Rational Numbers

Introduction

We have already studied about the integers, the fractions, the decimals and their operations. We know that for a given integer a and b, their addition, subtraction and multiplication are always an integer but the result of division of an integer by a non-negative integer may or may not be an integer. In this chapter we will study about a new number system which is formed by the division of two integers.

A number which is in the form of $\frac{a}{b}$ where and b are integers and $b \neq 0$ is called a rational number, for example $\frac{-1}{2}, \frac{3}{-14}, \frac{15}{7}$ are rational numbers.

Important Point Related to Rational Numbers

- Zero is a rational number because 0 can be written as $\frac{0}{a}$ where a $a \neq 0$
- Every natural number is a rational number but the rational number may or may not be a natural number, for example $\frac{2}{3}$ is a rational number which is not a natural number.
- Every whole number is a rational number but the rational number may or may not be a whole number, for example $\frac{-2}{3}$ is a rational number which is not a whole number.
- Every integers is a rational number but the rational number may or may not be an integer, for example $\frac{3}{5}$ is a rational number which is not an integer.

Types of Rational Number

Rational numbers are of two types.

- Positive rational numbers
- Negative rational numbers

Positive Rational Numbers

If the numerator and denominator of a rational number having same sign then it is said to be positive rational number, for example $\frac{3}{7}, \frac{-4}{-3}, \frac{15}{25}, \frac{-13}{-15}$ are positive rational numbers.

Negative Rational Number

If the numerator and denominator of a rational number having different sign then it is said to be negative rational number, for example $\frac{3}{7}, \frac{-4}{-3}, \frac{15}{25}, \frac{-13}{-15}$ are the negative rational numbers.

Note: 0 is non - negative and non- positive rational number, in other words we can say that it is neither negative nor positive rational number.

Properties of Rational Number

- A rational number remains unaltered if we multiply numerator or denominator by the same non-zero numbers .i.e $\frac{x}{y}$ remains same if we multiply the numerator and denominator by the same non-zero number "m" i.e. $\frac{x}{y} = \frac{m \times x}{m \times y}$

- A rational number remains same if we divide numerator or denominator by the same non - zero numbers .i.e. $\frac{x}{y}$ remains same if we divide numerator and denominator by the same non zero number "n" $\frac{x}{y}=\frac{x \div n}{y \div n}$.

Equivalent Rational Numbers

If we multiply or divide the numerator or denominator of a rational by the same none zero integers then we get equivalent rational number.

Illustrative EXAMPLE

Find the equivalent rational numbers of $\frac{p}{q}$.

Solution:

$\frac{2\times P}{2\times q},\frac{3\times p}{3\times q},\frac{4\times p}{4\times q},\frac{0.235\times p}{0.235\times q}$.

We can write infinite equivalent rational numbers of a rational number.

Lowest Form

Divide the numerator and denominator by the HCF of (Numerator, Denominator) by ignoring the sign of it, so that we get the new numerator and denominator which are co-prime.

Illustrative EXAMPLE

Find the lowest form of $\frac{-60}{96}$.

Solution:

The HCF of 60 and 96 is 12. Therefore, divide the numerator and denominator

By l2.We get $\frac{-60}{96}=\frac{-60\div 12}{96\div 12}=\frac{-5}{8}$

Standard Form of a Rational Number

In the standard form we always write the denominator as positive. For this if the denominator of the rational number is negative then we multiply the numerator and denominator of the rational number by (-1)

Illustrative EXAMPLE

Write the following rational numbers in standard form:

Solution:

In the given problem $\frac{-5}{7},\frac{7}{8}$ are in the standard form because its denominator is positive. But in the rational numbers $\frac{12}{-5},\frac{-3}{-4}$ are not in the standard form so we multiply the numerator and denominator by (-1).

So the standard form of the given rational numbers are $\frac{-12}{5},\frac{3}{4},\frac{-5}{7},\frac{7}{8}$

Comparison of Rational Numbers

Write the rational numbers in the standard form.

Every positive rational number is greater than the negative rational number.

We can compare two rational numbers in the following way:

- By making the denominators same
- By short-cut method

Comparing Rational Number by Making the Denominator Same

Step 1: Write the rational number in the standard form.

Step 2: Find the LCM of all the denominators.

Step 3: Make the denominator same for all the rational numbers.

Step 4: Write all the rational number on the same denominator.

Step 5: Compare the numerators so obtained.

Illustrative EXAMPLE

Arrange the following rational numbers in ascending order

$\frac{3}{-5}, \frac{5}{7}, \frac{-4}{15}, \frac{-7}{-15}$

Solution:

Step 1: Standard form of rational numbers is $\frac{-3}{5}, \frac{5}{7}, \frac{-4}{15}, \frac{7}{15}$

Step 2: The LCM of denominator 5, 7, 15, 15 is 105.

Step 3: Now all the rational number with denominator 105

$\frac{-3}{5} = \frac{(-3) \times 21}{5 \times 21} = \frac{-63}{105}; \frac{5}{7} = \frac{5 \times 15}{7 \times 15} = \frac{75}{105}$

$\frac{-4}{15} = \frac{(-4) \times 7}{15 \times 7} = \frac{-28}{105}; \frac{7}{15} = \frac{7 \times 7}{15 \times 7} = \frac{49}{105}$

Step 4: The numerators are $(-63), 75, (-28)$ and 49. The ascending order of numerators are $-63 < -28 < 49 < 75$.

Therefore $\frac{-63}{105} < \frac{-28}{105} < \frac{49}{105} < \frac{75}{105}$

The ascending order of the given rational numbers is $\frac{-3}{5} < \frac{-4}{15} < \frac{7}{15} < \frac{5}{7}$.

Short cut Method (for comparing two Rational Numbers)

Step 1: Write the rational number in the standard form.

Step 2: Multiply the numerator of first rational number with the denominator of the second and vice - versa.

For two rational numbers which are in the standard form $\frac{A}{B}$ and $\frac{C}{D}$

We find the product of A and D, similarly B and C and compare AD and BC

- If AD> BC then $\frac{A}{B} > \frac{C}{D}$
- If AD> BC then $\frac{A}{B} < \frac{C}{D}$

Illustrative EXAMPLE

Which one of the two fractions is greater $\frac{7}{-8}$ or $\frac{6}{-7}$?

Solution:

Step 1: The standard form of the given rational numbers are $\frac{-7}{8}$ and $\frac{-6}{7}$.

Step 2: The product of the numerator of first and denominator of second is $(-7)\times 7 = -49$ The product of numerator of second and denominator of first is $(-6)\times 8 = (-48)$

Here $(-49)\ < (-48)$. Therefore, $\frac{7}{-8} < \frac{6}{-7}$

Commonly Asked QUESTIONS

Arrange the following rational numbers in descending order

$\frac{17}{30}, \frac{-3}{-5}, \frac{4}{-15}, \frac{-7}{15}$

(a) $\frac{-3}{-5} < \frac{17}{30} < \frac{4}{-15}$

(b) $\frac{-3}{-5} > \frac{17}{30} > \frac{4}{-15} > \frac{-7}{15}$

(c) $\frac{17}{30} > \frac{-3}{-5} > \frac{4}{-15} > \frac{-7}{15}$

(d) $\frac{4}{-15} < \frac{-7}{15} < \frac{-3}{-5} < \frac{17}{30}$

(e) None of these

Answer: (b)

Explanation

The given rational numbers are:

$\frac{17}{30}, \frac{-3}{-5}, \frac{4}{-15}, \frac{-7}{15}$

Write the above rational numbers in standard form.

$\frac{17}{30}, \frac{-3}{-5}, \frac{4}{-15}, \frac{-7}{15}$

The LCM of denominators 30, 5,15,15 is 30.

Now, $\frac{17}{30}, \frac{-3}{-5}, \frac{4}{-15}, \frac{-7}{15}$

$\frac{-4}{15} = \frac{-4\times 2}{15\times 2} = \frac{-8}{30}; \frac{-7}{15} = \frac{-7\times 2}{15\times 2} = \frac{-14}{30}$

Now denominators are equal. The descending order is

$\frac{18}{30} > \frac{17}{30} > \frac{-8}{30} > \frac{14}{30}$

Or $\frac{3}{5} > \frac{17}{30} > \frac{-4}{15} > \frac{-7}{15}$

For three rational number x, y, z such that x > y and y < z. Which one of the following is true?

(a) x<z
(b) x>z
(c) y is the smallest rational number
(d) Both A and Bare correct
(e) None of these

Answer: (c)

Explanation

x > y and y < z => y is the smallest rational number among x, y, z.

Peter defines a rational number in the following ways. "It is or the form $\frac{p}{q}$ where q is the smallest whole number." This definition is:

(a) Always true
(b) Represents some rational number only
(c) Some as the definition of rational number
(d) Always false
(e) None of these

Answer: (d)

Which one of the following statement is true?

(a) The equivalent of $\left(\frac{p}{q}\right)$ is different from $\left(\frac{p}{q}\right)$

(b) $\frac{35p}{33q}$ is the equivalent of $\frac{p}{q}$

(c) There are only 5 equivalent rational numbers of the form $\frac{p}{q}$

(d) We can write infinite equivalent rational numbers of the form $\frac{p}{q}$

(e) None of these

Answer: (d)

Which one of the following statement is false?

(a) "0" is a positive rational number

(b) A positive rational number is in the form $\frac{p}{q}$, where p and q are integers of same sign and $q \neq 0$

(c) A negative rational number is of the form of $\frac{p}{q}$, where p and q are integer of opposite sign and $q \neq 0$

(d) A positive rational number is always greater than the negative rational number

(e) None of these

Answer: (a)

Operation on Rational Numbers

In this topic we will study about addition, subtraction, multiplication and division of rational numbers.

Addition of Rational Numbers

Step 1: Write the rational number in the standard form.
Step 2: Make the denominator same by taking the LCM of denominators.
Step 3: Write all the rational number with the same denominator.
Step 4: Add the numerators
Step 5: Write the numerator getting after addition on the denominator.
Step 6: Reduce the rational number to lowest term.

Illustrative EXAMPLE

Add $2\frac{3}{5}, \frac{-15}{13}, \frac{-13}{-15}, -4\frac{3}{5}$

Solution:

Step 1: The standard form of given rational numbers are
$\frac{13}{5}, \frac{-15}{13}, \frac{13}{15}, \frac{-23}{5}$

Step 2: LCM of 5, 13, 15, 5 is 195

Step3: $\frac{13}{5} = \frac{13 \times 39}{5 \times 39} = \frac{507}{195}$
$\frac{-15}{13} = \frac{-15 \times 15}{13 \times 15} = \frac{-225}{195}$
$\frac{13}{15} = \frac{13 \times 13}{15 \times 13} = \frac{169}{195}$
$\frac{-23}{5} = \frac{23 \times 39}{5 \times 39} = \frac{-897}{195}$

Step 4: The sum of numerators
$= 507 + (-225) + 169 + (-897) = -446$

Step 5: $\frac{-446}{195}$

Step 6: $-2\frac{56}{195}$

Therefore
$2\frac{3}{5} + \frac{-15}{13} + \frac{-13}{-15} + \left(-4\frac{3}{5}\right) = -2\frac{56}{195}$

Subtraction of Two Rational Numbers

Step 1: Write the rational numbers in standard form.
Step 2: Make the denominator same by taking LCM.
Step 3: Subtract numerators.
Step 4: Write the result with denominator and reduce it to lowest terms.

Illustrative EXAMPLE

If the sum of two rational numbers is $\frac{-5}{6}$ and if one of the number is $-\frac{9}{20}$ then find the other rational number.

Solution:

Sum of Rational number $=\frac{-5}{16}$

One number $=\frac{-9}{20}$, Then the other $=\frac{-5}{16}-\left(\frac{-9}{20}\right)$

$$=\frac{-25-(-36)}{80}=\frac{-25+36}{80}=\frac{11}{80}$$

Multiplication of Rational Numbers

Product of rational numbers

$=\frac{\textit{Product of the numerators}}{\textit{Product of the denominators}}$ and reduce it to the lowest terms.

Reciprocal of Rational Number

For non - zero rational numbers, a rational number is said to be reciprocal of other if the product is 1.

Find the product of $\frac{13}{6}\times\frac{-18}{91}\times\frac{-5}{9}\times\frac{72}{-125}$

Solution:

$$\frac{13}{6}\times\frac{-81}{91}\times\frac{-5}{9}\times\frac{72}{-125}=\frac{13\times(-18)\times(-5)\times72}{6\times91\times9(-125)}$$

$$=\frac{-3\times8}{7\times25}=\frac{-24}{175}$$

Illustrative EXAMPLE

Find the reciprocal of $\frac{4}{9}\times\frac{-7}{13}\times\frac{-3}{8}$

Solution:

$$=\frac{4}{9}\times\frac{-7}{13}\times\frac{-3}{8}=\frac{\cancel{4}^{1}}{\cancel{9}_{3}}\times\frac{-7}{13}\times\frac{-\cancel{3}^{1}}{\cancel{8}_{2}}=\frac{7}{78}$$

The reciprocal of $\left(\frac{7}{78}\right)$ is $\left(\frac{78}{7}\right)$ because $\frac{7}{78}\times\frac{78}{7}=1$

Division of Rational Numbers

If a and b are two rational numbers in such a way that $b \neq 0$, then $a \div b$ is same as the product of a and reciprocal of b.

Illustrative EXAMPLE

The product of two rational numbers is $\frac{-23}{88}$ If one of the number is $\frac{5}{22}$ then find the other rational number.

Solution:

Product of two number $= \frac{-23}{88}$ one number $= \frac{5}{22}$, So the other number $= \frac{-23}{88} \div \frac{5}{22} = \frac{-23}{88} \times \frac{22}{5} = \frac{-23}{20}$

Commonly Asked QUESTIONS

If $p = \frac{-4}{5} \times \left(\frac{10}{9}\right) \times \left(\frac{3}{4}\right)$ and $Q = (-6) \times \frac{7}{5} \times \frac{2}{3}$ then P + Q is equal to:

(a) $-2\frac{14}{15}$ (b) $-4\frac{14}{15}$

(c) $-3\frac{14}{15}$ (d) $-1\frac{14}{15}$

(e) None of these

Answer: (b)

Solution:

Here $P = \left(\frac{\overset{-1}{\cancel{-4}}}{\underset{1}{\cancel{5}}}\right) \times \left(\frac{\overset{-2}{\cancel{-10}}}{\underset{3}{\cancel{9}}}\right) \times \frac{\cancel{3}}{\cancel{4}} = \frac{2}{3}$ and Q $= \overset{2}{\cancel{6}} \times \frac{7}{5} \times \frac{2}{\cancel{3}} = \frac{-28}{5}$

$\Rightarrow P + Q = \frac{2}{3} + \left(\frac{-28}{5}\right) = \frac{10 + (-84)}{15} = \frac{-74}{15} = -4\frac{14}{15}$

Simplify: $\left(\frac{5}{13} \times \frac{6}{15}\right) \div \left(\frac{9}{12} \times \frac{4}{3}\right) - \left(\frac{3}{11} \times \frac{5}{6}\right)$

(a) $\frac{42}{286}$ (b) $\frac{109}{286}$

(c) $\frac{-21}{286}$ (d) $\frac{21}{286}$

(e) None of these

Answer: (c)

Explanation

$$\left(\frac{\cancel{5}}{13}\times\frac{\overset{2}{\cancel{6}}}{\underset{\underset{1}{\cancel{3}}}{\cancel{15}}}\right)\div\left(\frac{\overset{3}{\cancel{9}}}{\underset{3}{\cancel{12}}}\times\frac{\cancel{4}}{\cancel{3}}\right)-\left(\frac{\cancel{3}}{11}\times\frac{5}{\underset{2}{\cancel{6}}}\right) \qquad =\frac{2}{23}\div 1-\frac{5}{22}\Rightarrow\frac{2}{13}-\frac{5}{22}=\frac{44-65}{286}=\frac{-21}{286}$$

Find the value of $\frac{-1}{2}+\frac{1}{3}+\frac{-5}{6}+\frac{5}{3}+\frac{-7}{4}$.

(a) $\frac{23}{12}$ (b) $\frac{15}{12}$ (c) $\frac{-13}{12}$ (d) $\frac{-11}{12}$ (e) None of these

Answer: (c)

The value of $(17\times 12)^{-1}$ **Is equal to :**

(a) $17^{-1}\times 12^{1}$ (b) $17\times\left(\frac{1}{12}\right)^{-1}$

(c) $\left(\frac{1}{17}\right)^{-1}\times 12^{-1}$ (d) $\left(\frac{1}{17}\right)\times 12^{-1}$

(e) None of these

Answer: (d)

SUMMARY

- A number which is in the form of $\frac{a}{b}$, where a and b are integers and $b\neq 0$.
- Every positive rational number is greater than the negative rational number.
- For two rational numbers which are in the standard form $\frac{A}{B}$ and $\frac{C}{D}$

(I) if AD > BC Then $\frac{A}{B}>\frac{C}{D}$

(II) If AD < BC , Then $\frac{A}{B}<\frac{C}{D}$

You Must KNOW

- A continued fraction is expression of a number as the sum of an integer and a quotient, the denominator of which is the sum of an integer and a quotient, and so on. In general,

$$y=a_0+\cfrac{b_0}{a_1+\cfrac{b_1}{a_2+\cfrac{b_2}{a_3+\ldots\ldots}}}$$,where $a_o, a_1, a_2, \ldots$ and $b_o, b_1, b_2, \ldots$ are all integers.

- In a simple continued fraction, all the b; are equal to 1 and all the a. are positive integers.

Self Evaluation TEST

Duration 10 Minutes

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1. **If $(-24)^{-1}$ is divided by (-3^{1}) then the quotient will be:**

(a) $\frac{-1}{8}$ (b) (-8)

(c) 8 (d) $\frac{1}{8}$

(e) None of these

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2. **Simplify:** $\left[7^{-1}+\left(\frac{3}{2}\right)^{-1}\right]^{-1} \div \left[6^{-1}+\left(\frac{3}{2}\right)^{-1}\right]^{-1}$

(a) $\frac{34}{35}$ (b) $\frac{35}{34}$

(c) $\frac{21}{105}$ (d) $\frac{5}{121}$

(e) None of these

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3. **Which one of the following statement is false?**

(a) The product of 10 negative integers and 15 positive integers is always positive

(b) The product of 15 positive integers and 10 negative integers is always negative

(c) The product of rational number and its reciprocal is always 1

(d) If a is reciprocal of p then p will be reciprocal of a

(e) None of these

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4. **Steve has a negative rational number which is reciprocal of itself. If he multiplies the rational number with** $x=\left[\frac{1}{7}\times 4\frac{3}{5}+4\frac{3}{5}+7\frac{3}{5}+4\frac{9}{17}\div 8\frac{7}{17}\right]$ **then which one of the following is the correct statement?**

(a) When x multiplied by that number product will be reciprocal of x

(b) When x multiplied by that number product will be additive inverse of x

(c) x will be $\frac{3}{4}$ times

(d) x will be doubled

(e) None of these

5. Stephen has three boxes whose total weight is $\frac{121}{2}$ pound. If the first box weight is $3\frac{1}{2}$ pound more than the second box and third box weight $5\frac{1}{3}$ pound more than the first box, then the weight of second box is____.

(a) $\frac{289}{18}$ Pound
(b) $\frac{18}{289}$ Pound
(c) $\frac{279}{18}$ Pound
(d) $\frac{18}{279}$ Pound
(e) None of these

6. The height of a triangle is three fifth of it corresponding base. If the height increased by 4 % and base decreased by 2 % then area of triangle remains same. The base of triangle is:

(a) $\frac{45}{3}cm$
(b) $\frac{33}{15}cm$
(c) It is impossible
(d) $\frac{15}{3}cm$
(e) None of these

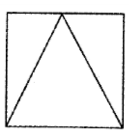
7. Find a number which is 28 greater than the average of its one third, quarter and one - twelfth.

(a) 36
(b) 33
(c) 72
(d) 90
(e) None of these

8. If the length of a rectangle is $3\frac{1}{3}$ cm more than the breadth which is equal to the side of a square whose perimeter is 24 cm then the area of rectangle is:

(a) $64cm^2$
(b) $59cm^2$
(c) $84cm^2$
(d) $56cm^2$
(e) None of these

9. Find the length of square in which an equilateral triangle formed as shown in figure whose half of the perimeter is $4\frac{5}{17}m$.



(a) $3\frac{12}{17}m$
(b) $\frac{412}{57}m$
(c) $2\frac{44}{51}m$
(d) $\frac{57}{216}m$
(e) None of these

10. The perimeter is rectangle is $3\frac{1}{3}$ more than the perimeter of square. A triangle whose perimeter is $\frac{1}{3}$ times of the circumference of the circle whose radius is 5 cm and is equal to the length of sides of the square. Find the perimeter of rectangle is:

(a) $23.\overline{3}cm$

(b) 15-$15\frac{1}{3}cm$ cm

(c)$15\frac{1}{9}cm$

(d) $25\frac{1}{9}cm$

(e) None of these

Answers – Self Evaluation Test																			
1.	A	**2.**	B	**3.**	A	**4.**	B	**5.**	A	**6.**	C	**7.**	A	**8.**	D	**9.**	C	**10.**	A

Self Evaluation Test
SOLUTIONS

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1. $(-24)^{-1} \div (3^{-1}) = \frac{1}{24} \div \frac{1}{3} = \frac{3}{-24} = \frac{-1}{8}$

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2. $\left\{\frac{1}{7}+\frac{2}{3}\right\}^{-1} \div \left\{\frac{1}{6}+\frac{2}{3}\right\}^{-1} = \left\{\frac{3+14}{21}\right\}^{-1} \div \left\{\frac{1+4}{6}\right\}^{-1}$

$\Rightarrow \frac{21}{17} \div \frac{6}{5} \Rightarrow \frac{21\times 5}{17\times 6} = \frac{105}{102} = \frac{35}{34}$

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4. There are only one negative rational number which is reciprocal of itself is -1.

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5. Let the weight of second box be x pound. Then according to question

Weight of 1st box $= \left(x+\frac{7}{2}\right)$ pounds

Weight of III rd box $= \left(x+\frac{7}{2}+\frac{16}{3}\right)$Pounds

Then, $x+x+\frac{7}{2}+x+\frac{7}{2}+\frac{16}{3} = \frac{121}{2}$

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6. Let the base of the triangle be x

Then the height of the triangle $= \frac{3}{5}x$

Area $= \frac{1}{2}\times\frac{3}{5}x\times x = \frac{3}{10}x^2$

Now new height

$= \frac{\cancel{4}}{\underset{250}{\cancel{1}00}} \times \frac{3}{5}x + \frac{3}{5}x = \frac{3}{5}x + \frac{3}{125}x$

$= \frac{75x+3x}{125} = \frac{78x}{125}$

Base $= x - \frac{2x}{100} = \frac{98x}{100}$

So we have , $\frac{1}{2}\times\frac{78x}{125}\times\frac{98x}{100} = \frac{3}{10}x^2$

Thus we cannot determine the value of $'x'$

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7. Let the number be x, then according to question

$$= \frac{1}{3}\left(\frac{x}{3}+\frac{x}{4}+\frac{x}{12}\right)+28=x$$

9. The perimeter of equilateral triangle $=3x$ side of the square.

10. Perimeter of square $=4\times 5cm$

Perimeter of rectangle $=20+3\frac{1}{3}$

$$23\frac{1}{3}=23.\bar{3}$$