

8. 2025 plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows and the number of plants in each row.
9. Find the smallest square number that is divisible by each of the numbers 4, 9 and 10.
10. Find the smallest square number that is divisible by each of the numbers 8, 15 and 20.

5.5.4 Finding square root by division method

When the numbers are large, even the method of finding square root by prime factorisation becomes lengthy and difficult. To overcome this problem we use Long Division Method.

For this we need to determine the number of digits in the square root. See the following table:

Number	Square	
10	100	which is the smallest 3-digit perfect square
31	961	which is the greatest 3-digit perfect square
32	1024	which is the smallest 4-digit perfect square
99	9801	which is the greatest 4-digit perfect square

So, what can we say about the number of digits in the square root if a perfect square is a 3-digit or a 4-digit number? We can say that, if a perfect square is a 3-digit or a 4-digit number, then its square root will have 2-digits.

Can you tell the number of digits in the square root of a 5-digit or a 6-digit perfect square?

The smallest 3-digit perfect square number is 100 which is the square of 10 and the greatest 3-digit perfect square number is 961 which is the square of 31. The smallest 4-digit square number is 1024 which is the square of 32 and the greatest 4-digit number is 9801 which is the square of 99.

THINK, DISCUSS AND WRITE

Can we say that if a perfect square is of n -digits, then its square root will have $\frac{n}{2}$ digits if n is even or $\frac{(n+1)}{2}$ if n is odd?



The use of the number of digits in square root of a number is useful in the following method:

- Consider the following steps to find the square root of 529.

Can you estimate the number of digits in the square root of this number?

Step 1 Place a bar over every pair of digits starting from the digit at one's place. If the number of digits in it is odd, then the left-most single digit too will have a bar.

Thus we have, $\overline{5} \overline{29}$.

Step 2 Find the largest number whose square is less than or equal to the number under the extreme left bar ($2^2 < 5 < 3^2$). Take this number as the divisor and the quotient with the number under the extreme left bar as the dividend (here 5). Divide and get the remainder (1 in this case).

$$\begin{array}{r} 2 \\ 2 \overline{) 529} \\ \underline{- 4} \\ 1 \end{array}$$

$$\begin{array}{r} 2 \\ 2 \overline{) 529} \\ \underline{-4} \\ 129 \end{array}$$

Step 3 Bring down the number under the next bar (i.e., 29 in this case) to the right of the remainder. So the new dividend is 129.

Step 4 Double the quotient and enter it with a blank on its right.

$$\begin{array}{r} 2 \\ 2 \overline{) 529} \\ \underline{-4} \\ 4 \overline{) 129} \end{array}$$

Step 5 Guess a largest possible digit to fill the blank which will also become the new digit in the quotient, such that when the new divisor is multiplied to the new quotient the product is less than or equal to the dividend.

In this case $42 \times 2 = 84$.

As $43 \times 3 = 129$ so we choose the new digit as 3. Get the remainder.

$$\begin{array}{r} 23 \\ 2 \overline{) 529} \\ \underline{-4} \\ 43 \overline{) 129} \\ \underline{-129} \\ 0 \end{array}$$

Step 6 Since the remainder is 0 and no digits are left in the given number, therefore, $\sqrt{529} = 23$.

• Now consider $\sqrt{4096}$

$$\begin{array}{r} 6 \\ 6 \overline{) 4096} \\ \underline{-36} \\ 4 \end{array}$$

Step 1 Place a bar over every pair of digits starting from the one's digit. ($\overline{40} \overline{96}$).

Step 2 Find the largest number whose square is less than or equal to the number under the left-most bar ($6^2 < 40 < 7^2$). Take this number as the divisor and the number under the left-most bar as the dividend. Divide and get the remainder i.e., 4 in this case.

$$\begin{array}{r} 6 \\ 6 \overline{) 4096} \\ \underline{-36} \\ 496 \end{array}$$

Step 3 Bring down the number under the next bar (i.e., 96) to the right of the remainder. The new dividend is 496.

$$\begin{array}{r} 6 \\ 6 \overline{) 4096} \\ \underline{-36} \\ 12 \overline{) 496} \end{array}$$

Step 4 Double the quotient and enter it with a blank on its right.

Step 5 Guess a largest possible digit to fill the blank which also becomes the new digit in the quotient such that when the new digit is multiplied to the new quotient the product is less than or equal to the dividend. In this case we see that $124 \times 4 = 496$.

So the new digit in the quotient is 4. Get the remainder.

$$\begin{array}{r} 64 \\ 6 \overline{) 4096} \\ \underline{-36} \\ 124 \overline{) 496} \\ \underline{-496} \\ 0 \end{array}$$

Step 6 Since the remainder is 0 and no bar left, therefore, $\sqrt{4096} = 64$.

Estimating the number

We use bars to find the number of digits in the square root of a perfect square number.

$$\sqrt{529} = 23 \quad \text{and} \quad \sqrt{4096} = 64$$

In both the numbers 529 and 4096 there are two bars and the number of digits in their square root is 2. Can you tell the number of digits in the square root of 14400?

By placing bars we get $\overline{144} \overline{00}$. Since there are 3 bars, the square root will be of 3 digit.

TRY THESE

Without calculating square roots, find the number of digits in the square root of the following numbers.

- (i) 25600 (ii) 100000000 (iii) 36864



Example 9: Find the square root of : (i) 729 (ii) 1296

Solution:

$$\begin{array}{r} \text{(i)} \quad \begin{array}{r} 27 \\ 2 \overline{) 729} \\ \underline{-4} \\ 47 \overline{) 329} \\ \underline{-329} \\ 0 \end{array} \end{array} \quad \text{Therefore } \sqrt{729} = 27$$

$$\begin{array}{r} \text{(ii)} \quad \begin{array}{r} 36 \\ 3 \overline{) 1296} \\ \underline{-9} \\ 66 \overline{) 396} \\ \underline{-396} \\ 0 \end{array} \end{array} \quad \text{Therefore } \sqrt{1296} = 36$$

Example 10: Find the least number that must be subtracted from 5607 so as to get a perfect square. Also find the square root of the perfect square.

Solution: Let us try to find $\sqrt{5607}$ by long division method. We get the remainder 131. It shows that 74^2 is less than 5607 by 131.

This means if we subtract the remainder from the number, we get a perfect square.

Therefore, the required perfect square is $5607 - 131 = 5476$. And, $\sqrt{5476} = 74$.

Example 11: Find the greatest 4-digit number which is a perfect square.

Solution: Greatest number of 4-digits = 9999. We find $\sqrt{9999}$ by long division method. The remainder is 198. This shows 99^2 is less than 9999 by 198.

This means if we subtract the remainder from the number, we get a perfect square.

Therefore, the required perfect square is $9999 - 198 = 9801$.

And, $\sqrt{9801} = 99$

Example 12: Find the least number that must be added to 1300 so as to get a perfect square. Also find the square root of the perfect square.

Solution: We find $\sqrt{1300}$ by long division method. The remainder is 4.

This shows that $36^2 < 1300$.

Next perfect square number is $37^2 = 1369$.

Hence, the number to be added is $37^2 - 1300 = 1369 - 1300 = 69$.

$$\begin{array}{r} \begin{array}{r} 74 \\ 7 \overline{) 5607} \\ \underline{-49} \\ 144 \overline{) 707} \\ \underline{-576} \\ 131 \end{array} \\ \begin{array}{r} 99 \\ 9 \overline{) 9999} \\ \underline{-81} \\ 189 \overline{) 1899} \\ \underline{-1701} \\ 198 \end{array} \\ \begin{array}{r} 36 \\ 3 \overline{) 1300} \\ \underline{-9} \\ 66 \overline{) 400} \\ \underline{-396} \\ 4 \end{array} \end{array}$$

5.6 Square Roots of Decimals

Consider $\sqrt{17.64}$

Step 1 To find the square root of a decimal number we put bars on the integral part (i.e., 17) of the number in the usual manner. And place bars on the decimal part

$$\begin{array}{r} 4 \\ 4 \overline{) 17.64} \\ \underline{-16} \\ 1 \end{array}$$

Step 2

(i.e., 64) on every pair of digits beginning with the first decimal place. Proceed as usual. We get $\overline{17.64}$.

$$\begin{array}{r} 4 \\ 4 \overline{) 17.64} \\ \underline{-16} \\ 164 \end{array}$$

Step 3

Now proceed in a similar manner. The left most bar is on 17 and $4^2 < 17 < 5^2$. Take this number as the divisor and the number under the left-most bar as the dividend, i.e., 17. Divide and get the remainder.

The remainder is 1. Write the number under the next bar (i.e., 64) to the right of this remainder, to get 164.

$$\begin{array}{r} 4. \\ 4 \overline{) 17.64} \\ \underline{-16} \\ 82 \overline{) 164} \end{array}$$

Step 4

Double the divisor and enter it with a blank on its right. Since 64 is the decimal part so put a decimal point in the quotient.

Step 5

We know $82 \times 2 = 164$, therefore, the new digit is 2. Divide and get the remainder.

Step 6 Since the remainder is 0 and no bar left, therefore $\sqrt{17.64} = 4.2$.

$$\begin{array}{r} 4.2 \\ 4 \overline{) 17.64} \\ \underline{-16} \\ 82 \overline{) 164} \\ \underline{-164} \\ 0 \end{array}$$

Example 13: Find the square root of 12.25.

Solution:

$$\begin{array}{r} 3.5 \\ 3 \overline{) 12.25} \\ \underline{-9} \\ 65 \overline{) 325} \\ \underline{325} \\ 0 \end{array}$$

Therefore, $\sqrt{12.25} = 3.5$

Which way to move

Consider a number 176.341. Put bars on both integral part and decimal part. In what way is putting bars on decimal part different from integral part? Notice for 176 we start from the unit's place close to the decimal and move towards left. The first bar is over 76 and the second bar over 1. For .341, we start from the decimal and move towards right. First bar is over 34 and for the second bar we put 0 after 1 and make $\overline{.3410}$.

$$\begin{array}{r} 48 \\ 4 \overline{) 2304} \\ \underline{-16} \\ 88 \overline{) 704} \\ \underline{704} \\ 0 \end{array}$$

Example 14: Area of a square plot is 2304 m^2 . Find the side of the square plot.

Solution: Area of square plot = 2304 m^2

Therefore, side of the square plot = $\sqrt{2304} \text{ m}$

We find that, $\sqrt{2304} = 48$

Thus, the side of the square plot is 48 m.

Example 15: There are 2401 students in a school. P.T. teacher wants them to stand in rows and columns such that the number of rows is equal to the number of columns. Find the number of rows.

Solution: Let the number of rows be x
 So, the number of columns = x
 Therefore, number of students = $x \times x = x^2$
 Thus, $x^2 = 2401$ gives $x = \sqrt{2401} = 49$
 The number of rows = 49.

$$\begin{array}{r}
 49 \\
 \hline
 4 \overline{) 2401} \\
 \underline{-16} \\
 89 \\
 \underline{801} \\
 801 \\
 \underline{0}
 \end{array}$$

5.7 Estimating Square Root

Consider the following situations:

1. Deveshi has a square piece of cloth of area 125 cm^2 . She wants to know whether she can make a handkerchief of side 15 cm. If that is not possible she wants to know what is the maximum length of the side of a handkerchief that can be made from this piece.
2. Meena and Shobha played a game. One told a number and other gave its square root. Meena started first. She said 25 and Shobha answered quickly as 5. Then Shobha said 81 and Meena answered 9. It went on, till at one point Meena gave the number 250. And Shobha could not answer. Then Meena asked Shobha if she could atleast tell a number whose square is closer to 250.

In all such cases we need to *estimate* the square root.

We know that $100 < 250 < 400$ and $\sqrt{100} = 10$ and $\sqrt{400} = 20$.

So $10 < \sqrt{250} < 20$

But still we are not very close to the square number.

We know that $15^2 = 225$ and $16^2 = 256$

Therefore, $15 < \sqrt{250} < 16$ and 256 is much closer to 250 than 225.

So, $\sqrt{250}$ is approximately 16.

TRY THESE

Estimate the value of the following to the nearest whole number.

(i) $\sqrt{80}$

(ii) $\sqrt{1000}$

(iii) $\sqrt{350}$

(iv) $\sqrt{500}$



EXERCISE 5.4

1. Find the square root of each of the following numbers by Division method.

(i) 2304	(ii) 4489	(iii) 3481	(iv) 529
(v) 3249	(vi) 1369	(vii) 5776	(viii) 7921
(ix) 576	(x) 1024	(xi) 3136	(xii) 900
2. Find the number of digits in the square root of each of the following numbers (without any calculation).

(i) 64	(ii) 144	(iii) 4489	(iv) 27225
(v) 390625			



3. Find the square root of the following decimal numbers.
 (i) 2.56 (ii) 7.29 (iii) 51.84 (iv) 42.25
 (v) 31.36
4. Find the least number which must be subtracted from each of the following numbers so as to get a perfect square. Also find the square root of the perfect square so obtained.
 (i) 402 (ii) 1989 (iii) 3250 (iv) 825
 (v) 4000
5. Find the least number which must be added to each of the following numbers so as to get a perfect square. Also find the square root of the perfect square so obtained.
 (i) 525 (ii) 1750 (iii) 252 (iv) 1825
 (v) 6412
6. Find the length of the side of a square whose area is 441 m^2 .
7. In a right triangle ABC, $\angle B = 90^\circ$.
 (a) If $AB = 6 \text{ cm}$, $BC = 8 \text{ cm}$, find AC (b) If $AC = 13 \text{ cm}$, $BC = 5 \text{ cm}$, find AB
8. A gardener has 1000 plants. He wants to plant these in such a way that the number of rows and the number of columns remain same. Find the minimum number of plants he needs more for this.
9. There are 500 children in a school. For a P.T. drill they have to stand in such a manner that the number of rows is equal to number of columns. How many children would be left out in this arrangement.

WHAT HAVE WE DISCUSSED?

1. If a natural number m can be expressed as n^2 , where n is also a natural number, then m is a **square number**.
2. All square numbers end with 0, 1, 4, 5, 6 or 9 at units place.
3. Square numbers can only have even number of zeros at the end.
4. **Square root** is the inverse operation of square.
5. There are two integral square roots of a perfect square number.

Positive square root of a number is denoted by the symbol $\sqrt{\quad}$.

For example, $3^2 = 9$ gives $\sqrt{9} = 3$

CHAPTER

6

Algebraic Expressions and Identities



6.1 What are Expressions?

In earlier classes, we have already become familiar with what algebraic expressions (or simply expressions) are. Examples of expressions are:

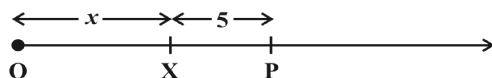
$$x + 3, 2y - 5, 3x^2, 4xy + 7 \text{ etc.}$$

You can form many more expressions. As you know expressions are formed from variables and constants. The expression $2y - 5$ is formed from the variable y and constants 2 and 5. The expression $4xy + 7$ is formed from variables x and y and constants 4 and 7.

We know that, the value of y in the expression, $2y - 5$, may be anything. It can be 2, 5, -3 , 0 , $\frac{5}{2}$, $-\frac{7}{3}$ etc.; actually countless different values. The value of an expression changes with the value chosen for the variables it contains. Thus as y takes on different values, the value of $2y - 5$ goes on changing. When $y = 2$, $2y - 5 = 2(2) - 5 = -1$; when $y = 0$, $2y - 5 = 2 \times 0 - 5 = -5$, etc. Find the value of the expression $2y - 5$ for the other given values of y .

Number line and an expression:

Consider the expression $x + 5$. Let us say the variable x has a position X on the number line;



X may be anywhere on the number line, but it is definite that the value of $x + 5$ is given by a point P , 5 units to the right of X . Similarly, the value of $x - 4$ will be 4 units to the left of X and so on.

What about the position of $4x$ and $4x + 5$?



The position of $4x$ will be point C ; the distance of C from the origin will be four times the distance of X from the origin. The position D of $4x + 5$ will be 5 units to the right of C .





TRY THESE

1. Give five examples of expressions containing one variable and five examples of expressions containing two variables.
2. Show on the number line x , $x - 4$, $2x + 1$, $3x - 2$.

6.2 Terms, Factors and Coefficients

Take the expression $4x + 5$. This expression is made up of two terms, $4x$ and 5 . **Terms are added to form expressions. Terms themselves can be formed as the product of factors.** The term $4x$ is the product of its factors 4 and x . The term 5 is made up of just one factor, i.e., 5 .

The expression $7xy - 5x$ has two terms $7xy$ and $-5x$. The term $7xy$ is a product of factors 7 , x and y . The numerical factor of a term is called its **numerical coefficient or simply coefficient**. The coefficient in the term $7xy$ is 7 and the coefficient in the term $-5x$ is -5 .

TRY THESE

Identify the coefficient of each term in the expression $x^2y^2 - 10x^2y + 5xy^2 - 20$.

6.3 Monomials, Binomials and Polynomials

Expression that contains only one term is called a **monomial**. Expression that contains two terms is called a **binomial**. An expression containing three terms is a **trinomial** and so on. In general, an expression containing, one or more terms with non-zero coefficient (with variables having non negative integers as exponents) is called a **polynomial**. A polynomial may contain any number of terms, one or more than one.

Examples of monomials: $4x^2$, $3xy$, $-7z$, $5xy^2$, $10y$, -9 , $82mnp$, etc.

Examples of binomials: $a + b$, $4l + 5m$, $a + 4$, $5 - 3xy$, $z^2 - 4y^2$, etc.

Examples of trinomials: $a + b + c$, $2x + 3y - 5$, $x^2y - xy^2 + y^2$, etc.

Examples of polynomials: $a + b + c + d$, $3xy$, $7xyz - 10$, $2x + 3y + 7z$, etc.



TRY THESE

1. Classify the following polynomials as monomials, binomials, trinomials.
 $-z + 5$, $x + y + z$, $y + z + 100$, $ab - ac$, 17
2. Construct
 - (a) 3 binomials with only x as a variable;
 - (b) 3 binomials with x and y as variables;
 - (c) 3 monomials with x and y as variables;
 - (d) 2 polynomials with 4 or more terms.

6.4 Like and Unlike Terms

Look at the following expressions:

$$7x, 14x, -13x, 5x^2, 7y, 7xy, -9y^2, -9x^2, -5yx$$

Like terms from these are:

- (i) $7x, 14x, -13x$ are like terms.
- (ii) $5x^2$ and $-9x^2$ are like terms.



- (iii) $7xy$ and $-5yx$ are like terms.
 Why are $7x$ and $7y$ not like?
 Why are $7x$ and $7xy$ not like?
 Why are $7x$ and $5x^2$ not like?

TRY THESE

Write two terms which are like

- (i) $7xy$ (ii) $4mn^2$ (iii) $2l$



6.5 Addition and Subtraction of Algebraic Expressions

In the earlier classes, we have also learnt how to add and subtract algebraic expressions. For example, to add $7x^2 - 4x + 5$ and $9x - 10$, we do

$$\begin{array}{r} 7x^2 - 4x + 5 \\ + \quad \quad 9x - 10 \\ \hline 7x^2 + 5x - 5 \end{array}$$

Observe how we do the addition. We write each expression to be added in a separate row. While doing so we write like terms one below the other, and add them, as shown. Thus $5 + (-10) = 5 - 10 = -5$. Similarly, $-4x + 9x = (-4 + 9)x = 5x$. Let us take some more examples.

Example 1: Add: $7xy + 5yz - 3zx$, $4yz + 9zx - 4y$, $-3xz + 5x - 2xy$.

Solution: Writing the three expressions in separate rows, with like terms one below the other, we have

$$\begin{array}{r} 7xy + 5yz - 3zx \\ + \quad \quad 4yz + 9zx - 4y \\ + \quad -2xy \quad - 3zx + 5x \quad \quad \quad \text{(Note } xz \text{ is same as } zx) \\ \hline 5xy + 9yz + 3zx + 5x - 4y \end{array}$$

Thus, the sum of the expressions is $5xy + 9yz + 3zx + 5x - 4y$. Note how the terms, $-4y$ in the second expression and $5x$ in the third expression, are carried over as they are, since they have no like terms in the other expressions.

Example 2: Subtract $5x^2 - 4y^2 + 6y - 3$ from $7x^2 - 4xy + 8y^2 + 5x - 3y$.

Solution:

$$\begin{array}{r} 7x^2 - 4xy + 8y^2 + 5x - 3y \\ 5x^2 \quad \quad - 4y^2 \quad \quad + 6y - 3 \\ (-) \quad \quad \quad (+) \quad \quad (-) \quad (+) \\ \hline 2x^2 - 4xy + 12y^2 + 5x - 9y + 3 \end{array}$$

Note that subtraction of a number is the same as addition of its additive inverse. Thus subtracting -3 is the same as adding $+3$. Similarly, subtracting $6y$ is the same as adding $-6y$; subtracting $-4y^2$ is the same as adding $4y^2$ and so on. The signs in the third row written below each term in the second row help us in knowing which operation has to be performed.



EXERCISE 6.1

- Identify the terms, their coefficients for each of the following expressions.
 - $5xyz^2 - 3zy$
 - $1 + x + x^2$
 - $4x^2y^2 - 4x^2y^2z^2 + z^2$
 - $3 - pq + qr - rp$
 - $\frac{x}{2} + \frac{y}{2} - xy$
 - $0.3a - 0.6ab + 0.5b$
- Classify the following polynomials as monomials, binomials, trinomials. Which polynomials do not fit in any of these three categories?
 $x + y$, 1000 , $x + x^2 + x^3 + x^4$, $7 + y + 5x$, $2y - 3y^2$, $2y - 3y^2 + 4y^3$, $5x - 4y + 3xy$, $4z - 15z^2$, $ab + bc + cd + da$, pqr , $p^2q + pq^2$, $2p + 2q$
- Add the following.
 - $ab - bc$, $bc - ca$, $ca - ab$
 - $a - b + ab$, $b - c + bc$, $c - a + ac$
 - $2p^2q^2 - 3pq + 4$, $5 + 7pq - 3p^2q^2$
 - $l^2 + m^2$, $m^2 + n^2$, $n^2 + l^2$,
 $2lm + 2mn + 2nl$
- Subtract $4a - 7ab + 3b + 12$ from $12a - 9ab + 5b - 3$
 - Subtract $3xy + 5yz - 7zx$ from $5xy - 2yz - 2zx + 10xyz$
 - Subtract $4p^2q - 3pq + 5pq^2 - 8p + 7q - 10$ from
 $18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q$

6.6 Multiplication of Algebraic Expressions: Introduction

- Look at the following patterns of dots.

Pattern of dots	Total number of dots
	4×9
	5×7

	$m \times n$	To find the number of dots we have to multiply the expression for the number of rows by the expression for the number of columns.
	$(m + 2) \times (n + 3)$	Here the number of rows is increased by 2, i.e., $m + 2$ and number of columns increased by 3, i.e., $n + 3$.

- (ii) Can you now think of similar other situations in which two algebraic expressions have to be multiplied?

Ameena gets up. She says, “We can think of area of a rectangle.” The area of a rectangle is $l \times b$, where l is the length, and b is breadth. If the length of the rectangle is increased by 5 units, i.e., $(l + 5)$ and breadth is decreased by 3 units, i.e., $(b - 3)$ units, the area of the new rectangle will be $(l + 5) \times (b - 3)$.

- (iii) Can you think about volume? (The volume of a rectangular box is given by the product of its length, breadth and height).

- (iv) Sarita points out that when we buy things, we have to carry out multiplication. For example, if

price of bananas per dozen = ₹ p

and for the school picnic bananas needed = z dozens,

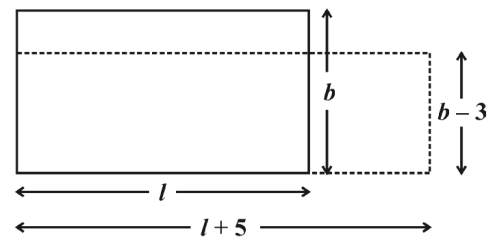
then we have to pay = ₹ $p \times z$

Suppose, the price per dozen was less by ₹ 2 and the bananas needed were less by 4 dozens.

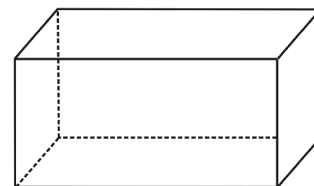
Then, price of bananas per dozen = ₹ $(p - 2)$

and bananas needed = $(z - 4)$ dozens,

Therefore, we would have to pay = ₹ $(p - 2) \times (z - 4)$



To find the area of a rectangle, we have to multiply algebraic expressions like $l \times b$ or $(l + 5) \times (b - 3)$.





TRY THESE

Can you think of two more such situations, where we may need to multiply algebraic expressions?

- [Hint: • Think of speed and time;
• Think of interest to be paid, the principal and the rate of simple interest; etc.]

In all the above examples, we had to carry out multiplication of two or more quantities. If the quantities are given by algebraic expressions, we need to find their product. This means that we should know how to obtain this product. Let us do this systematically. To begin with we shall look at the multiplication of two monomials.

6.7 Multiplying a Monomial by a Monomial

6.7.1 Multiplying two monomials

We begin with

$$4 \times x = x + x + x + x = 4x \text{ as seen earlier.}$$

$$\text{Similarly, } 4 \times (3x) = 3x + 3x + 3x + 3x = 12x$$

Now, observe the following products.

- (i) $x \times 3y = x \times 3 \times y = 3 \times x \times y = 3xy$
- (ii) $5x \times 3y = 5 \times x \times 3 \times y = 5 \times 3 \times x \times y = 15xy$
- (iii) $5x \times (-3y) = 5 \times x \times (-3) \times y$
 $= 5 \times (-3) \times x \times y = -15xy$

Notice that all the three products of monomials, $3xy$, $15xy$, $-15xy$, are also monomials.

Some more useful examples follow.

$$\begin{aligned} \text{(iv)} \quad 5x \times 4x^2 &= (5 \times 4) \times (x \times x^2) \\ &= 20 \times x^3 = 20x^3 \end{aligned}$$

$$\begin{aligned} \text{(v)} \quad 5x \times (-4xyz) &= (5 \times -4) \times (x \times xyz) \\ &= -20 \times (x \times x \times yz) = -20x^2yz \end{aligned}$$

Observe how we collect the powers of different variables in the algebraic parts of the two monomials. While doing so, we use the rules of exponents and powers.

Note that $5 \times 4 = 20$

i.e., coefficient of product = coefficient of first monomial $\times$ coefficient of second monomial;

and $x \times x^2 = x^3$

i.e., algebraic factor of product = algebraic factor of first monomial $\times$ algebraic factor of second monomial.

6.7.2 Multiplying three or more monomials

Observe the following examples.

$$\text{(i)} \quad 2x \times 5y \times 7z = (2x \times 5y) \times 7z = 10xy \times 7z = 70xyz$$

$$\begin{aligned} \text{(ii)} \quad 4xy \times 5x^2y^2 \times 6x^3y^3 &= (4xy \times 5x^2y^2) \times 6x^3y^3 = 20x^3y^3 \times 6x^3y^3 = 120x^3y^3 \times x^3y^3 \\ &= 120 (x^3 \times x^3) \times (y^3 \times y^3) = 120x^6 \times y^6 = 120x^6y^6 \end{aligned}$$

It is clear that we first multiply the first two monomials and then multiply the resulting monomial by the third monomial. This method can be extended to the product of any number of monomials.

TRY THESEFind $4x \times 5y \times 7z$ First find $4x \times 5y$ and multiply it by $7z$;
or first find $5y \times 7z$ and multiply it by $4x$.

Is the result the same? What do you observe?

Does the order in which you carry out the multiplication matter?

We can find the product in other way also.

$$4xy \times 5x^2y^2 \times 6x^3y^3$$

$$= (4 \times 5 \times 6) \times (x \times x^2 \times x^3) \times (y \times y^2 \times y^3)$$

$$= 120 x^6y^6$$

**Example 3:** Complete the table for area of a rectangle with given length and breadth.**Solution:**

length	breadth	area
$3x$	$5y$	$3x \times 5y = 15xy$
$9y$	$4y^2$	
$4ab$	$5bc$	
$2l^2m$	$3lm^2$	

Example 4: Find the volume of each rectangular box with given length, breadth and height.

	length	breadth	height
(i)	$2ax$	$3by$	$5cz$
(ii)	m^2n	n^2p	p^2m
(iii)	$2q$	$4q^2$	$8q^3$

Solution: Volume = length $\times$ breadth $\times$ heightHence, for (i) volume = $(2ax) \times (3by) \times (5cz)$

$$= 2 \times 3 \times 5 \times (ax) \times (by) \times (cz) = 30abcxyz$$

for (ii) volume = $m^2n \times n^2p \times p^2m$

$$= (m^2 \times m) \times (n \times n^2) \times (p \times p^2) = m^3n^3p^3$$

for (iii) volume = $2q \times 4q^2 \times 8q^3$

$$= 2 \times 4 \times 8 \times q \times q^2 \times q^3 = 64q^6$$

EXERCISE 6.2

1. Find the product of the following pairs of monomials.

(i) $4, 7p$

(ii) $-4p, 7p$

(iii) $-4p, 7pq$

(iv) $4p^3, -3p$

(v) $4p, 0$

2. Find the areas of rectangles with the following pairs of monomials as their lengths and breadths respectively.

$(p, q); (10m, 5n); (20x^2, 5y^2); (4x, 3x^2); (3mn, 4np)$

3. Complete the table of products.

First monomial → Second monomial ↓	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$	...	...	...	...	...
$-5y$	...	...	$-15x^2y$	...	...	...
$3x^2$	...	...	...	...	...	...
$-4xy$	...	...	...	...	...	...
$7x^2y$	...	...	...	...	...	...
$-9x^2y^2$	...	...	...	...	...	...

4. Obtain the volume of rectangular boxes with the following length, breadth and height respectively.

(i) $5a, 3a^2, 7a^4$ (ii) $2p, 4q, 8r$ (iii) $xy, 2x^2y, 2xy^2$ (iv) $a, 2b, 3c$

5. Obtain the product of

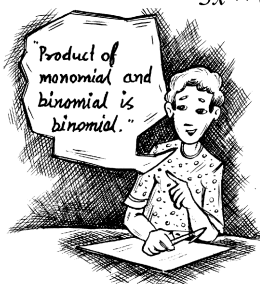
(i) xy, yz, zx (ii) $a, -a^2, a^3$ (iii) $2, 4y, 8y^2, 16y^3$
 (iv) $a, 2b, 3c, 6abc$ (v) $m, -mn, mnp$

6.8 Multiplying a Monomial by a Polynomial

6.8.1 Multiplying a monomial by a binomial

Let us multiply the monomial $3x$ by the binomial $5y + 2$, i.e., find $3x \times (5y + 2) = ?$

Recall that $3x$ and $(5y + 2)$ represent numbers. Therefore, using the distributive law,
 $3x \times (5y + 2) = (3x \times 5y) + (3x \times 2) = 15xy + 6x$



We commonly use distributive law in our calculations. For example:

$$\begin{aligned} 7 \times 106 &= 7 \times (100 + 6) \\ &= 7 \times 100 + 7 \times 6 && \text{(Here, we used distributive law)} \\ &= 700 + 42 = 742 \end{aligned}$$

$$\begin{aligned} 7 \times 38 &= 7 \times (40 - 2) \\ &= 7 \times 40 - 7 \times 2 && \text{(Here, we used distributive law)} \\ &= 280 - 14 = 266 \end{aligned}$$

Similarly, $(-3x) \times (-5y + 2) = (-3x) \times (-5y) + (-3x) \times (2) = 15xy - 6x$
 and $5xy \times (y^2 + 3) = (5xy \times y^2) + (5xy \times 3) = 5xy^3 + 15xy.$

What about a binomial $\times$ monomial? For example, $(5y + 2) \times 3x = ?$

We may use commutative law as : $7 \times 3 = 3 \times 7$; or in general $a \times b = b \times a$

Similarly, $(5y + 2) \times 3x = 3x \times (5y + 2) = 15xy + 6x$ as before.

TRY THESE

Find the product

(i) $2x (3x + 5xy)$

(ii) $a^2 (2ab - 5c)$



6.8.2 Multiplying a monomial by a trinomial

Consider $3p \times (4p^2 + 5p + 7)$. As in the earlier case, we use distributive law;

$$\begin{aligned} 3p \times (4p^2 + 5p + 7) &= (3p \times 4p^2) + (3p \times 5p) + (3p \times 7) \\ &= 12p^3 + 15p^2 + 21p \end{aligned}$$

Multiply each term of the trinomial by the monomial and add products.

Observe, by using the distributive law, we are able to carry out the multiplication term by term.

TRY THESE

Find the product:

$$(4p^2 + 5p + 7) \times 3p$$

Example 5: Simplify the expressions and evaluate them as directed:

- (i) $x(x - 3) + 2$ for $x = 1$, (ii) $3y(2y - 7) - 3(y - 4) - 63$ for $y = -2$

Solution:

(i) $x(x - 3) + 2 = x^2 - 3x + 2$

For $x = 1$, $x^2 - 3x + 2 = (1)^2 - 3(1) + 2$
 $= 1 - 3 + 2 = 3 - 3 = 0$

(ii) $3y(2y - 7) - 3(y - 4) - 63 = 6y^2 - 21y - 3y + 12 - 63$
 $= 6y^2 - 24y - 51$

For $y = -2$, $6y^2 - 24y - 51 = 6(-2)^2 - 24(-2) - 51$
 $= 6 \times 4 + 24 \times 2 - 51$
 $= 24 + 48 - 51 = 72 - 51 = 21$

Example 6: Add

- (i) $5m(3 - m)$ and $6m^2 - 13m$ (ii) $4y(3y^2 + 5y - 7)$ and $2(y^3 - 4y^2 + 5)$

Solution:

(i) First expression $= 5m(3 - m) = (5m \times 3) - (5m \times m) = 15m - 5m^2$

Now adding the second expression to it, $15m - 5m^2 + 6m^2 - 13m = m^2 + 2m$

(ii) The first expression $= 4y(3y^2 + 5y - 7) = (4y \times 3y^2) + (4y \times 5y) + (4y \times (-7))$
 $= 12y^3 + 20y^2 - 28y$

The second expression $= 2(y^3 - 4y^2 + 5) = 2y^3 + 2 \times (-4y^2) + 2 \times 5$
 $= 2y^3 - 8y^2 + 10$

Adding the two expressions,

$$\begin{array}{r} 12y^3 \quad + \quad 20y^2 - 28y \\ + \quad 2y^3 \quad - \quad 8y^2 \quad + 10 \\ \hline 14y^3 \quad + \quad 12y^2 - 28y \quad + 10 \end{array}$$

Example 7: Subtract $3pq(p - q)$ from $2pq(p + q)$.

Solution: We have $3pq(p - q) = 3p^2q - 3pq^2$ and

$$2pq(p + q) = 2p^2q + 2pq^2$$

Subtracting,

$$\begin{array}{r} 2p^2q \quad + \quad 2pq^2 \\ 3p^2q \quad - \quad 3pq^2 \\ - \quad \quad \quad + \quad \quad \quad \\ \hline -p^2q \quad + \quad 5pq^2 \end{array}$$



EXERCISE 6.3

1. Carry out the multiplication of the expressions in each of the following pairs.

- (i) $4p, q + r$ (ii) $ab, a - b$ (iii) $a + b, 7a^2b^2$ (iv) $a^2 - 9, 4a$
 (v) $pq + qr + rp, 0$

2. Complete the table.

	First expression	Second expression	Product
(i)	a	$b + c + d$	...
(ii)	$x + y - 5$	$5xy$	...
(iii)	p	$6p^2 - 7p + 5$	...
(iv)	$4p^2q^2$	$p^2 - q^2$	...
(v)	$a + b + c$	abc	...

3. Find the product.

- (i) $(a^2) \times (2a^{22}) \times (4a^{26})$ (ii) $\left(\frac{2}{3}xy\right) \times \left(\frac{-9}{10}x^2y^2\right)$
 (iii) $\left(-\frac{10}{3}pq^3\right) \times \left(\frac{6}{5}p^3q\right)$ (iv) $x \times x^2 \times x^3 \times x^4$

4. (a) Simplify $3x(4x - 5) + 3$ and find its values for (i) $x = 3$ (ii) $x = \frac{1}{2}$.

(b) Simplify $a(a^2 + a + 1) + 5$ and find its value for (i) $a = 0$, (ii) $a = 1$
 (iii) $a = -1$.

5. (a) Add: $p(p - q)$, $q(q - r)$ and $r(r - p)$

(b) Add: $2x(z - x - y)$ and $2y(z - y - x)$

(c) Subtract: $3l(l - 4m + 5n)$ from $4l(10n - 3m + 2l)$

(d) Subtract: $3a(a + b + c) - 2b(a - b + c)$ from $4c(-a + b + c)$

6.9 Multiplying a Polynomial by a Polynomial

6.9.1 Multiplying a binomial by a binomial

Let us multiply one binomial $(2a + 3b)$ by another binomial, say $(3a + 4b)$. We do this step-by-step, as we did in earlier cases, following the distributive law of multiplication,

$$(3a + 4b) \times (2a + 3b) = 3a \times (2a + 3b) + 4b \times (2a + 3b)$$

Observe, every term in one binomial multiplies every term in the other binomial.

$$\begin{aligned} &= (3a \times 2a) + (3a \times 3b) + (4b \times 2a) + (4b \times 3b) \\ &= 6a^2 + 9ab + 8ba + 12b^2 \\ &= 6a^2 + 17ab + 12b^2 \quad (\text{Since } ba = ab) \end{aligned}$$

When we carry out term by term multiplication, we expect $2 \times 2 = 4$ terms to be present. But two of these are like terms, which are combined, and hence we get 3 terms. **In multiplication of polynomials with polynomials, we should always look for like terms, if any, and combine them.**

Example 8: Multiply

(i) $(x - 4)$ and $(2x + 3)$

(ii) $(x - y)$ and $(3x + 5y)$

Solution:

$$\begin{aligned}
 \text{(i)} \quad (x - 4) \times (2x + 3) &= x \times (2x + 3) - 4 \times (2x + 3) \\
 &= (x \times 2x) + (x \times 3) - (4 \times 2x) - (4 \times 3) = 2x^2 + 3x - 8x - 12 \\
 &= 2x^2 - 5x - 12 \quad \text{(Adding like terms)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad (x - y) \times (3x + 5y) &= x \times (3x + 5y) - y \times (3x + 5y) \\
 &= (x \times 3x) + (x \times 5y) - (y \times 3x) - (y \times 5y) \\
 &= 3x^2 + 5xy - 3yx - 5y^2 = 3x^2 + 2xy - 5y^2 \quad \text{(Adding like terms)}
 \end{aligned}$$

Example 9: Multiply

(i) $(a + 7)$ and $(b - 5)$

(ii) $(a^2 + 2b^2)$ and $(5a - 3b)$

Solution:

$$\begin{aligned}
 \text{(i)} \quad (a + 7) \times (b - 5) &= a \times (b - 5) + 7 \times (b - 5) \\
 &= ab - 5a + 7b - 35
 \end{aligned}$$

Note that there are no like terms involved in this multiplication.

$$\begin{aligned}
 \text{(ii)} \quad (a^2 + 2b^2) \times (5a - 3b) &= a^2(5a - 3b) + 2b^2 \times (5a - 3b) \\
 &= 5a^3 - 3a^2b + 10ab^2 - 6b^3
 \end{aligned}$$

9.9.2 Multiplying a binomial by a trinomial

In this multiplication, we shall have to multiply each of the three terms in the trinomial by each of the two terms in the binomial. We shall get in all $3 \times 2 = 6$ terms, which may reduce to 5 or less, if the term by term multiplication results in like terms. Consider

$$\begin{aligned}
 \underbrace{(a + 7)}_{\text{binomial}} \times \underbrace{(a^2 + 3a + 5)}_{\text{trinomial}} &= a \times (a^2 + 3a + 5) + 7 \times (a^2 + 3a + 5) \quad \text{[using the distributive law]} \\
 &= a^3 + 3a^2 + 5a + 7a^2 + 21a + 35 \\
 &= a^3 + (3a^2 + 7a^2) + (5a + 21a) + 35 \\
 &= a^3 + 10a^2 + 26a + 35 \quad \text{(Why are there only 4 terms in the final result?)}
 \end{aligned}$$

Example 10: Simplify $(a + b)(2a - 3b + c) - (2a - 3b)c$.**Solution:** We have

$$\begin{aligned}
 (a + b)(2a - 3b + c) &= a(2a - 3b + c) + b(2a - 3b + c) \\
 &= 2a^2 - 3ab + ac + 2ab - 3b^2 + bc \\
 &= 2a^2 - ab - 3b^2 + bc + ac \quad \text{(Note, } -3ab \text{ and } 2ab \text{ are like terms)}
 \end{aligned}$$

and $(2a - 3b)c = 2ac - 3bc$

Therefore,

$$\begin{aligned}
 (a + b)(2a - 3b + c) - (2a - 3b)c &= 2a^2 - ab - 3b^2 + bc + ac - (2ac - 3bc) \\
 &= 2a^2 - ab - 3b^2 + bc + ac - 2ac + 3bc \\
 &= 2a^2 - ab - 3b^2 + (bc + 3bc) + (ac - 2ac) \\
 &= 2a^2 - 3b^2 - ab + 4bc - ac
 \end{aligned}$$



EXERCISE 6.4

1. Multiply the binomials.

- | | |
|--|-------------------------------|
| (i) $(2x + 5)$ and $(4x - 3)$ | (ii) $(y - 8)$ and $(3y - 4)$ |
| (iii) $(2.5l - 0.5m)$ and $(2.5l + 0.5m)$ | (iv) $(a + 3b)$ and $(x + 5)$ |
| (v) $(2pq + 3q^2)$ and $(3pq - 2q^2)$ | |
| (vi) $\left(\frac{3}{4}a^2 + 3b^2\right)$ and $4\left(a^2 - \frac{2}{3}b^2\right)$ | |

2. Find the product.

- | | |
|----------------------------|----------------------------|
| (i) $(5 - 2x)(3 + x)$ | (ii) $(x + 7y)(7x - y)$ |
| (iii) $(a^2 + b)(a + b^2)$ | (iv) $(p^2 - q^2)(2p + q)$ |

3. Simplify.

- | | |
|---|--------------------------------|
| (i) $(x^2 - 5)(x + 5) + 25$ | (ii) $(a^2 + 5)(b^3 + 3) + 5$ |
| (iii) $(t + s^2)(t^2 - s)$ | |
| (iv) $(a + b)(c - d) + (a - b)(c + d) + 2(ac + bd)$ | |
| (v) $(x + y)(2x + y) + (x + 2y)(x - y)$ | (vi) $(x + y)(x^2 - xy + y^2)$ |
| (vii) $(1.5x - 4y)(1.5x + 4y + 3) - 4.5x + 12y$ | |
| (viii) $(a + b + c)(a + b - c)$ | |

6.10 What is an Identity?

Consider the equality $(a + 1)(a + 2) = a^2 + 3a + 2$

We shall evaluate both sides of this equality for some value of a , say $a = 10$.

For $a = 10$, LHS = $(a + 1)(a + 2) = (10 + 1)(10 + 2) = 11 \times 12 = 132$

RHS = $a^2 + 3a + 2 = 10^2 + 3 \times 10 + 2 = 100 + 30 + 2 = 132$

Thus, the values of the two sides of the equality are equal for $a = 10$.

Let us now take $a = -5$

LHS = $(a + 1)(a + 2) = (-5 + 1)(-5 + 2) = (-4) \times (-3) = 12$

RHS = $a^2 + 3a + 2 = (-5)^2 + 3(-5) + 2$
 $= 25 - 15 + 2 = 10 + 2 = 12$

Thus, for $a = -5$, also LHS = RHS.

We shall find that for any value of a , LHS = RHS. **Such an equality, true for every value of the variable in it, is called an identity.** Thus,

$(a + 1)(a + 2) = a^2 + 3a + 2$ is an identity.

An equation is true for only certain values of the variable in it. It is not true for all values of the variable. For example, consider the equation

$$a^2 + 3a + 2 = 132$$

It is true for $a = 10$, as seen above, but it is not true for $a = -5$ or for $a = 0$ etc.

Try it: Show that $a^2 + 3a + 2 = 132$ is not true for $a = -5$ and for $a = 0$.

6.11 Standard Identities

We shall now study three identities which are very useful in our work. These identities are obtained by multiplying a binomial by another binomial.

Let us first consider the product $(a + b)(a + b)$ or $(a + b)^2$.

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b) \\ &= a(a + b) + b(a + b) \\ &= a^2 + ab + ba + b^2 \\ &= a^2 + 2ab + b^2 \quad (\text{since } ab = ba)\end{aligned}$$

Thus
$$(a + b)^2 = a^2 + 2ab + b^2 \quad \text{(I)}$$

Clearly, this is an identity, since the expression on the RHS is obtained from the LHS by actual multiplication. One may verify that for any value of a and any value of b , the values of the two sides are equal.

- Next we consider $(a - b)^2 = (a - b)(a - b) = a(a - b) - b(a - b)$

We have
$$= a^2 - ab - ba + b^2 = a^2 - 2ab + b^2$$

or
$$(a - b)^2 = a^2 - 2ab + b^2 \quad \text{(II)}$$

- Finally, consider $(a + b)(a - b)$. We have $(a + b)(a - b) = a(a - b) + b(a - b)$

$$= a^2 - ab + ba - b^2 = a^2 - b^2 \quad (\text{since } ab = ba)$$

or
$$(a + b)(a - b) = a^2 - b^2 \quad \text{(III)}$$

The identities (I), (II) and (III) are known as **standard identities**.

TRY THESE

1. Put $-b$ in place of b in Identity (I). Do you get Identity (II)?

- We shall now work out one more useful identity.

$$\begin{aligned}(x + a)(x + b) &= x(x + b) + a(x + b) \\ &= x^2 + bx + ax + ab\end{aligned}$$

or
$$(x + a)(x + b) = x^2 + (a + b)x + ab \quad \text{(IV)}$$



TRY THESE

1. Verify Identity (IV), for $a = 2, b = 3, x = 5$.
2. Consider, the special case of Identity (IV) with $a = b$, what do you get? Is it related to Identity (I)?
3. Consider, the special case of Identity (IV) with $a = -c$ and $b = -c$. What do you get? Is it related to Identity (II)?
4. Consider the special case of Identity (IV) with $b = -a$. What do you get? Is it related to Identity (III)?



We can see that Identity (IV) is the general form of the other three identities also.

6.12 Applying Identities

We shall now see how, for many problems on multiplication of binomial expressions and also of numbers, use of the identities gives a simple alternative method of solving them.

Example 11: Using the Identity (I), find (i) $(2x + 3y)^2$ (ii) 103^2

Solution:

$$(i) \quad (2x + 3y)^2 = (2x)^2 + 2(2x)(3y) + (3y)^2 \quad [\text{Using the Identity (I)}]$$

$$= 4x^2 + 12xy + 9y^2$$

We may work out $(2x + 3y)^2$ directly.

$$\begin{aligned} (2x + 3y)^2 &= (2x + 3y)(2x + 3y) \\ &= (2x)(2x) + (2x)(3y) + (3y)(2x) + (3y)(3y) \\ &= 4x^2 + 6xy + 6yx + 9y^2 \quad (\text{as } xy = yx) \\ &= 4x^2 + 12xy + 9y^2 \end{aligned}$$

Using Identity (I) gave us an alternative method of squaring $(2x + 3y)$. Do you notice that the Identity method required fewer steps than the above direct method? You will realise the simplicity of this method even more if you try to square more complicated binomial expressions than $(2x + 3y)$.

$$(ii) \quad (103)^2 = (100 + 3)^2$$

$$= 100^2 + 2 \times 100 \times 3 + 3^2 \quad (\text{Using Identity I})$$

$$= 10000 + 600 + 9 = 10609$$

We may also directly multiply 103 by 103 and get the answer. Do you see that Identity (I) has given us a less tedious method than the direct method of squaring 103? Try squaring 1013. You will find in this case, the method of using identities even more attractive than the direct multiplication method.

Example 12: Using Identity (II), find (i) $(4p - 3q)^2$ (ii) $(4.9)^2$

Solution:

$$(i) \quad (4p - 3q)^2 = (4p)^2 - 2(4p)(3q) + (3q)^2 \quad [\text{Using the Identity (II)}]$$

$$= 16p^2 - 24pq + 9q^2$$

Do you agree that for squaring $(4p - 3q)^2$ the method of identities is quicker than the direct method?

$$(ii) \quad (4.9)^2 = (5.0 - 0.1)^2 = (5.0)^2 - 2(5.0)(0.1) + (0.1)^2$$

$$= 25.00 - 1.00 + 0.01 = 24.01$$

Is it not that, squaring 4.9 using Identity (II) is much less tedious than squaring it by direct multiplication?

Example 13: Using Identity (III), find

$$(i) \quad \left(\frac{3}{2}m + \frac{2}{3}n\right)\left(\frac{3}{2}m - \frac{2}{3}n\right) \quad (ii) \quad 983^2 - 17^2 \quad (iii) \quad 194 \times 206$$

Solution:

$$(i) \quad \left(\frac{3}{2}m + \frac{2}{3}n\right)\left(\frac{3}{2}m - \frac{2}{3}n\right) = \left(\frac{3}{2}m\right)^2 - \left(\frac{2}{3}n\right)^2$$

$$= \frac{9}{4}m^2 - \frac{4}{9}n^2$$

$$(ii) \quad 983^2 - 17^2 = (983 + 17)(983 - 17)$$

$$[\text{Here } a = 983, b = 17, a^2 - b^2 = (a + b)(a - b)]$$

$$\text{Therefore, } 983^2 - 17^2 = 1000 \times 966 = 966000$$

Try doing this directly.
You will realise how easy
our method of using
Identity (III) is.

$$\begin{aligned} \text{(iii)} \quad 194 \times 206 &= (200 - 6) \times (200 + 6) = 200^2 - 6^2 \\ &= 40000 - 36 = 39964 \end{aligned}$$

Example 14: Use the Identity $(x + a)(x + b) = x^2 + (a + b)x + ab$ to find the following:

$$\text{(i)} \quad 501 \times 502$$

$$\text{(ii)} \quad 95 \times 103$$

Solution:

$$\begin{aligned} \text{(i)} \quad 501 \times 502 &= (500 + 1) \times (500 + 2) = 500^2 + (1 + 2) \times 500 + 1 \times 2 \\ &= 250000 + 1500 + 2 = 251502 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad 95 \times 103 &= (100 - 5) \times (100 + 3) = 100^2 + (-5 + 3) \times 100 + (-5) \times 3 \\ &= 10000 - 200 - 15 = 9785 \end{aligned}$$

EXERCISE 6.5



1. Use a suitable identity to get each of the following products.

$$\text{(i)} \quad (x + 3)(x + 3)$$

$$\text{(ii)} \quad (2y + 5)(2y + 5)$$

$$\text{(iii)} \quad (2a - 7)(2a - 7)$$

$$\text{(iv)} \quad \left(3a - \frac{1}{2}\right)\left(3a - \frac{1}{2}\right)$$

$$\text{(v)} \quad (1.1m - 0.4)(1.1m + 0.4)$$

$$\text{(vi)} \quad (a^2 + b^2)(-a^2 + b^2)$$

$$\text{(vii)} \quad (6x - 7)(6x + 7)$$

$$\text{(viii)} \quad (-a + c)(-a + c)$$

$$\text{(ix)} \quad \left(\frac{x}{2} + \frac{3y}{4}\right)\left(\frac{x}{2} + \frac{3y}{4}\right)$$

$$\text{(x)} \quad (7a - 9b)(7a - 9b)$$

2. Use the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$ to find the following products.

$$\text{(i)} \quad (x + 3)(x + 7)$$

$$\text{(ii)} \quad (4x + 5)(4x + 1)$$

$$\text{(iii)} \quad (4x - 5)(4x - 1)$$

$$\text{(iv)} \quad (4x + 5)(4x - 1)$$

$$\text{(v)} \quad (2x + 5y)(2x + 3y)$$

$$\text{(vi)} \quad (2a^2 + 9)(2a^2 + 5)$$

$$\text{(vii)} \quad (xyz - 4)(xyz - 2)$$

3. Find the following squares by using the identities.

$$\text{(i)} \quad (b - 7)^2$$

$$\text{(ii)} \quad (xy + 3z)^2$$

$$\text{(iii)} \quad (6x^2 - 5y)^2$$

$$\text{(iv)} \quad \left(\frac{2}{3}m + \frac{3}{2}n\right)^2$$

$$\text{(v)} \quad (0.4p - 0.5q)^2$$

$$\text{(vi)} \quad (2xy + 5y)^2$$

4. Simplify.

$$\text{(i)} \quad (a^2 - b^2)^2$$

$$\text{(ii)} \quad (2x + 5)^2 - (2x - 5)^2$$

$$\text{(iii)} \quad (7m - 8n)^2 + (7m + 8n)^2$$

$$\text{(iv)} \quad (4m + 5n)^2 + (5m + 4n)^2$$

$$\text{(v)} \quad (2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$$

$$\text{(vi)} \quad (ab + bc)^2 - 2ab^2c$$

$$\text{(vii)} \quad (m^2 - n^2m)^2 + 2m^3n^2$$

5. Show that.

$$\text{(i)} \quad (3x + 7)^2 - 84x = (3x - 7)^2$$

$$\text{(ii)} \quad (9p - 5q)^2 + 180pq = (9p + 5q)^2$$

$$\text{(iii)} \quad \left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2$$

$$\text{(iv)} \quad (4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$$

$$\text{(v)} \quad (a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = 0$$

6. Using identities, evaluate.

- | | | | |
|------------------------|-----------------------|----------------------|----------------|
| (i) 71^2 | (ii) 99^2 | (iii) 102^2 | (iv) 998^2 |
| (v) 5.2^2 | (vi) 297×303 | (vii) 78×82 | (viii) 8.9^2 |
| (ix) 10.5×9.5 | | | |

7. Using $a^2 - b^2 = (a + b)(a - b)$, find

- | | | |
|-----------------------|----------------------------|-----------------------|
| (i) $51^2 - 49^2$ | (ii) $(1.02)^2 - (0.98)^2$ | (iii) $153^2 - 147^2$ |
| (iv) $12.1^2 - 7.9^2$ | | |

8. Using $(x + a)(x + b) = x^2 + (a + b)x + ab$, find

- | | | | |
|----------------------|-----------------------|-----------------------|-----------------------|
| (i) 103×104 | (ii) 5.1×5.2 | (iii) 103×98 | (iv) 9.7×9.8 |
|----------------------|-----------------------|-----------------------|-----------------------|

WHAT HAVE WE DISCUSSED?

- Expressions are formed from **variables** and **constants**.
- Terms are added to form **expressions**. Terms themselves are formed as product of **factors**.
- Expressions that contain exactly one, two and three terms are called **monomials**, **binomials** and **trinomials** respectively. In general, any expression containing one or more terms with non-zero coefficients (and with variables having non-negative integers as exponents) is called a **polynomial**.
- Like** terms are formed from the same variables and the powers of these variables are the same, too. Coefficients of like terms need not be the same.
- While adding (or subtracting) polynomials, first look for like terms and add (or subtract) them; then handle the unlike terms.
- There are number of situations in which we need to multiply algebraic expressions: for example, in finding area of a rectangle, the sides of which are given as expressions.
- A monomial multiplied by a monomial always gives a monomial.
- While multiplying a polynomial by a monomial, we multiply every term in the polynomial by the monomial.
- In carrying out the multiplication of a polynomial by a binomial (or trinomial), we multiply term by term, i.e., every term of the polynomial is multiplied by every term in the binomial (or trinomial). Note that in such multiplication, we may get terms in the product which are like and have to be combined.
- An **identity** is an equality, which is true for all values of the variables in the equality. On the other hand, an equation is true only for certain values of its variables. An equation is not an identity.
- The following are the standard identities:

$(a + b)^2 = a^2 + 2ab + b^2$	(I)
$(a - b)^2 = a^2 - 2ab + b^2$	(II)
$(a + b)(a - b) = a^2 - b^2$	(III)
- Another useful identity is $(x + a)(x + b) = x^2 + (a + b)x + ab$ (IV)
- The above four identities are useful in carrying out squares and products of algebraic expressions. They also allow easy alternative methods to calculate products of numbers and so on.

CHAPTER

7

Practical Geometry



7.1 Introduction

You have learnt how to draw triangles in Class VII. We require three measurements (of sides and angles) to draw a unique triangle.

Since three measurements were enough to draw a triangle, a natural question arises whether four measurements would be sufficient to draw a unique four sided closed figure, namely, a quadrilateral.

DO THIS

Take a pair of sticks of equal lengths, say 10 cm. Take another pair of sticks of equal lengths, say, 8 cm. Hinge them up suitably to get a rectangle of length 10 cm and breadth 8 cm.

This rectangle has been created with the 4 available measurements.

Now just push along the breadth of the rectangle. Is the new shape obtained, still a rectangle (Fig 7.2)? Observe that the rectangle has now become a parallelogram. Have you altered the lengths of the sticks? No! The measurements of sides remain the same.

Give another push to the newly obtained shape in a different direction; what do you get? You again get a parallelogram, which is altogether different (Fig 7.3), yet the four measurements remain the same.

This shows that 4 measurements of a quadrilateral cannot determine it uniquely. Can 5 measurements determine a quadrilateral uniquely? Let us go back to the activity!

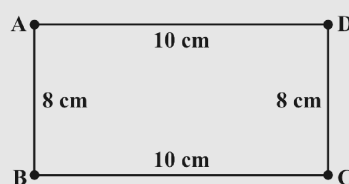


Fig 7.1

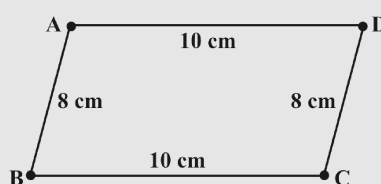


Fig 7.2

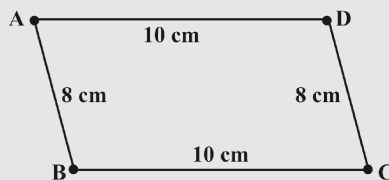


Fig 7.3

You have constructed a rectangle with two sticks each of length 10 cm and other two sticks each of length 8 cm. Now introduce another stick of length equal to BD and tie it along BD (Fig 7.4). If you push the breadth now, does the shape change? No! It cannot, without making the figure open. The introduction of the fifth stick has fixed the rectangle uniquely, i.e., there is no other quadrilateral (with the given lengths of sides) possible now.

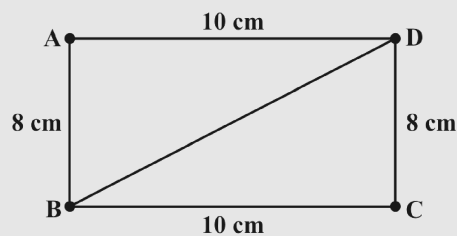


Fig 7.4

Thus, we observe that five measurements can determine a quadrilateral uniquely. But will any five measurements (of sides and angles) be sufficient to draw a unique quadrilateral?

THINK, DISCUSS AND WRITE

Arshad has five measurements of a quadrilateral ABCD. These are $AB = 5$ cm, $\angle A = 50^\circ$, $AC = 4$ cm, $BD = 5$ cm and $AD = 6$ cm. Can he construct a unique quadrilateral? Give reasons for your answer.

7.2 Constructing a Quadrilateral

We shall learn how to construct a unique quadrilateral given the following measurements:

- When four sides and one diagonal are given.
- When two diagonals and three sides are given.
- When two adjacent sides and three angles are given.
- When three sides and two included angles are given.
- When other special properties are known.

Let us take up these constructions one-by-one.

7.2.1 When the lengths of four sides and a diagonal are given

We shall explain this construction through an example.

Example 1: Construct a quadrilateral PQRS where $PQ = 4$ cm, $QR = 6$ cm, $RS = 5$ cm, $PS = 5.5$ cm and $PR = 7$ cm.

Solution: [A rough sketch will help us in visualising the quadrilateral. We draw this first and mark the measurements.] (Fig 7.5)

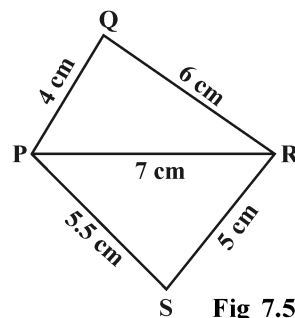


Fig 7.5

Step 1 From the rough sketch, it is easy to see that $\triangle PQR$ can be constructed using SSS construction condition. Draw $\triangle PQR$ (Fig 7.6).

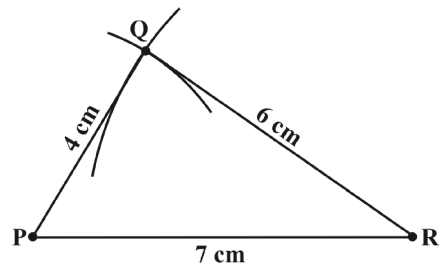


Fig 7.6

Step 2 Now, we have to locate the fourth point S. This 'S' would be on the side opposite to Q with reference to PR. For that, we have two measurements.

S is 5.5 cm away from P. So, with P as centre, draw an arc of radius 5.5 cm. (The point S is somewhere on this arc!) (Fig 7.7).

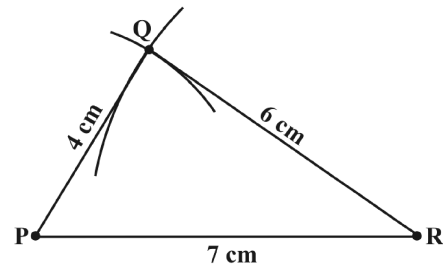


Fig 7.7

Step 3 S is 5 cm away from R. So with R as centre, draw an arc of radius 5 cm (The point S is somewhere on this arc also!) (Fig 7.8).

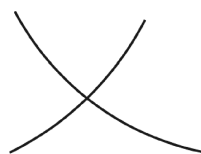
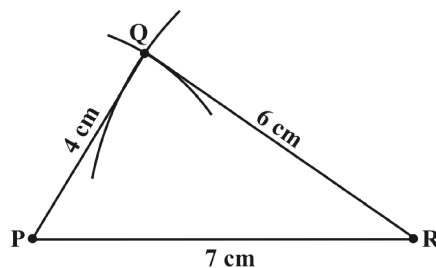


Fig 7.8



Step 4 S should lie on both the arcs drawn. So it is the point of intersection of the two arcs. Mark S and complete PQRS. PQRS is the required quadrilateral (Fig 7.9).

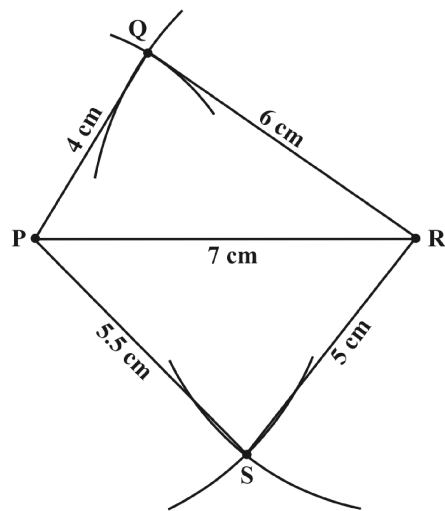


Fig 7.9



THINK, DISCUSS AND WRITE

- (i) We saw that 5 measurements of a quadrilateral can determine a quadrilateral uniquely. Do you think any five measurements of the quadrilateral can do this?
- (ii) Can you draw a parallelogram BATS where $BA = 5$ cm, $AT = 6$ cm and $AS = 6.5$ cm? Why?
- (iii) Can you draw a rhombus ZEAL where $ZE = 3.5$ cm, diagonal $EL = 5$ cm? Why?
- (iv) A student attempted to draw a quadrilateral PLAY where $PL = 3$ cm, $LA = 4$ cm, $AY = 4.5$ cm, $PY = 2$ cm and $LY = 6$ cm, but could not draw it. What is the reason?

[**Hint:** Discuss it using a rough sketch].



EXERCISE 7.1

1. Construct the following quadrilaterals.

- | | |
|--|--|
| <p>(i) Quadrilateral ABCD.</p> <p>$AB = 4.5$ cm</p> <p>$BC = 5.5$ cm</p> <p>$CD = 4$ cm</p> <p>$AD = 6$ cm</p> <p>$AC = 7$ cm</p> | <p>(ii) Quadrilateral JUMP</p> <p>$JU = 3.5$ cm</p> <p>$UM = 4$ cm</p> <p>$MP = 5$ cm</p> <p>$PJ = 4.5$ cm</p> <p>$PU = 6.5$ cm</p> |
| <p>(iii) Parallelogram MORE</p> <p>$OR = 6$ cm</p> <p>$RE = 4.5$ cm</p> <p>$EO = 7.5$ cm</p> | <p>(iv) Rhombus BEST</p> <p>$BE = 4.5$ cm</p> <p>$ET = 6$ cm</p> |

7.2.2 When two diagonals and three sides are given

When four sides and a diagonal were given, we first drew a triangle with the available data and then tried to locate the fourth point. The same technique is used here.

Example 2: Construct a quadrilateral ABCD, given that $BC = 4.5$ cm, $AD = 5.5$ cm, $CD = 5$ cm the diagonal $AC = 5.5$ cm and diagonal $BD = 7$ cm.

Solution:

Here is the rough sketch of the quadrilateral ABCD (Fig 7.10). Studying this sketch, we can easily see that it is possible to draw $\triangle ACD$ first (How?).

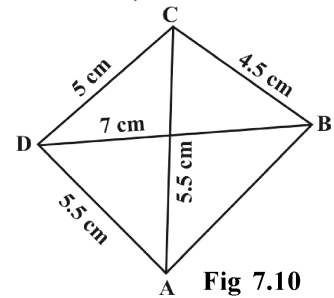


Fig 7.10

- Step 1** Draw $\triangle ACD$ using SSS construction (Fig 7.11).
(We now need to find B at a distance of 4.5 cm from C and 7 cm from D).

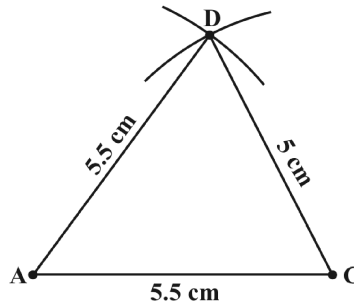


Fig 7.11

- Step 2** With D as centre, draw an arc of radius 7 cm. (B is somewhere on this arc) (Fig 7.12).

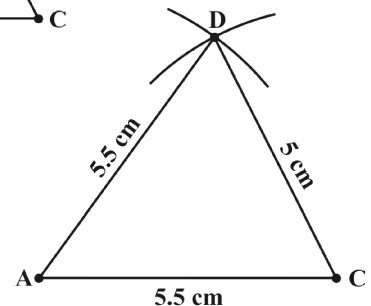


Fig7.12

- Step 3** With C as centre, draw an arc of radius 4.5 cm (B is somewhere on this arc also) (Fig 7.13).

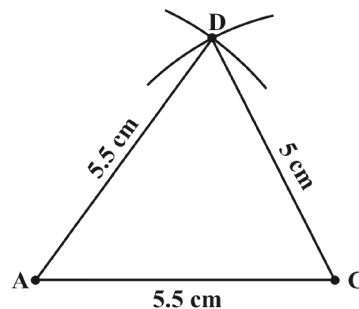


Fig 7.13

Step 4 Since B lies on both the arcs, B is the point intersection of the two arcs. Mark B and complete ABCD. ABCD is the required quadrilateral (Fig 7.14).

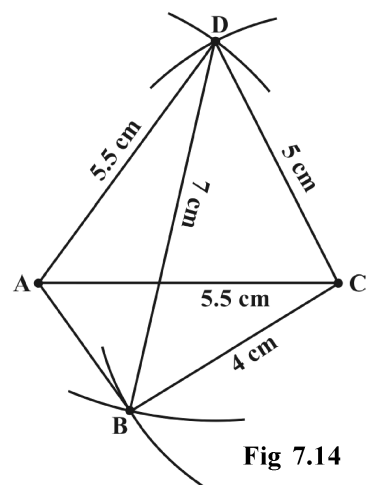


Fig 7.14



THINK, DISCUSS AND WRITE

1. In the above example, can we draw the quadrilateral by drawing $\triangle ABD$ first and then find the fourth point C?
2. Can you construct a quadrilateral PQRS with $PQ = 3$ cm, $RS = 3$ cm, $PS = 7.5$ cm, $PR = 8$ cm and $SQ = 4$ cm? Justify your answer.

EXERCISE 7.2

1. Construct the following quadrilaterals.

(i) quadrilateral LIFT

$$LI = 4 \text{ cm}$$

$$IF = 3 \text{ cm}$$

$$TL = 2.5 \text{ cm}$$

$$LF = 4.5 \text{ cm}$$

$$IT = 4 \text{ cm}$$

(iii) Rhombus BEND

$$BN = 5.6 \text{ cm}$$

$$DE = 6.5 \text{ cm}$$

(ii) Quadrilateral GOLD

$$OL = 7.5 \text{ cm}$$

$$GL = 6 \text{ cm}$$

$$GD = 6 \text{ cm}$$

$$LD = 5 \text{ cm}$$

$$OD = 10 \text{ cm}$$

7.2.3 When two adjacent sides and three angles are known

As before, we start with constructing a triangle and then look for the fourth point to complete the quadrilateral.

Example 3: Construct a quadrilateral MIST where $MI = 3.5$ cm, $IS = 6.5$ cm, $\angle M = 75^\circ$, $\angle I = 105^\circ$ and $\angle S = 120^\circ$.

Solution:

Here is a rough sketch that would help us in deciding our steps of construction. We give only hints for various steps (Fig 7.15).

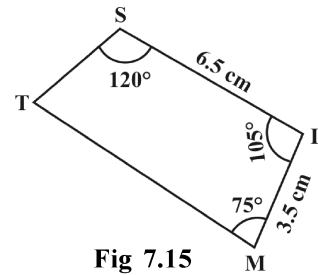


Fig 7.15

Step 1 How do you locate the points? What choice do you make for the base and what is the first step? (Fig 7.16)

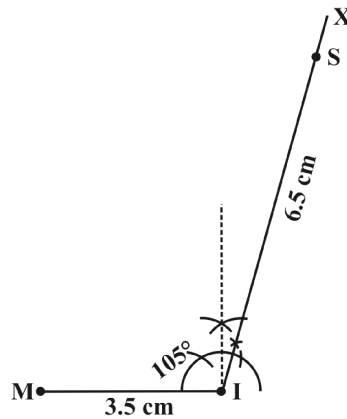


Fig 7.16

Step 2 Make $\angle ISY = 120^\circ$ at S (Fig 7.17).

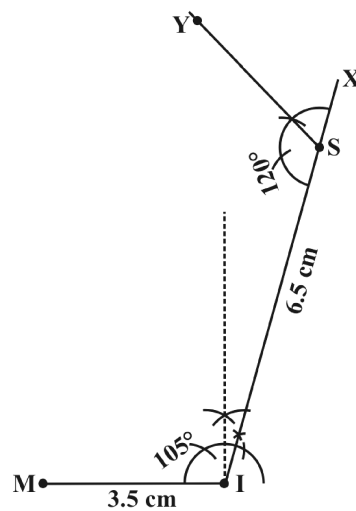


Fig 7.17



Step 3 Make $\angle IMZ = 75^\circ$ at M. (where will SY and MZ meet?) Mark that point as T. We get the required quadrilateral MIST (Fig 7.18).

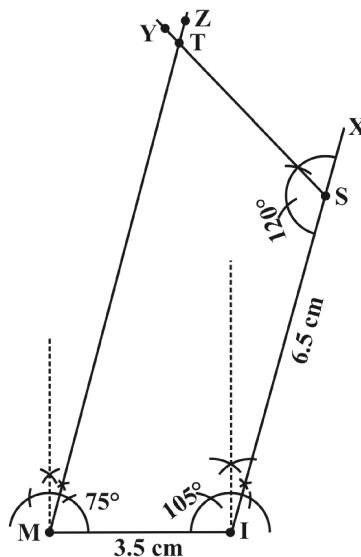


Fig 7.18



THINK, DISCUSS AND WRITE

1. Can you construct the above quadrilateral MIST if we have 100° at M instead of 75° ?
2. Can you construct the quadrilateral PLAN if $PL = 6$ cm, $LA = 9.5$ cm, $\angle P = 75^\circ$, $\angle L = 150^\circ$ and $\angle A = 140^\circ$? (**Hint:** Recall angle-sum property).
3. In a parallelogram, the lengths of adjacent sides are known. Do we still need measures of the angles to construct as in the example above?



EXERCISE 7.3

1. Construct the following quadrilaterals.

(i) Quadrilateral MORE

$$MO = 6 \text{ cm}$$

$$OR = 4.5 \text{ cm}$$

$$\angle M = 60^\circ$$

$$\angle O = 105^\circ$$

$$\angle R = 105^\circ$$

(iii) Parallelogram HEAR

$$HE = 5 \text{ cm}$$

$$EA = 6 \text{ cm}$$

$$\angle R = 85^\circ$$

(ii) Quadrilateral PLAN

$$PL = 4 \text{ cm}$$

$$LA = 6.5 \text{ cm}$$

$$\angle P = 90^\circ$$

$$\angle A = 110^\circ$$

$$\angle N = 85^\circ$$

(iv) Rectangle OKAY

$$OK = 7 \text{ cm}$$

$$KA = 5 \text{ cm}$$

7.2.4 When three sides and two included angles are given

Under this type, when you draw a rough sketch, note carefully the “included” angles in particular.

Example 4: Construct a quadrilateral ABCD, where $AB = 4$ cm, $BC = 5$ cm, $CD = 6.5$ cm and $\angle B = 105^\circ$ and $\angle C = 80^\circ$.

Solution:

We draw a rough sketch, as usual, to get an idea of how we can start off. Then we can devise a plan to locate the four points (Fig 7.19).

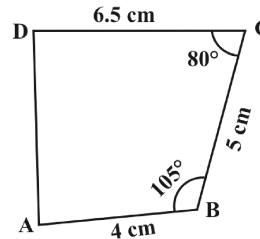


Fig 7.19

Step 1 Start with taking $BC = 5$ cm on B. Draw an angle of 105° along BX. Locate A 4 cm away on this. We now have B, C and A (Fig 7.20).

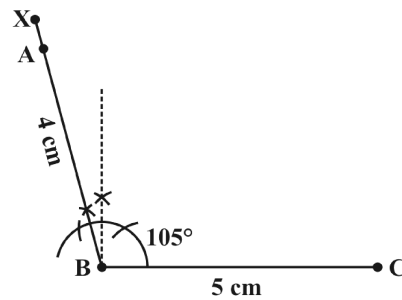


Fig 7.20

Step 2 The fourth point D is on CY which is inclined at 80° to BC. So make $\angle BCY = 80^\circ$ at C on BC (Fig 7.21).

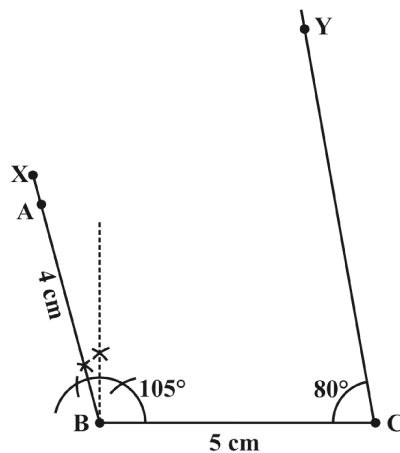


Fig 7.21



Step 3 D is at a distance of 6.5 cm on CY. With C as centre, draw an arc of length 6.5 cm. It cuts CY at D (Fig 7.22).

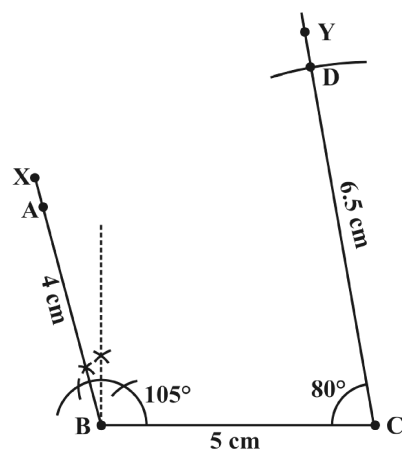


Fig 7.22

Step 4 Complete the quadrilateral ABCD. ABCD is the required quadrilateral (Fig 7.23).

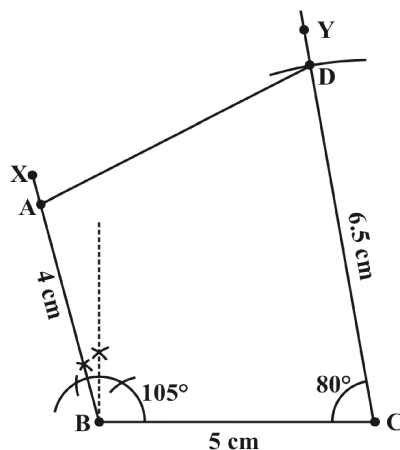


Fig 7.23



THINK, DISCUSS AND WRITE

1. In the above example, we first drew BC. Instead, what could have been the other starting points?
2. We used some five measurements to draw quadrilaterals so far. Can there be different sets of five measurements (other than seen so far) to draw a quadrilateral? The following problems may help you in answering the question.
 - (i) Quadrilateral ABCD with $AB = 5$ cm, $BC = 5.5$ cm, $CD = 4$ cm, $AD = 6$ cm and $\angle B = 80^\circ$.
 - (ii) Quadrilateral PQRS with $PQ = 4.5$ cm, $\angle P = 70^\circ$, $\angle Q = 100^\circ$, $\angle R = 80^\circ$ and $\angle S = 110^\circ$.

Construct a few more examples of your own to find sufficiency/insufficiency of the data for construction of a quadrilateral.

EXERCISE 7.4

1. Construct the following quadrilaterals.

(i) Quadrilateral DEAR

$$DE = 4 \text{ cm}$$

$$EA = 5 \text{ cm}$$

$$AR = 4.5 \text{ cm}$$

$$\angle E = 60^\circ$$

$$\angle A = 90^\circ$$

(ii) Quadrilateral TRUE

$$TR = 3.5 \text{ cm}$$

$$RU = 3 \text{ cm}$$

$$UE = 4 \text{ cm}$$

$$\angle R = 75^\circ$$

$$\angle U = 120^\circ$$



7.3 Some Special Cases

To draw a quadrilateral, we used 5 measurements in our work. Is there any quadrilateral which can be drawn with less number of available measurements? The following examples examine such special cases.

Example 5: Draw a square of side 4.5 cm.

Solution: Initially it appears that only one measurement has been given. Actually we have many more details with us, because the figure is a special quadrilateral, namely a square. We now know that each of its angles is a right angle. (See the rough figure) (Fig 7.24)

This enables us to draw $\triangle ABC$ using SAS condition. Then D can be easily located. Try yourself now to draw the square with the given measurements.

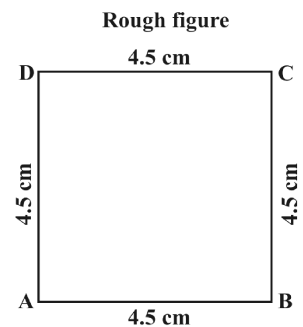


Fig 7.24

Example 6: Is it possible to construct a rhombus ABCD where $AC = 6 \text{ cm}$ and $BD = 7 \text{ cm}$? Justify your answer.

Solution: Only two (diagonal) measurements of the rhombus are given. However, since it is a rhombus, we can find more help from its properties.

The diagonals of a rhombus are perpendicular bisectors of one another.

So, first draw $AC = 7 \text{ cm}$ and then construct its perpendicular bisector. Let them meet at O. Cut off 3 cm lengths on either side of the drawn bisector. You now get B and D.

Draw the rhombus now, based on the method described above (Fig 7.25).

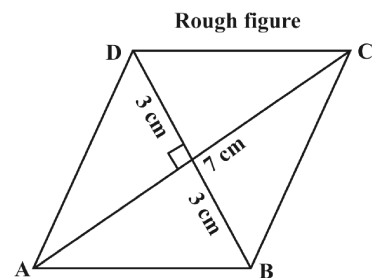


Fig 7.25

TRY THESE

- How will you construct a rectangle PQRS if you know only the lengths PQ and QR?
- Construct the kite EASY if $AY = 8 \text{ cm}$, $EY = 4 \text{ cm}$ and $SY = 6 \text{ cm}$ (Fig 7.26). Which properties of the kite did you use in the process?

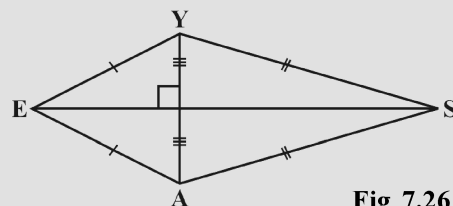


Fig 7.26



EXERCISE 7.5

Draw the following.

1. The square READ with $RE = 5.1$ cm.
2. A rhombus whose diagonals are 5.2 cm and 6.4 cm long.
3. A rectangle with adjacent sides of lengths 5 cm and 4 cm.
4. A parallelogram OKAY where $OK = 5.5$ cm and $KA = 4.2$ cm. Is it unique?

WHAT HAVE WE DISCUSSED?

1. Five measurements can determine a quadrilateral uniquely.
2. A quadrilateral can be constructed uniquely if the lengths of its four sides and a diagonal is given.
3. A quadrilateral can be constructed uniquely if its two diagonals and three sides are known.
4. A quadrilateral can be constructed uniquely if its two adjacent sides and three angles are known.
5. A quadrilateral can be constructed uniquely if its three sides and two included angles are given.



ANSWERS

1. PLAYING WITH NUMBERS

EXERCISE 1.1

1. $A = 7, B = 6$
2. $A = 5, B = 4, C = 1$
3. $A = 6$
4. $A = 2, B = 5$
5. $A = 5, B = 0, C = 1$
6. $A = 5, B = 0, C = 2$
7. $A = 7, B = 4$
8. $A = 7, B = 9$
9. $A = 4, B = 7$
10. $A = 8, B = 1$

EXERCISE 1.2

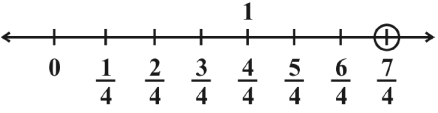
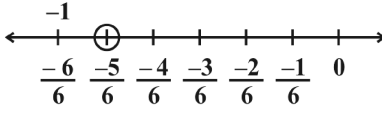
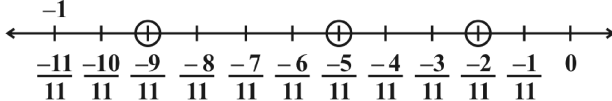
1. $y = 1$
2. $z = 0$ or 9
3. $z = 0, 3, 6$ or 9
4. $0, 3, 6$ or 9

2. RATIONAL NUMBERS

EXERCISE 2.1

1. (i) 2 (ii) $\frac{-11}{28}$
2. (i) $\frac{-2}{8}$ (ii) $\frac{5}{9}$ (iii) $\frac{-6}{5}$ (iv) $\frac{2}{9}$ (v) $\frac{19}{6}$
4. (i) $\frac{-1}{13}$ (ii) $\frac{-19}{13}$ (iii) 5 (iv) $\frac{56}{15}$ (v) $\frac{5}{2}$ (vi) -1
5. (i) 1 is the multiplicative identity (ii) Commutativity
(iii) Multiplicative inverse
6. $\frac{-96}{91}$
7. Associativity
8. No, because the product is not 1 .
9. Yes, because $0.3 \times 3\frac{1}{3} = \frac{3}{10} \times \frac{10}{3} = 1$
10. (i) 0 (ii) 1 and (-1) (iii) 0
11. (i) No (ii) $1, -1$ (iii) $\frac{-1}{5}$ (iv) x (v) Rational number
(vi) positive

EXERCISE 2.2

1. (i)  (ii) 
2. 
3. Some of these are $1, \frac{1}{2}, 0, -1, \frac{-1}{2}$
4. $\frac{-7}{20}, \frac{-6}{20}, \frac{-5}{20}, \frac{-4}{20}, \frac{-3}{20}, \frac{-2}{20}, \frac{-1}{20}, 0, \dots, \frac{1}{20}, \frac{2}{20}$ (There can be many more such rational numbers)
5. (i) $\frac{41}{60}, \frac{42}{60}, \frac{43}{60}, \frac{44}{60}, \frac{45}{60}$ (ii) $\frac{-8}{6}, \frac{-7}{6}, 0, \frac{1}{6}, \frac{2}{6}$ (iii) $\frac{9}{32}, \frac{10}{32}, \frac{11}{32}, \frac{12}{32}, \frac{13}{32}$
(There can be many more such rational numbers)
6. $-\frac{3}{2}, -1, \frac{-1}{2}, 0, \frac{1}{2}$ (There can be many more such rational numbers)
7. $\frac{97}{160}, \frac{98}{160}, \frac{99}{160}, \frac{100}{160}, \frac{101}{160}, \frac{102}{160}, \frac{103}{160}, \frac{104}{160}, \frac{105}{160}, \frac{106}{160}$
(There can be many more such rational numbers)

3. LINEAR EQUATIONS IN ONE VARIABLE**EXERCISE 3.1**

1. $x = 9$ 2. $y = 7$ 3. $z = 4$ 4. $x = 2$ 5. $x = 2$ 6. $t = 50$
 7. $x = 27$ 8. $y = 2.4$ 9. $x = \frac{25}{7}$ 10. $y = \frac{3}{2}$ 11. $p = -\frac{4}{3}$ 12. $x = -\frac{8}{5}$

EXERCISE 3.2

1. $\frac{3}{4}$ 2. length = 52 m, breadth = 25 m 3. $1\frac{2}{5}$ cm 4. 40 and 55
 5. 45, 27 6. 16, 17, 18 7. 288, 296 and 304 8. 7, 8, 9
 9. Rahul's age: 20 years; Haroon's age: 28 years 10. 48 students
 11. Baichung's age: 17 years; Baichung's father's age: 46 years;
 Baichung's grandfather's age = 72 years 12. 5 years 13. $-\frac{1}{2}$
 14. ₹ 100 → 2000 notes; ₹ 50 → 3000 notes; ₹ 10 → 5000 notes

15. Number of ₹ 1 coins = 80; Number of ₹ 2 coins = 60; Number of ₹ 5 coins = 20

16. 19

EXERCISE 3.3

1. $x = 18$ 2. $t = -1$ 3. $x = -2$ 4. $z = \frac{3}{2}$ 5. $x = 5$ 6. $x = 0$
 7. $x = 40$ 8. $x = 10$ 9. $y = \frac{7}{3}$ 10. $m = \frac{4}{5}$

EXERCISE 3.4

1. 4 2. 7, 35 3. 36 4. 26 (or 62)
 5. Shobo's age: 5 years; Shobo's mother's age: 30 years
 6. Length = 275 m; breadth = 100 m 7. 200 m 8. 72
 9. Grand daughter's age: 6 years; Grandfather's age: 60 years
 10. Aman's age: 60 years; Aman's son's age: 20 years

EXERCISE 3.5

1. $x = \frac{27}{10}$ 2. $n = 36$ 3. $x = -5$ 4. $x = 8$ 5. $t = 2$
 6. $m = \frac{7}{5}$ 7. $t = -2$ 8. $y = \frac{2}{3}$ 9. $z = 2$ 10. $f = 0.6$

EXERCISE 3.6

1. $x = \frac{3}{2}$ 2. $x = \frac{35}{33}$ 3. $z = 12$ 4. $y = -8$ 5. $y = -\frac{4}{5}$
 6. Hari's age = 20 years; Harry's age = 28 years 7. $\frac{13}{21}$

4. UNDERSTANDING QUADRILATERALS

EXERCISE 4.1

1. (a) 1, 2, 5, 6, 7 (b) 1, 2, 5, 6, 7 (c) 1, 2
 (d) 2 (e) 1
 2. (a) 2 (b) 9 (c) 0 3. 360° ; yes.
 4. (a) 900° (b) 1080° (c) 1440° (d) $(n-2)180^\circ$

5. A polygon with equal sides and equal angles.

- (i) Equilateral triangle (ii) Square (iii) Regular hexagon

6. (a) 60° (b) 140° (c) 140° (d) 108°

7. (a) $x + y + z = 360^\circ$ (b) $x + y + z + w = 360^\circ$

EXERCISE 4.2

1. (a) $360^\circ - 250^\circ = 110^\circ$ (b) $360^\circ - 310^\circ = 50^\circ$

2. (i) $\frac{360^\circ}{9} = 40^\circ$ (ii) $\frac{360^\circ}{15} = 24^\circ$

3. $\frac{360}{24} = 15$ (sides) 4. Number of sides = 24

5. (i) No; (Since 22 is not a divisor of 360)

(ii) No; (because each exterior angle is $180^\circ - 22^\circ = 158^\circ$, which is not a divisor of 360°).

6. (a) The equilateral triangle being a regular polygon of 3 sides has the least measure of an interior angle = 60° .

(b) By (a), we can see that the greatest exterior angle is 120° .

EXERCISE 4.3

1. (i) BC (Opposite sides are equal) (ii) $\angle DAB$ (Opposite angles are equal)

(iii) OA (Diagonals bisect each other)

(iv) 180° (Interior opposite angles, since $\overline{AB} \parallel \overline{DC}$)

2. (i) $x = 80^\circ$; $y = 100^\circ$; $z = 80^\circ$ (ii) $x = 130^\circ$; $y = 130^\circ$; $z = 130^\circ$

(iii) $x = 90^\circ$; $y = 60^\circ$; $z = 60^\circ$ (iv) $x = 100^\circ$; $y = 80^\circ$; $z = 80^\circ$

(v) $y = 112^\circ$; $x = 28^\circ$; $z = 28^\circ$

3. (i) Can be, but need not be.

(ii) No; (in a parallelogram, opposite sides are equal; but here, $AD \neq BC$).

(iii) No; (in a parallelogram, opposite angles are equal; but here, $\angle A \neq \angle C$).

4. A kite, for example 5. 108° ; 72° ; 6. Each is a right angle.

7. $x = 110^\circ$; $y = 40^\circ$; $z = 30^\circ$

8. (i) $x = 6$; $y = 9$ (ii) $x = 3$; $y = 13$; 9. $x = 50^\circ$

10. $\overline{NM} \parallel \overline{KL}$ (sum of interior opposite angles is 180°). So, KLMN is a trapezium.

11. 60° 12. $\angle P = 50^\circ$; $\angle S = 90^\circ$

EXERCISE 4.4

1. (b), (c), (f), (g), (h) are true; others are false.

2. (a) Rhombus; square. (b) Square; rectangle

3. (i) A square is 4-sided; so it is a quadrilateral.

- (ii) A square has its opposite sides parallel; so it is a parallelogram.
 (iii) A square is a parallelogram with all the 4 sides equal; so it is a rhombus.
 (iv) A square is a parallelogram with each angle a right angle; so it is a rectangle.
4. (i) Parallelogram; rhombus; square; rectangle.
 (ii) Rhombus; square (iii) Square; rectangle
5. Both of its diagonals lie in its interior.
6. $\overline{AD} \parallel \overline{BC}$; $\overline{AB} \parallel \overline{DC}$. So, in parallelogram ABCD, the mid-point of diagonal $\overline{AC}$ is O.

5. SQUARE AND SQUARE ROOTS

EXERCISE 5.1

1. (i) 1 (ii) 4 (iii) 1 (iv) 9 (v) 6 (vi) 9
 (vii) 4 (viii) 0 (ix) 6 (x) 5
2. These numbers end with
 (i) 7 (ii) 3 (iii) 8 (iv) 2 (v) 0 (vi) 2
 (vii) 0 (viii) 0
3. (i), (iii) 4. 10000200001, 100000020000001
5. 1020304030201, 101010101² 6. 20, 6, 42, 43
7. (i) 25 (ii) 100 (iii) 144
8. (i) $1 + 3 + 5 + 7 + 9 + 11 + 13$
 (ii) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21$
9. (i) 24 (ii) 50 (iii) 198

EXERCISE 5.2

1. (i) 1024 (ii) 1225 (iii) 7396 (iv) 8649 (v) 5041 (vi) 2116
2. (i) 6, 8, 10 (ii) 14, 48, 50 (iii) 16, 63, 65 (iv) 18, 80, 82

EXERCISE 5.3

1. (i) 1, 9 (ii) 4, 6 (iii) 1, 9 (iv) 5
2. (i), (ii), (iii) 3. 10, 13
4. (i) 27 (ii) 20 (iii) 42 (iv) 64 (v) 88 (vi) 98
 (vii) 77 (viii) 96 (ix) 23 (x) 90
5. (i) 7; 42 (ii) 5; 30 (iii) 7, 84 (iv) 3; 78 (v) 2; 54 (vi) 3; 48
6. (i) 7; 6 (ii) 13; 15 (iii) 11; 6 (vi) 5; 23 (v) 7; 20 (vi) 5; 18
7. 49 8. 45 rows; 45 plants in each row 9. 900 10. 3600

EXERCISE 6.4

1. (i) 48 (ii) 67 (iii) 59 (iv) 23 (v) 57 (vi) 37
 (vii) 76 (viii) 89 (ix) 24 (x) 32 (xi) 56 (xii) 30
2. (i) 1 (ii) 2 (iii) 2 (iv) 3 (v) 3
3. (i) 1.6 (ii) 2.7 (iii) 7.2 (iv) 6.5 (v) 5.6
4. (i) 2; 20 (ii) 53; 44 (iii) 1; 57 (iv) 41; 28 (v) 31; 63
5. (i) 4; 23 (ii) 14; 42 (iii) 4; 16 (iv) 24; 43 (v) 149; 81
6. 21 m 7. (a) 10 cm (b) 12 cm
8. 24 plants 9. 16 children

6. ALGEBRAIC EXPRESSIONS AND IDENTITIES**EXERCISE 6.1**

1.	Term	Coefficient
(i)	$5xyz^2$ $-3zy$	5 -3
(ii)	1 x x^2	1 1 1
(iii)	$4x^2y^2$ $-4x^2y^2z^2$ z^2	4 -4 1

(iv)	3 $-pq$ qr $-rp$	3 -1 1 -1
(v)	$\frac{x}{2}$ $\frac{y}{2}$ $-xy$	$\frac{1}{2}$ $\frac{1}{2}$ -1
(vi)	$0.3a$ $-0.6ab$ $0.5b$	0.3 -0.6 0.5

2. Monomials: 1000, pqr
 Binomials: $x + y$, $2y - 3y^2$, $4z - 15z^2$, $p^2q + pq^2$, $2p + 2q$
 Trinomials: $7 + y + 5x$, $2y - 3y^2 + 4y^3$, $5x - 4y + 3xy$
 Polynomials that do not fit in these categories: $x + x^2 + x^3 + x^4$, $ab + bc + cd + da$
3. (i) 0 (ii) $ab + bc + ac$ (iii) $-p^2q^2 + 4pq + 9$
 (iv) $2(l^2 + m^2 + n^2 + lm + mn + nl)$
4. (a) $8a - 2ab + 2b - 15$ (b) $2xy - 7yz + 5zx + 10xyz$
 (c) $p^2q - 7pq^2 + 8pq - 18q + 5p + 28$

EXERCISE 6.2

1. (i) $28p$ (ii) $-28p^2$ (iii) $-28p^2q$ (iv) $-12p^4$ (v) 0
2. pq ; $50mn$; $100x^2y^2$; $12x^3$; $12mn^2p$

3.

First monomial → Second monomial ↓	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$	$-10xy$	$6x^3$	$-8x^2y$	$14x^3y$	$-18x^3y^2$
$-5y$	$-10xy$	$25y^2$	$-15x^2y$	$20xy^2$	$-35x^2y^2$	$45x^2y^3$
$3x^2$	$6x^3$	$-15x^2y$	$9x^4$	$-12x^3y$	$21x^4y$	$-27x^4y^2$
$-4xy$	$-8x^2y$	$20xy^2$	$-12x^3y$	$16x^2y^2$	$-28x^3y^2$	$36x^3y^3$
$7x^2y$	$14x^3y$	$-35x^2y^2$	$21x^4y$	$-28x^3y^2$	$49x^4y^2$	$-63x^4y^3$
$-9x^2y^2$	$-18x^3y^2$	$45x^2y^3$	$-27x^4y^2$	$36x^3y^3$	$-63x^4y^3$	$81x^4y^4$

4. (i) $105a^7$ (ii) $64pqr$ (iii) $4x^4y^4$ (iv) $6abc$
 5. (i) $x^2y^2z^2$ (ii) $-a^6$ (iii) $1024y^6$ (iv) $36a^2b^2c^2$ (v) $-m^3n^2p$

EXERCISE 6.3

1. (i) $4pq + 4pr$ (ii) $a^2b - ab^2$ (iii) $7a^3b^2 + 7a^2b^3$
 (iv) $4a^3 - 36a$ (v) 0
 2. (i) $ab + ac + ad$ (ii) $5x^2y + 5xy^2 - 25xy$
 (iii) $6p^3 - 7p^2 + 5p$ (iv) $4p^4q^2 - 4p^2q^4$
 (v) $a^2bc + ab^2c + abc^2$
 3. (i) $8a^{50}$ (ii) $-\frac{3}{5}x^3y^3$ (iii) $-4p^4q^4$ (iv) x^{10}
 4. (a) $12x^2 - 15x + 3$; (i) 66 (ii) $-\frac{3}{2}$
 (b) $a^3 + a^2 + a + 5$; (i) 5 (ii) 8 (iii) 4
 5. (a) $p^2 + q^2 + r^2 - pq - qr - pr$ (b) $-2x^2 - 2y^2 - 4xy + 2yz + 2zx$
 (c) $5l^2 + 25ln$ (d) $-3a^2 - 2b^2 + 4c^2 - ab + 6bc - 7ac$

EXERCISE 6.4

1. (i) $8x^2 + 14x - 15$ (ii) $3y^2 - 28y + 32$ (iii) $6.25l^2 - 0.25m^2$
 (iv) $ax + 5a + 3bx + 15b$ (v) $6p^2q^2 + 5pq^3 - 6q^4$ (vi) $3a^4 + 10a^2b^2 - 8b^4$
 2. (i) $15 - x - 2x^2$ (ii) $7x^2 + 48xy - 7y^2$ (iii) $a^3 + a^2b^2 + ab + b^3$
 (iv) $2p^3 + p^2q - 2pq^2 - q^3$
 3. (i) $x^3 + 5x^2 - 5x$ (ii) $a^2b^3 + 3a^2 + 5b^3 + 20$ (iii) $t^3 - st + s^2t^2 - s^3$
 (iv) $4ac$ (v) $3x^2 + 4xy - y^2$ (vi) $x^3 + y^3$
 (vii) $2.25x^2 - 16y^2$ (viii) $a^2 + b^2 - c^2 + 2ab$

EXERCISE 6.5

1. (i) $x^2 + 6x + 9$ (ii) $4y^2 + 20y + 25$ (iii) $4a^2 - 28a + 49$
 (iv) $9a^2 - 3a + \frac{1}{4}$ (v) $1.21m^2 - 0.16$ (vi) $b^4 - a^4$

- (vii) $36x^2 - 49$ (viii) $a^2 - 2ac + c^2$ (ix) $\frac{x^2}{4} + \frac{3xy}{4} + \frac{9y^2}{16}$
- (x) $49a^2 - 126ab + 81b^2$
2. (i) $x^2 + 10x + 21$ (ii) $16x^2 + 24x + 5$ (iii) $16x^2 - 24x + 5$
- (iv) $16x^2 + 16x - 5$ (v) $4x^2 + 16xy + 15y^2$ (vi) $4a^4 + 28a^2 + 45$
- (vii) $x^2 y^2 z^2 - 6xyz + 8$
3. (i) $b^2 - 14b + 49$ (ii) $x^2 y^2 + 6xyz + 9z^2$ (iii) $36x^4 - 60x^2 y + 25y^2$
- (iv) $\frac{4}{9}m^2 + 2mn + \frac{9}{4}n^2$ (v) $0.16p^2 + 0.04pq + 0.25q^2$ (vi) $4x^2 y^2 + 20xy^2 + 25y^2$
4. (i) $a^4 - 2a^2 b^2 + b^4$ (ii) $40x$ (iii) $98m^2 + 128n^2$
- (iv) $41m^2 + 80mn + 41n^2$ (v) $4p^2 - 4q^2$ (vi) $a^2 b^2 + b^2 c^2$ (vii) $m^4 + n^4 m^2$
6. (i) 5041 (ii) 9801 (iii) 10404 (iv) 996004
- (v) 27.04 (vi) 89991 (vii) 6396 (viii) 79.21
- (ix) 99.75
7. (i) 200 (ii) 0.08 (iii) 1800 (iv) 84
8. (i) 10712 (ii) 26.52 (iii) 10094 (iv) 95.06

