Chapter 15th

Statistics

Exercise 15.1

Q. 1 Find the mean deviation about the mean for the data in Exercises.

4, 7, 8, 9, 10, 12, 13, 17

Answer:

The given data is

4, 7, 8, 9, 10, 12, 13, 17

Mean of the data, $\overline{x} = \frac{4+7+8+9+10+12+13+17}{8} = \frac{80}{8} = 10$

The respective absolute values of

The deviations from mean $\overline{x},$ i.e., $x_i - x$ are

6, 3, 2, 1, 0, 2, 3, 7

The required mean derivation about the mean is

M.D. $(\overline{x}) = \frac{\sum_{i=1}^{8}|x_i - \overline{x}|}{8} = \frac{6+3+2+1+0+2+3+7}{8} = \frac{24}{8} = 3$

Hence, the mean deviation about the mean for the given data is 3.

Q. 2 Find the mean deviation about the mean for the data in Exercises.

38, 70, 48, 40, 42, 55, 63, 46, 54, 44

Answer:

The given data is

38, 70, 48, 40, 42, 55, 63, 46, 54, 44

We first find the mean $(\overline{x})$ of the data

$\overline{x} = \frac{38+70+48+40+42+55+63+46+54+44}{10} = \frac{500}{10} = 50$

The respective absolute values of the deviations from mean$\overline{x}$, i.e., $x_i - x$ are

-12, 20, -2, -10, -8, 5, 13, -4, 4, -6

The absolute values of derivation, i.e. $|x_i - \overline{x}|$

12, 20, 2, 10, 8, 5, 13, 4, 4, 6

The required mean derivation about the mean is

$$M.D.\ (\overline{x}) = \frac{\sum_{i=1}^{10} |x_i - \overline{x}|}{10}$$

$$= \frac{12+20+10+2+8+5+13+4+4+6}{10} = \frac{84}{10} = 8.4$$

Hence, the mean deviation about the mean for the given data is 8.4.

Q. 3 Find the mean deviation about the median for the data in Exercises.

13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

Answer:

The given data is

13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

Here the number of observations is 12 which is even. Arranging the data into ascending order, we have 10, 11, 11, 12, 13, 13, 14, 16, 16, 17, 17, 18.

$$M = \frac{\left(\frac{12}{2}\right)^{th} observation + \left(\frac{12}{2}+1\right)^{th}}{2}$$

$$= \frac{6^{th} observation + 7^{th} observation}{2} = \frac{13+14}{2} = \frac{27}{2} = 13.5$$

The derivation of respective observation from the median, i.e. $x_i - M$

-3.5, -2.5, -2.5, 1.5, -0.5, -0.5, 0.5, 2.5, 2.5, 3.5 ,3.5, 4.5

The absolute values of the respective deviations from the median, i.e., |xi – M| are

3.5, 2.5, 2.5, 1.5, 0.5, 0.5, 0.5, 2.5, 2.5, 3.5, 3.5, 4.5

The required mean derivation about the median is

$$\text{M.D.(M)} = \frac{\sum_{i=1}^{12}|x_1 - M|}{12}$$

$$= \frac{3.5+2.5+2.5+1.5+0.5+0.5+0.5+2.5+2.5+3.5+3.5+4.5}{12}$$

$$= \frac{28}{12} = 2.33$$

Hence, the mean deviation about the median for the given data is 2.33.

Q. 4 Find the mean deviation about the median for the data in Exercises.

36, 72, 46, 42, 60, 45, 53, 46, 51, 49

Answer:

The given data is

36, 72, 46, 42, 60, 45, 53, 46, 51, 49

Here the number of observations is 10 which is even. Arranging the data into ascending order, we have 36, 42, 45, 46, 46, 49, 51, 53, 60, 72.

Now, Median

$$\text{Median } M = \frac{\left(\frac{10}{2}\right)^{th} observation + \left(\frac{10}{2}+1\right)^{th} observation}{2}$$

$$= \frac{5^{th} observation + 6^{th} observation}{2}$$

$$= \frac{46+49}{2} = \frac{95}{2} = 47.5$$

The derivation of the respective observations from the median, i.e. x_i - M

-11.5, -5.5, -2.5, -1.5, -1.5, -1.5, 3.5, 5.5, 12.5, 24.5

The absolute values of the respective deviations from the median, i.e., |xi – M| are

11.5, 5.5, 2.5, 1.5, 1.5, 1.5, 3.5, 5.5, 12.5, 24.5

$$\text{MD .(M)} = \frac{\sum_{i=1}^{10}|x_i - M|}{10} = \frac{11.5+5.5+2.5+1.5+1.5+1.5+3.5+5.5+12.5+24.5}{10}$$

$$= \frac{70}{10} = 7$$

Hence, the mean deviation about the median for the given data is 7.

Q. 5 Find the mean deviation about the mean for the data.

Answer:

We make the following table and add other columns after calculations

x_i	f_i	$f_i\, x_i$	$\lvert x_i - \overline{x}\rvert$	$f_i \lvert x_i - \overline{x}\rvert$
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6
20	3	60	6	18
25	5	125	11	55
	$\sum f_i = 25$	$\sum f_i x_i = 350$		$\sum f_i \lvert x_i - \overline{x}\rvert = 158$

So, we can calculate the absolute values of the deviations from the mean, i.e., |xi – x̄|, as shown in the table.

$$N = \sum_{I=1}^{5} f_i = 25$$

$$\sum_{i=1}^{5} f_i x_i = 350$$

$$\therefore \overline{x} = \frac{1}{N}\sum_{i=1}^{5} f_i x_i = \frac{1}{25} \times 350 = 14$$

$$\therefore \text{MD}(\overline{x}) = \frac{1}{N}\sum_{i=1}^{5} f_i\, |x_i - \overline{x}| = \frac{1}{25} \times 158 = 6.32$$

Hence, the mean deviation about the mean for the given data is 6.32.

Q. 6 Find the mean deviation about the mean for the data.

Answer:

We make the following table and add other columns after calculations.

x_i	f_i	f_i x_i	$\|x_i - \overline{x}\|$	fi $\|x_i - \overline{x}\|$
10	4	40	40	160
30	24	720	20	480
50	28	1400	0	0
70	16	1120	20	320
90	8	720	40	320
	$\sum f_i$= 80	$\sum f_i x_i$=4000		$\sum f_i \|x_i - \overline{x}\|$= 1280

So, we can calculate the absolute values of the deviations from the mean, i.e., $|xi - \overline{x}|$, as shown in the table.

N = $\sum_{i=1}^{5} f_i = 80, \sum_{i=1}^{3} f_i x_i = 4000$

$\therefore \overline{x} = \frac{1}{N} \sum_{i=1}^{5} f_i x_i = \frac{1}{80} \times 4000 = 50$

MD($\overline{X}$) $= \frac{1}{N} \sum_{i=1}^{5} f_i |x_i - \overline{x}| = \frac{1}{80} \times 1280 = 16$

Hence, the mean deviation about the mean for the given data is 16.

Q. 7 Find the mean deviation about the median for the data.

Answer:

The given observations are already in ascending order. We make a table for the given data, as shown below, adding other columns after calculations.

x_i	f_i	c.f.	$\|x_i - M\|$	f_i $\|x_i - M\|$
5	8	8	2	16

7	6	14	0	0
9	2	16	2	4
10	2	18	3	6
12	2	20	5	10
15	6	26	8	48

Now, N = 26 which is even.

Median is the mean of 13th and 14th observations. Both these observations lie in the cumulative frequency 14, for which the corresponding observation is 7.

the absolute values of the deviations from the median, i.e., $|xi - M|$, as shown in the table.

$\lvert x_i - M \rvert$	2	0	2	3	5	8
f_i	8	6	2	2	2	6
$f_i \lvert x_i - M \rvert$	16	0	4	6	10	48

$$\sum_{i=1}^{6} f_i \;= 26 \sum and \sum_{i=1}^{6} f_i|x_i - M| = 84$$

M.D.(M) $= \frac{1}{N} = \sum_{i=1}^{6} f_i\,|x_i - M| = \frac{1}{26} \times 84 = 3.23$

Hence, the mean deviation about the median for the given data is 3.23.

Q. 8 Find the mean deviation about the median for the data.

Answer:

The given observations are already in ascending order. We make a table for the given data, as shown below, adding other columns after calculations.

x_i	f_i	c.f.	$\lvert x_i - M \rvert$	$f_i \lvert x_i - M \rvert$

15	3	3	13.5	40.5
21	5	8	7.5	37.5
27	6	14	1.5	9
30	7	21	1.5	10.5
35	8	29	6.5	52
	$\sum f_i = 29$			$\sum fi \lvert xi - M \rvert = 149.5$

Now, N = 29 which is odd.

Median is the mean of 15th and 16th observations. We see that both the 15th as well as the 16th observations lie in the cumulative frequency 21, whose corresponding observation is 30.

the absolute values of the deviations from the median, i.e., |xi – M|, as shown in the table.

$\lvert x_i - M \rvert$	15	9	3	0	5
f_i	3	5	6	7	8
$f_i \lvert x_i - M \rvert$	45	45	18	0	40

$\sum_{i=1}^{5} f_i = 29, \sum_{i=1}^{5} f_i |x_i - M| = 148$

M.D.(M) $= \frac{1}{N} \sum_{I=1}^{5} f_i \, |x_{\,i} - M| = \frac{1}{29} \times 148 = 5.1$

Hence, the mean deviation about the median for the given data is 5.1.

Q. 9 Find the mean deviation about the mean for the data.

Income per day	0 – 100	100 – 200	200 – 300	300 – 400	400 – 500	500 – 600	600 – 700	700 – 800
Number of persons	4	8	9	10	7	5	4	3

Answer:

We make the following table and add other columns after calculations.

Income per day	Number of persons	Mid-point x_i	f_i x_i	$\|x_i - \overline{x}\|$	$f_i \|x_i - \overline{x}\|$
0 – 100	4	50	200	308	1232
100 – 200	8	150	1200	208	1664
200 – 300	9	250	2250	108	972
300 – 400	10	350	3500	8	80
400 – 500	7	450	3150	92	644
500 – 600	5	550	2750	192	960
600 – 700	4	650	2600	292	1168
700 – 800	3	750	2250	392	1176
	$\sum f_i = 50$		$\sum f_i x_i =$ 17900		$\sum f_i \|x_i - \overline{x}\|$

the absolute values of the deviations from the mean, i.e., |xi – x̄|, as shown in the table.

N = $\sum_{i=1}^{8} f_i = 50, \sum_{i-1}^{8} f_i x_i = 17900$

$\therefore \overline{x} = \frac{1}{N}\sum_{i=1}^{8} f_i x_i = \frac{1}{50} \times 17900 = 358$

M.D.$(\overline{x}) = \frac{1}{N}\sum_{i=1}^{8} f_i |x_i - \overline{x}| = \frac{1}{50} \times 7896 = 157.92$

Hence, the mean deviation about the mean for the given data is 157.92.

Q. 10 Find the mean deviation about the mean for the data.

Height in cms	95 – 105	105 – 115	115 – 125	125 – 125	135 – 145	145 – 155
Numbers of boys	9	13	26	30	12	10

Answer:

We make the following table and add other columns after calculations.

Height of cms	Numbers of boys f_i	Mid – point x_i	$f_i x_i$	$\lvert x_i - \bar{x}\rvert$	$f_i \lvert x_i - \bar{x}\rvert$
95 – 105	9	100	900	25.3	227.7
105 – 115	13	110	1430	15.3	198.9
115 – 125	26	120	3120	5.3	137.8
125 – 135	30	130	3900	4.7	141
135 – 145	12	140	1680	14.7	176.4
145 - 155	10	150	1500	24.7	247
	$\sum f_i$ = 100		$\sum f_i x_i$=12530		$\sum f_i \lvert x_i - \bar{x}\rvert$ = 1128.8

the absolute values of the deviations from the mean, i.e., |xi – x|,

N = $\sum_{i=1}^{6} f_i = 100, \sum_{i=1}^{6} f_i x_i$= 12530

$$\therefore \bar{x} = \frac{1}{N}\sum_{i=1}^{6} f_i x_i \times \frac{1}{100} \times 12530 = 125.3$$

M.D.($\bar{x}$) $= \frac{1}{N}\sum_{i=1}^{6} f_i \lvert x_i - \bar{x}\rvert = \frac{1}{100} \times 1128.8 = 11.28$

Hence, the mean deviation about the mean for the given data is 11.28.

Q. 11 Find the mean deviation about median for the following data:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Numbers of girls	6	8	14	16	4	2

Answer:

We make the following table and add other columns after calculations.

Marks	Number of girls f_i	Mid – point x_i	$f_i x_i$	$\lvert x_i - \bar{x}\rvert$	$f_i \lvert x_i - \bar{x}\rvert$

0 – 10	6	6	5	22.85	137.1
10 – 20	8	14	15	12.85.	102.8
20 – 30	14	28	25	2.85	39.9
30 – 40	16	44	35	7.15	114.4
40 – 50	4	48	45	17.15	68.6
50 – 60	2	50	55	27.15	54.3

The class interval containing or 25th item is 20-30. Therefore, 20-30 is the median class.

Now, we know that

Median $= l + \frac{\frac{N}{2}+C}{f} \times h$

Here, l = 20, C = 14, f = 14, h = 10 and N = 50

Median $= 20 + \frac{\frac{25-14}{2}}{14} \times 10 = 20 + \frac{110}{14} = 27.85$

So, we can calculate the absolute values of the deviations from the median, i.e., |xi – Med.|,

M.D.(M) $= \frac{1}{N}\sum_{i=1}^{6} f_i\,|x_i - M| = \frac{1}{50} \times 517.1 = 10.34$

Hence, the mean deviation about the median for the given data is 10.34.

Q. 12 Calculate the mean deviation about median age for the age distribution of 100 persons given below:

Age	16 – 20	21 – 25	26 - 30	31 – 35	36 – 40	41 – 45	46 – 50	51 – 55
number	5	6	12	14	26	12	16	9

[Hint Convert the given data into continuous frequency distribution by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit of each class interval]

Answer:

We first convert the given data into continuous frequency distribution by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit of each class interval and then form the following table adding the other columns after calculations.

Age	Number f_i	Cumulative frequency (c.f.)	Mid – point x_i	$\lvert x_i$ – med.$\rvert$	$f_i \lvert x_i$ – med.$\rvert$
15.5 – 20.5	5	5	18	20	100
20.5 – 25.5	6	11	23	15	90
25.5 – 30.5	12	23	28	10	120
30.5 – 35.5	14	37	33	5	70
35.5 – 40.5	36	63	38	0	0
40.5 – 45.5	12	75	43	5	60
45.5-50.5	16	91	48	10	160
50.5-55 .5	9	100	53	15	135
	$\sum f_i =$ 100				$\sum Fi\ \lvert xi - med\ .\rvert =$ 735

The class interval containing theory 50^{th} item is 35.5 – 40.5

Therefor, 35.5 - 40.5 is the median class

It is known that

Median $= l + \frac{\frac{N}{2} - c}{f} \times h$

Here l = 35.5, C = 37, f = 26, h = 5 and N = 100

Median $= 35.5 + \frac{\frac{50-37}{2}}{26} \times 5 = 35.5 + \frac{13 \times 5}{26} = 35.5 + 2.5 = 38$

Exercise 15.2

Q. 1 Find the mean and variance for each of the data.

6, 7, 10, 12, 13, 4, 8, 12

Answer:

We know that Mean,

Mean, $\overline{x} = \frac{\sum_{i=1}^{8} x_i}{n} = \frac{6+7+10+12+13+4+8+12}{8} = \frac{72}{8} = 9$

From the given data, we can form the table:

x_i	Deviation forms mean $\lvert x_i - \overline{x} \rvert$	$(x_i - \overline{x}\rvert^2$
6	6 – 9 = - 3	9
7	7 – 9 = -2	4
10	10 – 9 = 1	1
12	12 – 9 = 3	9
13	13 – 9 = 4	16
4	4 – 9 = - 5	25
8	8 – 9 = - 1	1
12	12 – 9 = 3	9
		$\sum \lvert x_i - \overline{x} \rvert^2 = 74$

We know that Variance, $\sigma^2 = \frac{1}{n} \sum_{i=1}^{8} (x_i - \overline{x})^2 = \frac{1}{8} \times 74 = 9.25$

Mean = 9 and Variance = 9.25

Q. 2 Find the mean and variance for each of the data.

First 10 multiples of 3

Answer:

The 10 multiples of 3 are:

3, 6, 9, 12, 15, 18, 21, 24, 27, 30

Here the number of observation, n = 10

We know that Mean,

Mean $\overline{x} = \frac{\sum_{i=1}^{10} x_i}{10} = \frac{165}{10} = 16.5$

From the given data, we can form the table:

x_i	Deviations from mean $(x_i - \bar{x})$	$(X_i - \bar{x})^2$
3	3 -16.5 = -13.5	182.25
6	6 -16.5 = -10.5	110.25
9	9 -16.5 = -7.5	56.25
12	12 -16.5 = -4.5	20. 25
15	15 -16.5 = -1.5	2. 25
18	18 -16.5 = 1.5	2. 25
21	21 -16.5 = 4.5	20.25
24	24 -16.5 = 7.5	56.25
27	27 -16.5 = 10.5	110.25
30	30 -16.5 = 13.5	182.25
		$\sum_{f=1}^{10} (x_i - \bar{x})^2 = 742.5$

We know that Variance, σ² = $1\frac{1}{n}\sum_{i=1}^{10}(x_i - \overline{x})^2 = \frac{1}{10} \times 742.5 = 74.25$

Ans. Mean = 16.5 and Variance = 74.25

Q. 3 Find the mean and variance for each of the data.

x_i	6	10	14	18	24	28	30
y_i	2	4	7	12	8	4	3

Answer:

Presenting the data in the tabular form, we get

x_i	f_i	$f_i x_i$	(x_i - $\bar{x}$)	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
6	2	12	6-19 = -13	169	338
10	4	40	10-19 = -9	81	324
14	7	98	14-19 =-5	25	175
18	12	2136	18 – 19 = -1	1	12
24	8	192	24-19 =5	25	200
28	4	112	28-19=9	81	324
30	3	90	30-19 = 11	121	363
	$\Sigma f_i = N = 40$	$\Sigma f_i x_i$=760			$\Sigma f_i(x_i - \bar{x})^2$=1736

We know that Mean,

Where N = 40

$\therefore \overline{x} = \frac{\sum_{i=1}^{7} f_i x_i}{N} = \frac{760}{40} = 19$

We know that Variance, $\sigma^2 = \frac{1}{n}\sum_{i=1}^{7}(x_i - \overline{x})^2 = \frac{1}{40} \times 1736 = 43.4$

Mean = 19 and Variance = 43.4

Q. 4 Find the mean and variance for each of the data.

x_i	92	93	97	102	104	109
f_i	3	2	3	6	3	3

Answer:

Presenting the data in the tabular form, we get

x_i	f_i	$f_i x_i$	$\|x_i - \overline{x}\|$	$\|x_i - \overline{x}\|^2$	$f_i \|x_i - \overline{x}\|^2$

92	3	276	92 – 100 = -8	64	192
93	2	186	93 – 100 = -7	49	98
97	3	291	97 – 100 = -3	9	27
98	2	196	98 – 100 = -2	4	8
102	6	612	102 -100 = 2	4	24
104	3	312	104 -100 = 4	16	48
109	3	327	109 -100 = 9	81	243

We know that Mean,

Where N = 22

$\therefore \overline{x} = \frac{1}{N}\sum_{i=1}^{7} f_i x_i = \frac{1}{22} \times 2200 = 100$

We know that Variance, $\sigma^2 = \frac{1}{n}\sum_{i=1}^{7}(x_i - \overline{x})^2 = \frac{1}{22} \times 640 = 29.09$

Mean = 100 and Variance = 29.09

Q. 5 Find the mean and standard deviation using short-cut method.

x_i	60	61	62	63	64	65	66	67	68
f_i	2	1	12	29	25	12	10	4	5

Answer:

Let the assumed mean, A = 64 and h = 1

We obtain the following table from the given data:

x_i	fi	$y_i = \frac{x_i - A}{h}$	y_1^2	$f_i y_i$	$f_i y_i^2$
60	2	-4	16	-8	32
61	1	-3	9	-3	9

62	12	-2	4	-24	48
63	29	-1	1	-29	29
64	25	0	0	0	0
65	12	1	1	12	12
66	10	2	4	20	40
67	4	3	9	12	36
68	5	4	16	20	80

We know that Mean,

∴Mean, $\overline{x}$ = A $\frac{\sum_{i=1}^{9} f_i x_i}{N} \times h = 64 + \frac{0}{100} \times 1 = 64 + 0 = 64$

We know that Variance σ2 =

∴ $\sigma^2 = \frac{h^2}{N^2}\left[N \sum_{i=1}^{9} f_i {x_i}^2 - \left(\sum_{i=1}^{9} f_i x_i\right)^2\right]$

$= \frac{1}{100^2}[100 \times 286 - 0]$

$= 2.86$

We know that Standard Deviation = σ

∴ $\sigma = \sqrt{2.86} = 1.691$

Ans. Mean = 64 and Standard Deviation = 1.691

Q. 6 Find the mean and variance for the following frequency distributions.

Classes	0-30	30-60	60-90	90-120	120-150	150-180	180-210
Frequencies	2	3	5	10	3	5	2

Answer:

classes	frequency f_i	Midpoint X_i	$f_i x_i$	$(X_i - \bar{X})$	$(X_i - \bar{X})^2$	$f_i(X_i - \bar{X})^2$

0-30	2	15	30	-92	8464	16928
30-60	3	45	135	-62	3844	11532
60-90	5	75	375	-32	1024	5120
90-120	10	105	1050	-2	4	40
120-150	3	135	405	28	784	2352
150-180	5	165	825	58	3364	16820
180-210	2	195	390	88	7744	15488
	Σf_i=N = 30		$\Sigma f_i X_i = 3210$			$\Sigma f_i (X_i - \bar{X})^2 = 68280$

Presenting the data in the tabular form, we get

We know that Mean,

Where N = 30

$\therefore \overline{x}$ = A + $\frac{\sum_{i=1}^{7} f_i y_i}{N} \times h = 105 + \frac{20}{30} \times 30 = 105 + 2 = 107$

We know that Variance, σ2 =

$= \frac{h^2}{N^2}\left[N \sum_{I=1}^{7} f_i {x_i}^2 - \left(\sum_{i=1}^{7} f_i x_i\right)^2\right]$

$= \frac{(30)^2}{(30)^2}[\ 30 \times 76 - (2)^2]$

= 2280- 4

= 2276

Ans. Mean = 107 and Variance = 2276

Q. 7 Find the mean and variance for the following frequency distributions.

Class	0-10	10-20	20-30	30-40	40-50
Frequencies	5	8	15	16	6

Answer:

Presenting the data in the tabular form, we get

classes	frequency f_i	Midpoint X_i	$f_i x_i$	$(X_i$-$\bar{X})$	$(X_i - \bar{X})^2$	$f_i(X_i - \bar{X})^2$
0-10	5	5	25	-22	484	2420
10-20	8	15	120	-12	144	1152
20-30	15	25	375	-2	4	60
30-40	16	35	560	8	64	1024
40-50	6	45	270	18	324	1944
	Σf_i=N = 50		$\Sigma f_i X_i = 1350$			$\Sigma f_i(X_i - \bar{X})^2$=6600

We know that Mean,

$$\therefore \overline{x} = A + \frac{\sum_{i-1}^{3} f_i y_i}{N} \times h = 25 + \frac{10}{50} \times 10 = 25 + 2 = 27$$

We know that Variance

$$\therefore \sigma^2 = \frac{h^2}{N^2}\left[N \sum_{i=1}^{5} f_i {y_i}^2 - \left(\sum_{i=1}^{5} f_i y_i\right)^2\right]$$

$$= \frac{(10)^2}{(50)^2}[50 \times 68 - (10)^2]$$

$$= \frac{1}{25}[3400 - 100] = \frac{3300}{25}$$

= 132

Ans. Mean = 27 and Variance = 132

Q. 8 Find the mean, variance and standard deviation using short-cut method

Height in cms	70-75	75-80	80-85	85-90	90-95	95-110	105-110	105-115
No. of children	3	4	7	15	9	6	6	3

Answer:

Let the assumed mean, A = 92.5 and h = 5

We obtain the following table from the given data:

Height (class)	Number of children (frequency) f_i	Midpoint X_i	$y_i = \frac{x_{1-A}}{h}$	${y_i}^2$	$f_i y_i$	$f_i {y_i}^2$
70-75	3	72.5	-4	16	-12	48
75-80	4	72.5	-3	9	-12	36
80-85	7	38.5	-2	4	-14	28
85-90	7	42.5	-1	1	-7	7
90-95	15	46.5	0	0	0	0
95-100	9	50.5	1	1	9	9
100-105	6	102.5	2	4	12	24
105-110	6	107.5	3	9	18	54
110-115	3	112.5	4	16	12	48
	$\Sigma f_i = N = 60$				$\Sigma f_i y_i = 6$	$\Sigma f_i {y_i}^2$=254

We know that Mean,

$$\therefore \overline{x} = A + \frac{\sum_{i=1}^{9} f_i y_i}{N} \times h = 92.5 + \frac{6}{60} \times 5 = 92.5 + 0.5 = 93$$

We know that Variance

$\therefore \sigma^2 = \frac{h^2}{N^2}\left[N\sum_{i=1}^{9} f_i {y_i}^2 - \left(\sum_{i=1}^{9} f_i y_i\right)^2\right]$

$= \frac{(5)^2}{(60)^2}[60 \times 254 - (6)^2]$

$= \frac{25}{3600}(15204) = 105.58$

We know that Standard Deviation = σ

$\therefore \sigma = \sqrt{105.58} = 10.275$

Ans. Mean = 93, Variance = 105.583 and Standard Deviation = 10.275

Q. 9 The diameters of circles (in mm) drawn in a design are given below:

Diameters	33-36	37-40	41-44	45-48	49-52
No. of circles	15	17	21	22	25

Calculate the standard deviation and mean diameter of the circles.

[Hint: First make the data continuous by making the classes as 32.5-36.5, 36.5-40.5,

40.5-44.5, 44.5 - 48.5, 48.5 - 52.5 and then proceed.]

Answer:

Let the assumed mean, A = 42.5 and h = 4

We obtain the following table from the given data:

Height (class)	Number of children (frequency)	Midpoint X_i	$y_i = \frac{x_{1-A}}{h}$	$f_i y_i$	$f_i {y_i}^2$
32.5-36.5	15	34.5	-2	-30	60

36.5-40.5	17	38.5	-1	-17	17
40.5-44.5	21	42.5	0	0	0
44.5-48.5	22	46.5	1	22	22
48.5-52.5	25	50.5	4	50	100
	$\Sigma f_i = N = 100$			$\Sigma f_i y_i = 25$	$\Sigma f_i {y_i}^2$=199

We know that Mean,

$\therefore \overline{x} = \text{A} + \frac{\sum_{i=1}^{5} f_i y_i}{N} \times h = 42.5 + \frac{25}{100} \times 4 = 43.5$

We know that Variance σ2 =

$\therefore \sigma^2 = \frac{h^2}{N^2}\left[N \sum_{i=1}^{5} f_i {y_i}^2 - \left(\sum_{i=1}^{5} f_i y_i\right)^2\right]$

$= \frac{16}{10000}[100 \times 199 - (25)^2]$

$= \frac{16}{10000}[19900 - 625]$

$= \frac{16}{10000}(19275)$

= 30.84

We know that Standard Deviation = σ

$\therefore \sigma = \sqrt{30.84}$= 5.553

Ans. Mean = 43.5, Variance = 30.84 and Standard Deviation = 5.553

Exercise 15.3

Q. 1 From the data given below state which group is more variable, A or B?

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Group A	9	17	32	33	40	10	9
Group B	10	20	30	25	43	15	7

Answer:

The group having higher coefficient of variation will be more variable

So, we will calculate

Where σ is standard deviation is mean

For group A

Marks	Group A f_i	Mid-point x_i	$y_i = \frac{x_i - A}{h}$	$(y_i)^2$	$f_i y_i$	$f_i (y_i)^2$
10-20	9	15	$\frac{15-45}{10}$ = -3	$(-3)^2$ =9	-27	81
20-30	17	25	$\frac{25-45}{10}$ = -2	$(-2)^2$ =4	-34	68
30-40	32	35	$\frac{35-45}{10}$ = -1	$(-1)^2$ =1	-32	32
40-50	33	45	$\frac{45-45}{10}$ = 0	$0^2 = 0$	0	0
50-60	40	55	$\frac{55-45}{10}$ = 1	$(1)^2$ =1	40	40
60-70	10	65	$\frac{65-45}{10}$ = 2	$(2)^2$ =4	20	40
70-80	9	75	$\frac{75-45}{10}$ = 3	$(3)^2$ =9	27	81
total	150				-6	342

Mean

Where A is assumed mean = 45

h = class size= 20- 10 = 10

Mean = A + $\frac{\sum_{i=1}^{7} x_i}{N} \times h = 45 + \frac{(-6)\times 10}{150} = 45 - 0.4 = 44.6$

Variance $\sigma_1^2 = \frac{h^2}{N^2}\left(N \sum_{i=1}^{7} f_i {x_i}^2 - \left(\sum_{i=1}^{7} f_i x_i\right)^2\right)$

$= \frac{100}{22500}[150 \times 342 - (-6)^2]$

$= \frac{1}{225} \times 51264$

= 227.84

Standard deviation σ1=$\sqrt{227.84} = 15.09$

The standard deviation of group B is calculated as follows.

Marks	Group B f_i	Mid-point x_i	$y_i = \frac{x_i - A}{h}$	$(y_i)^2$	$f_i y_i$	$f_i(y_i)^2$
10-20	10	15	$\frac{15-45}{10}$ = -3	9	-30	90
20-30	20	25	$\frac{25-45}{10}$ = -2	4	-40	80
30-40	30	35	$\frac{35-45}{10}$ = -1	1	-30	30
40-50	25	45	$\frac{45-45}{10}$ = 0	0	0	0
50-60	43	55	$\frac{55-45}{10}$ = 1	1	43	43
60-70	15	65	$\frac{65-45}{10}$ = 2	4	30	160
70-80	7	75	$\frac{75-45}{10}$ = 3	9	21	189
total	150				-6	592

Variance $\sigma_2{}^2 = \frac{h^2}{N^2}\left[N\sum_{i=1}^{7} f_i {x_i}^2 - \left(\sum_{i=1}^{7} f_i x_i\right)^2\right]$

$= \frac{100}{22500}[150 \times 366 - (-6)^2]$

$= \frac{1}{225}[54864] = 243.84$

Standard deviation = $\sqrt{243.84}$ = 15.61

Since the mean of both the group is same, the group with greater standard deviation will be more variable.

Thus, group B has more variability in the marks.

Q. 2 From the prices of shares x and y below, find out which is more stable in value:

X	35	54	53	56	58	52	50	51	49
Y	108	107	105	106	107	104	103	104	101

Answer:

The prices of the shares x are

35, 54, 52, 53, 56, 58, 52, 50, 51, 49

Here. The number of observation, N = 10

Mean $\overline{x} = \frac{1}{N}\sum_{i=1}^{10} x_i = \frac{1}{10} \times 510 = 51$

X (x_i)	Y(y_i)	${x_i}^2$	${y_i}^2$
35	108	1225	11664
54	107	2916	11449
52	105	2704	11025
53	105	2809	11025
56	106	8136	11236
58	107	3364	11449
52	104	2704	10816
50	103	2500	10609
51	104	2601	10816

49	101	2401	10201
Total = 510	1050	26360	110290

Variance $(\sigma_1{}^2) = \frac{1}{N}\sum_{i=1}^{10}(x_i - \overline{x})^2 = \frac{1}{10} \times 350 = 35$

Standard deviation (σ1) = $\sqrt{35} = 5.91$

C.V. (shares x) = $\frac{\sigma_1}{x} \times 100 = \frac{5.91}{51} \times 100 = 11.58$

The prices of shares Y are

108, 107, 105, 105, 106, 107, 104, 103, 104, 101

Mean $\overline{x} = \frac{1}{N}\sum_{i=1}^{10} y_i = \frac{1010}{11} \times 1050 = 105$

The following table is obtained corresponding to shares y.

y_i	$(y_i - \overline{y})$	$(y_i - \overline{y})^2$
108	3	9
107	2	4
105	0	0
105	0	0
106	1	1
107	2	4
104	-1	1
103	-2	4
104	-1	1
101	-4	16
		40

Variance $(\sigma_1{}^2) = \frac{1}{N}\sum_{i=1}^{10}(y_i - \overline{y})^2 = \frac{1}{10} \times 40 = 4$

Standard deviation $(\sigma_2) = \sqrt{4} = 2$

C.V. (shares Y) = $\frac{\sigma^2}{y} \times 100 = \frac{2}{105} \times 100 = 1.9$

C.V. of prices of shares X is greater than the C.V. of prices of shares y.

Thus, the prices of shares Y are more stable than the prices of shares X.

Q. 3 An analysis of monthly wages paid to workers in two firms A and B, belonging to the same industry, gives the following results:

	Firm A	Firm B
No of wage earners	586	648
Mean of monthly wages	$5253	$5253
Variance of the distribution of wages	100	121

(i) Which firm A or B pays larger amount as monthly wages?

(ii) Which firm, A or B, shows greater variability in individual wages?

Answer:

Here

(i) monthly wages of firm A = 5253

No. of wage earners = 586

Total amount paid = 586 × 5253 = 3078258

Mean monthly wages of firm B = 5253

No. of wage earners = 648

Total amount paid = 648 × 5253 = 340 3944

Thus, firm B pays the larger amount as monthly wages as the number of wage earners in firm B are more than the number of wage earners in firm A.

(ii) Variance of the distribution of wages in firm A ($\sigma_1{}^2$) = 100

Standard deviation of the distribution of wages in firm

A $(\sigma_1) = \sqrt{100} = 10$

Variance of the distribution of wages in firm B $(\sigma_2{}^2)$ = 121

Standard deviation of the distribution of wages in firm B $(\sigma_2{}^2)$ = $\sqrt{121} = 11$

The mean of monthly wages of both the firms is same i.e., 5253.

Therefore, the firm with greater

Thus, firm B has greater variability in the individual wages.

Q. 4 The following is the record of goals scored by team A in a football session:

No. of goals scored	0	1	2	3	4
No. of matches	1	9	7	5	3

For the team B, mean number of goals scored per match was 2 with a standard deviation 1.25 goal. Find which team may be considered more consistent?

Answer:

No. of goals scored x_i	No. of matches f_i	f_i x_i	x_i^2	fix_i^2
0	1	0	0	0
1	9	9	1	9
2	7	14	4	28
3	5	15	9	45
4	3	12	16	48
Total	$\sum f_i$ = 25	$\sum f_i x_i$ = 50		$\sum f_i x_i^2$ = 130

Mean = $\frac{\sum_{i=1}^{3} f_i x_i}{\sum_{i=1}^{3} f_i} = \frac{50}{25} = 2$

Thus, the mean of both the teams is same.

$\sigma = \frac{1}{N}\sqrt{N\sum f_i x_i{}^2 - (\sum f_i x_i)^2}$

$= \frac{1}{25}\sqrt{25 \times 130 - (50)^2}$

$= \frac{1}{25}\sqrt{750}$

$= \frac{1}{25} \times 27.38$

= 1.09

The standard deviation of team B is 1.25 goals.

The average number of goals scored by both the teams is same i.e., 2.

Therefore, the team with lower standard deviation will be more consistent.

Thus, team A is more consistent than team B.

Q. 5 The sum and sum of squares corresponding to length x (in cm) and weight y (in gm) of 50 plant products are given below:

Which is more varying, the length or weight?

Answer:

$$\sum_{i=1}^{50} x_1 = 212, \sum_{i=1}^{50} x_1{}^2 = 902.8$$

Here, N = 50

Mean, $\overline{x} = \frac{\sum_{i=1}^{50} y_i}{N} = \frac{212}{50} = 4.24$

Variance $(\sigma_1{}^2) = \frac{1}{N}\sum_{i=1}^{50}(x_i - \overline{x})^2$

$= \frac{1}{50}\sum_{i=1}^{50}(x_i - 4.24)^2$

$= \frac{1}{50}\sum_{i=1}^{50}[{x_i}^2 - 8.48x_i + 17.97]$

$= \frac{1}{50}\left[\sum_{i=1}^{50} {x_1}^2 - 8.48\sum_{i=1}^{50} x_i + 17.97 \times 50\right]$

$= \frac{1}{50}[902.8 - 8.48 \times (212) + 898.5]$

$= \frac{1}{50}[1801.3 - 1797.76]$

$= \frac{1}{50} \times 3.58$

$= 0.07$

Standard deviation, σ_1(Length) = $\sqrt{0.07} = 0.26$

C.V. (Length) $= \frac{Standard\ deviation}{Mean} \times 100 = \frac{0.26}{4.24} \times 100 = 6.13$

$\sum_{i=1}^{50} y_i = 261, \sum_{i=1}^{50} {y_1}^2 = 1457.6$

Mean, $\overline{y} = \frac{1}{N}\sum_{i=1}^{50} y_i = \frac{1}{50} \times 261 = 5.22$

Variance $({\sigma_2}^2) = \frac{1}{N}\sum_{i=1}^{50}(y_i - \overline{y})^2$

$= \frac{1}{N}\sum_{i=1}^{50}(y_i - 5.22)^2$

$= \frac{1}{50}\sum_{i=1}^{50}[{y_i}^2 - 10.44y_i + 27.24]$

$= \frac{1}{50}\left[\sum_{i=1}^{50} {y_i}^2 - 10.44\sum_{i=1}^{50} y_i + 27.24 \times 50\right]$

$= \frac{1}{50}[1457.6 - 10.44 \times (212) + 1362]$

$= \frac{1}{50}[2819.6 - 2724.84]$

$= \frac{1}{50} \times 94.76$

$= 1.89$

Standard deviation, σ_2(weight) = 1.37

C.V.(weight) $= \frac{Standard\ deviation}{Mean} \times 100 = \frac{1.37}{5.22} \times 100 = 26.24$

Thus, C.V. of weight is greater than the C.V. of lengths.

Therefore, weights very more than the lengths.

Miscellaneous Exercise

Q. 1 The mean and variance of eight observations are 9 and 9.25, respectively. If six of the observations are 6, 7, 10, 12, 12 and 13, find the remaining two observations.

Answer:

Let us assume the remaining two observations to be x and y respectively such that,

Observations: 6, 7, 10, 12, 12, 13, x, y.

Mean, $\overline{x} = \frac{6+7+10+12+2+13+x+y}{8} = 9$

$= 60 + x + y = 72$

$= x + y = 12$... (1)

Variance $= 9.25 = \frac{1}{n}\sum_{i=1}^{8}(x_i - \overline{x})^2$

$9.25 = \frac{1}{8}[(-3)^2 + (-2)^2 + (1)^2 + (3)^2 + (3)^2 + (4)^2 + x^2 + y^2 - 2 \times 9(x + y) + 2 \times (9)^2]$

$9.25 = \frac{1}{8}[9 + 4 + 1 + 9 + 9 + 16 + x^2 + y^2 - 18(12) + 162]$

$9.25 = \frac{1}{8}[48 + x^2 + y^2 - 216 + 162]$

$9.25 = \frac{1}{8}[x^2 + y^2 - 6]$

$x_2 + y_2 = 80$… (2)

From (1), we obtain

$x_2 + y_2 + 2xy = 144$… (3)

From (2) and (3), we obtain

$2xy = 64$… (4)

Substituting (4) from (2), we obtain

$x_2 + y_2 - 2xy = 80 - 64 = 16$

$= x - y = \pm 4 \ldots (5)$

Therefore, from (1) and (5), we obtain

$x = 8$ and $y = 4$, when $x - y = 4$

$x = 4$ and $y = 8$, when observations are 4 and 8.

Q. 2 The mean and variance of 7 observations are 8 and 16, respectively. If five of the observations are 2, 4, 10, 12, 14. Find the remaining two observations.

Answer:

Let us assume the remaining two observations be x and y

The given observations in the question are 2, 4, 10, 12, 14, x, y

Mean, $\overline{x} = \frac{2+4+10+12+14+x+y}{7} = 8$

$= 56 = 42 + x + y$

$= x + y = 14 \ldots (1)$

Variance $= 16 = \frac{1}{n}\sum_{i=1}^{7}(x_i - \overline{x})^2$

$16 = \frac{1}{7}[(-6)^2 + (-4)^2 + (x_i - \overline{x})^2 + (6)^2 + x^2 + y^2 - 2 \times 8\ (x + y) + 2 \times (8)^2]$

$16 = \frac{1}{7}[(-6)^2 + 4 + 16 + 36 + x^2 + y^2 - 16(14) + 2(64)]$

$16 = \frac{1}{7}[36 + 16 + 4 + 16 + 36 + x^2 + y^2 - 16(14) + 2(64)]$

$16 = \frac{1}{7}[108 + x^2 + y^2 - 224 + 128]$

$16 = \frac{1}{7}[12 + x^2 + y^2]$

$= x_2 + y_2 = 112 – 12 = 100$

$x_2 + y_2 = 100$ .... (2)

From (1), we obtain

$x^2 + y^2 + 2xy = 196$ … (3)

From (2) and (3), we obtain

$2xy = 196 - 100$

$= 2xy = 96$… (4)

Substituting (4) from (2), we obtain

$x_2 + y – 2xy = 100 - 96$

$= x – y = \pm 2$ …(5)

Therefore, from (1) and (5), we obtain

$x = 8$ and $y = 6$ when $x – y = 2$

$x = 6$ and $y = 8$ when $x – y = - 2$

Thus, the remaining observation are 6 and 8

Q. 3 The mean and standard deviation of six observations are 8 and 4, respectively. If each observation is multiplied by 3, find the new mean and new standard deviation of the resulting observations.

Answer:

Let us assume the observations be x_1, x_2, x_3, x_4, x_5 and x6

It is given in the question that,

Mean = 8 and Standard deviation = 4

Mean, $\overline{x} = \frac{x_1+x_2+x_3+x_4+x_5+x_6}{6} = 8$

Now, according to question if each observation is multiplied by 3 and the resulting observations are y_i then, we have:

$y_i = 3x_i$ *i.e.,* $x_i = \frac{1}{3} y_i$, for I = 1 to 6

New mean, $\overline{y} = \frac{y_1+y_2+y_3+y_4+y_5+y_6}{6}$

$= \frac{3(x_1+x_2+x_3+x_4+x_5+x_6)}{6}$

$= 3 \times 8$

$= 24$

Standard deviation, σ = $\sqrt{\frac{1}{n}\sum_{i=1}^{6}(x_i - \overline{x})^2}$

$$(4)^2 = \frac{1}{6}\sum_{i=1}^{6}(x_i - \overline{x})^2$$

$$\frac{1}{6}\sum_{i=1}^{6}(x_i - \overline{x})^2 = 96 \ldots (2)$$

From (1) and (2), it can be observed that,

$\overline{y} = 3\overline{x}$

$\overline{x} = \frac{1}{3}\overline{y}$

Substituting the value of xi and $\overline{x}$ in (2), we obtain

$$\sum_{i=1}^{6}\left(\frac{1}{3}y_i - \frac{1}{3}y\right)^2 = 96$$

$$= \quad \sum_{i=1}^{6}\left(\frac{1}{3}y_i - \frac{1}{3}y\right)^2 = 864$$

Therefore, variance of new observation = $(\frac{1}{6} \times 864) = 144$

Hence, the standard deviation of new observation is $\sqrt{144} = 12$

Q. 4 Given that is the mean and is the variance of n observations x_1, x_2, ..., x_n.

Prove that the mean and variance of the observations ax1, ax_2, ax_3, ..., a $_n$ is and, respectively,

Answer:

The given observations in the question are x_1, x_2, $\ldots x_n$

And, variance = σ^2

$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} y_i\,(x_i - \overline{x})^2 \ldots$ (1)

If each observation is multiplied by a and the new observation are Y_i, then

y_i = ax i.e., $X_i = \frac{1}{a} Y_i$

$\overline{Y} = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{1}{n}\sum_{i=1}^{n} ax_i = \frac{a}{n}\sum_{i=1}^{n} x_i = a\overline{x}$ $\left(\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i\right)$

Therefore, mean of the observation, ax_1, ax_2, $\ldots ax_n$, is

Substituting the value of x_i and $\overline{x}$ in (1), we obtain

$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{a}y_i - \frac{1}{a}y\right)^2$

$= a^2\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(y_i - \overline{y})^2$

Thus, the variance of the observation, ax_1, $ax_2 \ldots ax_n$, is $a^2\sigma^2$

Q. 5 A

The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation 8 was incorrect.

Calculate the correct mean and standard deviation in each of the following cases:

If wrong item is omitted

Answer:

Total number of observations (n) = 20

Also, incorrect mean = 20

And, incorrect standard deviation = 2

Thus, incorrect sum of observations = 200

Hence, correct sum of observations = 200 – 8

= 192

= 10.1

Thus,

= 2080 – 64

= 2016

= 2.02

Q. 5 B

The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation 8 was incorrect.

Calculate the correct mean and standard deviation in each of the following cases:

(i) If wrong item is omitted.

(ii) If it is replaced by 12

Answer:

(i) Number of observation (n) = 20
Incorrect mean = 10
Incorrect standard deviation = 2

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{20} x_i$$

$$10 = \frac{1}{20}\sum_{i=1}^{20} x_i$$

$$= \sum_{i-1}^{20} x_{i=200}$$

incorrect sum of observations = 200

Also, correct sum of observations = 200 – 8 = 192

Correct mean = $\frac{Correct\ sum}{19} = \frac{192}{19} = 10.1$

Standard deviation σ = $\sqrt{\frac{1}{n}\sum_{i=1}^{n} {x_i}^2 - \frac{1}{n^2}\left(\sum_{i=1}^{n}(x)\right)^2}$

$$= \sqrt{\frac{1}{n}\frac{1}{4}x^2 - (x)^2}$$

= 2 = $\sqrt{\frac{1}{20} Incorrect\sum_{i=1}^{n} {x_i}^2 - (10)^2}$

= 4 = $\frac{1}{20} Incorrect\sum_{k=1}^{n} x^2 - 100$

= Incorrect $\sum_{H=1}^{n} x^2 = 2080$

Correct $\sum {x_1}^2 = Incorret\sum_{k=1}^{n} {x_i}^2 - (8)^2$

= 2080- 64

= 2016

Correct standard deviation = $\sqrt{\frac{Correct\ \sum x^2}{n} - (Correct mean\ )^2}$

= $\sqrt{\frac{2016}{19} - (10.1)^2}$

= $\sqrt{1061.1 - 102.1}$

= $\sqrt{4.09}$

= 0.02

(ii) When 8 is replaced by 12,

Incorrect sum of observation = 200

Correct sum of observation = 200 – 8 + 12 = 204

Correct mean = $\frac{Correct\ sum}{20} = \frac{204}{20} = 10.2$

Standard deviation σ =

$$\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i{}^2 - \frac{1}{n^2}\left(\sum_{i=1}^{n} x_i\right)^2} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i{}^2 - (\overline{x})^2}$$

= $2 = \sqrt{\frac{1}{20} Incoeerct \sum_{i=1}^{n} x_i{}^2 - (10)^2}$

= $4 = \frac{1}{20} Incorrect \sum_{i=1}^{n} x_i{}^2 - 100$

= Incorrect $\sum_{i=1}^{n} x_i{}^2 = 2080$

Correct $\sum_{i=1}^{n} x_i{}^2 = Incorrect \sum_{i=1}^{n} x_i{}^2 - (8)^2 + (12)^2$

= 2080 – 64 + 144

= 2160

Correct standard deviation = $\frac{Correct\ \sum x_i{}^2}{n} - Correct\,mean\ {}^2$

= $\sqrt{\frac{2160}{20} - (10.2)^2}$

= $\sqrt{108 - 104.04}$

= 13.96

= 1.98

Q. 6 The mean and standard deviation of marks obtained by 50 students of a class in three subjects, Mathematics, Physics and Chemistry are given below:

Subject	Mathematics	Physics	Chemistry
Mean	42	32	40.9
Standard deviation	12	1520	

Which of the three subjects shows the highest variability in marks and which shows the lowest?

Answer:

Standard deviation of mathematics = 12

Also, standard deviation of physics = 15

And, standard deviation of chemistry = 20

The coefficient of variation (c.v.) is given by $\frac{Standard\ deviation}{Mean} \times 100$

C.V. (in mathematics) = $\frac{12}{42} \times 100 = 28.57$

C.V. (in physics) = $\frac{15}{32} \times 100 = 46.87$

C.V. (in chemistry) = $\frac{20}{40.9} \times 100 = 48.89$

The subject with greater C.V. is more variable than others.

Therefore, the highest variability in marks is in chemistry and the lowest variability in marks is in mathematics.

Q. 7 The mean and standard deviation of a group of 100 observations were found to be 20 and 3, respectively. Later on, it was found that three observations were incorrect, which were recorded as 21, 21 and 18. Find the mean and standard deviation if the incorrect observations are omitted.

Answer:

Total number of observations (n) = 100

Incorrect mean, $(\overline{x})$ = 20

And, Incorrect standard deviation (σ) = 3

$= 20 + \frac{1}{100}\sum_{i=1}^{100} x_i$

$= \sum_{i=1}^{100} x_i = 20 \times 100 = 2000$

Thus, incorrect sum of observations is 2000

Now, correct sum of observations = 2000 – 21 – 21 – 18= 2000 – 60= 1940

Correct mean $= \frac{Correct\ sum}{100-3} = \frac{1940}{97} = 20$

Standard deviation (σ) = $\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i - \frac{1}{n^2}\left(\sum_{i=1}^{n} x_i\right)^2}$

$$= \sqrt{\frac{1}{n}\sum_{i=1}^{n} {x_i}^2 - (\overline{x})^2}$$

$= 3 = \sqrt{\frac{1}{100} \times Incorrect \sum {x_i}^2 - (20)^2}$

= Incorrect $\sum {x_i}^2 = 100(9 + 400) = 40900$

Correct $\sum_{i=1}^{n} {x_t}^2 = Incorrect \sum_{i=1}^{n} {x_i}^2 - (21)^2 - (21)^2 - (18)^2$

= 40900 – 441 – 441 – 324

= 39694

Correct standard deviation = $\sqrt{\frac{Correct\ \sum {x_i}^2}{n} - (Correct\ mean\ )^2}$

$$= \sqrt{\frac{39694}{97} - (20)^2}$$

$$= \sqrt{9.216}$$

$$= 3.036$$