6} = -12$$

$$(13) s^5 + s^4 + 3s^3 + 3s^2 + 2s + 2 = 0$$

s^5	1	3	2
s^4	1	3	2
s^3	0(ϵ)	0(ϵ)	
s^2			
s^1			
s^0			

Difficulty no: -2

→ rows of zeros

- Whenever in the RH table row of zeros occurs we require to form an auxiliary eqⁿ by using row above the row of zero and differentiate the A.E. and replace zeros by co-efficient by differentiated A.E. and continue Routh's stability.

s^5	1	3	2
s^4	1	3	2
s^3	$\cancel{0}^4$	$\cancel{0}^6$	
s^2	$\frac{3}{2}$	2	
s^1	$\frac{2}{3}$		
s^0	2		

→ A.E. (auxiliary equation)

→ row of zeros

$$A.E. = s^4 + 3s^2 + 2 \Rightarrow 4^{\text{th}} \text{ order}$$

4 poles are symmetrical @ origin.

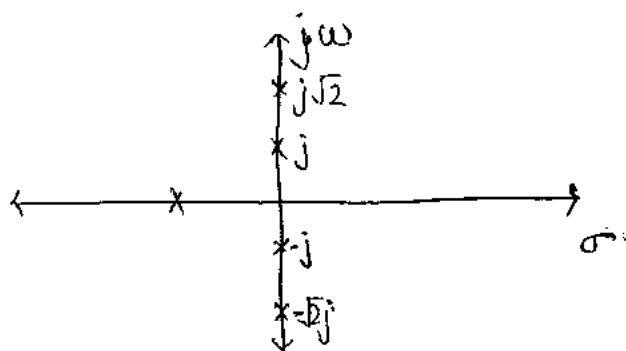
$$\frac{dA.E.}{ds} = 4s^3 + 6s$$

$$A.E. : s^4 + 3s^2 + 2 = 0$$

$$(s^2 + 2)(s^2 + 1) = 0$$

$$s^2 + 2 = 0 ; s^2 + 1 = 0$$

$$s = \pm j\sqrt{2} ; s = \pm j$$



MS

- In Routh's tabular form row of zero occurs means poles must be symmetrical about the origin.

- The auxiliary eqⁿ must consist of an even power of s term.
 bec. roots of A.E. must be symmetrical about origin.

Whenever in Routh's tabular form only one ROZ (rows of zeros) and all the co-efficient of 1st column are positive then system is marginal stable system because poles must lie on imaginary axis which are non-repeated.

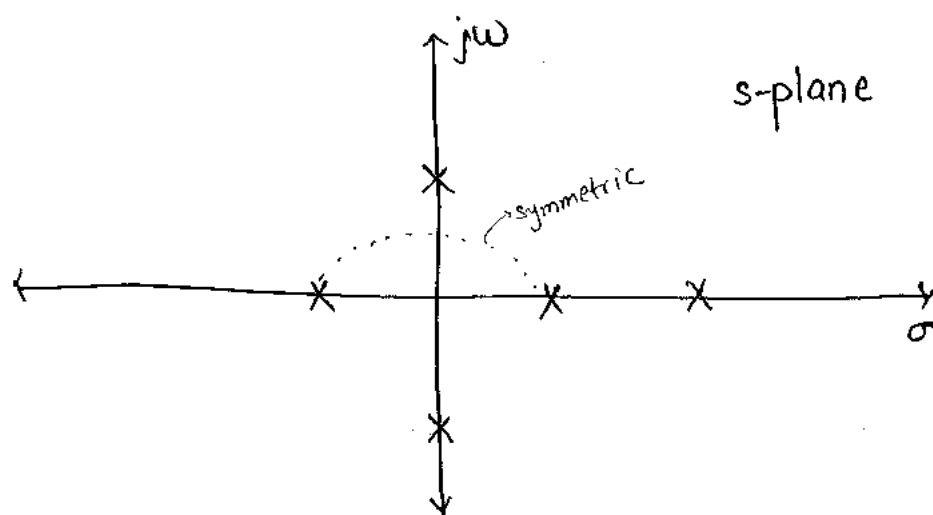
NOTE:

only once $\rightarrow 1 \rightarrow \text{ROZ} \rightarrow$ poles lie on $j\omega$ axis $\rightarrow$ non-repeated
 $\downarrow$
 marginal stable

more than 1 ROZ $\rightarrow$ poles lie on $j\omega$ axis $\rightarrow$ repeated

*Concept

s^5	-	① Non-symmetrical		$\rightarrow$ auxiliary eq ⁿ	4 th order AE
s^4	+				
s^3	0	0	0	$\rightarrow$ ROZ	4 poles are symmetrical @ origin
s^2	+				
s^1	+				
s^0	-	② Symmetrical			



NOTE:-

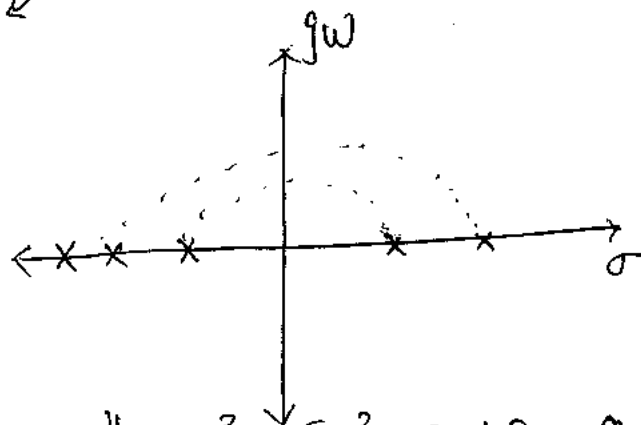
-The sign change occurs below the A.E. there must be a symmetrical pole in left to the pole placed in right

-The sign change occurs above the A.E. there must be non-symmetrical pole in ~~right~~ left to the pole placed in right.

Q:-

s^5	+	non-symmetric	
s^4	+		
s^3	0	0	0 $\rightarrow$ ROZ
s^2	+	symmetrical pole.	
s^1	-		
s^0	+		

A.E. $\therefore$ 4th order
4 poles are sym. @ origin



Q:- $s^6 + 3s^5 + 4s^4 + 6s^3 + 5s^2 + 3s + 2 = 0$

+	s^6	1	4	5	2
+	s^5	3	6	3	
+	s^4	2	4	2	
+	s^3	0	8	0	
+	s^2	2	2		
+	s^1	0	4	0	
+	s^0	2			

A.E. ① = $2s^4 + 4s^2 + 2$ 4th order
 $2s^4 + 4s^2 + 2$

$\frac{dA.E.}{ds} = 8s^3 + 8s$

② A.E. = $2s^2 + 2$ 2nd order

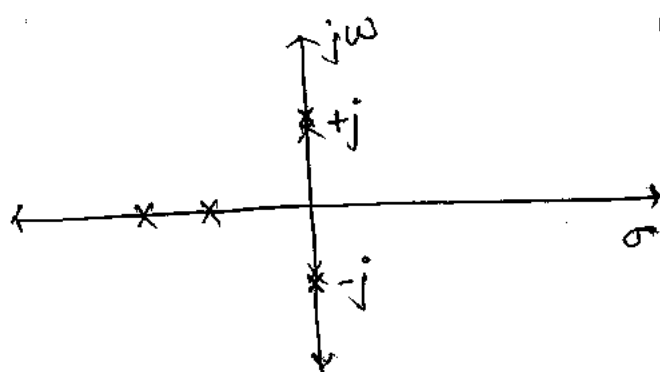
$\frac{dA.E.}{ds} = 4s$

For higher order A.E.

$\Rightarrow 4^{\text{th}}$ order

$$A.E. = 2s^4 + 4s^2 + 2 = 0$$

4^{th} order A.E. $\rightarrow$ 4 poles are symmetric



$\hookrightarrow$ 2 times repeated poles on imaginary axis bcz. 2 times Rows of zeros occurs

unstable

$$2s^4 + 4s^2 + 2 = 0$$

$$s^4 + 2s^2 + 1 = 0$$

$$(s^2 + 1)^2 = 0$$

$$s = \pm j, s = \pm j$$

-Whenever many times row of zero occurs & all the co-efficient in 1st column are positive then the system is unstable bcz. poles lies on ~~the~~ imaginary axis which are repeated.

$$Q:- s^4 + s^3 - s - 1 = 0$$

+	s^4	1	0	-1
+	s^3	1	-1	
+	s^2	1	-1	
+	s^1	1	2	
+	s^0	-1		

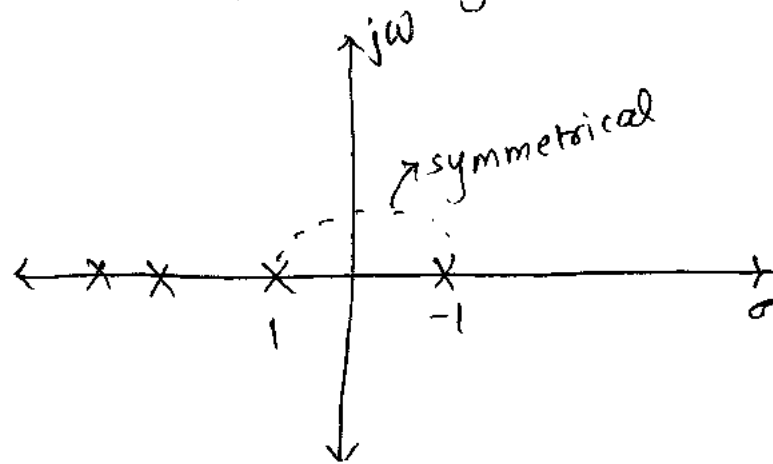
$$A.E._1 = s^2 - 1$$

$$\frac{dA.E._1}{ds} = 2s$$

$\rightarrow$ R.O.Z₁

A.E. $\rightarrow 2^{\text{nd}}$ order

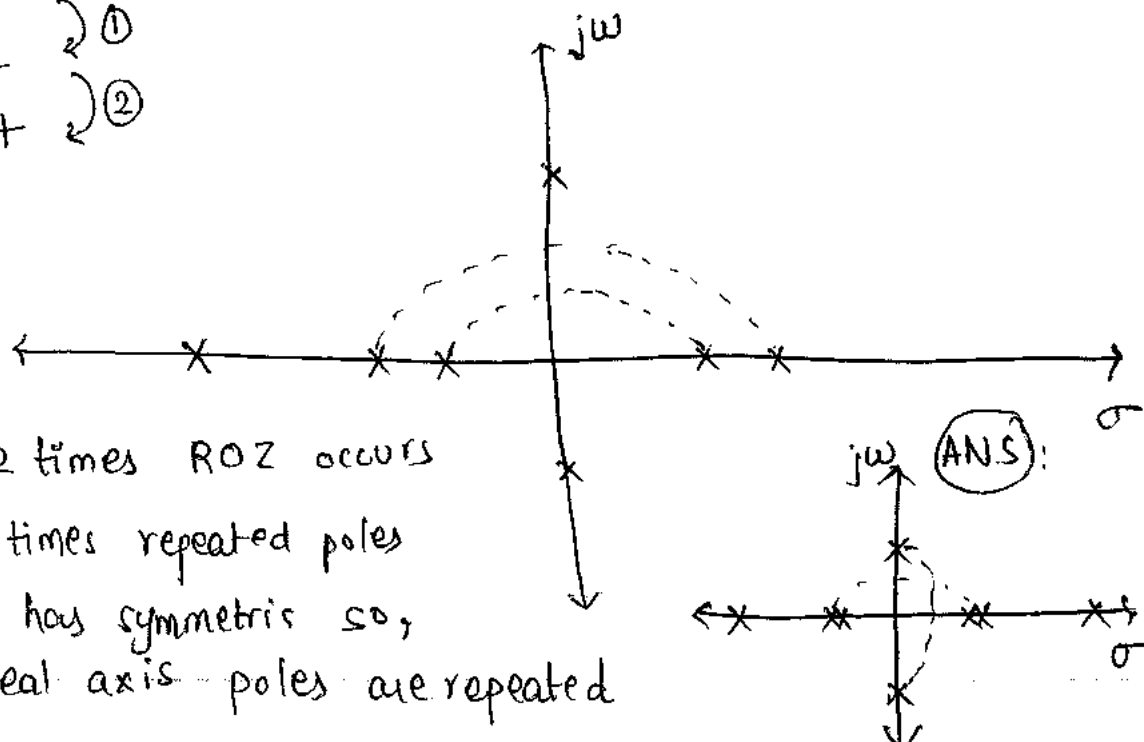
2 poles are symmetric @ origin



Q:- Identify the poles ^{lies} on real axis, imaginary axis, R.H. side & L.H. side to the given Routh's tabular form?

s^7	+				
s^6	+				
s^5	0	0	0	$\rightarrow$ ROZ ₁	
s^4	+				
s^3	0	0	$\rightarrow$ ROZ ₂		
s^2	+				
s^1	-				①
s^0	+				②

A.E.₁ = $s^6 \rightarrow 6^{\text{th}}$ order



Now, 2 times ROZ occurs

s^0 , 2 times repeated poles

has symmetric so,

on real axis poles are repeated

RHS $\rightarrow 2$

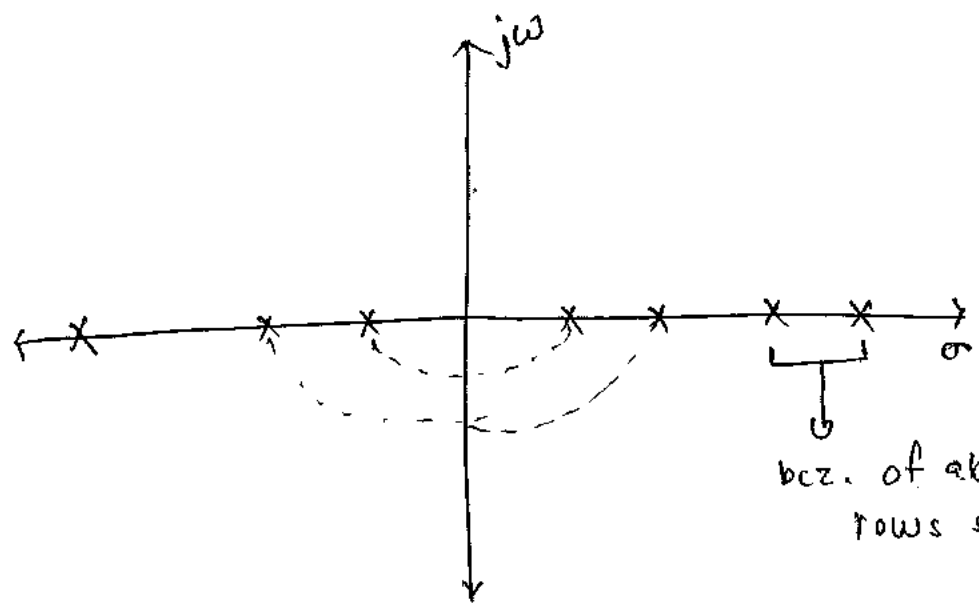
LHS $\rightarrow 3$

$j\omega$ axis $\rightarrow 2$

Q:-

s^7	+			
s^6	-			
s^5	+			
s^4	+			
s^3	0	0		
s^2	+			
s^1	-			
s^0	+			

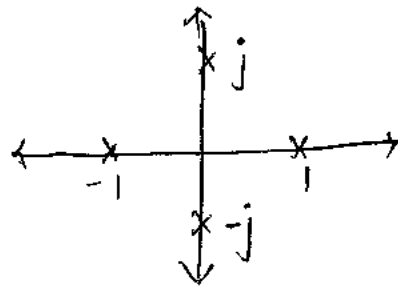
$s^4 \rightarrow 4^{\text{th}}$ order
4 poles are symmetric
w.r.t. origin



bcz. of above a.e.
rows sign changes

R.H.S. = 4
L.H.S. = 3
 $j\omega = 0$

Q:- Identify Routh's tabular form to given poles location in s-plane.



Sol:- 4 poles are symmetrical @ origin

- 4th order A.E.

- 1 time sign changes below ROZ

- No sign changes above ROZ

- 1 ROZ

s^4	+
s^3	0 0
s^2	+
s^1	+
s^0	-

Conventional method:-

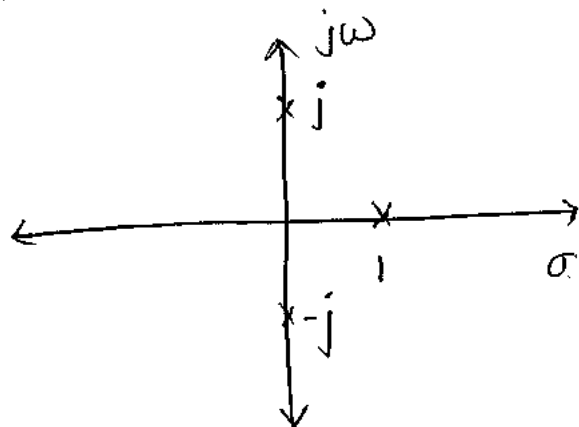
$$CE: (s+1)(s-1)(s^2+1) = 0$$

$$(s^2-1)(s^2+1) = 0$$

$$s^4 - 1 = 0$$

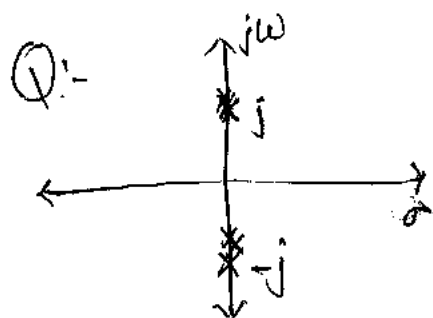
$+s^4$	1	0	-1	→ A.E.
$+s^3$	0	0	0	→ ROZ
$+s^2$	0	-1		
$+s^1$	$\frac{4}{E}$			
$-s^0$	-1			

Q:- Fill in the blank:



2 poles are symmetrical @ origin
2nd order of AE
0 time sign changes below AE
1 time sign changes above ROZ
1 ROZ

s^3	+
s^2	-
s^1	0
s^0	-



6 poles are symmetric @ origin
6th order of AE
0 time sign changes below AE
0 time sign changes above ROZ
3 ROZ

s^6	+
s^5	0 0 0 → ROZ
s^4	+
s^3	+
s^2	+
s^1	+
s^0	+

(unstable)

$$(s^2 + 1)^3 = 0$$

$$s^6 + 1 + 3s^4 + 3s^2 = 0$$

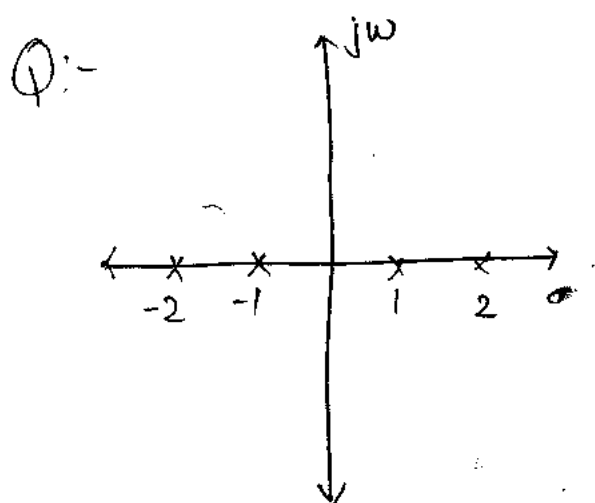
$$s^6 + 3s^4 + 3s^2 + 1 = 0$$

s^6	1	3	3	1	→ ROZ ₁
s^5	6	12	6	1	
s^4	1	2	1	1	
s^3	4	4	1	1	→ ROZ ₂
s^2	1	1	1	1	
s^1	2	1	1	1	→ ROZ ₃
s^0	1	1	1	1	

A.E. = 6th order

↓
6 poles are symmetrical @ origin

- 4 poles are symmetric about origin
- 4 order of A.E.
- 2 time sign changes below A.E.
- 0 time sign changes above ROZ
- 1 ROZ



unstable

Q:- Find the range of k value for system stability.
Find the k value for marginal system or undamped system. Find natural value of oscillation when system is marginal stable.

① $s^3 + 8s^2 + 4s + k = 0$

$+ s^3$	1	4
$+ s^2$	8	k
$+ s^1$	$\frac{32-k}{8}$	.
$+ s^0$	k	.

For stable system,

① $k > 0$	}	⑤ $0 < k < 32$
② $\frac{32-k}{8} > 0$		⑥ $k < 0 \text{ \& } k > 32$
$\Rightarrow k < 32$		⑦ $\frac{32-k}{8} = 0 \quad k_{\text{mar.}} = 32$

$$8s^2 + k_{mar.} = 0$$

$$8s^2 = -32$$

$$s^2 = -4$$

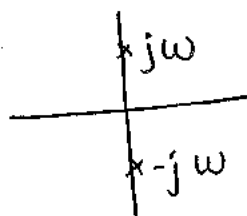
$$s = \pm 2j$$

$$j\omega = \pm j2$$

$$\boxed{\omega_n = 2 \text{ rad/sec.}}$$

$$s = \sigma + j\omega$$

$$s = j\omega$$



For sinusoidal, put $\sigma = 0$

$$\therefore s = j\omega$$

$$\textcircled{2} \quad 2s^3 + 5s^2 + 10s + (k+5) = 0$$

s^3	2	10
s^2	5	$k+5$
s^1	$\frac{50-2k-10}{5}$	
s^0	$k+5$	

For stable, $\frac{50-10-2k}{5} > 0$

$$-5 < k < 20$$

$$k+5 > 0$$

$$k > -5$$

$$40 > 2k$$

$$k < 20$$

For marginal stable,

$$50-2k-10=0$$

$$40-2k=0$$

$$2k=40$$

$$\boxed{k_{ma.} = 20}$$

For unstable system,

$$\frac{40-2k}{5} < 0, \text{ } k+5 \text{ be any value}$$

$$40 < 2k$$

$$k > 20$$

$$5s^2 + k+5 = 0$$

$$5s^2 + 20+5 = 0$$

$$5s^2 = -25$$

$$s^2 = -5$$

$$s = \pm \sqrt{5}j$$

$$j\omega_n = \pm \sqrt{5}j$$

$$\omega_n = +\sqrt{5} \text{ rad/sec.}$$

Q:- Repeat the above problem for finding k for system stability

$$GH(s) = \frac{k}{s(s+2)(s+4)(s+6)}$$

C.E. :- $1 + GH(s) = 0$

$$1 + \frac{k}{s(s+2)(s+4)(s+6)} = 0$$

$$s(s+2)(s+4)(s+6) + k = 0$$

$$(s^2+2s)(s^2+10s+24) + k = 0$$

$$s^4 + 10s^3 + 24s^2 + 2s^3 + 20s^2 + 48s + k = 0$$

$$s^4 + 12s^3 + 44s^2 + 48s + k = 0$$

s^4		1	44	k
s^3		12	48	
s^2		40	k	
s^1		$\frac{1920-12k}{40}$		
s^0		k		

$$k > 0 \quad 1920 - 12k > 0$$

$$12k < 1920$$

$$k < 160$$

For stable $0 < k < 160$

For ms, $k_n = 160$

$$40s^2 + k = 0$$

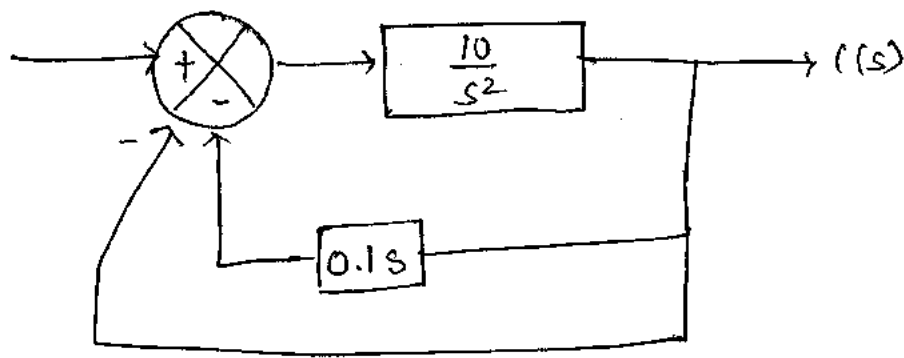
$$s^2 = -4$$

$$s = \pm 2j$$

$$j\omega_n = +2j$$

$$\omega_n = 2 \text{ rad/sec.}$$

Q:- Check the system stability for given block diagram



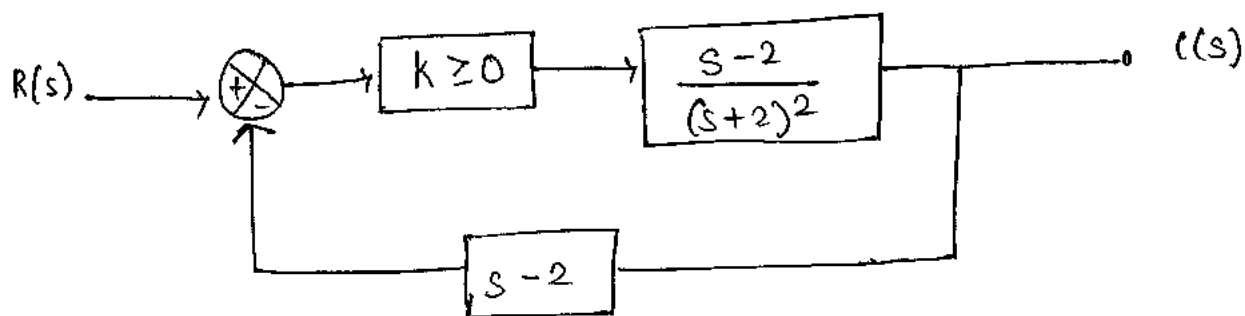
$$\Delta = 1 - \left[-\frac{10}{s^2} \cdot 0.1s - \frac{10}{s^2} \right] = 0$$

$$\Delta = 1 + \frac{1}{s} + \frac{10}{s^2} = 0 \rightarrow \text{C.F.}$$

$$s^2 + s + 10 = 0$$

s^2		1	10		stable
s^1		1			
s^0		10			

Q:- Find the range of k values to the given system for system stability.



$$\Delta = 1 + \frac{s-2}{(s+2)^2} (s-2) K = 0$$

$$(s+2)^2 \pm + K (s^2 - 4s + 4) = 0$$

$$s^2 + 4s + 4 + s^2 k - 4ks + 4k = 0$$

$$\cancel{s^2 k - 4ks + (4k+1)} = 0$$

$$s^2 - 4s + \frac{4k+1}{k} = 0$$

$$\begin{array}{c|cc} s^2 & 1 & 4 + 1/k \\ s^1 & -4 & \\ s^0 & 4 + 1/k & \end{array}$$

$$s^2(1+k) + 4(1-k)s + 4(k+1)$$

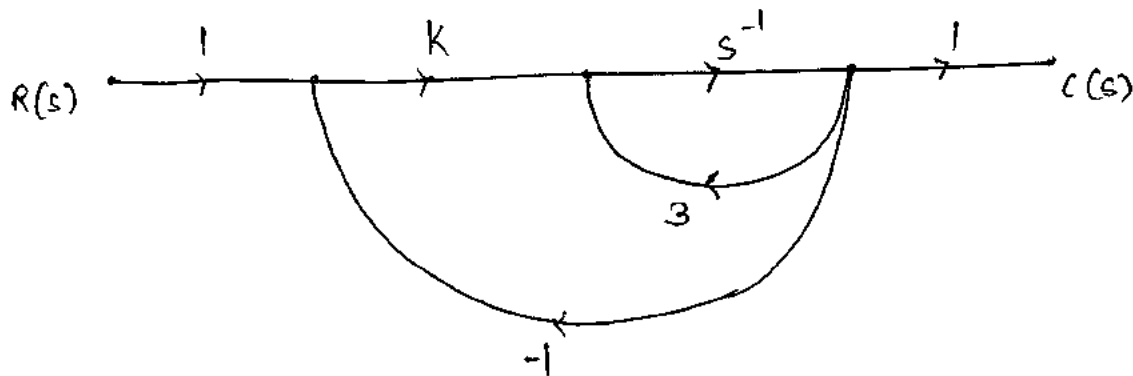
$$\begin{array}{c|cc} + s^2 & 1+k & 4(k+1) \\ + s^1 & 4(1-k) & \\ + s^0 & 4(k+1) & \end{array}$$

For stable,

$$\therefore \boxed{0 \leq k < 1}$$

$$\begin{array}{c|cc} 1+k > 0 & 4(1-k) > 0 & 4(k+1) > 0 \\ k > -1 & \times & 1 > k & k > -1 \\ & & k < 1 & \times \end{array}$$

Q:-



$$\Delta = 1 - 3s^{-1} + ks^{-1} = 0$$

$$1 - \frac{3}{s} + \frac{k}{s} = 0$$

$$s - 3 + k = 0$$

$$\begin{array}{c|c} + s^1 & 1 \\ + s^0 & k-3 \end{array}$$

For stable:-

$$\boxed{k > 3}$$

Q:- Find the k and b values so that $G_H(s) = \frac{k(s+1)}{s^3 + bs^2 + 3s + 1}$ oscillates with frequency 2 rad/s .

CE: $1 + G_H(s) = 0$

$$1 + \frac{k(s+1)}{s^3 + bs^2 + 3s + 1} = 0$$

$$s^3 + bs^2 + 3s + 1 + k(s+1) = 0$$

$$s^3 + bs^2 + (3+k)s + 1+k = 0$$

s^3	1	$3+k$
s^2	b	$1+k$
s^1	$\frac{3b+k b - 1 - k}{b}$	
s^0	$1+k$	

for f.o.o.

$$bs^2 + 1+k = 0$$

$$s^2 = -\frac{(1+k)}{b}$$

$$s = j\sqrt{\frac{k+1}{b}}$$

$$j\omega = j\sqrt{\frac{k+1}{b}}$$

$$\omega = \sqrt{\frac{k+1}{b}}$$

$$2 = \sqrt{\frac{k+1}{b}}$$

$$b4 = k+1$$

$$\boxed{b = \frac{k+1}{4}}$$

Now, for marginal stable

Inner product of CE = Outer product of CE

$$b(3+k) = 1+k$$

$$b = \frac{1+k}{3+k}$$

$$\frac{k+1}{4} = \frac{1+k}{3+k}$$

$$\boxed{k = 1}$$

$$\boxed{b = 1/2}$$

Q:- The loop gain of the system is

$GH(s) = \frac{k}{s(s+1)(s+2)}$. The value of k for which system just becomes unstable is _____.

$$1 + GH(s) = 0$$

$$1 + \frac{k}{s(s+1)(s+2)} = 0$$

$$s(s+1)(s+2) + k = 0$$

$$(s^2 + s)(s+2) + k = 0$$

$$s^3 + s^2 + 2s^2 + 2s + k = 0$$

$$s^3 + 3s^2 + 2s + k = 0$$

I.P. = O.P. (marginal)

$$\boxed{6 = k}$$

Q:- A system has $GH(s) = \frac{k}{s^3 + 8s^2 + 4s}$. For what value of k the system will produce continuous oscillation.

(sustained oscillation / continuous / damped oscillation $\rightarrow$ m.s.)

$$C.E. = 1 + GH(s) = 0$$

$$1 + \frac{k}{s^3 + 8s^2 + 4s} = 0$$

$$s^3 + 8s^2 + 4s + k = 0$$

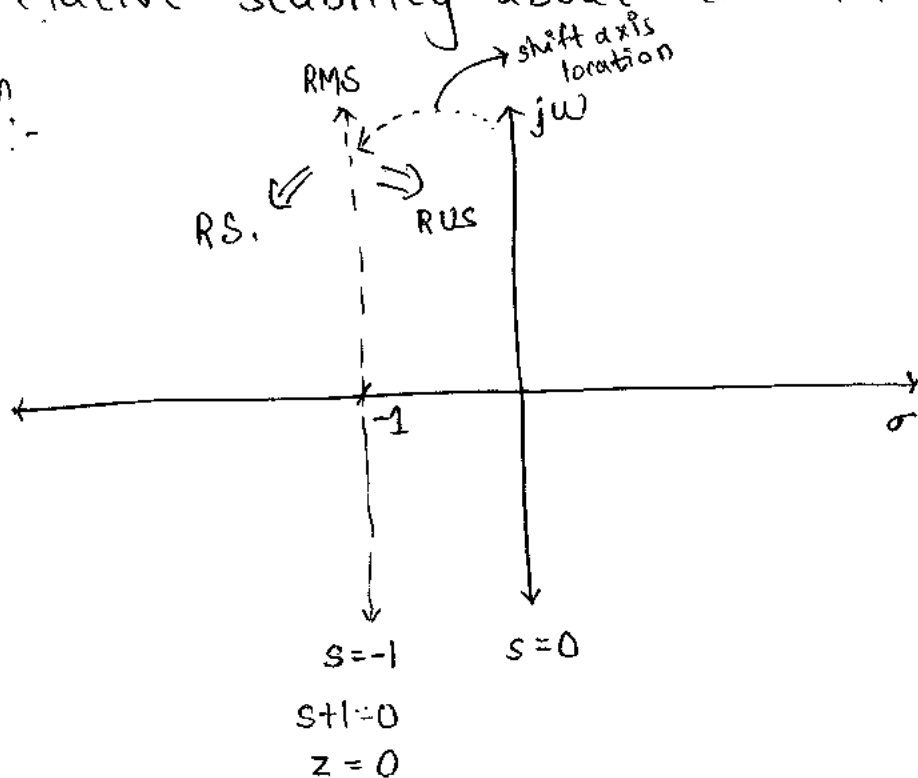
$$\boxed{k = 32} \quad (I.P. = O.P.)$$

*Relative stability

It is applicable for stable system. only. A system has $G_H(s) = \frac{2}{s(s+1)(s+2)}$ and $H(s) = 1$ Determine it's

relative stability about the line $s = -1$.

Sol:-



$$s = -1$$

$$s+1 = 0$$

$$z = s+1$$

$$G(s) = \frac{2}{s(s+1)(s+2)}$$

$$\text{C.E. : } 1 + G_H(s) = 0$$

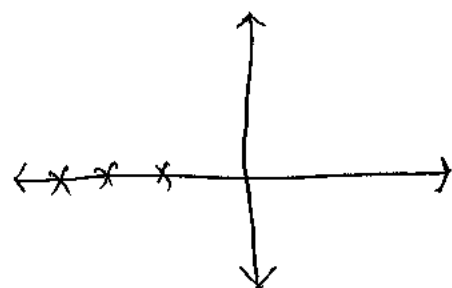
$$1 + \frac{2}{s(s+1)(s+2)} = 0$$

$$s^3 + 3s^2 + 2s + 2 = 0$$

$$\begin{array}{r|rr} +s^3 & 1 & 2 \\ +s^2 & 3 & 2 \\ +s^1 & 4/3 & \\ +s^0 & 2 & \end{array}$$

w.r.t. $s = 0$

stable



w.r.t. $z=0$

$$G_H(z) = ?$$

$$\text{Put } s = z - 1$$

$$G(z) = \frac{2}{(z-1)z(z+1)} = \text{OLTF}$$

$$H(z) = 1$$

$$\text{CE: } 1 + G_H(z) = 0$$

$$1 + \frac{2}{z(z-1)(z+1)} = 0$$

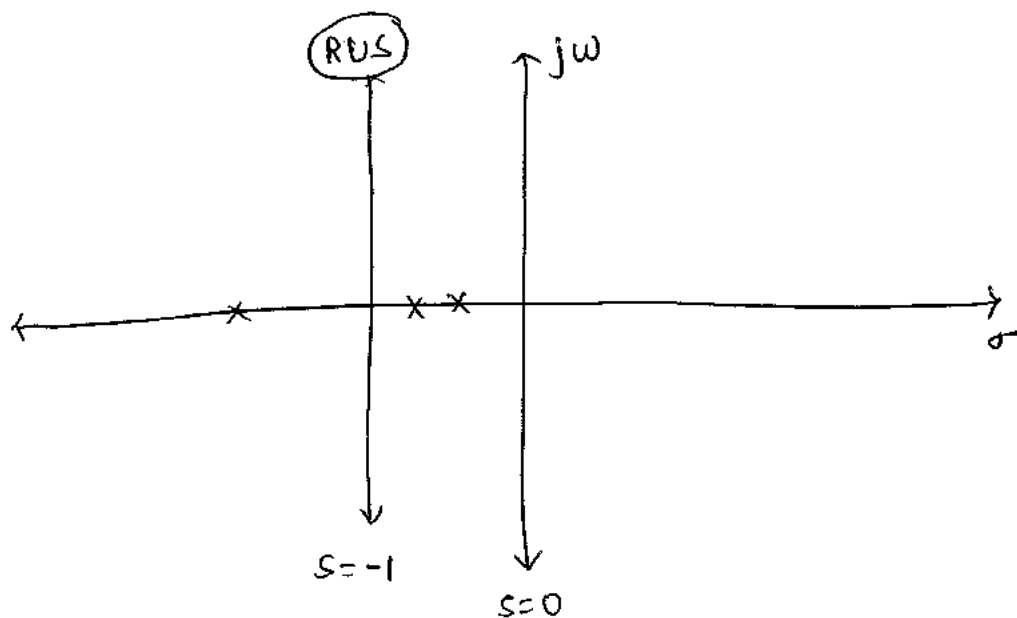
$$z(z^2 - 1) + 2 = 0$$

$$z^3 - z + 2 = 0$$

$$\begin{array}{r|rr} + & z^3 & 1 & -1 \\ + & z^2 & 0 & 2 \\ - & z^1 & -\frac{2}{\epsilon} & \frac{2}{\epsilon} \\ + & z^0 & 2 & \end{array}$$

unstable but relatively

2 poles R.H.S. $z=0$
 $s=-1$



Limitations:-

1. The exact location of poles can't be determined
2. RH criteria is not applicable for exponential, sine or cosine term because it gives infinite series
3. RH criteria is applicable to finite no. of terms

NOTE:-

By using RH criteria, we can get approximate solⁿ of the exponential term.

Q:- Find the value of k for the system stability to the given system.

$$GH(s) = \frac{k e^{-s\tau}}{s(s+1)}$$

$$CE: 1 + GH(s) = 0$$

$$1 + \frac{k e^{-s\tau}}{s(s+1)} = 0$$

$$s(s+1) + k e^{-s\tau} = 0$$

$$\text{From series expansion: } e^{-s\tau} = 1 - s\tau + \frac{(s\tau)^2}{2!} - \frac{(s\tau)^3}{3!} + \dots$$

Neglecting higher order terms

$$s^2 + s + k[1 - s\tau] = 0$$

$$s^2 + s(1 - k\tau) + k = 0$$

$$\begin{array}{c|cc} s^2 & 1 & k \\ s^1 & 1 - k\tau & \\ s^0 & k & \end{array}$$

$$1 - k\tau > 0, \quad k > 0$$

$$1 > k\tau$$

$$k < \frac{1}{\tau}$$

for stable system:

$$0 < k < 1/\tau$$