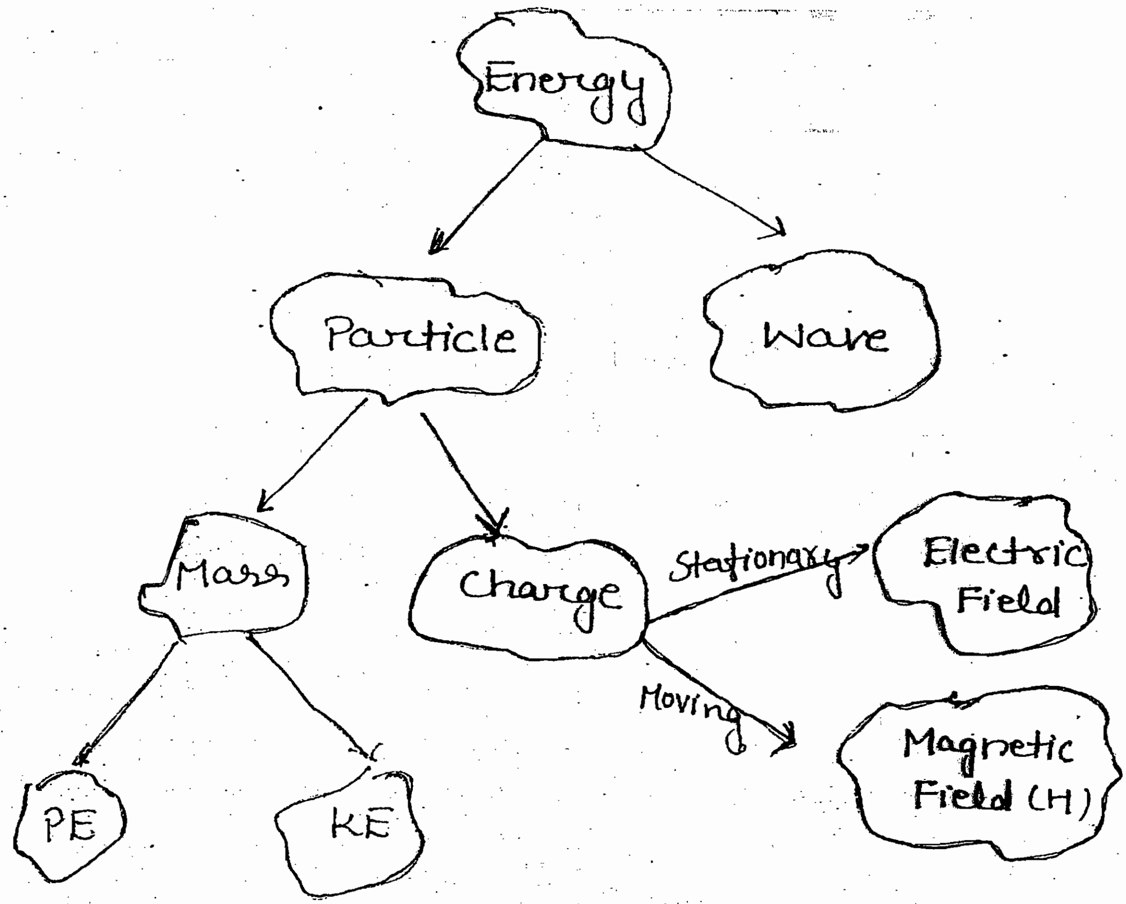
Lecture - I

Static Electromagnetic Fields



Electric Field :-

It is a format of energy that is all around a charge & influences other charges near by

Magnetic Field (H) :-

It is a format of energy that is all around a moving charge & influences other moving charges nearby.

NOTE :-

The word influence in electric fields is a linear accelerating attractive or repulsive force on a charge resulting in straight line path This is Coulomb's law and hence electric of charge particles

$$\boxed{\vec{F} = QE = ma} \longrightarrow \text{Linear path}$$

energised

The word influence in magnetic field is only on moving charges such that forces perpendicular to velocity and the field

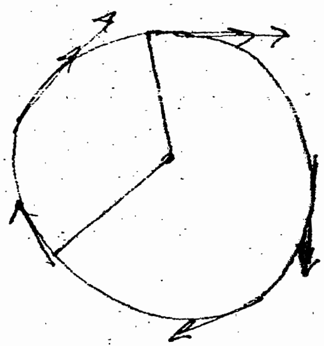
F $\perp$ displacement

So that workdone = 0

This is Lorentz law and particle acquire circular path

$$\boxed{\vec{F} = Q(v \times B) = \frac{mv^2}{r}} \longrightarrow \text{Circular path}$$

deflection



Summary :-

1) Charge is stationary then $\longrightarrow$ Electric Field (Static)

$\longrightarrow$ Time invariant

$\longrightarrow$ Space Varying

eg:- (i) accumulated charge

(ii) D.C voltage

Electric field is energy in the above discussion

2. Charge is moving / flowing. without accelerati or with constant velocity or linearly with time

$$Q = Kt$$

$$\frac{dQ}{dt} = K = I \quad (\text{DC current})$$

and DC current cause magnetic field whic is static in nature

→ Magnetic field is the energy in above discussion.

3. Charge is moving with acceleration which creates ~~energy~~ Electric field E(t) ~~an~~ as well as magnetic field H(t) which are power.

Basic Terms & Definitions :-

There is a measure of the field strength at any point in the field

(I) $\vec{E}$ → Electric field Intensity (V/m or N/C)

↓ per unit length

(II) $\vec{H}$ → Magnetic ~~flux density~~ field intensity (Amp/m)

(III) $\vec{B}$ → Magnetic flux density [Weber/m² or Tesla]

(IV) $\vec{D}$ → Electric flux density (Coulombs/m²)

↓ per unit area

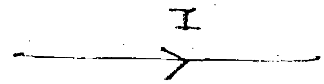
(V) ϵ → Permittivity of the medium

The ability to permit /allow/ hold electric field in that medium

(II) <u>Current Carrying Wire :-</u>

→ Scalar

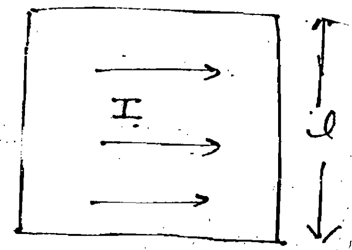
→ Amp



(III) <u>Surface current :-</u>

→ $\vec{K} = \frac{dI}{dl} = Amp/m$

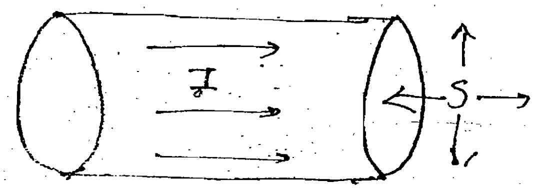
→ Vector



(IV) <u>Solid conductor (J) :-</u>

→ vector

→ $J = \frac{dI}{ds} = Amp/m^2$



Vector Calculus :-

It is a study of directional integrations & directional derivatives.

<u>Directional Integration :-</u>

It is the study of the total effects or cummulative effects of a phenomeno in a specific direction in a specific region

<u>Directional derivative :-</u>

It is the study of the instantaneous or rate of change analysis of a phenomenon in a specific direction in a specific region

eg:- Del - operator

$$\nabla \rightarrow \frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z$$

→ It is used to study the rate of change of various space varying quantities in 3d-space

→ Del is called as vector spatial derivative operators

∇ (Vector operator)

→ Scalar function $f \rightarrow \nabla f \rightarrow$ Gradient of Scalar $\downarrow$ vector form

→ Vector function $\vec{A}$
- $\nabla \cdot \vec{A} \rightarrow$ Divergence of vector (Scalar function)
- $\nabla \times \vec{A} \rightarrow$ curl of vector (Vector function)

→ $\nabla \rightarrow \nabla \cdot \nabla \rightarrow \nabla^2 \rightarrow$ Scalar Laplacian operator

NOTE :-

Vector Identities :-

$$\nabla \times \nabla f = 0$$

$$\nabla \times (\nabla f) = 0$$

curl (Grad. of scalar) = 0

$$\nabla \cdot (\nabla \times \vec{A}) = 0$$

Div. (curl of vector) = 0

$$\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$$

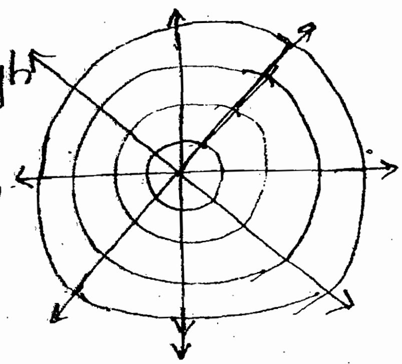
Divergence & Out-flow:–

1) Consider a cause or source which has effects spread outward from the cause

eg:-(I) light from a bulb

(II) air velocity from a punctured tyre

For all such phenomenon the strength dec. as the area of expansion inc. such that total outflow is same



The total outflow through any enclosing surface is always the same (constant) and this constant depends on central cause

Total outflow = ↑Strength × Area ↓

= constant. $\propto$ cause

The strength for all such phenomenon can be expressed as the constant per unit area or cause/area i.e. called as density.

NOTE:–

1) If the cause is Q coulomb charge then the repulsive force or attractive force is called as electric flux and the strength is called as flux density (D) such that

$$\oint D \cdot ds = \Psi_{e\,total} \propto Q$$

If proportionality constant is 1 then this called Gauss Law in integral form

$$\Rightarrow \boxed{\oint D \cdot ds = Q}$$

NOTE 1:-

→ If the surface is not completely enclosing then the effects are the partial flux crossing i.e.

$$\int D \cdot ds = \psi_e \neq \psi_{e\,\text{total}}$$

This is not a Gauss law

→ Every $\underset{\text{vector}}{\underline{\text{Closed surface}}}$ is identified by a $\underset{}{\overset{\text{scalar}}{\underline{\text{finite}}}}$ volume

eg:- $4\pi r^2 \xrightarrow{\text{sphere}} \frac{4}{3}\pi r^3$

$2\pi r h \xrightarrow{\text{cylinder}} \pi r^2 h$

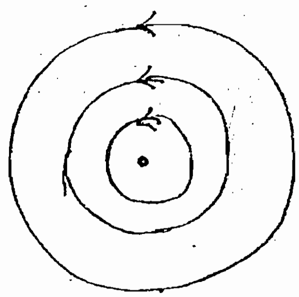
$6a^2 \xrightarrow{\text{cube}} a^3$

→ Closed surface having direction while finite volume doesn't having direction

Circulation and curl :-

Cause or Source →

Effects → around the cause



→ The total circulation in any closed length is always a constant and this constant depends on the central cause.

eg:- air velocity under the fan

Total circulation = strength ↓ × length ↑

= constant ∝ Cause

$$\text{Strength} = \frac{\text{Constant}}{\text{length}} \text{ or } \frac{\text{Cause}}{\text{length}}$$

→ Intensity

Cause or Source → I ampere current

Effects → Magnetic line field

Strength → Magnetic field intensity ($\vec{H}$)

$$\boxed{\oint_{\text{closed}} H \cdot dl = I}$$

This is ampere's law in integral form

Note -1 :-

Every closed line is identified with a finite area enclosed

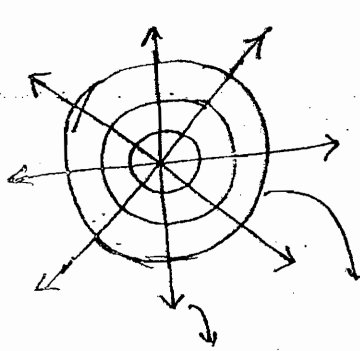
closed line → vector

area enclosed → vector

$2\pi r \xrightarrow{\text{circle}} \pi r^2$

$4a \xrightarrow{\text{square}} a^2$

Summary 1 :-



$\Psi_{e\,\text{total}} = Q$ i.e. outflow total = Strength × area = Cause

→ Electric flux has the units of coulomb's

ϑ is constant

ϑ's direction & direction of divergence

→ closed surface element

Strength around the cause $\vec{D} = \frac{dQ}{ds} = \frac{C}{m^2}$

where ds = any closed surface element

Strength at the cause $= \frac{dQ}{dv} = \frac{d}{dl}\left(\frac{dQ}{ds}\right)$

$= \nabla \cdot D$ = Divergence of D

$= \frac{\text{outflow}}{\text{volume}} = \rho_v$

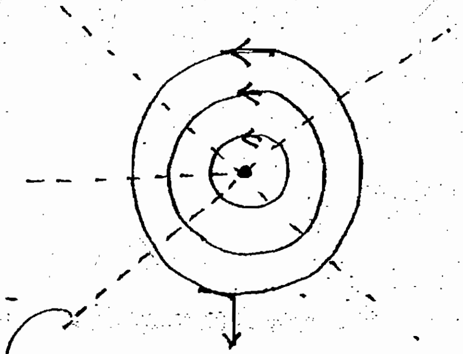
$\boxed{\nabla \cdot D = \rho_v}$ → Gauss law in point form

→ The (·) dot product signifies that D's change or derivative exists only when we move in direction of D. This is called as directional derivative

→ The (·) dot product indicates a surface (vector) derivative resulting in volume (scalar) derivative

Summary 2 :-

Total circulation = Strength x length
= current



perpendicular to H & direction of decrease

H's direction & direction in which H is constant

Total Circulation = strength x length

= Current

$$H = \text{Strength around the current} = \frac{dI}{dl} = \frac{\text{Amp}}{m}$$

where dl = any close line element

$$\text{Strength at the current} = \frac{d}{dl}\left(\frac{dI}{dl}\right) = \nabla \times H$$

= curl of H

$$= \frac{\text{circulation}}{\text{Area}} = J$$

$\boxed{\nabla \times H = J}$ → This is Ampere's law in point form

→ The (X) cross product signifies that H changes only when we move perpendicular to H

→ The (X) cross product implies a vector (line) derivative resulting a vector (surface) derivative