CHAPTER 08

Design for Torsion in Reinforced Concrete

8.1 Introduction

It is another important *limit state of collapse* just like shear and flexure. Torsion invariably occurs in most of the type of structures. In this chapter, we will define the two different types of torsion that exist in actual structures, the approach followed for the design of members subjected to combined bending/flexure, shear and torsion along with the concept of equivalent moment and equivalent shear.

8.2 Design for Torsion

Loads acting normal to the plane of bending in case of beams and slabs, gives rise to bending moments and shear force. However, when the loads act away from the plane of bending at an eccentricity then this will induce torsion in members. In reinforced concrete, the state of pure torsion rarely exists unlike the case of shafts where we encounter pure torsion. It generally occurs in combination with transverse shear and flexure. Behavior of reinforced concrete members under the combined influence of flexure, shear and torsion is quite complex owing to the fact that concrete is a non-homogeneous material. Presence of cracks in concrete members and composite nature of material i.e. concrete and steel adds to the complexity of torsion analysis. Research in this area is still going on. However, various design codes in the world describe very simplified procedure for torsion design of reinforced concrete members.

Torsional Moment v/s Flexural Moment

FLEXURAL MOMENT	TORSIONAL MOMENT
Flexural/bending moments are distributed among the connected members according to their distribution factors which are proportional to flexural stiffness $\frac{EI}{L}$.	Torsional moments are distributed among the connected members according to their distribution factors which are proportional to torsional stiffness $\frac{GJ}{L}$.

8.3 Mechanism of Torsion in Reinforced Concrete Structures

There are many mechanisms by which torsion can be induced in reinforced concrete members during load transfer stage (during the process of load transfer) in the structural system. Depending upon the situations which induce torsion in reinforced concrete, torsion can be classified as:

1. Equilibrium torsion (or primary torsion)
2. Compatibility torsion (or secondary torsion)

1. **Equilibrium Torsion:** This torsion gets induced in a reinforced concrete member due to eccentric loading w.r.t. shear centre of the cross section. Here, the equilibrium conditions are sufficient in itself to determine the twisting/torsional moments especially **in statically determinate structures**. This torsional component **must be considered in design (as per Cl. 41.4 of IS: 456-2000)** as it is a major component in statically determinate structures. Also, the ends of the member should be suitably designed to resist this type of torsion.

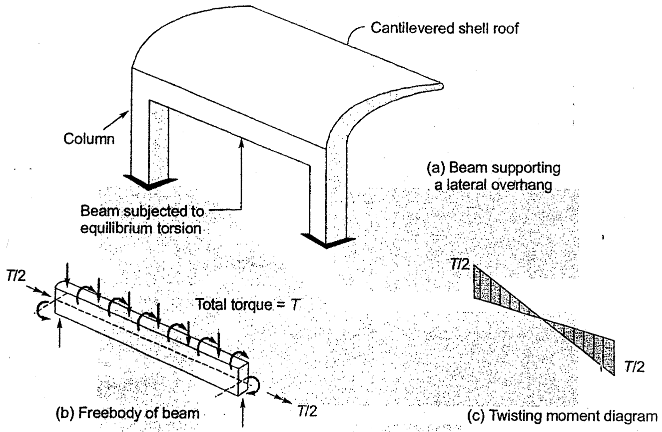


Fig. 8.1 *Concept of Equilibrium Torsion*

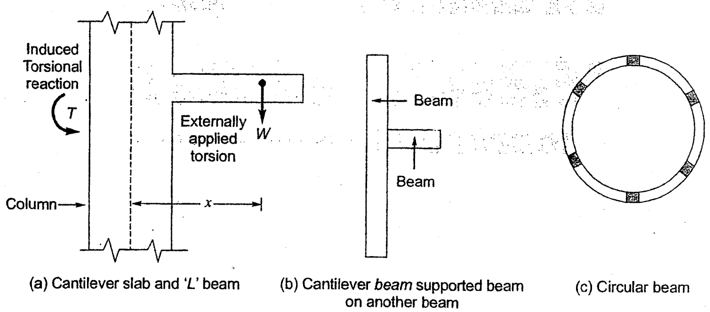


Fig. 8.2 *Simplified explanation of equilibrium torsion*

2. **Compatibility Torsion:** This type of torsion arises due to need of the member to undergo a certain angle of twist to maintain the compatibility conditions. Here, the twisting moments developed are dependent on torsional stiffness of the member and these twisting moments are **statically indeterminate**, minor amounts of which **can be ignored in design due to multiple load paths available in statically indeterminate structures.**

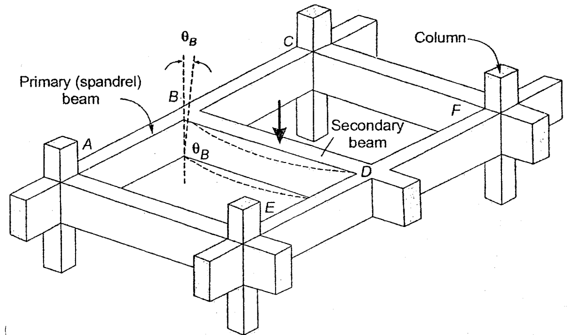


Fig. 8.3 *Concept of Compatibility Torsion*

IS 456: 2000 (Cl. 41.1) states that where the torsion can be eliminated by releasing the redundant restrains, there, no specific design for torsion is necessary provided torsional stiffness is not taken into the analysis of internal stresses.

Remember Just like *Rupture Moment* for flexural moment, twisting moment (or torsional moment) is associated with ***Cracking Torque*** which implies after the first time loading of plain concrete member, the cracks that develop prior to torsional cracking. A minimum torsional reinforcement is always provided in reinforced concrete members in order to control cracks and impart ductility to the member. Minimum stirrups requirement as per **Cl. 26.5.1.6 of IS 456: 2000** also reinforces the fact that some degree of torsional cracking can be controlled in concrete members due to compatibility torsion.

8.4 Plain Concrete Subjected to Torsion

From the principles of **Solid Mechanics**, it is known that torsion induces shear stresses and causes warping in non-circular sections. Fig. 8.5 shows the distribution of torsional shear stresses over a rectangular cross section which follows linear elastic behavior.

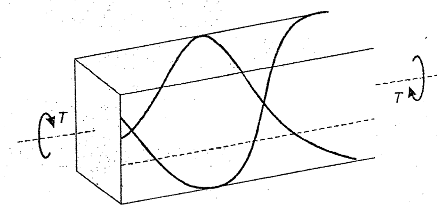


Fig. 8.4 *Plain concrete acted upon by torsional moment*

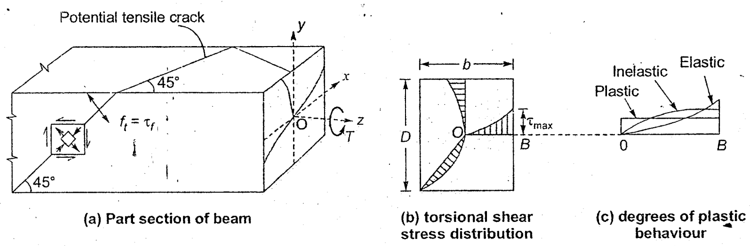


Fig. 8.5 *Distribution of Torsional Shear Stress in Elastic Rectangular Beam Section*

Maximum torsional shear stress occurs at the middle portion of the larger face of the section and is given by:

$$\tau_{tor,\,max} = \frac{T}{\alpha D b^2}$$

where T is the applied torsional moment, b and D are the cross section dimensions and α is a constant which depends on the ratio D/b.

Pure shear induces diagonal tensile and diagonal compressive stresses.

As shown in **Fig. 8.5**, principle tensile and compressive stress paths spiral around the beam in orthogonal directions at 45 degrees to the beam axis. When this diagonal tensile stress reaches the tensile strength of concrete then cracks start appearing on the section. Due to brittle nature of concrete, this crack penetrates rapidly in the inward direction from the exposed surface which nullifies the torsional resistance of the member.

Thus in plain concrete members, the diagonal torsional cracking leads to the failure of the entire section almost immediately.

The ultimate torsional resistance of the plain concrete can be assessed by measuring cracking torque (T_{cr}). The expression for the cracking torque (T_{cr}) can be computed from any of the following proposed theories viz.:

1. Elastic theory
2. Plastic theory
3. Skew bending theory
4. Theory of equivalent tube analogy

The cracking torque (T_{cr}) computed from any of the theory has to be verified and correlated experimentally with the actual tensile strength of concrete. IS 456: 2000 has adopted the **design shear strength of concrete (τ_c) (table 19 of IS 456: 2000)** as the measure of tensile strength of concrete.

8.5 Torsionally Reinforced Concrete Subjected to Torsion

As discussed in the previous section, failure of plain concrete in torsion occurs due to the diagonal tensile stresses and thus to prevent this failure, steel should be provided in the form of spirals around the member in the direction of principle tensile stresses.

Thus **to prevent a beam against torsional failure, torsional reinforcement should be provided in spirals along the direction of principal tensile stress which is in fact not a practical solution**. So, torsional reinforcement is provided in the form of longitudinal reinforcement (as longitudinal bars) and transverse reinforcement (as stirrups).

The twisting behaviour of torsionally reinforced concrete beam is similar to that of plain concrete beam until the formation of first crack (which corresponds to **torsional cracking moment T_{cr}**). After the occurrence of first crack, there is a large increase in twist at constant torque due to abrupt loss of torsional stiffness. After this the torsional behavior of reinforced concrete member depends on the amount of torsional reinforcement present.

Very small amount of torsional reinforcement may not be able to prevent the torsional cracks and thus no increase in strength is possible beyond T_{cr}. As the reinforcement is increased, torsional strength increases but this cannot be done indefinitely because ultimate failure occurs by crushing of concrete. Increasing **the torsional reinforcement increases the ductile behavior but this is felt at very large angle of twists.**

Consequences of Torsional Moment

1. Torsional moments are associated with shear stresses in beams since torsion causes diagonal tension thereby leading to spiral cracks.
2. Due to spiral cracks, reinforcement should be provided in the form of spirals along the direction of principal tensile stresses but this is often not possible. Thus the requirement of spiral reinforcement is converted to an equivalent longitudinal and transverse reinforcement. **Thus effect of torsion is split into equivalent shear and equivalent moment.**

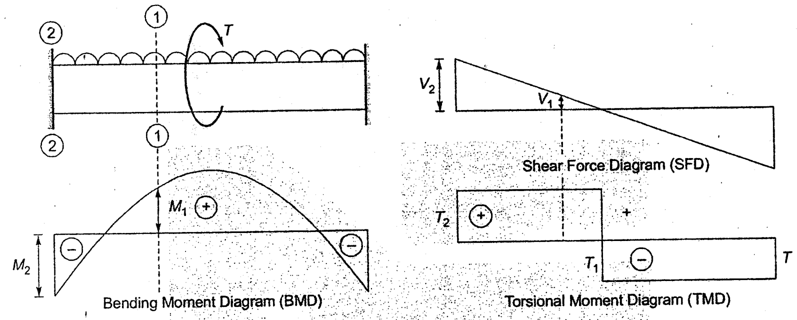


Fig. 8.6 *Variation of BM, SF and TM diagram in a fixed beam*

8.6 Analysis for Torsion

There are many methods available to understand the behavior of reinforced concrete members under torsion like **Skew Bending Theory, Space-Truss Analogy** etc.

8.7 Torsional Reinforcement

When torsional shear stress (τ_t) exceeds the design shear strength of concrete (τ_c) **(table 19 of IS 456: 2000)**, then torsional reinforcement is required to be provided in concrete members. As stated earlier, normally torsional shear acts in association with flexural shear (V_u) and in that case **equivalent shear (V_e)** has to be considered as per **Cl. 41.3.1 of IS 456: 2000.**

The expression for equivalent shear (V_e) is given as:

$$V_e = V_u + 1.6\left(\frac{T_u}{b}\right)$$

Remember: The flexural shear and torsional shear are additive only on one side of the beam and they act in opposite directions on the other side of the beam.

The equivalent nominal shear stress (τ_{ve}) is then given by:

$$\tau_{ve} = \frac{V_u + 1.6\left(\frac{T_u}{b}\right)}{bd}$$

If this equivalent nominal shear stress (τ_{ve}) exceeds the maximum shear strength of concrete ($\tau_{c\,max}$), then section has to be redesigned.

The strength of a member subjected to both torsion and flexure is described in terms of interaction of T_u/T_{uR} with M_u/M_{uR}. Where, T_{uR} and M_{uR} are respectively the strength of the member subjected to pure torsion and pure flexure.

Cl. 41.4.2 of IS 456: 2000 gives recommendations for the design of concrete members subjected to both flexure and torsion. This recommendation is based on simplified **Skew Bending Theory**.

Accordingly, the torsional moment can be converted to an equivalent flexural moment as:

$$M_t = \frac{T_u}{1.7}\left(1+\frac{D}{b}\right)$$

Here the M_t so calculated is combined with the flexural moment (M_u) to give equivalent bending moments M_{e1} and M_{e2} as:

$$M_{e1} = M_t + M_u$$
$$M_{e2} = M_t - M_u$$

Reinforcement is designed to resist the equivalent bending moment M_{e1} and the corresponding required steel is provided in the flexural tension zone.

Now when $M_t > M_u$ i.e. $M_{e2} > 0$, then a reinforcement for this equivalent moment (M_{e2}) has also to be designed and is provided in flexural compression zone.

When $M_u = 0$ i.e. in case of **pure torsion**, then $M_{e1} = M_{e2} = M_t$ and equal longitudinal reinforcement has to be provided in the both flexural tension zone and flexural compression zone.

8.8 IS 456: 2000 Provisions for the Design of Reinforcement in Members Subjected to Torsion

1. **Cl. 41.4.3 of IS 456: 2000** provides recommendations for the design of **2 legged**, closed transverse **stirrup reinforcement** the area of which is given by:

$$A_{sv} = \frac{T_u S_v}{0.87 f_y b_1 d_1} + \frac{V_u S_v}{2.5 d_1 (0.87 f_y)} = \frac{S_v}{0.87 f_y d_1}\left(\frac{\tau_u}{b_1} + \frac{V_u}{2.5}\right)$$

2. In addition to the above, **Cl. 41.4.3 of IS 456: 2000** specifies limit on minimum area of transverse reinforcement also as:

$$A_{sv} \geq \frac{(\tau_{ve} - \tau_c) b.S_v}{0.87 f_y} \quad \text{or} \quad \frac{A_{sv}}{bS_v} \geq \frac{(\tau_{ve} - \tau_c)}{0.87 f_y}$$

3. As per **Cl. 41.1 of IS 456: 2000,** in structures where torsion is required to maintain the equilibrium, members shall be designed for torsion. However where torsion can be eliminated by releasing the redundant restraints, no specific design for torsion is necessary provided torsional stiffness is neglected in the calculation of internal forces. There, adequate control of any torsional cracking is being taken care of by the required nominal shear reinforcement.

4. **Cl. 41.2 of IS 456: 2000** states that sections located at less than distance 'd' from the face of the support may be designed for the same torsion as computed at a distance 'd'.

5. **Cl. 26.5.1.7a of IS 456: 2000** specifies the distribution of torsional reinforcement in terms of maximum spacing of stirrups (s_v), in order to have sufficient post crack torsional resistance and to control crack widths.

$$s_v \leq \begin{cases} x_1 \\ \dfrac{x_1 + y_1}{4} \\ 300\,mm \end{cases}$$

where, $x_1 = b_1 + 2$ (longitudinal bar dia/2) + 2 (stirrups dia/2)
$y_1 = d_1 + 2$ (longitudinal bar dia/2) + 2 (stirrups dia/2)
shown in Fig.8.7

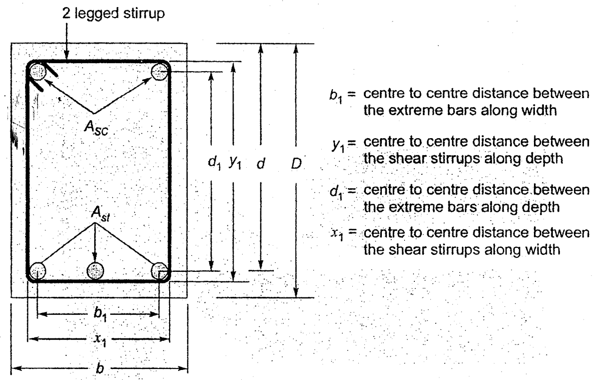


Fig. 8.7 *Stirrups in beams*

6. Cl. 26.5.1.7b of IS 456: 2000 states that *longitudinal reinforcement shall be placed as close as possible to the corners of the cross section and in all cases, there shall be atleast one longitudinal bar at each corner of ties.*

7. **If depth of the member subjected to torsion exceeds 450 mm,** then additional longitudinal bars are required to be provided at the faces with total area not less than 0.1% of web area. These side face bars are to be distributed equally on each face at a spacing not exceeding 300 mm or web thickness, whichever is small.

8.9 Design for Torsion as per Working Stress Method

1. **Shear and Torsion:** Cl. 41.3.1 states that equivalent shear taking into account the effect of torsion, is given by:

$$V_e = V + 1.6\left(\frac{T}{B}\right)$$

where, T = Torsional moment
V = Shear force
V_e = Equivalent shear force
B = Beam width

Thus equivalent nominal shear stress is given by:

$$\tau_{ve} = \frac{V_e}{Bd} \leq \tau_{c\,max}$$

2. **Longitudinal Reinforcement:** Longitudinal reinforcement must be designed to resist an equivalent bending moment M_{e1} given by:

$$M_{e1} = M + M_t$$

where, M = Flexural / Bending moment at the cross section under working conditions

$$M_t = \left(\frac{T}{1.7}\right)\left(1+\frac{D}{B}\right)$$

Case 1: When $M_t < M$

In this case, only tension reinforcement is designed without the requirement of any compression reinforcement.

Depth of the beam section is given by: $d = \sqrt{\frac{M_{e1}}{QB}}$

Area of tension steel (A_{st}) is given by: $A_{st} = \frac{M_{e1}}{\sigma_{st} \cdot jd} = \frac{M_{e1}}{\sigma_{st} \cdot \left(d - \frac{x_a}{3}\right)}$

Case 2: When $M_t > M$

In this case, longitudinal reinforcement shall be provided on flexural compression face, such that the beam can also withstand an equivalent moment M_{e2} (= $M_t - M$) where moment M_{e2} being taken as acting in the opposite sense to the moment M_x.

Depth of the beam section is given by: $d = \sqrt{\frac{M_{e1}}{QB}}$

Area of tension steel (A_{st}) is given by: $A_{st} = \frac{M_{e1}}{\sigma_{st} \cdot jd} = \frac{M_{e1}}{\sigma_{st} \cdot \left(d - \frac{x_a}{3}\right)}$

Area of compression steel (A_{sc}) is also designed for a moment M_{e2} (= $M_t - M$) taken in opposite sense to that of M_x.

(Design tension reinforcement for M_{e2} and provide as negative (–ve) reinforcement).

$$A_{st2} = \frac{M_{e2}}{\sigma_{st} \cdot jd} = \frac{M_{e2}}{\sigma_{st} \cdot \left(d - \frac{x_a}{3}\right)}$$

Thus, provide this much reinforcement at top.

As per **Cl. 28.5.1.2 of IS 456: 2000**, area of compression reinforcement should be less than or equal to 4% of gross sectional area of beam.

8.9.1 Shear/Transverse Reinforcement

IS 456: 2000 states that two legged closed hoops enclosing the corner longitudinal bars shall have an area of cross section A_{sv} given by:

$$A_{sv} = \frac{TS_v}{b_1 d_1 \sigma_{SV}} + \frac{VS_v}{2.5 d_1 \sigma_{SV}} = \frac{S_v}{d_1 \sigma_{SV}}\left[\frac{T}{b_1} + \frac{V}{2.5}\right]$$

But total reinforcement shall not be less than **minimum shear reinforcement which is** given by:

$$A_{sv} \geq \frac{(\tau_{ve} - \tau_c) b \cdot S_v}{\sigma_{SV}}, \quad \frac{A_{sv}}{bs_v} \geq \frac{(\tau_{ve} - \tau_c)}{\sigma_{SV}}$$

Spacing of shear reinforcement is given by:

$$S_V = \frac{A_{sv}\sigma_{SV} d_1}{\left[\frac{T}{b_1} + \frac{V}{2.5}\right]} = \frac{A_{sv}\sigma_{SV} d_1}{V_{seq}} \qquad \text{where, } V_{Seq} = \frac{T}{b_1} + \frac{V}{2.5}$$

where,
S_V = spacing of shear reinforcement
b_1 = centre to centre distance between corner bars in the direction of width
= **b – clear cover – 2 × stirrups dia – 2 (longitudinal bar dia/2)**
d_1 = centre to centre distance between corner bars
= **D – clear cover – 2 × stirrups dia – 2 (longitudinal bar dia/2)**
τ_{ve} = equivalent shear stress
τ_c = design shear strength of concrete as per **Table 19 of IS: 456-2000**

Maximum Spacing

$$S_v \leq \begin{cases} x_1 \\ \frac{x_1 + y_1}{4} \\ 300\ mm \end{cases}$$

where, $x_1 = b_1 + 2$ (longitudinal bar dia/2) + 2 (stirrups dia/2)
$y_1 = d_1 + 2$ (longitudinal bar dia/2) + 2 (stirrups dia/2)

Side face reinforcement

Side face reinforcement shall be provided as per **Cl. 26.5.1.3 and 26.5.17(6) of IS 456: 2000.**

1. Beam depth > 450 mm (if beam subjected to torsion)
2. Beam depth > 750 mm (if beam not subjected to torsion)
   Provide @ 0.1% of web area and distribute it equally on both side faces.

8.10 Design for Torsion as per Limit State Method

Factored bending moment = 1.5 $M = M_u$
Factored shear force = 1.5 $V = V_u$
Factored torsion moment = 1.5 $T = T_u$

Shear and torsion: Cl. 41.3.1 states that equivalent shear taking into account the effect of torsion, is given by:

$$V_e = V_u + 1.6\left(\frac{T_u}{B}\right)$$

where, T_u = Factored torsional moment, V_u = Factored shear
V_e = Equivalent shear, B = Beam width

Thus equivalent nominal shear stress is given by:

$$\tau_{ve} = \frac{V_e}{Bd} \leq \tau_{c\,max}$$

Longitudinal Reinforcement: **Cl. 41.4.2** states that longitudinal reinforcement must be designed to resist an equivalent bending moment M_{e1} given by:

$$M_{e1} = M_u + M_t$$

where, M_u = Factored bending moment at the cross section

Case 1: When $M_t < M_u$

In this case, only tension reinforcement is designed without the requirement of any compression reinforcement.

Depth of the beam section is given by: $d = \sqrt{\dfrac{M_{e1}}{QB}}$

Area of tension steel (A_{st}) is given by: $A_{st} = \dfrac{M_{e1}}{0.87 f_y \cdot jd}$

Case 2: When $M_t > M_u$ (Cl. 41.4.2.1 of IS 456: 2000)

In this case, longitudinal reinforcement shall be provided on flexural compression face, such that the beam can also withstand an equivalent moment M_{e2} (= $M_t - M_u$) where, moment M_{e2} being taken as acting in the opposite sense to the moment M_u.

Depth of the beam section is given by: $d = \sqrt{\dfrac{M_{e1}}{QB}}$

Area of tension steel (Ast) is given by: $A_{st} = \dfrac{M_{e1}}{0.87 f_y \cdot jd}$

Area of compression steel (A_{sc}) is also provided for M_{e2} (=$M_t - M_u$).

$$A_{st2} = \frac{M_{e2}}{0.87 f_y . jd}$$

Provide this much reinforcement at top.

As per **Cl. 26.5.1.2 of IS 456: 2000**, area of compression reinforcement area should not exceed 4% of gross sectional area of beam.

Shear/Transverse reinforcement: **Cl. 41.4.3 of IS 456: 2000** states that two legged closed hoops enclosing the corner longitudinal bars shall have an area of cross section A_{sv} given by:

$$A_{sv} = \frac{T_u S_v}{b_1 d_1 (0.87 f_y)} + \frac{V_u S_v}{2.5 d_1 .(0.87 f_y)} = \frac{S_v}{d_1 (0.87 f_y)}\left[\frac{T_u}{b_1} + \frac{V_u}{2.5}\right]$$

$$\Rightarrow \qquad S_v = \frac{A_{sv}(0.87 f_y) d_1}{\dfrac{T_u}{b_1} + \dfrac{V_u}{2.5}}$$

But total reinforcement shall not be less than **minimum shear reinforcement** which is given by:

$$A_{sv} \geq \frac{(\tau_{ve} - \tau_c) b \cdot S_v}{0.87 f_y}, \qquad \frac{A_{sv}}{b s_v} \geq \frac{(\tau_{ve} - \tau_c)}{0.87 f_y}$$

Spacing of shear reinforcement is given by:

$$S_v = \frac{A_{sv}(0.87 f_y) d_1}{\left[\dfrac{T_u}{b_1} + \dfrac{V_u}{2.5}\right]} = \frac{A_{sv}(0.87 f_y) d_1}{V_{seq}}$$

where $V_{seq} = \dfrac{T_u}{b_1} + \dfrac{V_u}{2.5}$ = Equivalent shear force

S_v = spacing of shear reinforcement

b_1 = centre to centre distance between corner bars in the direction of width

= b – clear cover – 2 x stirrups dia – 2 (longitudinal bar dia/2)

d_1 = centre to centre distance between corner bars

= D – clear cover – 2 x stirrups dia – 2 (longitudinal bar dia/2)

τ_{ve} = equivalent shear stress

τ_c = design shear strength of concrete as per **Table 19 of IS: 456-2000**

Maximum spacing

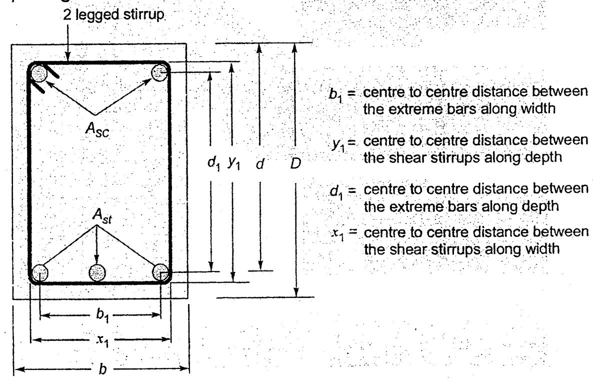


Fig. 8.8 *Stirrups in beams*

$$S_v \leq \begin{cases} x_1 \\ \dfrac{x_1 + y_1}{4} \\ 300\ mm \end{cases}$$

where, $x_1 = b_1$ + 2 (longitudinal bar dia/2) + 2 (stirrups dia/2)

$y_1 = d_1$ + 2 (longitudinal bar dia/2) + 2 (stirrups dia/2)

Example 8.1 Design a rectangular beam of size 350 x 750 mm which is acted upon by a factored twisting moment of 150 kNm in combination with an ultimate negative moment of 210 kNm and an ultimate shear force of 110 kN. Use M 30 grade of concrete and Fe 415 steel.

Solution:

Given,

b = 350 mm, D = 750 mm

f_{ck} = 30 N/mm², f_y = 415 N/mm²

T_u = 150 kNm, M_u = 210 kNm

V_u = 110 kNm

Let effective cover to the reinforcing bars = 50 mm

Thus effective depth, d = 750 mm – 50 mm = 700 mm

Equivalent bending moment due to torsion:

$$M_t = \frac{T_u}{1.7}\left(1+\frac{D}{b}\right) = 150/1.7 \times (1 + 750/350) = 277.31 \text{ kNm}$$

Bending moment for design:

$$M_e = M_t \pm M_u$$
$$= 277.31 \pm 210 = 487.31 \text{ kNm} \quad \text{(flexural tension at bottom)}$$
$$= 67.31 \text{ kNm} \quad \text{(flexural compression at top)}$$

Design of bottom reinforcement:

$$R_1 = \frac{M_{e1}}{bd^2} = 487.31 \times 10^6/(350) \times (700)^2 = 2.84146 \text{ N/mm}^2$$

and

$$\frac{M_{ulim}}{bd^2} = 0.138 \times 30 = 4.14 \text{ N/mm}^2 > 2.84145 \text{ N/mm}^2$$

Thus

$$\frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{f_{ck}}{2f_y}\left[1-\sqrt{1-4.598\frac{R_1}{f_{ck}}}\right] = 8.98798 \times 10^{-3}$$

$$p_t = 0.89\%$$

$$A_{st,reqd} = 2202.06 \text{ mm}^2$$

Provide 5-25 mm diameter bars at the bottom.

Thus $A_{st,\,provided}$ = 2454.37 mm²

Design of top reinforcement:

$$R_2 = M_{e2}/bd^2 = 67.31 \times 10^6/(350) \times (700)^2 = 0.392478 \text{ N/mm}^2$$

$$\frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{f_{ck}}{2f_y}\left[1-\sqrt{1-4.598\frac{R_2}{f_{ck}}}\right] = 1.10398 \times 10^{-3}$$

$$p_t = 0.11\ \%$$

$$A_{st,reqd} = \frac{0.110398}{100} \times 350 \times 700 = 270.48 \text{ mm}^2$$

Provide 2-16 mm diameter bars at the top.

Thus $A_{st,\,provided}$ = 402.12 mm²

Side face reinforcement:

Since depth of the beam (D) is greater than 450 mm, side face reinforcement @ 0.1 % of beam cross sectional area, for torsion is required to be provided.

Thus torsional reinforcement on each face = 0.05 % = $\left(\frac{0.05}{100}\right) \times 350 \times 750 = 131.25 \text{ mm}^2$

Provide 2-10 mm diameter bars on each face.

Side face reinforcement provided on each face = 157.08 mm²

Vertical spacing between the longitudinal bars should not exceed 300 mm.

Design of shear/transverse reinforcement:

Equivalent nominal shear stress = $\tau_{ve} = \dfrac{V_u + 1.6\frac{T_u}{b}}{bd}$

$$= \frac{110 \times 10^3 + 1.6\left(\frac{150 \times 10^6}{350}\right)}{350 \times 700} = 3.24 \text{ N/mm}^2$$

$\tau_{c,\,max}$ (for M30 concrete) = 3.5 N/mm² > 3.2478 N/mm² (OK)

Now for $p_t = \dfrac{2454.37 \times 100}{350 \times 700}$ = 1 % and M30 concrete, τ_c = 0.66 N/mm²

It can be seen that torsional shear stress (= 3.24 N/mm²) is much higher than design shear strength of concrete (= 0.66 N/mm²).

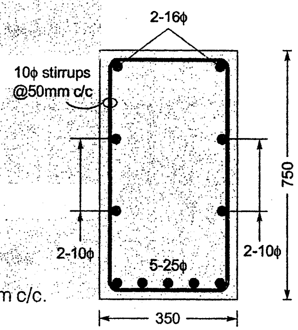
Using 2 legged 10 mm diameter bars as shear stirrups, A_{sv} = 2 × 78.54 mm² = 157 mm²

Since effective cover assumed is 50 mm.

So

$$d_1 = 750 \text{ mm} - 2 \times 50 \text{ mm} = 650 \text{ mm}$$
$$b_1 = 350 \text{ mm} - 2 \times 50 \text{ mm} = 250 \text{ mm}$$

$$s_v = \frac{0.87 f_y A_{sv}}{\frac{T_u}{b_1 d_1} + \frac{V_u}{2.5 d_1}} = \frac{0.87(415)157}{\frac{150 \times 10^6}{250 \times 650} + \frac{110 \times 10^3}{2.5 \times 650}}$$
$$= 57.21 \text{ mm}$$



Maximum shear spacing requirement:

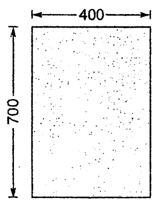
$$s_v \le \begin{cases} x_1 = 250 + 25 + 10 = 285 \text{ mm} \\ \frac{x_1 + y_1}{4} = \frac{285 + (650 + 12.5 + 8 + 10)}{4} \\ 300 \text{ mm} \end{cases}$$
$$= 241.375 \text{ mm}$$

Provide 2 legged 10 mm diameter bars as shear stirrups @ 50 mm c/c.

Check for clear cover:

Clear cover = 50 – 10 – 14 mm = 26 mm > 20 mm (O.K.)

Example 8.2 Determine the amount of reinforcement required for a beam of size 400 mm × 700 mm and subjected to a working moment of 150 kNm, twisting moment of 60 kNm and working shear force of 80 kN. Use M 20 concrete and Fe 415 steel.



Solution:

Let effective cover = 50 mm

∴ Effective depth (d) = 700 mm – 50 mm = 650 mm

Step 1: Check for adequacy of beam depth

$$\text{Equivalent shear force } (V_e) = V_u + 1.6\frac{T_u}{b}$$
$$= (1.5 \times 80) + 1.6\left(\frac{1.5 \times 60}{0.4}\right)$$
$$= 120 + 360 = 480 \text{ kN}$$

∴ Equivalent shear stress (τ_{ve}) $= \frac{V_e}{bd} = \frac{480\times10^3}{400\times650} = 1.846$ N/mm²

For M-20 concrete, $\tau_{max} = 2.8$ N/mm² (Table 20 of IS 456: 2000)

$> \tau_{ve} (= 1.846$ N/mm²) (OK)

Thus depth provided is OK.

Step 2: Check for requirement of shear reinforcement

Let percentage of tension reinforcement (p_t) = 0.5%

For M 20 concrete and p_t = 0.5%, design shear stress of concrete

$\tau_c = 0.48$ N/mm² (Table 19 of IS 456: 2000)

∵ $\tau_c < \tau_{ve} < \tau_{max}$

∴ Shear reinforcement is required.

Step 3: Tension reinforcement requirement

$$\text{Equivalent bending moment } (M_e) = M_u + M_t$$

$$= (1.5\times150) + \frac{T_u}{1.7}\left(1+\frac{D}{b}\right)$$

$$= 225 + \frac{1.5\times60}{1.7}\left(1+\frac{700}{400}\right) = 225 + 145.6 = 370.6 \text{ kNm}$$

$$\therefore \quad R = \frac{M_e}{bd^2} = \frac{370.6\times10^6}{400\times650^2} = 2.1929$$

$$\therefore \quad \frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{20}{2(415)}\left[1-\sqrt{1-4.598\times\frac{2.1929}{20}}\right] = 0.0071285$$

∴ p_t = 0.71% > 0.5% (assumed)

⇒ τ_c = 0.5472 N/mm² for M 20 concrete

and p_t = 0.71% (Table 19 of IS: 456 - 2000)

From here also, $\tau_c > \tau_{ve} < \tau_{max}$

⇒ Shear reinforcement is required.

Also A_{st} = 0.0071 × 400 × 650 = 1846 mm²

$$\therefore \quad \text{No. of 25 mm } \phi \text{ bars required} = \frac{1846}{\frac{\pi}{4}\times25^2} = 3.76 = 4 \text{ bars (say)}$$

$$\therefore \quad p_{t\,(provided)} = \frac{4\times\frac{\pi}{4}\times25^2}{400\times650}\times100 = 0.76\%$$

⇒ τ_c = 0.5624 N/mm² (Table 19 of IS: 456 - 2000)

Minimum reinforcement percentage $= \left(\frac{0.85}{f_y}\right)\times1000 = \left(\frac{0.85}{415}\right)\times100 = 0.20\% < 0.76\%$ (OK)

Maximum reinforcement percentage ($P_{t(max)}$) = 4% > 1.76% (OK)

Step 4: Compression reinforcement requirement

From Step-3 above, M_t = 145.6 kNm < M_u (= 225 kNm)

∴ No compression reinforcement is required.

However provide 2-12ϕ bars as hanger bars for holding the transverse reinforcement.

Step 5: Side face reinforcement requirement

∵ $D >$ 450 mm

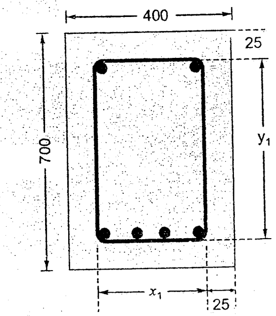
∴ Side face reinforcement is required @ 0.1% of bD

$= \frac{0.1}{100}\times400\times700 = 280$ mm² equally distributed on the side face

∴Side face reinforcement on one face $= \frac{280}{2}$ mm² = 140 mm²

Providing 2-10ϕ bars, area of side face reinforcement provided

$= 2\times\frac{\pi}{4}\times10^2 = 157$ mm² > 140 mm² (OK)



Step 6: Transverse reinforcement

Using 2 legged 10ϕ bars as transverse reinforcement

d_1 = 700 − 50 − 50 = 600 mm

b_1 = 400 − (25 + 10 + 12.5) − (25 + 10 + 12.5)

= 305 mm

$$\therefore \quad \frac{0.87 f_y A_{sv}}{s_v} = \frac{T_u}{b_1 d_1} + \frac{V_u}{2.5 d_1}$$

$$= \frac{1.5\times60\times10^6}{305\times600} + \frac{1.5\times80\times10^3}{2.5\times600}$$

$$= 491.8 + 80 = 571.8 \text{ N/mm} \quad \text{...}(i)$$

Also, $$A_{sv} \geq (\tau_{ve} - \tau_c)\frac{b.s_v}{0.87 f_y} \geq (1.846 - 0.5624)\frac{400\, s_v}{0.87(415)}$$

$$\Rightarrow \quad \frac{0.87 f_y A_{sv}}{s_v} \geq (1.846 - 0.5624)\,400 = 513.44 \text{ N/mm} \quad \text{...}(ii)$$

Comparing equations (i) and (ii), it is clear that equation (i) is governing equation.

$$\therefore \quad \frac{0.87 f_y A_{sv}}{s_v} = 571.8 \text{ N/mm}$$

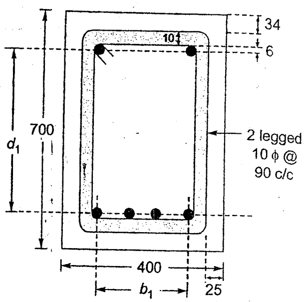
With 2 legged 10ϕ bars (as assumed),

$$A_{sv} = 2\times\frac{\pi}{4}\times10^2 = 157 \text{ mm}^2$$

$$\therefore \quad s_v = \frac{0.87 f_y A_{st}}{571.8} = \frac{0.87(415)157}{571.8} = 99.13 \text{ mm } c/c$$

Step 7: Check for adequacy of s_v

$$x_1 = b_1 + (12.5 + 12.5) + (5 + 5)$$
$$= 305 + 25 + 10 = 340 \text{ mm}$$
$$y_1 = d_1 + (6 + 12.5) + (5 + 5)$$
$$= 600 + 18.5 + 10 = 628.5 \text{ mm}$$

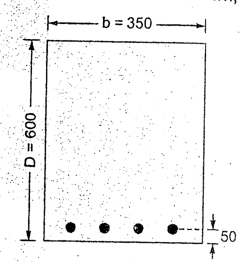


$$s_v \le \begin{bmatrix} x_1 \\ \frac{(x_1+y_1)}{4} \\ 300\text{mm} \end{bmatrix} = \begin{bmatrix} 340\text{mm} \\ \frac{340+628.5}{4} \\ 300\text{mm} \end{bmatrix} = 242.125\text{mm}$$

∴ $s_v \le 242.125$ mm

∴ Provide 2 legged 10ϕ stirrups @ 90 c/c.

Example 8.3 A rectangular beam as shown below is subjected to a moment of 174 kNm, torsion of 35 kNm and shear force of 88 kN at service load. Determine the reinforcement required for the beam. use M 20 and Fe 415.



Solution:

Given

Moment (M) = 174 kN

Torsion (T) = 35 kNm

SF (V) = 88 kN

At service load conditions

Thus

Factored moment (M_u) = 1.5 × 174 = 361 kNm

Factored torsion (T_u) = 1.5 × 35 = 52.5 kNm

Factored shear force (V_u) = 1.5 × 88 = 132 kN

Check for adequacy of beam depth

$$\text{Equivalent shear force } (V_e) = V_u + 1.6\left(\frac{T_u}{b}\right) = 132 + 1.6\left(\frac{52.5}{\frac{350}{1000}}\right) = 372 \text{ kN}$$

$$\text{Equivalent shear stress } (\tau_v) = \frac{V_e}{bd} = \frac{372\times1000}{350\times(600-50)} = 1.932 \text{ N/mm}^2$$

As per table 20 of IS 456: 2000, for M 20,

$\tau_{c\,max}$ = 2.8 N/mm2

> τ_v(= 1.932 N/mm²)

Thus there is no need to redesign the section.

Thus depth of beam is OK.

Requirement of shear reinforcement

Let percentage of tension reinforcement

(p_t) = 0.5 %

∴Design shear stress of M 20 concrete for

p_t = 0.5 %

as per Table 19 of IS 456: 2000, τ_c = 0.48 N/mm² < τ_v (1.932 N/mm²)

Thus shear reinforcement is required.

Requirement of flexural reinforcement

$$\text{Equivalent bending moment } (M_e) = M_u + M_t = M_u + \left(\frac{T_u}{1.7}\right)\left(1+\frac{D}{b}\right)$$

$$= 361 + \left(\frac{52.5}{1.7}\right)\left(1+\frac{0.6}{0.35}\right) = 261 + 83.82 = 344.82 \text{ kNm}$$

$$∴ \quad R = \frac{M_e}{bd^2} = \frac{344.82\times10^6}{350(550)^2} \doteq 3.25686 \text{ N/mm}^2$$

$$∴ \quad \frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{20}{2(415)}\left[1-\sqrt{1-\frac{4.598(3.25686)}{20}}\right] = 0.012018$$

∴ p_t = 1.2 %

⇒ A_{st} = 2313.5 mm²

But we assumed p_t = 0.5 % (initially)

Design shear strength of M 20 concrete for

p_t = 1.2 %

as per table 19 of IS 456: 2000, τ_c = 0.65 N/mm² < τ_v(=1.932 N/mm²)

Compression reinforcement

Here M_t = 83.82 kNm

< M_u(= 261 kNm)

Thus no compression reinforcement is required

Side face reinforcement

Depth of the beam (D) = 600 mm > 450 mm

So side face reinforcement is required

Total area of side face reinforcement ≥ 0.1 % of web area

$$\ge \frac{0.1}{100}(350)(600)$$

≥ 210 mm²

Provide two 10 ϕ bar on each face of the beam,

$$\text{Total area} = 4\times\frac{\pi}{4}\times10^2 = 314.16 \text{ mm}^2 > 210 \text{ mm}^2$$

Example 8.4 Design a rectangular reinforced concrete beam of width 400 mm subjected to the following at any particular section:

(i) Bending moment = 300 kNm (ii) Shear force = 200 kN (iii) Torsional moment = 100 kNm

Use M 30 concrete and Fe 415 steel.

Design the beam using WSM.

Solution:

By WSM

Width of the beam, B = 400 mm

Assume overall depth of the beam, D = 800 mm

and effective cover = 50 mm

Effective depth of the beam = 800 – 50 mm = 750 mm

Equivalent shear

$$V_e = V + \frac{1.6T}{B} = 200 + \frac{1.6\times100}{0.4} \text{ kN} = 600 \text{ kN}$$

Nominal shear stress, $\tau_{ve} = \dfrac{V_e}{Bd} = \dfrac{600\times10^3}{400\times750}$ N/mm^2

$= 2$ N/mm$^2 < \tau_{c\,max}$ (= 2.2 N/mm^2 for M 30 concrete)

Equivalent moment

$$M_e = M + M_T$$

$$= 300 + \frac{T}{1.7}\left(1+\frac{D}{B}\right) = 300 + 176.47 \text{ kNm}$$

$$= 300 + \frac{100}{1.7}\left(1+\frac{0.8}{0.4}\right)\text{kNm} = 476.47 \text{ kNm}$$

Now $M_T < M$

∴ Reinforcement on compression side is not required

Depth required

$$k = \frac{mc}{t+mc} = \frac{9\times10}{230+9\times10} = 0.28$$

$$j = 1-\frac{k}{3} = 1-\frac{0.28}{3} = 0.906$$

$$Q = \frac{1}{2}cjk = \frac{1}{2}\times10\times0.906\times0.28 = 1.27$$

$$d = \sqrt{\frac{M_e}{BQ}} = \sqrt{\frac{476.47\times10^6}{400\times1.27}} = 968.47 \text{ mm}$$

∴ $D = 968.47 + 50$ mm

$= 1018.47$ mm $\simeq 1020$ mm > 800 mm (Not OK)

Take $D = 1050$ mm

∴ $d = 1050 - 50$ mm $= 1000$ mm

$$M_e = M + M_T = M + \frac{T}{1.7}\left(1+\frac{D}{B}\right)$$

$$= 300 + \frac{100}{1.7}\times\left(1+\frac{1050}{400}\right)$$

$$= 300 + 213.23 \text{ kNm} = 513.23 \text{ kNm}$$

∴ $d = \sqrt{\dfrac{M_e}{QB}} = \sqrt{\dfrac{513.23\times10^6}{1.27\times400}}$ mm $= 1005$ mm

Provide $d = 1010$ mm

So that $D = 1010 + 50$ mm $= 1060$ mm

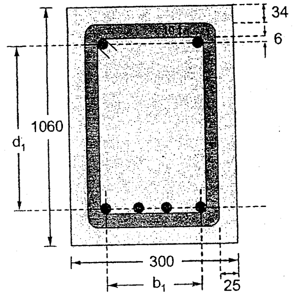
$$M_e = 300 + \frac{100}{1.7}\left(1+\frac{1.060}{0.4}\right) = 514.7 \text{ kNm}$$

∴ $d = \sqrt{\dfrac{M_e}{QB}} = \sqrt{\dfrac{514.7\times10^6}{1.27\times400}}$ mm $= 1006$ mm

Area of steel required

$$A_{st} = \frac{M_e}{\sigma_{st} jd} = \frac{514.7\times10^6}{230\times0.906\times1010}\text{mm}^2 = 2431.1 \text{ mm}^2$$

∴ No. of 25 mm dia. bars required $= \dfrac{2431.1}{\frac{\pi}{4}\times25^2} = 4.95 = 5$ (say)



Provide 5-25 mm dia. bars

$b_1 = B - 100 = 400 - 100 = 300$ mm

$d_1 = D - 100 = 1060 - 100 = 960$ mm

$x_1 = 300 + (25 + 5 + 5) = 300 + 35 = 335$ mm

$y_1 = 960 + (25 + 5 + 5) = 960 + 35 = 995$ mm

Design of shear reinforcement

Using 2-legged 8 mm dia. stirrups,

Spacing, $s_v = \dfrac{A_{sv}\sigma_{sv}d_1}{\left(\dfrac{V}{2.5}+\dfrac{T}{b_1}\right)} = \dfrac{\frac{\pi}{4}\times2\times8^2\times230\times960}{\left(\dfrac{200\times10^3}{2.5}+\dfrac{100\times10^6}{300}\right)} = 53.7$ mm

With 10 mm dia. bar

$$s_v = \frac{10^2}{8^2}\times53.7 = 83.9 \text{ mm}$$

With 2-legged 12 mm dia. stirrups

$$s_v = \frac{12^2}{8^2}\times53.7 = 120.8 \text{mm}$$

Minimum shear reinforcement

$$A_{sv} \not< \frac{(\tau_{ve}-\tau_c)Bs_v}{\sigma_{sv}}$$

$$\frac{A_{sv}}{Bs_v} \not< \frac{\tau_{ve}-\tau_c}{\sigma_{sv}}$$

$$\tau_{ve} = \frac{600\times10^3}{400\times1010} = 1.48 \text{ N/mm}^2$$

Percentage of steel, $p_t = \dfrac{5\times\frac{\pi}{4}\times25^2\times100}{400\times1010} = 0.60\%$

∴ $\tau_c = 0.334$ N/mm^2

∴ $s_v = \dfrac{A_{sv}\sigma_{sv}}{(\tau_{ve}-\tau_c)B} = \dfrac{2\times\frac{\pi}{4}\times12^2\times230}{(1.48-0.334)\times400} = 113.5$ mm

Maximum spacing

$$s_v \le \begin{cases} x_1 = 335 \text{ mm} \\ \dfrac{x_1+y_1}{2} = \dfrac{335+995}{4} = 332.5 \text{ mm} \\ = 300 \text{ mm} \end{cases}$$ (whichever is less.)

Provide 2-legged 12 mm dia. stirrup @ 110 mm c/c

Side face reinforcement

∵ $D > 450$ mm

∴ Side face reinforcement is required @ 0.1% $= \dfrac{0.1}{100}\times400\times1060 = 424$ mm^2

No. of 10 mm dia. bars required $= \dfrac{424}{\frac{\pi}{4}\times10^2} = 5.398 \approx 6$

Provide 3-10 mm dia. bars on each face

Spacing $= \frac{d_1}{4} = \frac{960}{4} = 240 \text{ mm} < 300 \text{ mm}$ (OK)

Objective Brain Teasers

Q.1 As per IS 456: 2000, which of the following type of torsion must be considered in the design of reinforced concrete members?
(a) Equilibrium torsion
(b) Compatibility torsion
(c) Both (a) and (b)
(d) No torsion need to be considered for design

Q.2 Minor amounts of which of the following torsion can be ignored in design due to the availability of multiple load paths?
(a) Equilibrium torsion
(b) Compatibility torsion
(c) Both (a) and (b)
(d) None of (a) or (b)

Q.3 Side face reinforcement shall be provided in beams if:
(i) Beam depth exceeds 750 mm without torsion
(ii) Beam depth exceeds 450 mm without torsion
(iii) Beam depth exceeds 450 mm with torsion
(iv) Beam depth exceeds 750 mm with torsion
Among the above statements, true one(s) is/are:
(a) (i) and (iii) (b) (i) and (ii)
(c) (ii) and (iv) (d) (iii) and (iv)

Q.4 The torsion moment in a reinforced concrete beam needs to be converted to an equivalent moment and equivalent shear because:
(a) It is economical to provide reinforcement for moment and shear than for torsion directly.
(b) It is recommended by the principles of Structural Analysis.
(c) It is difficult to provide inclined stirrups to check torsional cracks.
(d) All of the above

Q.5 In a reinforced concrete rectangular beam, the flexural shear and the torsional shear are:
(a) Additive only on one side of the beam
(b) Additive on both the sides of beam
(c) Additive on one or both the sides of beam depending on the type of loading
(d) Cannot be said with certainty

Q.6 Compression reinforcement in a beam subjected to torsion shall be provided when:
(a) Equivalent flexural moment due to torsion exceeds the flexural moment on beam.
(b) Equivalent flexural moment due to torsion is less than the flexural moment on beam.
(c) It is provided in all cases irrespective of the value of equivalent flexural moment due to torsion.
(d) None of the above

Q.7 The horizontal distance between two parallel reinforcing bars in a beam shall not be less than:
(a) Dia of larger bar
(b) Maximum aggregate size + 5 mm
(c) Twice the dia. of bar
(d) Maximum aggregate size

Q.8 Additional longitudinal bars shall be provided in beams subjected to torsion if depth of beam exceeds
(a) 750 mm (b) 450 mm
(c) 500 mm (d) None of these

Q.9 In a beam subjected to torsion, the equivalent torsion moment is less than the flexural moment. In this case
(a) steel is provided only on tension side
(b) steel is provided only on compression side
(c) steel is provided in both tension and compression side
(d) data insufficient

Q.10 Torsion in a reinforced concrete member gives rise to
(a) diagonal cracks (b) vertical cracks
(c) inclined cracks (d) spiral cracks

Q.11 State true or false
'In indeterminate structures, torsion can be eliminated by releasing the redundant reactions.'

Q.12 A RC beam is subjected to a bending moment of 200 kNm, shear force of 20 kN and a torque of 9 kNm. The size of the beam is 300 mm width × 425 mm overall depth. The effective cover is 25 mm. The equivalent shear force is
(a) 68 kN (b) 50 kN
(c) 79 kN (d) 55 kN

Q.13 The equivalent moment in the above question is ________
(a) 200 kNm (b) 250 kNm
(c) 213 kNm (d) 400 kNm

Q.14 Torsion resisting capacity of a given RC section
(a) increases with increase in stirrup and longitudinal steel
(b) does not depend on stirrup and longitudinal steel
(c) decreases with decrease in stirrup spacing
(d) decreases with increase in longitudinal bars

Answers

1. (a)	2. (b)	3. (a)	4. (c)	5. (a)
6. (a)	7. (a, b)	8. (b)	9. (a)	10. (d)
11. True	12. (a)	13. (c)	14. (a)	

Hints:

12. (a)

$$V_e = V + \frac{1.6T}{b} = 20 + \frac{1.6\times 9}{0.3} = 68 \text{ kN}$$

$$M_e = M + \frac{T}{1.7}\left(1+\frac{D}{b}\right)$$

$$= 200 + \frac{9}{1.7}\left(1+\frac{425}{300}\right) = 213 \text{ kNm}$$

Conventional Practice Questions

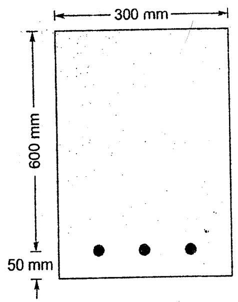
Q.1 A beam of size 300 mm × 450 mm is subjected to a torsional moment of 20 kNm along with a flexural moment of 100 kNm. The shear force acting on the beam is 105 kN at ultimate state of collapse. Design the beam using M25 concrete and Fe415 steel.

Q.2 Find the shear stress in a 300 mm × 500 mm sized beam which is subjected to a shear force of 25 kN and a torsional moment of 15 kNm at service loads. Assume 50 mm effective cover and tension reinforcement as 0.75%. Use M20 concrete and Fe415 steel.

Q.3 A rectangular beam of size 300 mm × 500 mm effective depth is reinforced with 0.5% tension steel of Fe415 grade. It is subjected to a shear force of 25 kN and a torsional moment of 10 kNm. Check if the beam needs torsional reinforcement. Use M20 concrete.

Q.4 A RC rectangular beam of size 350 mm × 450 mm effective depth is subjected to a flexural moment of 25 kNm, a shear force of 55 kN and a torsional moment of 40 kNm. Using M20 concrete and Fe415 steel design the reinforcement for the beam.

Q.5 A reinforced concrete beam is as shown. The section is subjected to a SF of 47 kN under service load conditions. Assuming the percentage tensile reinforcement as 0.5%, find the factored torsional moment that the section can resist when



(a) no torsional reinforcement is provided
(b) maximum steel for torsion is provided
Use M30 concrete and Fe500 steel.